

Semileptonic Decays of B Meson into Charmed Higher Resonances in the Heavy Quark Effective Theory

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Heavy quark effective theory is applied to semileptonic decays of B mesons into charmed higher resonances D_X , where subscript X denotes various spin-parity states of resonances. We calculate the differential decay rates and branching ratios of the decay $B \rightarrow D_X l \nu_l$. We find that the largest fraction of the semileptonic process $B \rightarrow D_X l \nu_l$ other than $D(1869)$ and $D^*(2010)$ is $D_2^*(1P$ state) whose value is 0.11%. It is also estimated that among the unidentified modes in the semileptonic decays of B meson other than $D(1896)$ and $D^*(2010)$ the total contribution of exclusive higher resonance modes to the branching ratios is 0.3%. The results are compared to the recent experimental data and the other theoretical analyses. We discuss also the unidentified modes in the semileptonic decays of B meson which cannot be reduced to the exclusive higher resonance modes.

§ 1. Introduction

Recently considerable experimental and theoretical effort has been devoted to understanding the dynamics of hadrons containing a heavy quark. In the case that a heavy quark Q has a enough large mass compared to the QCD scale parameter Λ_{QCD} (i.e., $m_Q \gg \Lambda_{\text{QCD}}$), it is good approximation that a heavy quark has no recoil by QCD interaction in the heavy flavored hadrons. Then the angular momenta of the heavy and light degrees of freedom decouple (due to the above approximation) in the limit of $m_Q \rightarrow \infty$.¹⁾ The heavy quark effective theory(HQET) has simplified the analysis of various phenomena of heavy flavored hadrons such as weak decays, mass spectra and so on.²⁾ HQET enables us to describe semileptonic decay processes by use of only a single form factor so called as the Isgur-Wise function $\xi(y)$,¹⁾ where y is the invariant product of four-velocities of initial and final heavy flavored mesons. This theory is very useful to determine the Kobayashi-Maskawa matrix element $|V_{cb}|$ with the normalization $\xi(1)=1$ for $B \rightarrow D^* l \nu_l$ even though the differential decay rate is singular at $y=1$.³⁾ In this paper we study the semileptonic decay processes of B meson with the final state including higher resonance of charmed meson D_X on the basis of HQET, where subscript X of D_X denotes various spin-parity states of charmed meson resonances.

The present experimental data for inclusive and exclusive branching ratios of B meson semileptonic decays are listed as⁴⁾

$$\text{Br}(B \rightarrow X e \nu_e) = 10.7 \pm 0.5\%, \quad \text{Br}(B \rightarrow X \mu \nu_\mu) = 10.3 \pm 0.5\% \quad (1)$$

and

$$\text{Br}(B^0 \rightarrow D^- l^+ \nu_l) = 1.8 \pm 0.5\%, \quad \text{Br}(B^+ \rightarrow \bar{D}^0 l^+ \nu_l) = 1.6 \pm 0.7\%,$$

$$\text{Br}(B^0 \rightarrow D^{*-} l^+ \nu_l) = 4.9 \pm 0.8\%, \quad \text{Br}(B^+ \rightarrow \bar{D}^{*0} l^+ \nu_l) = 4.6 \pm 0.7\%, \quad (2)$$

respectively, where l indicates electron or muon. The unidentified exclusive modes other than the above, whose fraction seems to be about 4%, are possibly reduced to charmed higher resonances D_X , direct production of two or more uncorrelated hadrons and non-charmed hadrons. The direct decay mode into non-charmed meson, however, is expected to be negligibly small due to the small Kobayashi-Maskawa matrix element $|V_{ub}|^{[5]}$

$$\frac{|V_{ub}|}{|V_{cb}|} = 0.05 - 0.10.$$

Therefore in order to understand whole of B meson semileptonic decays, from both theoretical and experimental sides it is important to study how the higher resonance modes are participated.

This paper is organized as follows. In § 2 we formulate $B \rightarrow D_X l \nu_l$ processes including kinematics by use of HQET. In § 3 we obtain the Isgur-Wise function on the basis of constituent quark model.^[6] Then we calculate the decay rates and the branching ratios of the processes $B \rightarrow D_X l \nu_l$. Finally we give brief discussion and a summary in § 4.

§ 2. Semileptonic decay $B \rightarrow D_X l \nu_l$

Heavy flavored mesons are classified by the angular momentum J_l and parity P of the light degrees of freedom denoted as J_l^P . According to heavy quark spin symmetry of HQET, for each J_l^P there are two heavy flavored meson states with $J^P = J_l^P \pm (1/2)$. These two states with spin-parity of the light degrees of freedom $J_l^P = (1/2^-, 1/2^+, 3/2^+, 3/2^-, \dots)$ are denoted by D_X with subscripts $X = C, E, F, G, \dots$, and the radially excited state of ground state C with $J_l^P = 1/2^-$ by D_{C_2} , etc., following the notation given by Ali, Ohl and Mannel.^[7] In this paper we study the semileptonic decays into the following ten charmed mesons:

$$C: D(1869)(1^1S_0), \quad D^*(2010)(1^3S_1); \quad E: D_0^*(1^3P_0), \quad D_{1E}(1^{(1,3)}P_1);$$

$$F: D_{1F}(1^{(1,3)}P_1), \quad D_2^*(2460)(1^3P_2); \quad G: D_1^*(1^3D_1), \quad D_2(1^{(1,3)}D_2);$$

$$C_2: D_{C_2}(2^1S_0), \quad D_{C_2}^*(2^3S_1).$$

In the parentheses the mesons are also assigned by the notation $n^{2S+1}L_J$. We investigate the decays into D , D^* and these eight charmed meson higher resonances. Here we notice that both of heavy flavored meson states D_{1E} and D_{1F} are the mixing states of 1^1P_1 and 1^3P_1 with the mixing angle to be about 35° , that is $|D_{1E}\rangle = \sqrt{1/3}|^1P_1\rangle - \sqrt{2/3}|^3P_1\rangle$, and $|D_{1F}\rangle = \sqrt{2/3}|^1P_1\rangle + \sqrt{1/3}|^3P_1\rangle$.^[8] The observed state $D_1(2420)$ should be assigned to either D_{1E} or D_{1F} .

Following the works by Falk et al.,^[9] we represent these D_X fields by the operator

$$\mathcal{H}_X(v) = \sqrt{m_X} \bar{l}_X(v) H_X(v) \frac{1 + \not{v}}{2} h_c(v), \quad (3)$$

Table I. The heavy flavored meson field.

	J_l^P	J^P	D_X	$H_X(v)$ in Eq. (3)
C	$\frac{1}{2}^-$	0^-	D	γ_5
		1^-	D^*	$\not{\epsilon}$
E	$\frac{1}{2}^+$	0^+	D_0^*	1
		1^+	D_{1E}	$\gamma_5 \not{\epsilon}$
F	$\frac{3}{2}^+$	1^+	D_{1F}	$-\sqrt{\frac{3}{2}}\gamma_5[\epsilon^\mu - \frac{1}{3}(\gamma^\mu + v^\mu)\not{\epsilon}]$
		2^+	D_2^*	$\gamma_\nu \epsilon^{\mu\nu}$
G	$\frac{3}{2}^-$	1^-	D_1^*	$\sqrt{\frac{3}{2}}[\epsilon^\mu - \frac{1}{3}(\gamma^\mu + v^\mu)\not{\epsilon}]$
		2^-	D_2	$\gamma_5 \gamma_\nu \epsilon^{\mu\nu}$
C_2	$\frac{1}{2}^-$	0^-	D_{C_2}	γ_5
		1^-	$D_{C_2}^*$	$\not{\epsilon}$

which annihilates the heavy flavored meson D_X with the four-velocity v , where $h_c(v)$ and $l_X(v)$ are the fields of the charm quark and the light degrees of freedom of the heavy flavored meson D_X with the four-velocity v , respectively. The matrices $H_X(v)$ for ten D_X states are given in terms of the γ matrices in Table I. These representations are very useful to calculate the transition elements with suitable form factors for the processes by using trace formalism as shown later.

Now we concentrate to the semileptonic decays $B \rightarrow D_X l \nu_l$. The momenta of B , l , ν_l and D_X are denoted as P , p_1 , p_2 and p_3 , respectively. In the rest frame of B meson, the decay rate of $B \rightarrow D_X l \nu_l$ is given by

$$d\Gamma(B \rightarrow D_X l \nu_l) = \frac{1}{2m_B} |A(B \rightarrow D_X l \nu_l)|^2 d\pi_3, \quad (4)$$

where the phase space

$$d\pi_3 = (2\pi)^4 \delta^{(4)}(P - p_1 - p_2 - p_3) \prod_{i=1}^3 \frac{d^3 p_i}{(2\pi)^3 2E_i} \quad (5)$$

is taken over all final state momenta and spin sum is assumed. The decay amplitude $A(B \rightarrow D_X l \nu_l)$ is given by a product of the leptonic current L^μ and the hadronic current H^μ with the Kobayashi-Maskawa matrix element V_{cb} as

$$A(B \rightarrow D_X l \nu_l) = \frac{G_F}{\sqrt{2}} V_{cb} L^\mu H_\mu. \quad (6)$$

In the following we put lepton masses to be zero, because the masses of muon and electron are negligibly small compared to the masses of these heavy flavored mesons. The leptonic and hadronic currents are given by

$$L^\mu = \bar{u}_l \gamma^\mu (1 - \gamma_5) v_{\nu_l}, \quad (7)$$

$$H^\mu = \langle D_X(p_3) | J^\mu(0) | B(P) \rangle, \quad (8)$$

where

$$J^\mu = V^\mu - A^\mu = \bar{c} \gamma^\mu (1 - \gamma_5) b.$$

The decay rate is Lorentz invariant and we split the phase space into Lorentz invariant pieces so that it takes on a particularly simple form:

$$\begin{aligned} d\pi_3 &= \frac{1}{(4\pi)^5} \frac{K}{m_B} dq^2 d\Omega_l d\tilde{\Omega}_{D_X}, \\ q^2 &\equiv (P - p_3)^2, \\ K^2 &\equiv \left(\frac{m_B^2 + m_{D_X}^2 - q^2}{2m_B} \right)^2 - m_{D_X}^2, \end{aligned} \quad (9)$$

where $d\Omega_l$ is the solid angle of the charged lepton in the $l\nu_l$ center of mass frame ($l\nu_l$ frame), $d\tilde{\Omega}_{D_X}$ is the solid angle of the final meson in the rest frame of B meson and K is the magnitude of the final meson momentum in the rest frame of B meson.¹⁰⁾ After summing over the spins of charged lepton and neutrino the matrix element squared is given by

$$|A(B \rightarrow D_X l \nu_l)|^2 = \frac{1}{2} G_F^2 |V_{cb}|^2 L^{\mu\nu} H_\mu H_\nu^\dagger, \quad (10)$$

where

$$L^{\mu\nu} = 8(p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - g^{\mu\nu} p_1 \cdot p_2 + i\epsilon^{\mu\nu\alpha\beta} p_{1\alpha} p_{2\beta}).$$

The matrix element of the hadronic current H_μ should be constructed by Lorentz invariant form factors and the four-velocities. Now we set the four-velocity v_μ and v'_μ by the convention of HQET,

$$P_\mu = m_B v_\mu,$$

$$p_{3\mu} = m_{D_X} v'_\mu,$$

and introduce an invariant variable $y \equiv v^\mu v'_\mu$. The momentum transfer squared quantity is

$$q^2 = (P - p_3)^2 = (p_1 + p_2)^2 = m_B^2 + m_{D_X}^2 - 2m_B m_{D_X} y. \quad (11)$$

Then the differential decay rate $d\Gamma/dy$ is given by

$$\frac{d\Gamma}{dy} = \frac{G_F^2 |V_{cb}|^2}{2(4\pi)^5} \frac{m_{D_X}}{m_B} \int KL^{\mu\nu} H_\mu H_\nu^\dagger d\Omega_l d\tilde{\Omega}_{D_X}. \quad (12)$$

Non-zero matrix elements of the vector and axialvector parts of H_μ in Eq. (8) are expressed in terms of form factors as follows:¹⁾

$$\frac{\langle D(v') | V_\mu | B(v) \rangle}{\sqrt{m_B m_D}} = \tilde{f}_+^c(y) (v + v')_\mu + \tilde{f}_-^c(y) (v - v')_\mu, \quad (13a)$$

$$\frac{\langle D^*(v') | V_\mu | B(v) \rangle}{\sqrt{m_B m_{D^*}}} = i\tilde{g}^c(y) \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} v'^\alpha v^\beta, \quad (13b)$$

$$\frac{\langle D^*(v') | A_\mu | B(v) \rangle}{\sqrt{m_B m_{D^*}}} = \tilde{f}^c(y)(y+1)\varepsilon_\mu^* - (\varepsilon^* \cdot v) \{ \tilde{a}_+^c(y)(v+v')_\mu + \tilde{a}_-^c(y)(v-v')_\mu \}, \quad (13c)$$

$$\frac{\langle D_0^*(v') | A_\mu | B(v) \rangle}{\sqrt{m_B m_{D_0^*}}} = \tilde{f}_+^E(y)(v+v')_\mu + \tilde{f}_-^E(y)(v-v')_\mu, \quad (13d)$$

$$\frac{\langle D_{1E,1F}(v') | V_\mu | B(v) \rangle}{\sqrt{m_B m_{D_1}}} = \tilde{f}^{1E,1F}(y)(y+1)\varepsilon_\mu^* - (\varepsilon^* \cdot v) \{ \tilde{a}_+^{1E,1F}(y)(v+v')_\mu + \tilde{a}_-^{1E,1F}(y)(v-v')_\mu \}, \quad (13e)$$

$$\frac{\langle D_{1E,1F}(v') | A_\mu | B(v) \rangle}{\sqrt{m_B m_{D_1}}} = i\tilde{g}^{1E,1F}(y)\varepsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}v'^\alpha v^\beta, \quad (13f)$$

$$\frac{\langle D_2^*(v') | V_\mu | B(v) \rangle}{\sqrt{m_B m_{D_2^*}}} = i\tilde{g}^{2F}(y)\varepsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu\rho}v_\rho v'^\alpha v^\beta, \quad (13g)$$

$$\frac{\langle D_2^*(v') | A_\mu | B(v) \rangle}{\sqrt{m_B m_{D_2^*}}} = \tilde{f}^{2F}(y)(y+1)\varepsilon_{\mu\nu}^* v^\nu - (\varepsilon_{\alpha\beta}^* v^\alpha v^\beta) \{ \tilde{a}_+^{2F}(y)(v+v')_\mu + \tilde{a}_-^{2F}(y)(v-v')_\mu \}, \quad (13h)$$

$$\frac{\langle D_1^*(v') | V_\mu | B(v) \rangle}{\sqrt{m_B m_{D_1^*}}} = i\tilde{g}^{1G}(y)\varepsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}v'^\alpha v^\beta, \quad (13i)$$

$$\frac{\langle D_1^*(v') | A_\mu | B(v) \rangle}{\sqrt{m_B m_{D_1^*}}} = \tilde{f}^{1G}(y)(y+1)\varepsilon_\mu^* - (\varepsilon^* \cdot v) \{ \tilde{a}_+^{1G}(y)(v+v')_\mu + \tilde{a}_-^{1G}(y)(v-v')_\mu \}, \quad (13j)$$

$$\frac{\langle D_2(v') | V_\mu | B(v) \rangle}{\sqrt{m_B m_{D_2}}} = \tilde{f}^{2G}(y)(y+1)\varepsilon_{\mu\nu}^* v^\nu - (\varepsilon_{\alpha\beta}^* v^\alpha v^\beta) \{ \tilde{a}_+^{2G}(y)(v+v')_\mu + \tilde{a}_-^{2G}(y)(v-v')_\mu \}, \quad (13k)$$

$$\frac{\langle D_2(v') | A_\mu | B(v) \rangle}{\sqrt{m_B m_{D_2}}} = i\tilde{g}^{2G}(y)\varepsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu\rho}v_\rho v'^\alpha v^\beta, \quad (13l)$$

where ε_μ^* and $\varepsilon_{\mu\nu}^*$ are polarization vector and tensor for final state meson respectively. The matrix elements for radially excited states D_{c_2} and $D_{c_2}^*$ are given by the same expressions with those of the ground states D and D^* by replacing masses and form factors.

The polarization sums for spin-1 and spin-2 mesons are

$$\begin{aligned} M_{\mu\nu}^{(1)}(v') &\equiv \sum_{\text{pol}} \varepsilon_\mu^* \varepsilon_\nu^* \\ &= -g_{\mu\nu} + v'_\mu v'_\nu, \end{aligned} \quad (14a)$$

$$M_{\mu\nu,\rho\sigma}^{(2)}(v') \equiv \sum_{\text{pol}} \varepsilon_{\mu\nu}^* \varepsilon_{\rho\sigma}^*$$

$$\begin{aligned}
&= \frac{1}{2} M_{\mu\rho}^{(1)}(v') M_{\nu\sigma}^{(1)}(v') + \frac{1}{2} M_{\mu\sigma}^{(1)}(v') M_{\nu\rho}^{(1)}(v') \\
&\quad - \frac{1}{3} M_{\mu\nu}^{(1)}(v') M_{\rho\sigma}^{(1)}(v'), \tag{14b}
\end{aligned}$$

respectively. In general the matrix elements of bilinear currents of two heavy quarks are calculated by taking the trace

$$\begin{aligned}
\frac{\langle D_X(v') | \bar{h}_c(v') \Gamma h_b(v) | B(v) \rangle}{\sqrt{m_B m_{D_X}}} &= \frac{\langle 0 | \mathcal{H}_X(v') \bar{h}_c(v') \Gamma h_b(v) \mathcal{H}_B(v)^\dagger | 0 \rangle}{\sqrt{m_B m_{D_X}}} \\
&= \text{Tr} \{ H_X(v') \frac{1+\not{v}'}{2} \Gamma \frac{1+\not{v}}{2} \gamma_5 \mathcal{M}_X(v, v') \}, \tag{15}
\end{aligned}$$

where $\mathcal{H}_B(v) = \sqrt{m_B} \bar{l}_B(v) \gamma_5 \frac{1+\not{v}}{2} h_b(v)$ is the annihilation operator of B meson and $\mathcal{H}_X(v')$ is also annihilation operator of D_X , which is given by Eq. (3) and Table I, and $\Gamma = \gamma_\mu (1 - \gamma_5)$. The matrix $\mathcal{M}_X(v, v')$ represents overlapping of the light degrees of freedom between the initial and final heavy mesons. Due to the heavy quark symmetry, for each doublet of spin-symmetry, these matrices $\mathcal{M}_X(v, v')$ can be expressed in terms of only one independent form factor called as the Isgur-Wise function $\xi_X(y)$.^{1),9)} For the processes $0^- \rightarrow X$ ($X = C, E$ and C_2)

$$\mathcal{M}_X(v, v') = \xi_X(y), \tag{16a}$$

and for $0^- \rightarrow X$ ($X = F$ and G)

$$\mathcal{M}_X(v, v') = \xi_X(y) v_\mu. \tag{16b}$$

The vector index μ of v_μ in Eq. (16b) is contracted with that of $\mathcal{H}_X(v')$ of the final excited mesons given in Table I. The function $\xi_X(y)$ generally depends on the states of the light degrees of freedom. Form factors in the matrix elements given by Eqs. (13) with the same J^P final state are related to the corresponding Isgur-Wise function $\xi_X(y)$. The relations among these form factors are given in Table II.

Through a straight forward calculation we get the following expression for the decay rate,

$$\frac{d\Gamma_X}{dy} = \frac{G_F^2 |V_{cb}|^2}{48\pi^3} m_B^2 \sqrt{y^2 - 1} m_{D_X}^3 W_X(y, r_X) \xi_X(y)^2, \tag{17}$$

where $W_X(y, r_X)$ is given by

$$\begin{aligned}
W_X(y, r_X) &= (y-1)(y+1)(1+r_X)^2 && \text{(for } B \rightarrow D \text{ and } D_{C_2}) \tag{18a}
\end{aligned}$$

$$\begin{aligned}
&= (y+1)\{(y+1)(1-r_X)^2 + 4y(1-2r_X y + r_X^2)\} \\
&\hspace{15em} \text{(for } B \rightarrow D^* \text{ and } D_{C_2}^*) \tag{18b}
\end{aligned}$$

$$\begin{aligned}
&= (y-1)(y+1)(1-r_X)^2 && \text{(for } B \rightarrow D_0^*) \tag{18c}
\end{aligned}$$

$$\begin{aligned}
&= (y-1)\{(y-1)(1+r_X)^2 + 4y(1-2r_X y + r_X^2)\} && \text{(for } B \rightarrow D_{1E}) \tag{18d}
\end{aligned}$$

Table II. The relation of form factors to $\xi_X(y)$.

X	$C(J_l^P = \frac{1}{2}^-)$		$E(J_l^P = \frac{1}{2}^+)$	
J^P	0^-	1^-	0^+	1^+
		$\bar{g}^C(y) = \xi_C(y)$		$\bar{g}^E(y) = -\xi_E(y)$
		$\bar{f}^C(y) = \xi_C(y)$		$\bar{f}^E(y) = \frac{1-y}{1+y} \xi_E(y)$
	$\bar{f}_+^C(y) = \xi_C(y)$	$\bar{a}_+^C(y) = \frac{1}{2} \xi_C(y)$	$\bar{f}_+^E(y) = 0$	$\bar{a}_+^E(y) = -\frac{1}{2} \xi_E(y)$
	$\bar{f}_-^C(y) = 0$	$\bar{a}_-^C(y) = -\frac{1}{2} \xi_C(y)$	$\bar{f}_-^E(y) = -\xi_E(y)$	$\bar{a}_-^E(y) = \frac{1}{2} \xi_E(y)$

X	$F(J_l^P = \frac{3}{2}^+)$		$G(J_l^P = \frac{3}{2}^-)$	
J^P	1^+	2^+	1^-	2^-
	$\bar{g}^{1F}(y) = \frac{-1-y}{\sqrt{6}} \xi_F(y)$	$\bar{g}^{2F}(y) = \xi_F(y)$	$\bar{g}^{1G}(y) = \frac{1-y}{\sqrt{6}} \xi_C(y)$	$\bar{g}^{2G}(y) = -\xi_C(y)$
	$\bar{f}^{1F}(y) = \frac{1-y}{\sqrt{6}} \xi_F(y)$	$\bar{f}^{2F}(y) = \xi_F(y)$	$\bar{f}^{1G}(y) = \frac{1-y}{\sqrt{6}} \xi_C(y)$	$\bar{f}^{2G}(y) = \frac{1-y}{1+y} \xi_C(y)$
	$\bar{a}_+^{1F}(y) = \frac{5-y}{2\sqrt{6}} \xi_F(y)$	$\bar{a}_+^{2F}(y) = \frac{1}{2} \xi_F(y)$	$\bar{a}_+^{1G}(y) = \frac{1-y}{2\sqrt{6}} \xi_C(y)$	$\bar{a}_+^{2G}(y) = -\frac{1}{2} \xi_C(y)$
	$\bar{a}_-^{1F}(y) = \frac{1+y}{2\sqrt{6}} \xi_F(y)$	$\bar{a}_-^{2F}(y) = -\frac{1}{2} \xi_F(y)$	$\bar{a}_-^{1G}(y) = \frac{5+y}{2\sqrt{6}} \xi_C(y)$	$\bar{a}_-^{2G}(y) = \frac{1}{2} \xi_C(y)$

X	$C_2(J_l^P = \frac{1}{2}^-)$			
J^P	0^-	1^-		
		$\bar{g}^{C_2}(y) = \xi_{C_2}(y)$		
		$\bar{f}^{C_2}(y) = \xi_{C_2}(y)$		
	$\bar{f}_+^{C_2}(y) = \xi_{C_2}(y)$	$\bar{a}_+^{C_2}(y) = \frac{1}{2} \xi_{C_2}(y)$		
	$\bar{f}_-^{C_2}(y) = 0$	$\bar{a}_-^{C_2}(y) = -\frac{1}{2} \xi_{C_2}(y)$		

$$= \frac{2}{3} (y-1)(y+1)^2 \{(y-1)(1+r_X)^2 + y(1-2r_X y + r_X^2)\} \quad (\text{for } B \rightarrow D_{1F}) \quad (18e)$$

$$= \frac{2}{3} (y-1)^2 (y+1) \{(y+1)(1-r_X)^2 + y(1-2r_X y + r_X^2)\} \quad (\text{for } B \rightarrow D_1^*), \quad (18f)$$

respectively and $r_X \equiv m_{D_X}/m_B$.

The exclusive decay rates for the processes $B \rightarrow D_X$ can be calculated also in terms of helicity decomposition method developed by Gilman and Singleton.¹⁰⁾ In the Appendix we show the helicity decomposition method for the spin-2 final mesons as well as for spin-1 mesons.

§ 3. Isgur-Wise functions and the constituent quark model

In order to estimate the decay rates for $B \rightarrow D_X \nu_l$, we have to determine these Isgur-Wise functions by use of specific model. Before the model dependent arguments, we point out the following property of the Isgur-Wise function. Form factor $\xi_c(y)$ of the transition between ground states of B meson and D or D^* meson is normalized as $\xi_c(1)=1$. In the heavy quark limit the other form factors $\xi_X(y)$ for $X = E, F, G, C_2$ are $\xi_X(1)=0$ due to the orthogonality of the wave functions of each light degrees of freedom.

In the following in order to obtain the Isgur-Wise functions we employ the method that is systematically analysed by Grinstein, Isgur, Scora and Wise (GISW model)⁶⁾ on the basis of the constituent quark model and that is developed by Ali, Ohi and Mannel to apply the rare decay process as $B \rightarrow KX\gamma$.⁷⁾ We assume that the constituent quarks are bounded by a potential given by

$$V(r) = -\frac{4\alpha_s}{3r} + c + br,$$

where α_s is QCD coupling constant and b and c are the parameters of the confinement potential. From the standpoint of HQET, heavy flavored hadrons are classified by spin-parity of the light degrees of freedom J_l^P as shown in § 2. However, since we employ the constituent quark model, the wave functions are chosen to be eigenfunction of the orbital angular momentum L .

We use the variational method, which is used in Ref. 6), to determine the above parameters. Here the trial wave functions of initial and final meson states are given by the harmonic oscillator wave functions,¹¹⁾

$$\Phi_I(\mathbf{x}) = Y_0^0\left(\frac{\mathbf{x}}{|\mathbf{x}|}\right) \phi_I(|\mathbf{x}|), \quad (19a)$$

$$\Phi_F(\mathbf{x}) = Y_L^m\left(\frac{\mathbf{x}}{|\mathbf{x}|}\right) \phi_F(|\mathbf{x}|) \quad (19b)$$

with the normalization

$$1 = \int d^3x \Phi_{I/F}^*(\mathbf{x}) \Phi_{I/F}(\mathbf{x}) = \int dr r^2 \phi_{I/F}^*(r) \phi_{I/F}(r).$$

The form factor $\xi(y)$ is defined by the overlap integral of wave functions of the initial and final mesons as

$$\xi(y) \equiv I(v \cdot v') = i^L \sqrt{2L+1} \int dr r^2 \phi_F^*(r) \phi_I(r) j_L(\Lambda r \sqrt{(v \cdot v')^2 - 1}), \quad (20)$$

where j_L is the spherical Bessel function of order L , $\Lambda = m_{D_X} m_q / (m_c + m_q)$ and $\Lambda v'$ is the momentum distributed to the light quark q . The relevant wave functions $\phi_F(r)$ are

$$\phi_F^{(1S)}(r) = \sqrt{4\pi} \frac{\beta_{SD}^{3/2}}{\pi^{3/4}} \exp[-\beta_{SD}^2 r^2 / 2], \quad (21a)$$

$$\phi_F^{(1P)}(r) = \sqrt{\frac{8}{3}} \frac{\beta_{PD}^{5/2}}{\pi^{1/4}} r \exp[-\beta_{PD}^2 r^2/2], \quad (21b)$$

$$\phi_F^{(1D)}(r) = \frac{4}{\sqrt{15}} \frac{\beta_{DD}^{7/2}}{\pi^{1/4}} r^2 \exp[-\beta_{DD}^2 r^2/2], \quad (21c)$$

$$\phi_F^{(2S)}(r) = \sqrt{4\pi} \sqrt{\frac{2}{3}} \frac{\beta_{SD}^{7/2}}{\pi^{3/4}} \left(\frac{3}{2} \beta_{SD}^{-2} - r^2 \right) \exp[-\beta_{SD}^2 r^2/2], \quad (21d)$$

where the parameters β_{SD} , β_{PD} and β_{DD} are obtained by using variational method. For $\phi_F^{(2S)}(r)$ in Eq. (21d) we use the same value of β_{SD} to keep the orthogonality between $\phi_F^{(1S)}(r)$ and $\phi_F^{(2S)}(r)$.⁶⁾ This condition does not give the stationary point for $\phi_F^{(2S)}(r)$ and its mass becomes a little bit larger than the mass at the stationary point. However this result does not affect the following analysis numerically. Since the initial state is the pseudoscalar B meson, its wave function $\phi_I(r)$ is

$$\phi_I(r) = \sqrt{4\pi} \frac{\beta_{SB}^{3/2}}{\pi^{3/4}} \exp[-\beta_{SB}^2 r^2/2], \quad (22)$$

where we take β_{SB} as a variational parameter. Finally we obtain the Isgur-Wise function $\xi_X(y)$ as, for example,

$$\xi_c(y) = \left(\frac{2\beta_{SD}\beta_{SB}}{\beta_{SD}^2 + \beta_{SB}^2} \right)^{3/2} \exp \left[\frac{-\Lambda^2}{2(\beta_{SD}^2 + \beta_{SB}^2)} (y^2 - 1) \right] \quad (23)$$

for $D^{(*)}$ modes.

In the following we use $\alpha_s = 0.5$ and $m_u = m_d = 0.33$ GeV for the constituent quark masses. To determine the other parameters of the potential and trial wave function and constituent mass of bottom and charm quark, we use the following values as the inputs:

$$m_D = 1.869 \text{ GeV}, \quad m_{D^*} = 2.010 \text{ GeV}, \quad m_{D_1} = 2.424 \text{ GeV},$$

$$m_{D_2^*} = 2.459 \text{ GeV}, \quad m_B = 5.279 \text{ GeV},$$

where we assign $D_1(2420)$ to $D_{1F}(J_l^P = 3/2^+)$. The adjusted values of variational parameters which reproduce these input data are

$$b = 0.158 \text{ GeV}^2, \quad c = -0.563 \text{ GeV},$$

$$m_c = 1.63 \text{ GeV}, \quad m_b = 4.98 \text{ GeV},$$

and we also get

$$\beta_{SD} = 0.373 \text{ GeV}, \quad \beta_{PD} = 0.316 \text{ GeV},$$

$$\beta_{DD} = 0.293 \text{ GeV}, \quad \beta_{SB} = 0.393 \text{ GeV}.$$

We compare the Isgur-Wise function $\xi_c(y)$ obtained by using these parameters with the experimental data. However, this function $\xi_c(y)$ gives too larger branching ratio than the experimental data in Eq. (2). This incoincidence could be attributed to the non-relativistic constituent quark model, so we should take the relativistic recoil effect in the form factor $\xi_X(y)$. Then we introduce the compensation factor κ to

account for this effect,⁶⁾ so that

$$\xi_c(y) = \left(\frac{2\beta_{SD}\beta_{SB}}{\beta_{SD}^2 + \beta_{SB}^2} \right)^{3/2} \exp \left[\frac{-\Lambda^2/\kappa^2}{2(\beta_{SD}^2 + \beta_{SB}^2)} (y^2 - 1) \right], \quad (24a)$$

$$\xi_{E,F}(y) = \frac{1}{\sqrt{2}} \left(\frac{2\beta_{PD}\beta_{SB}}{\beta_{PD}^2 + \beta_{SB}^2} \right)^{5/2} \exp \left[\frac{-\Lambda^2/\kappa^2}{2(\beta_{PD}^2 + \beta_{SB}^2)} (y^2 - 1) \right] \frac{\Lambda}{\beta_{SB}} \sqrt{y^2 - 1}, \quad (24b)$$

$$\xi_G(y) = \frac{1}{2\sqrt{3}} \left(\frac{2\beta_{DD}\beta_{SB}}{\beta_{DD}^2 + \beta_{SB}^2} \right)^{7/2} \exp \left[\frac{-\Lambda^2/\kappa^2}{2(\beta_{DD}^2 + \beta_{SB}^2)} (y^2 - 1) \right] \times \left(\frac{\Lambda}{\beta_{SB}} \sqrt{y^2 - 1} \right)^2, \quad (24c)$$

$$\xi_{c_2}(y) = \sqrt{\frac{2}{3}} \left(\frac{2\beta_{SD}\beta_{SB}}{\beta_{SD}^2 + \beta_{SB}^2} \right)^{3/2} \exp \left[\frac{-\Lambda^2/\kappa^2}{2(\beta_{SD}^2 + \beta_{SB}^2)} (y^2 - 1) \right] \times \left\{ \frac{3}{2} \sqrt{1 - \left(\frac{2\beta_{SD}\beta_{SB}}{\beta_{SD}^2 + \beta_{SB}^2} \right)^2} + \frac{1}{4} \left(\frac{2\beta_{SD}\beta_{SB}}{\beta_{SD}^2 + \beta_{SB}^2} \right)^2 \left(\frac{\Lambda}{\beta_{SB}} \sqrt{y^2 - 1} \right)^2 \right\}. \quad (24d)$$

We choose $\kappa=0.61$ so that the sum of the obtained branching ratios $\text{Br}(B \rightarrow D l \nu_l)$ and $\text{Br}(B \rightarrow D^* l \nu_l)$ are consistent with the experimental data $\text{Br}(B \rightarrow D l \nu_l) + \text{Br}(B \rightarrow D^* l \nu_l) = 6.7\%$.

The calculated form factors $\xi_x(y)$ in Eqs. (24) are shown in Fig. 1. For comparison we also show the function $\xi(y) = \exp[-\rho^2(y-1)]$, with $\rho=1.08$,¹³⁾ which is given by fitting the experimental data of the decay $B \rightarrow D^* l \nu_l$. From this figure it is shown that the Isgur-Wise functions for high angular momentum states are very small due to the little overlap integral and we can expect that contributions from the D_x states higher than D -wave (D_1^* and D_2) are negligibly small. Also the form factor for the first radially excited states $\xi_{c_2}(y)$ is small as shown in Fig. 1, and the form factors for the second or higher radial excitation states become negligibly small. It is noted that the conditions in the heavy quark limit, $\xi_c(1)=1(0.998)$ and $\xi_{c_2}(1)=0(0.06)$, are not satisfied in our calculations, where the values in the parentheses denote the numerical values in the present model. If we put $\beta_{SD}=\beta_{SB}$, we easily see that the conditions are fulfilled by using Eqs. (24a) and (24d). $\beta_{SD} \neq \beta_{SB}$ is attributed to the effect of finite heavy quark masses.

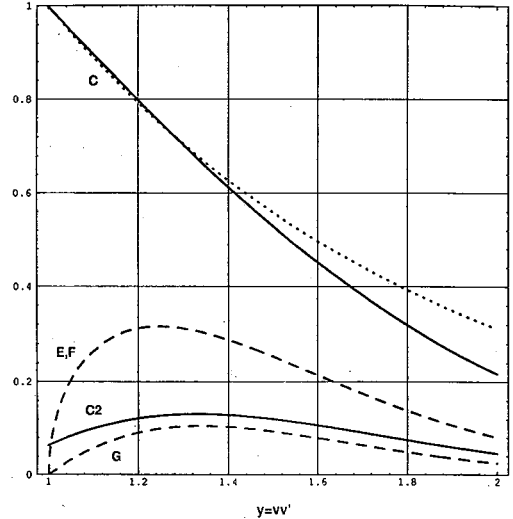


Fig. 1. Isgur-Wise function $\xi_x(y)$. ξ_c , $\xi_{E,F}$, ξ_G and ξ_{c_2} , which are calculated with the degenerate masses, are shown as the upper solid line, the upper dashed line, the lower dashed line and the lower solid line, respectively. For comparison, the dotted line is the Isgur-Wise function $\xi_c(y) = \exp[-\rho^2(y-1)]$ ($\rho=1.08$, $|V_{cb}|=0.045$) given by fitting the experimental data.¹³⁾

The calculated decay rates are listed in Table III. To calculate these decay rates we use the physical parameters

$$G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}, \quad |V_{cb}| = 0.042,^{5)} \quad m_B = 5.279 \text{ GeV},$$

the experimental values of mass for $X = C, F^{4)}$ and the degenerate masses of D_X ($X = E, G$ and C_2) calculated from the constituent quark model which are also listed in Table III. We also summarize the branching ratios of the decay modes $B \rightarrow D_X$ in Table IV, where we take the total decay rate of B meson to be $5.10 \times 10^{-13} \text{ GeV}$ from the mean lifetime $\tau_B = 12.9 \pm 0.5 \times 10^{-13} \text{ sec.}^{4)}$

As shown in Tables III and IV the dominant decay modes of the semileptonic decay $B \rightarrow D_X \nu_l$ are of course the decays to the ground states D and D^* which contribute about 6.7% of the total decay of the B meson and account for the major part of the inclusive semileptonic decay $B \rightarrow X \nu_l$. The contribution to the semile-

Table III. $\Gamma(B \rightarrow D_X \nu_l)$ in unit $|V_{cb}/0.042|^2 \text{ GeV}$.

		J_l^P	J^P	D_X	$m_{D_X}(\text{GeV})$	$\Gamma(B \rightarrow D_X \nu_l)$
1S	C	$\frac{1}{2}^-$	0^-	D	1.869 ^{a)}	8.60 $\times 10^{-15}$
			1^-	D^*	2.010 ^{a)}	25.54 $\times 10^{-15}$
1P	E	$\frac{1}{2}^+$	0^+	D_0^*	2.446	0.13 $\times 10^{-15}$
			1^+	D_{1E}	2.446	0.17 $\times 10^{-15}$
	F	$\frac{3}{2}^+$	1^+	D_{1F}	2.424 ^{a)}	0.40 $\times 10^{-15}$
			2^+	D_2^*	2.459 ^{a)}	0.57 $\times 10^{-15}$
1D	G	$\frac{3}{2}^-$	1^-	D_1^*	2.800	0.00083 $\times 10^{-15}$
			2^-	D_2	2.800	0.00085 $\times 10^{-15}$
2S	C_2	$\frac{1}{2}^-$	0^-	D_{C_2}	2.767	0.078 $\times 10^{-15}$
			1^-	$D_{C_2}^*$	2.767	0.19 $\times 10^{-15}$

a) These are experimental values.⁴⁾

Table IV. $\text{Br}(B \rightarrow D_X \nu_l)$ in unit $(\tau_B/1.29 \text{ ps})\%$. The values of GISW^{a)} are estimated for $\text{Br}(B \rightarrow D, D^* \nu_l) = 6.7\%$, and the relative ratio of each mode are referred from Ref. 13). The values of CNP¹²⁾ are estimated for $|V_{cb}| = 0.042$.

		J_l^P	J^P	D_X	$\text{Br}(B \rightarrow D_X \nu_l)$	GISW	CNP	$\text{Br}(D_X \rightarrow D^*)$
1S	C	$\frac{1}{2}^-$	0^-	D	1.69		1.54	—
			1^-	D^*	5.00		4.47	—
1P	E	$\frac{1}{2}^+$	0^+	D_0^*	0.025	0.11	0.06	0
			1^+	D_{1E}	0.033	0.21 ^{a)}	0.08	1
	F	$\frac{3}{2}^+$	1^+	D_{1F}	0.079	0.41 ^{b)}	0.11	1
			2^+	D_2^*	0.11	0.14	0.22	0.29
1D	G	$\frac{3}{2}^-$	1^-	D_1^*	0.00016			0.25
			2^-	D_2	0.00017			1
2S	C_2	$\frac{1}{2}^-$	0^-	D_{C_2}	0.015	0.07		1
			1^-	$D_{C_2}^*$	0.036	0.06		0.56

a) Value for 3P_1 state.

b) Value for 1P_1 state.

ptonic decay next to D and D^* mesons comes from $1P$ states D_2^* and D_{1F} which are 0.11% and 0.08%, respectively. Sum of the contributions of the semileptonic decays into D_X of the P -states ($X=E$ and F) and the D -states ($X=G$) are about 0.25% and 0.0003%. From these results we may conclude that the contributions from the higher resonance states of D meson with orbital angular momentum $L \geq 3$ will be completely negligible due to the small overlapping integrals and small phase volumes. Contributions from the decays into radially excited states also rapidly decrease with increase of the principal quantum number. The contribution of the $2S$ states seems to be less than 1/100 of that of the $1S$ states. Then the total contribution of the states except for $1S$ states is only 1/20 of that of the ground states.

The decay rates of semileptonic processes $B \rightarrow D_X l \nu_l$ are sensitive to the mass of D_X due to the form factor and the phase volume. If the masses of D_X are changed by ± 0.2 GeV with the other parameters fixed, then the decay rate into the P -wave resonant states will change by $\mp 160\%$ while into radially excited $2S$ states will change by $\mp 130\%$. The ratios of longitudinal to transverse, however, are rather insensitive to the mass of D_X . We can get $L/T=1.4$ for D_{1E} and $L/T=5.8$ for D_{1F} . The decay rate into multiplet G is negligibly small so that its uncertainty due to the masses of G states could not give sensitive effects to the branching ratios of the inclusive decay $B \rightarrow X l \nu_l$. From Table IV the total sum of the branching ratio of $B \rightarrow D_X l \nu_l$ other than the ground states D and D^* is 0.3%.

§ 4. Discussion and summary

In the preceding sections we evaluated the decay rate and the branching ratio for the exclusive process $B \rightarrow D_X l \nu_l$ on the basis of the HQET. As shown in Table IV, the result for branching ratio of exclusive semileptonic processes to the S -wave ground states are $\text{Br}(B \rightarrow D l \nu_l) = 1.7\%$ and $\text{Br}(B \rightarrow D^* l \nu_l) = 5.0\%$. These calculated branching ratios reproduce the experimental branching ratios. The sum of the branching ratios of the processes into the S -wave ground states is $\text{Br}(B \rightarrow S\text{-states}) = \text{Br}(B \rightarrow D l \nu_l) + \text{Br}(B \rightarrow D^* l \nu_l) = 6.7\%$, the one into P -wave states is $\text{Br}(B \rightarrow P\text{-states}) = 0.25\%$, and the one into D -wave states is estimated to be about 0.0003%. For the radially excited states ($2S$ -states) the sum of the branching ratios is $\text{Br}(B \rightarrow 2S\text{-states}) = 0.05\%$. From these results, as expected, the exclusive branching ratios $\text{Br}(B \rightarrow D_X l \nu_l)$ decrease rapidly as the orbital angular momentum and the radial quantum number of excited D_X states increase. So most part of the contribution of higher resonance to semileptonic branching ratio is sufficiently occupied by $1S$, $1P$, $1D$ and $2S$ states.

The sum of the calculated branching ratios of $\text{Br}(B \rightarrow D_X l \nu_l)$ other than the ground states D and D^* , $\text{Br}(B \rightarrow D_X l \nu_l; X \neq C) \cong 0.3\%$, is only one twentieth of the branching ratio into D and D^* mesons, $\text{Br}(B \rightarrow S\text{-states})$. This means that $\text{Br}(B \rightarrow X_c l \nu_l) \cong 7.0\%$ and the remains of $\text{Br}(B \rightarrow X l \nu_l) \cong 3.7\%$ may be continuum and non-charmed processes. Our result of the total branching fractions into charmed higher resonances is consistent with that given by GISW⁽⁶⁾ and by Colangelo Nardulli and Pavor.⁽¹²⁾ We obtain the ratio

$$R = \frac{\text{Br}(B \rightarrow D_X l \nu_l; X \neq C)}{\text{Br}(B \rightarrow X l \nu_l)} \simeq 0.03. \quad (25)$$

This value should be compared to the recent experimental value $R = 0.21 \pm 0.08$.⁵⁾ From our result in Table IV we can also evaluate the ratio between observed numbers of $N(D_X l^-)$ and $N(D^* l^-)$,

$$\frac{N(D_X l^-)}{N(D^* l^-)} = \sum_{X \neq C} \frac{\text{Br}(\bar{B}^0 \rightarrow D_X^+ l^- \bar{\nu}_l) \text{Br}(D_X^+ \rightarrow D^*) \varepsilon_X}{\text{Br}(\bar{B}^0 \rightarrow D^{*+} l^- \bar{\nu}_l)} \leq 0.04, \quad (26)$$

where D_X represents the higher resonance state other than D and D^* and $\text{Br}(D_X^+ \rightarrow D^*)$ is the decay branching ratio of D_X^+ into D^* via strong interaction which are listed in Table IV. Though this value 0.04 is given at the efficiency $\varepsilon_X = 1$, this is still very small in comparison with ARGUS's data 0.27.¹³⁾ On the other hand, as shown in Table IV, the ones resulted from branching fractions into charmed higher resonances calculated by GISW and CNP are 0.15 and 0.05, respectively. These values are also smaller than the ARGUS's data. Grinstein pointed out that the processes like $B \rightarrow D^{(*)} \pi l \nu_l$ could be an important correction to the inclusive semileptonic rate and it may be accounted for some of the anomalously large $D_X (X \neq C)$ contributions.¹⁴⁾ It is also discussed that the virtual B^* processes like $B \rightarrow (B^* \pi) \rightarrow D^{(*)} \pi l \nu_l$ give the large fraction of the continuum.¹⁵⁾

How can we interpret this discrepancy between the theoretical predictions and the experimental data about the contribution of the excited states D_X in semileptonic B decays? Solving this problem is important to check the applicability of HQET. We have to proceed with the theoretical investigation as well as to wait the further detailed experimental data.

In this paper we use the constituent quark model which requires $\xi_E(y) = \xi_F(y)$. This is caused from that the wave function of each meson is assigned to an eigenfunction of orbital angular momentum L , not J_l^P . The $1/m_Q$ correction may be important as the energy of light degrees of freedom increases. For example, in Ref. 12) the branching ratios for $1P$ states with the $1/m_Q$ correction are 2~3 times as the ones in the heavy quark limit. So it is meaningful to clarify the model-dependence and the $1/m_Q$ correction in the Isgur-Wise functions.

Appendix

In this appendix we formulate the helicity decomposition method, which is developed by Gilman and Singleton,¹⁰⁾ to obtain the decay rate for the D meson resonances with spin-2 as well as spin-0 and -1. The differential decay rate is given in terms of the helicity decomposition as:

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{cb}|^2}{96\pi^3} \frac{K}{m_B^2} q^2 (|\bar{H}_+|^2 + |\bar{H}_-|^2 + |\bar{H}_0|^2), \quad (27)$$

where subscripts of helicity amplitude denote the helicity of final meson state and K is defined in Eq. (9). In the ν_l frame the helicity amplitudes become

$$\bar{H}_{\pm}=0, \quad (28a)$$

$$\bar{H}_0=-2\frac{K}{\sqrt{q^2}}m_B f_+(q^2) \quad (28b)$$

for $J^P=0^-$ final meson, and

$$\bar{H}_{\pm}=f(q^2)\mp 2m_B K g(q^2), \quad (29a)$$

$$\bar{H}_0=\frac{m_B^2}{2m_{D_X}\sqrt{q^2}}\left\{\left[1-\frac{m_{D_X}^2}{m_B^2}-\frac{q^2}{m_B^2}\right]f(q^2)+4K^2 a_+(q^2)\right\} \quad (29b)$$

for $J^P=1^-$ final meson. The form factors $f_+(q^2)$, $g(q^2)$, $f(q^2)$ and $a_+(q^2)$ in above equations are related to those in Eqs. (13a)~(13c) by

$$f_+(q^2)=\frac{1}{2}\sqrt{\frac{m_B}{m_{D_X}}}\left\{\left(1+\frac{m_{D_X}}{m_B}\right)\tilde{f}_+^c(y)-\left(1-\frac{m_{D_X}}{m_B}\right)\tilde{f}_-^c(y)\right\}, \quad (30a)$$

$$f(q^2)=\sqrt{m_B m_{D_X}}(1+y)\tilde{f}^c(y), \quad (30b)$$

$$g(q^2)=\frac{1}{2\sqrt{m_B m_{D_X}}}\tilde{g}^c(y), \quad (30c)$$

$$a_+(q^2)=\frac{-1}{2\sqrt{m_B m_{D_X}}}\left\{\left(1+\frac{m_{D_X}}{m_B}\right)\tilde{a}_+^c(y)-\left(1-\frac{m_{D_X}}{m_B}\right)\tilde{a}_-^c(y)\right\}. \quad (30d)$$

Also for $J^P=0^+$, 1^+ final meson we obtain the decomposition the same as for $J^P=0^-$, 1^- final meson replacing the Isgur-Wise form factors by corresponding ones. Then using Eqs. (27)~(30d) and Table II we have the function $W_X(y, r_X)$ in the decay rate Eq. (17):

$$\frac{d\Gamma_X}{dy}=\frac{G_F^2|V_{cb}|^2}{48\pi^3}m_B^2\sqrt{y^2-1}m_{D_X}^3W_X(y, r_X)\xi_X(y)^2 \quad (31)$$

for each spin and the polarization (longitudinal and transverse), and $W_X(y, r_X)$ are listed in Table V.

We proceed to the final meson with spin $J^P=2^\pm$ by extending the method given in Ref. 10). In the expression of the decay amplitude squared Eq. (10) only the spacial components of $L^{\mu\nu}$ are non zero in the ν_l frame, because we neglect the mass of the leptons. So we only need the spacial components \mathbf{H} of H_μ in the ν_l frame,

$$\begin{aligned} \mathbf{H} &= 2i\sqrt{q^2}m_B g(q^2)(\boldsymbol{\varepsilon}^* \mathbf{P}) \times \mathbf{p}_3 - f(q^2)(\boldsymbol{\varepsilon}^* \mathbf{P}) - 2(\boldsymbol{\varepsilon}^* PP)a_+(q^2)\mathbf{p}_3, \\ (\boldsymbol{\varepsilon}^* \mathbf{P}) &\equiv (\varepsilon_{1\lambda}^* P^\lambda, \varepsilon_{2\lambda}^* P^\lambda, \varepsilon_{3\lambda}^* P^\lambda), \\ (\boldsymbol{\varepsilon}^* PP) &\equiv (\varepsilon_{\nu\lambda}^* P^\nu P^\lambda), \end{aligned} \quad (32)$$

where $g(q^2)$, $f(q^2)$ and $a_+(q^2)$ are form factors given as

$$\langle D_2(p_3) | J^\mu(0) | B(P) \rangle = -ig(q^2)\varepsilon_{\mu\nu\rho\sigma}(\varepsilon^{*\nu\lambda}P_\lambda)(P+p_3)^\rho(P-p_3)^\sigma + f(q^2)(\varepsilon_{\mu\lambda}^*P^\lambda)$$

Table V. Integrand $W_X(y, r_X)$. The subscripts X of r_X are omitted in the table. L and T represent longitudinal and transverse components, respectively.

	J^P	$J_{L,T}^P$	$W_X(y, r_X)$
C & C_2	$\frac{1^-}{2}$	0^-	$(y-1)(y+1)(1+r)^2$
		1_L^-	$(y+1)^2(1-r)^2$
		1_T^-	$4y(y+1)(1-2ry+r^2)$
E	$\frac{1^+}{2}$	0^+	$(y-1)(y+1)(1-r)^2$
		1_L^+	$(y-1)^2(1+r)^2$
		1_T^+	$4y(y-1)(1-2ry+r^2)$
F	$\frac{3^+}{2}$	1_L^+	$\frac{2}{3}(y-1)^2(y+1)^2(1+r)^2$
		1_T^+	$\frac{2}{3}y(y-1)(y+1)^2(1-2ry+r^2)$
		2_L^+	$\frac{2}{3}(y-1)(y+1)^3(1-r)^2$
		2_T^+	$2y(y-1)(y+1)^2(1-2ry+r^2)$
G	$\frac{3^-}{2}$	1_L^-	$\frac{2}{3}(y-1)^2(y+1)^2(1-r)^2$
		1_T^-	$\frac{2}{3}y(y-1)^2(y+1)(1-2ry+r^2)$
		2_L^-	$\frac{2}{3}(y-1)^3(y+1)(1+r)^2$
		2_T^-	$2y(y-1)^2(y+1)(1-2ry+r^2)$

$$+ a_+(\epsilon_{\nu\lambda}^* P^\nu P^\lambda)(P + p_3)_\mu + a_-(\epsilon_{\nu\lambda}^* P^\nu P^\lambda)(P - p_3)_\mu \quad (33)$$

for 2^- state. For 2^+ state, the sign of each term on r.h.s. of Eq. (33) is changed. To proceed further, we express the polarization tensor $\epsilon_{\nu\lambda}^*$ in the concrete form. Because the degree of freedom of polarization is five, we put

$$\epsilon^{*\mu\nu} = \sum_{a=1}^5 \alpha_a \lambda_a^{\mu\nu}, \quad \sum_{a=1}^5 \alpha_a^2 = 5,$$

where α_a is coefficient for each polarization tensor $\lambda_a^{\mu\nu}$ ($a=1, 2, \dots, 5$) which is transverse to p_3^μ , symmetric and traceless tensor and which is normalized as $g_{\mu\rho} g_{\nu\sigma} \lambda_a^{\mu\nu} \lambda_b^{\rho\sigma} = \delta_{ab}$. In the rest frame of final meson, $p_3^\mu = (m_{D^*}, 0, 0, 0)$, λ_a ($a=1, 2, \dots, 5$) are given by

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad \sqrt{\frac{2}{3}} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -1/2 & 0 & 0 \\ 0 & 0 & -1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \quad (34)$$

In the $l\nu_l$ frame, where we define the positive z axis as the direction of the $-\mathbf{p}_3$ and let $p \equiv |\mathbf{p}_3| = m_B K / \sqrt{q^2}$, the momenta of neutrino and the B meson p_3^μ and P_μ become as

$$\begin{aligned} p_3^\mu &= (E_{Dx}, 0, 0, -p), \\ P^\mu &= (E_B, 0, 0, -p), \end{aligned} \quad (35)$$

where

$$\begin{aligned} E_{Dx} &= \frac{m_B^2}{2\sqrt{q^2}} \left(1 - r_X^2 - \frac{q^2}{m_B^2} \right), \\ E_B &= \frac{m_B^2}{2\sqrt{q^2}} \left(1 - r_X^2 + \frac{q^2}{m_B^2} \right). \end{aligned} \quad (36)$$

In this frame the matrices λ_a are given by

$$\begin{aligned} & \frac{1}{\sqrt{2}m_{Dx}} \begin{bmatrix} 0 & p & 0 & 0 \\ p & 0 & 0 & -E_{Dx} \\ 0 & 0 & 0 & 0 \\ 0 & -E_{Dx} & 0 & 0 \end{bmatrix}, \quad \frac{1}{\sqrt{2}m_{Dx}} \begin{bmatrix} 0 & 0 & p & 0 \\ 0 & 0 & 0 & 0 \\ p & 0 & 0 & -E_{Dx} \\ 0 & 0 & -E_{Dx} & 0 \end{bmatrix}, \\ & \sqrt{\frac{2}{3}} \frac{p}{m_{Dx}^2} \begin{bmatrix} p & 0 & 0 & -E_{Dx} \\ 0 & -1/2 & 0 & 0 \\ 0 & 0 & -1/2 & 0 \\ -E_{Dx} & 0 & 0 & E_{Dx}^2/p \end{bmatrix}, \\ & \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \end{aligned} \quad (37)$$

So in the $l\nu_l$ frame

$$\begin{aligned} (\epsilon^* P) &= \frac{m_B K}{m_{Dx}} \left(\frac{\alpha_1}{\sqrt{2}}, \frac{\alpha_2}{\sqrt{2}}, -\alpha_3 \sqrt{\frac{2}{3}} \frac{E_{Dx}}{m_{Dx}} \right), \\ (\epsilon^* PP) &= \alpha_3 \sqrt{\frac{2}{3}} \left(\frac{m_B K}{m_{Dx}} \right)^2. \end{aligned} \quad (38)$$

Then in the same sense of Ref. 10) we have that for spin-2 final meson

$$\bar{H}_{\pm} = \frac{m_B K}{\sqrt{2} m_{D_X}} [f(q^2) \mp 2m_B K g(q^2)], \quad (39a)$$

$$\bar{H}_0 = \sqrt{\frac{2}{3}} \frac{m_B^3 K}{2m_{D_X}^2 \sqrt{q^2}} \left[\left(1 - r_X^2 - \frac{q^2}{m_B^2} \right) f(q^2) + 4K^2 a_+(q^2) \right]. \quad (39b)$$

The relation of the form factors $f(q^2)$, $g(q^2)$ and $a_+(q^2)$ to ours is

$$f(q^2) = \sqrt{\frac{m_{D_X}}{m_B}} \tilde{f}^X(y), \quad (40a)$$

$$g(q^2) = \frac{1}{2m_B \sqrt{m_B m_{D_X}}} \tilde{g}^X(y), \quad (40b)$$

$$a_+(q^2) = \frac{-1}{2m_B \sqrt{m_B m_{D_X}}} \{ (1 + r_X) \tilde{a}_+^X(y) - (1 - r_X) \tilde{a}_-^X(y) \}. \quad (40c)$$

Inserting Eqs. (39a)~(40c) into Eq. (27) we get the function $W_X(y, r_X)$ for each polarization (longitudinal and transverse) of spin-2 as in Table V.

We notice here that in the case of longitudinal component with spin-1 and -2 D_X states the factor q^2 in Eq. (27) is cancelled by $1/\sqrt{q^2}$ in \bar{H}_0 but in the case of transverse component with D_X the factor q^2 remains so that only transverse components have the factor $(1 - 2r_{XY} + r^2)$ as seen in Table V. Factor $1/\sqrt{q^2}$ of \bar{H}_0 is due to the Lorentz boost of polarization vector or tensor from the rest frame of final meson to the lv_l frame.

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