Auswertungsmethoden bei Experimenten mit Nachwirkungen und korrelierten Fehlern

(Analysis of experiments with carryover effects and correlated errors)

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Abstract

This dissertation deals with the analysis of crossover studies for the comparison of more than two treatments. The statistical analysis of such studies has to address small sample sizes, carryover effects of treatments and correlated observations. This work studies and improves several methods for analysis to this end.

One approach is based on randomization theory. Kunert and Sailer (2007, J. Stat. Plan. Infer. 137: 2045-2055) show that in the absence of carryover effects, randomizing experimental units and treatments justifies an analysis in the row-column model if regular generalized Youden designs are applied. The result does not hold if carryover effects are present and therefore carryover effects should be modelled explicitly whenever they are likely to occur.

In the model with carryover effects it is often assumed that the errors are uncorrelated. In such models, correlated errors lead to biased variance estimates. Kunert and Utzig (1993, J. Roy. Stat. Soc. B 55: 919-927) propose an analysis based on a model with uncorrelated errors but include a correction factor for the variance estimate of the treatment contrasts. The correction leads to a conservative variance estimate irrespective of the true correlation. They derive the exact correction factor based on the worst case covariance matrix for $p=3$ periods only and give an upper bound for $p>3$.

This dissertation considerably improves the method of Kunert and Utzig. The exact correction factor is derived for optimal neighbour-balanced designs with any number of experimental units, treatments and periods. Thus the variance estimates are less conservative if $p>3$, resulting in tests of larger power.

Bellavance, Tardif and Stephens (1996, Biometrics 52: 607-612) propose a further analysis in the model with carryover effects and uncorrelated errors. They estimate the unknown covariance matrix and correct the variance estimate by applying the Box correction (Box, 1952, Ann. Math. Stat. 25: 290-302) to their model. The question arises, whether the improvement to the method of Kunert and Utzig or the method by Bellavance et al. is to be preferred.

This work shows that the bias correction proposed by Kunert and Utzig is equal to the correction factor of Bellavance et al. if the covariance matrix is known. In the case where the covariance matrix is not known and the true covariance matrix is the worst-case covariance matrix with respect to the variance bias of treatment contrasts, even the estimated correction factor of Bellavance et al. equals the correction factor of Kunert and Utzig.

A simulation study is carried out to compare the test size and power of the two approaches. The method of Bellavance et al. holds the nominal test size reasonably well, while the improvement of the method by Kunert and Utzig is anti-conservative in the worst-case scenario. This is due to the fact, that the Box-correction also corrects the degrees of freedom while the method by Kunert and Utzig does not. Whenever both methods hold the test size, the method of Bellavance et al. has greater power. This means that it is better suited for the analysis of crossover designs with carryover effects and correlated errors.

Keywords: *crossover design, carryover effects, correlated errors, linear model, variance estimation, Box correction.*