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A Cross-Grade Study of Elementary School Students' Misconceptions Concerning Probability Judgment

1. Issues and Objectives

Misconceptions Concerning Probability Judgment

In cognitive studies, it has become that various biases and misunderstandings exist in people's probability judgment, including the misconception of random sequences, the conjunction fallacy, and disregard for sample size and prior probability (Fischbein & Schnarch, 1997). "Precipitation probability" is often cited as an instance of misunderstanding the meaning of probability. Suzuki (2018) points out that not only secondary school students but also adults fail to give accurate answers regarding the meaning of precipitation probability. As a concrete example of incorrect answers, Suzuki mentions the following: "When the precipitation probability is 30%, it means that, supposing 10 is the degree of actual rainfall, today's degree is approximately 3." A precipitation probability of 30% means that if the forecast was announced 100 times, there would be 1 or more mm of rainfall in the specified period at approximately 30 of those times. The indicator for the degree of rainfall is the amount of rainfall, which, from a mathematical standpoint, is defined as the "value of a random variable." Therefore, a high precipitation probability may not necessarily mean a large amount of rainfall (i.e., a high random variable value). This study focuses on the misconceptions arising due to the influence of the value of random variables.

Objectives

There have been no significant revisions to the syllabus or timing of teaching the probability field in Japanese schools since 1968. This is an issue because the status quo does not enable an adequate response to today's social context, where uncertainty is rising (Okabe, 2006). In particular, there is no clear focus on understanding the concept of probability in Japan's elementary education. Elsewhere, the timing of teaching the concept of probability has been discussed in foreign countries; probability is taught in elementary school in at least 20 countries, including Australia and Finland (Langrall, 2018). In this context, we may obtain insights toward developing a curriculum for teaching probability by clarifying when and how students' misconceptions about probability develop. Based on these arguments, this study set the following two objectives.

Objective 1. To clarify whether misconceptions of probability arise due to

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the value of random variables and the conditions in which these misconceptions arise.

Objective 2. To clarify when misconceptions of probability due to the influence of the value of random variables occur and their developmental characteristics.

2. Method

Target Group

The target group consisted of students from X Elementary School in Y Prefecture, Japan (first grade: 66, second grade: 64, third grade: 61, fourth grade: 64, fifth grade: 56, sixth grade: 66 students). None of the students had studied probability so far.

Worksheet

The worksheet used for the survey consisted of a sheet of A4-sized paper (excluding the cover sheet) containing three tasks that required comparing and judging probability levels ("probability tasks"). The question was "Which spinner is more likely to win (has a higher probability of winning)?" Each task had three choices: "Spinner A," "No difference," and "Spinner B." The author produced the following three types of tasks by combining three numerical values: probability ("P"), a random variable ("V"), and the expected value ("Ex").

Task 1 (V fixed):

V is fixed and P is varied (PA < PB, VA = VB, ExA < ExB).

Task 2 (P is fixed):

P is fixed and V is varied (PA = PB, VA > VB, ExA > ExB).

Task 3 (Conflict):

Both P and V are varied; Ex is bigger when V is higher and P is lower (PA < PB, VA > VB, ExA > ExB).

Procedure

The cross-grade survey targeted first- through sixth-grade students. Before beginning, the author read out instructions that applied to all grades, confirming the following four points: 1) Each spinner is divided into ten equal sections. 2) Turn the spinners ten times. (3) If the arrow stops at the colored section, it is a win, and the number on it shows how many candies will be won as a prize. Otherwise, it is not a win and no candies will be given. (4) Select the spinner that is most likely to get a win (has a higher probability of

winning). One answer was required for each question, and the survey proceeded to the next question after the author confirmed that all the students had finished writing their answers.

3. Results and Discussion

Objective 1 (Misconceptions of probability and conditions in which they arise)

The first objective of the study was whether misconceptions of probability exist among school children. The survey results reveal that, although the students correctly answered the task when V was fixed, many gave incorrect answers for the task when P was fixed and the conflict task. This suggests that the students made erroneous probability judgments due to the influence of the information about V, rather than because they did not understand the question. Therefore, the results clarify that misconceptions of probability are caused by the influence of the value of a random variable exist.

The next objective was investigating the conditions in which these misconceptions arise. The information about V in the question did not necessarily influence all children's judgments. In the task where V was fixed, for example, most students did not consider V as a piece of information used to solve it. However, when P was fixed and in the conflict task, many students could not help paying attention to V. In other words, it is clear that misconceptions of probability arise due to the influence of the value of a random variable arise when V is varied.

Objective 2 (when misconceptions of probability arise and their developmental characteristics)

The next objective was to clarify when students form misconceptions of probability. The survey results reveal that children have already formed misconceptions of probability in first grade. This suggests that misconceptions of probability due to the influence of the random variable may develop during early childhood as young children begin to form the concept of probability.

Next we focus on the developmental characteristics of misconceptions of probability. The survey results reveal that the misconceptions tend to be rectified from fourth through fifth grade. One factor for this improvement may be the introduction of lessons on percentages and ratios in the curriculum for fifth-grade arithmetic in Japan. It is possible that learning about these subjects that are closely related to the concept of probability helps children rectify their misconceptions of probability. Nonetheless, the misconceptions tend to increase slightly among sixth graders. At the primary education level, children's misconceptions are not fully rectified and correction may only be

temporary in nature.

4. Overall Discussion

This study reveals that children have already formed misconceptions of probability due to the influence of random variables in the first grade. In the current context in Japan, where the concept of probability is not clearly addressed at the primary education level, there is a risk that these misconceptions may become ingrained through their life experiences (Way & Ayres, 2002). For example, if a child thought, "Because the chance of rain is high, the amount of rainfall is high," and they then experienced a day when heavy rainfall coincided with a high chance of rain, their misconception would be instilled in their minds. This leads to concerns that they will incorporate the value of a random variable as an attribute to determine the level of probability. Therefore, it is highly probable that if children's misconceptions of probability are not appropriately corrected during their primary education, their study of probability in secondary school and college education would be hindered.

Regarding issues to consider in the future, it should be noted that this study presents insights gained from the analysis of tasks with spinners conducted under controlled conditions. It is necessary to achieve a more comprehensive grasp of children's misconceptions about the concept of probability. Additionally, further exploring the appropriate timing and effective strategies for the introduction of teaching the concept of probability is vital.

Literature

Langrall. C. W. (2018). The Status of Probability in the Elementary and Lower Secondary School Mathematics Curriculum: The Rise and Fall of Probability in School Mathematics in the United States. In Batanero. C, Chernoff,E. J. (Eds.), *Teaching and Learning Stochastics Advances in Probability Education Research* (pp.39-50). Springer International Publishing.

Masashi Suzuki. (2018). *A Study on Misconception of Probability Concept and Mathematics Curriculum, Bulletin of Soka Educational Studies, Vol.70*, pp.323-339.

Way, J. & Ayres, P. (2002). The instability of young students' probability notions, A. D, Cockburn & E. Nardi (Eds) *Proceedings of the 26th conference of the International Group for the Psychology of Mathematics Education, Vol.4*, pp.394-401.

Yasuyuki Okabe. (2014). A Study on probability education in early elementary school, *Japan Jounal of Mathematics Education and Related Fields, Vol.55*, pp.53-59.