HEALY, Lulu
King's College London, UK

Sensing and making sense of difference in mathematics learning

Abstract

This short paper summarises key issues raised in the keynote presentation. It offers reflections on some of the challenges involved in constructing an inclusive school mathematics: a school mathematics in which classroom practices and discourses are congruent with the diverse ways that different students sense and make sense of mathematical knowledge. It considers how discriminatory visions of students' potential for learning, compounded by the myth of independence and the artificial separation of cognition from emotion, can act as obstacles which deter or disallow some learners from engaging with mathematics. It outlines how our attempts, together with teachers and students, to resist and disrupt such exclusionary practices have led us to reject binary oppositions such as those that align independence with strength and dependence with weakness, or that value "abstract" above "concrete" representation, as we attempt to create classroom cultures aimed at legitimising and validating disabled embodiment and sensibility.

An ongoing journey: Towards an inclusive mathematics education

I would like to begin by thanking the conference organisers for their kind invitation to contribute to the 57th annual conference of the German Society for Mathematics Education (GDM). It is both and honour and a pleasure. It has also given me the opportunity to reflect back over more than 20 years of the research programme *Towards an Inclusive Mathematics Education*, which began in São Paulo, Brazil as a collaboration between a small group of researchers and mathematics teachers in 2002 studies with a shared interest in understanding the teaching and learning processes of mathematics students socially characterized as belonging to the target audience of Special Education. Over the years, the programme expanded into many other states in Brazil as well as crossing international borders.

When we began, the area of Inclusive Mathematics Education was relatively new, and we were perhaps a little naïve to the extent of its complexities. We were – and still are – motivated by two aspirations, both of which are strongly committed to social justice. The first focuses on creating learning situations that incorporate tools and pedagogies that might empower those who interact with them (be they typically developing or not) to participate successfully and meaningfully with school mathematics, rather than disabling them from doing so. The second focuses on the nature of school

In: P. Ebers, F. Rösken, B. Barzel, A. Büchter, F. Schacht & P. Scherer (Hrsg.),
Beiträge zum Mathematikunterricht 2024.
57. Jahrestagung der Gesellschaft für Didaktik der Mathematik. WTM.
https://doi.org/10.37626/GA9783959872782.0

mathematics itself. Is it a discipline that stimulates critical engagement with the mathematical practices that take place inside the classroom and with what happens outside of it? To what extent are existing school mathematics practices complicit in disabling certain groups of students? Is school mathematics something that students (if they could choose) would want to be included in?

Our overriding aim is to contribute to the development, characterisation and dissemination of classroom practices and discourses which are congruent with the diverse ways that different students sense and make sense of mathematical knowledge. This paper documents aspects of our journey over the last two decades: the theoretical influences which have shaped the interventions we have designed and our analyses of them, the methodological approaches we have adopted on different projects and what we have learnt along the way. It is divided into three parts. The first centres on learning, and how it is affected by the distinct forms through which we, differently, access and interpret the world. The second considers teaching and the challenge of questioning and resisting the ableist perspectives that currently permeate mathematics education. The third looks at inclusive design and argues for a need to move away from notions of normality when creating learning scenarios, curricula and assessments, privileging instead different ways of doing mathematics, as well as encouraging and validating multiple ways of being mathematical.

Learning: Difference as diversity

At the start, we worked closely with teachers and with disabled and neurodiverse learners, as well as those identified as experiencing difficulties in thriving mathematically in their current school environments, to collaboratively design mathematical activities that would enable them to explore chosen mathematical concepts. We first worked in small groups and videoed our interactions with the activities we created, before extending our research activities into inclusive mathematics classroom settings. Theoretical underpinning was drawn from Vygotsky's work with disabled learners in the 1920s and 30s (Vygotsky, 1993). His work is also infused with a concern for social justice and he argued that the key to empowering learners to develop capacities for a satisfying and constructive life is to treat disability as a potential strength rather than as an inherent deficit.

Our initial focus was on supporting inclusion in mathematics teaching and learning practices using what Pais and Valero (2011) have termed the micro-view and, as we sought to understand these processes, we made connections between aspects of Vygotsky's theory and more contemporary ideas from

embodied cognition (Barsalou, 2008; Gallese, 2010). As our journey progressed, we have sought also to revision the micro-view in the context of the macro-social conditions in which the teaching and learning practices occur, drawing in particular on constructs from the field of disability studies (Meekosha & Shuttlesworth, 2009; Vale & Connor, 2011; Goodley, 2020).

From the beginning, we recognised the situated and embodied nature of mathematical practices, the roles played material and semiotic tools in sensing and making sense of mathematics as well as the unity/integration of cognition and affect (Vygotsky, 1994). From our analyses of the videos we collected in the early years, we saw how students' senses of the mathematical concepts they explored were shaped not only by the material and semiotic tools available for use, but also by the constraints and affordances of their own bodies. In Healy and Fernandes (2011), for example, we report on how blind students use their hands rather than their eyes as tools for seeing and how this way of seeing serves as a means to create and express embodied abstractions associated with the identification or definition of general mathematical relationships (p. 171). Similarly, an example of how speaking with hands, using signed rather than spoken language affected the deaf students' senses of mathematics concepts, is their choice to use the sign for "secret" to refer to algebraic variables, which, in Brazilian sign language is expressed using a gesture that involves indicating something hidden in a sleeve (Fernandes and Healy, 2014).

Bringing together aspects of Vygotsky's views on tool mediation with more contemporary ideas from embodied cognition drew our attention to two important issues. First, the tool-dependent nature of mathematical – and indeed all human – activity. Disability scholars (e.g. Goodley, 2020) have pointed to a tendency of positioning independence as desirable and dependency as weakness. All our analyses pointed to the centrality of the tools used to the mathematical ideas our research participants expressed. Offering learners tools that are congruent with their bodies enabled them to sense and hence make sense of mathematical concepts. The tools also become part of the that sense (Healy & Fernandes, 2023). This has led us to suggest that confining, or even prohibiting access to such tools serves an obstacle to mathematics learning, as does positioning the use of certain tools – usually those deemed as "concrete – as less valuable or sophisticated than – "abstract" – others (Finesilver, Healy & Bauer, 2022).

Second, our analyses indicate a strong relationship between the notion of *simulation* from embodied cognition (Barsalou, 2008) and the cognition/affect unity proposed by Vygotsky (1994). We saw how learners acted in the present by drawing on their senses of past activities, as they re-enacted, or

simulated, the actions, emotions and sensations associated with them. As they reused a previously experienced concept, it was not just some decontextualized version of its meaning that was recalled. Also re-enacted, and refelt, were other aspects associated with their sense of the concept, the processes by which they came to know it and how they felt during these processes. Here again, we see connection between micro- and macro views of inclusion. The feelings evoked during mathematical activity become part of both the learners' senses of the concept in question and their sense of themselves as mathematics learners. When discourses and practices that disable rather than able participation frame their mathematics learning, they may experience feelings of inadequacy, failure or exclusion. Such feelings may hence become part of the contents' sense, and there is a danger they will be simulated each time it is met. Perhaps not surprising then that, in her interviews with young mathematics learners (9-11 years) identified as having difficulties in learning mathematics, Bauer (2020) found that none of the students expressed positive sentiments about doing mathematics. They all said that they felt confused, many felt scared, others sad, angry or embarrassed. Some said they felt like crying or actually cried.

How might such feelings be avoided? Partly perhaps by acknowledging that depending on the instruments and signs that we use when we see, talk, remember etc., the ways that we learn and perform can be expected to vary. Partly, by recognising tools are an integral part of thinking, and considering very carefully the learning tools we offer and legitimise, as well as the consequences of taking them away. And partly by becoming more aware of the disabling discourses and practices that the students we teach sense. The next section considers confronting teachers with this last point.

Affecting teachers: Difference as inspiration, expansion, disruption

Much of the work on our research programme has involved long-term collaborations with teachers and students (see, for example, Healy & Santos, 2014). In recent years, we have also been exploring ways of effectively sharing what we have learnt together more widely to practising and future teachers other than those who have acted as co-researchers. In the work on the CAPTeaM project (Challenging Ableist Perspectives on the Teaching of Mathematics), we have been researching ways of providing opportunities for practising and future teachers to reflect upon and to disrupt at least some of the ableist assumptions that currently affect our interpretations of mathematics teaching and learning. Our approach is to design tasks that attempt to share how positioning disability, or more generally difference, as a strength rather as deficit, and to allow participants to feel both the mathematics of

difference and how their students might experience the impact of current exclusionary discourses and practices.

Again, we have been influenced by the embodied perspective:

"Anytime we meet someone, we are implicitly aware of his/her similarity to us, because we literally embody it. The very same neural substrate activated when actions are executed or emotions and sensations are subjectively experienced, is also activated when the same actions, emotions and sensations are executed or experienced by others." (Gallese, 2010, p. 81).

For us this citation raises an important issue. It focusses on similarity, yet teaching involves learning to inhabit bodies that may be different to our own. Essentially, our aim was to explore the affects of inviting research participants to do just that. To this end, we devised two types of tasks. Type 1 tasks provide opportunities for teachers to reflect on episodes in which disabled students engage in interesting ways with mathematics. Type 2 tasks invite participants to engage in mathematical activities while they themselves are temporarily constrained from using some of the sensory or communication means that might usually be available to them. For example, one of the Type 1 tasks shared video clips of a blind student constructing a physical model of a square-based pyramid and then describing it as a pile of squares whose sides decrease until they form the point at the apex. In a Type 2 task, one participant might be asked to communicate, without speaking, a multiplication problem – say 305 x 67 – to another participant who has to keep their eyes shut, then assist them in effectuating the calculation, also without speaking.

Our results indicated that the Type 1 tasks did encourage teachers to imagine inhabiting the body of the student whose work they observed. One of the ways that this was evidenced was in a strong tendency to re-enact the gestures and strategies that the student employed. We also found that this task type encouraged the teachers to discern – and sense – the potency and creativity of these students' mathematical productions, to recognise that they are not a priori mathematically deficient and, on some occasions, even to transform their own thinking about the mathematical objects at play (Nardi, Healy, Biza and Fernandes, 2018). Type 2 tasks, on the other hand, motivated the participants to seek new ways of expressing themselves mathematically, to recognise (and re-evaluate) their own tool-dependencies and to develop productive interdependencies which disrupted aspects of the conventional roles of teachers and learners. Some participants became involved in questioning the privileging of solitary performance of mathematics and in rethinking the relationships between formal written methods and other forms of mathematical agency. Type 2 tasks also provoked deeply felt reflection

on the ways that difficulties in understanding each other's attempts to communicate might impact on the learners' and/or the teachers' sense of self (Healy, Nardi & Biza, 2024).

Design: Difference as renewal

This last section focuses on design. In addition to Vygotsky's principal of treating disability as a strength (mentioned earlier), here too we have been influenced by ideas from critical disabilities studies and by approaches to design (originally developed in the context of architecture) which resist use of *normate templates*.

"*Rather than accounting for diverse body types, sizes, and abilities, the normate template privileges a small group of individuals in mainstream design, giving these individuals the appearance of normalcy or universality due to their fit in the environment. The resulting built environment is precisely what the social model criticizes—a world built without considering all ranges of ability.*" (Hamrie, 2012; s/p)

We would argue that the normate template still characterises the way mathematics curricula and learning activities are currently developed – the starting point for design process is a view of "normal" or "typical" student. Hence, in educational institutions, tasks, curricula and assessment are designed without regards to the diverse bodies and abilities of those who are subjected to them. The idea, at best, being that only later might these be adapted for those deemed to have special needs. This seems completely counter-intuitive as a design paradigm. Surely at the stage of design, it makes no sense to plan something without thinking of the all the different people they are intended for, and only subsequently make adaptions? Apart from anything else, this seems like a very inefficient design strategy. Wouldn't it be more sensible to conceptualise inclusive structures that everyone can navigate from the beginning of the design process? We would argue that this is fundamental for any design paradigm aiming for an inclusive mathematics education. In our approach to design, no part of the process is based on normalisation. Instead, we endeavour to imagine things otherwise (Runswick-Cole, 2014) and create learning scenarios which from the outset validate "both a disabled embodiment and sensibility" (Overboe, 1999, p. 22).

The learning scenarios we have developed hence recognise and support many ways of doing mathematics and being mathematical:

"They offer multiple ways of acting with multiple ways of representing mathematical objects – using colour, sound, music, movement, textures and manipulatable tools so that mathematics can be experienced through different sensory canals, via the skin, fingers, ears and eyes, as well as through spoken and signed languages, and conventional mathematical representations." (Fernandes & Healy, 2020, p. 306)

During the many years in which we have had the opportunity to collaborate with learners labelled as different and with their teachers, we have seen glimmers of what is possible when we design otherwise. Yet, inclusion will continue to be partial and sporadic if such a paradigm is not extended beyond the micro-view of teaching and learning in the classrooms we work in to the macro-level and to the design of institutional structures such as mathematics curricula and high-status assessments.

In my view, this should be the focus of the next stages of the journey towards an inclusive mathematics education.

Acknowledgements

The research reported in this paper was supported by funding received in Brazil, from the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES), No. 23038.019444/2009-3, from the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), Nos. 303390/2010-6 and 311827/2013-6, and from the Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP), No. 2004/15109-9. CAPTeaM is an International Partnership and Mobility project between institutions in the UK and Brazil funded by the British Academy (Nos. PM140102 and PM160190).

References

Barsalou, L. W. (2008). Grounded cognition. *Annual Review of Psychology, 59*.

Bauer, A. (2020). *An exploration of feelings, memory and time in the mathematical education of pupils with special educational needs, including mathematical learning difficulties.* EdD Thesis. King's College, London.

Fernandes, S.H.H.A & Healy, L. (2014). Algebraic expressions of deaf students: Connecting visuo-gestural and dynamic digital representations. In: S. Oesterle, P. Liljedahl, C. Nicol, & D. Allan (Eds.). *Proceedings of the Joint Meeting of PME 38 and PME-NA 36* (Vol. 3 pp. 49-56). Vancouver, Canada: PME.

Fernandes, S.H.A.A & Healy, L. (2020). Mathematics Education in Inclusive, Plurilingual and Multicultural Schools. In: L. Leite, E. Oldham, A.S. Afonso, F. Viseu, L. Dourado, & M.H. Martinho (Eds). *Science and mathematics education for 21st century citizens: Challenges and ways forward* (pp. 301-321). New York: Nova Science Publishers, 2020.

Finesilver, C., Healy, L., & Bauer, A. (2022) Supporting diverse approaches to meaningful mathematics: from obstacles to opportunities. In Y.P. Xin, R. Tzur & H. Thouless (Eds.), *Enabling Mathematics Learning of Struggling Students: International Perspectives*. New York: Springer.

Gallese V. (2010). Embodied Simulation and its Role in Intersubjectivity. In: T. Fuchs, H.C. Sattel, P. Henningsen (Eds.) *The Embodied Self. Dimensions, Coherence and Disorders*. Stuttgart: Schattauer, pp. 78-92.

Goodley, D. (2020). *Disability and other human questions*. Emerald Publishing Limited.

Hamraie, A. (2012). Universal Design Research as a New Materialist Practice. *Disability Studies Quarterly 32*(4).

Healy, L., & Fernandes, S.H.A.A. (2011). The role of gestures in the mathematical practices of those who do not see with their eyes. *Educational Studies in Mathematics, 77*, 157–174.

Healy, L. & Fernandes, S.H.A.A. (2023). Investigando inclusão, exclusão e diferença na educação matemática: uma aventura inspirada em Vygotsky. *Revista Sergipana de Matemática e Educação Matemática, 8*(2), 1–23.

Healy, L., Nardi, E. & Biza, I. (2024). Interdependency, alternative forms of mathematical agency and joy as challenges to ableist narratives about the learning and teaching of mathematics. *ZDM Mathematics Education, 56*, 379–391.

Healy, L., & Santos, H.F. (2014). Changing perspectives on inclusive mathematics education: Relationships between research and teacher education. *Education as Change*, *18*(1), 121-136.

Meekosha, H., & Shuttlesworth, R. (2009). What's so 'critical' about critical disability studies? *Australian Journal of Human Rights, 15*, 47–75.

Nardi, E., Healy, L., Biza, I. & Fernandes, S. H. A. A. (2018). "Feeling" the mathematics of disabled learners: Supporting teachers towards valuing, attuning, integrating and resignifying in an inclusive mathematics classroom. In: R. Hunter; M. Civil; B. Herbel-Eisenmann; N. Planas; D. Wagner. (Eds.). *Mathematical Discourse that Breaks Barriers and Creates Space for Marginalized Learners*. (pp. 147-170). Rotterdam: Sense Publishers.

Overboe, J. (1999). Difference in Itself: Validating Disabled People's Lived Experience. *Body & Society, 5*(4), 17-29.

Pais, A., & Valero, P. (2011). Beyond disavowing the politics of equity and quality in mathematics education. In B. Atweh, M. Graven, W. Secada & P. Valero (Eds.), *Mapping equity and quality in mathematics education* (pp. 35-48): Springer.

Runswick-Cole, K. (2014). 'Us' and 'them': The limits and possibilities of a 'politics of neurodiversity' in neoliberal times. *Disability & Society, 29*(7), 1117-1129.

Valle, J., & Connor, D. (2011). *Rethinking disability: A disability studies approach to inclusive practices*. McGraw-Hill.

Vygotsky, L. S. (1993). *The fundamental problems of defectology. Collected Works of L.S. Vygotsky* (volume 2). Plenum.

Vygotsky, L. S. (1994). The problem of the environment. In: R. van der Veer, & J. Valsiner (Eds.), *The Vygotsky reader* (pp. 338–354). Blackwell.