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HEAVY MESON FORM FACTORS FROM QCD

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We calculate the leading QCD radiative corrections to the relations which follow from the decoupling of the heavy quark spin as the quark mass goes to infinity and from the symmetry between systems with different heavy quarks. One of the effects we calculate gives the leading q^2-dependence of the form factor of a heavy quark, which in turn dominates the q^2-dependence of the form factors of bound states of the heavy quark with light quarks. This, combined with the normalization of the form factor provided by symmetry, gives us a first principles calculation of the heavy meson (or baryon) form factors in the limit of very large heavy quark mass.

1. Introduction

In this paper, we calculate the leading QCD radiative corrections to the relations discussed by Voloshin and Shifman [1] and Isgur and Wise [2], which follow from the decoupling of the heavy quark spin as the quark mass goes to infinity [3–8] and from the symmetry between systems with different heavy quarks (after the kinematical dependence on the heavy quark mass has been removed) [2]. One of the effects we calculate gives the leading q^2-dependence of the form factor of a heavy quark, which in turn dominates the q^2-dependence of the form factors of bound states of the heavy quark with light quarks. This, combined with the normalization of the form factor provided by ref. [2], gives us a first principles calculation of the heavy meson (or baryon) form factors in the limit of very large heavy quark mass. We will discuss this in our concluding sect. 6.

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For definiteness, let us discuss the exclusive semi-leptonic B-decay,

$$B^- \to D^0 + e^- + \bar{\nu}, \qquad B^- \to D^{*0} + e^- + \bar{\nu}. \tag{1}$$

Suppose that the B-meson in the initial state is moving with four-velocity v'^{μ} carrying momentum (approximately) $m_b v'^{\mu}$, and that the D-meson in the final state is moving with four-velocity v^{μ} carrying momentum $m_c v^{\mu}$. The amplitudes for these processes are proportional to the matrix elements of the vector and axial vector currents between the meson states,

$$\langle D(v)|V_\mu|B(v')\rangle\,, \quad \langle D^*(v),\varepsilon|A_\mu|B(v')\rangle\,, \quad \langle D^*(v),\varepsilon|V_\mu|B(v')\rangle\,, \tag{2}$$

where ε is the polarization vector, $\varepsilon v = 0$.

Isgur and Wise showed that in lowest order in QCD and leading order in the inverse heavy quark masses, all of these matrix elements are determined in terms of a single function $\xi(v'v)$, which we call the Isgur–Wise function*. We will review this in sects. 2 and 3. Meanwhile, note that unlike the second paper of ref. [2], we have defined ξ to be a function of a dimensionless variable.

In the limit in which the b- and c-quark masses are very heavy, and after we have removed the kinematic dependence on the heavy quark masses, the hadronic part of the processes (1) involves no large dimensional parameter. There is a large momentum transferred to the leptons, and a change in the heavy quark velocities, but as far as the light quarks and antiquarks and the gluons are concerned, not much is happening. Of course, the spectator antiquark and its cloud of glue has to change direction along with the heavy quarks, but this does not introduce a large dimensional parameter. Instead, the scale is set by the QCD scale parameter Λ. The momentum carried by the light antiquark and the gluons is of order $\Lambda v'^{\mu}$ before the decay and of order Λv^{μ} afterwards, so the momentum transfer is of order Λ. It does not scale with the quark masses. Thus we cannot expect to be able to calculate the Isgur–Wise function (except where we can use symmetry arguments, as in the first paper in ref. [2]). However, by the same token, we expect that the only dependence of the Isgur–Wise function on the masses (or indeed the identities) of the heavy quarks comes from the renormalization group scaling of the appropriate operators from the heavy quark mass scales down to low energies. Thus up to calculable logarithmic mass dependence, the Isgur–Wise function $\xi(v'v)$ will be a universal function for the ground-state heavy mesons. In this sense, it is very much like a parton distribution function. We discuss the logarithmic mass dependence of ξ in sect. 4.

We also note that QCD effects in the full theory will produce calculable corrections to the relations of ref. [2]. We will calculate these in sect. 5. Finally, we discuss the phenomenological implications of our results in sect. 6.

*One of the authors, M.B.W., objected to this name, but the rest of us ganged up and overpowered him.

Throughout the paper, we concentrate on effects that can actually be calculated in QCD perturbation theory. Of course, there are important nonperturbative effects in the physical $B \to D$ processes. In the formalism of refs. [2,7,9], these correspond to the contributions of higher-dimension operators. Their effects were estimated in ref. [2], but they cannot be calculated using only perturbative tools. We also ignore effects that are suppressed by powers of m_c/m_b, some of which can reliably be calculated using perturbation theory. It is not at all obvious that these effects are smaller than some of the effects that we calculate in this paper. These issues will be discussed in ref. [10].

2. Effective field theory

We will do the calculations in the notation of ref. [9]. There it is shown that a Lorentz-invariant effective field theory description of systems containing heavy quarks can be obtained by integrating in heavy quark degrees of freedom to implement a velocity superselection rule for the heavy quarks. We will assume for simplicity that $m_b \gg m_c \gg \Lambda$. Then in this effective field theory language, the logarithmic mass dependence of ξ comes from the renormalization group running of the currents both in the intermediate energy region between m_b and m_c, in which the b-quark is regarded as heavy, and in the low-energy region below m_c, in which both quarks are heavy. The calculable corrections to the relations come from one-loop matching conditions at the boundaries between different effective theories.

In the low-energy theory, we redefine the heavy quark fields, $h = c$ or b, so that the mass dependence is entirely in the commutator of the momentum operator with the field,

$$[P^\mu, h_v(x)] = (-m_h \not{v} v^\mu - i\partial^\mu) h_v(x), \tag{3}$$

where v is the heavy quark four-velocity. The heavy quark lagrangian is built with a separate field for each velocity, to implement the velocity superselection rule,

$$\mathscr{L}_{\text{heavy}} = \sum_{h=c,b} \int \frac{d^3 v}{2v^0} \mathscr{L}_v , \tag{4}$$

where

$$\mathscr{L}_v = i\overline{h}_v \not{v} v_\mu \partial^\mu h_v . \tag{5}$$

If we are interested, as we are in this paper, only in quark (or antiquark) states, we can project onto them by focusing on the two component fields

$$h_v^{\pm} = \tfrac{1}{2}(1 \pm \not{v}) h_v , \tag{6}$$

in terms of which the lagrangian becomes

$$\mathscr{L}_v = i\overline{h_v^+}\, v_\mu \partial^\mu h_v^+ - i\overline{h_v^-}\, v_\mu \partial^\mu h_v^- \,. \tag{7}$$

This leads to the propagators and vertices used in sects. 4 and 5.

3. Spin symmetry

In the language of refs. [2, 9], the relations between the matrix elements (2) come about because of the separate spin symmetries of the b-quark and the c-quark. The states and the operators can all be classified into irreducible representations of the spin symmetry, and then the Wigner–Eckart theorem ensures that a single function determines them all. We make the natural assumption that it is the physical D^* which transforms as part of the same irreducible representation of spin symmetry as the D.

There is a simple way to see the consequences of this symmetry. Even though the states and fields are doublets under the spin symmetries, it is convenient to embed each of the spin symmetries in a four-dimensional space of the Dirac indices. Then we can think of the D-states and the B-states as 4×4 matrices,

$$|B(v')\rangle \to \tilde{B}(v')\,, \quad \langle D(v)| \to \tilde{D}(v)\,, \quad \langle D^*(v), \varepsilon| \to \tilde{D}^*(v, \varepsilon)\,. \tag{8}$$

One index in each matrix represents the spin degree of freedom of the heavy quark in the state, while the other represents all the rest of the Lorentz structure, so that under a Lorentz transformation, the matrices transform like (for example)

$$\tilde{D}(v) \to \mathscr{D}(\Lambda)^{-1} \tilde{D}(\Lambda^{-1} v) \mathscr{D}(\Lambda)\,, \tag{9}$$

where $\mathscr{D}(\Lambda) = \exp(i\varepsilon_{\mu\nu}\sigma^{\mu\nu})$ is the usual Dirac representation.

Now we can make use of the explicit Lorentz invariance of the formalism of ref. [9] to take

$$\begin{aligned} \tilde{B}(v') &= (1 + \not{v}')/2\,, \\ \tilde{D}(v) &= (1 + \not{v})/2\,, \\ \tilde{D}^*(v, \varepsilon) &= \gamma_5 \not{\varepsilon}^* (1 + \not{v})/2\,. \end{aligned} \tag{10}$$

This takes proper account of the fact that the states are built out of heavy quarks rather than antiquarks. Then if we have a quark–antiquark operator G,

$$G = \bar{c}\Gamma b\,, \tag{11}$$

where Γ is any fixed matrix in the space of the Dirac indices, we can calculate the matrix elements of G for any Γ in terms of a single function, as follows:

$$\langle D(v)|G|B(v')\rangle = \sqrt{m_c m_b}\,\xi(v'v)\mathrm{tr}\{\tilde{D}(v)\Gamma\tilde{B}(v')\},$$

$$\langle D^*(v),\varepsilon|G|B(v')\rangle = \sqrt{m_c m_b}\,\xi(v'v)\mathrm{tr}\{\tilde{D}^*(v,\varepsilon)\Gamma\tilde{B}(v')\}. \tag{12}$$

The factor of $\sqrt{m_c m_b}$ is another kinematic effect of the heavy quark mass. Thus in leading order

$$\langle D(v)|V_\mu|B(v')\rangle = \sqrt{m_c m_b}\,\xi(v'v)(v_\mu + v'_\mu),$$

$$\langle D^*(v),\varepsilon|A_\mu|B(v')\rangle = \sqrt{m_c m_b}\,\xi(v'v)\left[\varepsilon^*_\mu(1+v'v) - v_\mu v'\varepsilon^*\right],$$

$$\langle D^*(v),\varepsilon|V_\mu|B(v')\rangle = -\sqrt{m_c m_b}\,\xi(v'v)i\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}v'^\alpha v^\beta. \tag{13}$$

Up to trivial normalization, this agrees with Isgur and Wise [2].

4. Running

The renormalization group running of the current below m_c is determined by the diagram given in fig. 1, where v'^μ is the four-velocity of the b-quark and v^μ is the four-velocity of the c-quark. The double lines represent the heavy quark propagators. In general, the form for the propagator of a heavy quark or antiquark with velocity v carrying residual momentum p is

$$\frac{i\not{v}}{vp}. \tag{14}$$

However, we will always be interested in only heavy quark states, for which $\not{v} = 1$,

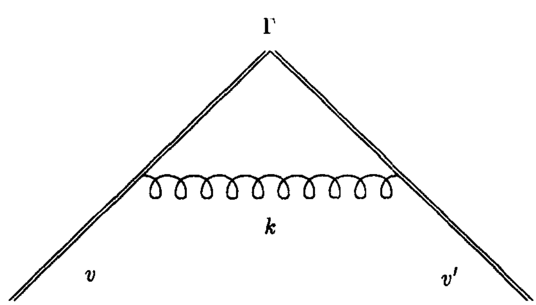

Fig. 1. Feynman diagram contributing to the one-loop matrix element of $\bar{b}\Gamma c$ in the low-energy effective theory.

so we can take the propagator to be simply

$$\frac{i}{vp}\,. \tag{15}$$

The gluon vertex for the heavy quark or antiquark is

$$igT_a \not{v} v^{\mu}\,, \tag{16}$$

where again, $\not{v}$ can be set equal to 1 in this case. Eqs. (15) and (16) are the momentum-space realization to the statement that in the effective field theory, the heavy quark propagates as a Wilson line [3, 5].

Thus the contribution of the diagram in fig. 1 is

$$D_1 = -i\int \frac{\mathrm{d}^4 k}{(2\pi)^4}\,\frac{4g^2}{3}\,\frac{v'v\Gamma}{vkk^2v'k}\,. \tag{17}$$

The renormalization group running of the current in the intermediate-energy region is determined by the diagram given in fig. 2. The contribution of this diagram is

$$D_2 = -i\int \frac{\mathrm{d}^4 k}{(2\pi)^4}\,\frac{4g^2}{3}\,\frac{\not{v}'(\not{k} + m_c\not{v} + m_c)\Gamma}{(k^2 + 2m_c vk)k^2v'k}\,. \tag{18}$$

The calculation of the running in the intermediate region has been done in refs. [1, 5, 7]. We will not repeat it here. The result is a scaling of the currents by a factor

$$\left(\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right)^{a_1}, \tag{19}$$

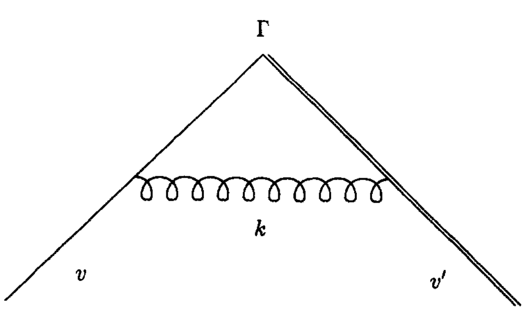

Fig. 2. Feynman diagram contributing to the one-loop matrix element of $\bar{b}\Gamma c$ in the intermediate-energy effective theory.

where

$$a_1 = -\tfrac{6}{25} = -6/(33-2f) \tag{20}$$

with $f=4$ the number of light flavors in the intermediate region. The calculation of the running in the low-energy region is more interesting, because the anomalous dimension of the operator depends on $v'v$. Evaluating the ultraviolet log divergent part of D_1, along with the appropriate self-energy diagrams, we find a velocity-dependent factor in scaling the currents from m_c down to a scale μ,

$$\left(\frac{\alpha_s(m_c)}{\alpha_s(\mu)}\right)^{a_L}, \tag{21}$$

where

$$a_L = \frac{8[v'vr(v'v)-1]}{27} = \frac{8[v'vr(v'v)-1]}{33-2f} \tag{22}$$

with

$$r(v'v) = \frac{1}{\sqrt{(v'v)^2-1}} \ln\left(v'v + \sqrt{(v'v)^2-1}\right) \tag{23}$$

and $f=3$. Note that the effect of the self-energy diagrams was to cancel the scaling of the current when $v'^{\mu} = v^{\mu}$, because $r(1)=1$. This had to happen because, as shown in ref. [2], the normalization of the current for $v'^{\mu} = v^{\mu}$ is fixed by a symmetry of the effective theory.

The "running" in (21) was overlooked in ref. [2]. However, this error does not effect their relations for the form factors in the semi-leptonic decays (1).

5. Matching conditions

The first-order correction to the matching of an operator G in the high-energy theory above the b-quark mass scale onto an operator in the intermediate-energy theory in which the b-quark is treated as heavy is determined by the Feynman graph in the high-energy theory given in fig. 3, where v'^{μ} is the four-velocity of the

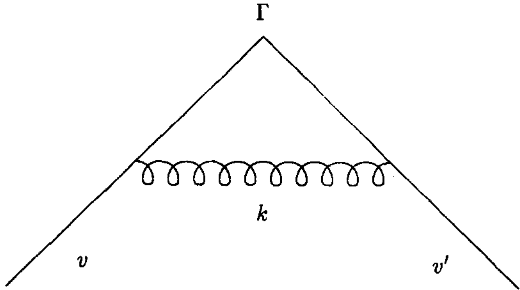

Fig. 3. Feynman diagram contributing to the one-loop matrix element of $\bar{b}\Gamma c$ in the high-energy theory.

b-quark and v^μ is the four-velocity of the c-quark. Because the c-quark is light at the b-scale, where this matching is done, the leading-order result is independent of the velocity of the c-quark. This is the matching calculation done by Voloshin and Shifman in ref. [1]. The result is that Γ is replaced by

$$\Gamma \to \left[\Gamma - \frac{g(m_b)^2}{24\pi^2} \gamma^\mu \not{v}' \Gamma \not{v}' \gamma_\mu \right]. \tag{24}$$

Note that we have ignored terms proportional to Γ which simply renormalize the lowest-order result. Any such renormalization of the lowest-order result is uninteresting at this level, because it depends on the precise renormalization prescription. To calculate this radiative correction in a meaningful way, we would have to do the calculation of the anomalous dimensions to two loops. However, as we have seen in sect. 4, the anomalous dimensions are completely spin independent because of the spin symmetries, and thus any matching contribution with a spin dependence difference from that of the lowest-order piece is well defined. Note also there may be another term in the matching (24) proportional to $\gamma^\mu \gamma^\nu \Gamma \gamma_\nu \gamma_\mu$, associated with the different behavior of scalar and vector operators in the high-energy theory. This term depends on renormalization prescription, but it contributes only a common renormalization of the vector and axial vector currents, so we ignore it here.

The first-order correction to the matching of an operator G in the theory with a heavy b-quark and a light c-quark onto an operator in the low-energy theory in which both the b- and the c-quarks are treated as heavy is determined by the Feynman graph in the high-energy theory, fig. 2. This should be compared to the corresponding diagram in the low-energy theory, with both quarks heavy, fig. 1. The matching contribution is the difference, $D_2 - D_1$. The piece of (18) proportional to $(m_c \not{v} + m_c)$ can be rewritten as

$$D_{\text{ir}} = -i \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{4g^2}{3} \frac{v' v 2 m_c \Gamma}{(k^2 + 2 m_c v k) k^2 v' k}. \tag{25}$$

This term cancels the infrared divergence of (17). Clearly, both this term and D_1 are proportional to Γ, thus they simply renormalize the lowest-order contribution.

The spin-dependent matching contribution is entirely in the difference

$$D_2 - D_1 = -i \int \frac{\mathrm{d}^4 k}{(2\pi)4} \frac{4g^2}{3} \frac{\not{v}' \not{k} \Gamma}{(k^2 + 2 m_c v k) k^2 v' k} + \dots, \tag{26}$$

where $\dots = D_{\text{ir}} - D_1$ is a renormalization of Γ. Combining denominators in eq.

(26) gives

$$-\frac{16ig^2}{3}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\int_0^1\mathrm{d}x\int_0^\infty\mathrm{d}\lambda$$

$$\times\frac{\not{v}'\not{k}\Gamma}{\left[(1-x)k^2+x\left(k^2+2m_c vk\right)+2\lambda v'k\right]^3}+\dots. \tag{27}$$

We shift to the integration variable

$$l^\mu=k^\mu+m_c x v^\mu+\lambda v'^\mu, \tag{28}$$

to get

$$D_2-D_1=\frac{16ig^2}{3}\int\frac{\mathrm{d}^4l}{(2\pi)^4}\int_0^1\mathrm{d}x\int_0^\infty\mathrm{d}\lambda\,\frac{m_c x\not{v}'\not{v}\Gamma}{\left[l^2-(m_c xv+\lambda v')^2\right]^3}+\dots. \tag{29}$$

Doing the l-integral gives

$$D_2-D_1=\frac{g^2}{6\pi^2}\int_0^1\mathrm{d}x\int_0^\infty\mathrm{d}\lambda\,\frac{m_c x\not{v}'\not{v}\Gamma}{\left[(m_c xv+\lambda v')^2\right]}+\dots \tag{30}$$

$$=-\frac{g^2}{6\pi^2}\int_0^1\mathrm{d}x\int_0^\infty\mathrm{d}\lambda\,\frac{m_c x\not{v}'\Gamma}{\left[(m_c xv+\lambda v')^2\right]}+\dots, \tag{31}$$

where we have used the fact that $\not{v}=1$ when acting on the c-state. Thus

$$D_2-D_1=-\frac{g(m_c)^2}{6\pi^2}r(v'v)\not{v}'\Gamma+\dots, \tag{32}$$

where $r(v'v)$ is given by eq. (23). Thus the matching at the m_c-scale is accomplished by the replacement

$$\Gamma\to\left[1-\frac{g(m_c)^2}{6\pi^2}r(v'v)\not{v}'\right]\Gamma. \tag{33}$$

6. Phenomenology

Let us now return to the matrix elements (12) and (13), and consider the effects of the radiative corrections that we have calculated in the previous sections. The matching corrections add or α_s terms to the Γ in eq. (12). We will discuss these below. The more interesting result comes from the velocity-dependent running. This is a leading logarithmic correction, which becomes more important than the matching corrections for a very heavy quark. For B → D decays, the running starts at m_b. Combining (19) and (21), we have

$$\xi(v'v) = \Xi(v'v, m_b, m_c, \mu)\xi_0(v'v, \mu), \tag{34}$$

where

$$\Xi(v'v, m_b, m_c, \mu) = \left(\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right)^{a_I}\left(\frac{\alpha_s(m_c)}{\alpha_s(\mu)}\right)^{a_L}. \tag{35}$$

The function $\xi_0(v'v, \mu)$ is the universal, nonperturbative contribution to the Isgur–Wise function. We cannot calculate it*. However, we expect that if we choose $\mu \approx \Lambda$, there will be no large parameters in the function $\xi_0(v'v, \mu)$. On the other hand, if $\log(\alpha_s(\mu)/\alpha_s(m_c))$ is large, the $v'v$-dependence of (35) is rapid. Furthermore, we know that $\xi_0(1, \mu) = 1$, from ref. [2]. Thus in the region of the Dalitz plot in which $v'v$ is close to 1, we can reliably estimate the matrix elements by ignoring ξ_0 and including only Ξ.

For the particular case of b → c decay, the approximations discussed above are of very questionable validity. $v'v$ is related to the four-momentum transfer squared, q^2, of the leptonic system by

$$v'v = \frac{m_b^2 + m_c^2 - q^2}{2m_b m_c}. \tag{36}$$

Thus the maximum $v'v$ occurs at $q^2 = 0$,

$$v'v|_{\max} - 1 = \frac{(m_b - m_c)^2}{2m_b m_c} \approx 1 \tag{37}$$

and

$$\log\left(\frac{1}{\alpha_s(m_c)}\right) \approx 1, \tag{38}$$

while what we need is for (38) to be much greater than 1.

In fig. 4, we plot Ξ for B → D decay versus q^2 of the leptons in GeV, with two different choices for $\alpha_s(\mu)$. The upper dotted line corresponds to $\alpha_s(\mu) = 1$ and the lower to $\alpha_s(\mu) = 3$.

As discussed in ref. [2], we can also predict matrix elements of $\bar{b}\Gamma b$ currents between B-states and $\bar{c}\Gamma c$ currents between D-states. In ref. [2], the velocity-

*However, it is calculable, in principle, using nonperturbative techniques such as lattice gauge theory.

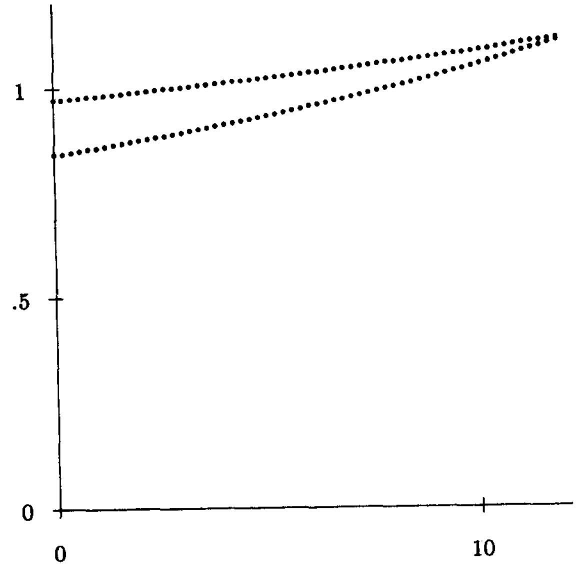

Fig. 4. The function Ξ versus q^2 in GeV^2 for B → D decay. The upper and lower dotted lines correspond to $\alpha_s(\mu) = 1$ and 3, respectively.

dependent running is ignored, but it is easy to include, just by incorporating the appropriate dimensional factors and the right mass dependence in Ξ. The result is

$$\begin{aligned}\langle B(v)|G|B(v')\rangle &= m_b \Xi(v'v, m_b, m_b, \mu)\xi_0(v'v, \mu)\mathrm{tr}\{\tilde{B}(v)\Gamma\tilde{B}(v')\},\\ \langle B^*(v), \varepsilon|G|B(v')\rangle &= m_b \Xi(v'v, m_b, m_b, \mu)\xi_0(v'v, \mu)\mathrm{tr}\{\tilde{B}^*(v,\varepsilon)\Gamma\tilde{B}(v')\},\end{aligned} \tag{39}$$

and

$$\begin{aligned}\langle D(v)|G|D(v')\rangle &= m_c \Xi(v'v, m_c, m_c, \mu)\xi_0(v'v, \mu)\mathrm{tr}\{\tilde{D}(v)\Gamma\tilde{D}(v')\},\\ \langle D^*(v), \varepsilon|G|D(v')\rangle &= m_c \Xi(v'v, m_c, m_c, \mu)\xi_0(v'v, \mu)\mathrm{tr}\{\tilde{D}^*(v,\varepsilon)\Gamma\tilde{D}(v')\}.\end{aligned} \tag{40}$$

Note that, as promised, the nonperturbative ξ_0 is universal. Note further that our formulas look slightly different from those in ref. [2], both because we have included the velocity-dependent running, and because, as mentioned in sect. 1, our Isgur–Wise function is defined differently, as a function of the dimensionless variable $v'v$.

We now return to the matching conditions for the semileptonic B-decays*. Combining (24) and (33), we can summarize the results by calculating the matrix elements using eq. (12), but replacing

$$\gamma^\mu \to (1+\kappa)\gamma^\mu + (\lambda_b - \lambda_c(v'v))\not{v}'\gamma^\mu \tag{41}$$

for the vector current and

$$\gamma^\mu\gamma_5 \to (1+\kappa)\gamma^\mu\gamma_5 - (\lambda_b + \lambda_c(v'v))\not{v}'\gamma^\mu\gamma_5 \tag{42}$$

for the axial vector current, where

$$\lambda_b = \frac{\alpha_s(m_b)}{3\pi}, \qquad \lambda_c(v'v) = \frac{2\alpha_s(m_c)}{3\pi} r(v'v), \tag{43}$$

and where κ is an unknown constant of order α_s, which summarizes the unknown order α_s renormalization of the currents. Relations that are valid to order α_s are those which do not involve κ.

In general, the order α_s corrections are not very illuminating. None of them is anomalously large, and we do not expect them to dominate over Λ/m_c corrections which we have neglected. However, it is worth nothing that there is a perturbative correction to the amplitude for the axial current, in the rest frame of the D*-meson, to product a B-meson in a D-wave. Using the notation of ref. [2], this amplitude is proportional to $a_+ + a_-$, and we find

$$\frac{a_+ + a_-}{a_+} = -4\frac{m_c}{m_b}\left\{\frac{\alpha_s(m_b)}{3\pi} + \frac{2}{3}\frac{\alpha_s(m_c)}{\pi} r(v'v)\right\}. \tag{44}$$

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*Note that the matching contributions for eqs. (39) and (40) will be different.

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