

Damjan KOBAL, University of Ljubljana

Ancient mathematical ideas and their applicability in technology and teaching

We present two simple but very applicable mathematical ideas. The idea of an arithmetic mean, is used in car differential (differential gear). The idea of continuous and discrete functions is applied in modern electronic communications. Both ideas are very useful to present and motivate the interdisciplinary approach and intuitive value of abstract mathematical ideas in teaching.

The arithmetic mean and differential gear

We all know, that a car is powered by a motor. But how? How is the power (the rotation) of the motor transferred to the wheels that make the car move. On a bicycle, we use a chain that transfers the rotation of the pedal to the back wheel. Is it not done very similarly for a car, just that the source of power is a motor and not our muscles? Well, the very first cars were truly done that way. By the use of a chain the rotation was transferred from the motor to the (back) axle. So both, right and left wheel rotated simultaneously and made the vehicle to move.

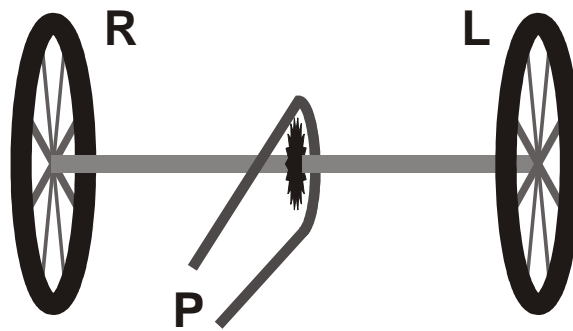


Figure 1: Right and left wheels are attached to the same axes of rotation

With a look on the picture, disregarding possible transmission ratio and denoting ‘power’ (engine rotation) by P , right wheel rotation by R and left wheel rotation by L , we get a very simple equation:

$$P = R = L$$

But does a car work like that? Well, the very first cars did function like that and as a consequence, the steering was very hard. In a left turn the left wheel slows down and the right speeds up a bit. Their average speed

remains unchanged. If P is their average speed, R is the speed of the right and L is the speed of the left wheel, then their moving is described by a simple equation:

$$P = \frac{R+L}{2}$$

Could this simple formula be mechanically realized for powering right and left wheels of a car? But how? It might be surprising, but a positive answer to this question has been known for over 2000 years. In fact, a mechanical realisation of the formula for arithmetic mean is surprisingly simple.

Imagine first, that equal powering of the right and left wheels is achieved by a 'rotating handle on a disc', which is attached to the right and left disc, that are welded at the end of the right and left wheel axes, as shown on the bellow picture.

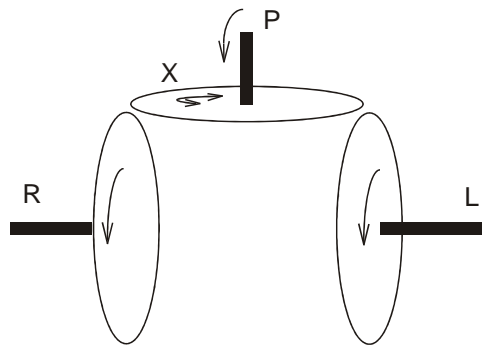


Figure 2: Right, left and freely rotatable powering discs

Instead of discs we can imagine cogs. It is obvious, that with a help of such a mechanism, a rotation of 'the handle P ' would imply an equal rotation of the left and right wheel, thus $P=R=L$. This seems like still far away from the desired arithmetic mean equation. But it is not. Denoting

$$\frac{L-R}{2} = X,$$

we have

$$R = P - X \text{ and } L = P + X.$$

The value of X can be understood as a free parameter in the relation of three variables within the arithmetic mean equation.

Namely, if we allow that our 'power disc' in the above picture, is freely rotatable (free variable X) around the 'handle', as shown in the picture, we already have a model of a differential gear.

It is a nice exercise to think about the different values of the variable X in different situations, like when we drive straight, when turning left or right, when one of the wheels is blocked.

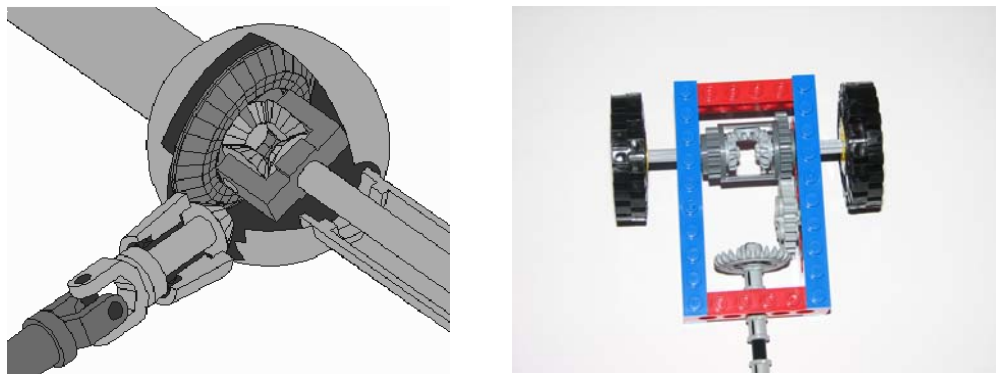


Figure 6: Drawing of a differential gear with universal joint and Lego model of differential

Today, one can easily experiment with the idea, as even Lego (Technics) provides sophisticated but yet simple models of devices like differential gear. And with such a model one can ask many interesting questions related to practical 'car driving' issues (like snow driving) and relate them to the abstract mathematical meanings of the arithmetic mean formula. A creative teacher can further find many interesting historical findings which are related to the invention and development of these ideas in car technology.

The concept of a discrete function in a modern communication technology

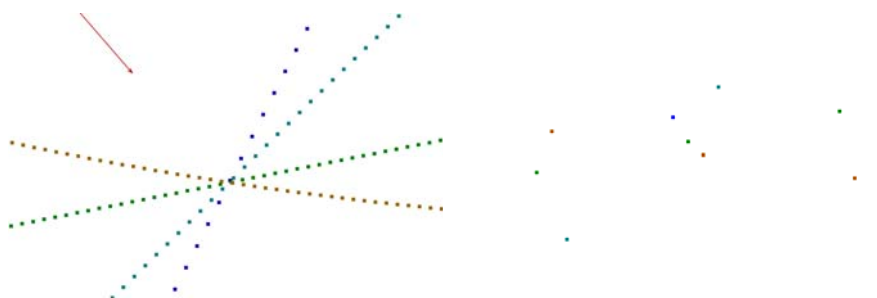
Sounds can be represented by functions. Many modern computer programs (like *Mathematica*) can not only *draw* but also *play* functions. For example by *playing* the function $\sin(2\pi 440 t)$ one gets a pure 'A tone'.

Let us look at the graph of four different functions. We made the functions to intersect (at a point indicated by red arrow) just to make it easier to explain our idea. It seems we have four different and 'precisely described functions'. It is clear that if we think of functions as sounds, this picture

could easily be transformed (imagine colour filter) into four different (clearly heard) sounds. And this is basically the trick of digital technology. People talking on a phone and real life users and customers of audio technology have very limited ear sensitivity and can be fooled to truly hear four different sounds from the bellow picture. In fact we can say, that our eyes were fooled to see four different functions bellow, while mathematically (that is precisely) speaking, we even do not have one function defined at the whole visible interval.



To see and comprehend what we are talking about, let us focus to the intersection point indicated by red arrow. On the left picture bellow one can still recognize the above functions, while on the right picture even stronger focus shows us, that ‘we used only a very small portion of a function’ to transmit ‘four functions with limited but sufficient resolution’.



How many functions like that can be 'squeezed' into one discrete function? How many phone conversation can be squeezed into a single phone line? Of course it depends on the quality of the sound required (density of discrete points, which represent particular functions) and on the ability of technology to 'listen' and 'record' ever shorter bits of time.

With modern computer technology this wonderful and simple idea can easily be simulated by dynamic presentation of functions, when 'zoom in' and 'zoom out' can nicely and intuitively visualize how relative to human eye and ear a discrete or continuous looking functions can be.

Literature:

[1] Kopal D. (2007); ScienceMath: <http://uc.fmf.uni-lj.si/com/>