A Multiplicative Concept for Random Utility

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ABSTRACT

The paper introduces a new type of random utility model. Here the random term is formulated as a multiplicative factor to express the random effects of the utility together with deterministic utility aspects. Some arguments are given why this multiplicative approach is preferable over the additive solution which is more common in transportation planning. It is shown using the Weibull distribution, that this concept can be transformed into rather simple equations for the probability of the selection of different modes by individual travellers. More mathematical derivations show that the new concept has close relations to the Logit model. It can, however, provide one more degree of freedom and thus, better flexibility to adjust the model to reality. Also with other assumptions for the random utility term, like the lognormal distribution, this basic concept can be developed. For the model, a method of parameter calibration based on individual traveller's decisions by maximum likelihood has been developed. The whole concept is demonstrated with real world data to demonstrate its usability.

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1. INTRODUCTION

In transportation planning disaggregated choice models play an important role. They try to model decisions of individual travellers by means of mathematical equations. These could be decisions about the travel mode on a trip from A to B (modal split). This could also be decisions between different destinations of their activities (traffic distribution) or between different routes (traffic assignment). Disaggregated models try to represent these decisions based on individual traveller's behaviour. Most popular are the so-called Logit-models and Probit-models and different kinds of model approaches derived from these two basic types. All of these models can be summarised under the term "random utility" solutions since they are based on the determination of the utility which different travel modes provide to the individual traveller. Here the utility is divided into one deterministic term and a second term which has properties of randomness. This random term traditionally is treated as an additional term to the deterministic utility where the mean of the random utility is usually regarded as sero. The different formulations of models are a consequence of different statistical distributions which are applied for the random utility term.

In this paper the random term is defined as a multiplicative factor to the deterministic utility. This requires a different set of mathematical derivations to make the model ready for application. These derivations together with some examples are given below. Moreover, the implications of the model in comparison to classical solutions are discussed.

2. DEFINITIONS AND TERMINOLOGY

We look at an individual traveller (index t) who has the choice between different modes or alternatives to travel on a trip from A to B . He/she (in this paper „he“ stands for „he/she“) has to decide which mode from a set of J alternatives (index i) he is going to select. For his selection he applies a set of K attributes (index k) which characterise the usefulness of the alternative i for the traveller t . Each attribut can obtain different specific values ($x_{i\,t\,k}$) which depend on the individual t and on the alternative i . Such an attribut e.g. could be the travel time or costs. Moreover, an attribut could also be established by a socio-economic parameter of the individual itself like e.g. income or car-ownership. Each attribut k can have different importance for the individual . Therefore, each attribut k is weighted by a specific factor B_k which we call the parameter of attribut k .

The individual expects a characteristic utility from his decision. Of course, this utility is different for each alternative i and it depends on the special situation of the individual t . We call this utility: U_{it} . Now we assume that the utility is combined by two different components:

- a deterministic utility V_{it} which can be derived from objective values of the attributes and
- a random component ε_{it} .

The random component - like in other random utility models - comprises all disturbances which cannot be described precisely by a mathematical model plus all the uncertainties which the traveller has to suffer within his decision making process.

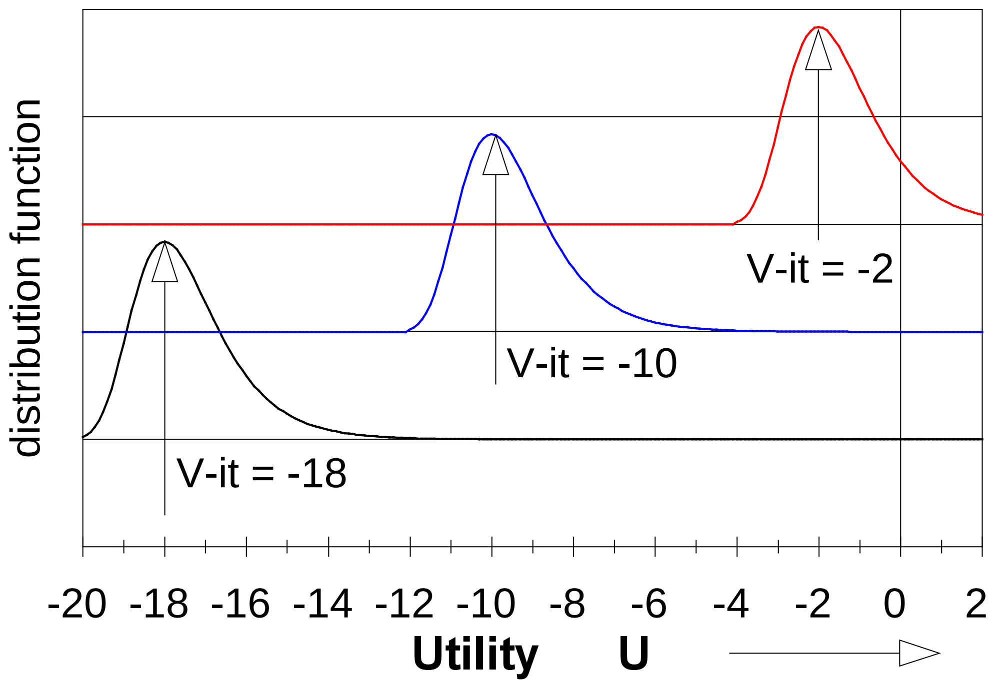


Fig. 1: The figure illustrates that the Logit model using an additive Gumbel distributed random term always implies an identical variance independent of the magnitude of the deterministic utility V.

Therefore, the random component ε_{it} should be treated as a disturbance term by which the deterministic utility V_{it} has to be corrected to obtain the true utility U_{it} for the individual t and mode i. Due to these reasons of uncertainty it is realistic to assume that the random component can be modelled as a multiplicative disturbance term attached to the deterministic utility. This multiplicative assumption seems to be more meaningful than to use an additive term to describe the uncertainties. By the multiplicative term, larger attribut values are weighted with a larger absolute value of uncertainty than smaller values. This seems to be more realistic. E.g. this example could help for further illustration: If a traveller likes to get a rough imagination about his travel time using his car he might be wrong by 1 or 2 minutes if the total travel time is around 10 minutes. But for a longer trip of 1 hour with the same degree of precision the error could be between 6 and 12 minutes. It can not be expected that in this larger range of total travel time a margin for estimation precision of 1 minute is realistic. Exactly this, however, is assumed by the conventional additive random utility models like Logit. This is illustrated in figure 1. Independent from the magnitude of the deterministic utility V_{it} the variance of the total utility U_{it} remains always the same using the Logit model.

Therefore, we assume for our model that the random component of the utility should be proportional to the absolute value of the utility. Thus we define:

$$U_{it} = V_{it} \cdot \varepsilon_{it} \tag{1}$$

where ε_{it} is a positive random variable.

This multiplicative usage of the random component has been called the Rubit-concept in a former paper (Brilon, Bondzio, 1996). In this paper we enhance this basic concept.

In our further derivations we propose a linear model for the deterministic utility, i.e.

$$V_{it} = B_0 + \sum_{k=1}^{K} B_k \cdot X_{itk} \tag{2}$$

This assumption corresponds to the other conventional types of random utility models. It is the most simple format for the utility function. To enhance the model, of course, also non-linear components might be included, an improvement which would not complicate the model solution. It should be noted that with real world data the V_{it} which result from eq. 2 are usually below sero. This is assumed to be fulfilled in each case for the following derivations. This assumption demands for attributes X_{itk} which should always be adverse to the selection of an alternative like travel time, distance, or costs. Attributes X_{itk} which with increasing values argue towards selecting that alternative, should be reformulated such that the assumption mentioned before is fulfilled (e.g. $1 / X_{itk}$ or $1- X_{itk}$).

For the random utility term ε_{it} we have to assume a useful distribution function. This distribution function in our paper is denoted by

- $f_\varepsilon(x) = f(x)$ for the statistical density function,
- $F_\varepsilon(x) = F(x)$ for the cumulative distribution function .

It is reasonable to assume that the expectation $E(\varepsilon)$ of the random utility term ε_{it} is equal to 1 irrespective of the individual t and the alternative i . The distribution function should fulfil this requirement. That means: we assume that the individuals, on average, make a suitable estimation of their utility situation in the selection process.

We now assume that the individual makes his decision based on rational considerations. That means: he selects alternative i instead of any of the other alternatives j if

$$U_{it} > \max_{j \neq i} U_{jt} \tag{3}$$

for all alternatives j . With eq. 1 this can be expressed as

$$V_{it} \cdot \varepsilon_{it} > \max_{j \neq i} (V_{jt} \cdot \varepsilon_{jt}) \tag{4}$$

Taking into account that $V_{it} < 0$ (for each i and t) this inequality can be transformed via

$$\frac{V_{it}}{V_{jt}} < \frac{\varepsilon_{jt}}{\varepsilon_{it}} \quad \text{(for all alternatives } j \neq i \text{)}$$

into

$$\frac{\varepsilon_{it}}{\varepsilon_{jt}} < \frac{V_{jt}}{V_{it}} \quad \text{(for all alternatives } j \neq i \text{)} \tag{5}$$

This means: The alternative i is chosen by the individual t if inequation 5 is fulfilled for all alternatives j other than i . Therefore, the probability that alternative i is chosen, is p_i with:

$$p_i = p\left(\frac{\varepsilon_{it}}{\varepsilon_{jt}} < z_{ij} \right) \quad \text{(for all alternatives } j \neq i \text{) .} \tag{6}$$

where $z_{ij} = \dfrac{V_{jt}}{V_{it}}$

For further derivations the index t for the individual can be omitted. Eq. 6 can be transformed as follows

$$p_i = p\left(\frac{\varepsilon_i}{\varepsilon_j} < z_{ij} \quad \forall \quad j \neq i \right) \ . \tag{7}$$

where „$\forall \quad j \neq i$ " stands for „for all alternatives j except i "

With the derivations in annex 1 we get:

$$p_i = \int p\left(\frac{\varepsilon_i}{z_{ij}} < \varepsilon_j \quad \forall \quad j \neq i \mid \varepsilon_i = x \right) f(x)\, dx \tag{8}$$

$$p_i = \int p\left(\frac{x}{z_{ij}} < \varepsilon_j \quad \forall \quad j \neq i \right) f(x)\, dx \tag{9}$$

$$p_i = \int f(x) \cdot \prod_{i \neq j} \left(1 - F\left(\frac{x}{z_{ij}} \right) \right) dx \tag{10}$$

where f(x) = statistical density function for the random variables ε_i and ε_j

F(x) = corresponding distribution function

Eq. 10 is accessible for analytical or numerical solutions if

- the type of distribution function F(x) (and f(x) = dF/dx) is given and if
- all the V_i for each of the alternatives i (included in z_{ij}) are known numerically.

3. WEIBULL DISTRIBUTED RANDOM UTILITY

Let us now assume as a specific type of the distribution function for the random utility term ε_i :

$$f_\varepsilon(x) = \alpha \cdot \beta \cdot x^{\alpha-1} \cdot e^{-\beta \cdot x^\alpha} \tag{11}$$

This is the density function of the Weibull-distribution (cf. Plate, 1993). The form of the distribution function F(x) then is:

$$F_\varepsilon(x) = 1 - e^{-\beta \cdot x^\alpha} \tag{12}$$

The function is only defined for x > 0 . The shape of this distribution can obtain various forms (cf. fig. 2 and 3). For $\alpha = 1$ the Weibull distribution coincides with the well known exponential function whereas for larger α we get skewed functions with shapes comparable to Erlang or Log-normal or other distributions. Thus the function can figure out quite differently shaped distributions. One advantage is that for large α , the distribution is rather concentrated on an area along the x-axis in a close vicinity around 1 (cf. fig. 3). The Rayleigh distribution which has been used in a former approach to the problem (Brilon, Bondzio, 1996) is just a special case (for $\alpha = 2$ and $\beta = \pi/4$) of this more general distribution.

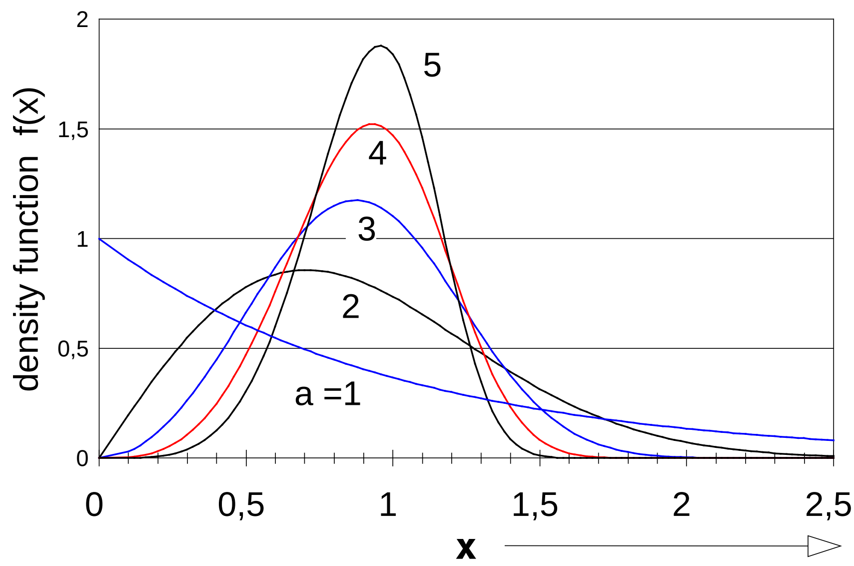


Fig. 2: Distribution density function (eq. 11) for the Weibull-distribution for different parameters a = α . In this example β is always = 1 .

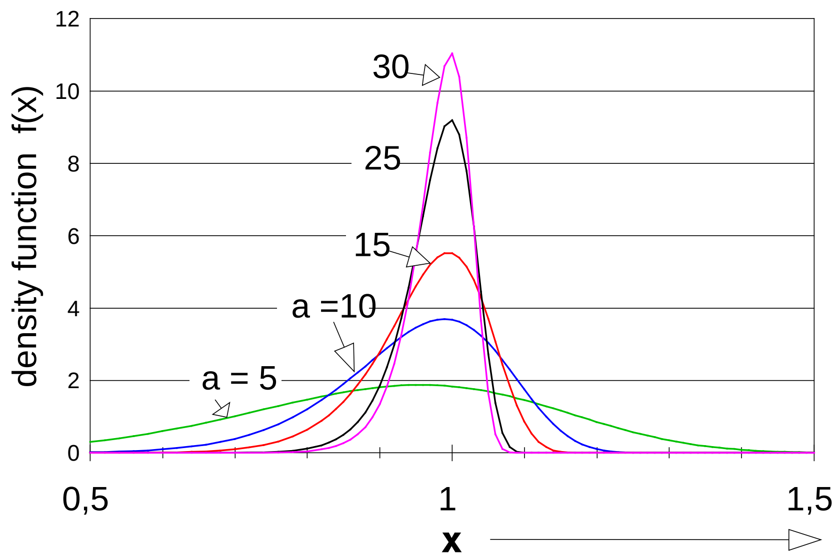


Fig. 3: Distribution density function (eq. 11) for the Weibull-distribution for large parameters a = α . In this example β has been adjusted such that E(ε) = 1.

The parameters of this function are :

$$E(X) = \frac{1}{\beta^{\frac{1}{\alpha}}} \cdot \Gamma\left(\frac{1}{\alpha}+1\right)$$
$$\mathrm{var}(X) = \frac{1}{\beta^{\frac{2}{\alpha}}} \cdot \left[\Gamma\left(\frac{2}{\alpha}+1\right) - \Gamma^2\left(\frac{1}{\alpha}+1\right)\right] \tag{13}$$

where Γ is the Gamma-function.

With E (X) = 1 we get

$$\beta = \Gamma\left(\frac{1}{\alpha}+1\right)^{\alpha} \tag{13a}$$

From eq. 10 using the Weibull distribution for f(x) and F(x) we get

$$p_i = \alpha \cdot \beta \cdot \int_0^{\infty} x^{\alpha-1} \cdot e^{-\beta \cdot x^{\alpha}} \cdot \prod_{i \neq j} \left(e^{-\beta\left(\frac{x}{z_{ij}}\right)^{\alpha}} \right) dx \tag{14}$$

Combining all the exponential terms in eq. 14 into one exponent and with $z_{ii} = 1$ we get

$$p_i = \alpha \cdot \beta \cdot \int_0^{\infty} x^{\alpha-1} \cdot e^{-\beta \cdot \sum_{j=1}^{J} \frac{1}{z_{ij}^{\alpha}} \cdot x^{\alpha}} \, dx$$
$$p_i = \alpha \cdot \beta \cdot \int_0^{\infty} x^{\alpha-1} \cdot e^{-\gamma \cdot x^{\alpha}} \, dx \tag{15}$$

where $\gamma = \beta \cdot \sum_{j=1}^{J} \frac{1}{z_{ij}^{\alpha}}$

Solving the integral in eq. 15 we obtain:

$$p_i = \left\{ \sum_{j=1}^{J} \left(\frac{V_i}{V_j} \right)^{\alpha} \right\}^{-1} \tag{16}$$

Here the parameter β of the model is not of importance whereas the parameter α introduces another degree of freedom into the model structure. This formula allows an easy calculation of the probability (p_i) that alternative i is selected by the individual under consideration, if the deterministic utilities V_i are known for each alternative i , which is an important advantage similar to the Logit model. Moreover, this formula provides better flexibility compared to Logit due to the additional parameter α . α is a representation for the variance of the random component ε_i of the utility.

We finally note that in the case of only J = 2 alternatives we get

$$p_1 = \frac{V_2^{\alpha}}{V_1^{\alpha} + V_2^{\alpha}}$$
$$p_2 = \frac{V_1^{\alpha}}{V_1^{\alpha} + V_2^{\alpha}} = 1 - p_1 \tag{17}$$

4. RELATIONS BETWEEN THE MULTIPLICATIVE APPROACH AND LOGIT

It is worthwhile to mention, that our approach also contains the classical Logit model (cf. Ben Akiva, Lerman, 1987). But it allows more freedom with respect to the distributional assumptions for the random component. To be precise, define

$$W_{it} = \ln(-V_{it})$$

$$\eta_{it} = \ln(-\varepsilon_{it})$$

$$T_{it} = \ln(-U_{it})$$

Then the multiplicative model (eq. 1) can be rewritten as an addititive model

$$T_{it} = W_{it} + \eta_{it} \tag{18}$$

If ε_{it} is Weibull-distributed (eq. 11) then the random variable η_{it} is distributed according to a Gumbel distribution with density

$$f(t) = \alpha \cdot \beta \cdot e^{\alpha \cdot t} \cdot e^{-\beta \cdot e^{\alpha \cdot t}} \tag{19}$$

and distribution function

$$F(t) = 1 - \cdot e^{-\beta \cdot e^{\alpha \cdot t}} \tag{20}$$

The decission process in eq. 3 to 6 is equivalent to the well known derivation of the Logit model. Here the probability p_i that alternative i is selected is given by

$$p_i = p(W_{it} + \eta_{it} < W_{jt} + \eta_{jt} \quad \forall \ j \neq i)$$

$$p_i = p\left(\ln \frac{\varepsilon_{it}}{\varepsilon_{jt}} < \ln \frac{W_{it}}{W_{jt}} \ \forall \ j \neq i \right)$$

$$p_i = \left(\sum_{J=1}^{J} \left[\frac{V_{it}}{V_{jt}} \right]^{\alpha} \right)^{-1}$$

$$p_i = \frac{e^{-\alpha \cdot W_{it}}}{\sum_{j=1}^{J} e^{-\alpha \cdot W_{jt}}} \tag{21}$$

Thus the model can be transformed into the same form as the Logit model. However, there are important differences between the method proposed in this paper and the Logit approach:

(1) Note that the linear representation of the utility

$$W_{it} = B_0 + \sum_{k=1}^{K} B_k \cdot X_{itk}$$

shows that the parameter α in eq. 21 can not be included into the Logit model and can be assumed as $\alpha = 1$ without loss of generality. On the other hand in the Rubit model the parameter α in eq. 16 is identifiable because of the non-linearity of the transformation $x \to x^{\alpha}$. Therefore the Rubit concept introduces an additional parameter, which allows for further flexibility and for a better fit of the model to real world data.

(2) Secondly, the introduction of the additional parameter β for the Weibull distribution (eq. 11) allows an easy adjustment to the condition $E[\varepsilon_{it}] = 1$ in the Rubit

model without loosing any degree of freedom. Note that in principle this is also possible in the Logit model, by solving

$$E(\eta_{it}) = \frac{\ln \beta + \gamma}{\alpha} = 0 \tag{22}$$

where γ denotes Euler's constant, i.e. $\beta = \varepsilon^{-\gamma}$. However, α is not identifiable and therefore the adjustment according to eq. 22 eliminates all degrees of freedom in the distributional assumptions for the random component.

5. LOG-NORMAL DISTRIBUTED RANDOM UTILITY

The Weibull distribution must not be the only assumed function for the distribution of the ε_i . A set of other function has been tested (cf. Brilon, Bondzio, 1996). Here the Raleigh distribution turns out as a special case of the Weibull function. Also the Gamma function would provide possibilities to further develop the model.

However, as the only solution which is worth to be mentioned besides the Weibull solution the log-normal distribution (Aitchison, Brown, 1957) can be considered for the distribution of the ε_i. Then the distribution function f(x) has to be written as (cf. Plate, 1993):

$$f_\varepsilon(x) = \frac{1}{x \cdot \sigma_y \cdot \sqrt{2\pi}} \cdot e^{-\frac{(\ln x - \mu_y)^2}{2 \cdot \sigma_y^2}} \qquad \text{for } x > 0 \tag{23}$$

where

y = ln x : logarithmic transformation of the original scale x for the ε_{it}

$$\mu_y = \frac{1}{2} \ln\left(\frac{\mu_x^2}{1 + C_x^2}\right) = E(\ln X) \tag{24}$$

= expectation of the transformed variable

$$\sigma_y^2 = \ln(C_x^2 + 1) = \mathrm{var}(\ln X) \tag{25}$$

= variance of the transformed variable

$$C_x = \frac{\sigma_x}{\mu_x}$$

$$\mu_x = e^{\left(\mu_y + \frac{\sigma_y^2}{2}\right)} \tag{26}$$

= expectation of the original variable ε_{it} on the x-scale

$$\sigma_x^2 = \mu_x^2 \cdot (e^{\sigma_y^2} - 1) \tag{27}$$

= variance of the original variable ε_{it} on the x-scale

It should be noted that for these derivations the variance σ_x^2 is assumed to be identical for each alternative i . Then the cumulative distribution function of the ε_i is

$$F_\varepsilon(x) = \int_0^x f_\varepsilon(u)\, du \tag{28}$$

As we have shown in connection to eq. 1 and 2 the expectation of the ε_i should equal 1 ; i.e.: we assume that on average the individuals can estimate their utility correctly. As a consequence from this consideration we get $\mu_X = 1$ and further on:

$$\mu_y = \frac{1}{2}\ln\left(\frac{1}{1+\sigma_x^2}\right) \tag{29}$$

$$\sigma_y^2 = \ln(\sigma_x^2 + 1) \tag{30}$$

where μ_y always is negative.

With the Log-normal distribution we can only treat the two-dimensional case; i.e. that the number J of alternatives is 2 . We now look at the variable Q (cf. eq. 6) :

$$Q = \frac{\varepsilon_{1t}}{\varepsilon_{2t}}$$

Taking the logarithm of Q we obtain: $\ln Q = \ln(\varepsilon_1) - \ln(\varepsilon_2)$. Since $\ln(\varepsilon_1)$ and $\ln(\varepsilon_2)$ are both assumed to be independent normal distributed variables with their respective parameters $(\mu_{y,1}, \sigma_{y,1})$ and $(\mu_{y,2}, \sigma_{y,2})$ the new variable $\ln(Q)$ is also normal distributed with parameters

$$\left.\begin{aligned} \mu_z &= \mu_{y,1} - \mu_{y,2} \\ \sigma_z^2 &= \sigma^2_{y,1} + \sigma^2_{y,2} \end{aligned}\right\} \tag{31}$$

This means: Also Q is a log-normal distributed variable with the distribution function

$$f(x) = \frac{1}{x \cdot \sigma_z \sqrt{2\pi}} \cdot e^{-\frac{(\ln x - \mu_z)^2}{2\sigma_z^2}} \tag{32}$$

For the case that both ε_1 and ε_2 have the same variance σ_X the parameters of the distribution for Q can be written using eq. 29 and 30 as

$$\left.\begin{aligned} \mu_Z &= 0 \\ \sigma_Z^2 &= 2\ln(\sigma_x^2 + 1) \end{aligned}\right\} \tag{33}$$

For abbreviation we now use the parameter

$$R^2 = \sigma_Z^2 = 2\ln\left(\sigma_x^2 + 1\right) \tag{34}$$

as the new parameter of the model. Thus the variance of the ε_i is expressed by R using the equation

$$\sigma_x^2 = e^{\frac{R^2}{2}} - 1 \tag{35}$$

Eq. 32 then can be written as

$$f_Z(x) = \frac{1}{x \cdot R\sqrt{2\pi}} \cdot e^{-\frac{(\ln x)^2}{2R^2}} \tag{36}$$

Using as another transformation

$$Y = \frac{Q}{R} \qquad \text{or} \qquad Q = R \cdot Y \tag{37}$$

we get a N(0,1) - distribution for the variable Y . For practical application we can obtain solutions for the cumulative distribution $F_Y(x)$ from each tabulation of the N(0,1) - distribution which is available from statistical textbooks.

At this point we should once again state (cf. eq. 6):

$$p_1 = \Phi\left(\frac{1}{R}\ln\left(\frac{V_{2t}}{V_{1t}}\right)\right) \tag{38}$$

where

$\Phi(x)$ = value of the cumulative distribution function of the N(0,1) - distribution at point x

R = parameter of the model which is related via eq. 34 and 35 to the variance σ_X which the individuals apply to recognise their utility

This means: the application of the Rubit-model for the log-normal case becomes quite easy. The only equation which has to be used for practical application is eq. 38 where R is treated as the parameter of the model. Once again it should be noted that the Log-normal distribution allows only the treatment of the two-dimensional case. Other derivations by Brilon, Bondzio (1995) were based on an insufficient assumption of statistical independence.

6. ESTIMATION OF PARAMETERS IN THE MODEL

The parameters of the model B_k (cf. eq. 2) and α (for the Weibull approach) or R (for the Log-normal approach) have to be evaluated based on empirical data. Let us assume that we have information from a sufficient sample of travellers about their travel behaviour. Let the sample size be T individuals. For each of these individuals we know

- the true value for each of the attributes which we consider, e.g. travel time, costs, length of the distance to be covered, availability of modes. Also socio-economic parameters of the person itself could be used as attributes, e.g. income, size of the household, access to specific travel modes. These attributes are known for every individual and here for each of the possible alternatives.
- the type of decision which has been made by the individual (cf. y_{it} below).

Then the likelihood function can be written as

$$L^*(parameters) = \prod_{t=1}^{T}\left[\sum_{j=1}^{J}\left(p_{jt}\cdot y_{jt}\right)\right] \tag{39}$$

with:

j = index for the J alternatives

t = index for the T individuals in the sample

$$y_{i,t} = \begin{cases} 1 & \text{if individual } t \text{ selects alternative } i \\ 0 & \text{elsewhere} \end{cases}$$

T = no. of individuals in the sample

For y_{it} we introduce the known values obtained from the sample. For the p_{ij} the formulas of the model (eq. 17 or 38) are introduced. They contain the deterministic utilities V_{it} (eq. 2) calculated from the given attributes of travellers and their possible alternatives observed in the sample. Then L* is only a function of the parameters in the model.

We now have a look on the logarithmic transformation of eq. 39:

$$L(parameters) = \ln\left[L^*(parameters)\right] \tag{40}$$

$$L(parameters) = \sum_{t=1}^{T} \ln\left[\sum_{j=1}^{J} p_{jt} * y_{jt}\right] \tag{41}$$

For the two-dimensional case this equation can be written as

$$L(parameters) = \sum_{t=1}^{T} \ln\left[p_{1,t}\left(y_{1,t} - y_{2,t}\right) + y_{2,t}\right] \tag{42}$$

The function L^* has to be maximized to obtain a maximum likelihood estimation for the parameters B_0, B_1, ... , B_K of the model plus the parameters:

- α for the Weibull-approach
- R for the log-normal approach.

L has its maximum at the same parameter values as L^*. Due to numerical reasons it is easier to maximise L (eq. 41 or 42). An analytical solution for the maximum by determining the derivatives of L towards the parameters seems to be rather complicated. A solution of the maximisation problem for L is, however, possible using maximisation facilities in spreadsheet programs such as Quattro-Pro or Excel. Of course, some more mathematical derivations might be useful to prove the existence of the maximum. However, using a spreadsheet, solutions for the maximum of L could always be found for the examples so far.

7. IIA - PROPERTIES OF THE MODEL

The model in the form of eq. 16 has also the IIA property (IIA = Irrelevance of independent alternatives) like the Logit model which can be shown easily applying eq. 16 for J = 2 and J = 3. This is not a desired property of a random utility model. However, all the methods to overcome this problem might be used for the multiplicative model as they are applied for the Logit solution. Among these we should first mention the nesting of alternatives. This might be studied in a further phase of the model's development.

8. APPLICATION

As a realistic example we use the modelling of the selection process of air passengers who have to decide which from two accessible airports they use for starting their trip. Bertram (1995) has performed a survey by interviewing air passengers waiting in the departure lounge of two smaller airports in Germany. These are Muenster/Osnabrueck (MOS) and Dortmund (DTM) . For each passenger a set of parameters were evaluated from the interviews. Our evaluations for this example are concentrated on two attributes:

1. the travel times of the travellers from the origin of their trips to both of the two airports (in minutes of car travel time; the proportion of travellers using public transport accessing the airport is negligible) and
2. the availability of flight connections to the desired destination of the travellers. A linear model for the deterministic utility has been used:

$$V_{it} = B_0 + B_1 \cdot T_{it} + B_2 \cdot a_{id} \tag{43}$$

where

T_{it} = travel time of traveller t to airport i (in minutes of car travel time)

a_{id} = availability of flights from airport i to the desired destination d (in number of flights per normal workday; max. value in the sample = 5). The evaluation only took into account those destinations d which were provided by both airports with at least 1 connection per day.

B_0 , B_1 and B_2 = parameters of the model

The parameters of the model obtained by a maximum likelihood estimation both for a conventional Logit-model and the multiplicative Rubit-model are given in table 1. Also the frequencies of differences between observed behaviour and model estimations are indicated. Applying the different models it turned out by t-tests that in the sample the frequency of flights was not significant for the results. This may be caused by the fact that - if a destination was provided by both airports - nearly the same number of flights were scheduled at both of the airports. Basis is a sample size of 521 interviewed persons. From table 1 we can obtain that for each of the models the number of wrong estimations was around 5 % . These 24 cases where the model did not explain real behaviour were identical for each of the models.

	Logit	Rubit Weibull	Rubit Log-normal
B_0	0.092	-49	-1.865
B_1	-0.1276	-0.774	-0.0314
B_2	not significant		
parameter of the model		$\alpha = 14$	R = 0.1360
no. of wrong estimations	24	24	24

Table 1: Parameters of the binary choice models for the example

From the rather large constant parameter B_0 we can obtain that in the Weibull-based Rubit-model the attributes are not rather important for the selection probabilities developed from the model.

The model concept has also been applied on a sample for modal split behaviour of students at the Ruhr-University. Zöllner (1997) has investigated this behaviour based on a sample of 122 interviewed students and employees. For the example we like to focus on modelling modal split. Our evaluations for this example are concentrated on the attributes:

1. the travel times
2. estimated travel costs

A linear model for the deterministic utility has been used:

$$V_{it} = B_0 + B_1 \cdot T_{it} + B_2 \cdot C_{it} \tag{44}$$

where

T_{it} = travel time of traveller t from the origin of his trip to the university using mode i (in minutes)

C_{it} = the estimated travel costs using mode i in DM (assumed to be 0 for bike and walking) .

B_0 , B_1 and B_2 = parameters of the model

Modes: 1 = car 2 = public transit 3 = motor bike

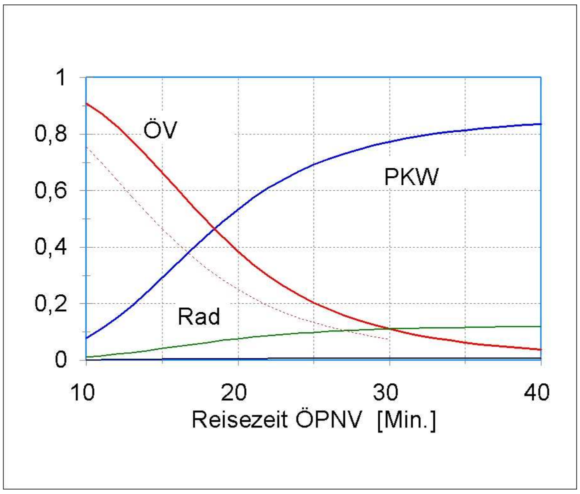
4 = bicycle 3 = walking

The parameters of the model obtained by a maximum likelihood estimation for the multiplicative Rubit-model are given in table 2.

	Rubit Weibull
B_0	0,006466
B_1	-3,83
B_2	-4,37
parameter of the model	$\alpha = 4$

Table 2: Parameters of the modal split model for the example

As an example for the application of the example fig. 4 illustrates the consequences of increased travel times in public transit on the modal split. Here we look on the case that the car travel time is 15 minutes and the cycle travel time is 30 minutes. Travel costs for car usage is treated as a constant value of 3,oo German marks (DM) whereas the transit costs are 0, since meanwhile each student has a ticket included into his university inscription fees, which was not the case at the time of the data collection. We see that with increasing transit travel time the acceptance of transit is reduced towards increased car usage. The dotted line applies for the case that also transit passengers have to pay the price for a single ticket (3,oo DM). The example makes it clear that the model is able to produce useful results.



ÖV = public transit (= ÖPNV)
Rad = cycle
PKW = passenger car
Reisezeit = travel time [minutes]

Fig. 4: Example for the use of the model according to eq. 44.

9. CONCLUSION

This paper has shown that it can be useful to apply a multiplicative term ε_{it} as the component of randomness in a random utility model. We call this the Rubit concept. There is some evidence that this basic concept is more meaningful then other more conventional model outlines. Like in the traditional case where ε_{it} is added to the deterministic utility (i.e. Logit or Probit) also here different assumptions for the distribution of the random variable ε_{it} can be introduced.

Here the Weibull distribution has been found to provide the easiest solution for the estimation of selection probabilities. These probabilities can be obtained from a rather simple equation (eq. 16). Also the multinominal case can be treated without any complication. This model concept is rather closely related to the traditional Logit model. It provides, however, an improved flexibility.

Another solution is provided by using the log-normal distribution for the random utility term ε_{it}. Then still an acceptable applicability is reached by referring the solution to the standardised normal distribution Φ. This case has so far only been solved for the two-dimensional case of binary choice.

Also the parameter estimation from given samples based on a maximum likelihood technique can be solved. Here up to now only an estimation procedure which can be run on a spreadsheet has been proposed. Examples for the application of the model concept in transportation planning are given. They showed the usefulness of the concept.

Of course, some further investigations on the model concept would be useful if the basic outline is acceptable for further application. Among the possible unsolved questions the following items should be studied in further detail:

- development of iterative numerical procedures for the computation of maximum likelihood estimators,
- study of the model performance using more data provided by transportation studies from the real world in comparison with other models,
- expanding the model by nesting alternatives for the multinominal case.

Nevertheless, also the version and procedures of the model being presented here provide direct applicability in practice.

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Appendix I

Equivalence between (7) and (8). Without loss of generality assume $i = 1$. Because $\varepsilon_1, \dots, \varepsilon_g$ are independent, the conditional density of $\varepsilon_1, \dots, \varepsilon_g$ given $\varepsilon_1 = x$ is given by

$$g(x_2, \dots, x_y \mid x) = \prod_{j=2}^{y} f(x_j)$$

while the joint density is

$$g(x_1, \dots, x_y) = \prod_{j=1}^{y} f(x_j) .$$

Let $I\{A\}$ denote the function which is equal to 1 if A is satisfied and 0 otherwise, then

$$P_1 = \int P\left(\frac{\varepsilon_i}{z_{1j}} < \varepsilon_j \quad \forall \quad j \neq 1 \mid \varepsilon_1 = x_1\right) f(x_1)\, dx_1$$

$$= \int \dots \int I\left\{\frac{x_1}{z_{1j}} < x_j \ \forall\ j \neq 1\right\} g(x_2; \dots; x_y \mid x_1)\, f(x_1)\, dx_1 \dots dx_y$$

$$= \int \dots \int I\left\{\frac{x_1}{z_{1j}} < x_j \ \forall\ j \neq 1\right\} \prod_{j=1}^{y} f(x_j)\ dx_1 \dots dx_y$$

$$= \int \dots \int I\left\{\frac{x_1}{z_{1j}} < x_j \ \forall\ j \neq 1\right\} g(x_1; \dots; x_y)\ dx_1 \dots dx_y$$

$$= P\left(\frac{\varepsilon_1}{z_{1j}} < \varepsilon_j \ \forall\ j \neq 1\right)$$