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Testing AI Companion for Solving the Fermi Problems: Implications for the Solution Categorization Framework

Introduction

Fermi Problems (FPs) are open-ended tasks that are open to modeling the estimation-based situations considering realistic assumptions. Such problems carry potentials to promote higher level mathematical thinking and even encourage STEM-based thinking (Ärlebäck & Albarracín, 2023). Prior research investigated solution methods of a wide range of learners and categorized these solution methods based on the way of thinking they possessed (e.g., Albarracín & Gorgorió, 2014). Although we have known a set of empirically founded solution methods for FPs, we have not known yet how Artificial Intelligence (AI) can support learners in solving FPs. To understand this phenomenon, we first need to explore the range of solution methods that an AI can produce and the extent of usability that an AI could identify for FPs with different characteristics. Given that AI has been considered recently as one of the fundamental educational supports for improving teaching and learning mathematics (Mao et al., 2024), this study addresses the following research questions:

- RQ1: What are the solution methods that AI, particularly ChatGPT, can produce for three pairs of FPs?
- RQ2: How does the AI, particularly ChatGPT, identify the usefulness of the solution methods considering the characteristics of the FPs in each pair (i.e., requiring educated assumptions vs. external search of the resources)?

Theoretical Framework

Due to their nature, FPs are open to multiple educated assumptions, rather than guesses, and they present the opportunity of several solution methods to the problem solvers. As a result of an empirical study, Albarracín and Gorgorió (2014) presented a comprehensive solution categorization for the FPs:

- Informed decision based on external source: Gathering some reliable information from external source such as responsible staff or internet.
- Exhaustive count: Attempting to count the items although there may be different strategies for counting.

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- Problem reduction of and use of proportion: Reducing the problem into a smaller scale and then applying a scaling factor to solve the problem proportionally.
- Use of concentration measures: Concentrating on a part of the whole and finding the ratio of number of objects in the part, which then be scaled up to the whole.
- Iteration of the base (reference) unit: Estimating the number of elements in a reference unit and dividing the whole by the unit measurement.
- Grid distribution: Organization of the elements in rows and columns and then finding the Cartesian product of the measures.

Although this categorization included several ways of approaching the FPs, all are not efficient in all FPs. The researchers compiled this list of strategies from several problem-solvers' solution plans for several FPs. Yet, this list of methods was comprehensive enough to categorize problem-solvers' solutions. However, whether AI can provide us another solution method to expand this list and help us to decide which method is particularly useful for which FP is an intriguing question to explore in this study.

Methods

This study involves three pairs of Fermi problems (see Fig. 1). Each pair indicates a different characteristic of a Fermi problem, yet each pair has one problem with an unknown size to be filled with some elements (i.e., regular, semi-regular or irregular) and one with a size that was either specified (i.e., 1 meter-cube safe box) or could be reached out through internet search (i.e., area of the Central park and volume of an Olympic pool). The methodological conjecture asserts that the usefulness of the solution method may change depending on whether the size of a quantity could be reached through external sources or whether it requires some legitimate assumptions.

Those FPs were used in empirical studies earlier and their solution methods were already categorized; however, whether AI may approach each FP in the pairs differently and present additional solution methods drove this research. Therefore, the following prompt was inserted in GPT-4o chatbot (without activating internet search option) for each pair of the FPs: *Please list all the solution methods for each problem and then compare the usability of these methods considering the problem contexts and geometric properties involved.* The outputs drawn between Dec. 30, 2024 and Jan. 4, 2025 were analyzed to identify the solution methods and compared with the ones proposed in Albarracín and Gorgorió's study (2014).

Pair 1 involves irregular elements (people and trees) to be placed in a place that involves assumptions to make sense with (1a – School courtyard) and a place that exact measures could be found based on an internet search (1b – Central Park).	**1a. School Courtyard Problem:** How many people could we fit in a school courtyard? (Brunet-Biarnes & Albarracín, 2024)	**1b. Central Park Problem:** How many trees are there in Central Park? (Ärlebäck & Albarracín, 2023)
Pair 2 involves volume estimation in a container that involves assumptions (2a – Bucket) and a known size of volume (2b – Olympic pool).	**2a. Bucket Problem:** How many glasses of water do we need to fill a whole bucket? (Albarracín & Gorgorió, 2014)	**2b. Olympic Pool Problem:** How many glasses of water do we need to fill an Olympic pool?" (adapted from Albarracín & Gorgorió, 2014)
Pair 3 involves semi-regular elements (3a – cars) and regular elements (3b – Coin) to be placed in a regular 2-dimensional shape with an unknown size that involves assumptions to make sense with (3a – Parking garage of a shopping mall) and in a semi-regular 3-dimensional known size (3b – 1-m^3 safe box).	**3a. Cars in a Parking Garage Problem:** How many cars can fit in a parking garage of a shopping mall? (adapted from Segura & Ferrando, 2023)	**3b. Coins in a Safe-Box Problem:** How many one-euro coins fit in a 1-m3 safe-box? (Albarracín & Gorgorió, 2014)

Fig. 1: Fermi Problem Pairs

Findings

The AI companion for solving FPs produced several solution methods, some of which were already listed by Albarracín and Gorgorió (2014) (see the methods in black in Fig. 2). In addition, some were different adaptions of the solution methods listed in the framework (marked in blue in Fig. 2). Specifically, direct assumption was close to direct counting but actually the assumptions were not based on counting, rather based personal knowledge. Sampling and extrapolation method was similar to reduction of the problem and concentration measure, but also different from those because it involves a small scale experimenting and then scaling up. Historical/statistical data was similar to reliance on external source, but different because it involves checking particularly the statistics of the situation not any relevant information. Lastly, packing efficiency adjustment was completely new and involved realistic considerations such as it is not possible to fill the entire volume of a safe box with coins, where empty spaces must be considered. Fig. 2 also shows that not all solution methods were found useful for all FPs. For instance, historical/statistical data could be more appropriate for the Central Park problem because such information could be drawn from internet. Another example is that grid distribution could be more useful for Cars in a Parking Garage problem due to regularity of the geometric elements albeit structural or spacing constraints.

Solution Methods	FP Pair 1		FP Pair 2		FP Pair 3	
	1a School Courtyard Problem	1b Central Park Problem	2a Bucket Problem	2b Olympic Pool Problem	3a Cars in a Parking Garage Problem	3b Coins in a Safe-Box Problem
Iteration of the base (reference) unit	*	*	*	*	*	*
Problem reduction of and use of proportion	*	*	*	*	*	*
Grid Distribution					*	
Direct Assumptions	*		*	*	*	
Sampling and Extrapolation	*	*	*	*	*	*
Historical/ Statistical Data		*				*
Packing Efficiency Adjustments						*

Fig. 2: Solution methods for FPs produced by AI

Conclusion

The analysis of the ChatGPT's output not only produced some new forms of the existing solution methods (c.f. Albarracín & Gorgorió, 2014; Segura & Ferrando, 2023) but also suggested appropriate solution methods based on the problem characteristics. This finding is important in twofold. First, the study showed that AI companion can help us to expand existing theoretical framework of solution methods through introducing new solution methods. This conclusion suggests investigating the ways of which AI suggested methods could be explored by the problem solvers themselves. Second, the AI could identify more useful solution methods considering the characteristics of the FPs, which is a competency that mathematics problem solvers need to develop. This conclusion consolidates the claims about the supporting role of the AI in mathematics education (Mao et al., 2024). In this regard, AI could be a company for problem solvers in evaluating the solution efficiency. This study presents an attempt to test the AI support in the case of FPs and suggests further explorations with FPs characterized differently.

References

Albarracín, L., & Gorgorió, N. (2014). Devising a plan to solve Fermi problems involving large numbers. *Educational studies in mathematics, 86*, 79-96.

Ärlebäck, J. B., & Albarracín, L. (2023). Fermi problems as a hub for task design in mathematics and stem education. *Teaching Mathematics and its Applications: An International Journal of the IMA, 43*(1), 25-37.

Mao, J., Chen, B., & Liu, J. C. (2024). Generative artificial intelligence in education and its implications for assessment. *Tech Trends, 68*(1), 58-66.

Segura, C., & Ferrando, I. (2023). Pre-service teachers' flexibility and performance in solving Fermi problems. *Educational Studies in Mathematics, 113*(2), 207-227.