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Semi-leptonic B decays into higher charmed resonances

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Abstract

We apply HQET to semi-leptonic B meson decays into a variety of excited charm states. Using three realistic meson models with fermionic light degrees of freedom, we examine the extent that the sum of exclusive single charmed states account for the inclusive semi-leptonic B decay rate. The consistency of form factors with the Bjorken and Voloshin sum rules is also investigated.

1 Introduction

The role of semi-leptonic B-decay in determining the CKM matrix element is well known. An important ingredient in this role is the understanding of the hadronic form factor, or Isgur-Wise (IW) function [1]. For $B \to D^{**} e \bar{\nu}_e$ decays the heavy-light limit is an excellent starting point.[1] Heavy quark effective theory (HQET) [2] is useful in organizing the descriptions of such processes. In this limit the wave functions and energies of the light degrees of freedom (LDF) are sufficient to define all IW functions [3].

As the number of measured B decays increases, other questions related to semi-leptonic decay can be addressed. Among these are the $B \to D^*$ spectrum shape, and the ratio of D^* to D rates. Of particular interest here are the inelastic processes $B \to D^{**} X e \bar{\nu}_e$, where D^{**} could be D_1 or D_2^* (or their radial excitations), and X are any non-charmed hadrons. Interest in these inelastic decays stems partly from experimental data [4, 5, 6] for exclusive inelastic decays, and partly from the relation between the exclusive decay rates and measured and theoretical results for the inclusive semi-leptonic rate.

An additional motivation for considering inelastic exclusive channels concerns sum rules. The Bjorken sum rule [7, 8] relates the slope of the elastic IW function (at the zero recoil point) to inelastic IW form factors describing S to P-wave semi-leptonic B decays. A less known sum rule involving the S to P-wave form factors has been derived by Voloshin [9]. It is the analog of the "optical" sum rule for the dipole scattering of light in atomic physics. It is not immediately obvious whether a particular model yields form factors which are consistent with both of these two sum rules.

Since the explicit IW functions are defined in terms of a relativistic LDF wave functions with a fermionic light quark, we have considered three appropriate hadronic models, based on the Dirac or the Salpeter equations. The models considered are the Dirac equation with scalar confinement, the Salpeter equation with time component vector confinement, and a relativistic flux tube model. In each case the strong coupling constant and the

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[1]We use the symbol D^{**} to denote any charmed meson.

various quark masses are chosen to best fit the heavy-light data. The resulting form factor predictions are quite similar and the variety of different models used provides an assessment of the confidence one has in predictions of this sort.

In Section 2 we briefly review our previous exclusive form factor results, and in Section 3 we outline the three Dirac-like hadronic models that are used for our numerical results. In Section 4 predictions for elastic and inelastic branching ratios into a single charmed hadron are compared with other results that can be found in the literature. A discussion of theoretical and experimental results of fractional semi-leptonic decay rates is found in Section 5. By considering fractional inclusive rates it is plausible that much of the uncertainty associated with the V_{cb} value, quark masses, and QCD corrections cancels out. The consistency of form factors with the Bjorken and Voloshin sum rules is considered in Section 6. A summary of the results and our conclusions are given in Section 7.

2 Isgur-Wise form factors and semi-leptonic B decays in the heavy-quark limit

In the $m_Q \rightarrow \infty$ limit the angular momentum of the LDF decouples from the spin of the heavy quark, and both are separately conserved by the strong interaction. Therefore, total angular momentum j of the LDF is a good quantum number. For each j there are two degenerate heavy meson states ($J = j \pm \frac{1}{2}$), and we can label states as J_j^P.

In HQET the covariant trace formalism [10, 11, 12] is the most convenient for keeping track of the relevant Clebsch-Gordan coefficients and for counting the number of indepen- dent form factors. Using the notation of [13], the lowest lying mesonic states are labeled as follows: C and C^* denote $0_{\frac{1}{2}}^-$ and $1_{\frac{1}{2}}^-$ states ($L = 0$ doublet), (E, E^*) and (F, F^*) denote the two $L = 1$ doublets ($0_{\frac{1}{2}}^+$, $1_{\frac{1}{2}}^+$) and ($1_{\frac{3}{2}}^+$, $2_{\frac{3}{2}}^+$), respectively, and G and G^* denote $1_{\frac{3}{2}}^-$ and $2_{\frac{3}{2}}^-$ states ($L = 2$ doublet). These states are described by 4×4 matrices, and matrix elements of the heavy quark currents are calculated by taking the corresponding

traces [12]. In this way, one can write the decay rate for $B \to D^{**}e\bar{\nu}_e$ in the form

$$\frac{d\Gamma^{**}}{d\omega} = \frac{G_F^2|V_{cb}|^2}{48\pi^3}m_B^2 m_{D^{**}}^3 \sqrt{\omega^2 - 1}|\xi^{**}(\omega)|^2 f^{**}(\omega, r^{**}) .\tag{1}$$

Here $\omega = v \cdot v'$ denotes velocity transfer, $r^{**} = m_{D^{**}}/m_B$, and the function f^{**} is given by

$$f_C(\omega, r_C) = (\omega^2 - 1)(1 + r_C)^2 ,\tag{2}$$

$$f_{C^*}(\omega, r_{C^*}) = (\omega + 1)[(\omega + 1)(1 - r_{C^*})^2 + 4\omega(1 - 2\omega r_{C^*} + r_{C^*}^2)] ,\tag{3}$$

$$f_E(\omega, r_E) = (\omega^2 - 1)(1 - r_E)^2 ,\tag{4}$$

$$f_{E^*}(\omega, r_{E^*}) = (\omega - 1)[(\omega - 1)(1 + r_{E^*})^2 + 4\omega(1 - 2\omega r_{E^*} + r_{E^*}^2)] ,\tag{5}$$

$$f_F(\omega, r_F) = \frac{2}{3}(\omega - 1)(\omega + 1)^2[(\omega - 1)(1 + r_F)^2 + \omega(1 - 2\omega r_F + r_F^2)] ,\tag{6}$$

$$f_{F^*}(\omega, r_{F^*}) = \frac{2}{3}(\omega - 1)(\omega + 1)^2[(\omega + 1)(1 - r_{F^*})^2 + 3\omega(1 - 2\omega r_{F^*} + r_{F^*}^2)] ,\tag{7}$$

$$f_G(\omega, r_G) = \frac{2}{3}(\omega - 1)^2(\omega + 1)[(\omega + 1)(1 - r_G)^2 + \omega(1 - 2\omega r_G + r_G^2)] ,\tag{8}$$

$$f_{G^*}(\omega, r_{G^*}) = \frac{2}{3}(\omega - 1)^2(\omega + 1)[(\omega - 1)(1 + r_{G^*})^2 + 3\omega(1 - 2\omega r_{G^*} + r_{G^*}^2)] .\tag{9}$$

The above expressions can be found in [14]-[17].

The only unknown quantity in the expression (1) is the appropriate IW form factor $\xi^{**}(\omega)$. Since these form factors cannot be calculated from first principles, one has to rely on some model of strong interactions. By comparing the wave function approach of [18] with the covariant trace formalism of [10, 11, 12], and performing the necessary integrations in the modified Breit frame (as suggested in [15]), one finds the expressions for the unknown form factors in terms of the LDF rest frame wave functions and energies [3]. These expressions include transitions from the ground state into radially excited states, and are given as

$$\xi_C(\omega) = \frac{2}{\omega + 1}\langle j_0(ar)\rangle_{00} \qquad (C \to C, C^* \text{ transitions}) ,\tag{10}$$

$$\xi_E(\omega) = \frac{2}{\sqrt{\omega^2 - 1}}\langle j_1(ar)\rangle_{10} \qquad (C \to E, E^* \text{ transitions}) ,\tag{11}$$

$$\xi_F(\omega) = \sqrt{\frac{3}{\omega^2 - 1}}\frac{2}{\omega + 1}\langle j_1(ar)\rangle_{10} \qquad (C \to F, F^* \text{ transitions}) ,\tag{12}$$

$$\xi_G(\omega) = \frac{2\sqrt{3}}{\omega^2 - 1}\langle j_2(ar)\rangle_{20} \qquad (C \to G, G^* \text{ transitions}) .\tag{13}$$

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In the above formulae a is defined in terms of the initial (E_q) and final (E_q') energies of the LDF as

$$a = (E_q + E_q')\sqrt{\frac{\omega - 1}{\omega + 1}} \ . \tag{14}$$

The expectation values appearing in (10)-(13) are defined as

$$\langle F(r)\rangle_{j'j}^{\alpha'\alpha} = \int r^2 dr [f_{\alpha'j'}^{*k'}(r)f_{\alpha j}^k(r) + g_{\alpha'j'}^{*k'}(r)g_{\alpha j}^k(r)]F(r) \ . \tag{15}$$

This follows from the form of the wave function in the Dirac-like models with spherical symmetry,

$$\phi_{j\lambda_j}^{(\alpha k)}(\mathbf{x}) = \left(\begin{array}{c} f_{\alpha j}^k(r)\mathcal{Y}_{j\lambda_j}^k(\Omega) \\ ig_{\alpha j}^k(r)\mathcal{Y}_{j\lambda_j}^{-k}(\Omega) \end{array} \right) \ , \tag{16}$$

where $\mathcal{Y}_{j\lambda_j}^k$ are the usual spherical spinors, $k = l$ $(l = j + \frac{1}{2})$ or $k = -l - 1$ $(l = j - \frac{1}{2})$, and α denotes all other quantum numbers.

3   Modelling the light degrees of freedom

In order to obtain a reasonable estimate of the model dependence of our results, we employ three qualitatively different realistic models to describe heavy-light mesons: the Dirac equation with scalar confinement (DESC), the Salpeter equation with vector confinement (SEVC), and the relativistic flux tube confinement (RFTC). All three models involve a short range Coulomb potential. With wave function of the form (16), it can be shown [19, 20] that all three models satisfy a radial equation of the form

$$(E_q \mathbb{1} - \mathbb{H}_0 - \mathbb{L}\mathbb{I}\mathbb{L}) \left(\begin{array}{c} f_j^k(r) \\ g_j^{-k}(r) \end{array} \right) = 0 \ , \tag{17}$$

where

$$\mathbb{H}_0 = \left(\begin{array}{cc} m_q & -D_- \\ D_+ & -m_q \end{array} \right) \ , \tag{18}$$

and

$$D_\pm = \pm\frac{k}{r} + (\frac{\partial}{\partial r} + \frac{1}{r}) \ . \tag{19}$$

The 2×2 matrices $\mathbb{L}$ and $\mathbb{I}$ will be defined when the specific models are discussed. The numerical methods used to deal with these three models are described in [19, 20].

3.1   Dirac equation with scalar confinement (DESC)

Scalar confinement is the only type of confinement potential that has correct sign of the spin-orbit coupling. In the Dirac equation it also yields linear Regge trajectories. This model also assumes a time component vector short range Coulomb interaction. Specifically, we have

$$\mathbb{L} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} , \tag{20}$$

$$\mathbb{I} = \begin{pmatrix} -\frac{4\alpha_s}{3r} + br & 0 \\ 0 & -\frac{4\alpha_s}{3r} - br \end{pmatrix} . \tag{21}$$

The parameter values chosen to give an excellent fit (see Table 1) to the heavy-light spin averaged data [21] are

$$
\begin{aligned}
m_{u,d} &= 0.300 GeV \quad \text{(fixed)} , \\
m_s &= 0.465 GeV , \\
m_c &= 1.357 GeV , \\
m_b &= 4.693 GeV , \\
\alpha_s &= 0.462 , \\
b &= 0.284 GeV^2 \quad \text{(fixed)} .
\end{aligned}
\tag{22}
$$

The quality of fit is insensitive to the value of $m_{u,d}$ and the confinement tension b was chosen to yield the universal Regge slope [17].[2]

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[2]The slope of the Regge trajectories in the heavy-light case is expected to be exactly twice the slope in the light-light case [19, 22], i.e. $\alpha'_{HL} = 2\alpha'_{LL}$. The observed Regge slope for the light-light states is $\alpha'_{LL} = 0.88 \ GeV^{-2}$ [23].

3.2 Salpeter equation with vector confinement (SEVC)

The instantaneous version of the Bethe-Salpeter equation [24, 25] (usually referred to as the Salpeter equation [26]) is widely used for the discussion of bound state problems. It is also equivalent [27] to the so called "no-pair" equation [28], which was introduced in order to avoid the problem of mixing of positive and negative energy states that occurred in the Dirac equation for the helium atom. A similar problem also occurs for a single fermionic particle moving in the confining Lorentz vector potential. For a very long time [29] it has been known that there are no normalizable solutions to the Dirac equation in this case.

It has been shown analytically for the heavy-light case [19], and numerically for the case of fermion and antifermion with arbitrary mass [30, 31], that in this type of model linear scalar confinement does not yield linear Regge trajectories. We have therefore used time component vector confinement with short range Coulomb interaction, even though it is well known that this model gives the wrong sign of the spin-orbit coupling.

In this model matrices $\mathbb{L}$ and $\mathbb{I}$ are given by

$$\mathbb{L} = \begin{pmatrix} \lambda_+ & -\frac{1}{2E_0^+}D_- \\ D_+\frac{1}{2E_0^+} & \lambda_- \end{pmatrix} , \tag{23}$$

$$\mathbb{I} = \begin{pmatrix} -\frac{4\alpha_s}{3r} + br & 0 \\ 0 & -\frac{4\alpha_s}{3r} + br \end{pmatrix} , \tag{24}$$

where

$$\lambda_\pm = \frac{E_0^\pm \pm m_q}{2E_0^\pm} , \tag{25}$$

and

$$E_0^\pm = \sqrt{m_q^2 - D_\mp D_\pm} . \tag{26}$$

Again we fix the confinement tension b to yield the universal Regge slope, and choose our parameters as

$$m_{u,d} = 0.300 GeV \quad \text{(fixed)} ,$$

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$$
\begin{aligned}
m_s &= 0.598 GeV \ , \\
m_c &= 1.406 GeV \ , \\
m_b &= 4.741 GeV \ , \\
\alpha_s &= 0.539 \ , \\
b &= 0.142 GeV^2 \quad \text{(fixed)} \ ,
\end{aligned}
\tag{27}
$$

to obtain an excellent fit to the spin averaged heavy-light states, as shown in Table 1.

3.3   Relativistic flux tube confinement (RFTC)

In the RFTC model formalism for fermionic quark confinement is unusual in that the confinement is introduced into the kinetic rather than in the usual interaction term. The flux tube contributes to both energy and momentum, so it makes little sense to consider it as a "potential" type interaction. By a covariant substitution we add the tube to the quark momentum and energy. We may equivalently view this as a "minimal substitution" of a vector interaction field. The result nicely reduces to the Nambu string in the limit in which the quark moves ultra-relativistically. This physically motivated generalization of the potential model incorporates many aspects of QCD [20].

In this model the $\mathbb{L}$ matrix is the same as in (23), while the interaction matrix $\mathbb{I}$ is given by

$$
\mathbb{I} = \begin{pmatrix} -\frac{4\alpha_s}{3r} + H_t^k & T_t \\ T_t & -\frac{4\alpha_s}{3r} + H_t^{-k} \end{pmatrix} \ .
\tag{28}
$$

In the above T_t is defined as

$$
T_t = \frac{1}{2} \left[\frac{(1-k)}{\sqrt{-k(1-k)}} p_t^{-k} - \frac{(1+k)}{\sqrt{k(1+k)}} p_t^k \right] \ .
\tag{29}
$$

The flux tube energy and momentum, obtained by symmetrization of the classical expressions [32, 33], are defined by ($\{A, B\} = AB + BA$)

$$
H_t^{\pm k} = \frac{a}{2} \{ r, \frac{\arcsin v_\perp^{\pm k}}{v_\perp^{\pm k}} \} \ ,
\tag{30}
$$

$$
p_t^{\pm k} = = a \{ r, F(v_\perp^{\pm k}) \} \ ,
\tag{31}
$$

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with $(\gamma_\perp = 1/\sqrt{1 - v_\perp^2})$

$$F(v_\perp) = \frac{1}{4v_\perp}\left(\frac{\arcsin v_\perp}{v_\perp} - \frac{1}{\gamma_\perp}\right) . \tag{32}$$

The only unknown operators in the above expressions are $v_\perp^{\pm k}$. These are determined from the heavy-light orbital angular momentum equation as in the spinless RFT model [33]. With the definition $W_r = \sqrt{p_r^2 + m^2}$, these equations are [20]

$$\left[\frac{\sqrt{k(k+1)}}{r} = \tfrac{1}{2}\{W_r, \gamma_\perp^k v_\perp^k\} + a\{r, F(v_\perp^k)\}\right] f_j^k(r)\mathcal{Y}_{jm}^k(\hat{\mathbf{r}}) , \tag{33}$$

$$\left[\frac{\sqrt{-k(-k+1)}}{r} = \tfrac{1}{2}\{W_r, \gamma_\perp^{-k} v_\perp^{-k}\} + a\{r, F(v_\perp^{-k})\}\right] g_j^k(r)\mathcal{Y}_{jm}^{-k}(\hat{\mathbf{r}}) . \tag{34}$$

The numerical technique used to solve for $v_\perp$ is discussed in detail elsewhere [33].

Theoretical predictions of the model with parameters

$$
\begin{aligned}
m_{u,d} &= 0.300 \ GeV \ \text{(fixed)} , \\
m_s &= 0.580 \ GeV , \\
m_c &= 1.350 \ GeV , \\
m_b &= 4.685 \ GeV , \\
b &= 0.181 \ GeV^2 \ \text{(fixed)} , \\
\alpha_s &= 0.508 ,
\end{aligned}
\tag{35}
$$

are shown in Table 1. Again, the agreement with spin-averaged experimental masses is very good.

We conclude this section by noting that all of the above models have been used for the calculation of the elastic IW form factor [19, 20], and the predicted IW functions ($\xi_C(\omega)$) were all quite consistent with the experimental data [34, 35].

4 Branching ratios and comparison with other results

Our results for branching ratios obtained from the three different models discussed in Section 3 are shown in Table 2. We have assumed here that $V_{cb} = 0.040$ and $\tau_B = 1.5ps$.

Table 3 contains a comparison of our DESC calculation with calculation of Scora et al. (ISGW2 model) [36], the one of Suzuki et al. (SISM model) [16], with the QCD sum rule approach of Colangelo et al. (CNP model) [37], and with the model of Sutherland et al. (SHJL model) [38]. It is worth noting that results quoted for SISM and CNP are also obtained in the heavy quark limit. Table 4 contains ratios of partial widths for B decays into members of the same D^{**} doublet.

For calculation of branching ratios we have used experimental meson masses wherever possible. In those cases the only model dependent inputs were the appropriate IW form factors. For decays where the D^{**} mass is unknown, we have used spin-averaged masses obtained in a specific model. Based on the available information on the splitting between D and D^* (or D_1 and D_2^*), one could estimate mass splitting in other excited doublets, and use that together with model dependent spin-averaged mass to obtain separate prediction for the mass of each member of that doublet. Meson masses obtained in this way could then be used in the calculation of the branching ratio for the corresponding decay. However, we have found that this procedure does not significantly affect the results. For example, using spin-averaged mass of 1974 MeV for $D(1867)$ and $D^*(2009)$, instead of their experimental masses, in the case of DESC yields branching ratios of 2.235% and 6.773% instead of 2.401% and 6.615%, which are given in Table 2. For the higher states this effect is even less noticeable.

Let us first discuss $B \to D$ and $B \to D^*$ transitions. Recent results from CLEO [39],

$$BR(B^- \to D^0 e^- \bar{\nu}_e) = (1.95 \pm 0.55)\% , \qquad (36)$$

$$BR(B^- \to D^{*0} e^- \bar{\nu}_e) = (5.13 \pm 0.84)\% , \qquad (37)$$

and results given in Tables 2 and 3 imply that all models we used, as well as the ISGW2 and SHJL models, require V_{cb} slightly lower than 0.040. In our models values range from about 0.036 for DESC, to about 0.038 for RFTC and SEVC. On the other hand, ISGW2 is consistent with 0.035, SHJL gives about 0.036, and SISM and CNP models agree with

V_{cb} of about 0.040. From Table 3 it can also be found that[3]

$$\frac{BR(B^- \to D^0 e^- \bar{\nu}_e)}{BR(B^- \to D^{*0} e^- \bar{\nu}_e)} = \begin{cases} 0.48 \ , & \text{for ISGW2} \\ 0.34 \ , & \text{for SISM} \\ 0.33 \ , & \text{for CNP} \\ 0.31 \ , & \text{for SHJL} \end{cases} \ . \tag{38}$$

On the other hand, our calculation with three different models yields (see Table 4)

$$\frac{BR(B^- \to D^0 e^- \bar{\nu}_e)}{BR(B^- \to D^{*0} e^- \bar{\nu}_e)} = \begin{cases} 0.35 \ , & \text{for RFTC} \\ 0.36 \ , & \text{for DESC} \\ 0.35 \ , & \text{for SEVC} \end{cases} \ . \tag{39}$$

The results quoted in (36) and (37) imply an experimental ratio of

$$\frac{BR(B^- \to D^0 e^- \bar{\nu}_e)}{BR(B^- \to D^{*0} e^- \bar{\nu}_e)} = 0.38 \pm 0.17 \ . \tag{40}$$

It is interesting to note that ratio of polarization states of D and D^* is 0.33.

Individual contributions of P-wave $j = \frac{3}{2}$ states to the total semi-leptonic decay rate is another interesting point. From Table 2 it can be seen that the total semi-leptonic branching ratio for $B \to D_1$ and $B \to D_2^*$ is expected to be

$$BR(B^- \to D_1^0 e^- \bar{\nu}_e) + BR(B^- \to D_2^{*0} e^- \bar{\nu}_e) = \begin{cases} 0.84 \left| \frac{V_{cb}}{0.040} \right|^2 \frac{\tau_B}{1.50ps} \% \ , & \text{for RFTC} \\ 0.69 \left| \frac{V_{cb}}{0.040} \right|^2 \frac{\tau_B}{1.50ps} \% \ , & \text{for DESC} \\ 0.79 \left| \frac{V_{cb}}{0.040} \right|^2 \frac{\tau_B}{1.50ps} \% \ , & \text{for SEVC} \end{cases} \ . \tag{41}$$

These results are slightly larger than the ISGW2 result of $0.65 \left| \frac{V_{cb}}{0.040} \right|^2 \frac{\tau_B}{1.50ps}\%$, and significantly disagree with SISM, CNP, and SHJL models (0.20, 0.37, and 0.46 $\left| \frac{V_{cb}}{0.040} \right|^2 \frac{\tau_B}{1.50ps}\%$, respectively). However, as one can see from Table 4, our ratios of these two P wave decays are qualitatively different from the one obtained in ISGW2 and SHJL, and agree

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[3]It is worth noting that the calculation of [16] used form factor definitions which are not consistent with the covariant trace formalism.

with SISM and CNP models. We find

$$\frac{BR(B^- \to D_1^0 e^- \bar{\nu}_e)}{BR(B^- \to D_2^{*0} e^- \bar{\nu}_e)} = \begin{cases} 0.60 , & \text{for RFTC} \\ 0.63 , & \text{for DESC} \\ 0.60 , & \text{for SEVC} \end{cases} ,$$ (42)

while the other models yield

$$\frac{BR(B^- \to D_1^0 e^- \bar{\nu}_e)}{BR(B^- \to D_2^{*0} e^- \bar{\nu}_e)} = \begin{cases} 2.00 , & \text{for ISGW2} \\ 0.70 , & \text{for SISM} \\ 0.50 , & \text{for CNP} \\ 1.54 , & \text{for SHJL} \end{cases} .$$ (43)

Again, it is interesting to note that the ratio of number of polarization states of D_1 and D_2^* is 0.6. It remains to be seen whether this discrepancy between our results (which are obtained in the heavy-light limit) and ISGW2 and SHJL[4] models can be explained with large $\frac{1}{m_c}$ effects [40].

As already mentioned, within the HQET framework the only model dependent input for the decays $C \to C, C^*$ and $C \to F, F^*$ are the unknown IW functions. For these decays the uncertainty introduced by using a particular model is about 10%. For other decays, D^{**} mass is not known, so that in the calculation of decay rates we used predictions of a particular model. Therefore, one should expect larger discrepancies between different models. From Table 2 one can see that this is indeed the case. However, results obtained from the three different models are not significantly different. For example, for the decays $C \to E, E^*$ uncertainty the introduced by using a specific model is about 20%. Also, as one can see from Table 4, the ratios of the two exclusive decay widths for members of the same doublet are all consistent, which is the consequence of the application of HQET.

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[4]In [38] one can also find results obtained in the heavy quark limit. These are in general much smaller than our results, but the ratios of partial widths for the B decays into the members of the same D^{**} doublet agree much better with our predictions.

5 Fractional semi-leptonic decay rates

The exclusive decay rates discussed earlier suffer from a variety of theoretical oversimplifications. Some of the things which were not taken into account are QCD corrections, spectator effects, and deviations from exact heavy quark symmetry. In addition, there are several parameters which need to be specified before definite predictions can be made. Among these are the CKM parameter V_{cb}, the b-quark lifetime, and the quark masses.

Many of the above problems can be reduced by considering fractions of the inclusive $b \to c e \bar{\nu}_e$ rate. In particular, V_{cb} exactly cancels. Also, since the sum of the exclusive rates equals the inclusive rate, and since the inclusive calculation is structurally similar to the exclusive ones, there should be some cancellation of the QCD, spectator corrections, and heavy quark mass dependence. Since the inclusive rate has been measured, one can directly compare these fractional predictions with experiment in several cases.

The inclusive spectator model decay rate for $b \to c e \bar{\nu}_e$ is [41, 42]

$$\Gamma(b \to c e \bar{\nu}_e) = \frac{G_F^2 m_b^5 |V_{cb}|^2}{192\pi^3} I\left(\frac{m_c^2}{m_b^2}, 0, 0\right) , \tag{44}$$

where [43]

$$I(x, 0, 0) = (1 - x^2)(1 - 8x + x^2) - 12x^2 \ln x . \tag{45}$$

If for the moment we ignore the oversimplifications of the above inclusive model and assume $V_{cb} = 0.040$, the b-quark lifetime of $\tau_b = 1.5 \times 10^{-12} s$, and the quark masses of the three realistic models given in (22), (27), and (35), we find a total branching ratio

$$BR(b \to c e \bar{\nu}_e) = \begin{cases} 10.27 \, |V_{cb}/0.040|^2 \, (\tau_B/1.50 ps)\% , & \text{for RFTC} \\ 10.31 \, |V_{cb}/0.040|^2 \, (\tau_B/1.50 ps)\% , & \text{for DESC} \\ 10.53 \, |V_{cb}/0.040|^2 \, (\tau_B/1.50 ps)\% , & \text{for SEVC} \end{cases} . \tag{46}$$

The experimental branching ratio, [44]

$$BR(B^- \to X e^- \bar{\nu}_e) = (10.49 \pm 0.46)\% , \tag{47}$$

is in excellent agreement with the above numbers. However, one should keep in mind that the predicted value is very sensitive to the choice of V_{cb}, τ_b, and quark masses.

One plausibly assumes that ratio of the exclusive branching ratios to the inclusive one,

$$R^{**} = \frac{BR(B \to D^{**}e\bar{\nu}_e)}{BR(b \to ce\bar{\nu}_e)} \; , \tag{48}$$

will be more accurate than either of these separately. We first apply this idea to $B \to D$ and $B \to D^*$ decays. From (36), (37) and (47) we find experimental fractions,

$$R_D^{exp} = 0.19 \pm 0.06 \; , \tag{49}$$

$$R_{D^*}^{exp} = 0.49 \pm 0.10 \; . \tag{50}$$

From Table 2 one can see that our three models predict

$$R_D^{th} = \begin{cases} 0.20 \; , & \text{for RFTC} \\ 0.23 \; , & \text{for DESC} \\ 0.20 \; , & \text{for SEVC} \end{cases} \tag{51}$$

and

$$R_{D^*}^{th} = \begin{cases} 0.58 \; , & \text{for RFTC} \\ 0.64 \; , & \text{for DESC} \\ 0.58 \; , & \text{for SEVC} \end{cases} \tag{52}$$

The predicted values are reasonably consistent with measurement in all cases.

The fraction of semi-leptonic decay into final states other than D or D^* is by (49) and (50)

$$R^{exp}(D^{**} \text{ other than } D \text{ and } D^*) = 1 - (R_D^{exp} + R_{D^*}^{exp})$$

$$= 0.32 \pm 0.16 \; . \tag{53}$$

From Table 2 we see that three models discussed in this paper imply

$$R^{th}(D^{**} \text{ other than } D \text{ and } D^*) = 1 - (R_D^{th} + R_{D^*}^{th})$$

$$= \begin{cases} 0.22 \; , & \text{for RFTC} \\ 0.13 \; , & \text{for DESC} \\ 0.22 \; , & \text{for SEVC} \end{cases} \tag{54}$$

It is interesting to observe that single excited charmed states alone are nearly consistent with accounting for the entire inclusive semi-leptonic decay fraction. As a more direct way of seeing this note the total fractional percentage in Table 3. The predicted fraction into all D^{**} states lies at 89% or above for the three models discussed in this paper.

6 Consistency with sum rules

6.1 The Bjorken sum rule

The Bjorken sum rule [7, 8] relates the derivative of the elastic form factor to the values of inelastic S- to P-wave form factors at the zero recoil point. In our notation,

$$- \xi_C'(1) = \frac{1}{4} + \frac{1}{4} \sum_i \left| \xi_E^{(i)}(1) \right|^2 + \frac{2}{3} \sum_j \left| \xi_F^{(j)}(1) \right|^2 . \tag{55}$$

Since the S- to P-wave form factor normalizations at the zero recoil point are not fixed (as is the case of the elastic form factor ξ_C), but instead depend on the energies and wave functions of the LDF [3, 17], it is not immediately obvious that form factors obtained from the three different models will also satisfy the Bjorken sum rule. In particular, we observe from (55) the manifestly valid inequality

$$- \xi_C'(1) \geq \frac{1}{4} . \tag{56}$$

On the other hand, it follows from (10) that [3, 15]

$$- \xi_C'(1) \geq \frac{1}{2} . \tag{57}$$

Combined together, these two bounds imply

$$\frac{1}{4} \sum_i \left| \xi_E^{(i)}(1) \right|^2 + \frac{2}{3} \sum_j \left| \xi_F^{(j)}(1) \right|^2 \geq \frac{1}{4} . \tag{58}$$

One may ask here whether the above two bounds for $-\xi_C'(1)$ are consistent, or can one devise a model for which the S- to P-wave form factors in (58) come to less than $\frac{1}{4}$ [3].

With this in mind we have evaluated numerically both sides of (55) in order to check self consistency of the three models used in this paper. In the sum of the right-hand side

of (55) we included the lowest P-waves plus the first two radial excitations. For the Dirac equation with scalar confinement, with parameters given in (22), the direct evaluation yields

$$- \xi'_C(1) \simeq 0.86 \ , \tag{59}$$

while the sum rule approach (the right-hand side of (55)) gives

$$- \xi'_C(1) = 0.25 + (0.535 + 0.026 + 0.004 + \ldots) \simeq 0.82 \ . \tag{60}$$

For the Salpeter equation with vector confinement, with parameters given in (27), we obtained after direct evaluation

$$- \xi'_C(1) \simeq 1.26 \ , \tag{61}$$

and the sum rule result was

$$- \xi'_C(1) \simeq 1.03 \ . \tag{62}$$

Similarly, the relativistic flux tube model confinement, with parameters given in (35), yields after the direct evaluation

$$- \xi'_C(1) \simeq 1.14 \ , \tag{63}$$

while the sum rule approach gives

$$- \xi'_C(1) \simeq 1.09 \ . \tag{64}$$

It is also worthwhile noting here the CLEO result for derivative of the renormalized form factor [35]

$$- \hat{\xi}'_C(1) = 0.84 \pm 0.12 \pm 0.08 \ . \tag{65}$$

Since all the terms on the right-hand side of (55) are positive definite, and since we are neglecting nonresonant contributions to final states containing a pion (even if those were small), one might argue that in a self-consistent model result for $-\xi'_C(1)$ obtained by direct calculation should be smaller than the one obtained in the sum rule approach. Indeed, this is what happens in all three of the heavy-light models used in this paper.

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Furthermore, for all three models (and especially for the DESC and RFTC) the sum rule is very close to being saturated by the resonant contributions.

However, we must also mention at this point that a similar calculation was also performed in [38]. These authors find (also in the heavy quark limit)

$$- \xi_C'(1) = 1.28 \ , \tag{66}$$

after the direct calculation, and

$$- \xi_C'(1) = 0.25 + 0.21 + \ldots \simeq 0.46 \ , \tag{67}$$

in the sum rule approach (using only the lowest P-wave mesons). The above result shows that the sum rule is far from being saturated by resonances. The difference between the two approaches was in [38] explained as being mainly due to nonresonant contributions to final states containing a pion.

6.2   The Voloshin sum rule

The Voloshin sum rule [9] is the analog of the "optical" sum rule for the dipole scattering of light in atomic physics. In terms of our form factors and energies of the LDF ($E_{D^{**}} = m_{D^{**}} - m_c$), it can be written in the form

$$\frac{1}{2} = \frac{1}{4} \sum_i (\frac{E_{D^{**}}^{(i)}}{E_D} - 1) \left| \xi_E^{(i)}(1) \right|^2 + \frac{2}{3} \sum_j (\frac{E_{D^{**}}^{(j)}}{E_D} - 1) \left| \xi_F^{(j)}(1) \right|^2 \equiv \Delta \ . \tag{68}$$

Here $E_{D^{**}}^{(i)}$ and $E_{D^{**}}^{(j)}$ denote energies of the LDF of the $j = \frac{1}{2}$ and $j = \frac{3}{2}$ P-wave mesons, respectively. In this sum rule the dependence on the unknown charm mass (or the LDF energy) is even stronger than in the case of the Bjorken sum rule, where it is contained only implicitly through form factors. Therefore, one could expect that model calculations used with (68) are less satisfactory than in the case of (55). Still, one could again make the same arguments as in the case of the Bjorken sum rule, and conclude that any self-consistent model calculation of the right-hand side of (68) should yield result smaller than 0.5.

To evaluate the right-hand side of (68), we have used the spin-averaged energies of the LDF in D and D^* and other D^{**} doublets (because the spin-averaged experimental masses were used in numerical calculations which determined quark masses). As before, we have used the lowest P-waves plus the first two radial excitations for both, $j = \frac{1}{2}$ and $j = \frac{3}{2}$ mesons.

For the Dirac equation with scalar confinement (DESC), with parameters given in (22), we find

$$\Delta = 0.395 + 0.033 + 0.007 + \ldots \simeq 0.44 , \tag{69}$$

which is smaller than the predicted value of 0.5. On the other hand, the Salpeter equation with vector confinement (SEVC), with parameters given in (27), results in

$$\Delta \simeq 0.63 , \tag{70}$$

while the relativistic flux tube model confinement (RFTC), with parameters given in (35), yields

$$\Delta \simeq 0.60 . \tag{71}$$

These two results are about 20% higher than the predicted value of 0.5. Again, we emphasize that in the case of the Voloshin sum rule numerical results are much more dependent on the unknown charm quark mass and other parameters of the model. Unfortunately, we are not aware of any other model which has used the Voloshin sum rule as a test of self-consistency, so that we cannot compare our result with the literature. Nevertheless, we do have to say that two out of three heavy-light models used here (SEVC and RFTC) appear to be inconsistent as far as Voloshin sum rule is concerned. Based on the above calculations, one might also argue that the DESC model predictions are the most reliable of all the results presented in this paper.

7 Conclusion

We have examined the role of semi-leptonic B decay into higher charmed mesons. Within a HQET framework we have evaluated branching ratios for $B \to D^{**} e \bar{\nu}_e$, where the D^{**} are all S- and P−wave mesons, D−wave mesons with $j = \frac{3}{2}$, and some of their radial excitations. Our numerical calculations are based upon three realistic models. In each case a light fermion interacts with a fixed source. A short distance vector Coulomb interaction is used, and at large distances the fermion is confined by a scalar, time-component vector, or a flux tube.[5]

An important aim here has been to determine how much of the semi-leptonic decay rate ends up as one of the higher charmed resonances. Considered as a fraction of the inclusive $b \to c e \bar{\nu}_e$ rate we find that between 90 and 95% of B mesons decay semi-leptonically into a single excited charmed resonance.

Another area which we have explored in this paper is the consistency of form factors obtained from three different models with the Bjorken and Voloshin sum rules. One might suspect that there may be an inconsistency between the Bjorken expression for the slope of the Isgur-Wise function at the zero recoil point, and the expression for the same quantity which is obtained in the wave function approach and depends only on the elastic (S- to S-wave) transition. The two methods yield different upper bounds for the IW slope. However, using numerical values for the lowest two S- to P-wave form factors (and also for the derivative of elastic form factor) at the zero recoil point, we find that all three models yield results consistent with the Bjorken sum rule. The Voloshin sum rule provides another sensitive test of model calculations. There, the dependence on the unknown charm quark mass is even stronger than in the case of the Bjorken sum rule, and one might expect that $\frac{1}{m_c}$ effects play an even more significant role. All three of the heavy-light models used in this paper give results which are slightly higher than the sum

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[5]Results based on spinless models such as semi-relativistic quark model (also used in [17]), or spinless relativistic flux tube model (used in [45]) are not significantly different from the results based on the models discussed in this paper.

rule prediction. Nevertheless, given the large qualitative differences between models itself, and the fact that their predictions are very similar in all cases, we believe that all the results presented in this paper are trustworthy.

We have also compared our results with other calculations available in the literature. We find significant disagreements with [36] and [38] in ratios of decay widths for B decays into the members of the same D^{**} doublet. On the other hand, our results are in general significantly larger than the ones obtained in [16] and [37].

ACKNOWLEDGMENTS

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Table 1: Heavy-light states. Spin-averaged experimental masses are calculated in the usual way, by taking $\frac{3}{4}$ ($\frac{5}{8}$) of the triplet and $\frac{1}{4}$ ($\frac{3}{8}$) of the singlet mass for the $S(P)$ waves). Theoretical results are obtained from the three heavy-light models discussed in the text. Model parameters are given in (22) (DESC), (27) (SEVC), and (35) (RFTC). Theoretical errors with respect to the spin-averaged experimental masses are shown in parentheses.

State		J_j^P	k	$^{2S+1}L_J$	Mass (MeV)	RFTC (MeV)	DESC (MeV)	SEVC (MeV)
$c\bar{u},\ c\bar{d}$ quarks								
$D(1867)$	C	$0_{\frac{1}{2}}^-$	-1	$^1S_0$	$1S\ (1974)$	$1981(+7)$	$1977(+3)$	$1980(+6)$
$D^*(2009)$	C^*	$1_{\frac{1}{2}}^-$		$^3S_1$				
$D_1(2425)$	F	$1_{\frac{3}{2}}^+$	-2	$^1P_1/^3P_1$	$1P\ (2446)$	$2439(-7)$	$2444(-2)$	$2440(-6)$
$D_2^*(2459)$	F^*	$2_{\frac{3}{2}}^+$		$^3P_2$				
$c\bar{s}$ quarks								
$D_s(1969)$	C	$0_{\frac{1}{2}}^-$	-1	$^1S_0$	$1S\ (2076)$	$2071(-5)$	$2074(-2)$	$2072(-4)$
$D_s^*(2112)$	C^*	$1_{\frac{1}{2}}^-$		$^3S_1$				
$D_{s1}(2535)$	F	$1_{\frac{3}{2}}^+$	-2	$^1P_1/^3P_1$	$1P\ (2559)$	$2564(+5)$	$2560(+1)$	$2564(+5)$
$D_{s2}^*(2573)$	F^*	$2_{\frac{3}{2}}^+$		$^3P_2$				
$b\bar{u},\ b\bar{d}$ quarks								
$B(5279)$	C	$0_{\frac{1}{2}}^-$	-1	$^1S_0$	$1S\ (5314)$	$5316(+2)$	$5313(-1)$	$5316(+2)$
$B^*(5325)$	C^*	$1_{\frac{1}{2}}^-$		$^3S_1$				
$b\bar{s}$ quarks								
$B_s(5374)$	C	$0_{\frac{1}{2}}^-$	-1	$^1S_0$	$1S\ (5409)$	$5407(-2)$	$5410(+1)$	$5407(-2)$
$B_s^*(5421)$	C^*	$1_{\frac{1}{2}}^-$		$^3S_1$				

Table 2: Exclusive partial widths for decays $B \to D^{**}e\bar{\nu}_e$ obtained from the three different models discussed in the paper. Γ is given in units of $\left[\left|\frac{V_{cb}}{0.040}\right|^2 10^{-15}GeV\right]$, BR is in units of $\left[\left|\frac{V_{cb}}{0.040}\right|^2 \frac{\tau_B}{1.50ps}\%\right]$, while the ratio $R = \frac{BR(B \to D^{**}e\bar{\nu}_e)}{BR(b \to ce\bar{\nu}_e)}$ is given in [%]. Numerical values of $BR(b \to ce\bar{\nu}_e)$ for a particular model can be found in (46).

State		RFTC			DESC			SEVC		
D^{**}	J_j^P	Γ	BR	R	Γ	BR	R	Γ	BR	R
C	$0^-_{\frac{1}{2}}$	9.026	2.057	20.03	10.54	2.401	23.29	9.420	2.147	20.40
C^*	$1^-_{\frac{1}{2}}$	26.09	5.946	57.89	29.03	6.615	64.14	26.80	6.108	58.03
E	$0^+_{\frac{1}{2}}$	0.211	0.048	0.468	0.303	0.069	0.670	0.204	0.047	0.442
E^*	$1^+_{\frac{1}{2}}$	0.299	0.068	0.663	0.419	0.096	0.926	0.289	0.066	0.626
F	$1^+_{\frac{3}{2}}$	1.383	0.315	3.069	1.161	0.265	2.565	1.294	0.295	2.802
F^*	$2^+_{\frac{3}{2}}$	2.307	0.526	5.119	1.854	0.423	4.097	2.152	0.490	4.659
G	$1^-_{\frac{3}{2}}$	0.016	0.004	0.036	0.023	0.005	0.051	0.016	0.004	0.035
G^*	$2^-_{\frac{3}{2}}$	0.016	0.004	0.036	0.023	0.005	0.051	0.016	0.004	0.035
C_2	$0^-_{\frac{1}{2}}$	0.225	0.051	0.499	0.067	0.015	0.148	0.215	0.049	0.466
C_2^*	$1^-_{\frac{1}{2}}$	0.460	0.105	1.021	0.131	0.030	0.289	0.437	0.100	0.946
E_2	$0^+_{\frac{1}{2}}$	0.009	0.002	0.020	0.018	0.004	0.040	0.022	0.005	0.048
E_2^*	$1^+_{\frac{1}{2}}$	0.011	0.003	0.024	0.025	0.006	0.055	0.028	0.006	0.061
F_2	$1^+_{\frac{3}{2}}$	0.068	0.016	0.151	0.029	0.007	0.064	0.085	0.019	0.184
F_2^*	$2^+_{\frac{3}{2}}$	0.101	0.023	0.224	0.045	0.010	0.099	0.128	0.029	0.277
total		40.22	9.17	89.24	43.67	9.95	96.49	41.11	9.37	89.01

Table 3: Our results with the DESC model for $B \to D^{**} e \bar{\nu}_e$ compared to predictions of ISGW2 [36], SISM [16], CNP [37], and SHJL [38]. Γ is given in units of $\left[\left|\frac{V_{cb}}{0.040}\right|^2 10^{-15} GeV\right]$ and BR in units of $\left[\left|\frac{V_{cb}}{0.040}\right|^2 \frac{\tau_B}{1.50ps}\%\right]$.

State		ISGW2		SISM		CNP		SHJL		DESC	
D^{**}	J_j^P	Γ	BR	Γ	BR	Γ	BR	Γ	BR	Γ	BR
C	$0_{\frac{1}{2}}^-$	12.53	2.860	7.800	1.778	7.616	1.736	9.478	2.160	10.54	2.401
C^*	$1_{\frac{1}{2}}^-$	26.12	5.950	23.17	5.279	23.39	5.331	30.646	6.980	29.03	6.615
E	$0_{\frac{1}{2}}^+$	0.316	0.072	0.118	0.027	0.272	0.062	0.295	0.067	0.303	0.069
E^*	$1_{\frac{1}{2}}^+$	0.316	0.072	0.154	0.035	0.381	0.087	0.421	0.096	0.419	0.096
F	$1_{\frac{3}{2}}^+$	1.896	0.432	0.363	0.083	0.544	0.124	1.232	0.281	1.161	0.265
F^*	$2_{\frac{3}{2}}^+$	0.948	0.216	0.517	0.118	1.088	0.248	0.800	0.182	1.854	0.423
G	$1_{\frac{3}{2}}^-$	not given		0.001	0.000	not given		0.022	0.005	0.023	0.005
G^*	$2_{\frac{3}{2}}^-$	not given		0.001	0.000	not given		0.003	0.001	0.023	0.005
C_2	$0_{\frac{1}{2}}^-$	0.000	0.000	0.071	0.016	not given		0.579	0.132	0.067	0.015
C_2^*	$1_{\frac{1}{2}}^-$	0.632	0.144	0.172	0.039	not given		not given		0.131	0.030
E_2	$0_{\frac{1}{2}}^+$	not given		not given		not given		not given		0.018	0.004
E_2^*	$1_{\frac{1}{2}}^+$	not given		not given		not given		not given		0.025	0.006
F_2	$1_{\frac{3}{2}}^+$	not given		not given		not given		not given		0.029	0.007
F_2^*	$2_{\frac{3}{2}}^+$	not given		not given		not given		not given		0.045	0.010
total		42.76	9.74	32.36	7.38	33.29	7.59	43.49	9.91	43.67	9.95

Table 4: Ratios of partial widths for the B decays into the members of the same D^{**} doublet obtained from the three different models discussed in this paper (second to fourth columns), compared to ISGW2 [36], SISM [16], CNP [37], and SHJL [38] results.

Doublet	RFTC	DESC	SEVC	ISGW2	SISM	CNP	SHJL
C/C^*	0.35	0.36	0.35	0.48	0.34	0.33	0.31
E/E^*	0.71	0.72	0.71	1.00	0.76	0.71	0.70
F/F^*	0.60	0.63	0.60	2.00	0.70	0.50	1.54
G/G^*	1.00	1.00	1.00	not given	0.98	not given	7.00
C_2/C_2^*	0.49	0.50	0.49	0.00	0.41	not given	not given
E_2/E_2^*	0.82	0.72	0.79	not given	not given	not given	not given
F_2/F_2^*	0.67	0.64	0.66	not given	not given	not given	not given