S - estimators in the linear regression model with long - memory error terms [1]

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Abstract

We investigate the behaviour of S - estimators in the linear regression model, when the error terms are long - memory Gaussian processes. It turns out that under mild regularity conditions S - estimators are still normally distributed with a similar variance - covariance structure as in the i.i.d. case. This assertion holds for the parameter estimates as well as for the scale estimates. Also the rate of convergence is for S - estimators the same as for the least squares estimator and for the BLUE.

KEY WORDS: Linear regression model; long - range dependence; robustness

1 Introduction

Consider the linear regression model

$$y_i = x_i^T \theta + e_i \qquad i = 1, \ldots, n, \tag{1}$$

where y_i is the dependent variable, x_i is a p - dimensional vector of possibly stochastic regressors with distribution function G, θ is the p - dimensional parameter vector and e_i is an error process independent of x_i. Here we consider the case where e_i is a long - memory stationary process.

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Defining $R(k) := Cov(e_i, e_{i+k})$ long - memory time series can be modeled as stationary processes satisfying

$$\frac{R(k)}{L(k)|k|^{2H-2}} \to 1,$$

where $\frac{1}{2} < H < 1$, $L(k)$ is a slowly varying function and $k \to \infty$. $R(k)$ has the form

$$R(k) = \frac{\sigma_e^2(|k+1|^{2H} - 2|k|^{2H} + |k-1|^{2H})}{2}.$$

The equation $var(\bar{e}_n) = \sigma_e^2 n^{2H-2}$ also holds so that we have the convergence $n^{2-2H} L_{Var}^{-1}(n) var(\bar{e}_n)$ to 1, where $L_{Var}(n) = L(n)/(H(2H-1))$.

So the main property of long - memory processes is the slow decay of the correlations. This is a big problem for applied statisticians, because the standard assumption of independence is often a bad approximation. Small but slowly decaying correlations are dangerous for experimental statisticians, because they can often not be detected by standard test but they have a strong effect. For example the standard error of the arithmetic mean $\sigma n^{-1/2}$ has to be replaced by $n^{-\alpha}$ for $0 < \alpha < 1/2$. This phenomenon has been observed by many applied statisticians. For a review see Cox(1984), Beran(1992) or Beran(1994).

For the situation of independence Rousseeuw/Yohai(1984) proposed S - estimators and their asymptotic properties. Maronna/Yohai(1981) established the asymptotic properties of regression M - estimates. Davies(1987) considered the asymptotic properties of S - estimators of multivariate location parameters and Davies(1991) extended the results of Rousseuw/Yohai to S - estimators with a smooth ρ - function.

However, nothing is known about the behaviour of robust estimators in the linear regression model with long - memory error terms. For this model Yajima(1988,

1991) shows that the least - squares estimator is no longer efficient relative to the BLUE. But it is well known that neither the least - squares estimator nor the BLUE is robust anyhow.

So the aim of this article is to extend the asymptotic normality of S - estimators as established by Rousseeuw/Yohai(1984) to the linear regression model with long - memory error terms.

S - estimators are of special interest because of their good asympotic properties and their high breakdown point. Here the breakdown point introduced by Hampel(1971) is used as the measure of robustness of an estimator in the presence of outliers. The empirical breakdown point, which is used most of the time in the literature, is defined as the smallest fraction of contaminated data that can cause the estimator to take values arbitrarily far from the values obtained when the data are not contaminated. For example the least squares estimator has an empirical breakdown point of $1/n$. But some robust methods have a low breakdown point as well. For instance, Huber's M - estimator with monotonous ψ - function has an empirical breakdown point of $1/n$, too, which means that one outlier can break down the estimation.

S - estimators have an asymptotical breakdown point of $1/2$, which is the best possible asymptotic breakdown point. Moreover there are other good robustness properties, such as the exact fit property, which are fulfilled by S - estimators.

The idea of the S - estimators is based on a scale M - estimation, but in contrary to M - estimators the S - estimators first estimate the scale and subsequently, the regression parameter. So this estimator is scale invariant in contrary to M - estimators.

To define the S - estimators, let ρ be a real function satisfying the following assumptions:

1. ρ is symmetric, continuously differentiable and $\rho(0) = 0$;
2. there exists a $c > 0$, such that ρ is monotonously increasing in $[0, c]$ and constant in $[c, \infty)$.

For every set $\{e_1, \ldots, e_n\}$ the scale estimator $s(e_1, \ldots, e_n)$ is then defined as a solution of the equation

$$\frac{1}{n} \sum_{i=1}^{n} \rho(\frac{e_i}{s}) = K, \tag{2}$$

where the constant K is given by $E_\Phi[\rho] = K$ and Φ denotes the standard normal distribution. If (2) has more than one solution, $s(e_1, \ldots, e_n)$ is the supremum of all the solutions. If there is no solution, $s(e_1, \ldots, e_n) = 0$.

The S - estimator $\hat{\theta}$ of the regression parameter θ is defined as

$$\hat{\theta} = \min_{\theta} \{s[e_1(\theta), \ldots, e_n(\theta)]\} \tag{3}$$

and the scale estimator $\hat{\sigma}$ is

$$\hat{\sigma} = s[e_1(\hat{\theta}), \ldots, e_n(\hat{\theta})]. \tag{4}$$

2 Asymptotic normality

Unlike the consistency and the breakdown point of the estimator, the asymptotic normality depends on the properties of the error term. So we consider the linear regression model (2) and assume in what follows that the error term is a Gaussian long - memory process.

To investigate the asymptotic behaviour of S - estimators we need the Hermite rank of a function:

Definition (Hermite rank)

Let X be a standard normal random variable. A function $G : \mathrm{I\!R} \to \mathrm{I\!R}$ with $E[G(X)] = 0$ and $E[G^2(X)] < \infty$, is said to have Hermite rank m, if $E[G(X)P_q(X)] = 0$ for all Hermite polynomials $P_q, q = 1, \ldots, m-1$ and $E[G(X)P_m(X)] := J_G(m) \neq 0$.

The function $J_G(l)$ is defined by

$$J_G(l) := E[G(X)P_l(X)], \qquad l \in \mathrm{I\!N}. \tag{5}$$

Hermite polynomials, normalized by $q!$ provide an orthonormal basis in the L_2 - space with respect to the standard normal distribution. So every such function $G(\cdot)$ can be expanded into a series

$$G(X) = \sum_{q=m}^{\infty} J_G(q) \frac{P_q(X)}{q!}.$$

For $G(X)$ with Hermite rank 1, Taqqu(1975) showed that the normalized sum

$$n^{-H} L_{Var}^{-\frac{1}{2}}(n) \sum_{i=1}^{n} \frac{G(X_i)}{J_G(1)}$$

is asymptotically standard normal. Before proving the asymptotic normality of S - estimators we establish the consistency. Let X be the $n \times p$ matrix with rows $x_1^T, \ldots, x_n^T$. We have the following Lemma:

Lemma (consistency)

Let $\hat{\theta}_n$ and $\hat{\sigma}_n$ be the S - estimators for the regression parameter and scale respectively, in the linear regression (1) with long memory errors and let ρ be a function satisfying the assumptions 1) and 2) above with derivative $\rho' = \psi$. If $\sigma_0 > 0$ and

1. *$\frac{\psi(u)}{u}$ is nonincreasing for $u > 0$;*
2. *$E_H[\|X\|] < \infty$, and H has a density,*

then

$$\begin{aligned} \hat{\theta}_n &\to \theta_0 \qquad a.\ s. \\ \hat{\sigma}_n &\to \sigma_0. \end{aligned}$$

Proof

The consistency of the S - estimators is independent of the long - memory property of the error term. So the proof follows directly from Rousseeuw/Yohai(1984), Theorem 2. $\diamondsuit$

In the following theorem denote by ψ the derivative of the function ρ from (2).

Thus, we obtain a limit theorem for S - estimators:

Theorem (asymptotic normality)

Let $\hat{\theta}_n$ and $\hat{\sigma}_n$ again be the S - estimators for the regression parameter and scale respectively, in the linear regression (1) with long - memory errors. Then

if the conditions of the Lemma hold and

1) $\sum_{i=1}^{n} x_i \psi(\frac{y_i - x_i^T \hat{\theta}_n}{\hat{\sigma}_n}) = 0,$

where ψ is odd, continuously differentiable with absolutely continuous derivative ψ' and bounded second order derivative ψ'', which fulfill $\psi'(b) - \psi'(a) = \int_a^b \psi''(x)dx$;

2) $E_H(|x_{ij}x_{kl}x_{rs}|) < \infty \qquad \forall i, k, r = 1, \ldots, n; \qquad j, l, s = 1, \ldots, p;$

3) $E_\Phi \psi' \neq 0$ *for all* $\sigma > 0$.

Then we have:

$$\begin{aligned}
n^{1-H} L_{Var}^{-\frac{1}{2}}(n) J_{\psi(Q)}(1)^{-1}(\hat{\theta}_n - \theta_0) &\xrightarrow{d} N(0, \sigma_0^2 \frac{E_\Phi \psi^2}{(E_\Phi \psi')^2} E_G[X^T X]^{-1}) \\
n^{1-H} L_{Var}^{-\frac{1}{2}}(n) J_{\chi(T)}(1)^{-1}(\hat{\sigma}_n - \sigma_0) &\xrightarrow{d} N(0, \sigma_0^2 E_\Phi((\rho - K)^2)((E_\Phi e^T e \psi)^2)^{-1}),
\end{aligned}$$

where the function χ is defined as $\chi(x) = \rho(x) - K$.

Proof

We first prove the first relation. From assumption 1) we have

$$\begin{aligned}
\mathbf{0} &= n^{-H} \sum_{i=1}^{n} x_i \psi(\frac{y_i - x_i^T \hat{\theta}_n}{\hat{\sigma}_n}) \\
&= n^{-H} \sum_{i=1}^{n} x_i \psi(\frac{y_i - x_i^T \theta_0 - x_i^T \hat{\theta}_n + x_i^T \theta_0}{\hat{\sigma}_n})
\end{aligned}$$

$$= \quad n^{-H} \sum_{i=1}^{n} x_i \psi(\frac{e_i - x_i^T(\hat{\theta}_n - \theta_0)}{\hat{\sigma}_n}), \tag{6}$$

where $\mathbf{0}$ is the p - dimensional vector $(0, \ldots, 0)^T$.

For the function ψ we have the Taylor expansion

$$\psi(y+h) = \psi(y) + h \int_0^1 \psi'(y+th)dt. \tag{7}$$

Applying relation (7) twice to ψ we obtain

$$\psi(y+h) = \psi(y) + h \int_0^1 \psi'(y)dt + h^2 \int_0^1 \int_0^t \psi''(y+sh)dsdt. \tag{8}$$

Applying Fubini's theorem to the right - hand side of equation (8) gives

$$\psi(y+h) = \psi(y) + h[\psi'(y) + h \int_0^1 (1-s)\psi''(y+sh)ds]. \tag{9}$$

Setting $h_i := x_i^T(\hat{\theta}_n - \theta_0)/\hat{\sigma}_n$, relation (9) gives for (6)

$$\frac{1}{n}\sum_{i=1}^{n}[\psi'(\frac{e_i}{\hat{\sigma}_n}) - h_i \int_0^1 (1-s)\psi''(\frac{e_i}{\hat{\sigma}_n} - sh_i)ds]x_i x_i^T n^{1-H}\frac{\hat{\theta}_n - \theta_0}{\hat{\sigma}_n} = \\ n^{-H}\sum_{i=1}^{n}\psi(\frac{e_i}{\hat{\sigma}_n})x_i. \tag{10}$$

We now have to prove two assertions:

$$\mathbf{1)} \quad \frac{1}{n}\sum_{i=1}^{n}[\psi'(\frac{e_i}{\hat{\sigma}_n}) - \frac{x_i^T(\hat{\theta}_n - \theta_0)}{\hat{\sigma}_n}\int_0^1 (1-s)\psi''(\frac{e_i - s x_i^T(\hat{\theta}_n - \theta_0)}{\hat{\sigma}_n})ds]x_i x_i^T \\ \xrightarrow{P} \quad E_\Phi \psi' E_G[X^T X] \tag{11}$$

and

$$\mathbf{2)} \quad n^{-H}\sum_{i=1}^{n}\psi(\frac{e_i}{\hat{\sigma}_n})x_i \quad \xrightarrow{d} \quad N(0, E_\Phi[\psi^2]E_G[X^TX]). \tag{12}$$

Let us prove assertion 1) first.

To this end, we show first:

$$|\frac{1}{n}\sum_{i=1}^{n}\frac{x_i^T(\hat{\theta}_n-\theta_0)}{\hat{\sigma}_n}\int_0^1(1-s)\psi''(\frac{e_i-sx_i^T(\hat{\theta}_n-\theta_0)}{\hat{\sigma}_n})dsx_{ij}x_{il}| \xrightarrow{P} 0. \tag{13}$$

For this expression the following inequality holds, where $||f||_\infty$ denotes the ∞ - norm of the function f:

$$\begin{aligned} |\frac{1}{n}\sum_{i=1}^{n}\frac{x_i^T(\hat{\theta}_n-\theta_0)}{\hat{\sigma}_n}\int_0^1(1-s)\psi''(\frac{e_i-sx_i^T(\hat{\theta}_n-\theta_0)}{\hat{\sigma}_n})dsx_{ij}x_{il}| &\leq \\ |\frac{1}{\hat{\sigma}_n}|\,|\hat{\theta}_n-\theta_0|\,||\psi''||_\infty\,\frac{1}{n}\sum_{i=1}^{n}|x_i|\,|x_{ij}x_{il}| &\leq \\ |\frac{1}{\hat{\sigma}_n}|\,|\hat{\theta}_n-\theta_0|\,||\psi''||_\infty\frac{1}{n}\sum_{i=1}^{n}(\max_j|x_{ij}|)^3. & \end{aligned} \tag{14}$$

Because of the consistency of $\hat{\theta}_n$ and because the other terms on the right - hand side are restricted by assumptions 1) and 2), the right - hand side converges to zero. So relation (13) is verified.

To show (11) the first term on the left - hand side of this equation has to be considered. We have:

$$|\frac{1}{n}\sum_{i=1}^{n}x_{ij}x_{il}[\psi'(\frac{e_i}{\hat{\sigma}_n})-\psi'(\frac{e_i}{\sigma_0})]| \to 0. \tag{15}$$

This follows from the following inequality, which holds because of assumption 1):

$$
\begin{aligned}
|\frac{1}{n}\sum_{i=1}^{n} x_{ij}x_{il}[\psi'(\frac{e_i}{\hat{\sigma}_n}) - \psi'(\frac{e_i}{\sigma_0})]| &\leq |\frac{1}{\hat{\sigma}_n} - \frac{1}{\sigma_0}|\|\psi''\|_\infty \frac{1}{n}\sum_{i=1}^{n} |x_{ij}x_{il}||e_i| \\
&\xrightarrow{P} |\frac{1}{\hat{\sigma}_n} - \frac{1}{\sigma_0}|\, \|\psi''\|_\infty\, E|X^T X|\, E|e| \\
&\longrightarrow 0. \qquad (16)
\end{aligned}
$$

The right - hand side also converges to 0 because of the assumed consistency of $\hat{\sigma}_n$ and because the other terms are limited. It was also assumed that $\sigma_0 > 0$.

To prove the second assertion (12) let $Q : \mathsf{IR} \rightarrow \mathsf{IR}$ be a function defined as follows:

$$
Q(\eta) := \frac{\eta}{\hat{\sigma}_n}. \qquad (17)
$$

This function has Hermite rank 1 (see Taqqu(1975)). Define $J_{\psi(Q)}(1)$ as in (5).

In view of the limit theorem 5.1 from Taqqu(1975) we see that

$$
n^{-H} L_{Var}^{-\frac{1}{2}}(n) J_{\psi(Q)}(1)^{-1} \sum_{i=1}^{n} \psi(Q(e_i)) x_i
$$

is a normal random variable. To compute the mean and the covariance of this variable we get from ψ odd and because e_i is Gaussian:

$$
\begin{aligned}
E\psi(\frac{e_i}{\sigma_0}) &= -E\psi(-\frac{e_i}{\sigma_0}) \\
&= -E\psi(\frac{e_i}{\sigma_0}). \qquad (18)
\end{aligned}
$$

Hence

$$E\psi(\frac{e_i}{\sigma_0}) = 0 \tag{19}$$

and consequently by the independence of the e_i and x_i

$$E(\psi(\frac{e_i}{\sigma_0})x_i) = \mathbf{0}. \tag{20}$$

Again because the e_i are independent of the x_i, we have

$$Cov(\psi(\frac{e_i}{\sigma_0})x_i) = E_\Phi(\psi^2(\frac{e_i}{\sigma_0}))E_G[x_i x_i^T]. \tag{21}$$

Altogether we obtain from (11):

$$n^{1-H}L_{Var}^{-\frac{1}{2}}(n)J_{\psi(Q)}(1)^{-1}(\hat{\theta}_n - \theta_0) \xrightarrow{d} N(0, \sigma_0 \frac{E_\Phi \psi^2}{(E_\Phi \psi')^2} E_G[X^T X]^{-1}), \tag{22}$$

which is the first part of the theorem.

The proof of the second assertion is similar to the proof above. From (2) we have

$$n^{-H}\sum_{i=1}^{n} \rho(\frac{e_i}{\hat{\sigma}_n}) - K = 0. \tag{23}$$

For the function $\rho(e_i/\hat{\sigma}_n)$ we can write:

$$\begin{aligned}
\rho(\frac{e_i}{\hat{\sigma}_n}) &= \rho(\frac{e_i}{\hat{\sigma}_n} - \frac{e_i}{\sigma_0} + \frac{e_i}{\sigma_0}) \\
&= \rho\left(\frac{(\sigma_0 - \hat{\sigma}_n)e_i}{\sigma_0 \hat{\sigma}_n} + \frac{e_i}{\sigma_0}\right) \\
&= \rho(\frac{e_i}{\sigma_0} - \frac{(\hat{\sigma}_n - \sigma_0)e_i}{\hat{\sigma}_n \sigma_0}).
\end{aligned} \tag{24}$$

The function χ is now defined by

$$\chi(x) = \rho(x) - K. \tag{25}$$

Similar to the first part of the proof, χ fulfills the following equation:

$$\chi(y+h) = \chi(y) + h[\chi'(y) + h\int_0^1 (1-s)\chi''(y+sh)ds]. \tag{26}$$

Because of (24) we set

$$y := \frac{e_i}{\sigma_0} \tag{27}$$

and

$$h_i := -\frac{(\hat{\sigma}_n - \sigma_0)e_i}{\hat{\sigma}_n \sigma_0}. \tag{28}$$

From (26) we obtain:

$$\frac{1}{n}\sum_{i=1}^{n} e_i[\chi'(\frac{e_i}{\sigma_0}) - h_i\int_0^1 (1-s)\chi''(\frac{e_i}{\sigma_0} - s\frac{(\hat{\sigma}_n - \sigma_0)e_i}{\hat{\sigma}_n\sigma_0})ds]n^{1-H}\frac{\hat{\sigma}_n - \sigma_0}{\hat{\sigma}_n\sigma_0} = \\ n^{-H}\sum_{i=1}^{n}\chi(\frac{e_i}{\sigma_0}) \tag{29}$$

Similar to the first part of the proof we have to prove two assertions:

$$\mathbf{1)} \quad \frac{1}{n}\sum_{i=1}^{n} e_i[\chi'(\frac{e_i}{\sigma_0}) - h_i\int_0^1 (1-s)\chi''(\frac{e_i}{\sigma_0} - s\frac{(\hat{\sigma}_n - \sigma_0)e_i}{\hat{\sigma}_n\sigma_0})ds] \\ \xrightarrow{P} \quad E_\Phi[e^T e\psi] \tag{30}$$

and

$$\mathbf{2})\quad n^{-H}\sum_{i=1}^{n}\chi(\frac{e_i}{\sigma_0}) \quad\xrightarrow{d}\quad N(0,E_\Phi\chi^2). \tag{31}$$

Once again, we show assertion 1) first. To do this we prove:

$$|\frac{1}{n}\sum_{i=1}^{n}\frac{\hat{\sigma}_n-\sigma_0}{\hat{\sigma}_n\sigma_0}\int_0^1(1-s)\chi''(\frac{e_i}{\sigma_0}-s\frac{(\hat{\sigma}_n-\sigma_0)e_i}{\hat{\sigma}_n\sigma_0})dse_i^2| \xrightarrow{P} 0. \tag{32}$$

In view of assumptions 2) and 3), this can be seen as follows:

$$\begin{aligned}|\frac{1}{n}\sum_{i=1}^{n}\frac{\hat{\sigma}_n-\sigma_0}{\hat{\sigma}_n\sigma_0}\int_0^1(1-s)\chi''(\frac{e_i}{\sigma_0}-s\frac{(\hat{\sigma}_n-\sigma_0)e_i}{\hat{\sigma}_n\sigma_0})dse_i^2| \quad &\leq \\ |\hat{\sigma}_n-\sigma_0||\frac{1}{\hat{\sigma}_n\sigma_0}|\|\chi''\|_\infty\frac{1}{n}\sum_{i=1}^{n}|e_i^2| \quad &\xrightarrow{P} \quad 0.\end{aligned} \tag{33}$$

The first term after the $\leq$ converges to zero because of the consistency of $\hat{\sigma}_n$, the second term converges to $1/\sigma_0^2$ and the other parts of this side are bounded. Note that again $\sigma_0 > 0$.

The following result also holds

$$\frac{1}{n}\sum_{i=1}^{n}e_i\chi'(\frac{e_i}{\sigma_0}) \xrightarrow{P} E[e\chi']. \tag{34}$$

So the first assertion follows from $\chi'(e_i/\sigma_0) = e_i\psi(e_i/\sigma_0)$ because of (25).

Once again, the assertion 2) follows from Taqqu's limit theorem (Taqqu 1975 Theorem 5.1). To see this, let $T : \mathsf{IR} \to \mathsf{IR}$ be defined as

$$T(\eta) := \frac{\eta}{\sigma_0}. \tag{35}$$

The function T has Hermite rank 1. Define $J_{\chi(T)}(1)$ as in (5).

Then Taqqu's limit theorem gives

$$n^{-H} L_{Var}^{-\frac{1}{2}}(n) J_{\chi(T)}(1)^{-1} \sum_{i=1}^{n} \chi(T(e_i)) \xrightarrow{d} N(0, E_\Phi \chi^2). \tag{36}$$

With $\chi(x) = \rho(x) - K$ the assertion follows from (36). $\diamondsuit$

The theorem above establishes the asymptotic normality of S - estimators, when the error terms have long - memory. It also establishes a good asymptotic efficiency of S - estimators as compared to the least - squares estimator and the BLUE, because S-estimators have the same rate of convergence compared to the least-squares estimator and the BLUE.

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