A Comparison of Point Data Selection Schemes for Evolutionary Surface Reconstructions

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Abstract:
This article presents a study of the application of Computational Intelligence (CI) methods to the problem of optimal surface reconstruction using triangulations and NURBS (**N**on-**U**niform **R**ational **B**-**S**plines) surface approximations on digitized point data.
In mechanical engineering surface reconstructions are used to transform physical objects into mathematical representations for computer aided design purposes. In order to record the geometrical shape of the objects, tactile or optical sensors generate point sets with a huge number of sample points. The number and distribution of these points are decisive for the quality and computational efficiency of the numerical surface representations. Triangulations and NURBS are widely used in CAD/CAM-applications, because they belong to a class of very flexible discrete interpolation and approximation methods.
In order to verify the suitability of surface model independent point selection schemes and to find model dependent sampling point distributions, optimal surface reconstructions are used.

Keywords: Surface reconstruction, point selection schemes, evolutionary algorithms, triangulation, NURBS surface approximation.

Introduction

Optical and tactile scanners are used to record the surface of physical objects to represent their structure in a computer processable way. The digitization of surfaces is widely used in manufacturing processes. The applications reach from studies of manually designed car bodies over the feedback of tool and workpiece geometry changes for the machine adjustments to final quality assurance purposes.

The aim of surface reconstruction is to find efficient, exact and easy to use CAD descriptions on the basis of discrete 3D-point data. Depending on the dimension of the surface, the digitizing system and the sampling strategy, often hundreds of thousands of three-dimensional coordinate triples are sampled during one scan. Common CAD/CAM systems have difficulties to process, store and manipulate the data [Chen 1994]. Depending on the mathematical model and the desired accuracy, the number of points is often much too high to be efficiently processable.

In the following, points that are not needed for an exact and complete description of a surface will be called *redundant with respect to the surface model*, otherwise *relevant*. The purpose of point selection schemes is to find relevant points in arbitrarily distributed or weakly structured sampling point sets in order to yield easy to use and realistic CAD surface representations.

There exist two ways to reduce point data sets: A *generalistic* approach uses selection schemes that generate point distributions that can be used for a large variety of surface models. The redundancy of the point set may not be optimal regarding special surface types but fast and good enough for practical applications. This class of point selection schemes, called *sur-*

face model independent, use heuristics that are often motivated by linear or higher order approximation theory.

Schemes that are *surface model dependent* exploit mathematical properties of the surface models that are intended to be used for the surface reconstruction purposes. Typical surface models are piecewise linear interpolations or approximating splines. In the field of spline or polynome approximation, approximation theory [Hämmerlin 1989] gives hints for necessary sampling point distributions with respect to tolerable approximation errors. The advantage of this approach lies in its ability to find a minimum of relevant points. The disadvantage of this strategy comes from the fact that the complete point set has to be reconstructed as exactly as possible before the relevant points can be selected.

In the following sections both point reduction approaches will be discussed. Due to the fact that human designers finally have to decide weather a reconstruction is good enough for a practical application, the results have to be inspected optically. Here, global optimization strategies are used to allow practical comparisons of smooth triangulations and best fit NURBS surface reconstructions.

Point Selection Schemes

Generalizing Scan Line Schemes

Cord deviation filters are widely used in post-processing of digitized point data. These filters make use of the a-priori known 3D-structure of the digitized point data. The points are ordered according to the physical scanning process.

The cord deviation filters use linear (sometimes higher order) one-dimensional comparison models M and a tolerance value $\epsilon \geq 0$. All points that fit into the space around M are defined to be redundant. Only the points that are needed to specify M are relevant and will be selected.

Depending on the model M and the distance criterion (metric) $d : \mathbb{R}^3 \times \mathbb{R}^3 \rightarrow \mathbb{R}$ a family of cord deviation filters can be defined.

The **TDS** (*tangential distance strategy* [Aretz 1992]) uses a linear model to select relevant points. A line $\mathbf{v} \in \mathbb{R}^3$ through the first $\mathbf{p}_0 := (p_{0x}, p_{0y}, p_{0z})^T$ and second sample point $\mathbf{p}_1 := (p_{1x}, p_{1y}, p_{1z})^T$ is used to estimate a tangent line of the digitized surface. All points that fit into the cylinder with radius ϵ along $\mathbf{v}$ are redundant. Only $\mathbf{p}_0$ and the last point $\mathbf{p}_\nu$ that still fits into the cylinder will be selected as relevant points. Starting with p_ν the scheme is iterated until either the last point of a scan line is reached or no more points are to be processed. The metric d measures the minimum (orthogonal) distance between $\mathbf{v}$ and each consecutive sample point $(x, y, z)^T := \mathbf{p}_i$. The TDS is very efficient regarding its time and space complexity ($O(1)$). Because the first and second sampling point are used to define $\mathbf{v}$, the TDS is the more measurement error prone the smaller the distance between the digitized points becomes.

The following two variations of the TDS both have one degree of freedom. Here, only $\mathbf{p}_0$ is kept fix. $\mathbf{v}$ is the straight line between $\mathbf{p}_0$ and the last point read $\mathbf{p}_i, i = 1, 2, \ldots$. A point $\mathbf{p}_{i-1}$ joins the set of redundant elements iff. all points $\mathbf{p}_j, j = 1, \ldots, i-1$ are within the cylinder M along $\mathbf{v}$ with radius $\epsilon \geq 0$. If any point between the $\mathbf{p}_0$ and $\mathbf{p}_i$ has a distance $d > \epsilon$, $\mathbf{p}_0$ and $\mathbf{p}_{i-1}$ are defined to be relevant. The algorithm is iterated re-starting with point $\mathbf{p}_{i-1}$ as long as the end of the scan line is not reached. The **ATDSO** (*A*daptive *TDS* with *o*rthogonal distance) uses the orthogonal (minimum) distance d of an arbitrary point $\mathbf{p}_j$ from $\mathbf{v}$. **ATDSZ** measures the difference of the z-components of $\mathbf{v}$ and $\mathbf{p}_j$. The additional space needed to store the points $\mathbf{p}_0$ to $\mathbf{p}_i$ is linearly increasing with i. Due to the constantly varying orientation of $\mathbf{v}$, each point between $\mathbf{p}_0$ and $\mathbf{p}_i$ has to be tested with every new iteration. Let $\mathbf{p}_k$ be the last point for which the criterion that all points fit into the cylinder M still holds, the algorithm needs $O(k)$ space and $O(k^2)$ time. The value of k depends on the tolerance ϵ and the curvature of the digitized surface.

The **ULREG** (*u*nivariate *l*inear *r*egression) increases the freedom of the orientation of $\mathbf{v}$ to two dimensions. This flexibility is necessary especially for noisy data. The first two points $\mathbf{p}_0$ and $\mathbf{p}_1$ are used to define an initial $\mathbf{v}$. In the ULREG the orientation of $\mathbf{v}$ is then calculated for each consecutive point $\mathbf{p}_j = (p_{xj}, p_{yj}, p_{zj})^T, j = 2, 3, \ldots$ of the scan line using the classical regression equations:

$$
\begin{aligned}
\bar{x} &= \tfrac{1}{j} \textstyle\sum_{i=0}^{j} p_{xi} \\
\bar{z} &= \tfrac{1}{j} \textstyle\sum_{i=0}^{j} p_{zi} \\
s_x^2 &= \tfrac{1}{j-1} \textstyle\sum_{i=0}^{j} (p_{xi} - \bar{x})^2 \\
s_z^2 &= \tfrac{1}{j-1} \textstyle\sum_{i=0}^{j} (p_{zi} - \bar{z})^2 \\
m_{xz} &= \tfrac{1}{j-1} \textstyle\sum_{i=0}^{j} (p_{xi} - \bar{x})(p_{zi} - \bar{z}) \\
r &= \tfrac{m_{xz}}{s_x s_z} \\
v(x,z) &= \bar{z} + r \tfrac{s_z}{s_x} (x - \bar{x})
\end{aligned}
\qquad (1)
$$

If s_x or s_z are zero, then r is set to 1. The iteration is terminated iff. (a) either the border of the point set is reached, (b) all points are processed or (c) the difference $\mid r \mid -1$ of the absolute correlation coefficient is greater than a certain $\gamma \geq 0$. r describes the correlation of the sample points with the model M. Values near ± 1 indicate that the points analyzed diverge only little from a straight line. These points are not relevant and can be reduced. If the divergence of the absolute correlation coefficient r from 1 becomes larger than γ, then the first and the last point read are defined to be relevant. Any other points near or on $\mathbf{v}$ could have been chosen as well. The scheme is restarted with the last point read and is continued until all points are processed.
Let k be the highest difference of the suffixes of all relevant points, then the ULREG has $O(k)$ space and $O(k^2)$ time complexity.

Generalizing 3D Point Cloud Schemes

All strategies discussed so far make use of the line structure of the digitized point data. The difference between line and plane based schemes can be demonstrated, when they are applied to a surface like a half-cylinder. Let the cylinder be digitized either (a) parallel to the main axis and (b) orthogonal to the main axis. In case (a) the line based strategies yield, independent from any $\epsilon > 0$, only the borders of the surface. In case (b) the number of relevant points depends strongly on the value of ϵ.

The following multiple schemes are based on the idea of the univariate strategies applied to higher dimensions. The algorithms do not depend on any consecutive order of the sample points. In the examples discussed here, tactile digitized objects are used that do not have undercuts. This restriction is necessary for the strategies described next.

The basic idea of the **2DTDS**-algorithm is to use a box with height $2\epsilon \geq 0$ instead of a cylinder like in the traditional TDS [Aretz 1992]. The algorithm discussed in this article differs in many points from Aretz's original version. It uses a recursive flood fill algorithm [Foley 1992] that selects all points that are "cut out of the surface" by a box M. This model can be replaced by any other classifier.
The 2DTDS marks all points to be redundant with respect to M that (a) are not a part of the border of the complete digitized area, (b) belong to one surface patch that has not already been analyzed, (c) fit exactly into the volume M, and (d) do not lie on the intersection line of M and the digitized surface. Each redundant point is marked and will not be inspected a second time. Any border point can be used as a seed for a new search for relevant points. The distances are measured using normal vectors on S. The orientation of S is given by three relevant neighboring points that do not lie on a common line. Singular points or pairs that are left during the search and cannot be used to define the plane S are defined to be relevant. The algorithm scans every surface point and is iterated as long as all points are marked either redundant or relevant, respectively.

The **MLREG** strategy (*m*ultiple *l*inear *r*egression) is the multiple pendant of the univariate ULREG algorithm. Instead of a straight line that approximates a subset of points along a scan line, a plane is used. The index structure of the underlying point set is irrelevant for this algorithm. The sample points should not have undercuts or 90° edges, because the regression measures the distance form the plane to the sample points parallel to the z-axis.

The multiple regression of $k+1$ features $Y, X_1, X_2, ..., X_k$ describes the linear functional coherence by: $y(x_1, x_2, .., x_k) = \alpha + \sum_{j=1}^{k} \beta_j x_j$. Let $y_i, x_{1i}, ..., x_{ki}, \quad i = 1, ..., n$ be n fix realizations of n independent random variables. The estimators for β_j, $j = 1, ..., k$ and a can be calculated by solving the following equation system:

$$
\begin{array}{llll}
b_1 SQ_{x_1} & + b_2 SP_{x_1x_2} & + \ldots + b_k SP_{x_1x_k} & = SP_{x_1y} \\
b_1 SP_{x_1x_2} & + b_2 SQ_{x_2} & + \ldots + b_k SP_{x_2x_k} & = SP_{x_2y} \\
\vdots & \vdots & \vdots & \vdots \\
b_1 SP_{x_1x_k} & + b_2 SP_{x_2x_k} & + \ldots + b_k SQ_{x_k} & = SP_{x_ky}
\end{array}
$$

with

$$
\begin{aligned}
SQ_{x_j} &= \sum_{i=1}^{n} (x_{ji} - \bar{x}_j)^2 \\
SP_{x_jx_{j'}} &= \sum_{i=1}^{n} (x_{ji} - \bar{x}_j)(x_{j'i} - \bar{x}_{j'}) \qquad (2) \\
SP_{x_jy} &= \sum_{i=1}^{n} (y_i - \bar{y})(x_{ji} - \bar{x}_j) \quad \text{for} \quad j, j' = 1, ..., k. \\
a &= \bar{y} - b_1\bar{x}_1 - \ldots - b_k\bar{x}_k.
\end{aligned}
$$

The *multiple correlation value* $|R| \in [0, 1]$ is defined by

$$R^2 = B_{y,(x_1,..,x_k)} = 1 - \frac{\sum_{i=1}^{n} (y_i - a - \sum_{j=1}^{n} b_j x_{ji})^2}{\sum_{i=1}^{n} (y_j - \bar{y})^2}.$$

Values with a bar on top denote arithmetic averages.
In three dimensions k equals 2 and the sample points are realizations of the features (Y, X_1, X_2).

The MLREG algorithm uses a *quadtree strategy* [Foley 1992] to find the points that are to be tested against the "plane hypothesis". The quadtree algorithm recursively divides squares in the (x, y)-plane into four smaller squares. Digitized points with (x, y)-coordinates that lie within a square k form a subset S_k. A correlation coefficient R_k is calculated for each set S_k. As long as $|R_k| - 1 > \gamma$, where $\gamma > 0$ is an external (tolerance) parameter, each S_k is divided and tested recursively. If the termination criterion $|R_k| - 1 \leq \gamma$ of the recursion is reached, a (not unique) relevant point has to be selected. Here, the center of gravity of the points contained within a square is chosen.

Surface Model Dependent Schemes

Generalizing schemes do not depend on the surface model that is used to represent the object after the point selection process. If the properties of the mathematical models are known, points can be selected more specifically from the original point cloud. This allows to keep the approximation error within predefined limits. For this approach optimal surface reconstructions are necessary.

Using Optimized Triangulations

Triangulations belong to the most common surface descriptions for digitized point data. A decomposition of a polygon into triangles by a maximal set of non-intersecting diagonals is called a *triangulation* of the polygon [de Berg 1997]. *Terrain triangulations* are special 3D-triangulations without undercuts. They can be generated by adding a height component to the vertices of 2D triangulations.

Triangulations are usually not unique. In the context of surface reconstruction a terrain triangulation is called optimal if the reconstructed surface fits the digitized points tightly. Optimal *tightness* is reached if the *total absolute curvature* (TAC) [van Damme 1994] of the surface is minimal. This property can be transformed into an optimization task. Due to the fact that a deterministic solution of this optimization problem may take a lot of time or may suffer from premature stagnation in local optima, robust and globally optimizing strategies have to be used. **Evolutionary algorithms** (EA = Evolution Strategy (ES) $\cup$ Genetic Algorithm (GA) $\cup$ Evolutionary Programming (EP) $\cup$ Genetic Programming (GP)) [Bäck 2000] are a class of algorithms with very promising properties. In EA a population of possible solutions of a problem is used. Each individual represents a point in the search space and undergoes a set of genetic operators, e.g. recombination and mutation, that generate new individuals in an appropriate way. A new generation is found by selecting the fittest individuals using an appropriate selection scheme (e.g. truncation selection, fitness proportional selection, tournament selection, etc.). An EA is iterated until an appropriate termination criterion holds.

In the case of triangulations, each individual encodes an undirected, non-intersecting graph that interpolates all sample points. Here, initial individuals are generated using a Delaunay triangulation [de Berg 1997]. Mutation transforms regular triangulations into slightly varied triangulations by flipping single edges. An edge that is to be swapped is determined randomly. If the flip yields to a regular triangulation, the swap is realized in the graph. The number of flips per generation is adjusted by a hyperbolically decreasing function [Weinert 1998]. The fitness of each individual is determined by the calculation of the TAC of each 3D graph structure.

On the one hand optimal surface reconstructions using triangulations give a proper means to compare generalizing point selection schemes. On the other hand the data calculated during optimal triangulations can be used to find relevant points. These points fit more specific to the surface model than the points selected by generalizing schemes. The simple deterministic **DTPSS** algorithm (*D*eteministic *T*riangulation-based *P*oint *S*election *S*cheme) selects the relevant data depending on their local TAC value. If the largest absolute curvature of one point is larger than $2\pi - \epsilon$, the area around the point roughly approximates a plane. This point is defined to be redundant and will be excluded from the set of sampling points. More sophisticated optimal triangulation based on point selection schemes are matter of current research.

Optimized NURBS Approximations

NURBS surfaces (*N*on-*U*niform *R*ational *B*-*S*pline surfaces) are widely used in CAD systems [Piegl and Tiller 1997]. These models are very flexible, smooth and intuitive. NURBS are a superset of Bézier polynomials and non-rational B-splines. They are efficient regarding space and time complexity and numerically stable. International CAD interface standards like IGES or STEP support NURBS surface descriptions. Many objects with free form characteristics were originally designed using NURBS, thus, utilizing these models in reverse engineering can yield very realistic reconstructions.

NURBS surfaces are parametric tensor products that map the vector $(u, v)^T \in \mathbb{R}^2$ to

$(x,y,z)^T \in \mathbb{R}^3$. NURBS surfaces of order (p,q) are composed of the following components:

- the control point net with $n \times m$ vertices $\mathbf{P} = \{\mathbf{P}_{i,j} \in \mathbb{R}^3, i = 1, ..., n, j = 1, ..., m\}$,
- the knot vector U and V, where $\mathbf{U} = (\underbrace{0,\ldots,0}_{p+1}, u_{p+1}, \ldots, u_{r-p-1}, \underbrace{1,\ldots,1}_{p+1})^T$, $\mathbf{V} = (\underbrace{0,\ldots,0}_{q+1}, v_{q+1}, \ldots, v_{s-q-1}, \underbrace{1,\ldots,1}_{q+1})^T$, and $r = n+p+1$ and $s = m+q+1$. The u_i and v_j have an ascending or descending order, respectively.
- the weight vector $\mathbf{W} = \{w_{i,j} \in \mathbb{R}^+, i = 1, ..., n, j = 1, ..., m\}$.

The parametric surface function is defined over the domain $(u,v) \in [0,1] \times [0,1]$ and expressed by

$$\mathbf{S}(u,v) = \frac{\sum_{i=0}^{n}\sum_{j=0}^{m} N_{i,p}(u) N_{j,q}(v) w_{i,j} \mathbf{P_{i,j}}}{\sum_{i=0}^{n}\sum_{j=0}^{m} N_{i,p}(u) N_{j,q}(v) w_{i,j}}. \tag{3}$$

$N_{i,p}(u) : \mathbb{R} \to \mathbb{R}$ are the ith basis functions of order p for the parameter $u \in \mathbb{R}$ computed on a knot vector $(u_0, ..., u_m)^T$. The basis functions $N_{i,p}(u)$ can be defined recursively by

$$N_{i,0}(u) = \begin{cases} 1 & \text{if } u_i \le u \le u_{i+1} \\ 0 & \text{otherwise} \end{cases} \tag{4}$$
$$N_{i,p}(u) = \frac{(u-u_i)N_{i,p-1}(u)}{u_{i+p}-u_i} + \frac{(u_{i+p+1}-u)N_{i+1,p-1}(u)}{u_{i+p+1}-u_{i+1}}$$

and evaluated efficiently by e.g. the *Cox - de Boor* algorithm. The parameters of the control net vertices, knot vectors and the weight vectors can be joined to one vector $\varphi = (\mathbf{P}, \mathbf{T}, \mathbf{W})$ of the space $\mathbb{R}^\gamma$ with dimension $\gamma = 4nm + (n+p+m+q)$.

The Evolution Strategy (ES) [Schwefel 1994] is a robust global optimization method for real valued vector spaces and, thus, it perfectly suits the problem of discrete NURBS surface approximation. It can be shown [Weinert 2000] that an ES can find good surface reconstructions for singular surface patches using the sum of squared distances as a fitness criterion.

Analogous to the DTPSS, optimal NURBS surface reconstructions may be used as a reference model for deterministic or evolutionary point selection. The long computation times that are needed to find optimal NURBS surface reconstructions (several hours depending on the complexity of the surface) are an obstacle for practical applications. Mathematical estimations of the point density for given approximation errors are often restricted to B-Splines [Hämmerlin 1989]. Due to the fact, that there is still no deterministic strategy that finds optimal NURBS approximation schemes for arbitrary surface structures, the evolutionary NURBS surface approximation may be one way to create point selection schemes that correspond with sentences from *function* approximation theory.

Here, the optimized NURBS surfaces are used to test the influence of the generalizing selection schemes on the shape and approximation error of the reconstructions.

Experiments and Results

In the experiments the surface of an artificially designed piston top with diameter $79.9\,mm$ (Figure 1) has been digitized to get a set of $11,314$ sample points. The surface was measured by a tactile scanning digitizing machine (Renishaw Cyclone). The piston was selected to demonstrate the behavior of the point selection schemes on point sets from real objects with a complex surface.

Fig. 1: Photo of the top of a piston.

A LINUX Pentium III PC with 450 MHz computed all tasks from point selection to surface reconstruction. All programs are written in C++.

The TDS, ATDSO, and the ATDSZ reduced the original point set depending on the tolerance ϵ within a maximum time of $0.03\,s$ (TDS) and $0.04\,s$ (ATDSZ and ATDSO). Figure 2 shows the number of reduced points with respect to the given tolerances $\epsilon \in [0.0001, 0.001, 0.01, 0.05, 0.1, 0.2, 0.3]$. The increased reduction factor of the ATDSZ and ATDSO compared to the TDS can be seen in the picture. ATDSZ and ATDSO usually perform quite similar. On the average, the ATDSO is a little more efficient than the ATDSZ.

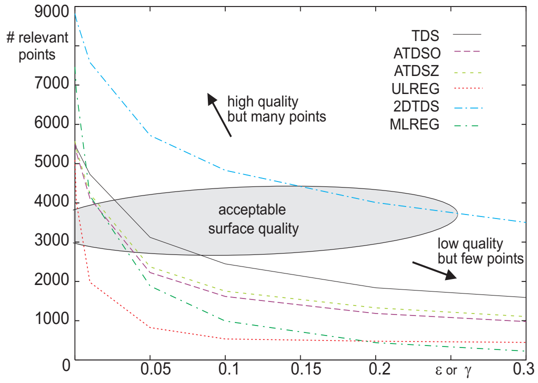

Fig. 2: Number of relevant points vs. tolerances.

The surfaces generated using a Delaunay triangulation on TDS points with $\epsilon \leq 0.01$ $(4,728)$ have a high quality. Triangulations with $\epsilon \in [0.1, 0.2]$ $(2,446$ and $1,839$ points) are only smooth enough after optimization. Larger tolerances yield surfaces with a visible triangle structure, although, the overall shape of the original object can still be estimated. The quality of the triangulations using ATDSO and ATDSZ are compatible with the reconstructions using TDS reduced points with the same $\epsilon \leq 0.01$ $(4,116$ and $4,227$ points, respectively). Triangulations with larger tolerances soon become very poor. In the example the relevant points found by the TDS variants cover the surface quite uniformly (see Figure 3). This simplifies the triangulation of the object, because the optimal triangulation of spherical shapes can be found directly by the Delaunay algorithm. In cylindric parts long triangles can be generated by optimization.

With increasing tolerance the ULREG algorithm selects – like the other selection schemes – an exponentially decreasing number of points from the original point set. The maximum calculation time was $0.04\,s$. A comparison of the triangulations generated by the ULREG algorithm and the TDS variants on point sets with the same number of points shows that the qualities of the reconstructions are simular. A $\gamma \leq 0.001$ $(4,022$ points) yields smooth Delaunay triangulations without optimization. The application of the EA is recommended for $\gamma \leq 0.0001$, because steep edges become a lot smoother after optimization. Optimized triangulations are sufficiently good with $\gamma \leq 0.005$ $(2,630$ points). Higher γ values yield point distributions that should not be used for smooth surface triangulations. These sets may still be useful for edge detection (e.g. $\gamma \in [0.01, 0.05]$), because the curvature of the surface is reflected by contour lines containing the relevant points.

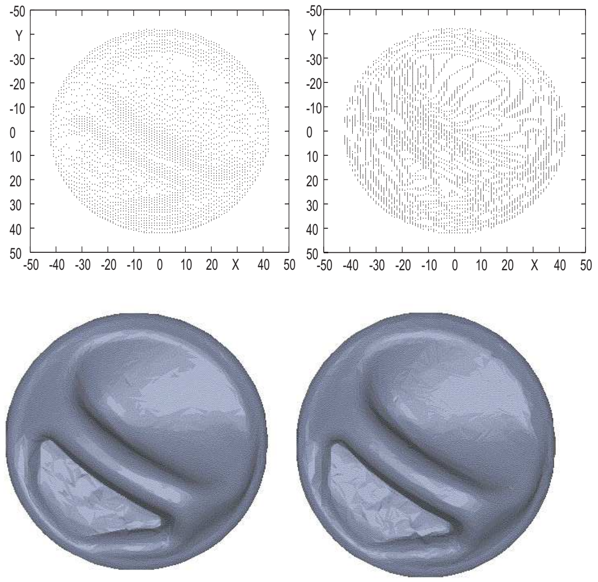

Fig. 3: Reconstructions generated by the TDS (left, $3,122$ points) and the 2DTDS (right, $4,829$ points).

Figure 3 illustrates different point distributions generated by the conventional TDS $(3,122$ points) and the 2DTDS $(4,829$ points). The parameters were selected to get reconstructions with similar quality. The reconstructions look similar to the rendered triangulation in Figure 4. The complex pattern of the relevant points selected by the 2DTDS is typical for that strategy. This net structure is the result of the combination of the intersection lines of the approximating planes and the surface. Both optimized surface reconstructions are comparably smooth with TAC values of 19.17 and 25.67, respectively. Although, in this example the 2DTDS needed $1,707$ points more to reach the quality of the TDS, the dependency of the TDS on the scanning strategy should be the reason to prefer the more robust 2DTDS. Starting from a Delaunay triangulation optimization helps to reduce the number of relevant points by 10% to 20% and to increase the smoothness in the same time. The TAC of the 2DTDS-points was increased from 76.05 (Delaunay) to 25.69 (optimized) and the TAC of the TDS-points from 58.71 (Delaunay) to 19.18 (optimized). In the example the optimization process and the application of the 2DTDS $(0.05\,s)$ together needed about three minutes.

The MLREG strategy generates points that are distributed on a non-uniformly spaced grid. Smooth reconstructions can be found after optimization of triangulations with $\gamma \leq 0.03$ $(\geq 2,555$ points, see Figure 4). Higher tolerances quickly lead to triangulations that approximate the shape of the original object only very roughly. A comparison of the 2DTDS and the MLREG using triangulation with $4,010$ and $4,180$ points, respectively, shows a much smoother surface generated by the MLREG. This comes from the fact, that the quadtree algorithm scans the surface with recursively quartered square patches. Each relevant point is the center of gravity of a square patch that has a correlation coefficient that is high enough to approximate the surface by a square plane in $\mathbb{R}^3$ with a certain tolerance. The recursive structure of the MLREG algorithm implies a high space and time complexity. The reduction of $11,314$ points to $4,180$ points needed $10.0\,s$. The slow quadtree algorithm enables the MLREG strategy to select points from unordered point sets with small errors. This makes the MLREG algorithm a quite robust 3D point selection scheme for terrain triangulations.

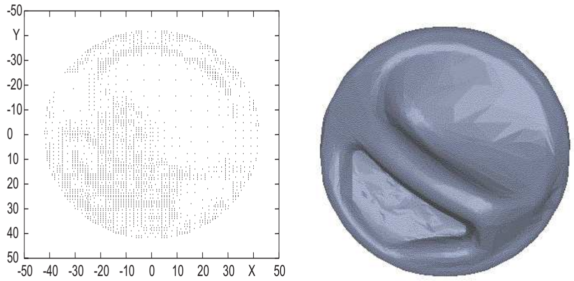

Fig. 4: Point set distribution and optimized triangulation using $2,555$ *points selected by the MLREG strategy.*

The DTPSS algorithm selects points depending on the TAC values of each point of the optimized triangulation. With small tolerance values ϵ the algorithm reduces redundant points in areas with small curvature. The procedure of point reduction and TAC calculation using optimal triangulations is repeated each time a point is deleted from the point sets. This helps to provide the reduction of complete sets of point in regions where the curvature values are similar and within the ϵ tolerance (e.g. in the case of spheres). In the example of the piston, the points in the large bowl shaped area (upper left part in Figure 5) are reduced first. With increasing tolerance the points in the area in the lower right part in Figure 5 are reduced. Due to the fact, that optimized triangulations are used, smooth reconstructions can be created over a wide range of tolerances. The necessity to have an initial triangulation is a problem for large point sets, because the calculation of these triangulations usually needs a lot of time and space.

Fig. 5: Optimized triangulation using DTPSS ($7,095$ *points).*

NURBS surface reconstructions are quite robust with respect to the digitized point distributions. Usually only a few percent of the complete point set is really needed to yield good NURBS surface reconstructions. This increases the speed of the expensive optimization process a lot. Points selected by the 2DTDS or the MLREG are a basis for good approximations. The demand for a high density of points in regions with high curvature is supported by these algorithms. This also matches to statements from approximation theory for B-Splines [Hämmerlin 1989]. Furthermore, the ability of NURBS to use equally spaced points is supported (locally) by the MLREG strategy, too. The exact number of necessary points depends on the accuracy, the curvature of the surface and the NURBS model. The lower the dimension of the vector φ, which is used in the ES, the fewer sampling points are needed. In the experiments the surface of the piston was approximated with a NURBS surface with variable weights, 8×8 control points and $p = q = 3$ using a $(50 + 200)$-ES [Weinert 2000]. Using $3,000$ points, the approximation error is 0.0179. This is equivalent to an error per point of $0.12\,mm$ on average. Increasing the number of sample points in areas with very large curvature may also yield better results, because the algorithm tries to satisfy the locally increased (relative) approximation error.

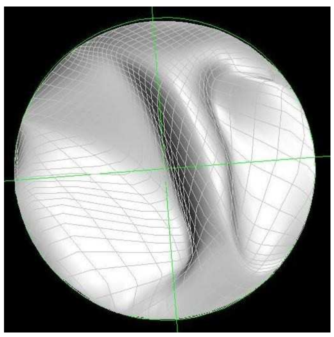

Fig. 6: CAD-NURBS surface reconstruction using IGES.

Conclusions and Outlook

Scan line based and multiple point selection schemes have been compared. The chordal deviation strategies ATDSO and ATDSZ perform better than the classic TDS algorithm. All three strategies are very fast. The univariate regression strategy ULREG has a high measurement error robustness. This excels the ULREG as a good smoothing filter. The multiple strategies 2DTDS and MLREG are slower than the scan line based algorithms but generate point distributions that are based on higher dimensional models. As generalizing schemes these point selection strategies adapt better to the surfaces used in reverse engineering. Surface model depending point selection strategies need precalculated optimal surface reconstructions and, thus, they are time and space consuming. Because they are excellently adapted to the surface model, they can give ideas to find good heuristics for fast generalizing selection schemes. In order to compare the results of point selection schemes optimized triangulations and NURBS surfaces are used. They demonstrate that adequate surface models allow efficient and practical reconstructions that also need less sample points. A significant threshold value that characterizes the relevance of a point set depending on a given tolerance exists (see Fig. 2) but is still not formulated explicitely. This is a challenge of current research activities.

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