Hyperfine Effects and Large $1/m_Q^2$ Corrections

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Abstract

Effects associated with hyperfine mass splitting between pseudoscalar and vector mesons may induce surprisingly large $1/m_Q^2$ corrections in the heavy quark expansion. We demonstrate this in a relativistic quark model by calculating to all orders in $1/m_Q$, with and without hyperfine effects, and comparing to the first order results. Total corrections of 30% or more are quite possible in the decay rates for $B \to Dl\nu$ and $B \to D^*l\nu$ near zero recoil.

The subject of semileptonic weak decays of mesons containing a heavy quark has recently attracted much attention. In the limit in which the heavy quark mass m_Q becomes infinite the theoretical treatment of these decays is greatly simplified,[1] potentially reducing the uncertainty in the extraction of the Kobayashi-Maskawa elements. But more insight into the magnitude of the corrections to this limit is required.

We present here a relativistic quark model which provides a representation of the decay amplitudes to all orders in $1/m_Q$. In [2] we show that the model correctly incorporates all heavy quark symmetry relations at zeroth and first order in the $1/m_Q$ expansion. We will use the model to study a source of corrections to the heavy quark limit which has not yet been seriously considered. This is the spin symmetry breaking effects due to gluon exchange between the heavy and light quarks. These hyperfine effects split the vector and pseudoscalar masses, but they will also cause a distortion of the light quark wave function in a way which depends on the spin of the heavy quark.

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Our relativistic model does not deal with gluon exchange directly, but it relates the distortions in light quark "wave functions" to the meson mass splitting. In [2] we explored the sensitivity of amplitudes describing $B \to Dl\nu$ and $B \to D^*l\nu$ to this hyperfine splitting at first order in the heavy quark expansion. Hyperfine splitting had no effect on any of the amplitudes at zero recoil. This gives some motivation for the consideration of $\mathcal{O}(1/m_Q^2)$ effects, in addition to the fact that the main source of corrections affecting the extraction of V_{cb} occurs at this order.

A systematic analysis of $\mathcal{O}(1/m_Q^2)$ corrections in the heavy quark effective theory has recently been made in [3]. Some attempt was also made to estimate the quantities relevant to semileptonic decay at zero recoil, and to do this the authors appealed to the ISGW nonrelativistic quark model.[4] But the ISGW meson wave functions are "spin averaged"; hyperfine effects are not included. The conclusion in [3] is therefore consistent with ours: if hyperfine effects are ignored then $\mathcal{O}(1/m_Q^2)$ corrections to semileptonic decay rates at zero recoil are small. But we are led to a very different result when hyperfine effects are included.

Our model Lagrangian will contain terms which couple the heavy meson fields to the heavy and light quark fields.

$$-\overline{Q(x)}\left\{i\gamma_5 P(x)F_P(x-y)+\gamma_\mu V^\mu(x)F_V(x-y)\right\}q(y)+\text{h.c.} \tag{1}$$

Q and q are the heavy and light quarks contained in the heavy pseudoscalar and vector mesons P and V. $F_{P,V}(x-y)$ are damping factors which will suppress large momentum flow into the light quark. This is the essential physical effect of the light quark wave function. In momentum space we take

$$F_{P,V}(k)=\frac{Z_{P,V}^2}{-k^2+\Lambda_{P,V}^2} \tag{2}$$

where k is the momentum of the light quark. The quantities $Z_{P,V}$ and $\Lambda_{P,V}$ are completely determined for each meson in a manner to be described below. In particular the fact that Λ_P is not equal to Λ_V reflects the spin dependent distortion of "wave functions" mentioned above, and $\Lambda_{P,V}$ will be fixed by the physical, spin dependent meson masses. Such a connection between the distortion and the meson mass splitting will be present in any quark model of heavy mesons, whether or not the model explicitly relates hyperfine effects to gluon exchange.

In [2] we considered the effect of raising $F_{P,V}$ in (2) to powers n other than unity. We found that meson decay constants were the physical quantities most sensitive to the high energy behavior of the vertex factors, and we found that they decrease for increasing n. To obtain realistic meson decay constants, values of n close to but greater than 1 are preferred. Although $n=1$ yields a divergent vector meson decay constant, the $n=1$ case modified by inverse powers of logarithms would probably also be acceptable. Such modifications would have minimal impact on the quantities of interest in this paper, and we shall focus on the pure $n=1$ case as given by (2).

The rest of our model Lagrangian consists of standard kinetic and mass terms for the quarks. The heavy mesons are composed of quarks, and thus to avoid double counting of dynamical degrees of freedom the model Lagrangian does not contain meson kinetic terms. Neither does it contain mass terms or meson self-couplings; all these terms are to be modeled via the quark loop. Various other fields may be coupled to the quarks, depending on the application. For example to describe semileptonic decay we will couple the heavy quarks to the appropriate external gauge fields. Processes involving pions or kaons may be described by coupling these fields to the light quarks.

The quark masses are the only parameters of the model. For the heavy quarks we use the values $m_b = 4.8$ GeV and $m_c = 1.44$ GeV. For the light quark we use its constituent mass m_q; but here there is some uncertainty. Since we expect that the actual momentum dependent constituent quark mass has fallen somewhat at momenta typical in the loop, we take a (momentum independent) $m_q \approx 250$ MeV. We will also explore the sensitivity of results to changes in m_q.

The free quarks may be integrated out of the theory and this yields an effective theory of heavy mesons. The form of such a theory has been well described in the literature.[5] This is a theory of heavy mesons propagating on mass shell or close to mass shell. It does not describe diagrams with closed loops of heavy mesons. It may describe pion loop corrections with an internal heavy meson line since the effective cutoff on such a loop is far below the heavy meson mass.

Here we note a peculiar feature of our model. It turns out that upon carrying out the procedure just described, the resulting effective heavy meson Lagrangian has an overall nonstandard minus sign. But this sign, which would be disastrous for a normal quantum field theory, turns out to be an unphysical sign for this effective theory. This is due to the absence of heavy meson loops. With each heavy meson vertex and each heavy meson propagator having an additional minus sign, the result is that a diagram receives a sign $(-1)^j$ where j is the number of heavy meson lines running through the diagram. Such a sign is of no consequence for physical quantities calculated from these amplitudes.[1] The amplitudes of interest here are determined by the tree level two-meson-external-gauge-field vertices with heavy mesons on shell. In the following we will simply omit the minus sign to coincide with standard conventions.

We denote the proper pseudoscalar and vector self-energy graphs by $i\Gamma_P(p^2)$ and $-ig_{\mu\nu}\Gamma_V(p^2) + \ldots$, where the ellipsis denotes $p_\mu p_\nu$ terms. Because the model treats the quarks as free, the $\Gamma_{P,V}$ become complex above the threshold at $p^2 = (m_Q + m_q)^2$. We will ignore this consequence of free quarks and drop all such imaginary pieces of quark loop graphs.[2] The meson masses are defined by $\Gamma_{P,V}(m_{P,V}^2) = 0$. The mass

[1] One may wonder about the existence of negative energies. But terms of higher order in derivatives and heavy quark mass would be relevant for the Hamiltonian.

[2] The imaginary parts may in fact be kept for all quantities we calculate; the result would be a consistent description of mesons coupling to free quarks. Our dropping the imaginary parts in the end is an effort to incorporate confinement. In this connection we note that the confinement physics

P	m_P	Λ_P	Z_P	V	m_V	Λ_V	Z_V
B	5.279	0.6214	1.1026	B^*	5.325	0.7015	1.2896
D	1.869	0.5367	0.7446	D^*	2.010	0.7623	1.1927
B	5.31	0.6661	1.1920	B^*	5.31	0.6789	1.2432
D	1.97	0.6739	0.9808	D^*	1.97	0.7045	1.0878

Table 1: Input meson masses and resulting values of $\Lambda_{P,V}$ and $Z_{P,V}$ with (top half of table) and without (bottom half) hyperfine splitting (in GeV).

functions Γ_{B,B^*} are plotted versus $\sqrt{p^2}$ in Fig. 1. The zeros lie between the threshold kink and singularities at $p^2 = (m_Q + \Lambda_{P,V})^2$. (We therefore obtain sensible meson masses only if $\Lambda_{P,V} > m_q$.) The $\Lambda_{P,V}$ are determined by requiring that the zeros of the meson mass functions coincide with the experimental meson masses.

The constants $Z_{P,V}$ are determined by normalizing in either of two equivalent ways. The Ward identities relate the slopes of the respective mass functions at their zeros to the $q^2 = 0$ matrix elements of the heavy quark vector current between on-shell mesons. Normalizing these matrix elements to unity is equivalent to the condition $\Gamma'_{P,V}(m^2_{P,V}) = 1$, where the prime denotes differentiation with respect to p^2. Thus,

$$\Gamma_{P,V}(p^2) \approx p^2 - m^2_{P,V} \tag{3}$$

in the neighborhood of $p^2 = m^2_{P,V}$, corresponding to standard meson kinetic terms. For the B and D sectors, the values of the input meson masses and the resulting values of $\Lambda_{P,V}$ and $Z_{P,V}$ are shown in the top half of Table 1.

We are interested in the effects associated with hyperfine mass splitting, $m_V - m_P$. In the model this originates largely in the different values of Λ_V and Λ_P. (There is also a little mass splitting even in the case $\Lambda_V = \Lambda_P$, with the vector mass slightly *less* than the pseudoscalar mass.) We will be comparing our results based on a fit to physical meson masses with results in the case of no hyperfine splitting. We will take the latter case to correspond to the values of Λ_P and Λ_V which yield both the pseudoscalar and vector masses equal to $\frac{1}{4}m_P + \frac{3}{4}m_V$. The amounts by which the physical masses are shifted from this common mass is characteristic of the effect of hyperfine mass splitting. We display for comparison purposes the various input parameters for the case of no hyperfine splitting in the bottom half of Table 1.

We are also interested in comparing to results obtained at first order in the heavy quark expansion.[2] The heavy quark limit of the model is described by the conditions

$$\Lambda_{P,V} \to \Lambda \ , \ \Lambda/m_Q \to 0 \ , \ \Lambda/m_q \text{ fixed.} \tag{4}$$

of QCD occurs at a lower momentum scale than the scale characterizing our model, $\Lambda_{P,V} \approx 650-700$ MeV.

This common Λ is the same for all heavy quark flavors. At zeroth order in an expansion in Λ/m_Q, quantities depend only on the ratio Λ/m_q and are independent of the way in which $\Lambda_{P,V} \to \Lambda$. At first order in the expansion we write

$$\Lambda_{P,V} = \Lambda \left(1 - (\delta_{P,V} h + g)\frac{\Lambda}{m_Q}\right) \tag{5}$$

where $\delta_P = 3$ and $\delta_V = -1$. In [2] we study the dependence of first order corrections on the parameters Λ/m_q, g and h.[3]

The parameter h is responsible for most of the hyperfine mass splitting. We may choose a set of parameters in the first order model which produces an optimal meson mass spectrum. For $m_q = 250$ MeV such a set is $\Lambda = 667$ MeV, $g = -0.13$ and $h = 0.19$ and it yields (B, B^*) and (D, D^*) masses within 0.2% and 2% respectively of the physical masses.

The form factors for the semileptonic decays $B \to D$ and $B \to D^*$ are defined as follows.

$$\langle P_2(v_2)\left|V_\mu\right|P_1(v_1)\rangle = \sqrt{M_{P_1}M_{P_2}}\left[h_+(\omega)(v_1+v_2)_\mu + h_-(\omega)(v_1-v_2)_\mu\right] \tag{6}$$

$$\langle V_2(v_2)\left|V_\mu\right|P_1(v_1)\rangle = \sqrt{M_{P_1}M_{V_2}}h_V(\omega)\varepsilon_{\mu\nu\rho\sigma}\varepsilon_2^{*\nu}v_2^\rho v_1^\sigma \tag{7}$$

$$\begin{aligned}&\langle V_2(v_2)\left|A_\mu\right|P_1(v_1)\rangle = \\ &\quad -i\sqrt{M_{P_1}M_{V_2}}\left[(\omega+1)h_{A_1}(\omega)\varepsilon^*_{2\mu} - \left(h_{A_2}(\omega)v_{1\mu} + h_{A_3}(\omega)v_{2\mu}\right)\varepsilon_2^*\cdot v_1\right]\end{aligned} \tag{8}$$

The v's are meson velocities, and $\omega = v_1 \cdot v_2$.

At zeroth order in the heavy quark expansion,

$$h_+(\omega) = h_V(\omega) = h_{A_1}(\omega) = h_{A_3}(\omega) = \xi(\omega)\ ,\ h_-(\omega) = h_{A_2}(\omega) = 0. \tag{9}$$

$\xi(\omega)$ is the Isgur-Wise function. The first order corrections to the $h_i(\omega)$ require the introduction of only four additional universal functions.[6] In [2] we show that this is also true for our model, for any choice of parameters. This is a nontrivial test for any model of heavy mesons.

We may calculate the $h_i(\omega)$ in the full model from the appropriate three point quark loop graphs. Our results are displayed in Fig. 2. For comparison we also display $\xi(\omega)$ emerging at zeroth order in the model (using $\Lambda = 667$ MeV). We find that $h_{V,A_3,A_1,+}(\omega)$ display substantial shifts of varying amounts from $\xi(\omega)$, although the shape of these functions is fairly similar. In addition $h_{-,A_2}(\omega)$ deviate significantly from zero.

We collect the full zero recoil values $h_i(1)$ in the first row of Table 2, and in the second row we collect the zero recoil values with no hyperfine splitting, $h_i^{\text{no hyp}}(1)$.

[3]Our definition of g and h here differs from that in [2].

i	V	A_3	A_1	$+$	$-$	A_2
$h_i(1)$	1.401	1.307	1.155	1.107	$-.138$	$-.249$
$h_i^{\text{no hyp}}(1)$	1.232	1.149	1.001	0.994	$-.108$	$-.274$
$h_i^{1\text{st}}(1)$	1.228	1.152	1	1	$-.124$	$-.270$
$h_i'(1)$	-2.21	-1.95	-1.71	-1.83	0.20	0.51
$(h_i^{\text{no hyp}})'(1)$	-1.76	-1.54	-1.32	-1.53	0.14	0.51
$(h_i^{1\text{st}})'(1)$	-1.69	-1.49	-1.29	-1.54	0.13	0.46

Table 2: Zero recoil values of form factors h_i (top half of table) and derivatives h_i' (bottom half) with hyperfine splitting, without hyperfine splitting, and at first order.

Most of the $h_i(1)$ deviate significantly further from their values in the heavy quark limit than do the $h_i^{\text{no hyp}}(1)$. Especially interesting are the quantities $h_+(1)$ and $h_{A_1}(1)$ which are both still equal to unity in the heavy quark expansion at first order (Luke's Theorem [6]). We see from Table 2 that at higher orders, in the absence of hyperfine splitting, $h_+(1)$ and $h_{A_1}(1)$ remain very close to unity. But the higher order corrections *with* hyperfine splitting cause significant deviation in these quantities away from the heavy quark limit.

The zero recoil values from the first order model, $h_i^{1\text{st}}(1)$, are given in the third row of Table 2. In [2] we found that these values are independent of g and h and thus independent of hyperfine splitting. The substantial difference between $h_i(1)$ and $h_i^{\text{no hyp}}(1)$ is therefore an $\mathcal{O}(1/m_Q^2)$ effect. It is also interesting that the $h_i^{1\text{st}}(1)$ are very close to the $h_i^{\text{no hyp}}(1)$. This shows that were it not for the hyperfine effects, the corrections beyond first order would be very small. It appears that hyperfine effects completely dominate the $\mathcal{O}(1/m_Q^2)$ corrections.

In the bottom half of Table 2 we give the values of the slopes of the form factors at zero recoil. By comparing the first two rows we see that the effect of hyperfine splitting is to make the slopes more negative. And the effect of hyperfine splitting at first order (that is, the shift in the $(h_i^{1\text{st}})'(1)$ caused by nonzero h) is no more than about 1%. Thus again the main effect of hyperfine splitting occurs at $\mathcal{O}(1/m_Q^2)$.

In Table 3 we show how the $h_i(1)$ change when the light quark mass m_q is varied. Increasing it to 300 MeV increases the corrections even further. And the corrections remain substantial even if m_q is lowered arbitrarily.

We consider further the two quantities which are protected from first order corrections at zero recoil. The following relations are derived in [3].

$$h_+(1) = 1 + \left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)^2 \ell_1(1) + \mathcal{O}\left(\frac{1}{m_Q^3}\right) \tag{10}$$

$$h_{A_1}(1) = 1 + \left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)\left[\frac{1}{2m_c}\ell_2(1) - \frac{1}{2m_b}\ell_1(1)\right] + \frac{\Delta}{4m_c m_b} + \mathcal{O}\left(\frac{1}{m_Q^3}\right) \tag{11}$$

m_q(MeV)	V	A_3	A_1	$+$	$-$	A_2
300	1.481	1.390	1.225	1.156	$-.150$	$-.259$
200	1.363	1.263	1.123	1.086	$-.132$	$-.242$
10	1.334	1.193	1.103	1.082	$-.124$	$-.208$

Table 3: Dependence of $h_i(1)$ on light quark mass m_q.

In a nonrelativistic quark model $\ell_1(1)$ and $\ell_2(1)$ are negative since they may be related to the deficit in a wave function overlap integral.[4] This is not the case for our relativistic quark model. Our meson-q-$\overline{Q}$ vertex factors are analogous to wave functions, but they are inserted into a relativistic quark loop calculation. Our results explicitly show that as the two "wave functions" are distorted by hyperfine effects and made different from each other, the result is a positive correction. We have numerically isolated the $1/m_Q^2$ corrections in (10,11) and find $\Delta \approx (1.6\,\mathrm{GeV})^2$, $\ell_1(1) \approx (1.0\,\mathrm{GeV})^2$, and $\ell_2(1) \approx (0.6\,\mathrm{GeV})^2$. The total magnitude of the $1/m_Q^2$ corrections in (10,11) amounts to about 2/3 of the corrections to all orders.

The Δ term in (11) may be written as $1.5\Lambda^2/m_b m_c$; the dimensionless coefficient here does not appear to be unreasonably large. In [3] Δ is written as

$$\Delta = \frac{4}{3}\lambda_1 + 2\lambda_2 + \cdots \tag{12}$$

where the ellipsis represents corrections resulting from a double insertion of the chromo-magnetic operator. The λ's are defined by

$$\lambda_1 = -\frac{1}{4}(\delta m_P^2 + 3\delta m_V^2) \quad \text{and} \quad \lambda_2 = -\frac{1}{4}(\delta m_P^2 - \delta m_V^2) \tag{13}$$

where

$$m_M - m_Q = \overline{\Lambda} + \frac{\delta m_M^2}{2m_Q} + \cdots \tag{14}$$

We find in our model that $\overline{\Lambda} = 0.50$ GeV, $\lambda_1 = -(0.27\,\mathrm{GeV})^2$ and $\lambda_2 = (0.29\,\mathrm{GeV})^2$. These λ's are small contributions which largely cancel in Δ, and thus the bulk of contribution to Δ must originate in the double insertions of the chromo-magnetic operator. In [3] it was assumed that these contributions could be neglected.

We now turn to the two processes which, by observing at zero recoil, have been proposed [8] as fairly model independent methods for extracting V_{cb}. In the differential decay rate at zero recoil for the process $B \to Dl\nu$ the following factor appears

$$\left| h_+(1) - \frac{m_B - m_D}{m_B + m_D} h_-(1) \right|^2 \tag{15}$$

[4] We find from [4] $\ell_1(1) = \ell_2(1) \approx -\frac{3}{4}m_q^2$, i.e. 1/4 times those in [3].

The deviation of this quantity from unity gives the correction to the heavy quark limit. This might be expected to be small since the only first order $1/m_Q$ correction occurs in $h_-(1)$ and this is suppressed by the prefactor. But inserting values from Table 2 gives a positive 38% correction with hyperfine splitting and a 9% correction without hyperfine splitting. For the decay $B \to D^* l\nu$ the factor of interest is $|h_{A_1}(1)|^2$ which has no first order $1/m_Q$ corrections. Here the correction is 33% with hyperfine splitting, to be compared with essentially no correction for no hyperfine splitting.

These numerical values for the corrections should be considered as illustrative. They depend not only on the light constituent mass, but also on the c and b quark masses we have chosen. And the corrections obviously reflect the simple form we have chosen for the meson vertex factors in (2). But the existence of significant corrections from hyperfine effects, with signs as given, seems to be generic to this class of relativistic quark models.

In this paper we have argued by way of an explicit model that corrections to the heavy quark limit may be larger than previously thought. This is seen most graphically in Fig. 2 which depicts the form factors for semileptonic decay of heavy mesons. At zero recoil the larger than expected corrections appear at $\mathcal{O}(1/m_Q^2)$ in the heavy quark expansion. We have traced the origin of these corrections to the physics associated with hyperfine mass splitting. We hope that these results will encourage further study of hyperfine effects in nonrelativistic quark models and sum rule calculations.

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Figure Captions

Figure 1. Normalized B and B^* mass functions for $m_q = 250$ MeV.

Figure 2. $B \to D$ and $B \to D^*$ form factors $h_i(\omega)$ and Isgur-Wise function $\xi(\omega)$.

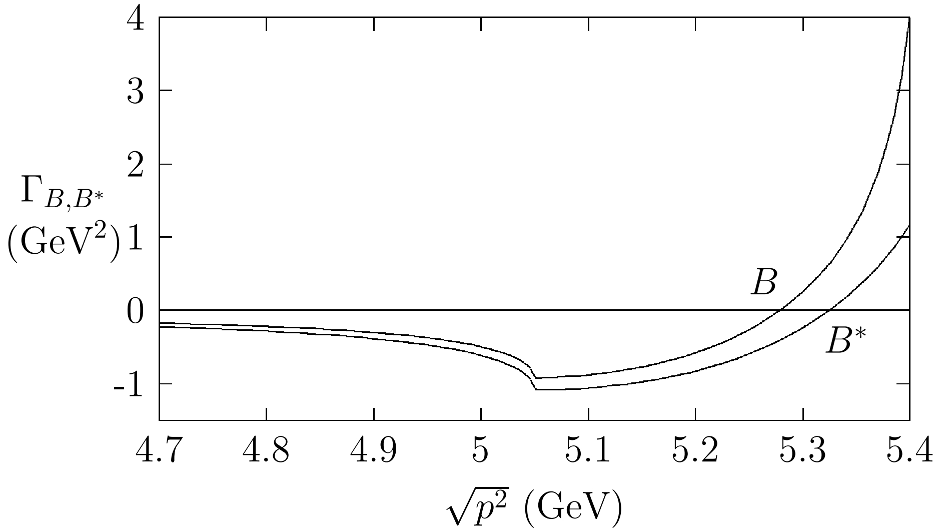

Figure 1

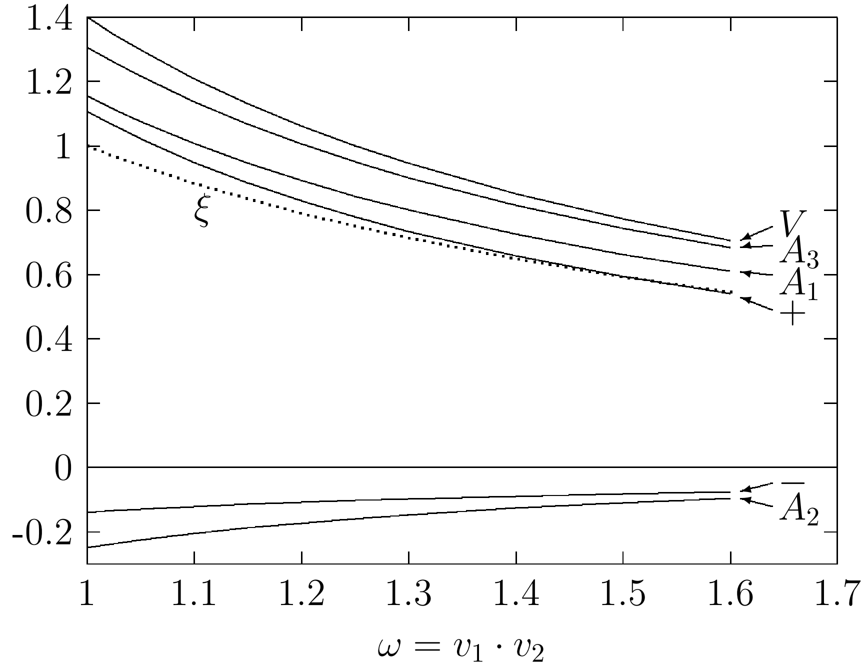
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
$\omega = v_1 \cdot v_2$
ξ
V
A_3
A_1
$+$
$-$
A_2

Figure 2