UND-HEP-00-BIG09
hep-ph/0011124

New Exact Heavy Quark Sum Rules

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Abstract

Considering nonforward scattering amplitude off the heavy quark in the Small Velocity limit two exact superconvergent sum rules are derived. The first sum rule leads to the lower bound $\varrho^2 > 3/4$ for the slope of the Isgur-Wise function. It also provides the rationale for the fact that the vector heavy flavor mesons are heavier than the pseudoscalar ones. A spin-nonsinglet analogue $\overline{\Sigma}$ of $\overline{\Lambda} = M_B - m_b$ is introduced.

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Heavy quark symmetry and the heavy quark expansion have played an important role in understanding weak decays of heavy flavors. Recent years witnessed significant success in quantifying strong nonperturbative dynamics in a number of practically important problems via application of Wilson Operator Product Expansion (OPE). It is fair to note, however, that with the analytic solution of QCD still missing, the effect of nonperturbative low-scale domain is more parametrized than computed from the first principles. Therefore, any model-independent constraint on the nonperturbative parameters playing the role in heavy quark decays, is an asset. A number of such relations come from the heavy quark sum rules, in particular for transitions between two sufficiently heavy quarks with no change of velocity (zero-recoil sum rules), or where velocity changes by a small amount (small velocity, or SV sum rules). The unified derivation of the sum rules in the field-theoretic OPE is described in detail in the dedicated papers [1], with their quantum mechanical interpretation elucidated. A more pedagogical derivation can be found in recent reviews [2, 3].[1]

In this letter the standard OPE approach is extended to the nonforward SV scattering amplitude of weak currents off the heavy quark. This leads to two new exact heavy quark sum rules related to the spin of the light constituents of the heavy flavor hadron.

We consider the SV scattering amplitude

$$T(q_0;\vec{v},\vec{u}) = \frac{1}{2M_{H_Q}}\langle H_Q(\vec{u})|\int \mathrm{d}^3x\,\mathrm{d}x_0\;\mathrm{e}^{\,i\vec{q}\vec{x}-iq_0x_0}\;iT\{J_0^\dagger(0),\;J_0(x)\}|H_Q(0)\rangle, \qquad (1)$$

where $J_\mu = \bar{Q}\gamma_\mu Q$ and $\vec{q} \equiv m_Q\vec{v}$; likewise $m_Q\vec{u}$ is the momentum of the final heavy flavor hadron. We will also denote $\vec{v}\,' = \vec{v}-\vec{u}$. Note that J_μ is the nonrelativistic current of heavy quarks, and the field $Q(x)$ entering it includes only the operator of annihilation of the heavy quark contained in H_Q, but not the creation of $\tilde{Q}$ which is present in $Q(x)$ from $J_0^\dagger$. Therefore, $J_\mu^\dagger$ is different from J_μ and only the product in the order $J_0^\dagger(0)J_0(x)$ contributes (we adopt the convention where H_Q contains heavy quark and not the antiquark). This can be visualized considering the nondiagonal $b \to c$ transitions with $J_\mu = \bar{c}\gamma_\mu b$, $J_0^\dagger = \bar{b}\gamma_\mu c$, however we assume that both m_b, $m_c \to \infty$ and, for simplicity, put $m_b = m_c$.

In the SV limit we retain only terms through second order in $\vec{v}$ and $\vec{u}$ and take the energy variable ϵ according to

$$\epsilon = q_0 - \left(\sqrt{\vec{q}^{\,2}+m_Q^2}-m_Q\right) \simeq q_0 - \frac{m_Q\vec{v}^{\,2}}{2}\;; \qquad (2)$$

the elastic transitions for a free quark would then correspond to $\epsilon = 0$. The amplitude $T(\epsilon;\vec{v},\vec{v}-\vec{v}\,')$ can be decomposed into symmetric and antisymmetric in $\vec{v}$, $\vec{v}\,'$ parts $h_+(\epsilon)$ and $h_-(\epsilon)$; the latter is present if H_Q has nonzero spin of light degrees of freedom j correlated with its total spin, *viz.* $h_- \sim i\epsilon_{kln}v_kv'_l\langle H_Q|j_n|H_Q\rangle$.

[1] An interesting introduction to heavy quarks in QCD can be found in lectures [4], with the references therein to the more conventional reviews.

At large (complex) $\epsilon \gg \Lambda_{\rm QCD}$ the amplitude (1) can be expanded in inverse powers of ϵ. Simultaneously we assume that $\epsilon \ll m_Q$ and can discard all terms suppressed by powers of m_Q. In this limit the OPE simplifies and takes the form

$$\begin{aligned}
-T(\epsilon;\vec{v},\vec{u}\,) = {} & \frac{1}{\epsilon}\;\frac{1}{2M_{H_Q}}\langle H_Q(\vec{u}\,)|\,\bar{Q}(1-\tfrac{\vec{v}^{\,2}}{4}+\tfrac{\vec{v}\vec{\gamma}}{2}\,)Q(0)\,|H_Q(0)\rangle \\
& +\frac{1}{\epsilon^2}\;\frac{1}{2M_{H_Q}}\langle H_Q(\vec{u}\,)|\,\bar{Q}(i\vec{D}\vec{v}\,)Q(0)|H_Q(0)\rangle \\
& +\sum_{k=0}^{\infty}\frac{(-1)^k}{\epsilon^{k+3}}\,\frac{1}{2M_{H_Q}}\langle H_Q(\vec{u}\,)|\,\bar{Q}[(i\vec{D}\vec{v}\,)\pi_0^k(i\vec{D}\vec{v}\,)-\pi_0^{k+1}(i\vec{D}\vec{v}\,)]Q(0)|H_Q(0)\rangle . \quad (3)
\end{aligned}$$

Here π_0 is nonrelativistic energy, $\pi_0 = iD_0 - m_Q$.

On the other hand, the dispersion relation equates the coefficients in the $1/\epsilon$ series to the moments of the absorptive part of $T(\epsilon;\vec{v},\vec{u}\,)$ (nonforward structure functions of the hadron H_Q):

$$-T(\epsilon;\vec{v},\vec{u}\,) = \sum_{k=0}^{\infty}\frac{1}{\epsilon^{k+1}}\,\frac{1}{2\pi}\int {\rm d}\omega\,\omega^k\,{\rm Im}\,T(\omega;\vec{v},\vec{u}\,)\;; \qquad (4)$$

the latter is given by the product of the SV transition amplitudes into the ground and excited stated. T is analytic at negative ϵ and has a cut at positive ϵ. The elastic transitions lead to the pole located at $\epsilon \simeq -\frac{\overline{\Lambda}\vec{v}^{\,2}}{2}$. Equating $1/\epsilon^k$ terms in Eqs. (3) and (4) we get the sum rules. We illustrate them on the example of B mesons ($j=\frac{1}{2}$) taking B^* as H_Q. With $j=\frac{1}{2}$ the only nontrivial symmetric structure in h_+ is proportional to $(\vec{v}\vec{v}\,')\cdot\langle H_Q'|H_Q\rangle$. Therefore, it is convenient to introduce

$$\begin{aligned}
-T(\epsilon;\vec{v},\vec{v}\!-\!\vec{v}\,') \; = \; & \left[\frac{1-a(\vec{v}^{\,2}+\vec{v}\,'^2)-b(\vec{v}-\vec{v}\,')^2}{\epsilon} - \frac{c\,\vec{v}^{\,2}}{\epsilon^2} + (\vec{v}\,\vec{v}\,')\,h_+(\epsilon)\right]\frac{\langle H_Q'|H_Q\rangle}{2M_{H_Q}} \\
& - \; i\epsilon_{jkl}\,v_j\,v_k'\,h_-(\epsilon)\,\frac{\langle H_Q'|J_l|H_Q\rangle}{2M_{H_Q}} \; + \; \mathcal{O}(\vec{v}^{\,3}) \qquad (5)
\end{aligned}$$

and

$$W_+(\epsilon) = \frac{1}{2\pi}{\rm Im}\,h_+(\epsilon)\;, \qquad W_-(\epsilon) = \frac{1}{2\pi}{\rm Im}\,h_-(\epsilon) \qquad (6)$$

with $\vec{J}$ denoting the angular momentum of H_Q; constants a, b and c are associated with the elastic transition.

The SV transitions can proceed to $j=\frac{1}{2}$ (scalar S and axial A) and to $j=\frac{3}{2}$ (axial D_1 and tensor D_2) "P-wave" states. Following the notations of Ref. [5], the corresponding nonrelativistic amplitudes are given by

$$\begin{aligned}
\langle S(\vec{v}_2)|J_0|B^*(\varepsilon,\vec{v}_1)\rangle &= \quad \tau_{1/2}\,(\vec{\varepsilon}\,(\vec{v}_2-\vec{v}_1))\;, \\
\langle A(e,\vec{v}_2)|J_0|B^*(\varepsilon,\vec{v}_1)\rangle &= -\tau_{1/2}\,i\epsilon_{jkl}\,e_j^*\,\varepsilon_k\,(v_2-v_1)_l \qquad (7)
\end{aligned}$$

and

$$\begin{aligned}
\langle D_1(e,\vec{v}_2)|J_0|B^*(\varepsilon,\vec{v}_1)\rangle &= -\frac{1}{\sqrt{2}}\,\tau_{3/2}\,i\epsilon_{jkl}\,e_j^*\,\varepsilon_k\,(v_2-v_1)_l\;, \\
\langle D_2(e,\vec{v}_2)|J_0|B^*(\varepsilon,\vec{v}_1)\rangle &= \quad \sqrt{3}\,\tau_{3/2}\,e_{jk}^*\,\varepsilon_j(v_2-v_1)_k\;. \qquad (8)
\end{aligned}$$

Here ε, e are nonrelativistic 3-vectors of polarization. In the case of D_2, however e is the symmetric rank-2 tensor, with the sum over polarizations

$$\sum_\lambda e_{ij}^{(\lambda)} e_{kl}^{(\lambda)} = -\frac{1}{3}\delta_{ij}\delta_{kl} + \frac{1}{2}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \; . \tag{9}$$

The elastic transition to this order can proceed only to B^*, and the amplitude is given by

$$\langle B^*(\vec\varepsilon_2, \vec v_2)|J_0|B^*(\vec\varepsilon_1, \vec v_1)\rangle = 2M_{B^*} \cdot \xi\Big((\vec v_2 - \vec v_1)^2\Big)\left(1+\tfrac{\vec v_1^{\,2}+\vec v_2^{\,2}}{4}\right)(\vec\varepsilon_2^{\,*}\vec\varepsilon_1) + \mathcal{O}(\vec v^{\,3}) \; , \tag{10}$$

where ξ is the Isgur-Wise function (its slope $\varrho^2 = -2\xi'(0)$ will be used later) and $\vec\varepsilon$ are the rest-frame polarizations. The latter are related to the polarization 4-vectors ϵ via $\epsilon_0 = (\vec v\vec\varepsilon)$, $\vec\epsilon = \vec\varepsilon + \frac{1}{2}\vec v(\vec v\vec\varepsilon)$, up to terms cubic in velocity.

The explicit computation yields the following structures for the contributions of the $\frac{1}{2}$ and $\frac{3}{2}$ multiplets,

$$\begin{aligned} &|\tau_{1/2}|^2 \,\{\; (\vec\varepsilon^{\,\prime *}\vec\varepsilon)(\vec v^{\,\prime}\, \vec v) - [(\vec\varepsilon^{\,\prime *}\vec v\,)(\vec\varepsilon\,\vec v^{\,\prime}\,) - (\vec\varepsilon^{\,\prime *}\vec v^{\,\prime})(\vec\varepsilon\,\vec v\,)]\} \, , \\ &|\tau_{3/2}|^2 \,\{2(\vec\varepsilon^{\,\prime *}\vec\varepsilon)(\vec v^{\,\prime}\, \vec v) + [(\vec\varepsilon^{\,\prime *}\vec v\,)(\vec\varepsilon\,\vec v^{\,\prime}\,) - (\vec\varepsilon^{\,\prime *}\vec v^{\,\prime})(\vec\varepsilon\,\vec v\,)]\} \, , \end{aligned} \tag{11}$$

respectively, where $\vec\varepsilon^{\,\prime}$ is the polarization vector of $B^*(\vec u\,)$. We also need the following matrix elements to evaluate the OPE part:

$$\frac{1}{2M_{B^*}}\langle H_Q(\varepsilon', \vec u\,)|\bar Q Q(0)|H_Q(\varepsilon, 0)\rangle = \xi(\vec u^{\,2})(1+\tfrac{\vec u^{\,2}}{4})\,(\vec\varepsilon^{\,\prime *}\vec\varepsilon) \; , \tag{12}$$

$$\frac{1}{2M_{B^*}}\langle H_Q(\varepsilon', \vec u\,)|\bar Q\gamma_i Q(0)|H_Q(\varepsilon, 0)\rangle = \tfrac{1}{2}u_i\,(\vec\varepsilon^{\,\prime *}\vec\varepsilon) + \tfrac{1}{2}\left[(\vec\varepsilon^{\,\prime *}\vec u\,)\varepsilon_i - \varepsilon_i^{\prime *}(\vec\varepsilon\,\vec u\,)\right] \tag{13}$$

and

$$\frac{1}{2M_{B^*}}\langle H_Q(\varepsilon', \vec u\,)|\bar Q iD_j Q(0)|H_Q(\varepsilon, 0)\rangle = -\frac{\overline\Lambda}{2}\,u_j(\vec\varepsilon^{\,\prime *}\vec\varepsilon) + \frac{\overline\Sigma}{2}\Big\{(\vec\varepsilon^{\,\prime *}\vec u\,)\varepsilon_j - \varepsilon_j^{\prime *}(\vec\varepsilon\,\vec u\,)\Big\} + \mathcal{O}\Big(\vec u^{\,2}\Big). \tag{14}$$

The higher order in $\vec u$, $\vec v$ terms have been omitted from all the expressions. The last matrix element introduces the new hadronic parameter $\overline\Sigma$ which can be viewed as the spin-nonsinglet analogue of $\overline\Lambda = \lim_{m_b\to\infty} M_B - m_b$. The part proportional to u_j in Eq. (14) amounts to $-\frac{\overline\Lambda}{2}$; this follows from the nonrelativistic equation of motion $(v_\mu iD_\mu)Q(x) = m_Q Q(x)$ which holds in the static limit $m_Q \to \infty$ for the heavy quark moving with velocity v:

$$M_{H_Q}(u - u^{(0)})_\mu u_\mu \langle H_Q(\vec u\,)|\bar Q Q(0)|H_Q(0)\rangle = -(u_\mu iD_\mu)\,\langle H_Q(\vec u\,)|\bar Q Q(x)|H_Q(0)\rangle\Big|_{x=0} =$$

$$\langle H_Q(\vec u\,)|m_Q \bar Q Q(0) - \bar Q(u_\mu iD_\mu)Q(0)|H_Q(0)\rangle$$

($u^{(0)} = (1, \vec 0)$ is the restframe four-velocity) which to order $\vec u^{\,2}$ leads to

$$\Big(M_{H_Q} - m_Q\Big)\,\frac{\vec u^{\,2}}{2}\,\langle H_Q|\bar Q Q(0)|H_Q\rangle = -u_k\langle H_Q(\vec u\,)|\bar Q iD_k Q(0)|H_Q(0)\rangle \; . \tag{15}$$

Considering the forward scattering amplitude where $\vec{u}=0$ and $\vec{v}\,'=-\vec{v}$ selects the symmetric structure function $W_+(\epsilon)$ and yields the Bjorken [6] and Voloshin [7] sum rules for the zeroth and first moments:

$$\varrho^2-\frac{1}{4} \;=\; 2\sum_m \;|\tau_{3/2}^{(m)}|^2 \;+\; \sum_n |\tau_{1/2}^{(n)}|^2\,, \tag{16}$$

$$\frac{\overline{\Lambda}}{2} \;=\; 2\sum_m \epsilon_m|\tau_{3/2}^{(m)}|^2 \;+\; \sum_n \epsilon_n|\tau_{1/2}^{(n)}|^2\,, \tag{17}$$

with $\epsilon_k = M^{(k)} - M_B$ denoting the mass gap between the P-wave and the ground state. Setting $\vec{v}=0$ or $\vec{v}'=0$ fixes $a=\varrho^2/2$, $b=-\frac{1}{4}$ and $c=\frac{\overline{\Lambda}}{2}$ in Eq. (5), as expected.

The new sum rules emerge for the zeroth and first moments of the antisymmetric structure function W_-:

$$\frac{1}{4} \;=\; \sum_m \;|\tau_{3/2}^{(m)}|^2 \;-\; \sum_n \;|\tau_{1/2}^{(n)}|^2\,, \tag{18}$$

$$\frac{\overline{\Sigma}}{2} \;=\; \sum_m \epsilon_m|\tau_{3/2}^{(m)}|^2 \;-\; \sum_n \epsilon_n|\tau_{1/2}^{(n)}|^2\,. \tag{19}$$

The higher moments yield the known sum rules [2] for μ_G^2, ρ_{LS}^3, ... which can be obtained, for instance, considering the usual zero-recoil structure functions of spacelike components of vector currents appearing at order $1/m_Q^2$ [1].

It is often convenient to work in the static limit $m_Q \to \infty$ assuming that Q are spinless; B and B^* in this case are different components of a single spin-$\frac{1}{2}$ particle, Ψ_0. The derivation of the sum rules in this case proceeds similarly; the antisymmetric structure would be absent from the OPE, but the elastic transition yield it with the opposite sign for the zeroth moment. The $D=4$ matrix element in this case takes the form

$$\langle\Psi_0(\vec{u})|\bar{Q}iD_jQ(0)|\Psi_0(0)\rangle = -\frac{\overline{\Lambda}}{2}\,u_j\,\Psi_0^\dagger\Psi_0 - i\frac{\overline{\Sigma}}{2}\,\epsilon_{jkl}\,u_k\,\Psi_0^\dagger\sigma_l\Psi_0 + \mathcal{O}\left(\vec{u}^{\,2}\right)\,. \tag{20}$$

The exact magnitude of the nonperturbative hadronic parameter $\overline{\Sigma}$ is not known at the moment, but can be estimated by means of QCD sum rules; or it can be directly measured on the lattices. Comparing the sum rules (18), (19) with the sum rule for the chromomagnetic expectation value μ_G^2 [2]

$$\frac{\mu_G^2}{6} = \sum_m \epsilon_m^2|\tau_{3/2}^{(m)}|^2 \;-\; \sum_n \epsilon_n^2|\tau_{1/2}^{(n)}|^2\,, \tag{21}$$

$$\mu_G^2 = \frac{1}{2M_{H_Q}}\langle B|\bar{b}\tfrac{i}{2}\sigma_{\mu\nu}G^{\mu\nu}b(0)|B\rangle \simeq \frac{3}{4}\,(M_{B^*}^2 - M_B^2) \approx 0.4\,\mathrm{GeV}^2\,,$$

we expect $\overline{\Sigma}$ to be about $0.25\,\mathrm{GeV}$.

The first sum rule (18) which is independent of the strong dynamics at first may look surprising. In the quark models the $\frac{1}{2}$ and $\frac{3}{2}$ states are differentiated only by spin-orbital interaction. The latter naively can be taken arbitrarily small if the light quark in the meson is nonrelativistic. To resolve this apparent paradox we note that in the nonrelativistic case τ^2 are large scaling like inverse square of the typical velocity of the light quark, $\tau^2 \sim 1/\vec{v}_{\rm sp}^{\,2}$, and the relativistic spin-orbital effects must appear at the relative level $\sim \vec{v}_{\rm sp}^{\,2}$ because spin ceases to commute with momentum to this accuracy. These relativistic corrections lead to the terms of order 1 in the first sum rules Eqs. (16), (18). The constant $\frac{1}{4}$ comes in the latter case from the $1/m^2$ LS-term; this is easy to show using the commutation relations between the momentum, coordinate and the nonrelativistic Hamiltonian.

The heavy quark sum rules lead to a number of exact inequalities in the static limit [2, 3]. The most familiar one is the Bjorken bound $\varrho^2 > 1/4$. The sum rule (18) gives us the stronger dynamical bound $\varrho^2 > 3/4$. Comparison to the QCD sum rule evaluation $\varrho^2 = 0.7 \pm 0.1$ [8] suggests that this bound can be nearly saturated. We also have the bound $\overline{\Lambda} > 2\overline{\Sigma}$.

The sum rule (18) provides the rationale for the experimental fact that vector mesons B^*, D^* are heavier than their hyperfine pseudoscalar partners B, D. Indeed, if the sum rule for μ_G^2 is dominated by the low-lying states then μ_G^2 must be of the same sign as the constant in Eq. (18), which dictates the negative energy of the heavy quark spin interaction in B and positive in B^*.

Let us note that the full matrix element in Eqs. (14), (20) is not well defined in the limit $m_Q \to \infty$ due to ultraviolet divergences in the static theory, and therefore the spin-independent part proportional to $\overline{\Lambda}$ depends on regularization. Nevertheless, the antisymmetric part proportional to $\overline{\Sigma}$ is well defined and finite. The new sum rules (18) and (19) are convergent and not renormalized by perturbative corrections; this distinguishes them from all other heavy quark sum rules.

At large $\epsilon \gg \Lambda_{\rm QCD}$ the sums over the excited states are dual to the quark-gluon contribution computed in perturbation theory [9, 3]:

$$2\sum_m \epsilon_m^2 |\tau_{3/2}^{(m)}|^2 + \sum_n \epsilon_n^2 |\tau_{1/2}^{(m)}|^2 \;\to\; \frac{8}{9}\frac{\alpha_s(\epsilon)}{\pi}\,\epsilon\,{\rm d}\epsilon \;, \tag{22}$$

$$\sum_m \epsilon_m^2 |\tau_{3/2}^{(m)}|^2 - \sum_n \epsilon_n^2 |\tau_{1/2}^{(n)}|^2 \;\to\; -\frac{3\alpha_s(\epsilon)}{2\pi}\,\frac{{\rm d}\epsilon}{\epsilon}\left\{\sum_{\epsilon_m<\epsilon} \epsilon_m^2 |\tau_{3/2}^{(m)}|^2 - \sum_{\epsilon_n<\epsilon} \epsilon_n^2 |\tau_{1/2}^{(n)}|^2\right\}, \tag{23}$$

where the embraced expression simply amounts to $\frac{1}{6}\mu_G^2(\epsilon)$ if the normalization point is implemented as the cutoff in the energy ϵ. Equation (22) can be immediately extended to higher orders in α_s, this amounts to using the so-called dipole coupling $\alpha_s^{(d)}(\epsilon)$ introduced in Ref. [10]:

$$\alpha_s^{(d)}(\epsilon) = \bar{\alpha}_s\left({\rm e}^{-5/3+\ln 2}\epsilon\right) - \left(\frac{\pi^2}{2} - \frac{13}{4}\right)\frac{\alpha_s^2}{\pi} + \mathcal{O}(\alpha_s^3)\;. \tag{24}$$

Using Eq. (23) we can estimate the contribution of the high-energy states in the sum rules (18) and (19):

$$\sum_{\epsilon_m<\mu} |\tau_{3/2}^{(m)}|^2 - \sum_{\epsilon_n<\mu} |\tau_{1/2}^{(m)}|^2 \;\simeq\; \frac{1}{4} + \frac{\alpha_s(\mu)}{8\pi}\,\frac{\mu_G^2(\mu)}{\mu^2}\;, \tag{25}$$

$$\sum_{\epsilon_m<\mu} \epsilon_m |\tau_{3/2}^{(m)}|^2 - \sum_{\epsilon_n<\mu} \epsilon_n |\tau_{1/2}^{(m)}|^2 \;\simeq\; \frac{\overline{\Sigma}}{2} + \frac{\alpha_s(\mu)}{4\pi}\,\frac{\mu_G^2(\mu)}{\mu}\;; \tag{26}$$

they are power suppressed and presumably small in the perturbative domain.

The transition amplitudes with change of the heavy quark velocity acquire ultraviolet divergences when $m_Q\to\infty$, being suppressed by the nonrelativistic analogue of the Sudakov formfactor

$$S \;\sim\; \mathrm{e}^{-\frac{4}{9\pi}(\Delta\vec{v}\,)^2 \int \frac{\mathrm{d}\omega}{\omega}\,\alpha_s^{(d)}(\omega)}\;, \tag{27}$$

where the upper limit of integration is set by m_Q. The nonforward scattering amplitude has the similar universal suppression. This does not affect our consideration since these effects are given only by symmetric combinations of $\vec{v}$ and $\vec{v}\,'$.

Even the convergent sum rules can, in principle, get a finite perturbative renormalization as it happens, for example, with the Bjorken or Ellis-Jaffe sum rules in DIS where it comes from gluon momenta scaling like $\sqrt{Q^2}$. Since we defined the SV transition amplitudes (τ's) via the flavor-diagonal vector current, such perturbative renormalization is absent from the sum rules. Alternatively, if the axial and/or flavor-nondiagonal currents are used, there will be the overall factor η^2 with η the short-distance renormalization of the corresponding zero recoil weak current which would enter the right hand sides of Eqs. (7) and (8).

The heavy quark sum rules can also be considered for other heavy flavor hadrons. In Λ_b the spin of light degrees of freedom vanishes, and no nontrivial relations are obtained with the scattering amplitudes at $\vec{u}\neq 0$. They are informative for the Σ_b-type states with $j=1$ and can be derived directly applying the quoted equations to the corresponding states. Although for spin-1 light cloud an additional, spin-dependent amplitude is present in the elastic transitions at order $\vec{v}\,^2$, it does not yield the antisymmetric in $\vec{v},\vec{v}\,'$ structure. Since the nonforward amplitude has three tensor structures bilinear in $\vec{v},\vec{v}\,'$, there are three sum rules for the contributions of the three types of P-wave states proportional to $|\tau_0|^2$, $|\tau_1|^2$ and $|\tau_3|^2$, respectively, which constrain the slope of the Isgur-Wise function and the second formfactor at zero recoil.

The sum rule (18) can be applied in atomic physics; there it is a sum rule for spin-orbital interaction in dipole transitions. There is a difference between the transitions in atom and B meson, though: in the latter case we are studying the amplitudes mediated by the heavy quark currents, whereas in atoms photons are emitted through their interaction with electrons. The two amplitudes are directly related only in the nonrelativistic approximation. In particular, additional relativistic effects emerge due to explicit corrections in the electromagnetic current of electrons.

The various sum rules we discuss for heavy flavor transitions are the operator relations for the equal time commutators of infinitely heavy quark currents (the lowest sum rules), or their time derivatives, for higher sum rules. The OPE allows straightforward calculation of these commutators including possible Schwinger terms.

To summarize, applying the OPE to the nonforward heavy quark scattering amplitude two new superconvergent sum rules are derived intrinsically connected to the spin of light cloud in the heavy flavor hadron. The first sum rule leads to the nontrivial bound $\varrho^2 > \frac{3}{4}$ for the slope of the IW function, and suggests why B^* is heavier than B. The second sum rule bounds the difference $M_B - m_b$ from below. These sum rules are exact in the heavy quark limit and can help to constrain a number of nonperturbative parameters in heavy flavor hadrons.

Acknowledgments: Useful discussion with I. Bigi and, in particular, M. Eides, M. Shifman and A. Vainshtein are gratefully acknowledged. This work was supported in part by NSF under grant number PHY96-05080 and by RFFI under grant No. 99-02-18355.

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