A method for including a-priori-preferences in Multicriteria Optimization

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Abstract

In this paper a method for including a-priori preferences of the decision makers into Multicriteria Optimization (MCO) problems is presented. A set of Pareto-optimal solutions is determined via desirability functions of the objectives which reveal experts' preferences regarding different objective regions. By applying the method to the Binh-problem it proofs to be very effective for focussing on different parts of the Pareto front respectively the objective space of the original problem.

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1 Introduction

Multicriteria optimization (MCO) aims at simultaneously optimizing a set of conflicting objectives which are dependent on a set of influence factors. The key problem is to find the factor levels which result in optimal objective values. Opposed to single-objective optimization problems however there is no unique definition of "optimum" as a ranking in a high-dimensional space is required. Therefore, decision makers who provide an expert's opinion always are an important element in MCO procedures.
In general MCO problems can be classified regarding at which stage preferences of the decision makers are integrated. A-priori optimization methods (e.g. Desirability Index (Harrington (1965)), utility functions (Weihs and Jessenberger (1999), p 174)) are based on experts' knowledge, which has to be specified upfront, and generate a respective unique

solution. A-posteriori methods however create a set of solutions (Pareto set) from which one solution has to be chosen after the optimization step. These solutions are called Pareto-optimal, i.e. one objective cannot be improved without deteriorating at least one of the remaining ones.

The proposed method is a mixture of the a-priori- and the a-posteriori-approach. Recently Multiobjective Evolutionary Algorithms (MOEAs) were found to be an important means for solving MCO problems based on the a-posteriori-approach. Those are designed to generate a preferably uniform spread of Pareto-optimal solutions covering the whole Pareto front, which is formed by the objective values of the Pareto set. In most practical cases the experts however will have at least a vague idea in which region of the objectives the "optimum" can be found or in which region the Pareto-optimal solutions to choose from are desired. For this purpose so-called desirability functions (DFs) originating from the concept of the Desirability Index (DI) can be specified a-priori, which transform the objectives onto a unitless scale between zero and one according to their desired values. The Pareto-optimal solutions are then determined for the DFs instead of the objective functions.

The solutions then are concentrated in the desired region which facilitates the solution selection process and prevents MOEAs from finding non-relevant solutions. In addition, despite of the focus on the desired region, the MOEA search can be carried out without any restrictions or additional parameters which have to be included. Furthermore, the application in practice is straightforward. Once the experts have agreed on the parameters of the DFs, which is the key step, MOEAs can be applied on the transformed objective space without modification.

So far a few concepts for integrating preferences into the a-posteriori MCO-approach exist but they rather focus on a weighting of the objectives than on individual objective preference regions, i.e. the objectives are seen to be not equally important (Branke and Deb (2004)). The Guided Dominance Principle requires the specification of maximally acceptable trade-offs for each pair of objectives. On the so-transformed objective space evolutionary algorithms can be applied without modification. The latter is a similarity to the approach proposed here and can be seen as an important advantage as the methods are applicable quite easy. A method specifically designed for the NSGA-II - algorithm

(Deb (2002)) is the alteration of the distance method in the sharing step.

A detailed method description and a review of the concept of DFs can be found in Chapter 2. Afterwards the proposed approach is illustrated by means of the Binh-Problem (Collette and Siarry (2003), p. 49) in Chapter 2.1. Finally a summary and an outlook on further research fields completes the results in Chapter 3.

2 Optimizing Desirability Functions of Objectives

The classical (unconstrained) multiobjective optimization problem, without loss of generality of minimization type, is defined as follows:

Definition 1 (MCO Problem)

$$\begin{aligned} \textit{Minimize} \quad & Y = f(X) = (f_1(X), \ldots, f_k(X)), \\ \textit{where} \quad & X = (X_1, \ldots, X_n) \in \mathcal{X} \qquad \textit{(decision vector)}, \\ & Y = (Y_1, \ldots, Y_k) \in \mathcal{Y} \qquad \textit{(objective vector)}. \end{aligned}$$

In this paper we integrate so-called desirability functions, originating from an a-priori MCO approach, into the MCO problem:

Definition 2 (Desirability MCO Problem)

$$\begin{aligned} \textit{Minimize} \quad & -d(Y) = -d[f(X)] = -(d_1[f_1(X)], \ldots, d_k[f_k(X)]), \\ \textit{where} \quad & X = (X_1, \ldots, X_n) \in \mathcal{X} \qquad \textit{(decision vector)}, \\ & Y = (Y_1, \ldots, Y_k) \in \mathcal{Y} \qquad \textit{(objective vector)}, \\ & d_i(Y_i) : Y_i \to [0,1],\ i = 1, \ldots, k \quad \textit{(desirability function)}. \end{aligned}$$

The concept of desirability was introduced by Harrington (1965) in the context of multiobjective industrial quality control. A univariate Desirability Index combines so-called desirability functions (DFs) into one overall univariate quality measure, which has to be maximized. Details can be found in Trautmann (2004).

A DF most generally is defined as any function $d : Y \to [0, 1]$ specifying how desirable different regions of the objective space are.
Harrington (1965) introduced two types of DFs. One aims at maxi- or minimization

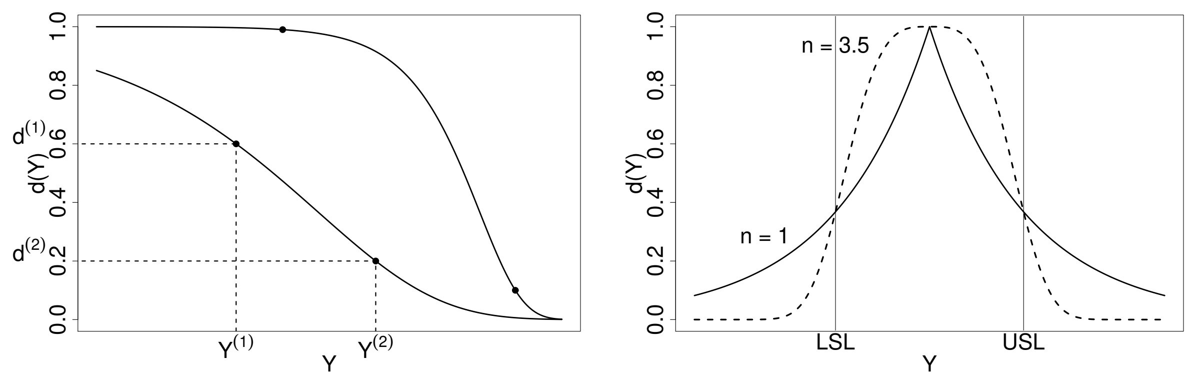


Figure 1: Harrington's one (left)- and two-sided Desirability Functions (right)

(one-sided specification) whereas the other one reflects a target value problem (two-sided specification) (see Fig. 1). Concerning the latter the transformation requires two specification limits (LSL_i, USL_i) for an objective Y_i symmetrically around the target value, which are associated with a desirability of $1/e$. Then the DF is defined as

$$d_i(Y_i') \;=\; e^{-|Y_i'|^{n_i}} \quad \text{with} \quad Y_i' = \frac{2Y_i - (USL_i + LSL_i)}{USL_i - LSL_i}, \quad i = 1, \ldots, k. \tag{1}$$

The parameter $n_i > 0$ is to be chosen so that the resulting kurtosis of the function adequately meets the expert's preferences.

The one-sided DF uses a special form of the Gompertz-Curve, where the kurtosis of the function is determined by the solution (b_{0i}, b_{1i}) of a system of two linear equations that require two values of Y_i and related values of d_i; $(y_i^{(1)}, d_i^{(1)})$ and $(y_i^{(2)}, d_i^{(2)})$.

$$d(Y') \;=\; e^{-e^{-Y'}} \text{ with } Y' = b_0 + b_1 Y, \tag{2}$$

$$b_1 \;=\; (-\log(-\log(d^{(2)})) - \log(-\log(d^{(1)}))/(y^{(2)} - y^{(1)}) \text{ and} \tag{3}$$

$$b_0 \;=\; -\log(-\log(d^{(1)})) - b_1 y^{(1)}. \tag{4}$$

After the DFs have been set up by the decision makers, these are used as new (transformed) objectives in the MCO problem. We will concentrate on Harrington's DFs (1) and (2) in the following. The procedure itself however is independent of the chosen type of DF.

The aim of the a-posteriori MCO approach is to find a set of Pareto-optimal solutions. A realization of objectives $Y = (Y_1, \ldots, Y_k)^T$ is said to be Pareto-optimal if it cannot be improved without deteriorating at least one objective. A corresponding factor setting

$X = (X_1, \ldots, X_n)^T$ then is Pareto-optimal in decision space $\mathcal{X}$ if the corresponding objective vector Y is Pareto-optimal in objective space $\mathcal{Y}$. The set of Pareto-optimal vectors Y is called the Pareto front, whereas the corresponding set of vectors X are called the Pareto set.

The solutions of the Desirability MCO problem, most likely generated by a MOEA, then do not cover the whole Pareto front of the original MCO problem, but rather lie concentrated in specific regions with respect to the specified DFs. Theorem 3 shows that in case that one-sided (minimization) DFs of Harrington-type are used these solutions always result in points on the Pareto front resulting from the MCO Problem in Definition 1.

Theorem 3 The Pareto-optimal solutions of $\max_{i=1,\ldots,k} d_i(Y_i) \hat{=} \min_{i=1,\ldots,k} -d_i(Y_i)$ for one-sided (minimization-) DFs of Harrington-type are also Pareto-optimal solutions of $\min_{i=1,\ldots,k} Y_i$.

Proof:
Let X_{DF}^{opt} be Pareto-optimal for $\min_{i=1,\ldots,k} -d_i(Y_i)$.

$$\begin{aligned} \Rightarrow \nexists X^*_{DF} \quad : \quad & -d_i(Y_i|X^*_{i_{DF}}) < -d_i(Y_i|X^{opt}_{i_{DF}}) \\ & -d_j(Y_j|X^*_{j_{DF}}) \leq -d_j(Y_j|X^{opt}_{j_{DF}}), \quad j \neq i \end{aligned}$$

$$\begin{aligned} \Leftrightarrow \nexists X^*_{DF} \quad : \quad & -\exp(-\exp(-(Y_i'|X^*_{i_{DF}}))) < -\exp(-\exp(-(Y_i'|X^{opt}_{i_{DF}}))) & (5) \\ & -\exp(-\exp(-(Y_j'|X^*_{j_{DF}}))) < -\exp(-\exp(-(Y_j'|X^{opt}_{j_{DF}}))), \quad j \neq i & (6) \end{aligned}$$

$$\begin{aligned} (5) \Leftrightarrow \quad & -\exp(-(Y_i'|X^*_{i_{DF}})) > -\exp(-(Y_i'|X^{opt}_{i_{DF}})) \\ \Leftrightarrow \quad & \exp(-(Y_i'|X^*_{i_{DF}})) < \exp(-(Y_i'|X^{opt}_{i_{DF}})) \\ \Leftrightarrow \quad & -(Y_i'|X^*_{i_{DF}}) < -(Y_i'|X^{opt}_{i_{DF}}) \\ \Leftrightarrow \quad & -b_{0i} - b_{1i}(Y_i|X^*_{i_{DF}}) < -b_{0i} - b_{1i}(Y_i|X^{opt}_{i_{DF}}) \\ \Leftrightarrow \quad & b_{1i}(Y_i|X^*_{i_{DF}}) > b_{1i}(Y_i|X^{opt}_{i_{DF}}) \\ \Leftrightarrow \quad & (Y_i|X^*_{i_{DF}}) < (Y_i|X^{opt}_{i_{DF}}) \text{ as } b_{1i} < 0 \text{ for minimization problems} & (7) \end{aligned}$$

$$(6) \Leftrightarrow \quad \ldots (Y_j|X^*_{j_{DF}}) \leq (Y_j|X^{opt}_{j_{DF}}), \quad j \neq i. \tag{8}$$

$\Rightarrow$ The proposition holds as equations (7) and (8) form the condition for Pareto-optimal solutions of $\min_{i=1,\dots,k} Y_i$. $\square$

In case a mixture of one- and two-sided DFs this is not guaranteed. Whereas for this case in most cases the Pareto-optimal solutions of the MCO- and the Desirability-MCO-problem overlap this will rarely be the case when exclusively two-sided DFs are used. Unless the DF specifications are totally out of bounds and inadequate this is not a problem however as Pareto-optimal solutions with regard to the desirabilities of the applying expert are determined. Therefore it is not possible to improve the desirability of one objective without simultaneously decreasing the desirability of at least one other objective.

The effect of different specifications of DFs is examined by means of the Binh problem in the next chapter.

2.1 Exemplary Results for Harrington's DFs and the Binh Problem

The **Binh problem** (Collette and Siarry (2003), p. 49) consists of two objective functions f_1 and f_2 which are dependent on two parameters x_1 and x_2:

$$\text{minimize} \quad f_1(x_1, x_2) = x_1^2 + x_2^2 \tag{9}$$

$$\text{minimize} \quad f_1(x_1, x_2) = (x_1 - 5)^2 + (x_2 - 5)^2 \tag{10}$$

$$\text{with} \quad -5 \leq x_1 \leq 10, \quad -5 \leq x_1 \leq 10. \tag{11}$$

The Pareto front is known to be convex and is shown in figure 2.

The aim is to integrate experts' knowledge to focus well directed on parts of the Pareto front. For the Binh-problem simulations were carried out as follows. At first the objective space consisting of the values of f_1 and f_2 was sampled with step width 0.1 for all possible combinations of the criteria X_1 and X_2 with regard to their constrained region (11). Afterwards the NSGA-II algorithm (Deb (2002)) was used to determine the Pareto fronts for the original Binh-problem and the Desirability-Binh-problem as indicated in Definition 2. For this purpose it was reverted to the respective C-code from KANGAL (Kanpur Genetic Algorithms Laboratory). Table 1 lists the parameter settings of the algorithm, which were chosen with respect to the default settings in the code and the

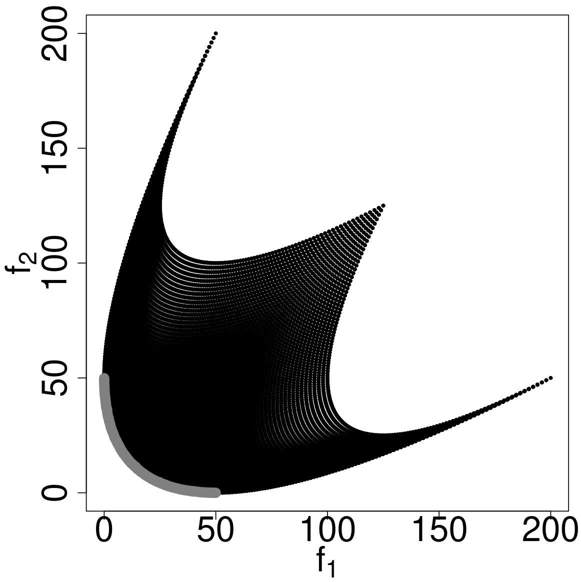


Figure 2: Pareto front of the Binh problem (grey)

Population Size	100
No. of generations	250
Cross-over probability	0.9
Mutation probability	1
Distribution index for crossover	20
Distribution index for mutation	20
Random seed	0.4

Table 1: Parameters of the NSGA-II-algorithm

recommendations in Deb (2002).
In order to be able to compare the results of both approaches the Pareto-optimal solutions are both shown in the objective space spanned by f_1 and f_2. Separately for two one-sided, a mixture of one- and two-sided and two two-sided DFs different test cases are considered. In each case the specification of one DF is kept constant whereas the parameters of the remaining one are varied in order to show the possible location changes of the Pareto-optimal solutions resulting from the Desirability-Binh-problem.

For two one-sided DFs (2) Table 2 lists the two test cases considered. For the first case both objectives are assumed to have to be minimized with zero having the highest desirability. It becomes obvious that a compromise between the two objectives has to be found

No.	$y_1^{(1)}$	$d_1^{(1)}$	$y_1^{(2)}$	$d_1^{(2)}$	$y_2^{(1)}$	$d_2^{(1)}$	$y_2^{(2)}$	$d_2^{(2)}$
1 a)	0	0.99	50	0.01	0	0.99	**5**	0.01
b)	0	0.99	50	0.01	0	0.99	**10**	0.01
c)	0	0.99	50	0.01	0	0.99	**20**	0.01
...	...	...	...	...	...	...	**...**	...
g)	0	0.99	50	0.01	0	0.99	**60**	0.01
2 a)	**5**	0.9	50	0.01	0	0.99	10	0.01
b)	**10**	0.9	50	0.01	0	0.99	10	0.01
c)	**20**	0.9	50	0.01	0	0.99	10	0.01
d)	**30**	0.9	50	0.01	0	0.99	10	0.01
e)	**40**	0.9	50	0.01	0	0.99	10	0.01

Table 2: Test Cases for the Binh-problem and one-sided DFs (2)

as the maximum desirability cannot be achieved for both. For f_1 an exceeding of 50 is totally undesirable whereas for f_2 this limit is varied from 5 to 60. Observing the results in Figure 3 the effect of this variation can clearly be seen. In 1a), having assigned a value of 0.1 to an f_2-value of 5, all solutions fall below this value. For 1d) the f_2-value of 40 is not exceeded which matches perfectly with the DF specification. Analogously this is true for 1g) where nearly the whole Pareto front of the original problem is covered.

In test case 2 the DF for f_2 is fixed to the value of 1b). The first DF is varied such that the f_1-value corresponding to a desirability of 0.9 is increased bit by bit, i.e in the extreme case all f_1 values below 40 are assigned a nearly equal high desirability. The first observation to be made when looking at Figure 4 is that the f_2-value of 10 is very seldom exceeded as they have a desirability below 0.01. Only in 2a) two solutions exceed this value as in this case the minimization task of f_1 is strongest. Furthermore the concentration of the solutions changes. The higher $y_1^{(1)}$ the more equally the solutions are spread in the region bordered by $f_2 = 10$.

In addition also two-sided DFs can be relevant in case a specific objective basically has to be minimized, but only up to a fixed value. For objective realizations below this value the desirability decreases. Table 3 shows test cases for a mixture of a one-and a two-sided DF. As already mentioned in Chapter 2 the solutions of the Desirability-Binh-problem might not correspond to Pareto-optimal solution of the original Binh-problem. In the test cases 3 to 4 however this is the case.

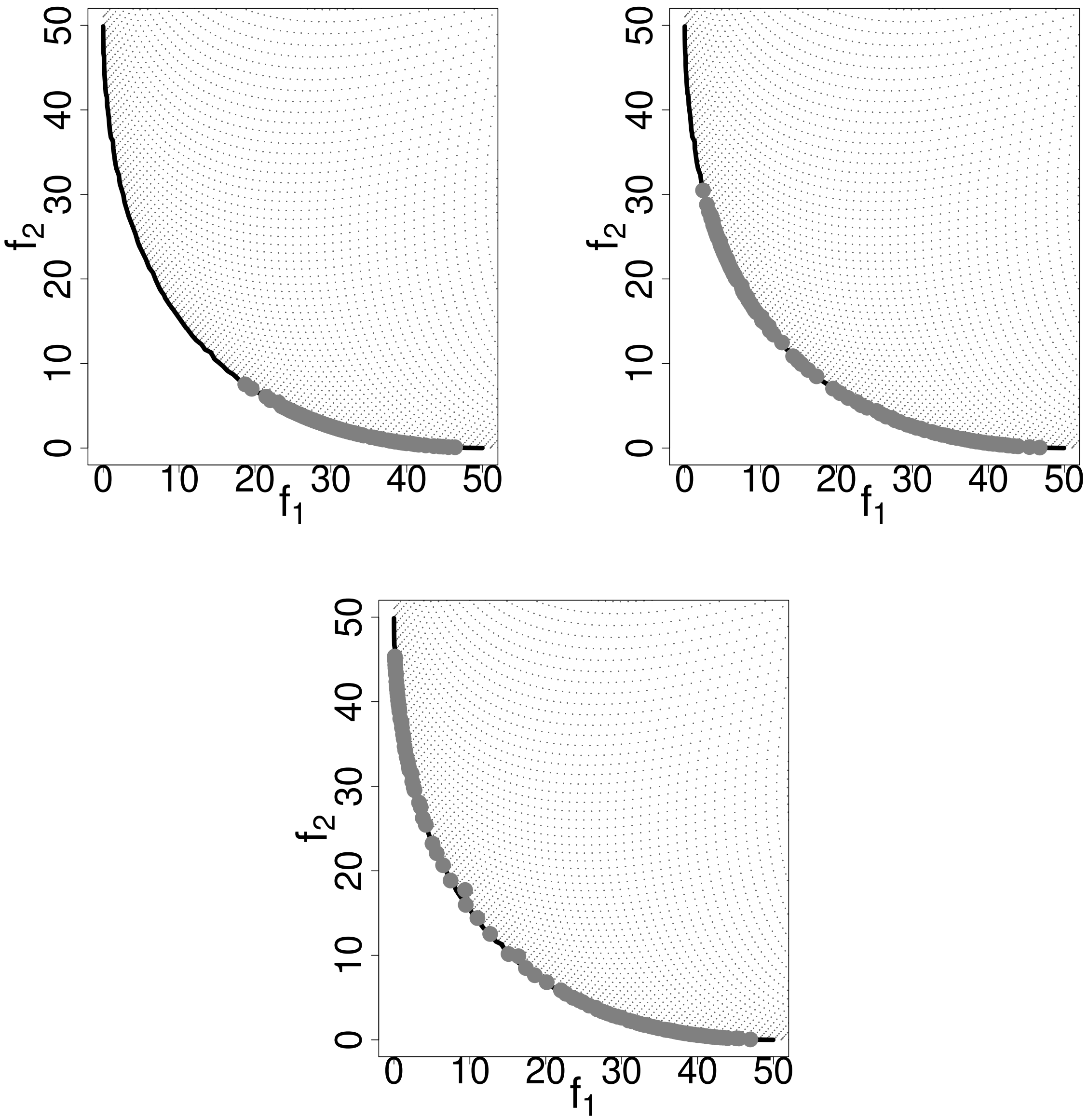


Figure 3: Pareto front of test cases 1a),1d) and 1g) without (black) and with DFs (grey)

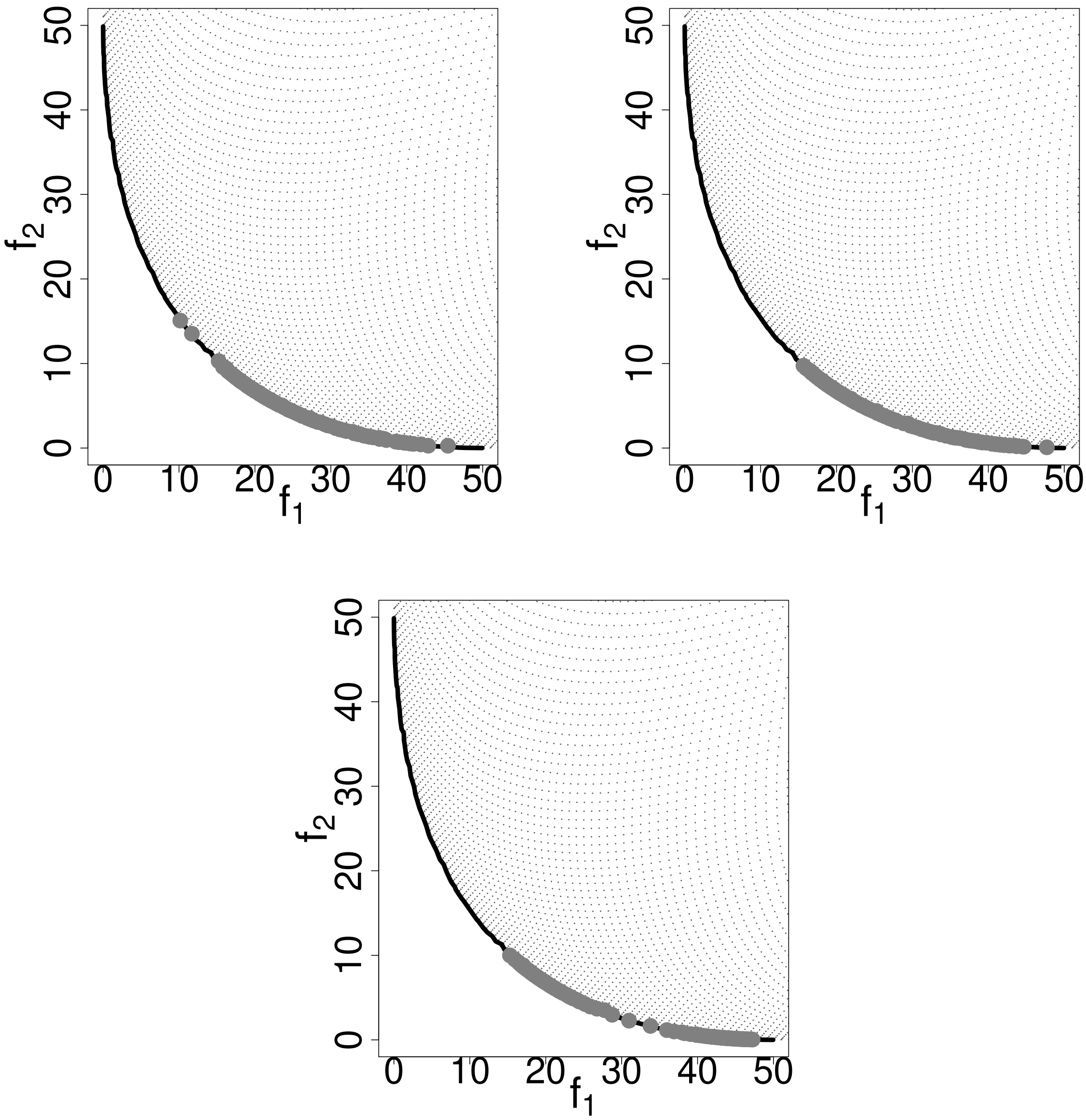


Figure 4: Pareto front of test cases 2a),2c) and 2f) without (black) and with DFs (grey)

No.	$y_1^{(1)}$	$d_1^{(1)}$	$y_1^{(2)}$	$d_1^{(2)}$	LSL_2	USL_2	n_2
3 a)	0	0.99	50	0.01	0	**10**	1
b)	0	0.99	50	0.01	0	**20**	1
c)	0	0.99	50	0.01	0	**30**	1
d)	0	0.99	50	0.01	0	**40**	1
4 a)	0	0.99	50	0.01	**10**	**30**	1
b)	0	0.99	50	0.01	**20**	**40**	1
c)	0	0.99	50	0.01	**30**	**50**	1
d)	0	0.99	50	0.01	**40**	**60**	1
5	0	0.9	30	0.2	50	90	1
6	0	0.9	30	0.2	90	120	1

Table 3: Test Cases for the Binh-problem and a mixture of one-(2) and two-sided DFs (1)

As an experience from the simulations one can say roughly that if the DF parameters of the two-sided function are specified reasonably, i.e. in the region of the Pareto front, the solutions will result in Pareto-optimal solutions of the original Binh-problem.

For case 3 the upper specification limit USL_2 is increased from 10 to 40 with LSL_2 equal to zero. The variation of USL_2 has the effect that the solutions seem to make an "upward movement" starting with the target value $T_2 := (LSL_2 + USL_2)/2$ of f_2. As the first objective has to be minimized over the whole Pareto front the solutions tend to concentrate on the left part of the figure so that as a compromise values below T_2 are not relevant.
An analogous but more extreme "upward movement" of the solutions is shown for test case 4 in Figure 6. In addition to ULS_2 also LSL_2 is varied. Again the solutions tend to concentrate on low f_1-values.

Cases 5 and 6 are examples in which there is no overlap between the solutions of the original Binh- and the Desirability-Binh-problem (see Figure 7). This phenomenon results due to the fact that the DF parameters of f_2 are specified such that the desired region lies outside the original Pareto front. This does not necessarily have to be a drawback – especially if the experts chose this settings knowingly – as the obtained solutions are Pareto-optimal for the desirability approach. I.e. the desirability of one objective cannot be improved without deteriorating the desirability of the other.

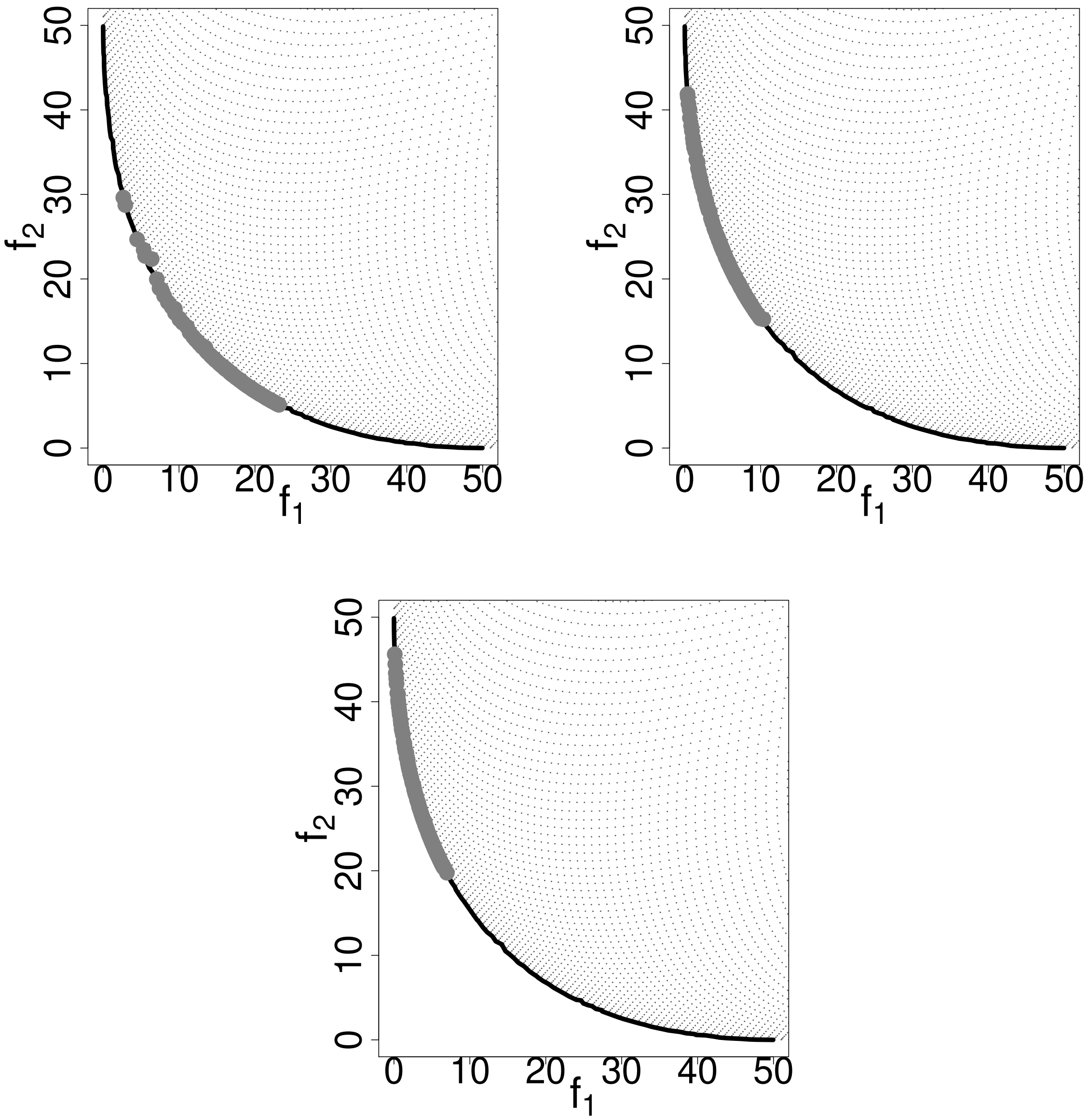


Figure 5: Pareto front of test cases 3a),3c) and 3d) without (black) and with DFs (grey)

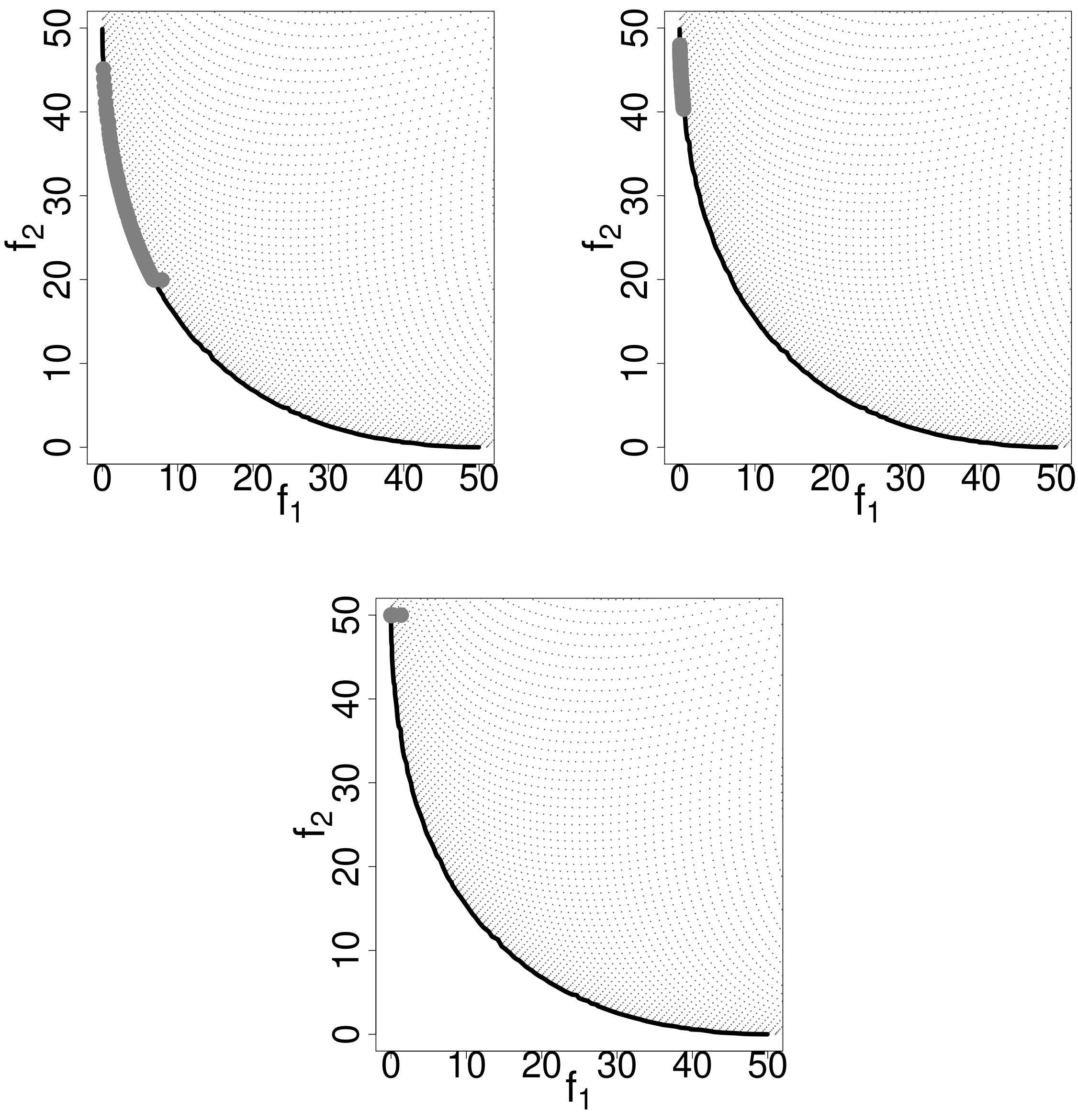


Figure 6: Pareto front of test cases 4a),4c) and 4d) without (black) and with DFs (grey)

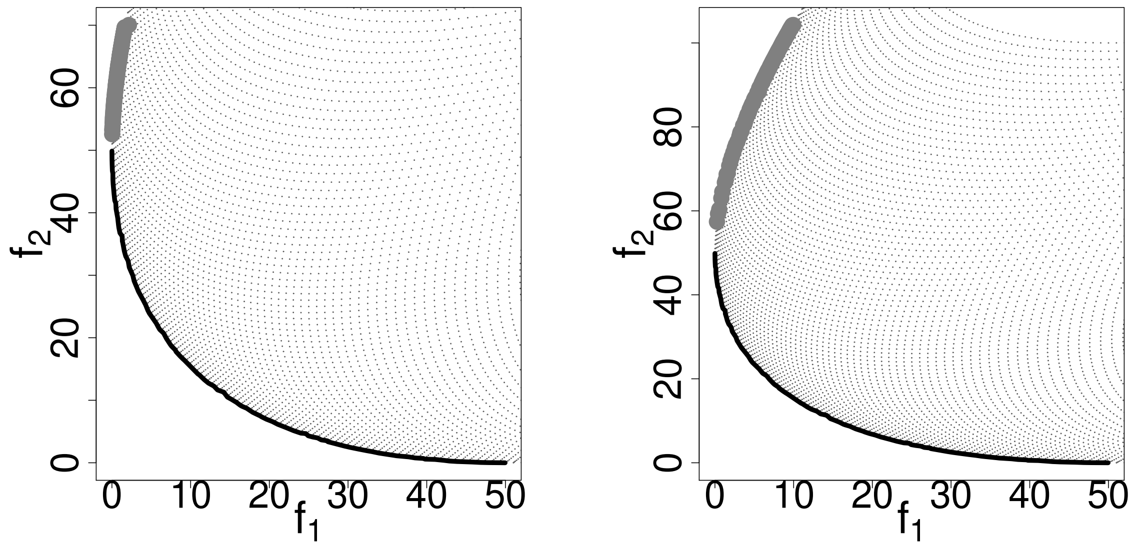
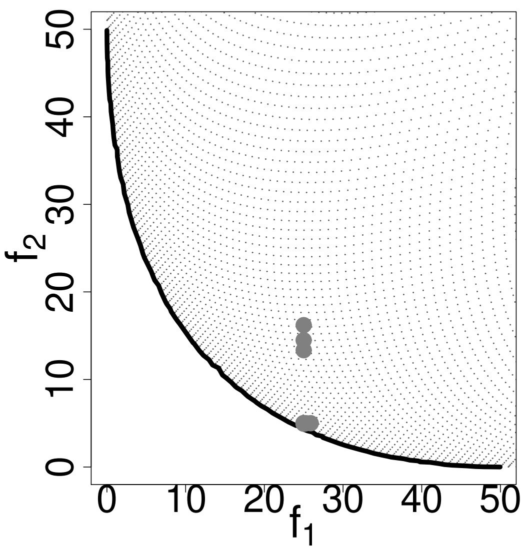
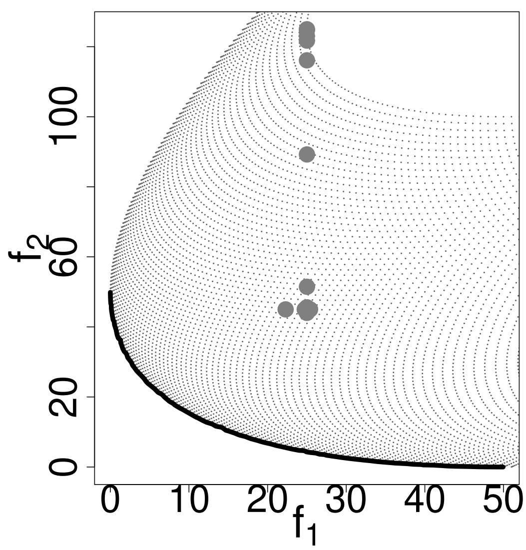


Figure 7: Pareto front of test cases 5) and 6) without (black) and with DFs (grey)

No.	LSL_1	USL_1	n_1	LSL_2	USL_2	n_2
7 a)	10	40	1	**0**	**10**	1
b)	10	40	1	**10**	**20**	1
...	10	40	1	**...**	**...**	1
f)	10	40	1	**50**	**60**	1
8 a)	**0**	**10**	1	30	40	1
b)	**10**	**20**	1	30	40	1
...	**...**	**...**	1	30	40	1
e)	**40**	**50**	1	30	40	1

Table 4: Test Cases for the Binh-problem and two two-sided DFs (1)





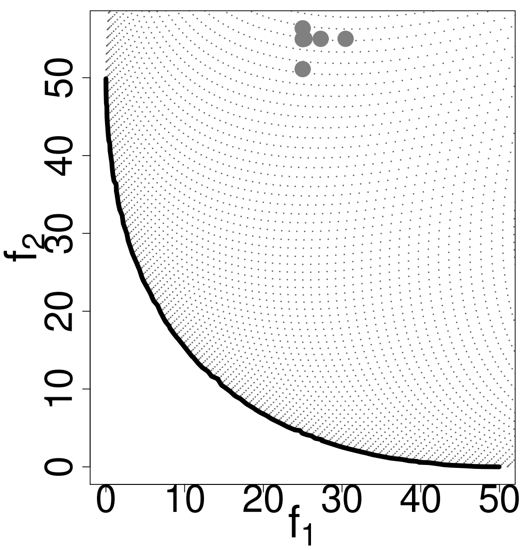


Figure 8: Pareto front of test cases 7a), 7e) and 7f) without (black) and with DFs (grey)

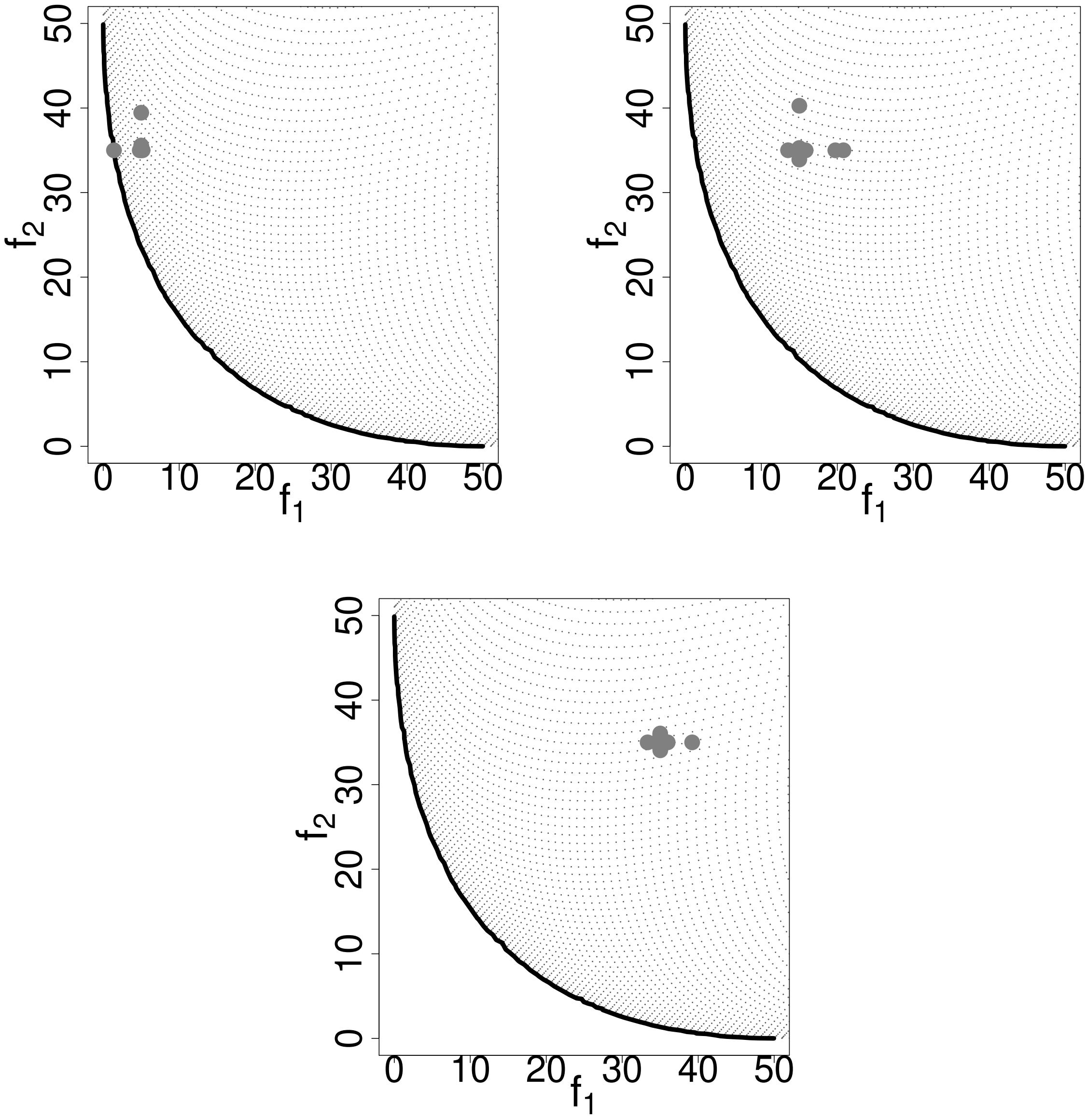


Figure 9: Pareto front of test cases 8a), 8b) and 8d) without (black) and with DFs (grey)

In case two two-sided DFs are specified it will not often be the case that the solutions of the Desirability-Binh-problem lie on the Pareto front of the original problem. Cases 8 and 9 show how various regions of the whole objective space can be focussed by altering the DF parameters.
But once again it has to be kept in mind that the obtained solutions are Pareto-optimal for the Desirability-Binh-problem. The expert gets optimal solutions with regard to his preferences. As explained earlier, if those preferences are not totally unreasonable it does not matter that there is no overlap of both Pareto fronts.
As the example shows the proposed approach is a very effective method for including a-priori-preferences in the a-posteriori-MCO-approach.

3 Summary and Outlook

In this paper a method for including a-priori preferences of the decision makers into the a-posteriori MCO-approach, where a set Pareto-optimal solutions is determined, is presented. For each objective a desirability function (DF) is set up which reflects the expert's preferences with regard to different objective regions. The Pareto-optimal solutions are then determined for the transformed objective space, i.e. for the DFs.
By means of the Binh-problem the method is shown to be very effective for focussing on different parts of the Pareto front if one-sided (minimization-) or a mixture of one- and two-sided DFs are used. In case two two-sided DFs, which reflect target value problems, are utilized, the focus is on different regions of the objective space. The key feature is the Pareto-optimality with regard to the experts' preferences, i.e. the desirability of one objective cannot be improved without deteriorating the desirability of at least one other objective.

For future work a combination of the method proposed here and the approaches of Branke and Deb (2004) seems to be promising. In addition to the preferences regarding each objective also a weighting of the objectives could be included. Furthermore the effect of different kinds of desirability functions and the behaviour in presence of constraints will be investigated.

Acknowledgements

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