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Lattice measurement of the Isgur-Wise functions $\tau_{1/2}$ and $\tau_{3/2}$

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Abstract

We propose a method to compute the Isgur-Wise form factors $\tau_{1/2}(1)$ and $\tau_{3/2}(1)$ for the decay of B mesons into orbitally excited (P wave) D^{**} charmed mesons on the lattice in the static limit. We also present the result of an exploratory numerical simulation which shows that the signal/noise ratio allows for a more dedicated computation. We find $\tau_{1/2}(1) = 0.38(5)$ and $\tau_{3/2}(1) = 0.53(8)$, with yet unknown systematic errors. These preliminary numbers agree fairly well with theoretical expectation.

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Introduction

The scalar heavy-light mesons and more generally the first orbital excitations D^{**} have attracted attention since years and they still remain somehow mysterious. The recent discovery of a $c\bar{s}$-scalar meson significantly lighter than expected has renewed the interest in these states [1, 2]. There have been several lattice studies of this spectrum [3, 4] and a recent rather complete one compares quenched and unquenched [5] computations. Recently the $H_0^* \to H\pi$ transition (scalar-pseudoscalar-pion) have also been considered [6, 7].

The transitions of the type $B \to D^{**}l\nu$ raise a serious problem. In the infinite mass limit these decays are described by the Isgur-Wise form factors $\tau_{1/2}$ and $\tau_{3/2}$ [8]. To make a long story short, a series of sum rules [9]-[14] have been derived from QCD, all indicating that $\tau_{3/2}$ should be significantly larger than $\tau_{1/2}$. These sum rules relate the τ_j form factors, as well as form factors related to excitations, to derivatives of the ground state Isgur-Wise function ξ and allow to bound the latter derivatives in an efficient and useful way [15]-[17]. Not only does the slope of ξ verify $\rho^2 > 3/4$ but also the curvature and even higher derivatives are bound. The limit in which $\tau_{1/2} = 0$ has been baptised "BPS" by Uraltsev [18]-[19] and was proven to provide intersting hints.

However the theoretical prediction that $\tau_{3/2}^{(0)} > \tau_{1/2}^{(0)}$ and hence that the decay $B \to D_2^*$ should be significantly larger than the $B \to D_0^*$ is not verified by experiment [2, 20]. This is the '$1/2 > 3/2$' paradox [21]. One might incriminate the corrections to the infinite mass limit. Another possibility could be that the sum rules are fulfilled by higher excitations and that the ground state obeys an opposite hierarchy i.e. $\tau_{3/2}^{(0)} < \tau_{1/2}^{(0)}$ [1].

To answer to this question one needs to compute directly $\tau_{3/2}^{(0)}$ and $\tau_{1/2}^{(0)}$. Here we propose a lattice method to do that. We will work in the static quark limit, $m_{b,c} \to \infty$, with the four vectors $v' = v = (1, 0, 0, 0)$, and we will exhibit we will exhibit operators whose matrix elements allow to measure these form factors.

This letter is meant to propose this new method and to make a feasibility study. We do not intend at this stage to provide accurate results for these form factors but merely to describe the principle of the method and to show with preliminary simulations that there is good hope to make the precision calculation.

1 Principle of the calculation

We are concerned with the matrix element of an electroweak current between a pseudoscalar or vector heavy-light meson $H^{(*)}$ and an orbitally excited one H^{**}. However, in the conditions of the infinite mass limit on the lattice with the heavy quarks at rest, both in the initial and final state ($v_\mu = v'_\mu$), this matrix element vanishes.

The way out is to use a series of relations derived in ref. [22]. In that paper it has been shown that in the case of a matrix element which vanishes linearly in the difference $v' - v$, when $v' \to v$, there are non-vanishing forward matrix elements (for $v' = v$) involving the

[1] There is no mathematical impossibility for the sum rules to be fulfilled with an reversed hierarchy for the ground state, but it does not seem very likely and is not seen in models.

covariant derivative operator D_μ. These matrix elements are proportional to the infinite mass limit form factors $\tau_{\frac{1}{2}}(1)$ or $\tau_{\frac{3}{2}}(1)$.

Let us summarise their proof using different notations: For simplicity we take $v' = (1,0,0,0)$, and $v = v' + v_\perp$, where $v_\perp$ is spatial up to higher orders in the difference $v' - v$. We assume that for some Dirac matrix Γ_l

$$\langle H^{**}(v')|\bar{h}(v')\Gamma_l h(v)|H^{(*)}(v)\rangle = t_l^m v_{\perp\, m}\tau_j(w) + \cdots\,, \tag{1}$$

where $w \equiv v \cdot v'$, $j = 1/2,\ 3/2$, and $l, m = 1, 3$ are spatial indices, t^{lm} is a tensor which depends on the final state (H^{**}) and the initial state (H^* or H). The dots represent higher orders in $v' - v$. From translational invariance in the time direction,

$$\begin{aligned} -i\partial_0\langle H^{**}(v')|\bar{h}(v')\Gamma_l h(v)|H^{(*)}(v)\rangle &= \\ -i\langle H^{**}(v')|\bar{h}(v')\left[\Gamma_l\, \overrightarrow{D}^0 + \overleftarrow{D}^0\, \Gamma_l\right] h(v)|H^{(*)}(v)\rangle &= t_l^m v_{\perp\, m}\tau_j(w)\left(M_{H^{**}} - M_H\right) + \cdots\,. \end{aligned} \tag{2}$$

The authors of ref. [22] use the field equation: $(v \cdot D)h(v) = 0$, which implies that

$$D^0 h(v') = 0, \qquad D^0 h(v) = -(D \cdot v_\perp)h(v), \tag{3}$$

whence from (2)

$$i\langle H^{**}(v')|\bar{h}(v')\Gamma_l(D \cdot v_\perp)h(v)|H^{(*)}(v)\rangle = t_l^m v_{\perp\, m}\tau_j(w)\left(M_{H^{**}} - M_H\right) + \cdots\,, \tag{4}$$

which has a finite limit when $v_\perp \to 0$ namely

$$i\langle H^{**}(v)|\bar{h}(v)\Gamma_l D^m h(v)|H^{(*)}(v)\rangle = t_l^m \tau_j(1)\left(M_{H^{**}} - M_H\right). \tag{5}$$

Applying eq. (1) to the $J = 0$ H_0^* state we get from ref. [8]:

$$\langle H_0^*(v')|A_i|H(v)\rangle \equiv -\tau_{\frac{1}{2}}(w)v_{\perp\, i}, \tag{6}$$

where A_i is the axial current in the spatial direction i and where our states normalisation is $1/\sqrt{2M}$ times the one used in ref. [8]. From eq. (6) it results that

$$\langle H_0^*(v)|A_i D_j|H(v)\rangle = i\, g_{ij}\left(M_{H_0^*} - M_H\right)\tau_{\frac{1}{2}}(1). \tag{7}$$

Analogously for the $J = 2$ H_2^* state we have

$$\langle H_2^*(v')|A_i|H(v)\rangle \equiv \sqrt{3}\,\tau_{\frac{3}{2}}(w)\,\epsilon_i^{*\,j} v_{\perp\, j} + \cdots\,, \tag{8}$$

where ϵ_{ij}^* is the polarisation tensor, whence

$$\langle H_2^*(v)|A_i D_j|H(v)\rangle = -i\sqrt{3}\left(M_{H_2^*} - M_H\right)\tau_{\frac{3}{2}}(1)\epsilon_{ij}^*\,. \tag{9}$$

2 Lattice calculations

To compute the matrix elements in eqs.(7) and (9) on the lattice we first need a discretized expression for the covariant derivative. We choose the symmetrised form

$$D_i(\vec{x},t) \to \frac{1}{2a}\left(U_i(\vec{x},t) - U_i^\dagger(x-\hat{\imath},t)\right), \tag{10}$$

where $U_i(\vec{x},t)$ is the link variable of the lattice.

2.1 Interpolating fields

The interpolating fields for orbitally excited states have been studied in ref. [24]. Smearing is used not only to improve the signal/noise ratio by better isolating the ground state, but also to produce convenient interpolating fields for the 0^-, 0^+ and 2^+ states. Inspired by ref. [25] we replace the quark fields $q(x)$ by

$$\begin{aligned} q(x) \quad \to \quad & \sum_{r=0}^{R_{max}} (r+\tfrac{1}{2})^2 \phi(r) \sum_{i=x,y,z} \left\{ \left[\prod_{k=1}^{r} U_i^F(x+(k-1)\hat{\imath})\right] q(x+r\hat{\imath}) \begin{matrix} (r\hat{\imath})\ \delta_{il} \\ 1 \end{matrix} \right. \\ + \quad & \left. \left[\prod_{k=1}^{r} U_i^{F\dagger}(x-k\hat{\imath})\right] q(x-r\hat{\imath}) \begin{matrix} (-r\hat{\imath})\ \delta_{il} \\ 1 \end{matrix} \right\}, \end{aligned} \tag{11}$$

where the upper (lower) expressions generate negative (positive) parity smearing functions. The vector $(\pm r\hat{\imath})\ \delta_{il}$ is introduced to generate an orbital excitation in the direction l. The wave function $\phi(r)$ is a radial function chosen to optimise the overlap with the ground state. We take $\phi(r) = e^{-r/R_b}$, where R_b is a parameter which is fixed by requiring the smearing to be optimal. Note that it is not necessary to normalize the wave function since the normalisation factors cancel in the computation of matrix elements. The smearing also includes the so-called fuzzing, see ref. [23]. For convenience, we will use the following notation for the interpolating fields:

$$\{\bar{h}(x) \begin{matrix} \gamma_k\, r^l(x) \\ \Gamma \end{matrix} q(x+\vec{r})\}, \tag{12}$$

where r^l indicates the presence of $(\pm r\hat{\imath})\ \delta_{il}$ in eq. (11), $\vec{r}$ is a generic vector for the distance between the light and heavy quark field and $\Gamma = 1$ ($\Gamma = \gamma_5$) for the 0^+ (0^-) meson.

Using the smeared quark fields from eq. (11) we now define the interpolating fields. We concentrate on the 0^+ (2^+) states which correspond to $j = 1/2$ ($j = 3/2$). The 0^+-state can be described according to two distinct interpolating fields:

$$a) \quad \bar{h}(x)\, q(x+\vec{r}) \quad \text{and} \quad b) \quad \frac{1}{\sqrt{3}}\, \bar{h}(x)\ (\vec{\gamma}\cdot\vec{r})\ q(x+\vec{r}). \tag{13}$$

These two interpolating fields differ in that the Dirac matrix is diagonal (antidiagonal) for 1 $(\vec{\gamma}\cdot\vec{r})$ inducing the coupling of the heavy quark to the "small" ("large") component of the light quark field. The latter is just the quark model combination of quark-spin 1 with orbital momentum 1 to generate $J = 0^+$.

Concerning the 2^+ states, the same duality of interpolating fields exists. In this letter we only consider the quark-model type interpolating fields. This gives the five $J = 2$ states [24] which we may write as follows:

$$\begin{aligned}
&a) \qquad -\frac{1}{\sqrt{2}}\bar{h}(x)\ [\gamma_i \cdot r_j(x) + \gamma_j \cdot r_i(x)]\ q(x+\vec{r}), \quad i \neq j, \\
&b) \qquad -\frac{1}{\sqrt{2}}\bar{h}(x)\ [\gamma_1 \cdot r_1(x) - \gamma_2 \cdot r_2(x)]\ q(x+\vec{r}), \\
&c) \qquad \frac{1}{\sqrt{6}}\bar{h}(x)\ [\gamma_1 \cdot r_1(x) + \gamma_2 \cdot r_2(x) - 2\gamma_3 \cdot r_3(x)\,]\, q(x+\vec{r}).
\end{aligned} \tag{14}$$

These are, as expected, symmetric traceless tensors.

2.2 Two-point Green functions and $1/2 - 3/2$ mass splitting

The 0^+ two-point Green function is written as

$$C^1_{2;0} = \left\langle \sum_{\vec{x}} \mathrm{Tr}\left[P^{\vec{0}}_{0,t_x}\frac{1+\gamma_0}{2}S(\vec{r}(0),0;x+\vec{r}(x);U)\right]\right\rangle_U , \tag{15}$$

when we use the interpolating field in eq. (13,a) [2]. P^0_{0,t_x} is a temporal Wilson line [3] corresponding to the Eichten-Hill action for the static quark [26]:

$$P^{\vec{x}}_{t_x,t_y} = \delta(\vec{x}-\vec{y})\prod_{t_z=t_x}^{t_y-1} U^{\mathrm{hyp}}_t(x+t_z\,\hat{t}), \tag{16}$$

using hypercubic blocking [27] - [29].

The two-point Green functions with $\gamma_i r_j$ interpolating fields allow an interesting comparison between the $j = 1/2$ and the $j = 3/2$ cases. They will contain terms of the general form

$$C^{ijkl}_{2;J}(0,t_x) = \left\langle \sum_{\vec{x}} \mathrm{Tr}\left[\gamma_i r_j(0)\,P^{\vec{0}}_{0,t_x}\,\frac{1+\gamma_0}{2}\gamma_k r_l(x)S(\vec{r}(0),0;x+\vec{r}(x);U)\right]\right\rangle_U , \tag{17}$$

where $(i,j) = (k,l)$ for the case of interpolating field (14,a) or $i = j, k = l$ for the case (14,b), (14,c) and for the 0^+ case. J stands for the total angular momentum ($J = 0, 2$).

After some simple algebra we can write

$$-C^{ijkl}_{2;J}(0,t_x) = \left\langle \sum_{\vec{x}} \mathrm{Tr}\,P^{\vec{0}}_{0,t_x}\left[\left(\delta_{jl} \pm i\epsilon^{jlm}\sigma_m\right) r_j(0)\frac{1-\gamma_0}{2}r_l(x)S(\vec{r}(0),0;x+\vec{r}(x);U)\right]\right\rangle_U . \tag{18}$$

[2]The superindex 1 ($\vec{\gamma}\cdot\vec{r}$) refers to the use of the a (b) interpolating field in eq. (13).

[3]Indeed we compute the two point function using the interpolating fields in eqs. (13) and (14) properly shifted in space so as to have the light propagator ending at the origin.

Let us define $C_{2,\delta(r_j(0),r_j(x))}$ and $C_{2,\epsilon^m(r_j(0),r_l(x))}$ respectively the two terms in eq. (18). It is easy to see from the second interpolating field in eq. (13) that the 0^+ two point Green function writes as

$$
\begin{aligned}
-C_{2;0}^{\vec{\gamma}\cdot\vec{r}} &= \frac{1}{3}\sum_{j=1,3} C_{2,\delta(r_j(0),r_j(x))} + \frac{i}{3}\sum_{i,j,k}\left[C_{2,\epsilon^k(r_i(0),r_j(x))} + C_{2,\epsilon^k(r_j(0),r_i(x))}\right] \\
&= C_{2,\delta(r_1(0),r_1(x))} + i\left[C_{2,\epsilon^3(r_1(0),r_2(x))} - C_{2,\epsilon^3(r_2(0),r_1(x))}\right], \qquad (19)
\end{aligned}
$$

where i, j, k are in cyclic order and where we have taken advantage of the hypercubic symmetry in the r.h.s.

Taking now any of the 2^+ meson interpolating fields and using the again the cubic symmetry we get

$$
-C_{2;2} = C_{2,\delta(r_1(0),r_1(x))} - \frac{i}{2}\left[C_{2,\epsilon^3(r_1(0),r_2(x))} - C_{2,\epsilon^3(r_2(0),r_1(x))}\right]. \qquad (20)
$$

The difference between the j=1/2 and the j=3/2 state is thus related to the relative sign and coefficient of the ϵ-term compared to the direct one. The effective energy is obtained by taking minus the time derivative of the logarithm of the two-point function. The energy difference between $j = 1/2$ and $j = 3/2$ is thus proportional to

$$
-i\left\langle [r_1(0)\dot{r}_2(x) - r_2(0)\dot{r}_1(x)]\ \mathrm{Tr}\, P^{\vec{0}}_{0,t_x}\left[\sigma_3\frac{1-\gamma_0}{2}S(\vec{r}(0),0;x+\vec{r}(x);U)\right]\right\rangle_U . \qquad (21)
$$

In a non-relativistic limit $\dot{\vec{r}} = i\vec{p}/m$. This imaginary velocity comes from the derivation versus the imaginary time. Then the expression in eq. (21) is reminiscent of a LS-term: $(\vec{r}\times\vec{p})\cdot\vec{\sigma}$ except that the operators $\vec{r}$ and $\vec{p}$ are not taken at the same time. σ in eq. (21) acts on the heavy quark but the trace will make it also act on the light quark. It is interesting that the coefficients of the last terms in eqs. (19) and (20) are in the ratio $(1), (-1/2)$ which is exactly the ratio of the LS-eigenvalues for $j = 1/2, 3/2$, built up from the combination of $L = 1$ and $s = 1/2$:

$$
2 < \vec{L}\cdot\vec{S} >= j(j+1) - \frac{3}{4} - 2 = \begin{matrix} -2 \\ 1 \end{matrix} \quad \text{for} \quad j = \begin{matrix} 1/2 \\ 3/2 \end{matrix} \quad . \qquad (22)
$$

From eqs. (19) and (20) it is obvious that if these LS-type terms did vanish the two-point correlators $C_{2;0}$ and $C_{2;2}$ would be equal, which would then imply $M_{H_2^*} = M_{H_0^*}$. In this limit the normalisation of the interpolating fields in eqs. (13) and (14) has further ensured the equality of the multiplicative constants $\mathcal{Z}_{2;0}$ and $\mathcal{Z}_{2;2}$, where the $\mathcal{Z}_{2;J}$ are defined from

$$
C_{2;J}(t_x) = (\mathcal{Z}_{2;J})^2\, e^{-M_{H_J^*}\, t_x}, \qquad (23)
$$

at large time t_x.

2.3 Three-point Green functions and $\tau_{\frac{1}{2}}$-$\tau_{\frac{3}{2}}$ splitting

The three point Green functions of the axial current using interpolating fields with $\gamma_i r_j$ will contain terms of the general form

$$C^{ijkl}_{3,JA_k5}(0,t_y,t_x) = \frac{1}{2}\Big\langle \sum_{\vec{x},\vec{y}} \mathrm{Tr}\Big[\gamma_i r_j(0)\, P^{\vec{0}}_{0,t_y}\, \frac{1+\gamma_0}{2}\gamma_k\gamma_5 \tag{24}$$

$$\Big\{U_l(0,t_y) P^{\hat{l}}_{t_y,t_x} S(\vec{r}(0),0;x+\vec{r}(x)+\hat{l};U) - U^\dagger_l(-\hat{l},t_y) P^{-\hat{l}}_{t_y,t_x} S(\vec{r}(0),0;x+\vec{r}(x)-\hat{l};U)\Big\}\,\gamma_5\Big]\Big\rangle_U \,,$$

where we have used eq. (10) in units of a. Writing for short the term in the curly bracket as $D_l(y)\cdots\gamma_5$, and we see can write

$$-C^{ijkl}_{3,JA_k5}(0,t_y,t_x) = \frac{1}{2}\left\langle \sum_{\vec{x},\vec{y}} \mathrm{Tr}\left[\left(\delta_{jl} \pm i\epsilon^{jlm}\sigma_m\right)\gamma_5 r_j(0)\, P^{\vec{0}}_{0,t_y}\, \frac{1+\gamma_0}{2} D_l(y)\cdots\gamma_5\right]\right\rangle_U \,, \tag{25}$$

where either $(i,j) = (k,l)$ for the $0^- \to 2^+$ transition, eq. (14,a), or $i=j, k=l$ for the $0^- \to 0^+$ one and $0^- \to 2^+$ with eqs. (14,b) and (14,c). Let us define $C_{3,\delta(r_j(0),D_j(x))}$ and $C_{3,\epsilon^m(r_j(0),D_l(x))}$, respectively the two terms in eq. (25). From the interpolating field in eq. (13,b), using the fact that $i=j$ and $k=l$, and choosing for simplicity $k=3$, one can derive,

$$-C^{\vec{\gamma}\cdot\vec{r}}_{3;05} \;=\; \frac{1}{\sqrt{3}} C_{3,\delta(r_3(0),D_3(x))} + \frac{i}{\sqrt{3}}\left[C_{3,\epsilon^1(r_2(0),D_3(x))} + C_{3,\epsilon^2(r_1(0),D_3(x))}\right] . \tag{26}$$

The axial matrix element is then given in Euclidean metric by

$$< H^*_0|A_3 D_3|H >= \frac{\mathcal{Z}_{2;0}\ \mathcal{Z}_{2;5}\ C_{3;05}(0,t_y,t_x)}{C_{2;0}(0,t_y)\ C_{2;5}(0,t_x-t_y)} = \left(M_{H^*_0} - M_H\right)\tau_{\frac{1}{2}}(1)\,, \tag{27}$$

where we have used eq. (7). This leads, using cubic symmetry, to

$$\left(M_{H^*_0} - M_H\right)\ \sqrt{3}\ \tau_{\frac{1}{2}}(1) = \mathcal{Z}_{2;0}\ \mathcal{Z}_{2;5}\frac{C_{3,\delta(r_3(0),D_3(x))} + 2\,i\,C_{3,\epsilon^1(r_2(0),D_3(x))}}{C_{2;0}(0,t_y)\ C_{2;5}(0,t_x-t_y)}\,, \tag{28}$$

plus all terms deduced by cubic symmetry.

From eq. (9), in Euclidean metric, we get

$$< H^*_2|\frac{A_1D_2 + A_2D_1}{2}|H >= \frac{\mathcal{Z}_{2;2}\ \mathcal{Z}_{2;5}\ C_{3;25}(0,t_y,t_x)}{C_{2;2}(0,t_y)\ C_{2;5}(0,t_x-t_y)} = \sqrt{\frac{3}{2}}\ \left(M_{H^*_2} - M_H\right)\ \tau_{\frac{3}{2}}(1)\,, \tag{29}$$

where we have used the polarisation tensor

$$\epsilon = \begin{matrix} 0 & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 \end{matrix}\,, \tag{30}$$

for the 2^+ state in eq. (14,a) with $(i,j)=(1,2)$. From the interpolating field in eq. (14,a) using the fact that $(i,j)=(k,l)=(1,2)$ one can derive,

$$-C_{3;25} \;=\; -\frac{1}{2\sqrt{2}}\sum_{l=1,2} C_{3,\delta(r_l(0),D_l(x))} + \frac{i}{2\sqrt{2}}\left[C_{3,\epsilon^3(r_1(0),D_2(x))} + C_{3,\epsilon^3(r_2(0),D_1(x))}\right]\;, \qquad (31)$$

and using cubic symmetry and eq. (29)

$$\left(M_{H_2^*}-M_H\right)\;\sqrt{3}\,\tau_{\frac{3}{2}}(1) = \mathcal{Z}_{2;2}\;\mathcal{Z}_{2;5}\;\frac{C_{3,\delta(r_1(0),D_1(x))} - i\,C_{3,\epsilon^1(r_2(0),D_3(x))}}{C_{2;2}(0,t_y)\;C_{2;5}(0,t_x-t_y)}\;. \qquad (32)$$

It can be checked that all the other states in eq. (17) lead to the same formula (32) up to a cubic rotation. The numerators in equations (28) and (32) exhibit identical $C_{3,\delta}$ terms and $C_{3,\epsilon}$, the latter differing only by multiplicative coefficients which, once more, are proportional to the LS eigenvalues given in eq. (22). Combining the results of eqs. (19), (20), (28) and (32) we may conclude that, if the $C_{2,\epsilon}$ and $C_{3,\epsilon}$ terms did vanish, we would get $M_{H_2^*}=M_{H_0^*}$ and $\tau_{3/2}=\tau_{1/2}$.

3 Condition and results of the simulation

Results presented for Isgur-Wise functions $\tau_{\frac{1}{2}}(1)$ and $\tau_{\frac{3}{2}}(1)$ are obtained from the quenched simulation on a $16^3\times 40$ lattice at $\beta=6.0$. We collected 580 independent $SU(3)$ gauge configurations in the quenched approximation using the non perturbatively $\mathcal{O}(a)$ improved Wilson fermion action with $C_{SW}=1.769$. The light-quark propagator is computed with the hopping parameter $\kappa=0.1334$, which corresponds to a pseudoscalar "light-meson" mass of 800 MeV. For the static quark we use the "hyp" links as written previously. In fig. 1 we plot the binding energy for the scalar and the pseudoscalar meson and in fig. 2 we plot the binding energy for the 2^+ heavy-light meson.

The scalar meson has been computed using the interpolating field [4] in eq. (13,a): $\bar{h}(x)\,q(x+\vec{r})$. The tensor meson has been computed using the properly averaged interpolating fields in eq. (14). We get $\Delta\equiv m_{H_0^*}-m_H=400(12)-411(16)$ MeV at $\beta=6.0$. Only statistical errors are considered. It agrees reasonably with ref. [5] where $\Delta\sim 400(40)$ MeV. Our present signal for the tensor-meson effective mass is still very poor, $m_{H_2^*}-m_H=0.50(8)$ GeV, which leads to $m_{H_2^*}-m_{H_0^*}=0.10(8)$ GeV. The large relative error reflects the poor quality of the plateau in fig. 2. Clearly a more refined simulation is needed here. In particular we have not yet optimised the wave function for the smearing of the tensor meson. Our result agrees with the result of [5] where we read from table 2 (Q_3) : $m_{H_2^*}-m_H=0.48(2)$ GeV, and $m_{H_2^*}-m_{H_0^*}=0.08(4)$ GeV.

Experimentally the situation is not yet clear: whereas Belle [2] reported $m_{D_2^*}-m_{D_0^*}=153(36)$ MeV [5], FOCUS [30] finds $m_{D_2^*}-m_{D_0^*}=61(41)$ MeV. Anyway large $1/m_c$ corrections are expected.

[4] This choice shows up a better signal than the one using eq. (13,b). A comparison of these signals has been performed in [7].

[5] Note however than in Belle experiment the narrow and broad $J^P=1^+$ resonances, usually interpreted as $j=3/2$, $j=1/2$ respectively, are practically degenerate in mass: $m_{D_1^0}=2421(2)$ MeV and $m_{D_1'^0}=2427(50)$

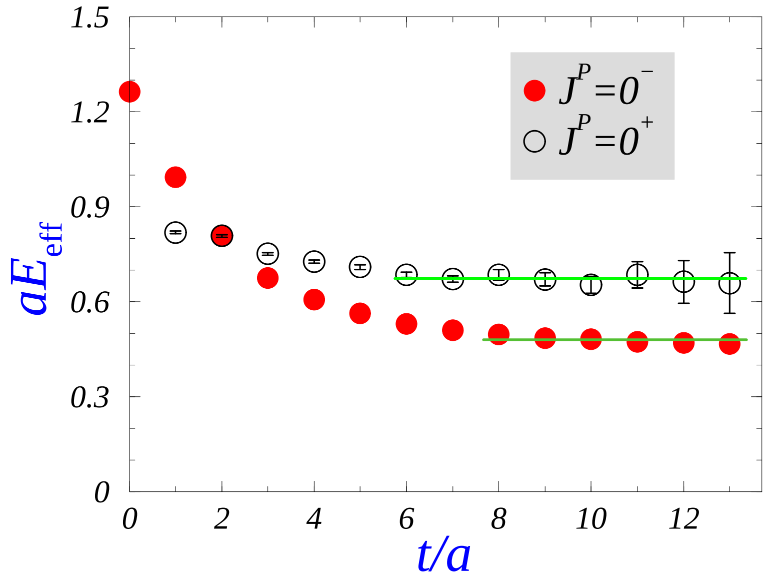

Figure 1: *Signals for the effective binding energies for the pseudoscalar and the scalar heavy-light mesons.*

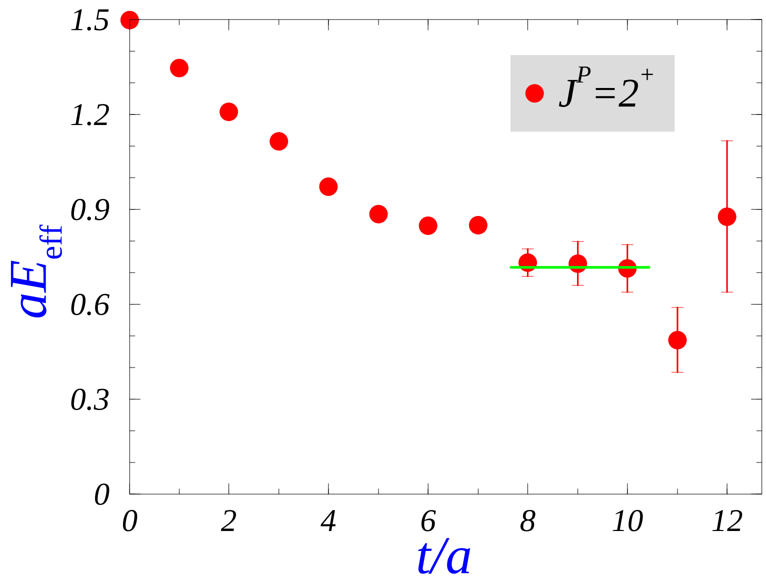

Figure 2: *Signal for the effective binding energies* $E_{\rm eff}$ *for the* 2^+ *heavy-light mesons.*

In fig 3, we plot the ratios

$$
\begin{aligned}
\tau_{\frac{1}{2}}(1) &= \frac{1}{(M_{H_0^*} - M_H)} \frac{\mathcal{Z}_{2;0}\ \mathcal{Z}_{2;5}\ C_{3;05}(0, t_y, t_x)}{C_{2;0}(0, t_y)\ C_{2;5}(0, t_x - t_y)} , \\
\tau_{\frac{3}{2}}(1) &= \sqrt{\frac{2}{3}} \frac{1}{(M_{H_2^*} - M_H)} \frac{\mathcal{Z}_{2;2}\ \mathcal{Z}_{2;5}\ C_{3;25}(0, t_y, t_x)}{C_{2;2}(0, t_y)\ C_{2;5}(0, t_x - t_y)} ,
\end{aligned}
\tag{33}
$$

where the source operator has been fixed at $t_x = 13a$. The equality is valid on the plateau.

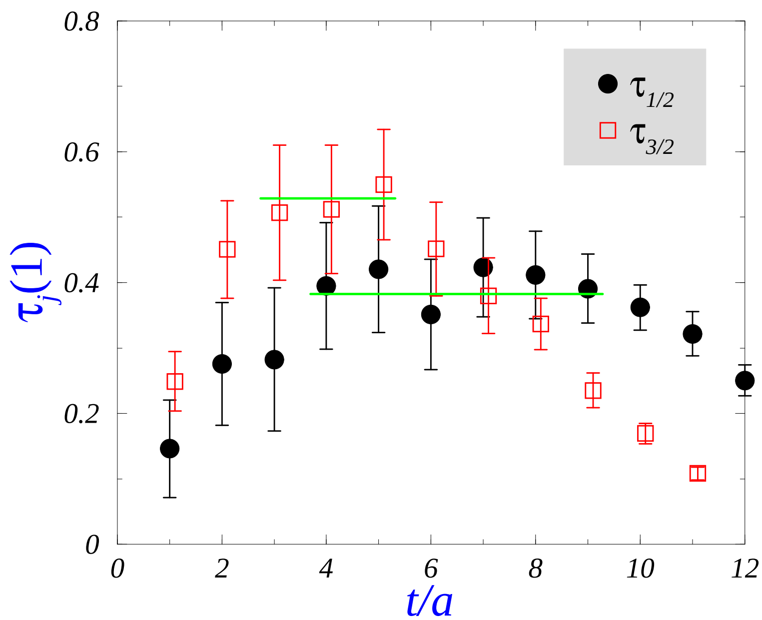

Figure 3: *Signals for the ratios defined in eq. (33); from the fit in* $t/a \in [4, 9]$ *and* $t/a \in [3, 5]$ *respectively, we obtain the value of* $\tau_{\frac{1}{2}}$ *and* $\tau_{\frac{3}{2}}$.

4 Results, Discussion and Conclusions

In this letter we propose a method to compute on the lattice, at the infinite mass limit, the zero recoil Isgur-Wise form factors $\tau_{\frac{1}{2}}(1)$ and $\tau_{\frac{3}{2}}(1)$ relevant to the decay of a heavy pseudoscalar meson into orbitally excited states. The main feature of the method is contained in eqs. (7) and (9). It uses matrix elements of the axial current multiplied by covariant derivatives.

We have also performed an exploratory lattice study in order to estimate if this method is practically usable. We find that the signal/noise ratio is encouraging if one considers that there is still room for improvement.

Our results are

$$\tau_{\frac{1}{2}}(1) = 0.38(4)(?) \quad \text{and} \quad \tau_{\frac{3}{2}}(1) = 0.53(8)(?) \ , \tag{34}$$

where the question mark represent yet unknown systematic errors. We have also:

$$\tau_{\frac{3}{2}}(1)^2 - \tau_{\frac{1}{2}}(1)^2 = 0.13(8)(?) \tag{35}$$

Within 1.5σ it saturates the Uraltsev sum rule [10]: $\sum_n |\tau^{(n)}_{\frac{3}{2}}(1)|^2 - |\tau^{(n)}_{\frac{1}{2}}(1)|^2 = \frac{1}{4}$. Note that an approximate saturation of the sum rule by the ground states is seen in several models [31, 33] although there is no strong theoretical reason for that.

The result for $\tau_{\frac{1}{2}}(1) = 0.38(4)(?)$ is presumably more reliable than the one on $\tau_{\frac{3}{2}}(1)$ since the two-point signal for the 0^+ meson is much better than the one for the 2^+ meson, see figs. 1 and 2. Our result for $\tau_{\frac{1}{2}}(1)$ is somewhat larger than the predictions of the covariant quark models a la Bakamjian-Thomas (BT) [31] which predict, for the preferred potentials [6], $\tau_{\frac{1}{2}}(1) \in [0.1, 0.23]$ and $\tau_{\frac{3}{2}}(1) \in [0.43, 0.54]$. The latter agrees well with eq. (34). Both numbers of eq. (34) are compatible with a recent calculation based on a covariant light-front approach with simple harmonic oscillator wave functions which are not derived from a potential [7]: $\tau_{\frac{1}{2}}(1) = 0.31$ and $\tau_{\frac{3}{2}}(1) = 0.61$ [33]. It is interesting to not that both in ref. [31] and in [33], $\tau_{\frac{3}{2}}(1) - \tau_{\frac{1}{2}}(1) \simeq 0.3$, which might be a general feature of BT covariant quark models (see eq. (5.1) in ref. [31]). This is somewhat larger than the difference between central values of (34). We also agree with an older QCD sum rule estimate [32]: $\tau_{\frac{1}{2}}(1) = 0.35(8)$.

The quantity we compute on the lattice is a physical quantity: the product of a mass difference times the form factors $\tau_{\frac{1}{2}}(1)$ and $\tau_{\frac{3}{2}}(1)$. We thus expect no multiplicative renormalisation to be needed. A closer scrutiny of this question is underway, in particular to understand if the hypercubic treatment of the Wilson line can have some effect on the discretized covariant derivative we use. Of course a complete control of systematic effects is also needed: finite volume, mass of the light quark, finite lattice spacing. In section 2.1 a duality of interpolating fields has been pointed out. A systematic comparison of the their predictions is still missing.

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[6]It should be stressed that the BT method provides a framework in which different potentials can be used. The physics prediction depends, of course, on the chosen potential.

[7]The covariant light-front framework is equivalent, in the infinite mass limit, to the BT one. The practical predictions depend, however, on the chosen parameters and shape of the wave function. We worry if the use of gaussian wave functions, which is frequent, is a good one as it neglects the short distance potential.

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