

# Constitutive relation development through the FIRE test

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## Abstract

*The importance of well-developed constitutive models for predicting deformation behavior of materials at high strain rates cannot be overstated. The study and development of these constitutive models is pertinent to several fields, yet the test methods utilized to probe this high strain-rate realm are limited in both number and standardization. In an effort to augment current high rate tests, new technologies have been leveraged to revive an old, under-utilized test method – the axisymmetric expanding ring. The combination of Photon Doppler Velocimetry (PDV) and one of several ring launch techniques allows the successful testing and instrumentation of samples loaded in tension without wave effects at strain rates exceeding  $10^4 \text{ s}^{-1}$ . Design and construction of the embodiment of this test at OSU, dubbed the Fully Instrumented Ring Expansion (FIRE) system, will be discussed. The key difficulties to implementation of the test are examined, along with our efforts to overcome them and preliminary results.*

## Keywords

High Strain-Rate, Tensile Test, Ring Expansion

## 1 Introduction

The ring expansion test is an interesting technique that has undergone several revisions since its inception many decades ago. Inherently this method does not suffer from end effects, and has the unique ability to test samples in uniform simple tension at very high rates of strain. Other high rate tests are typically limited to some extent by the unavoidable limit of plastic wave propagation velocity, given by the Von Karman velocity,  $v_{VK}$ :

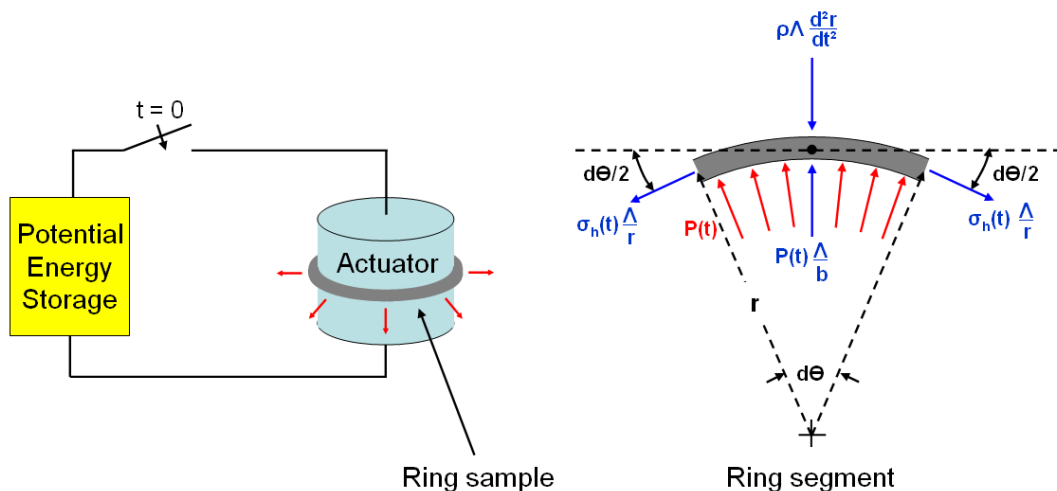
$$v_{VK} = \sqrt{\frac{\partial \sigma / \partial \varepsilon}{\rho}} \quad (1)$$

The relation determined by Theodore Von Karman is effectively a general case solution to the simpler issue of elastic wave velocity, and incorporates dissipative effects associated with plastic flow. For conventional tensile testing, the Von Karman velocity represents a critical threshold – above which localized fracture occurs near the loaded end. This effect arises because each portion of the sample has inertia, and the end closest to the load has the largest amount of sample to pull along with it. Hence the

stresses here reach the failure point first; following fracture a release wave propagates through the material and unloads the rest of the sample before additional deformation can be accomplished. In the case of real materials, this critical tensile impact velocity takes on values from less than 10 m/s for metals like lead and bismuth, to values approaching 200 m/s for high strength steels and superalloys [1].

The critical impact velocity does not represent the same type of threshold for all cases of one dimensional loading. While the simple tension test with one fixed & one moving crosshead is limited by flow localization near the loaded end for crosshead velocities greater than  $v_{VK}$ , compressive loading is instead limited by buckling in long geometries, and shear banding in cases with large frictional end effects. Buckling is actually less of an issue at high test velocities due to inertial effects, while shear banding is exacerbated at high rates due to adiabatic effects. Adiabatic effects can also plague high rate torsional tests, by causing runaway thermal softening which promotes heterogeneous deformation (also note that torsion is equivalent to biaxial balanced tension/compression, not as simple as 1-D tension or compression).

One way to realize materials deformation at high rates while avoiding the effects of abrupt one dimensional loading is to apply strain through a uniform velocity gradient. It is possible to establish such a gradient by expanding (or contracting) any surface of revolution about its axis. Uniformity of the velocity gradient is ensured if all angular segments have the same velocity at any given time, since the gradient in this case is due solely to the rotation of the outward facing surface normal. For such expanding surfaces of revolution with constant radius (tubes and rings), the entire problem thus reduces to a single dimension – coincident with the radial dimension of the test. This type of test, called the axisymmetric expanding ring test is therefore capable of testing ring shaped samples of material in uniform, simple tension at arbitrarily high strain rates.



**Figure 1:** Left – simplified schematic of a ring expansion experimental setup. Right – free body diagram for an angular ring segment;  $t$  is time,  $P(t)$  is the time varying pressure exerted by the actuator,  $r$  is the current ring radius,  $\theta$  is the included angle,  $\sigma_h(t)$  is the hoop stress as a function of time,  $\rho$  is material density, and  $\Lambda$  is the volume of the differential segment (given as  $\Lambda = (b \cdot h) \cdot r d\theta$  with  $h$  and  $b$  denoting current ring height and thickness respectively).

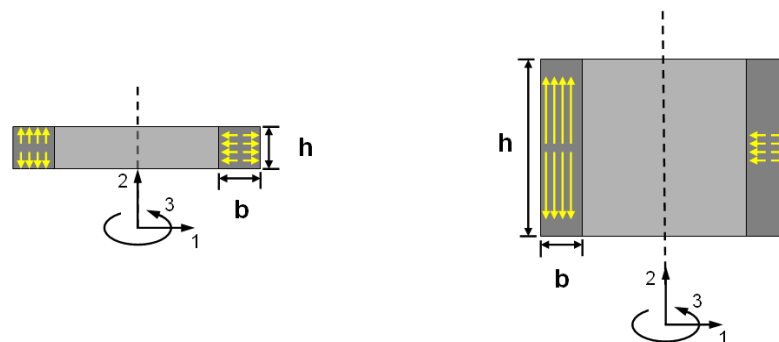
Upon examination of the free body diagram, one can see that initially the outward driving force ( $P(t)r d\Theta$ ) is balanced by the sum of the hoop tension  $\sigma_h$  and the inertial forces ( $\rho\Lambda d^2r/dt^2$ , where  $d^2r/dt^2$  is positive since the sample is accelerating). If  $P(t)$  goes to zero at some time greater than  $t = 0$ , the term  $d^2r/dt^2$  goes through zero and becomes negative such that the outward force is subsequently produced only by the sample's inertia. In this case, called the *freely* expanding ring, the deceleration is caused solely by the flow stress of the ring. Thus if a suitable method of measuring  $d^2r/dt^2$  is employed, the flow curve of the ring material can be unambiguously identified. These results can be summarized mathematically as shown below in equations (2) through (4). Please note that small  $d\Theta$  and conservation of volume have been assumed in the formulation of the following:

$$\rho\Lambda \frac{d^2r}{dt^2} = P(t) \frac{\Lambda}{b} - \sigma_h(t) \frac{\Lambda}{r} \quad ; \text{ in which:} \quad (2)$$

$$\Lambda = bhrd\theta \quad (3)$$

$$\sigma_h(t) = -r\rho \frac{d^2r}{dt^2} \quad ; \text{ For } P(t) = 0. \quad (4)$$

Additionally, since the ring sample is continuous in the direction of tensile stress there are no end effects. The only wave effects that are encountered are through the thickness direction and are correspondingly small. Furthermore the stress state can be adjusted from plane stress for a short ring (cross section aspect ratio of approximately 1), to plane strain in the case of a very tall ring or tube sample. This effect is due to the axial inertia of the sample – as the relative amount of material above and below a given segment increase, so does the constraint along the axial direction. In the limit of long tubes (cross section aspect ratio  $\gg 1$ ), very little strain is accomplished in the axial direction and a state of plane strain can be assumed as in Figure 2.



**Figure 2:** Left - ring sample with small aspect ratio, a state of uniform plane stress exists in the sample; i.e.  $\epsilon_1 = \epsilon_2 = -\epsilon_3/2$ . Right – tube sample with aspect ratio  $> 1$ , inertial constraint (denoted with yellow arrows) causes  $\epsilon_2$  to decrease such that in the limit of very long tubes a state of plane strain is achieved; i.e.  $\epsilon_2 = 0$ ,  $\epsilon_1 = -\epsilon_3$ .

Another inertial effect arises in the expanding ring test that is the source of some debate in the literature concerned with the effect of multiple necking on ductility [2]. From a strictly academic point of view, uniform elongation is not improved in the expanding ring geometry; i.e. the strain accumulated up to the point of instability remains very close to what would be observed in a quasi-static situation. However, instability propagation occurs at a finite speed and so as a result multiple necks can be formed in a sample – each necked region still participates in accomplishing post uniform elongation, and as a result the total elongation to failure is seen to increase monotonically with expansion velocity [3].

Significant work in the field of fragmentation has been done with the expanding ring as a research tool [4]. As a general rule these studies are fairly qualitative, with the number and mass distribution of fragments as primary interests. Fragmentation of rings with defects has been studied also (though to a much lesser extent) [5]. The idea of purposefully introducing notches or holes into the ring sample could also potentially be used to establish fracture toughness as a function of strain rate. This however, is a much more quantitative and difficult task than fragmentation studies – much like developing high rate flow curves. The success of such quantitative ring expansion testing hinges on two aspects of the experimental setup: accurate high speed velocimetry capabilities and precise, repeatable actuators.

## 2 Enabling Technologies

### 2.1 Velocimetry Equipment

Early efforts focused on determining the acceleration history ( $d^2r/dt^2$  vs.  $t$  for the entire experiment) by using high rate framing cameras or streak photography. These methods present the difficulty of double differentiating displacement data in the temporal domain. Since the time scales for these experiments are so short (typically less than 30  $\mu$ s) any error in velocity difference will be magnified when normalized by time. It is therefore desirable to eliminate one or both differentiation steps to enhance accuracy and repeatability.

Revisions of the ring expansion test in the 1980's by Gourdin [6] tackled this issue through the use of the Velocity Interferometer System for Any Reflector (or VISAR). While this was an important step forward, VISAR systems were and remain to be very difficult and finicky beasts to build and operate successfully. As such, the improved ring expansion test was still left as an academic curiosity. While a comparatively small amount of work happened on the issue of ring expansion in the following two decades, a vast amount of technology was being developed and proliferated for the telecommunications industry. In 2004 individuals working with VISAR systems for other high rate projects hit on the idea of utilizing cutting edge telecom technology to create a new velocimetry system that is more robust, less ambiguous and easier to use than the VISAR predecessors [7]. This new system is commonly referred to as Photon Doppler Velocimetry, or PDV.

Since that time, PDV has been adopted for the resurrected ring expansion test, both at The Ohio State University and the University of Texas [8]. The OSU team's PDV setup is an example of a compact, portable system with a minimalist set of components. Details may be found elsewhere [9] on the component selection for this system, but the overall capabilities include direct velocity measurement for speeds of up to approximately 1 km/sec, temporal resolution on a nanosecond scale and spatial resolution on the order of microns. This is coupled with the ability to simultaneously measure up to 4 channels, each

corresponding to the velocity history of a single point on a target's surface (actually an area of  $\sim 0.01 \text{ mm}^2$  is interrogated by the probe, from which multiple velocities and/or a velocity distribution can be extracted). Typical data analyses will process the raw PDV data with a fast Fourier transform, and frequency information from the signal is then directly converted to velocity data, as in the following spectral density surface plot obtained with the *spectrogram* command in MatLab.

Doppler shifted frequency:

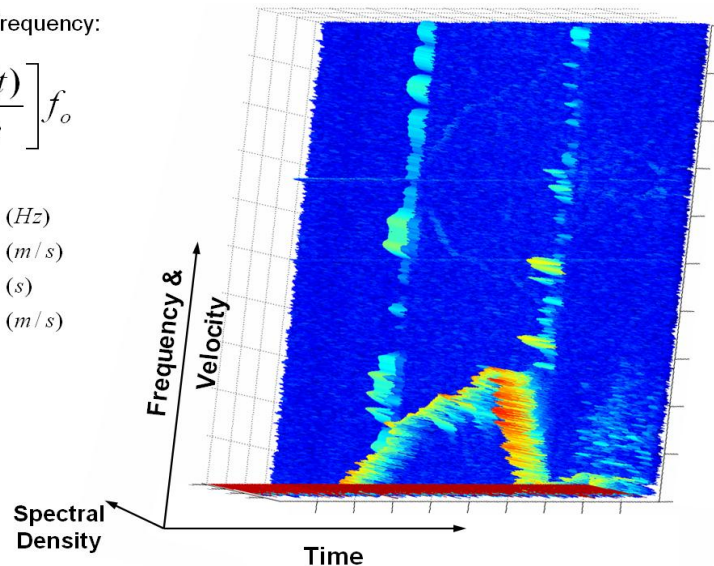
$$f(t) = 2 \left[ \frac{v(t)}{c} \right] f_o$$

$f$  = frequency (Hz)

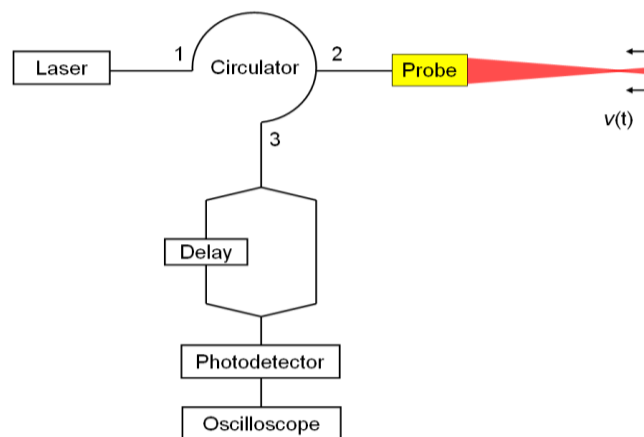
$v$  = velocity (m/s)

$t$  = time (s)

$c = 2.998 \cdot 10^8$  (m/s)



**Figure 3:** Example spectral density surface plot generated with MatLab's "spectrogram" function. The plot is essentially a color indexed Short Time Fourier Transform (STFT) of the raw PDV data. The dominant signal component can be extracted from a plot such as this and converted directly into a velocity history.



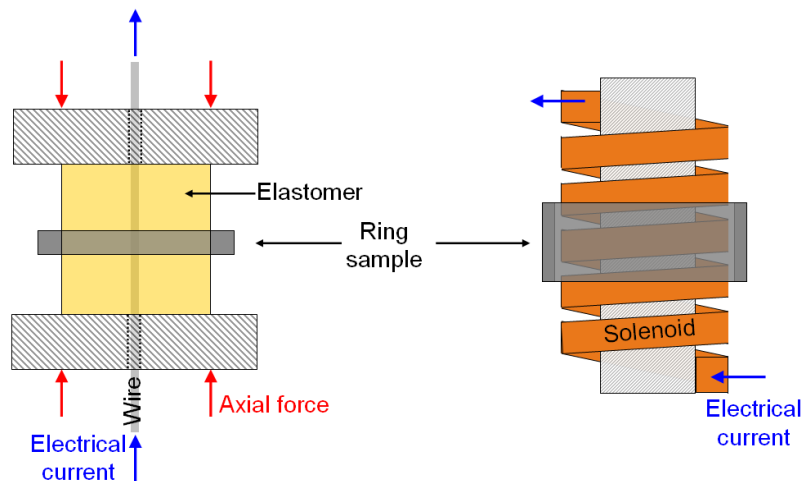
**Figure 4:** PDA concept. Probe must be a non-back reflecting type, i.e. -60 dB return loss. The delay is provided by a known length of fiber optic cable, and patched into the optical path with two 50/50 splitters as shown. In this fashion, Doppler shifted light from time  $t$  is mixed with shifted light from time  $t-t_{\text{delay}}$ . The resultant beat frequency is then indicative of the difference in velocity at the two times.

A promising new technique that leverages the same technology as traditional PDV has emerged very recently. Initially discussed at the 4<sup>th</sup> International PDV conference in

2009, the idea of utilizing PDV with appropriate fiber optic delay lines to measure acceleration directly has been dubbed the Photon Doppler Accelerometer, or PDA (the name PDVISAR has been suggested too, due to the similarity of the techniques). The concept is schematically illustrated above in Figure 4.

## 2.2 Actuators

The actuator is the active element that is used to drive a ring or tube sample to high radial velocities. Several variants have been demonstrated, these can largely be separated into categories including electromagnetically launched systems, explosively driven setups (both chemical explosives and exploding wire), and devices utilizing an incompressible media and high speed piston. Any of these techniques may be used in some cases with a driver or backing ring, and in other cases used directly to drive the sample ring. The first two cases are found to be the most useful so far and are illustrated below in Figure 5.



**Figure 5:** Schematic representation of the two main types of actuator, the Exploding Wire (EW) style – left, and the Electro-Magnetic (EM) style – right. EW actuators are capable of driving nearly any sample, while the EM type requires samples that are good electrical conductors (or the addition of a conductive driving ring). Not shown are optional catcher rings for stripping of driver ring from sample.

The Electro-Magnetic (EM) actuator has been in fairly wide use both in the manufacturing arena and elsewhere, including electromagnetic metal forming and induction heating. As such, the physics that accompany the design of solenoids and similar coils are well established. The basic incarnation of an EM actuator is a simple helical solenoid, though other geometries exist as well [10]. Materials that are highly conductive can be directly driven quite efficiently with an electromagnetic actuator; other materials require a conductive driver ring. The resulting composite sample can be analyzed by subtracting the effects of a well characterized driver, or by using an oversized driver that can be stripped away from the sample by catcher rings. In the latter case, a freely expanding ring analysis may be used, while the former requires a more sophisticated reduction technique. This is due to the fact that magnetic pressures are highly sensitive to small variation in geometric factors of the actuator and sample – resulting from the rapid change in mutual inductance with these dimensional variations.

Exploding Wire (EW) type actuators typically consist of a disposable elastomeric pressure medium, concentrically surrounding a sacrificial wire. Passage of large electrical

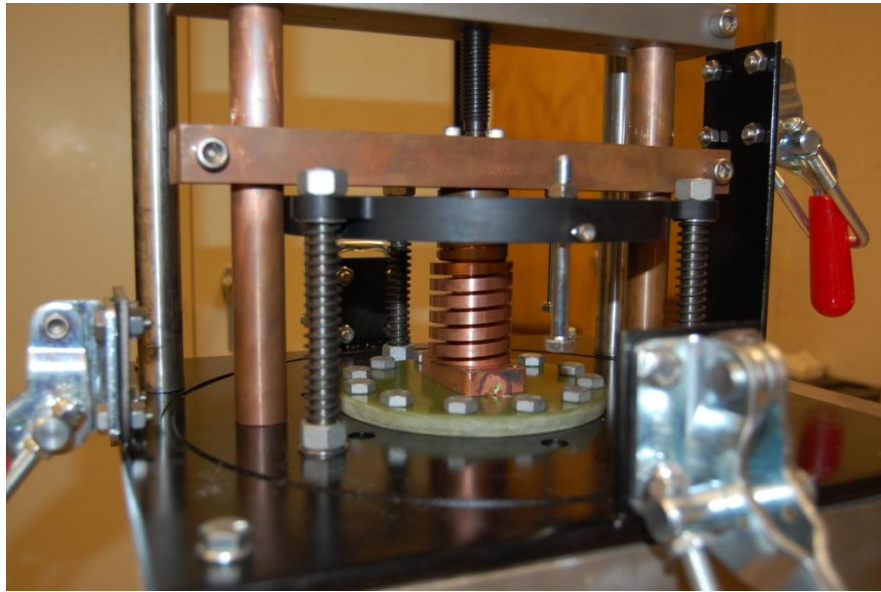
currents through the wire cause it to vaporize very rapidly and the expanding vapor/plasma pocket is harnessed to do work on the ring or tube sample. As the expanding gases take the path of least resistance, it is important to provide axial confinement on the pressure medium in order to ensure high velocity radial deformation. EW actuators are generally only utilized to produce a freely expanding ring. Here detailed knowledge of the time varying pressure  $P(t)$  is not always possible, hence the focus on the less demanding analysis route. EW actuators have the further advantage of possessing no definite upper limit for their power density capabilities. This is in contrast to an EM actuator, which has a definite limit on the amount of energy it can process in a given time frame without failure. Typical rigid EM actuators experience the same pressure as the sample or driver ring does (at a minimum), placing a further restriction on their ability to expand materials of high strength. The cost of overcoming these issues with EW actuators is primarily associated with their disposable nature.

### 2.3 The FIRE System

With the above discussion points in mind, a modular, multi-purposed Fully Instrumented Ring Expansion (FIRE) system has been developed at OSU. The central component in the system is a pulse discharge capacitor bank made by Maxwell Magneform. This source can supply up to 16 kJ of energy while discharging its 480  $\mu\text{F}$  capacitance into the actuator at currents of up to 200 kA. Interchangeable EW and EM actuators have been designed for ring and tube sizes from 0.25" to 2" inside diameter, and heights from 0.04" to 12". The actuator components are housed inside a safety chamber which provides shrapnel protection and electromagnetic shielding for test energies of up to at least the rated 16 kJ. Concentrically located with the actuator are provisions for 1 to 6 radial PDV or PDA channels (separated by  $60^\circ$ ), each independently adjustable in axial height and angular alignment. Machine tool accuracy has been maintained throughout the construction of all components, maximizing repeatability and accuracy.

One important feature of the FIRE system is the symmetric ground path. This 3-bar design, as shown below in Figure 6 provides balanced magnetic forces on the wire and plasma channel of an EW actuator, and also minimal disturbance of the field surrounding an EM actuator. The primary current path and nearly all of the other conductive components have been machined from C18150, a copper-chrome-zirconium alloy with excellent electrical and mechanical properties (Cu plus 1 wt% Cr, 0.15 wt% Zr; C18150 exhibits conductivity greater than 75% IACS while maintaining a yield stress of at least 400 MPa) [11]; alloy 304 stainless steel was chosen for most of the remaining components. Another feature of the present design is the use of periscope probes – where the focusing probe body is placed outside of the blast guard (shining straight up into the chamber through a small hole) and the light is subsequently redirected through  $90^\circ$  via a first surface mirror inside the chamber to the expanding ring target. In this setup, the relatively expensive probes are out of harm's way.

Also included in the current suite of instrumentation is measurement of the real-time current and voltage provided by the capacitor bank. This is done with a Rogowski type current probe and capacitive voltage divider (Tektronix p6015a). A reconfigurable port into the blast chamber also offers the option of adding other optical, thermal or magnetic transducers to the list of possible setups for data acquisition. This will become important in the near future, when elevated temperature testing capability is added to the FIRE system.



**Figure 6:** View of the FIRE system with the cover open. All visible copper is C18150, electrical feedthrough plates (center) are machined from high strength composite (G-11), and the PDV probe support ring is black acetal polymer. Not shown is the 12" diameter cylindrical stainless steel blast cover (above), or high current busbars and secondary PDV support fixturing (below).

### 3 Experimental results

#### 3.1 Polycarbonate

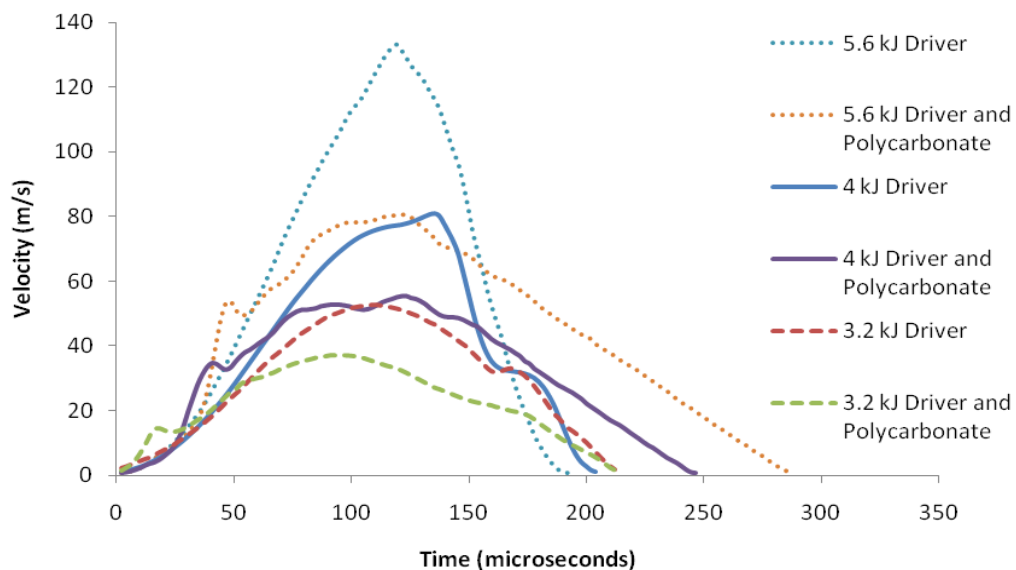
Polymers lack the highly ordered structure common to crystalline substances. While their long molecular chains may indeed exhibit in some cases a degree of crystallinity, the structures are relatively open in comparison to their close packed counterparts and they generally possess a lower degree of symmetry. However, unlike amorphous ceramics and metals the glass transition temperature of most engineering polymers is sufficiently low enough to promote ductile deformation at room temperature. In contrast to the deformation of crystalline solids, much of the "plastic" strain can be recovered since the strain is often accommodated through entropy driven anelastic processes [12]. Also, polymer structures and the associated deformation barriers change rapidly with temperature (due primarily to changes in free volume). This often makes the high rate behavior of polymers very interesting and/or counterintuitive.

To demonstrate a ring expansion technique appropriate for non-conductive materials such as polymers, an EM type actuator was used to indirectly expand polycarbonate rings. Rings were obtained by machining short segments of extruded polycarbonate tubing. Samples were cut from 2" inside diameter clear polycarbonate tube to provide a square cross section of 0.12" x 0.12". The actuator consisted of a 4.5 turn spiral stripline type coil [10], having an outside diameter of 1.5" and height of 1.25", potted inside a high strength biaxial fiber composite tube (G-10) with an outside diameter of 1.875". Half inch tall driver tubes were machined from an extruded 6061-O aluminum tube of 2" outside diameter and 0.049" wall thickness. Sample rings were then lightly press fit onto the drivers. Also at this point small squares of retro-reflective tape (made by 3M) were placed on the surface of the polycarbonate sample rings to maximize PDV return



signal (the tape did not come off during the experiment). Focusing probes of 10 cm focal length and -15 dB return loss were utilized to generate velocity histories.

Results from six shots are shown below in Figure 7; these represent two shots at each of 3 energy levels – 3.2, 4, and 5.6 kJ. One shot at each energy corresponds to a bare driver ring, and the other to the response of the composite driver and sample. Peak strain rates in the polycarbonate samples are 1500, 2165, and 3150 s<sup>-1</sup>, respectively. All polycarbonate samples remained intact, even to the largest strains of 0.4. Deconvolution of the velocity data to stress-strain information is quite involved for this case, since  $P(t)$  is significant throughout the duration of the experiment, and due to the fact that rings are being expanded with a driver. However, even this relatively qualitative comparison demonstrates both the excellent high rate properties of polycarbonate and the ability of a FIRE type system to rapidly compare material behavior under varying dynamic conditions.



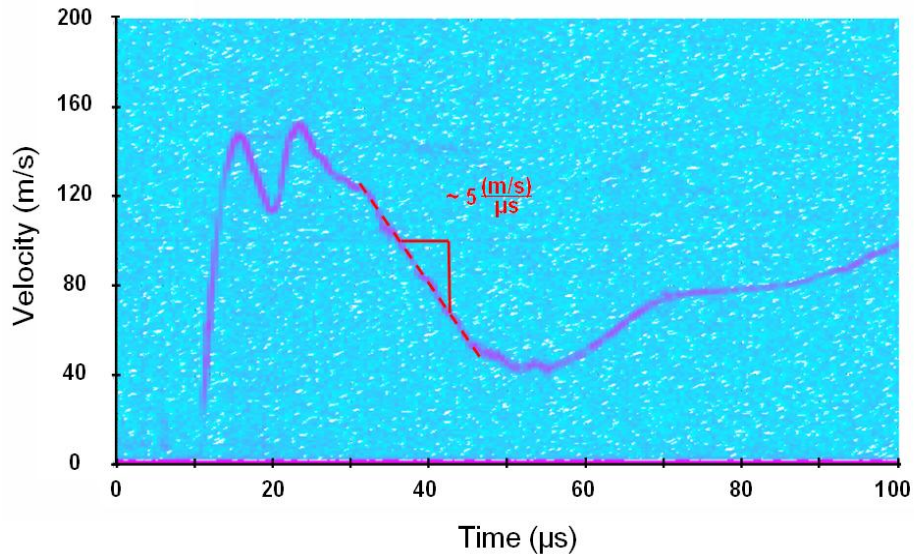
**Figure 7:** Radial velocity histories for bare drivers and drivers loaded with polycarbonate ring samples.

### 3.2 AISI 4130 Steel

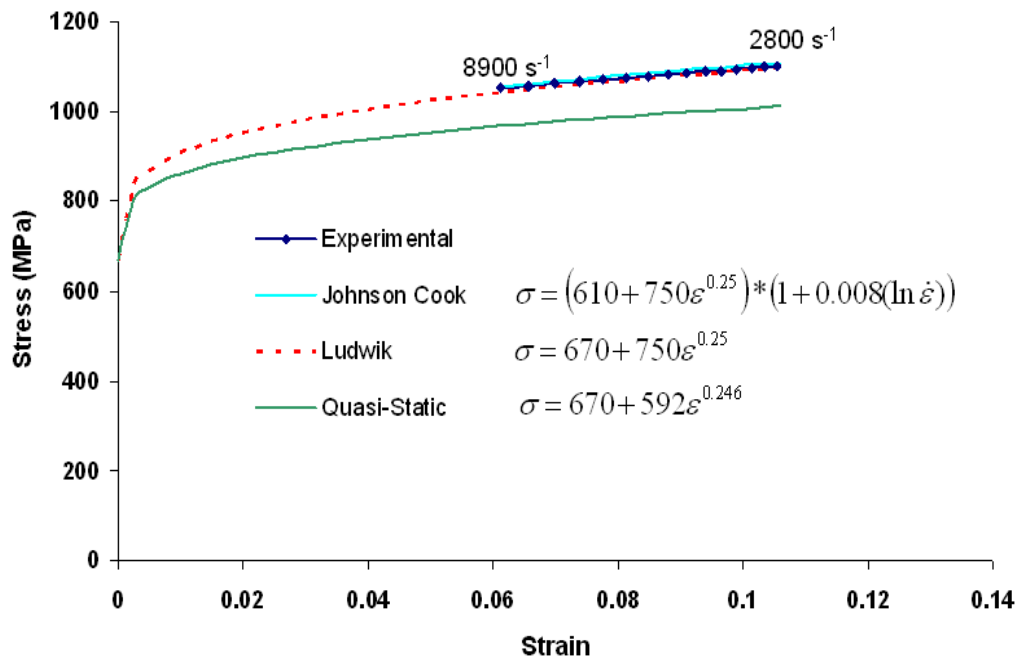
Much of the EW actuator technology here at OSU has been developed in the interest of expanding rings made from high strength materials. Here such an example is shown for expansion of a HSLA steel ring cut from a seamless DOM tube. The as received tube was 1.027" inside diameter with 0.049" wall thickness; the sample was obtained by cutting a 0.375" length of tube by parting on the lathe. The slight radial gap of 0.013" between the polyurethane and the steel ring is easily taken up by the polyurethane actuator when the confining axial force is applied to the pressure medium.

The EW actuator used for this experiment was constructed with a 1" outer diameter, 1" length of extruded polyurethane round stock with a hardness of 80 Shore A. The wire utilized was a 3 inch length of 1100 series aluminum welding wire with a diameter of 0.060". The wire was vaporized with a 6.4 kJ capacitor discharge which produced a non-ringing electrical oscillation with 20  $\mu$ s rise time and 70 kA peak current. Again a focusing type probe was used with -15 dB return loss, but here a 30 cm focal length was used. The PDV data from the shot is presented below in Figure 8, following this is the corresponding

stress-strain plot is depicted in Figure 9. Free flight of the ring starts around 25  $\mu\text{s}$  and fracture of the sample occurs at approximately 45  $\mu\text{s}$ . The release wave from fracture imparts a tangential momentum to the fragment of interest which causes rotation and a subsequent increase in velocity along the radial direction. This can be thought of much like cracking a whip, where the tip attains a velocity much greater than the driven end due to geometrical factors (plus conservation of momentum).



**Figure 8:** Velocity history spectrogram used for construction of Figure 9. A constant deceleration is assumed throughout the region marked with the red dotted line – this is the segment corresponding to the analysis for the following flow curve.



**Figure 9:** Flow curves for AISI 4130 steel. Both Johnson-Cook and rate insensitive Ludwik models were fit to the experimental data with good results. Quasi-static data adapted from Park et al [13], where 1" gage length samples were tested in an Instron load frame at a strain rate of  $10^{-3} \text{ s}^{-1}$ .

## 4 Conclusions

It is possible to accurately measure the high strain rate constitutive properties of ductile materials in tension with the ring expansion test. Accuracy and repeatability are ensured through the careful design and construction of actuators, and velocimetry fixtures, along with attention to overall system topology.

In the case of a ring expanding solely under its own momentum determining flow stress as a function of radial displacement is straightforward, provided that the experimental instrumentation is of high enough resolution. Establishing flow stress in cases of rings expanding under the influence of a time varying pressure and/or composite samples including a driver ring is less straightforward. Here detailed knowledge of the pressure variation with time is required for a full analysis.

Furthermore, it has been demonstrated that the same basic setup can be adapted to the expansion of many different materials through the use of Exploding Wire (EW) and Electro-Magnetic (EM) actuator technology. Also, Photon Doppler Velocimetry (PDV) has proved to be an indispensable tool in the development of the Fully Instrumented Ring Expansion (FIRE) system. Future revisions of the test will include the ability to perform testing at elevated temperatures and will also likely include modified ring geometries capable of testing situations other than simple tension. It is hoped that through these efforts the ring expansion test will become a standardized test methodology for probing the high rate response of materials.

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