Deriving a statistical model for the prediction of spiralling in BTA deep-hole-drilling from a physical model

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Abstract. One serious problem in deep-hole drilling is the occurrence of a dynamic disturbances called spiralling. A common explanation for the occurrence of spiralling is the coincidence of time varying bending eigenfrequencies of the tool with multiples of the spindle rotation frequency. We propose a statistical model for the estimation of the eigenfrequencies derived from a physical model. The major advantage of the statistical model is that it allows to estimate the parameters of the physical model directly from data measured during the process. This represents an efficient way of detecting situations in which spiralling is likely and of deriving countermeasures.

1 Introduction

Deep hole drilling methods are used for producing holes with a high length to diameter ratio, good surface finish and straightness. For drilling holes with a diameter of 20 mm and above, the BTA deep hole machining principle is usually employed (VDI (1974)). The necessarily slender tools, consisting of a boring bar and head, have low dynamic stiffness properties. Therefore deep-hole-drilling processes are at a high risk of dynamic disturbances such as spiralling, which causes a multi-lobe-shaped deviation of the cross section of the hole from absolute roundness, see fig. 1.

As the deep hole drilling process is often applied during the last production phases of expensive workpieces, process reliability is of prime importance. Prediction and prevention of spiralling are therefore highly desirable.

By using a finite elements model to determine drilling depth dependent bending eigenfrequencies of the tool, spiralling was shown to reproducibly occur when one of its slowly varying eigenfrequencies intersects with an uneven multiple of the tool rotational frequency (Gessesse et al. (1994)). This suggests preventing spiralling by avoiding these critical situations. Unfortunately the practical application of the finite elements model is limited as it has to be calibrated using experimentally determined eigenfrequencies.

Earlier investigations demonstrated that the courses of the bending eigenfrequencies clearly show in spectrograms of the structure borne sound of the

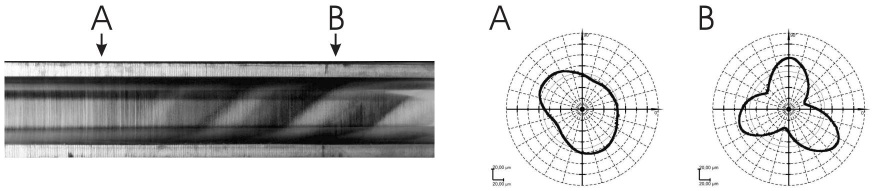


Fig. 1. Left: Longitudinal section of a bore hole showing marks resulting from spiralling. Right: Associated roundness charts.

boring bar, which can be recorded during the process (Raabe et al. (2004)). In this paper this signal is used to statistically estimate the parameters of a physical model of the bending eigenfrequencies. A lumped mass model is used to calculate the tools bending eigenfrequencies. It includes the physical parameters of the process allowing to directly calculate the influences of their variations on the eigenfrequency courses. However, this model contains some unknown parameters and naturally the measurement is subject to random error. It is therefore combined with a statistical model allowing the estimation of the unknown parameters by the Maximum Likelihood method.

The work presented in this paper is based on experiments carried out on a CNC deep hole drilling machine type Giana GGB 560 (see Szepannek et al. (2006) for technical details). Self excited torsional vibrations were prevented through the application of a Lanchester-damper. The damper was moved at feed speed together with the boring bar, implying a constant axial position of the damper relative to the tool. In order to detect bending vibrations occurring during the process, time series of the lateral acceleration of the boring bar were recorded. The experiments were carried out with stationary tool and rotating workpiece. The experimental setup is illustrated in fig. 2.

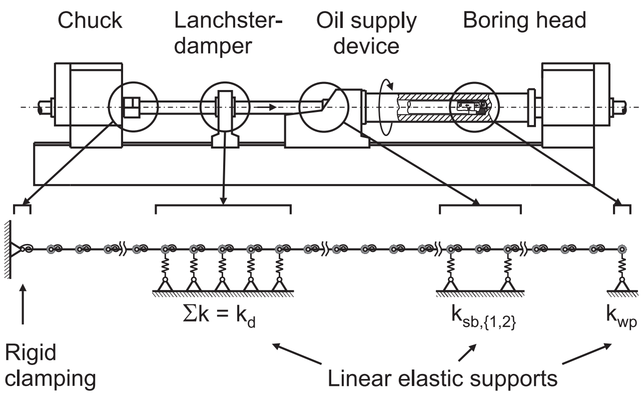


Fig. 2. Experimental setup (top) and proposed modelling approach (bottom).

2 Physical Model

For formulating the model the BTA system was reduced to its most important components. These are the tool, the Lanchester damper, two oil seal rings within the oil supply device and the workpiece, again see fig. 2, top. Under operating conditions, the latter components act as lateral elastic constraints of the boring bar. While the damper stays in the same location relative to the boring bar, the oil supply device is kept at constant distance relative to the workpiece and therefore moves at feed speed relative to the boring bar during the process. The workpiece permanently acts on the tip of the tool.

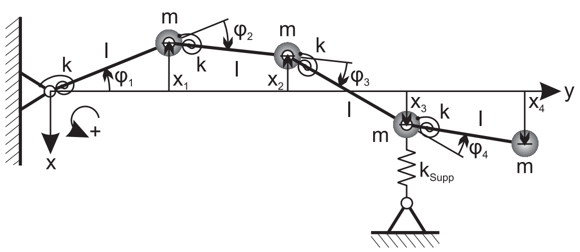


Fig. 3. Detailed modeling principle: Regular linear elastic chain with additional linear elastic support.

As illustrated in fig. 3 by an exemplary system with 4 degrees of freedom, the bar is subdivided into N elements of identical length l for constructing the lumped mass model. These elements are linked to form a regular linear elastic chain comprising N identically spaced and elastically linked masses. Additional linear elastic supports represent the constraints resulting from the supporting elements. Adopting the x-coordinates as generalized coordinates and assuming only small deflections we can write the homogenous equations of motion of the system as

$$[M]\{\ddot{x}\} + [K(l_B)]\{x\} = \{0\} \quad \text{with} \quad [K(l_B)] = [K_{Tool}] + [K_{Supp}(l_B)],$$

where $[M]_{N\times N}$ and $[K(l_B)]_{N\times N}$ are the mass and stiffness matrices of the system and l_B represents the actual drilling depth. The stiffness-matrix can be decomposed into the stiffness matrix $[K_{Tool}]$ of the boring bar and a matrix $[K_{Supp}(l_B)]$ containing the stiffness influences of the supporting elements. $[K_{Tool}]$ is time constant and can be computed from the physical and geometrical properties of the tool (see Szepannek et al. (2006)), whereas $[K_{Supp}(l_B)]$ changes stepwise with increasing drilling depth due to the movement of the oil supply device relative to the boring bar. Furthermore, $[K_{Supp}(l_B)]$ generally is unknown. More precisely, all elements of $[K_{Supp}(l_B)]$ are zero except of these elements on the main diagonal that correspond to a supporting element in the setup. The values of these matrix entries are the unknown parameters of the model.

The stiffness influences of the workpiece and the two seals of the stuffing box within the oil supply device are each assumed to act pointwise and are therefore modelled by one single parameter each (k_{wp}, $k_{sb\{1,2\}}$). The Lanchester-damper contacts the boring bar within a region of nominal length l_d. It is assumed, that this region may be reduced, e. g. by wear. So two parameters $\delta l_{d,r}$ and $\delta l_{d,l}$ representing a right- and left-hand truncation of l_d are added. The stiffness influence of the damper (k_d) is equally distributed over the elements within the remaining region of length $l_d - \delta l_{d,r} - \delta l_{d,l}$.

The stiffness constants k_{wp}, $k_{sb\{1,2\}}$, k_d together with $\delta l_{d,r}$, $\delta l_{d,l}$, which define the matrix $[K_{Supp}(l_B)]$, are a priori unknown and cannot be measured directly. These parameters therefore have to be estimated. For calculating the eigenfrequencies from the model the homogeneous equations of motion of the system (see above) have to be solved for each regarded value of the drilling depth l_B. This leads to the following eigenvalue-problem

$$\left([K(l_B)] - \omega^2[M]\right)\{x\}e^{i\omega t}.$$

The solution of this problem consists of the eigenvalues ω^2_{r,l_B}, the N squared eigenfrequencies of the model, and the eigenvectors $\{\Psi\}_{r,l_B}$, the corresponding N mode shape vectors.

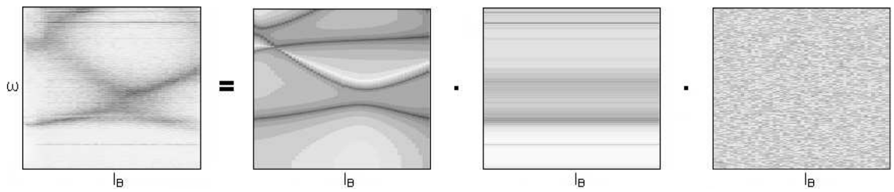
3 Statistical Model

For the estimation of the unknown parameters a statistical approach using the already introduced structure borne sound is proposed. In the following the data measured in a location between damper and oil supply device is exemplarily used.

To provide a basis for statistical estimation of the unknown parameters p, the following statistical model is proposed

$$S_k(\omega, l_B; p) = |\alpha_{jk}(\omega, l_B; p)|^2 \cdot \left|\alpha^*_j(\omega)\right|^2 \cdot S_\epsilon(l_B).$$

For each value of the hole depth l_B the term $S_k(\omega, l_B; p)$ presents the periodogram of the structure borne sound measured at a location corresponding to element k. Due to the discreteness of the physical model l_B changes stepwise and so the periodograms are computed based on non-overlapping time-windows. The model writes these periodograms $S_k(\omega, l_B; p)$ as the product of a systematic component $|\alpha_{jk}(\omega, l_B; p)|^2 \cdot \left|\alpha^*_j(\omega)\right|^2$ (the spectral density of the process) and a stochastic exciting component $S_\epsilon(l_B)$, the periodogram of a white noise process. The systematic component consists of the frequency response function (FRF) series $\alpha_{jk}(\omega, l_B; p)$ and the time constant $\alpha^*_j(\omega)$, which transforms the white noise process into the excitation in element j. In a first attempt $\alpha^*_j(\omega)$ for each frequency ω is set to its mean observed amplitude value. Refinements like fitting $\alpha_j^*(\omega)$ and p alternately are imaginable in later investigations. For a better impression fig. 4 gives a graphical representation of the proposed statistical model.



$$S_k(\omega, l_B; p) \quad = \quad |a_{jk}(\omega, l_B; p)|^2 \quad \cdot \quad \left|a_j^*(\omega) \cdot \sigma\right|^2 \quad \cdot \quad S_\epsilon(l_B)$$

Fig. 4. Visualization of the statistical model.

3.1 FRF Computation

For the computation of a FRF damping has to be included. The most straightforward way of doing this is assuming proportional damping, implying the damping matrix $[C(l_B)] = \beta[K(l_B)] + \gamma[M]$. This leads to the two further model parameters β and γ. Therefore the list of model parameters reads

$$p = (k_{wp}, k_{sb\{1,2\}}, k_d, \delta l_{d,r}, \delta l_{d,l}, \beta, \gamma).$$

Computation of the FRF necessitates the definition of the points of excitation j and response k. The excitation point j was chosen to be the last element N, because at this position the cutting process takes place. Element k naturally corresponds to the point at which the considered signal is recorded. The FRF can then be computed by

$$\alpha_{jk}(\omega, l_B; p) = \omega^2 \sum_{r=1}^{N} \frac{\Psi_{jr,l_B} \Psi_{kr,l_B}}{k_{rr,l_B} - \omega^2 m_{rr} + i\omega c_{rr,l_B}},$$

where Ψ_{jr,l_B} denotes the j-th element of the r-th mode shape vector $\{\Psi\}_{r,l_B}$, and k_{rr}, m_{rr} and c_{rr} are the r-th diagonal elements of the modal stiffness-, mass- and damping matrices, respectively. These matrices can directly be derived from quadratic forms of the mode shape vectors and the stiffness- and mass matrices $[M]$ and $[K(l_B)]$ (Ewins (2000)). Finally, the eigenfrequencies ω'_{r,l_B} of the proportionally damped system are given by

$$\omega'_{r,l_B} = \omega_{r,l_B} \sqrt{1 - (\beta\omega_{r,l_B}/2 + \gamma/\left[2\omega_{r,l_B}\right])^2}.$$

As for this description once a specific p is chosen, the corresponding eigenfrequencies can be determined.

3.2 Maximum Likelihood Estimation

The parameters of the systematic model part can be estimated using the Maximum Likelihood method. The Likelihood-function can be derived by connecting the following well known results.

1. The periodogram $I_x(\lambda)$ of each stationary process X_t with a moving-average representation
$$X_t = \sum_{u=-\infty}^{\infty} \beta_u \epsilon_{t-u} \quad \text{with} \sum_{u=-\infty}^{\infty} (1+|u|)|\beta_u| < \infty$$
implying the spectral density $f_x(\lambda) = \left|\sum_u \beta_u e^{i2\pi\lambda u}\right|^2$ has an exponential distribution at each Fourier frequency λ with parameter $1/f_x(\lambda)$. Then periodogram ordinates at different Fourier frequencies are asympotically independent (Schlittgen and Streitberg (1999), p. 364).
2. Each stationary process X_t with continuous and for all λ non-negative spectral density $f_x(\lambda)$ has an infinite moving-average representation (Schlittgen and Streitberg (1999), p. 184).

Assumption 2 can be seen as fulfilled, as all inspected spectrograms clearly show values different from zero for all frequencies and time points. Assumption 2 substantially implies assumption 1, so for each Fourier frequency ω and hole depth l_B the distribution function of $S(\omega, l_B; p)$ is approximatively given by
$$d.f.(s) = f(\omega, l_B; p)^{-1} e^{f(\omega, l_B; p)^{-1} s},$$
where $f(\omega, l_B; p) = |\alpha_{jk}(\omega, l_B; p)|^2 \cdot |\alpha_j^*(\omega)|^2$.

Using the asymptotical independence of periodogram ordinates of different frequencies and assuming independence for different hole depths, the Log-Likelihood-function is given by

$$LL(p) = \sum_{l_B} \sum_{\omega} \left[ln \frac{1}{f(\omega, l_B; p)} - \frac{S(\omega, l_B; p)}{f(\omega, l_B; p)} \right]$$

The ML-estimators are the set p_{ML} of parameters maximizing this function. With these parameters the estimated eigenfrequencies can be derived as illustrated in the last two sections.

The introduced model has been successfully fitted to different experiments by using the search-based method by Nelder and Mead (1965) for the maximization of the Log-Likelihood-function. Fig. 5 shows an exemplary comparison between an acceleration spectrogram and the bending eigenfrequencies computed from the fitted model for a process without spiralling.

Even though the second and third eigenfrequency seem to over-estimate the area of elevated amplitudes the pattern in the spectrogram is clearly represented by the fit. So apart from possible model refinements, these results,

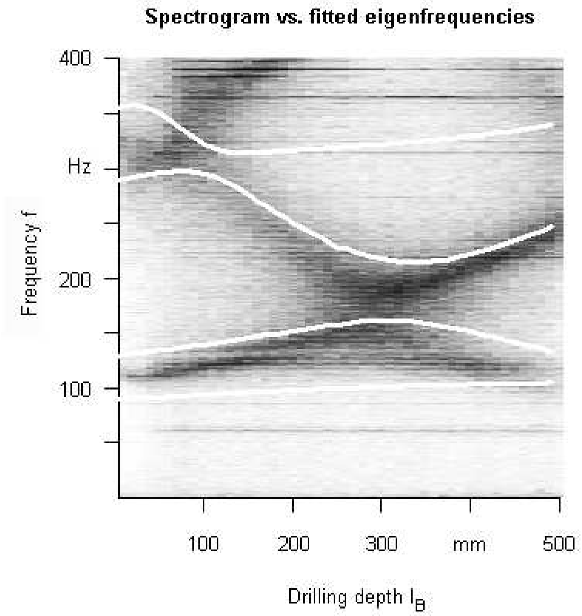


Fig. 5. Comparison between acceleration spectrogram and fitted eigenfrequencies.

which are similar for all other experiments investigated up to now, support the connection of the physical model with the statistical model as a basis for estimating its parameters from spectrogram data.

4 Summary and Outlook

The presented paper shows that a connection of a physical and a statistical model helps to estimate the bending eigenfrequencies of a deep-hole-drilling tool from data available during the process. Bending eigenfrequencies are known to cause spiralling when crossing multiples of the spindle rotational frequency. By supervising the estimated eigenfrequencies, shifts in the process dynamics can be detected and crucial situations can be predicted.

The supervision may be possible within a batch production, where after each completely drilled workpiece the eigenfrequencies are checked and the necessity of parameter changes is decided. In the actual form fitting the model is too time extensive to allow online intervention. But if the model can be simplified implying faster fitting procedures, strategies such as control charts for the eigenfrequencies could be feasible as well.

Simplifications of the physical model are possible by concentrating on the relevant regions of the spectra or modifications of the discretization. The statistical model may be simplified by estimating the spectra for frequency bands instead of Fourier frequencies using consistent estimates as introduced in Schlittgen and Streitberg (1999). Furthermore for a more efficient way of estimating the eigenfrequencies historical data may be used in connection with the physical model.

In future experiments roundness errors of the drilled workpieces will be measured at different equally spaced hole depth points. These measurements represent a quantization of the effect of spiralling over time and so help to investigate the development of spiralling more closely in different situations. Here main features of interest are whether spiralling starts rapidly or develops slowly and if the magnitude of spiralling depends on how quickly the frequency crossing takes place. As the measurement of roundness errors is not possible in production the potentials of estimating these from the spectrogram data will be checked as well.

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