**Long-memory in volatilities of German stock returns** [1]

by

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**Abstract**

We show that there is strong evidence of long-range dependence in the volatilities of several German stock returns. This will be done by estimating the memory parameter of the absolute returns with classical log-periodogram regression as well as by employing the tapered periodogram. Both estimators give similar values for the memory parameter what indicates long-memory.

KEY WORDS: Long-memory, volatilities, log-periodogram estimation

# 1 Introduction

It is an intensively discussed problem whether or not stock returns themselves and the squared or absolute returns exhibit long-range dependence (Ding et al.(1993), Baillie et al.(1996), Lobato/Savin(1998) and many others). Even

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though Willinger et al.(1999) again found evidence for long-memory in stock returns it is a widely accepted thesis that stock returns themselves do not follow a long-memory process. This holds true also for the German data considered in this paper (Krämer et al.(2001)).

It is still an open problem, if there is long-memory in the absolute or squared returns. Standard methodology indicates a strong evidence of long-range dependence. But this long-memory might be an effect artificially produced by trends or structural breaks.

Slowly decaying trends and structural breaks can easily be confused with long-range dependence by using standard methodology.

Krämer/Sibbertsen(2000) showed in this context that tests on structural breaks reject the hypothesis of no structural break with a probability tending to one if there is only long-memory present in the data. On the other hand Giraitis et al.(2000) showed that $R/S$-based estimators of the memory parameter estimate a long-memory effect if the data consists only of structural breaks or slowly decaying trends. For an overview see Sibbertsen(2001a).

This problem does not hold only for $R/S$-methodology. Also standard log-periodogram based estimators of the memory parameter are strongly biased if there are slowly decaying trends or structural breaks in the data.

Sibbertsen(2001b) showed by Monte Carlo that employing the tapered periodogram when estimating the memory parameter reduces the bias when trends are present in the data. The tapered periodogram is much more robust against trends and other non-stationarities as the classical periodogram.

The idea of this paper is to employ log-periodogram based estimators for the memory parameter to absolute returns of various German stocks. We apply the classical periodogram based estimator introduced by Geweke/Porter-Hudak(1983) as well as this estimator based on the tapered periodogram. Following the results of Sibbertsen(2001b) we can conclude that the data exhibits long-range dependence or at least no trends or structural breaks if both estimators give a similar estimation of the memory parameter. If the tapered periodogram based estimator gives a smaller parameter value this result would indicate a trend and no long-range dependence.

This paper is organized as follows. In the next section long-memory and log-periodogram regression is introduced. Section three gives our results for various German stocks and section four concludes.

# 2 The Estimation Procedures

In this section long-memory and log-periodogram regression is defined. Long-range dependence was first observed by the hydrologist Hurst(1951) while building the Aswan dam. Hurst considered the minimal water flow of the Nile river and found evidence for long term dependencies. In the meantime it turned out that the water flow of many other rivers exhibit long-range dependencies (see Lohre/Sibbertsen(2001)). But also many economic data show evidence of long-memory. This is especially the case for exchanges rates and volatilities of stock returns. For an overview see Baillie(1996).

We say a time series $X_t$, $t = 1, \ldots N$ exhibits long-memory or long-range dependence when the correlation function $\rho(k)$ behaves for $k \to \infty$ as

$$\lim_{k \to \infty} \frac{\rho(k)}{c_\rho k^{2d-1}} = 1. \tag{1}$$

Here $c_\rho$ is a constant and $d \in (0, 0.5)$ denotes the memory parameter. This means that observations far away from each other are still strongly correlated. The correlations of a long-memory process decay slowly that is with a hyperbolic rate.

An equivalent definition to (1) uses the spectral density of the time series. In this context a time series $X_t$ is said to exhibit long-memory if the spectral density $f(\lambda)$ behaves for $\lambda \to 0$ as

$$\lim_{\lambda \to 0} \frac{f(\lambda)}{c_f |\lambda|^{-2d}} = 1. \tag{2}$$

Here $c_f$ is a positive constant and again $d \in (0, 0.5)$ denotes the memory parameter. That is the spectral density has a pole at the origin.

For details concerning long-memory time series see for example Beran(1994) or Sibbertsen(1999).

The long term dependence structure of a long-memory time series allows for long term forecasts. Having in mind that the volatilities as a measure of risk are the only quantity concerning the stock having an influence on the price of a stock option the possibility of long term forecasts of the squared returns would result in a different valuation of the option. This would allow arbitrage. Thus the question whether volatilities do or do not exhibit long-range dependence is of strong consequences for evaluating stock options. The behaviour of the option price when considering a long-memory behaviour of the volatilities is considered in Bollerslev/Mikkelsen(1996). In some situations including long-memory doubles the price compared to the situation neglecting it.

On the other hand it is a well known fact that structural breaks or slowly decaying trends can easily be misspecified as long-memory as described in the last section.

Sibbertsen(2001b) showed that log-periodogram based estimators for the memory parameter provide a possibility for distinguishing both of these phenomena. Log-periodogram based estimators are popular in practice because of their simplicity. Whereas small trends do not influence these estimators they are strongly biased in case of slowly decaying trends or structural breaks.

It can also be shown that applying the tapered periodogram reduces the effect of trends and structural breaks. Thus comparing standard log-periodogram regression with log-periodogram regression based on the tapered periodogram gives an indicator whether the data exhibits long-memory or not.

Log-periodogram regression was introduced by Geweke/Porter-Hudak(1983) and is denoted as GPH-estimator in what follows. For defining the estimator denote with

$$I_X(\lambda_j) := \frac{1}{2\pi N} \left| \sum_{t=1}^{N} X_t \exp\left(\frac{-it2\pi j}{N}\right) \right|^2$$

the periodogram of the process $X_t$.

The GPH-estimator is based on the special shape of the spectral density (2). It is defined as the least-squares estimator of $d$ based on the regression equation

$$\log I_X(\lambda_j) = \log c_f - 2d \log \lambda_j + \log \xi_j, \tag{3}$$

where $\lambda_j$ denotes the $j-th$ Fourier frequency, that is $\lambda_j = 2\pi j/n$ and the $\xi_j$ are identically distributed errors with $E[\log \xi_j] = -0.577$, known as Euler constant.

Hurvich et al. (1998) showed that under some regularity conditions the GPH-estimator is asymptotically normal. The optimal number of frequencies wich should be used for the regression (3) is proportional to $N^{4/5}$.

Besides the problem of choosing the number of frequencies used for the estimation the GPH-estimator has several advantages. Because of its semiparametric structure no further knowledge of the underlying distribution of the data or eventual short-range dependencies is necessary.

But it is strongly influenced by slowly decaying trends or structural breaks resulting in a huge bias. Even though the underlying noise process is only white noise the GPH-estimator can be biased into the non-stationary region if there are trends in the data.

This estimator can be modified by using the tapered periodogram instead of the standard periodogram for estimating the spectral density. This modification provides more robustness against trends and structural breaks in the data.

The periodogram of the tapered process $w_t X_t$ is defined by

$$I_{T,X}(\lambda_j) = \frac{1}{2\pi \sum w_t^2} | \sum_{t=0}^{N-1} w_t X_t e^{-i\lambda_j t}|^2.$$

Here $\lambda_j$ again denotes the j-th Fourier frequency and $w_t$ denotes the taper. We use in this paper the full cosine bell taper given by

$$w_t = \frac{1}{2}[1 - \cos(\frac{2\pi(t+0.5)}{N})].$$

The taper is a smoothing function weighting down the influence of the low frequencies and thus of non-stationarities. So the idea is that the tapered periodogram will reduce the influence of trends or structural breaks on the estimation of the memory parameter.

In the case of no trends the tapered log-periodogram estimator is a consistent estimator for the memory parameter. But of course tapering the periodogram increases the variance of the estimator.

Sibbertsen(2001b) showed that comparing both of these estimators gives an indicator whether the data exhibits long-range dependence or not. If the estimated parameter is much smaller when applying the tapered periodogram based estimator compared to the GPH-estimator based on the standard periodogram this indicates a trend or structural break. On the other hand if both estimations give a nearly similar value this indicates long-memory.

In the following this method will be applied to volatilities of German stock returns.

# 3 Empirical Results

In this section we apply the method described in the last section to volatilities of German stock returns. We therefore consider daily returns of BASF, BMW, Daimler, DAX, Deutsche Bank, Dresdner Bank and Hoechst beginning from 4. 1. 1960 up to 30. 4. 1998. Thus we have approximately 9590 observation for each stock. We consider the absolute returns in this paper. The dependence structure of the absolute returns is similar to this of squared return but the long-memory effect is better visible by considering absolute returns. This is why absolute returns are used in this paper.

Standard analysis, that means considering the autocorrelations and the periodogram, show clear evidence of long-range dependence (see figure 1 in the Appendix). For simplicity we show only the autocorrelations of the series. But the results hold also true by using spectral analysis and $R/S$-methodology (see also Krämer et al.(2001)) .

Estimating the memory parameter with the GPH-estimator and tapered GPH-estimator (TGPH) result in

**Table I** *GPH- and tapered GPH-estimator for daily absolute returns of 7 German stocks*

| | GPH | TGPH |
|---|---|---|
| BASF | 0.235 | 0.24 |
| BMW | 0.225 | 0.247 |
| Daimler | 0.27 | 0.275 |
| DAX | 0.285 | 0.3 |
| Deutsche Bank | 0.235 | 0.24 |
| Dresdner Bank | 0.241 | 0.214 |
| Hoechst | 0.22 | 0.224 |

From Table I it can be seen that the standard GPH-estimator and the tapered GPH-estimator are almost same. In each case except Dresdner Bank the tapered GPH-estimator is slightly larger than the standard estimator. This clearly indicates long-range dependence. Thus trends or structural breaks seem not to be responsible for the observed long-memory effect. Long-range dependencies seem to be present in the absolute returns of German stocks.

The residuals show no more evidence of any dependence structure. The autocorrelations are shown in Figure 2 in the Appendix.

The number of frequencies used for the estimators is computed by using a plug-in estimator provided in Hurvich/Deo(1999). This choice is MSE-optimal.

Thus altogether there is clear indication of long-range dependence in the absolute returns of these German stocks. Structural changes or trends do not effect the estimations of the memory parameter.

# 4 Conclusion

Absolute daily returns of seven German stocks are considered. All of them show evidence of long-memory by using standard methodology. The aim of this paper is to prove whether this long-range dependence is an artefact of trends or structural breaks or if there is real evidence of long-memory.

This is done by comparing standard log-periodogram regression for the memory parameter with tapered log-periodogram regression. Tapering the periodogram reduces the effect of non-stationarities to the estimator. Thus in case that both estimators differ this would indicate trends or structural breaks instead of long-memory. On the other hand if the estimated values are equal this would indicate long-range dependence.

Absolute daily returns of BASF, BMW, Daimler, DAX, Deutsche Bank, Dresdner Bank and Hoechst are considered. For all of them the standard GPH-estimator and the tapered GPH-estimator estimate similar values for the memory parameter. In all cases except Dresdner Bank the tapered estimator is slightly larger than the standard GPH-estimator. This indicates real long-range dependence in the data not structural breaks or trends. The slightly larger value of the tapered GPH-estimator can be explained with its higher variance.

After eliminating the long-memory structure in the data the residuals show at most short-term dependencies. Thus the long-term structure of the data is eliminated. This also indicates long-range dependence.

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# Appendix

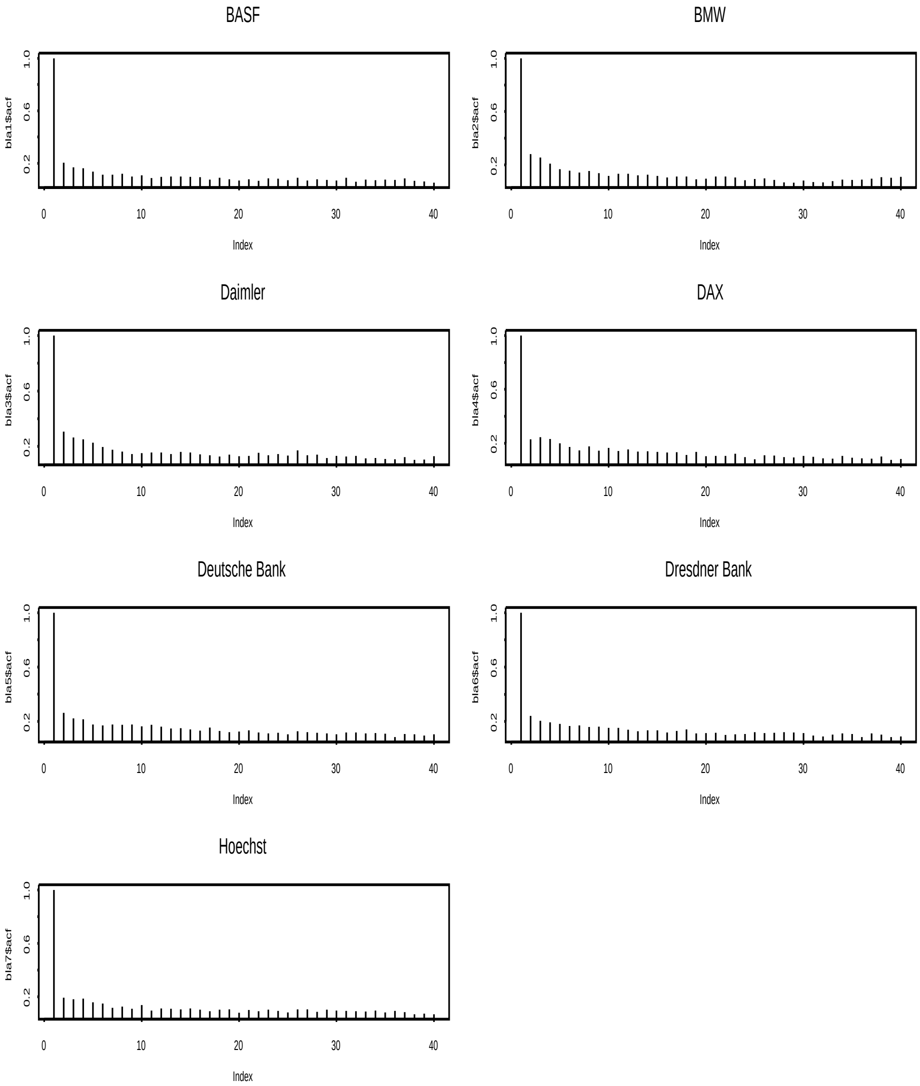


Figure 1: Autocorrelations of absolute daily returns of seven German stocks

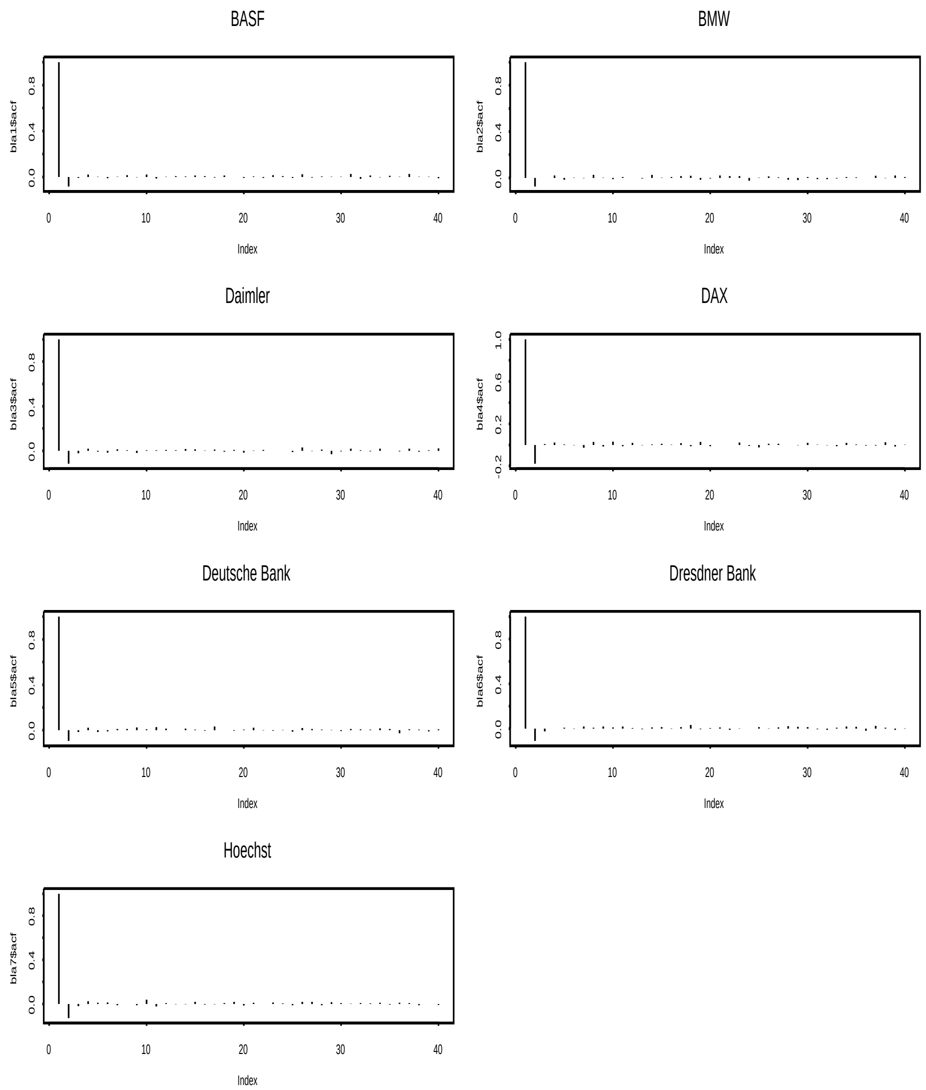


Figure 2: Autocorrelations of the residuals of absolute daily returns of seven German stocks