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Model independent results for $B \to D_1(2420)\,\ell\,\bar\nu$ and $B \to D_2^*(2460)\,\ell\,\bar\nu$ at order $\Lambda_{\rm QCD}/m_{c,b}$

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Abstract

Exclusive semileptonic B decays into D_1 and D_2^* mesons are investigated including order $\Lambda_{\rm QCD}/m_{c,b}$ corrections using the heavy quark effective theory. At zero recoil, the $\Lambda_{\rm QCD}/m_{c,b}$ corrections can be written in terms of the leading Isgur-Wise function for these transitions, τ, and known meson mass splittings. We obtain an almost model independent prediction for the shape of the spectrum near zero recoil, including order $\Lambda_{\rm QCD}/m_{c,b}$ corrections. We determine $\tau(1)$ from the measured $B \to D_1\,\ell\,\bar\nu$ branching ratio. Implications for B decay sum rules are discussed.

The use of heavy quark symmetry [1] resulted in a dramatic improvement in our understanding of the spectroscopy and exclusive semileptonic decays of mesons containing a single heavy quark. In the infinite mass limit, the spin and parity of the heavy quark and that of the light degrees of freedom are separately conserved. Light degrees of freedom with quantum numbers $s_l^{\pi_l}$ yield a doublet of meson states with total angular momentum $J = s_l \pm \frac{1}{2}$ and parity $P = \pi_l$. All semileptonic decay form factors of B mesons into either member of such a heavy quark spin symmetry doublet are given by just one function of $w = v \cdot v'$. Here v is the four-velocity of the B and v' is that of the charmed meson. Moreover, for the $B \to D^{(*)}$ ground state to ground state transitions (these states have $s_l^{\pi_l} = \frac{1}{2}^-$), this universal function is normalized to unity at zero recoil [1–4]. Corrections to these model independent predictions, suppressed by powers of $\Lambda_{\rm QCD}/m_{c,b}$, can be systematically investigated using the heavy quark effective theory (HQET) [5].

In this letter we discuss semileptonic B meson decays into excited charmed mesons. Surprisingly, we find model independent predictions that hold even including order $\Lambda_{\rm QCD}/m_{c,b}$ corrections. We concentrate on the doublet corresponding to $s_l^{\pi_l} = \frac{3}{2}^+$, which contains the $D_1(2420)$ and the $D_2^*(2460)$ mesons with widths around 20 MeV. States in the $s_l^{\pi_l} = \frac{1}{2}^+$ doublet can decay into $D^{(*)}\pi$ in an s-wave, and so they should be much broader than the D_1 and D_2^* which can only decay in a d-wave. (An s-wave decay amplitude for the D_1 is forbidden by heavy quark spin symmetry [6].) $B \to D_1\,\ell\,\bar\nu$ and $B \to D_2^*\,\ell\,\bar\nu$ account for sizable fractions of semileptonic B decays [7–9], and are probably the only three-body semileptonic B decays, other than $B \to D^{(*)}\,\ell\,\bar\nu$, whose differential decay distributions will be precisely measured.

The measured masses of various meson states containing a bottom or charm quark already give important information on HQET matrix elements. The $D_2^* - D_1$ mass splitting is only 37 MeV, suggesting that for $s_l^{\pi_l} = \frac{3}{2}^+$ states matrix elements involving the chromomagnetic operator are smaller than for the ground state ($m_{D^*} - m_D = 140$ MeV). The parameters $\bar\Lambda$ and λ_1 for the ground state multiplet can be related to the analogous parameters for D_1 and D_2^*, which we denote by $\bar\Lambda'$ and λ_1'. We define the spin averaged masses $\overline{m}_D = (3m_{D^*}+m_D)/4$

and $\overline{m}'_D = (5m_{D_2^*} + 3m_{D_1})/8$, and similarly for analogous mesons containing a bottom quark. Using the measured hadron masses [10], and identifying $\overline{m}'_B = 5.70\,\mathrm{GeV}$ as the mass of the $B_J^*(5732)$ state, we find

$$\begin{aligned}
\lambda'_1 - \lambda_1 &= 2m_c m_b\,(\overline{m}'_B - \overline{m}_B - \overline{m}'_D + \overline{m}_D)/(m_b - m_c) \simeq -0.34\,\mathrm{GeV}^2\,, \\
\bar\Lambda' - \bar\Lambda &= \overline{m}'_D - \overline{m}_D + (\lambda'_1 - \lambda_1)/(2m_c) \simeq 0.35\,\mathrm{GeV}\,.
\end{aligned} \tag{1}$$

The matrix elements of the vector and axial currents ($V^\mu = \bar c\,\gamma^\mu\, b$ and $A^\mu = \bar c\,\gamma^\mu\gamma_5\, b$) between B mesons and D_1 or D_2^* mesons can be parametrized as

$$\begin{aligned}
\langle D_1(v',\epsilon)|\, V^\mu\, |B(v)\rangle &= \sqrt{m_{D_1} m_B}\, [f_{V_1}\, \epsilon^{*\mu} + (f_{V_2} v^\mu + f_{V_3} v'^\mu)\, (\epsilon^*\cdot v)]\,, \\
\langle D_1(v',\epsilon)|\, A^\mu\, |B(v)\rangle &= \sqrt{m_{D_1} m_B}\, i\, f_A\, \varepsilon^{\mu\alpha\beta\gamma} \epsilon^*_\alpha v_\beta v'_\gamma\,, \\
\langle D_2^*(v',\epsilon)|\, A^\mu\, |B(v)\rangle &= \sqrt{m_{D_2^*} m_B}\, [k_{A_1}\, \epsilon^{*\mu\alpha} v_\alpha + (k_{A_2} v^\mu + k_{A_3} v'^\mu)\, \epsilon^*_{\alpha\beta}\, v^\alpha v^\beta]\,, \\
\langle D_2^*(v',\epsilon)|\, V^\mu\, |B(v)\rangle &= \sqrt{m_{D_2^*} m_B}\, i\, k_V\, \varepsilon^{\mu\alpha\beta\gamma} \epsilon^*_{\alpha\sigma} v^\sigma v_\beta v'_\gamma\,.
\end{aligned} \tag{2}$$

Here f_i and k_i are functions of w. The differential decay rates for $B \to D_1\,\ell\,\bar\nu$ and $B \to D_2^*\,\ell\,\bar\nu$ decays in terms of these form factors are, respectively ($r_1 = m_{D_1}/m_B$ and $r_2 = m_{D_2^*}/m_B$)

$$\begin{aligned}
\frac{\mathrm{d}\Gamma_1}{\mathrm{d}w} &= \frac{G_F^2\, |V_{cb}|^2\, m_B^5\, r_1^3}{48\pi^3}\, \sqrt{w^2-1}\, \Big\{ 2(1 - 2wr_1 + r_1^2)\, \Big[f_{V_1}^2 + (w^2-1)\, f_A^2 \Big] \\
&\quad + \Big[(w - r_1)\, f_{V_1} + (w^2-1)\, (f_{V_3} + r_1 f_{V_2}) \Big]^2 \Big\}\,, \qquad (3a) \\
\frac{\mathrm{d}\Gamma_2}{\mathrm{d}w} &= \frac{G_F^2\, |V_{cb}|^2\, m_B^5\, r_2^3}{144\pi^3}\, (w^2-1)^{3/2}\, \Big\{ 3(1 - 2wr_2 + r_2^2)\, \Big[k_{A_1}^2 + (w^2-1)\, k_V^2 \Big] \\
&\quad + 2\Big[(w - r_2)\, k_{A_1} + (w^2-1)\, (k_{A_3} + r_2\, k_{A_2}) \Big]^2 \Big\}\,. \qquad (3b)
\end{aligned}$$

The form factors f_i and k_i can be parametrized by a set of Isgur-Wise functions at each order in $\Lambda_{\rm QCD}/m_{c,b}$. It is simplest to calculate the matrix elements in Eq. (2) using the trace formalism [11]. The fields P_v and $P_v^{*\mu}$ that destroy members of the $s_l^{\pi_l} = \frac12^-$ doublet with four-velocity v are in the 4×4 matrix

$$H_v = \frac{1 + \not v}{2}\, \Big[P_v^{*\mu} \gamma_\mu - P_v\, \gamma_5 \Big]\,, \tag{4}$$

while for $s_l^{\pi_l} = \frac32^+$ the fields P_v^ν and $P_v^{*\mu\nu}$ are in

$$F_v^\mu = \frac{1+\rlap{/}v}{2}\Big\{P_v^{*\mu\nu}\gamma_\nu - \sqrt{\frac{3}{2}}\,P_v^\nu\gamma_5\Big[g_\nu^\mu - \frac{1}{3}\gamma_\nu(\gamma^\mu - v^\mu)\Big]\Big\}. \tag{5}$$

The matrices H and F satisfy $\rlap{/}v H_v = H_v = -H_v\rlap{/}v$, $\rlap{/}v F_v^\mu = F_v^\mu = -F_v^\mu\rlap{/}v$, $F_v^\mu\gamma_\mu = 0$, and $v_\mu F_v^\mu = 0$.

To leading order in $\Lambda_{\rm QCD}/m_{c,b}$ and α_s

$$\bar c\,\Gamma\,b = \bar h_{v'}^{(c)}\,\Gamma\,h_v^{(b)} = \tau\ {\rm Tr}\left\{v_\sigma\bar F_{v'}^\sigma\,\Gamma\,H_v\right\}, \tag{6}$$

for matrix elements between the states destroyed by the fields in H_v and $F_{v'}^\sigma$. Here τ is a dimensionless function of w, and $h_v^{(Q)}$ is the heavy quark field in the effective theory. This matrix element vanishes at zero recoil for any Dirac structure Γ and for any value of $\tau(1)$, since the B meson and the $(D_1,\,D_2^*)$ mesons are in different heavy quark spin symmetry multiplets, and the current at zero recoil is related to the conserved charges of the spin-flavor symmetry. Eq. (6) leads to the $m_{c,b}\to\infty$ predictions for the form factors f_i and k_i given in Ref. [12].

At order $\Lambda_{\rm QCD}/m_{c,b}$ there are corrections originating from the matching of the $b\to c$ flavor changing current onto the effective theory, and from order $\Lambda_{\rm QCD}/m_{c,b}$ corrections to the HQET Lagrangian. To leading order in α_s, the current $\bar c\,\Gamma\,b$ is represented in HQET by

$$\bar c\,\Gamma\,b = \bar h_{v'}^{(c)}\left(\Gamma - \frac{i}{2m_c}\overleftarrow{\rlap{/}D}\,\Gamma + \frac{i}{2m_b}\,\Gamma\overrightarrow{\rlap{/}D} + \ldots\right)h_v^{(b)}\,. \tag{7}$$

For matrix elements between the states destroyed by the fields in $F_{v'}^\sigma$ and H_v, the new order $\Lambda_{\rm QCD}/m_{c,b}$ operators in Eq. (7) are

$$\begin{aligned}
\bar h_{v'}^{(c)}\,i\overleftarrow{D}_\lambda\,\Gamma\,h_v^{(b)} &= {\rm Tr}\left\{\mathcal{S}_{\sigma\lambda}^{(c)}\,\bar F_{v'}^\sigma\,\Gamma\,H_v\right\},\\
\bar h_{v'}^{(c)}\,\Gamma\,i\overrightarrow{D}_\lambda\,h_v^{(b)} &= {\rm Tr}\left\{\mathcal{S}_{\sigma\lambda}^{(b)}\,\bar F_{v'}^\sigma\,\Gamma\,H_v\right\}.
\end{aligned} \tag{8}$$

Unlike $B\to D^{(*)}$ decays, since the $(D_1,\,D_2^*)$ mesons and the B are in different multiplets, there is no relation between $\mathcal{S}^{(c)}$ and $\mathcal{S}^{(b)}$. The most general form for these quantities is

$$\mathcal{S}_{\sigma\lambda}^{(Q)} = v_\sigma\Big[\tau_1^{(Q)}\,v_\lambda + \tau_2^{(Q)}\,v'_\lambda + \tau_3^{(Q)}\,\gamma_\lambda\Big] + \tau_4^{(Q)}\,g_{\sigma\lambda}\,. \tag{9}$$

The functions τ_i depend on w, and have mass dimension one. They are not all independent. The equation of motion for the heavy quarks, $(v\cdot D)\, h_v^{(Q)}=0$, implies

$$
\begin{aligned}
w\,\tau_1^{(c)}+\tau_2^{(c)}-\tau_3^{(c)}&=0\,,\\
\tau_1^{(b)}+w\,\tau_2^{(b)}-\tau_3^{(b)}+\tau_4^{(b)}&=0\,.
\end{aligned}
\tag{10}
$$

Using $i\partial_\nu\,(\bar h_{v'}^{(c)}\,\Gamma\,h_v^{(b)})=(\bar\Lambda v_\nu-\bar\Lambda' v'_\nu)\,\bar h_{v'}^{(c)}\,\Gamma\,h_v^{(b)}$, valid for matrix elements between the states in $F_{v'}^\sigma$ and in H_v, together with the equation of motion for the heavy quarks, and the constraints in Eq. (10), we obtain

$$
\begin{aligned}
(w-1)\,(\tau_1^{(c)}-\tau_2^{(c)})-\tau_4^{(c)}&=(w\bar\Lambda'-\bar\Lambda)\,\tau\,.\\
(w-1)\,(\tau_1^{(b)}-\tau_2^{(b)})-\tau_4^{(b)}&=(w\bar\Lambda-\bar\Lambda')\,\tau\,.
\end{aligned}
\tag{11}
$$

At zero recoil, $\tau_4^{(b)}(1)=-\tau_4^{(c)}(1)=(\bar\Lambda'-\bar\Lambda)\,\tau(1)$.

Next we consider the terms originating from the order $\Lambda_{\rm QCD}/m_{c,b}$ corrections to the HQET Lagrangian,

$$
\delta\mathcal{L}=\frac{1}{2m_Q}\,\bar h_v^{(Q)}\Big[(iD)^2+\frac{g_s}{2}\,\sigma_{\alpha\beta}G^{\alpha\beta}\Big]h_v^{(Q)}\,.
\tag{12}
$$

These corrections modify the heavy meson states compared to their infinite heavy quark mass limit. For example, they cause the mixing of the D_1 with the $J^P=1^+$ member of the $s_l^{\pi_l}=\frac12^+$ doublet. (This is a very small effect, since the D_1 is not any broader than the D_2^*.) The kinetic energy operator does not violate heavy quark spin symmetry, and therefore in $B\to D_1, D_2^*$ semileptonic decays its effect can be absorbed into a redefinition of τ, which is used hereafter. For matrix elements between the states destroyed by the fields in $F_{v'}^\sigma$ and H_v, the time ordered products of the chromomagnetic term in $\delta\mathcal{L}$ with the leading order currents are

$$
\begin{aligned}
i\int {\rm d}^4x\, T\Big\{\Big[\bar h_{v'}^{(c)}\,\frac{g_s}{2}\,\sigma_{\alpha\beta}\,G^{\alpha\beta}\,h_{v'}^{(c)}\Big](x)\,\Big[\bar h_{v'}^{(c)}\,\Gamma\,h_v^{(b)}\Big](0)\Big\}&={\rm Tr}\,\Big\{\mathcal{R}_{\sigma\alpha\beta}^{(c)}\,\bar F_{v'}^\sigma\,i\sigma^{\alpha\beta}\,\frac{1+\rlap/v'}{2}\,\Gamma\,H_v\Big\}\,,\\
i\int {\rm d}^4x\, T\Big\{\Big[\bar h_{v}^{(b)}\,\frac{g_s}{2}\,\sigma_{\alpha\beta}\,G^{\alpha\beta}\,h_{v}^{(b)}\Big](x)\,\Big[\bar h_{v'}^{(c)}\,\Gamma\,h_v^{(b)}\Big](0)\Big\}&={\rm Tr}\,\Big\{\mathcal{R}_{\sigma\alpha\beta}^{(b)}\,\bar F_{v'}^\sigma\,\Gamma\,\frac{1+\rlap/v}{2}\,i\sigma^{\alpha\beta}H_v\Big\}\,.
\end{aligned}
\tag{13}
$$

The most general parametrizations of $\mathcal{R}^{(c,b)}$ are

$$\mathcal{R}^{(c)}_{\sigma\alpha\beta} = \eta_1^{(c)}\, v_\sigma\gamma_\alpha\gamma_\beta + \eta_2^{(c)}\, v_\sigma v_\alpha\gamma_\beta + \eta_3^{(c)}\, g_{\sigma\alpha}v_\beta\,,$$
$$\mathcal{R}^{(b)}_{\sigma\alpha\beta} = \eta_1^{(b)}\, v_\sigma\gamma_\alpha\gamma_\beta + \eta_2^{(b)}\, v_\sigma v'_\alpha\gamma_\beta + \eta_3^{(b)}\, g_{\sigma\alpha}v'_\beta\,. \tag{14}$$

Only the part of $\mathcal{R}^{(c,b)}_{\sigma\alpha\beta}$ antisymmetric in α and β contributes, when inserted into Eq. (13). The functions η_i depend on w, and have mass dimension one. (Note that $g_{\sigma\alpha}\gamma_\beta$ is dependent on the tensor structures included in Eq. (14).) All contributions arising from the time ordered products in Eq. (13) vanish at zero recoil, since $v_\sigma\bar{F}_v^\sigma = 0$, and $v_\alpha(1+\not{v})\sigma^{\alpha\beta}(1+\not{v}) = 0$.*

Using Eqs. (4)–(14), it is straightforward to express the form factors f_i and k_i parametrizing $B \to D_1\,\ell\,\bar\nu$ and $B \to D_2^*\,\ell\,\bar\nu$ semileptonic decays in terms of Isgur-Wise functions. We use the constraints in Eqs. (10) and (11) to eliminate τ_3 and τ_4. The form factors in Eq. (2) depend on $\tau_i^{(b)}$ and $\eta_i^{(b)}$ only through the linear combinations $\tau_b = (2w+1)\,\tau_1^{(b)} + \tau_2^{(b)} + 2(\bar\Lambda' - w\bar\Lambda)\,\tau$ and $\eta_b = 6\,\eta_1^{(b)} - 2(w-1)\,\eta_2^{(b)} + \eta_3^{(b)}$. Denoting $\varepsilon_{c,b} = 1/(2m_{c,b})$ and dropping the superscript on $\tau_i^{(c)}$ and $\eta_i^{(c)}$, we obtain for the $B \to D_1\,\ell\,\bar\nu$ form factors

$$\begin{aligned}
\sqrt6\, f_A &= -(w+1)\tau - \varepsilon_b[(w-1)\tau_b + (w+1)\eta_b] \\
&\quad +\varepsilon_c[3(w-1)(\tau_1-\tau_2) + (w+1)(2\eta_1+3\eta_3) - 4(w\bar\Lambda' - \bar\Lambda)\tau], \\
\sqrt6\, f_{V_1} &= (1-w^2)\tau - \varepsilon_b(w^2-1)(\tau_b+\eta_b) \\
&\quad +\varepsilon_c[(w^2-1)(3\tau_1 - 3\tau_2 + 2\eta_1 + 3\eta_3) - 4(w+1)(w\bar\Lambda' - \bar\Lambda)\tau], \\
\sqrt6\, f_{V_2} &= -3\tau - 3\varepsilon_b(\tau_b+\eta_b) - \varepsilon_c[(4w-1)\tau_1 + 5\tau_2 + 10\eta_1 + 4(w-1)\eta_2 - 5\eta_3], \\
\sqrt6\, f_{V_3} &= (w-2)\tau + \varepsilon_b[(2+w)\tau_b - (2-w)\eta_b] + \varepsilon_c[(2+w)\tau_1 + (2+3w)\tau_2 \\
&\quad -2(6+w)\eta_1 - 4(w-1)\eta_2 - (3w-2)\eta_3 + 4(w\bar\Lambda' - \bar\Lambda)\tau],
\end{aligned} \tag{15}$$

The analogous formulae for $B \to D_2^*\,\ell\,\bar\nu$ are

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*Order $\Lambda_{\rm QCD}/m_c$ corrections were also analyzed in Ref. [13]. We find that τ_4 (denoted ξ_4 in [13]) does contribute in Eq. (8) for $\Gamma = \gamma_\lambda\widetilde\Gamma$, and corrections to the Lagrangian are parametrized by more functions than in [13].

$$
\begin{aligned}
k_V &= -\tau - \varepsilon_b(\tau_b + \eta_b) - \varepsilon_c(\tau_1 - \tau_2 - 2\eta_1 + \eta_3), \\
k_{A_1} &= -(1+w)\tau - \varepsilon_b[(w-1)\tau_b + (1+w)\eta_b] \\
&\qquad -\varepsilon_c[(w-1)(\tau_1 - \tau_2) - (w+1)(2\eta_1 - \eta_3)], \\
k_{A_2} &= -2\varepsilon_c(\tau_1 + \eta_2), \\
k_{A_3} &= \tau + \varepsilon_b(\tau_b + \eta_b) - \varepsilon_c(\tau_1 + \tau_2 + 2\eta_1 - 2\eta_2 - \eta_3).
\end{aligned} \tag{16}
$$

The allowed kinematic range for $B \to D_1\,\ell\,\bar\nu$ decay is $1 < w < 1.32$, while for $B \to D_2^*\,\ell\,\bar\nu$ decay it is $1 < w < 1.31$. Since these ranges are fairly small, it is useful to expand the differential decay rates in Eq. (3) simultaneously in powers of $w-1$ and $\Lambda_{\rm QCD}/m_{c,b}$.

$$
\begin{aligned}
\frac{{\rm d}\Gamma_{1,2}}{{\rm d}w} &\simeq \frac{G_F^2\,|V_{cb}|^2\,m_B^5\,r_{1,2}^3}{48\pi^3}\,\sqrt{w^2-1} \\
&\quad\times \left[x_{1,2}^{(0)} + x_{1,2}^{(1)}\,(w-1) + x_{1,2}^{(2)}\,(w-1)^2\right]\tau^2(1)\,,
\end{aligned} \tag{17}
$$

In $x_{1,2}^{(i)}$ $(i=0,1,2)$, we keep terms up to order $(\Lambda_{\rm QCD}/m_{c,b})^{2-i}$. Eqs. (3), (15), and (16) yield

$$
\begin{aligned}
x_1^{(0)} &= 32\,\varepsilon_c^2\,(\bar\Lambda' - \bar\Lambda)^2\,(1-r_1)^2\ , \\
x_1^{(1)} &= (8/3)\,[(1-r_1)^2 + 4\varepsilon_c\,(\bar\Lambda' - \bar\Lambda)\,(3 - 4r_1 + r_1^2) \\
&\qquad - 2(1-r_1)^2\,(2\varepsilon_c\eta_1 + 3\varepsilon_c\eta_3 - \varepsilon_b\eta_b)/\tau]\,, \\
x_1^{(2)} &= (8/3)\,[(3 - 4r_1 + 3r_1^2) + 2(1-r_1)^2\,\tau'/\tau]\,, \\
x_2^{(0)} &= 0\,, \\
x_2^{(1)} &= (40/3)\,(1-r_2)^2\,[1 - 2(2\varepsilon_c\eta_1 - \varepsilon_c\eta_3 - \varepsilon_b\eta_b)/\tau]\,, \\
x_2^{(2)} &= 8\,(3 - 8r_2 + 3r_2^2) + (80/3)\,(1-r_2)^2\,\tau'/\tau\,.
\end{aligned} \tag{18}
$$

Here τ, $\tau' = {\rm d}\tau/{\rm d}w$, and η_i are evaluated at $w=1$. The values of η_i that occur in Eq. (18) are not known. Since the $D_2^* - D_1$ mass splitting is very small, and model calculations indicate that the analogous functions parametrizing time ordered products of the chromomagnetic operator for $B \to D^{(*)}\,\ell\,\bar\nu$ decays are tiny [14], hereafter we neglect the corrections parametrized by η_i.

The value of $\tau(1)$ can be determined from the experimental measurement of the

$B \to D_1\,\ell\,\bar\nu$ branching ratio. We use the average of the ALEPH [7] and CLEO [8] results, $\text{Br}(B \to D_1\,\ell\,\bar\nu) = (6.1 \pm 1.1) \times 10^{-3}$, to obtain

$$\tau(1) = 0.55 \pm 0.05\,. \tag{19}$$

(We used $\bar\Lambda' - \bar\Lambda = 0.35\,\text{GeV}$ from Eq. (1), $\tau_B = 1.6\,\text{ps}$, $|V_{cb}| = 0.04$, and $m_c = 1.4\,\text{GeV}$.) To get Eq. (19) we assumed $\tau'(1)/\tau(1) = -0.8$. $\tau(1)$ has little sensitivity to this choice. Allowing $-1.2 < \tau'(1)/\tau(1) < -0.5$ only affects the central value in Eq. (19) by ± 0.01. The ISGW nonrelativistic constituent quark model predicts $\tau(1) = 0.54$ in surprising agreement with Eq. (19) [15,12].

The ALEPH and CLEO analyses assume that $B \to D_1\,\ell\,\bar\nu\,X$ is dominated by $B \to D_1\,\ell\,\bar\nu$, and that D_1 decays only into $D^*\pi$. If the first assumption turns out to be false then $\tau(1)$ will decrease, if the second assumption is false then $\tau(1)$ will increase compared to Eq. (19).

Even though $\varepsilon_c(\bar\Lambda' - \bar\Lambda) \simeq 0.12$ is small, the term proportional to it comes with a large coefficient, and dominates the value of $x_1^{(1)}$. Numerically, this Λ_{QCD}/m_c correction to $x_1^{(1)}$ is 1.8, while the part that survives in the $m_{c,b} \to \infty$ limit is 0.8. Note that the part of the $\Lambda_{\text{QCD}}^2/m_c^2$ correction to $x_1^{(1)}$ that involves $\bar\Lambda'$, $\bar\Lambda$, and $\tau'(1)$ is only $0.27 {\pm} 0.10$ for the previously mentioned range of $\tau'(1)$ (using $\bar\Lambda = 0.4\,\text{GeV}$ [16]). The order Λ_{QCD}/m_c terms that involve $\bar\Lambda'$ and $\bar\Lambda$ change $x_1^{(2)}$ by less than a third of its leading order value for $-1.2 < \tau'(1)/\tau(1) < -0.5$.

After Eq. (12), we absorbed into τ the form factor that parametrizes time ordered products of the kinetic energy operator with the leading order currents. While λ_1' is quite large (see Eq. (1)), this is probably a consequence of the D_1 and D_2^* being p-waves in the quark model, and does not necessarily imply that the heavy quark kinetic energies significantly distort the overlap of wave functions that yield the form factors. If we had explicitly included the time ordered product involving the charm quark kinetic energy, the leading term in $x_1^{(1)}$ would change from $(1-r_1)^2$ to $(1-r_1)^2\,(1+2\eta_{\text{ke}}\,\varepsilon_c)$. Even taking $\eta_{\text{ke}} = \pm\bar\Lambda'$ changes the extracted value of $\tau(1)$ by less than 0.04. So this Λ_{QCD}/m_c correction to $x_1^{(1)}$ is likely to be much smaller than the term proportional to $\varepsilon_c(\bar\Lambda' - \bar\Lambda)$ explicitly shown in Eq. (18). It is important to have experimental data on the w-spectrum of $B \to D_1\,\ell\,\bar\nu$ decay to test the hypothesis that the effects of the kinetic energy and $\Lambda_{\rm QCD}^2/m_c^2$ corrections are not large.

The order $\Lambda_{\rm QCD}/m_{c,b}$ corrections calculated in this letter are also important for the prediction of $R \equiv {\rm Br}(B \to D_2^*\, \ell\, \bar\nu)/{\rm Br}(B \to D_1\, \ell\, \bar\nu)$. As R is sensitive to $\tau'(1)/\tau(1)$, we shall explicitly display the dependence on $\tau'(1)$. In the $m_{c,b} \to \infty$ limit, expanding to linear order in $\tau'(1)/\tau(1)$, we obtain $R = 1.89 + 0.51\,\tau'(1)/\tau(1)$. Including the $\Lambda_{\rm QCD}^2/m_c^2$ correction to $x_1^{(0)}$, and the $\Lambda_{\rm QCD}/m_c$ correction to $x_1^{(1)}$, and expanding again to linear order in $\tau'(1)/\tau(1)$, yields $R = 0.79 + 0.30\,\tau'(1)/\tau(1)$. This suppression of R compared to the infinite mass limit is consistent with experimental data. (It is possible that part of the reason for ${\rm Br}(B \to D_2^*\, \ell\, \bar\nu\, X) \times {\rm Br}(D_2^* \to D^{(*)}\pi) \lesssim (1.5 - 2.0) \times 10^{-3}$ [7] is a suppression of ${\rm Br}(D_2^* \to D^{(*)}\pi)$ compared to ${\rm Br}(D_1 \to D^*\pi)$.)

Our results are important for sum rules that relate inclusive $B \to X_c\, \ell\, \bar\nu$ decays to the sum of exclusive channels. The Bjorken sum rule [17,12] for the slope of the $B \to D^{(*)}\, \ell\, \bar\nu$ Isgur-Wise function becomes $\rho^2 \equiv -{\rm d}\xi/{\rm d}w|_{w=1} > 0.25 + \tau^2(1) = 0.55$. Note that $2\tau^2(1)/3$ arises from the D_1, D_2^* doublet, while $\tau^2(1)/3$ is due to the broad $s_l^{\pi_l} = \frac12^+$ doublet, $(D_0^*,\, D_1^*)$. (Using the equality of the leading Isgur-Wise functions for these multiplets in the quark model, valid for any spin-orbit independent potential.) A class of zero recoil sum rules were considered in Ref. [18]. The axial sum rule, which bounds the $B \to D^*$ form factor that measures $|V_{cb}|$, receives no corrections from either the $\frac32^+$ or the $\frac12^+$ doublets. The $J^P = 1^+$ states contribute to the vector sum rule, which bounds the λ_1 parameter of HQET. This bound is strongest in the limit $m_c \gg m_b \gg \Lambda_{\rm QCD}$, where the D_1 state does not contribute. The equality of Isgur-Wise functions for the $\frac32^+$ and $\frac12^+$ doublets in the quark model implies that $B \to D_1^*$ transition modifies the bound to $\lambda_1 < -3\lambda_2 - 3(\bar\Lambda^* - \bar\Lambda)^2\, \tau^2(1) \simeq -3\lambda_2 - 0.09$. (Here $\bar\Lambda^* \simeq \bar\Lambda + 0.31\,{\rm GeV}$ [19] is the analogue of $\bar\Lambda$ for the $\frac12^+$ states.) Perturbative corrections to these bounds can be found in [20].

In this letter we analyzed $B \to D_1(2420)\, \ell\, \bar\nu$ and $B \to D_2^*(2460)\, \ell\, \bar\nu$ decay form factors at order $\Lambda_{\rm QCD}/m_{c,b}$. At zero recoil, all $\Lambda_{\rm QCD}/m_{c,b}$ corrections can be written in terms of the $m_{c,b} \to \infty$ Isgur-Wise function for these transitions, and known meson mass splittings. With some model dependent assumptions, we predicted the shape of the spectrum near zero recoil, including order $\Lambda_{\rm QCD}/m_{c,b}$ corrections. Testing these predictions will constitute an interesting check on our understanding of exclusive semileptonic decays based on the HQET. Similar results hold for semileptonic B decay into the broad $s_l^{\pi_l} = \frac{1}{2}^+$ charmed meson multiplet, and for the semileptonic Λ_b decays into excited charmed baryons. These will be presented in a separate publication. Perturbative QCD corrections and nonleptonic decays (using factorization) will also be considered there.

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