ROBUST TESTS ON FRACTIONAL COINTEGRATION[1]

by

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Abstract

Cointegration describes the pattern that pairs of time series keep together in long run, although they diverge in short run. A generalisation of this behaviour is the fractional cointegration. Two statistical tests, the M– and ML–test are formulated for fractional cointegration in different situations. It turns out that the robust M–test reaches almost the same power as the maximum likelihood test under certain assumptions. In contrast to this, the power of the M–test is much higher than that of the ML–test if the examined time series is contaminated following the general replacement model.

KEY WORDS:

Fractional Cointegration, Maximum Likelihood Estimation, Robustness, Long Memory

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1 Introduction

Cointegration, defined by [5] and [3], describes the pattern that pairs of time series do not diverge from each other in the long run, although each time series can wander extensively. The economic theory proposes forces which tend to keep such time series together. Examples might be capital appropriations and expenditures or household income and expenditures. A generalised notion of cointegration, called fractional cointegration is examined. Some tests on (fractional) cointegration have been developed. [13] employ a t–test based on [4] estimates of the long memory parameter of the regression residuals. [7] summarises different semi–parametric and nonparametric tests on long–memory, e.g. tests based on the trimmed Whittle Likelihood (see [11]) or based on Robinson's estimator, see [10]. Several other estimators for long–memory and tests like the R/S test, or a Maximum Likelihood estimator (proposed by [2]) are described in [12]. Most of these tests have the disadvantage that they are sensitive against outliers. A well known example of a situation in which outliers occur are crashes at the stock market. A Maximum Likelihood estimation of the memory parameter of such a time series would be forced to be close to $\frac{1}{2}$. Thus a test statistic created out of such an estimation would have a poor power. Hence it follows that a robust estimator against outliers is required. In this paper a robust and a non robust test for parametric stationary Gaussian models based on the finite Maximum Likelihood estimator and the M–estimator proposed by [1] are developed and their power properties are compared by using Monte Carlo. The hypothesis of classical cointegration vs. fractional cointegration is examined.

The paper is organised as follows. In the next section the setting of fractional cointegration is introduced. Section 3 formulates the considered test statistics, section 4 describes the used algorithm to compute them and its difficulties. The simulational results are provided in Section 5. We conclude in Section 6. The appendix contains the tables.

2 Fractional Cointegration

Even if one single economic variable regarded as a time series can wander extensively, there exist pairs of time series which do not diverge from each other in the long run. This phenomena for example occurs by considering the logarithm of daily returns of BASF and Bayer stocks, see [9]. This behaviour of time series is called cointegration.

To reach cointegration each time series has to be integrated with a certain memory parameter d. [3] defined an integrated time series of order d, denoted $x_t \sim I(d)$ as follows:

$$x_t \sim I(d) \iff (1-B)^d x_t = \epsilon_t, \tag{1}$$

where ϵ_t is white noise. Thus an integrated time series of order $d = 0$ is stationary. Thus two time series, which do not diverge in the long run have a "difference" which is $I(0)$. More generally [5] defined cointegration of two time series x_t and y_t of order d, b (denoted $(x_t, y_t)^\top \sim CI(d,b)$) as the behaviour that

$$(x_t, y_t)^\top \sim CI(d,b) \iff \begin{cases} x_t, y_t & \sim \quad I(d) \quad \wedge \\ \exists\, \alpha \neq 0 & : \quad z_t := x_t - \alpha y_t \sim I(d-b), \end{cases} \tag{2}$$

where $b > 0$. The parameter α is called the cointegration parameter.

Furthermore two cointegrated time series with $0 < b < 1$ are called fractional cointegrated and two cointegrated time series with $b = 1$ are called classical cointegrated.

In the following paper tests on cointegration are considered and their properties in power are examined. The hypothesis of classical cointegration vs. fractional cointegration is examined, i.e.

$$H_0 \quad : \quad z_t = x_t - \alpha y_t \sim I(0) \tag{3}$$

$$vs.$$

$$H_1 \quad : \quad z_t = x_t - \alpha y_t \sim I(d), \text{where}\, d \in (0,1). \tag{4}$$

3 M- and ML-Test

The examined tests are based on the autoregressive representation of a fractional ARIMA(p, d, q)-process. [1] proposed a maximum likelihood estimator and a class of M-estimators for estimating the memory parameter and the AR- and MA-parameters simultaneously. These estimators are used to create corresponding test statistics.

Let x_t be a Gaussian long-memory process with finite variance σ^2 and mean μ. Following [8] it has the one-sided autoregressive representation

$$\sum_{j=0}^{\infty} \psi_j(\eta) x_{t-j} = \varepsilon_t(\eta). \tag{5}$$

The spectral density is assumed to be characterised by the true but unknown parameter-vector $\theta^0 = (\theta_1^0, \ldots, \theta_m^0)$ where θ_1^0 is the scale parameter, θ_2^0 is the memory-parameter and $\theta_3^0, \ldots, \theta_m^0$ describe the short-run behaviour of the process. In the following denote $\theta = (\theta_1, \eta)$, where $\eta = (\theta_1, \ldots, \theta_m)$. Define

$$\begin{array}{rclrcl} e_t(\eta) & := & \sum_{j=0}^{t-1} \psi_j(\eta) x_{t-j}, & e_t'(\eta) & := & \left(\frac{\partial}{\partial \eta_1} e_t(\eta), \ldots, \frac{\partial}{\partial \eta_{m-1}} e_t(\eta)\right)^\top \\ \text{and} & & & & & \\ r_t(\theta) & := & \frac{e_t(\eta)}{\sqrt{\theta_1}}, & r_t'(\theta) & := & \left(\frac{\partial}{\partial \theta_1} r_t(\theta), \ldots, \frac{\partial}{\partial \theta_m} r_t(\theta)\right)^\top \end{array} \tag{6}$$

as estimations of

$$\begin{array}{rclrcl} \varepsilon_t(\eta) & := & \sum_{j=0}^{\infty} \psi_j(\eta) x_{t-j}, & \varepsilon_t'(\eta) & := & \left(\frac{\partial}{\partial \eta_1} \varepsilon_t(\eta), \ldots, \frac{\partial}{\partial \eta_{m-1}} \varepsilon_t(\eta)\right)^\top \\ \text{and} & & & & & \\ \nu_t(\theta) & := & \frac{\varepsilon_t(\eta)}{\sqrt{\theta_1}}, & \nu_t'(\theta) & := & \left(\frac{\partial}{\partial \theta_1} \nu_t(\theta), \ldots, \frac{\partial}{\partial \theta_m} \nu_t(\theta)\right)^\top . \end{array} \tag{7}$$

A class of M-estimators is proposed by solving the following equation:

$$\sum_{t=2}^{n} \Upsilon(r_t(\theta), r_t'(\theta)) = 0. \tag{8}$$

The weight function Υ is a function with a unique zero at the origin. In this paper we consider Υ as proposed by [2]. That is:

$$\Upsilon(r_t(\theta), r_t'(\theta)) = (\Upsilon_1(r_t(\theta), r_t'(\theta)), \ldots, \Upsilon_m(r_t(\theta), r_t'(\theta))^\top,$$

where

$$\Upsilon_i(r_t(\theta), r_t'(\theta)) := \Upsilon_{i1}(r_t\theta)\Upsilon_{i2}(r_t'(\theta_i)), \text{ with } i = 1, \ldots, m.$$

Because of $e_t(\eta)/\sqrt{\theta_1} = r_t(\theta)$ the scale parameter θ_1 can be estimated separately, for more details see [1]. Furthermore [1] proposed a robust M-estimator with weight function

$$\Upsilon_{11}(r_t(\theta)) = r_t(\theta)^2(\gamma^2 - r_t(\theta)^2)1_{[-\gamma,\gamma]}(r_t(\theta)) - c(\gamma), \qquad \Upsilon_{12}(r_t'(\theta)) = 1, \tag{9}$$

where $\gamma > 0$ and $c(\gamma)$ is such that $E[\Upsilon_{11}(Z)] = 0$ for a standard normal variable Z. For $i > 1$:

$$\Upsilon_{i1}(r_t(\theta)) = u(r_t(\theta); \alpha_1, \beta_1), \qquad \Upsilon_{i2}(r_t'(\theta)) = u(r_t'(\theta); \alpha_2, \beta_2). \tag{10}$$

Furthermore for $r_t(\theta) \geq 0$ let:

$$u(r_t(\theta); \alpha, \beta) = x1_{(-\infty,\alpha)}(r_t(\theta)) + \alpha\left(1 - \frac{r_t(\theta) - \alpha}{\beta - \alpha}\right)1_{(\alpha,\beta]}(r_t(\theta)), \tag{11}$$

with $0 < \alpha \leq \beta$. For $r_t(\theta) < 0$:

$$u(r_t(\theta); \alpha, \beta) = -u(-r_t(\theta); \alpha, \beta)\,. \tag{12}$$

This weight function Υ is re-descending, thus the described class of M-estimators has a breakdown point of 1/2.
The Maximum Likelihood estimator can be treated as a special case of M-estimators, defined by choosing

$$\begin{aligned} \Upsilon_{11}(r_t(\theta)) &= r_t^2(\theta) - 1, & \Upsilon_{12}(r_t'(\theta)) &= 1, \\ \Upsilon_{i1}(r_t(\theta)) &= r_t(\theta), & \Upsilon_{i2}(r_t'(\theta)) &= r_t'(\theta) \end{aligned} \tag{13}$$

for ($i > 1$), for more details see [1]. Combining (13) and (8), the ML-estimator can be written as a solution of

$$\theta_1 \;=\; \tfrac{1}{(n-1)}\textstyle\sum_{t=2}^n e_t^2(\eta) \quad \text{and } 0 \;=\; \textstyle\sum_{t=2}^n e_t(\eta)e_t'(\eta)\,. \tag{14}$$

This class of M-estimators is consistent and asymptotic normal with variance $n^{-1}W$, where $W = B^{-1}A(B^{\top})^{-1}$. A and B are defined by

$$\begin{array}{rclcl} A & = & E[\xi_t(\theta^0)\xi_t^{\top}(\theta^0)] & = & E[\xi_1(\theta^0)\xi_1^{\top}(\theta^0)] \\ B & = & E[\xi_t'(\theta^0)] & = & E[\xi_1'(\theta^0)], \end{array} \tag{15}$$

and $\xi_t(\theta) = \Upsilon(\nu_t(\theta), \nu_t'(\theta))$ and $\xi_t'(\theta) = \dfrac{\partial \xi_t'(\theta)}{\partial \theta}$.

In this paper only the ARFIMA(0, d, 0) model is examined. This model seems sensible because a fractional ARIMA(0, d, 0)-process has similar long-memory properties as an ARFIMA(p, d, q)-process, see [8]. The parameter under test is the memory parameter $d = H - 0.5$. The autoregressive representation of this model is given in (5). The coefficients there are defined by

$$\psi_k = \frac{\Gamma(k + \frac{1}{2} - H)}{\Gamma(k+1)\Gamma(\frac{1}{2} - H)}. \tag{16}$$

The spectral density is

$$f(\lambda) = \frac{\sigma_\varepsilon^2}{2\pi}\left(2\sin(\frac{\lambda}{2})\right)^{-2H+1}. \tag{17}$$

Hence it follows that the spectral density is characterised by $\theta = (\theta_1, H)$ where $\theta_1 = \sigma_\varepsilon^2$. Based on these facts the ML-test statistics is derived. Following the results of [1] the asymptotic variance of the ML-estimator is $6/(n\sigma_\varepsilon^2)$. Therefore the test statistic is

$$T_{ML} = \frac{\hat{H}_{ML} - H}{\sqrt{6/(n\pi^2)}}. \tag{18}$$

The test statistic (18) is asymptotically standard normal distributed. The variance of the robust M-estimator given a fractional ARIMA(0, d, 0)-process and $\alpha_1 = \alpha_2 = \beta_1/2 = \beta_2/2$ is:

$$var(\hat{H}_M) \quad = \quad \frac{1}{n}\frac{6}{\pi^2}s(\alpha) + o\left(\frac{1}{n}\right), \tag{19}$$

where

$$s(\alpha) = \frac{h(\alpha)h(\alpha\sqrt{6}/\pi)}{g(\alpha)g(\alpha\sqrt{6}/\pi)}$$

and

$$
\begin{aligned}
g(x) &= 4x\varphi(2x), 2(1+4x^2)\Phi(2x) - 8x^2\Phi(x) - 1 \\
h(x) &= 4\Phi(x) - 2\Phi(2x) - 1.
\end{aligned}
$$

For details see [1]. Hence it follows the asymptotically normal distributed M-test statistic is:

$$
T_M = \frac{\hat{H}_M - H^0}{\sqrt{(6/(n\pi^2))s(\alpha)}}. \tag{20}
$$

To compute the estimations for the memory parameter H an algorithm proposed by [1] has been used.

4 Algorithm and Difficulties

An algorithm proposed by [1] has been used to compute the estimates for the memory parameter H.

1. Calculation of $e_1(H), \ldots, e_n(H)$ where $H = k * (0.05)$ and the grid has to cover at least the true memory parameter H.

2. The derivatives $e_1'(H), \ldots, e_n'(H)$ are calculated by discrete differences with step size $\Delta H = 10^{-7}$.

3. Obtain $\hat{\theta}_1 = \hat{\theta}_1(H)$ from $e_1(H), \ldots, e_n(H)$ by an approximation of the solution of (9) in case of the M-test. For the ML-test the estimation of $\hat{\theta}_1$ is the empirical variance, see (14).

4. Set

$$
r_t(\hat{\theta}_1, H) = \frac{e_t(H)}{\sqrt{\hat{\theta}_1(H)}} \quad \text{and}\, r_t'(\hat{\theta}_1, H) = \frac{e_t'(H)}{\sqrt{\hat{\theta}_1(H)}},
$$

for $t = 1, \ldots, n$ and for the described grid of k.

5. Evaluate

$$\Upsilon(r_t(\hat{\theta}_1, H), r_t'(\hat{\theta}_1, H)) = \sum_{t=2}^{n} \Upsilon_{21}(r_t(\hat{\theta}_1, H))\Upsilon_{22}(r_t'(\hat{\theta}_1, H)).$$

6. Find the grid point H^* such that the signs of H^* and $H^* + 0.5$ are different. Define $\hat{H}$ as the point where a straight line between H^* and $H^* + 0.5$ intersects with zero.

In the calculation $k \in \mathbb{N}$ has to be chosen to such an extend that the grid covers the important area. In the executed simulation the biggest k was such that the grid covers a value which is 0.2 bigger than the true value. That means that if for example the known memory parameter H is 0.8, k should be as big that the area $(0.5, 1)$ is covered. Another difficulty is that $\psi_k(H)$ is not defined for $H = 0.5$ and $H = 1.5$. Therefore $\psi_k(H)$ is not calculated for these values. Hence it follows that estimations for H in time series with a true memory parameter of $H = 0.5$ or $H = 1.5$ can be less exact than estimations of the memory parameter in time series with another true H.

5 Simulational Results

In this chapter the behaviour of the power of the two tests is examined in different situations. At first standard normally distributed residuals are considered. Afterwards the behaviour of the tests is simulated under different contamination models.

5.1 Standard Normal Residuals

1000 standard normally distributed time series with length 1000 are simulated with a true memory parameter $d \in \{0, 0.1, \ldots, 1\}$. The rejection probability of the M- and ML-test is computed. The assumptions, described in chapter 3 are fulfilled, thus the test statistics are asymptotically normal. The given significance level is 5%. It turns out that the difference in power between the observed tests is quite small. The highest difference occurs at $d = 0.6$, the rejection probability of the M-test is 99%, the ML-test has a power of 100%. The simulated type I error of both tests is 5%. For details see table 1.

5.2 The General Replacement Model

In the next step of the simulation study the general replacement model is used. This model looks as follows:

$$Y_t = (1 - Z_t)X_t + Z_t W_t, \tag{22}$$

where the random variable X_t is here an ARFIMA(0, d, 0)-process, W_t is the contaminating process and Z_t is Bernoulli distributed with success probability p, i.e. $P(Z_t = 1) = p$. Define the contaminating process $W_t := cV_t$ where c is a constant and V_t is a t-distributed random variable with 2 degrees of freedom. Since the asymptotic distribution is not achieved here any more, empirical critical values are computed. Three degrees of contamination are considered, $c = 0, c = 10, c = 100$ (see tables 2 – 4).
In the case of no contamination ($c = 0$), the ML-test achieves a higher or equal power than the M-test. The differences between these two tests are small, similar to the case of the standard normal residuals. The highest difference occurs at $d = 0.1$ where the M–test has a rejection probability of 92% and the ML-test has one of 95%. For a memory parameter bigger than $d = 0.1$ there is no difference in power. This is what is expected by the theory, because under the ideal model the ML-test should be superior the robust test. For a higher contamination ($c = 10$), the advantage of the M-test becomes obvious. Simulational results are displayed in Figure 1. The power of the ML-test is lower than 90% for a true memory parameter of $d \in [0, 0.5)$, that means the power is poor in the stationary region. In contrast to this, the M-test achieves a power of 99% at a memory parameter of $d = 0.3$. The maximal difference of the power between these tests is achieved at a memory parameter of $d = 0.3$ with a value of 87%. Figure 7 shows the behaviour in power for the case of contaminated residuals with $c = 100$. In the stationary region, both tests have

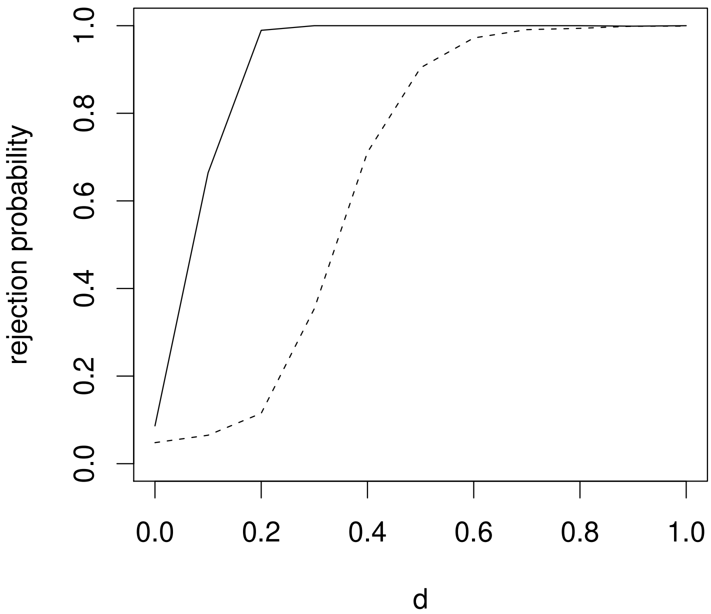

Figure 1: Power of the M- (solid) and ML-test (dashed), residuals following the general replacement model with $c = 10$

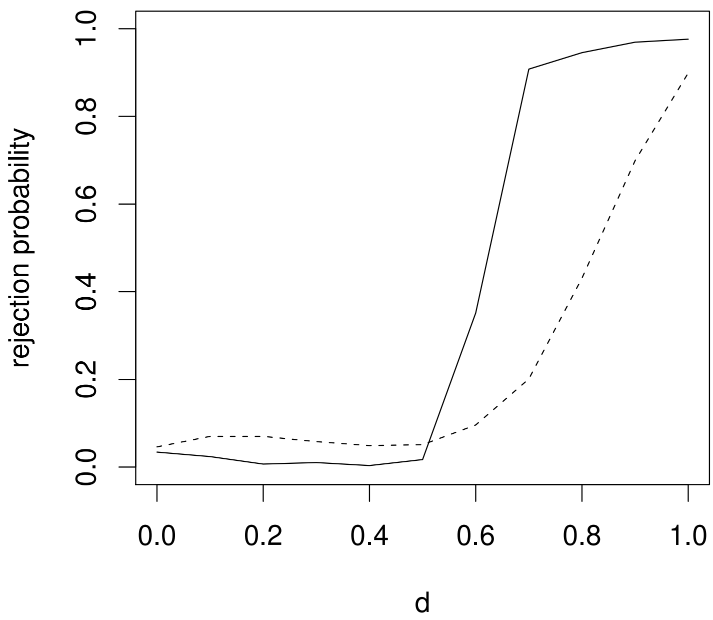

Figure 2: Power of the M- (solid) and ML-test (dashed), residuals following the general replacement model with $c = 100$

a very small power. The maximum of power for $d \in (0, 0.5)$ is achieved of the ML-test at $d = 0.2$ with a value of 7 %. The simulation results are given in tables 5 – 7. The result that the non-robust test achieves a higher power in the stationary region was not expected. For the non-stationary region, the increase of power of the M-test is much higher than the of the ML-test. All in all the M-test has a higher rejection probability in the non-stationary region.

5.3 Estimating the long-memory Parameter after doing linear Regression

The tests have been examined in the situation of estimated residuals. The Monte Carlo study is carried out in three steps:

1. Simulation of a cointegration system.
2. Estimation of the cointegration coefficient with linear regression.
3. Application of the two tests on the estimated residuals.

In the first step a cointegration system is needed. [13] formulates it as follows:

$$x_t + \beta y_t = u_{1t} \qquad u_{1t} = \varepsilon_{1t} + u_{1t-1} \tag{23}$$

$$x_t + \alpha y_t = u_{2t} \qquad \varepsilon_{2t} = (1 - B)^d u_{2t}\,, \tag{24}$$

where u_{2t} is integrated of order $d < 1$ and u_{1t} is a random walk, i.e. integrated of order 1. x_t and y_t can be written as linear combinations of u_{1t} and u_{2t}, hence it follows that $y_t, x_t \sim CI(1, 1)$. The parameters are set as $\alpha = 2$, $\beta = 1$ and $d \in [0, 1]$ in the Monte Carlo Study. In the second step a linear regression has to be carried out. In case of the ML-test the ordinary least-squares regression is chosen. This procedure seems not useful in the case of the robust M-test since the least-squares estimator is highly non-robust to outliers. Therefore instead of using least-squares estimation the regression parameter are estimated by a regression M-estimator which uses Tukey's biweight function. For details about robust regression estimates see for example [6]. Again the tests are analysed

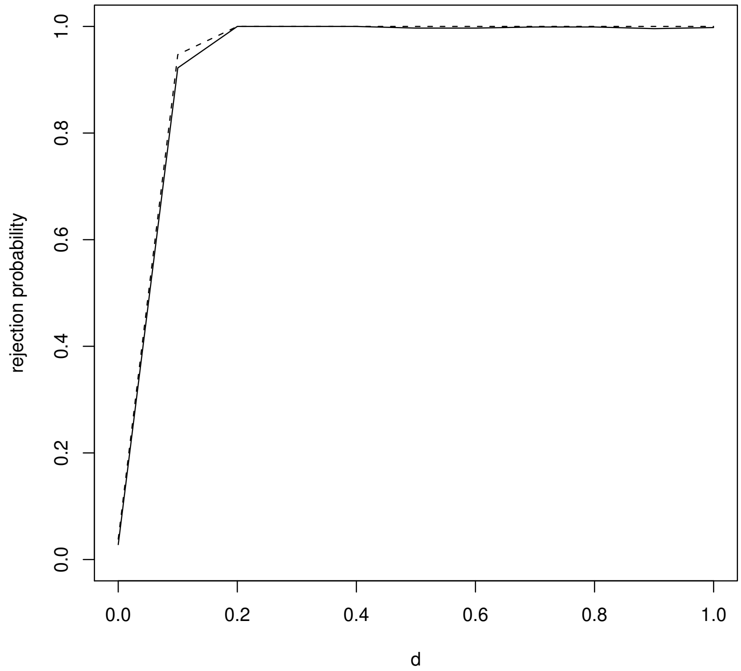

Figure 3: Power of the M- (solid) and ML-test (dashed) with standard normal distributed residuals after doing a linear regression

in two situations. First u_{2t} is assumed to be Gaussian distributed and in the second part of the simulation the general replacement model with $c = 10$ is used. Since the asymptotic distribution of the test statistics is not achieved after doing the linear regression, empirical critical values are computed for both situations (see tables 8, 9). In the first part of the simulations with estimated residuals, u_{2t} is Gaussian distributed.

Figure 10 displays the power of the two tests. Again the difference in power is very small. At $d = 0.2$ the ML-test has a higher rejection probability than the M-test. In case of the other memory parameters the power of the tests is almost equal (see table 10). Using the general replacement model with a degree of contamination of $c = 10$ the M-test applied on robust estimated residuals has a much higher rejection probability than the ML-test.

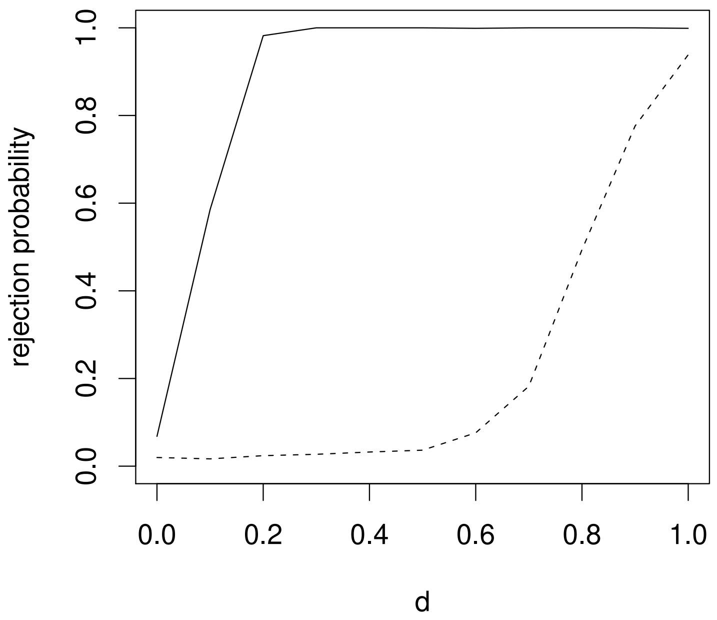

Figure 4: Power of the M- (solid) and ML-test (dashed) with contaminated residuals ($c = 10$) after linear regression.

The loss of power of the ML-test is very high in contrast to that of the M-test. The ML-test achieves a rejection probability greater than 90% only at a true memory parameter of $d = 1$, furthermore is the difference in power between M- and ML-test greater than 90% for $d \in [0.2, 0.7]$. For details see table 11.

6 Conclusion

Two tests on fractional cointegration namely the ML-test and the robust M-test are examined. It is expected that the asymptotically efficient maximum likelihood test achieves a higher power than the robust M-test under the ideal model. In contrast to this it is expected that the M-test behaves robust against contaminated residuals following the general replacement model. Simulational results show that the differences in power between these two tests is negligible under the ideal model. The loss of power by using the general replacement model to simulate the residuals is very high in case of the ML-test and much smaller in case of the M-test thus the robust test performs much better. An unexpected simulational result is achieved by regarding the power of the M- and ML-test using a high degree of contamination. In the stationary region, the ML-test has a higher power than the M-test. This relation turns round in the non-stationary region.
Advantages and disadvantages of the M-test become more obvious when doing a linear regression before applying the tests in the estimated residuals. Setting the original residuals as standard normal distributed the ML-test achieves a higher rejection probability. The difference to the M-test is again small. Using original residuals following the general replacement model, the M-test achieves a high rejection probability and the power of the ML-test is small.
Altogether the simulational results show, that the M-test seems to be an appropriate alternative to the ML-test under the ideal model since the differences in power are negligible. Estimating cointegration of two time series when outliers occur, it seems useful to use the robust M-test.

7 Appendix

Table 1: Power of the M- and ML-test, $\alpha = 0.05$, standard normal distributed residuals

$d = H - \frac{1}{2}$	M-test	ML-test
0.00	0.05	0.05
0.10	0.98	0.98
0.20	1.00	1.00
0.30	1.00	1.00
0.40	1.00	1.00
0.50	1.00	1.00
0.60	0.99	1.00
0.70	1.00	1.00
0.80	1.00	1.00
0.90	1.00	1.00
1.00	1.00	1.00

Table 2: Critical values for the M- and ML-test residuals from the general replacement model with c=0.

	$k_{1-\alpha}$ of the M-test	$k_{1-\alpha}$ of the ML-test
$\alpha = 0.010$	2.3109	2.3040
$\alpha = 0.025$	1.9284	1.9351
$\alpha = 0.050$	1.6387	1.6794
$\alpha = 0.075$	1.4637	1.5129
$\alpha = 0.100$	1.3236	1.3801
$\alpha = 0.125$	1.1957	1.2645
$\alpha = 0.150$	1.1004	1.1677
mean	0.0540	0.1252
$\sqrt{(var)}$	1.0232	1.0258

Table 3: Critical values for the M- and ML-test residuals from the general replacement model with c=10.

	$k_{1-\alpha}$ of the M-test	$k_{1-\alpha}$ of the ML-test
$\alpha = 0.010$	1.9018	2.4218
$\alpha = 0.025$	1.5356	1.7384
$\alpha = 0.050$	1.2376	1.3905
$\alpha = 0.075$	1.0759	1.1845
$\alpha = 0.100$	0.9376	1.0576
$\alpha = 0.125$	0.8220	0.9609
$\alpha = 0.150$	0.7260	0.8754
mean	-0.2332	0.1525
$\sqrt(var)$	1.6801	0.9053

Table 4: Critical values for the M- and ML-test residuals from the general replacement model with c=100.

	$k_{1-\alpha}$ of the M-test	$k_{1-\alpha}$ of the ML-test
$\alpha = 0.010$	13.5475	2.4229
$\alpha = 0.025$	13.1604	1.7478
$\alpha = 0.050$	12.8418	1.3466
$\alpha = 0.075$	12.6233	1.1488
$\alpha = 0.100$	12.4521	1.0183
$\alpha = 0.125$	12.3107	0.9182
$\alpha = 0.150$	12.1850	0.8276
mean	5.1855	0.1317
$\sqrt{(var)}$	6.5653	0.9208

Table 5: Power of the M- and ML-test, $\alpha = 0.05$, residuals from the general replacement model with c=0

$d = H - \frac{1}{2}$	M-test	ML-test
0.00	0.06	0.06
0.10	0.96	0.97
0.20	1.00	1.00
0.30	1.00	1.00
0.40	1.00	1.00
0.50	1.00	1.00
0.60	1.00	1.00
0.70	1.00	1.00
0.80	0.99	1.00
0.90	1.00	1.00
1.00	1.00	1.00

Table 6: Power of the M- and ML-test, $\alpha = 0.05$, residuals from the general replacement model with c=10

$d = H - \frac{1}{2}$	M-test	ML-test
0.00	0.09	0.05
0.10	0.66	0.07
0.20	0.99	0.12
0.30	1.00	0.35
0.40	1.00	0.71
0.50	1.00	0.90
0.60	1.00	0.97
0.70	1.00	0.99
0.80	1.00	0.99
0.90	1.00	1.00
1.00	1.00	1.00

Table 7: Power of the M- and ML-test, $\alpha = 0.05$, residuals from the general replacement model with c=100

$d = H - \frac{1}{2}$	M-test	ML-test
0.00	0.03	0.05
0.10	0.02	0.07
0.20	0.01	0.07
0.30	0.01	0.06
0.40	0.00	0.05
0.50	0.02	0.05
0.60	0.35	0.10
0.70	0.91	0.20
0.80	0.95	0.43
0.90	0.97	0.70
1.00	0.98	0.90

Table 8: Critical values for the M- and ML-test with normal distributes residuals, after doing a linear regression.

	$k_{1-\alpha}$ of the M-test	$k_{1-\alpha}$ of the ML-test
$\alpha = 0.010$	2.8029	2.812
$\alpha = 0.025$	2.4327	2.4208
$\alpha = 0.050$	2.0939	2.0894
$\alpha = 0.075$	1.8677	1.8982
$\alpha = 0.100$	1.7277	1.7786
$\alpha = 0.125$	1.6118	1.6825
$\alpha = 0.150$	1.5089	1.5976
mean	0.5189	0.5917
$\sqrt(var)$	1.0001	1.0046

Table 9: Critical values for the M- and ML-test residuals (c=10), after doing a linear Regression.

	$k_{1-\alpha}$ of the M-test	$k_{1-\alpha}$ of the ML-test
$\alpha = 0.010$	2.1457	15.6341
$\alpha = 0.025$	1.8687	12.5305
$\alpha = 0.050$	1.5363	10.8050
$\alpha = 0.075$	1.3592	9.7903
$\alpha = 0.100$	1.2190	9.0437
$\alpha = 0.125$	1.1040	8.3911
$\alpha = 0.150$	1.0120	7.9078
mean	-0.0396	4.9666
$\sqrt(var)$	1.7743	3.2003

Table 10: Power of the M- and ML-test, after linear regression, $\alpha = 0.05$, standard normal distributed residuals

$d = H - \frac{1}{2}$	M-test	ML-test
0.00	0.03	0.04
0.10	0.92	0.95
0.20	1.00	1.00
0.30	1.00	1.00
0.40	1.00	1.00
0.50	1.00	1.00
0.60	1.00	1.00
0.70	1.00	1.00
0.80	1.00	1.00
0.90	1.00	1.00
1.00	1.00	1.00

Table 11: Power of the M- and ML-test, after linear regression, $\alpha = 0.05$, residuals from the general replacement model with c=10

$d = H - \frac{1}{2}$	M-test	ML-test
0.00	0.07	0.02
0.10	0.59	0.02
0.20	0.98	0.02
0.30	1.00	0.03
0.40	1.00	0.03
0.50	1.00	0.04
0.60	1.00	0.08
0.70	1.00	0.18
0.80	1.00	0.49
0.90	1.00	0.78
1.00	1.00	0.94

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