Two connected models for varying amplitudes in BTA-deep-hole-drilling

Winfried Theis, Laurie Davies and Claus Weihs

September 23, 2005

Abstract

Two models are proposed to roughly approximate the observed behaviour of the amplitude of the drilling torque in the BTA-deep-hole-drilling process. It is shown that these models are closely connected.

1 Introduction

BTA-deep-hole-drilling is a process for the production of holes with a high length to diameter proportion. In our case the boring tool has a diameter of 60mm and the holes are 500mm long. The process produces holes of high quality with respect to straightness and roughness of the hole-wall. But because of the flexibility of the boring tool/toolbar assembly the process is vulnerable to dynamic disturbances such as chatter and spiralling. For more details see e.g. Weinert et al. (2002).

In this paper we focus on the modelling of chatter. We do not try to explicitly model this phenomenon, but compare different approaches to describe the phenomena observed in the drilling torque of experiments in which chatter was observed. Figure 1 gives two examples of time series of the drilling torque. From these series it can be seen that there exists more than one state in the process.

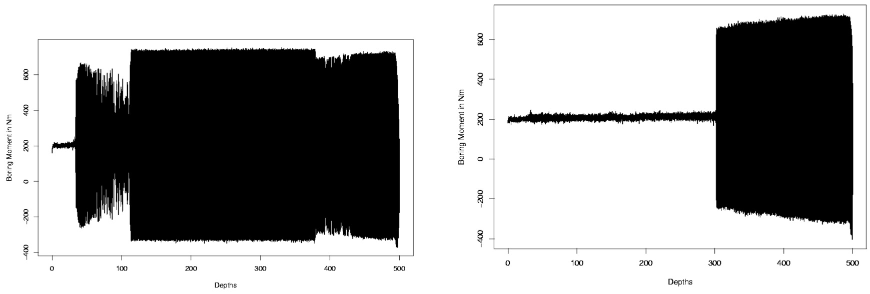

Figure 1: Two time series (Exp. 1 and 21a) of the drilling torque from experiments at the same parameters $f = 0.185 mm/rev$, $v_c = 90 m/min$ and $\dot{V} = 300 l/min$

The next figure shows the spectrograms of the time series above. These spectrograms show clearly that the chatter is dominated by single frequencies, which led to the idea of modelling the variation of these frequencies.

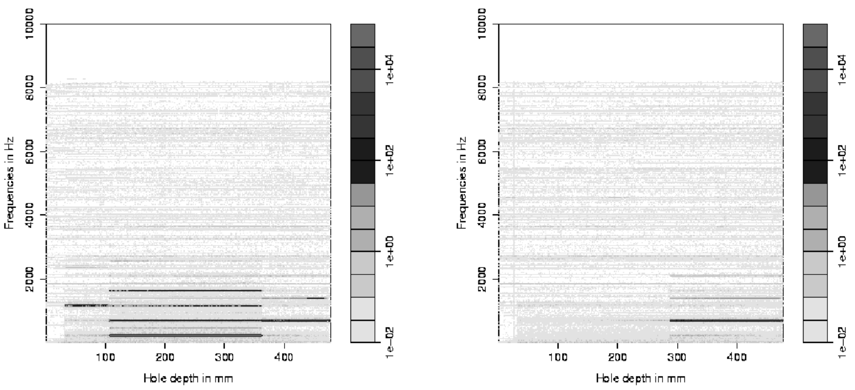

Figure 2: The spectrograms of the time series in Figure 1

To give an impression of the development of the amplitudes over time the most prominent frequencies in these spectrograms are plotted over time in Figure 3.

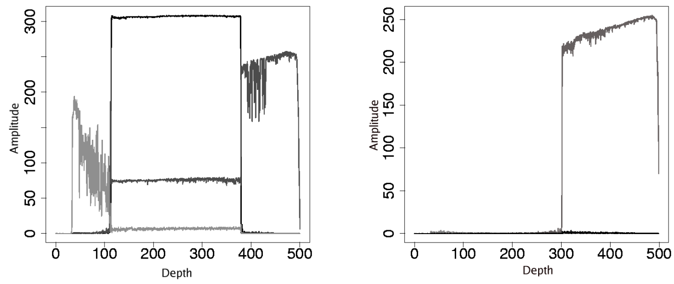

Figure 3: The amplitudes of most prominent frequencies 234Hz (black; high in the middle), 703Hz (dark grey; high in the last part) and 1182Hz (light grey; high in the beginning) plotted over time

2 Approximating the amplitudes by a two-sided logistic model

The main goal in this part of the analysis is to describe the main features of the variation of the amplitudes. From Figure 3 it is obvious that the function must allow for a very steep ascent and then staying on a certain level for some time and then a similar steep descent. Another form observed in the data is a small jump and the a long descent. Both features had to be included in the function to approximate the data. The basic functional form for the model is the following:

$$g(x; a, \mathbf{m}, \mathbf{d}) = \frac{a}{1 + \exp\left(\frac{-x+m_1}{d_1}\right) + \exp\left(\frac{x-m_2}{d_2}\right) + \exp\left(-\frac{(d_2-d_1)x+d_1m_2-d_2m_1}{d_1d_2}\right)} \tag{1}$$

The parameters in the function have the following effects: a determines the maximal value of the function if $m_1 \ll m_2$ which determine the position of the middle of the rise or fall, resp., d_1 and d_2 determine the slopes in m_1 and m_2.

Figure 4 shows two parameter settings for this function, which exhibit the requested behaviour.

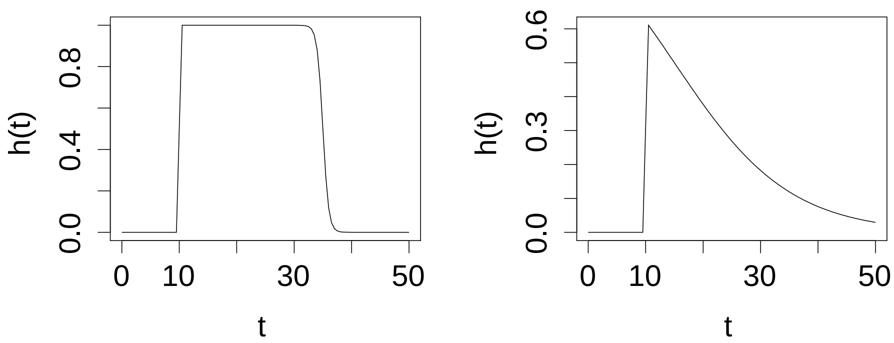

Figure 4: $g(x; a, \mathbf{m}, \mathbf{d})$ for $m_1 = 10$, $m_2 = 40$, and $d_1 = 2 = d_2$ on the left-hand side, and $m_{1,2} = 10$ and $d_1 = 2$ and $d_2 = 1000$ on the right-hand side.

Since in some experiments several changes in the chatter frequencies were observed, several functions of this form give a complete description of the amplitudes over time. Figure 5 shows fits of these functions to amplitudes from Figure 3.

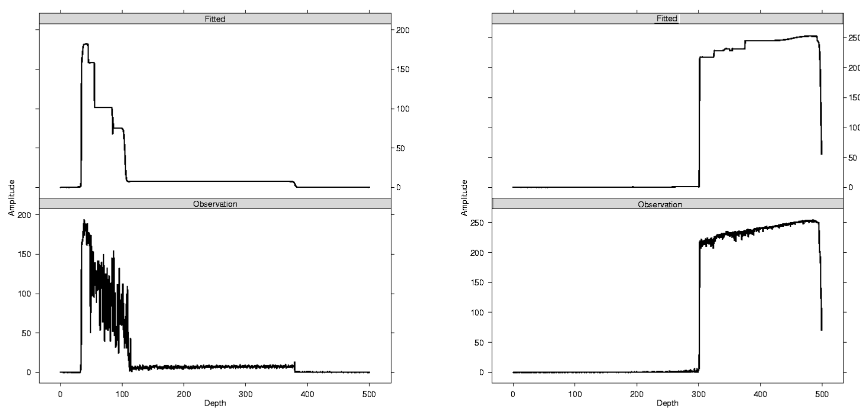

Figure 5: Upper Panels: Fits of sums of the basic function to the amplitudes; Lower Panels: Observations; Left: 1182Hz from the first Experiment; Right: 703Hz from second Experiment.

3 A dynamic model

In Weinert et al. (2002) the following differential equation was proposed as a general model for the description of the chatter:

$$\frac{d^2M(t)}{dt^2} + h(t)(b^2 - M(t)^2)\frac{dM(t)}{dt} + \omega^2 M(t) = W(t) \tag{2}$$

$t \in [0, \infty)$ and $200 \leq \omega \leq 2500$ in our application, $h : \mathbb{R} \to \mathbb{R}$ an integrable function, $b \in \mathbb{R}$, and $W(t)$ a white noise process. In a first step, this equation is considerably simplified, when $M(t)$ is taken as a harmonic process. Let

$$\begin{aligned}
M(t) &:= g(t)\cos(\omega t + \phi). \\
\frac{dM(t)}{dt} &= \frac{dg(t)}{dt}\cos(\omega t + \phi) - \omega g(t)\sin(\omega t + \phi), \\
\frac{d^2M(t)}{dt^2} &= \frac{d^2g(t)}{dt}\cos(\omega t + \phi) - 2\omega\frac{dg(t)}{dt}\sin(\omega t + \phi) - \omega^2 g(t)\cos(\omega t + \phi)
\end{aligned}$$

First, we note that the term $\omega^2 g(t)\cos(\omega t + \phi)$ is eliminated by substituting the derivations in (2). Second, $\frac{dg(t)}{dt}$ does not contain high-frequency-components and thus, the following holds

$$\int \frac{dg(t)}{dt}\cos(\omega t + \phi)dt \approx 0.$$

This means that for the solution of the differential equation the terms not containing the frequency ω have no effect because they are eliminated by integration. Therefore, (2) can be replaced by

$$-\omega\sin(\omega t + \phi)\left(2\frac{dg(t)}{dt} + h(t)g(t)(b^2 - g(t)^2\cos^2(\omega t + \phi))\right) = W(t). \tag{3}$$

Now $\cos^2(\omega t + \phi) = (1 - cos(2\omega t + 2\phi))/2$, and $\cos(2\omega t + 2\phi))$ has mean 0. So we get:

$$-\omega\sin(\omega t + \phi)\left(2\frac{dg(t)}{dt} + h(t)g(t)(b^2 - \frac{g(t)^2}{2})\right) = W(t). \tag{4}$$

Multiply (4) with $\sin(\omega t + \phi)$, and note that $W(t)\sin(\omega t + \phi)$ behaves similar to white noise. Moreover, note that $\sin^2(\omega t + \phi) \approx 1/2$. It follows for the amplitude that

$$\frac{dg(t)}{dt} + \frac{h(t)}{2}g(t)(b^2 - \frac{g(t)^2}{2}) = \frac{W(t)}{\omega}. \tag{5}$$

This is the amplitude-equation for the differential equation in (2), if there is only one frequency in the process.

4 Connection of the differential equation to the logistic function

Now assume that the logistic function from section 2 is the right form for $g(t)$. Then it has to be shown that there is a function $h(t)$ so that equation (5) has a solution. To show this, white noise is replaced by 0 to make the calculations more straightforward. Furthermore we reduce the problem to the upward jump in the function for symmetry reasons.

Set

$$g(t) := \frac{a}{1+\exp\left(-\frac{t-t_0}{d}\right)},$$

it follows

$$\frac{dg(t)}{dt} = \left(-\frac{\exp\left(-\frac{t-t_0}{d}\right)}{d}\right)\left(-\frac{a}{\left(1+\exp\left(-\frac{t-t_0}{d}\right)\right)^2}\right).$$

Inserting these formulas into (5) we get

$$2\frac{\exp\left(-\frac{t-t_0}{d}\right)a}{d\left(1+\exp\left(-\frac{t-t_0}{d}\right)\right)^2} + h(t)\left(\frac{ab^2}{1+\exp\left(-\frac{t-t_0}{d}\right)} - \frac{a^3}{2\left(1+\exp\left(-\frac{t-t_0}{d}\right)\right)^3}\right) = 0$$

Subtracting the first term on both sides:

$$h(t)\left(\frac{2ab^2\left(1+\exp\left(-\frac{t-t_0}{d}\right)\right)^2 - a^3}{2\left(1+\exp\left(-\frac{t-t_0}{d}\right)\right)^3}\right) = -2\frac{\exp\left(-\frac{t-t_0}{d}\right)a}{d\left(1+\exp\left(-\frac{t-t_0}{d}\right)\right)^2}.$$

Because the term in brackets on the left hand side is never 0, it is possible to divide by it. It follows

$$h(t) = -2\frac{\exp\left(-\frac{t-t_0}{d}\right)\left(1+\exp\left(-\frac{t-t_0}{d}\right)\right)}{d\left(b^2\left(1+\exp\left(-\frac{t-t_0}{d}\right)\right)^2 - \frac{a^2}{2}\right)}$$

This solution is well-defined for $t \in \mathbb{R}$. So (g, h) is a pair of functions, which solves equation (5).

5 Discretisation of the amplitude equation and simulations

The discretisation of equation (5) is given by

$$g(t+1) - g(t) = -\frac{b^2 h(t)}{2}g(t) + \frac{h(t)}{4}g(t)^3 + \varepsilon_t \quad (6)$$

$$\Leftrightarrow g(t+1) = \left(1 - \frac{b^2 h(t)}{2}\right)g(t) + \frac{h(t)}{4}g(t)^3 + \varepsilon_t. \quad (7)$$

Here ε is a discrete white noise process, e.g. normal white noise.

The latter relation is used to estimate $h(t)$ from the amplitudes for frequency 703 Hz in the experiment displayed in Figure 3 in the right-hand panel. Taking 21 successive observations of the amplitude and omitting the cubic term in (6), the slope – which is essentially $h(t)$ – displays the behaviour shown in Figure 6.

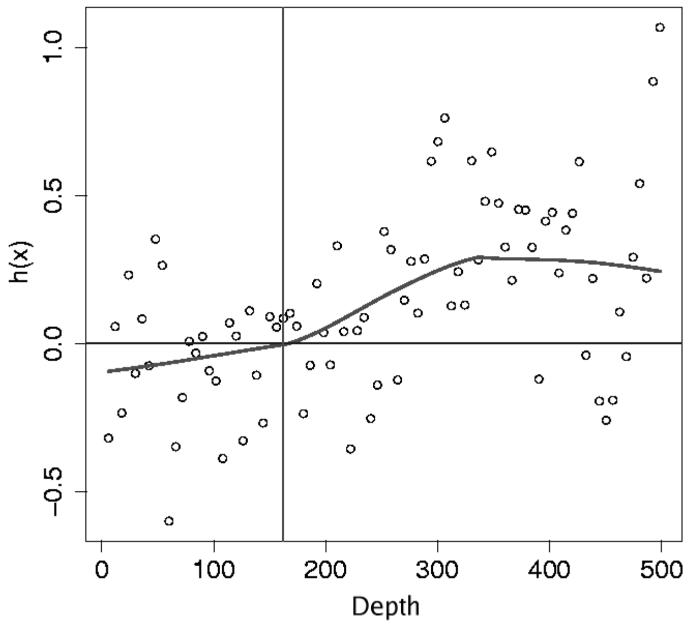

Figure 6: Slopes of the linear part in (6) over time

The horizontal grey curve in Figure 6 shows the result of the loess-smoother (Cleveland et al. (1992)) including 75% of the data in the fit of each point. This curve is similar to a logistic function like the function $h(t)$ found in the last section. The sign change indicated by the vertical grey line marks the turnover from stable to instable behaviour of this recursive formula. Obviously it is a lot earlier – at about 170mm – than the actual rise of the amplitude which does not happen until 300mm. This makes this estimate a candidate for an alarm signal for chatter.

As shown by Davies (1983) the actual shift from one state of the system to another is postponed by noise. In Figure 7 this can be seen for simulated data knowing that the change point was set to $t_0 = 160mm$ compared to the actual rise at $t = 400mm$. Furthermore, Figure 7 demonstrates that the behaviour of the simulated data is not far from the real data although it does not capture the obvious trend towards the end of the process.

All these observations combined show the appropriateness of the model and the possibility to detect a change in state of the system long before it truely has an impact on the output of the system.

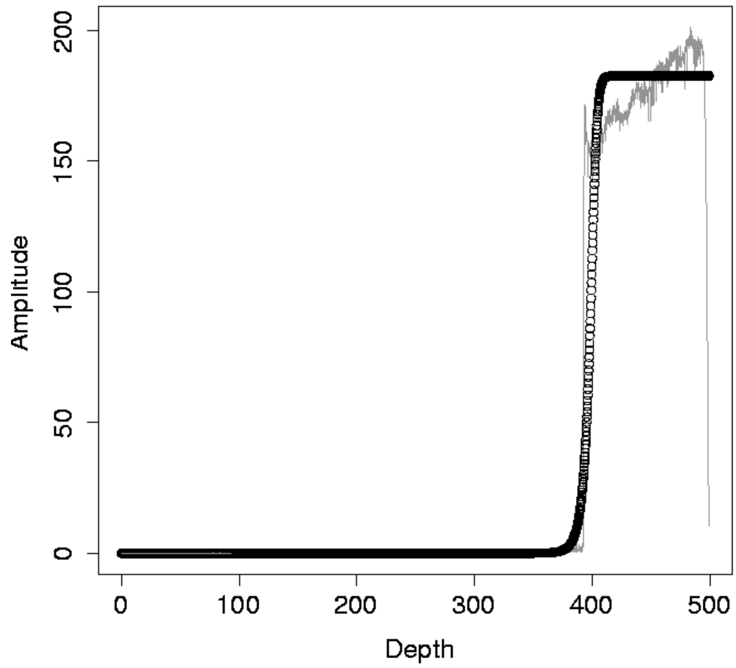

Figure 7: Comparison of a simulated amplitude (black) to a real amplitude (grey)

When $h(t)$ is changed to the function corresponding to the two-sided logistic function from equation (1) so that it returns below 0 after some time the process will return to the stable state again with a small delay as can be seen in Figure 8.

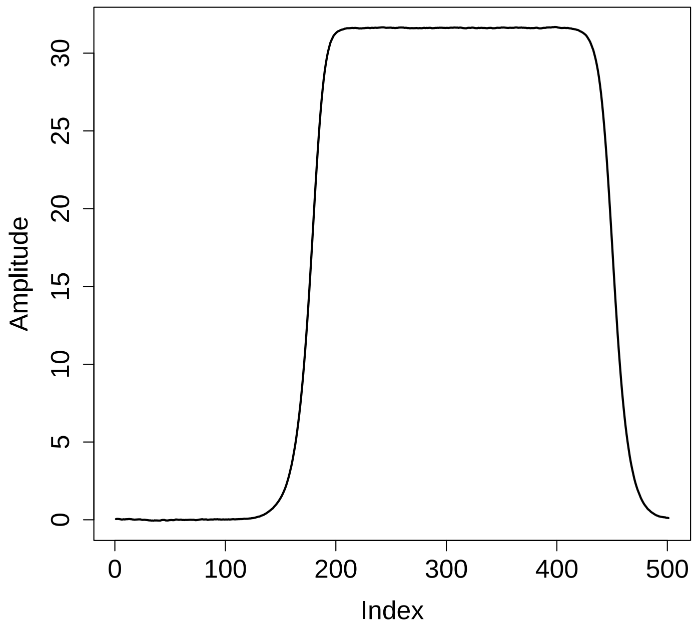

Figure 8: Simulation of recursive formula with $h(t)$ returning below 0; Start of instable process $t_0 = 100$, end $t_1 = 400$

Using the calculated (one-sided) $h(t)$ from Section 4 with the parameter values estimated on the real data (again on the data from Figure 3), we get e.g. the result shown in Figure 9.

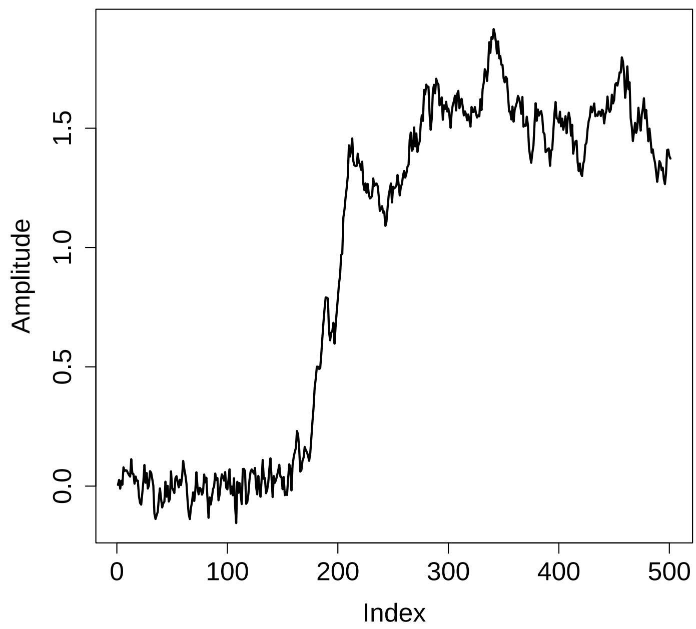

Figure 9: Example for a result of a simulation with the calculated $h(t)$

From Figure 9 it is obvious that the simulated data does not reach the maximal amplitude of the observed data. We tested a broad range of possible values for the free parameter b in (5) between $[10^{-6}, 10^{6}]$ but did not find a better result. This shows that an appropriate choice for the parameters in the recursive formula – and therefore in the differential equation – cannot be obtained by approximating the observed curve by the (one-sided) logistic function (or the function $g(x; a, \mathbf{m}, \mathbf{d})$ from equation (1)) and using the parameters from this approximation as estimates.

A major drawback of the first simulated data was its total smoothness when reaching its maximum. The real data displays a lot more variation when the chattering state is reached. Two ways were considered to include this feature into the simulation function. One way was to change the distribution of the noise to a Γ-distribution with a changing expected value and variance which was motivated by the distribution of periodogram ordinates determined in Theis (2004). This turned out to be absolutely inpredictable compared to the first approach with normal distributed noise. The second approach was to postulate that the variance is subject to a shift, as well, when the system changes from stable to instable.

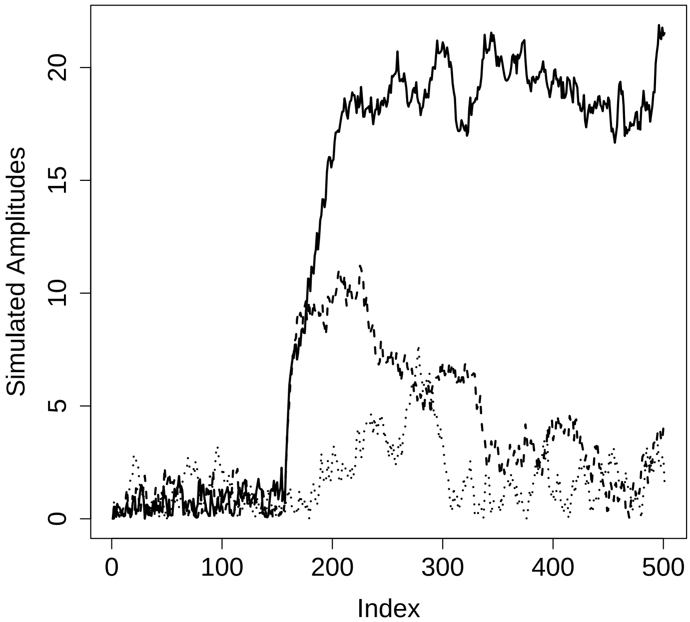

Figure 10: Simulated amplitudes with time-varying variance of the noise, all from the same parameters of the simulation function.

Figure 10 gives an impression of different possible behaviours of data with changing variance. The solid line displays the sought-after higher variance in the upper part but again does not reach the needed height and the other lines show clearly that this increased variance may eliminate the effect of the changed state completely.

6 Conclusion

It was shown that the chosen method for the approximation of the variation of the amplitude is directly connected to the proposed phenomenological model.

Furthermore, a possible way to estimate the time of the shift from stable to instable behaviour from the observations of the amplitudes was found by estimating the parameter of the linear part of the amplitude equation from windows of the observed data. The smoothed development of this parameter could be used as another alarm signal for a chattering state.

The simulations showed that the derived amplitude equation with the calculated function h is not yet appropriate for the approximation of the observed behaviour of the amplitudes. Extensions in the stochastic part of the model were tested to incorporate the fact that the observations display a higher variability in the chattering state. Two ways of inclusion of this feature of the data were tested. On the one hand a Γ-distribution was used for the disturbances, which led to an inadequate behaviour of the simulated series. On the other hand a change in the variance parallel to the change of the stability parameter was introduced, which looked slightly more appropriate but also did not reach the goal completely.

Acknowledgments

This work has been supported by the Collaborative Research Centre "Reduction of Complexity in Multivariate Data Structures" (SFB 475) of the German Research Foundation (DFG).

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