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Students' use of strategies when working on multiplication tasks with array representations: An eye-tracking study

Students' understanding of multiplication, especially the development of basic ideas ("Grundvorstellungen"), is an important aspect of primary mathematics education (KMK, 2022). Understanding basic ideas is vital to developing an understanding of multiplication. The focus of this study is on the spatial-simultaneous basic idea, often supported by array representations (AR) (Barmby et al., 2019). Through the use of eye tracking, this study aims to investigate students' strategies and their use of these strategies when working on multiplication tasks with AR of 163 fifth-graders. By analyzing eye-tracking videos, three kinds of strategies were identified, including (A) *Enumerating dots in rows and/or columns (quasi-) simultaneously*, (B) *Counting dots in rows/columns or unitizing*, and (C) *Counting dots one by one*, which echoes the findings of Barmby et al. (2009) and Bolden et al. (2015). The results indicate that the fifth-graders used *Counting dots one by one* in almost 20% of the tasks, i.e., they did not use multiplicative structures, indicating difficulties in their understanding of multiplication. Eye tracking allowed us to analyze students' strategies and their use of these strategies for a large number of students. This study highlights the need to support students' understanding of multiplication, even at the beginning of secondary school.

Acknowledgement

This study was coordinated by Prof. Dr. Maike Schindler and funded by the Federal Ministry of Education and Research as a part of the program KI-ALF (01NV2123). Responsibilities for the content of this publication remain with the authors.

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Students' use of strategies when working on multiplication tasks with array representations: An eye-tracking study

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Abstract

Students' understanding of multiplication, especially the development of basic ideas ("Grundvorstellungen"), is an important aspect of mathematics education. The focus of this study is on the spatial-simultaneous basic idea of multiplication, often supported by using array representations (AR). Through eye tracking (ET), we investigated the strategies used by 163 fifth-graders on multiplication tasks with AR. We found that the students used a counting-all strategy in almost 20% of the tasks, i.e., they did not use multiplicative structures, indicating difficulties in their understanding of multiplication.

Theoretical background

Multiplication of natural numbers

- Developing basic ideas of multiplication is crucial for students to acquire a comprehensive understanding of multiplication (vom Hofe et al., 2005).
- Basic ideas bridge the realm of mathematics and individuals' thinking, providing various interpretations of mathematical ideas (vom Hofe et al., 2005).
- Recognizing multiplicative structures in spatial representations and understanding that spatial representations stand for multiplicative structures are the key aspects of the spatial-simultaneous basic idea (Bolden et al., 2015; Mulligan, 2002).
- Unitizing refers to grouping elements into units and then operating these units (Singh, 2000). In multiplication, the process of operating with units of more than one requires an understanding of the multiplier and the multiplicand and the ability to think multiplicatively. 4 · 3 is then interpreted as 4 sets of 3 (Götze & Baiker, 2021).

Array representations (AR)

- AR are widely used to develop the spatial-simultaneous understanding of multiplication.
- AR are rectangular grids of discrete elements (e.g., dots) arranged in rows and columns. The rows and columns are representatives of the multiplier and the multiplicand, which allows multiplicative interpretations (Barmby et al., 2009; Mulligan, 2002).
- A widely used AR both in school and mathematics education research is the 100-dot field (e.g., Schindler et al., 2019).

Introduction

Multiplication of natural numbers is fundamental in primary mathematics education and crucial for understanding advanced concepts (KMK, 2022; Downton & Sullivan, 2017). Yet, students transitioning from primary to secondary school often struggle with the understanding of multiplication, emphasizing the need for insights into their strategy use (Downton & Sullivan, 2017). Understanding basic ideas is vital to developing an understanding of multiplication, e.g., the spatial-simultaneous basic idea typically developed by AR (Barmby et al. , 2009). Research showed that students used various strategies in solving multiplication tasks with AR, reflecting different understandings of multiplication. Yet, there is limited knowledge on the strategies and their use.

Aim

Investigate students' strategies and their use of these strategies when working on multiplication tasks with array representations

Research Questions

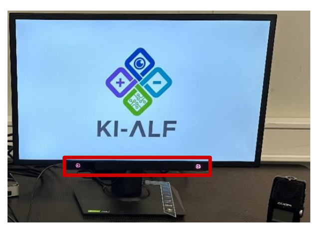
1) What strategies do fifth graders working on multiplication tasks with array representations use?
2) How often are these strategies used by fifth graders for different multiplication tasks with array representations?

Related studies of strategy use in multiplication tasks with AR

- For AR with structural spacings between the fifth and sixth column/row, Barmby et al. (2009) analysed audio-visual recordings of students from grade 4 and 6. They found that students used counting strategies (in ones or small groups), distributive strategies (distributive properties based on groups of 25), rearranging strategies (moving parts of the array to make the calculation easier), and completing strategies (first complete a bigger array then subtract) to solve the multiplication tasks.
- Tasks of AR without structural spacing between fifth and sixth column/row were the focus of a study by Bolden et al. (2015), where they analysed eye movements of nine fifth graders using ET. Findings revealed that fifth graders either counted each element in the AR, or limited their counting only in the rows and/or columns.
- In previous research findings, strategies such as counting all suggest that students have not adequately understood the spatial-simultaneous basic idea of multiplication (Mulligan, 2002).

Eye tracking in mathematics education

ET provides a valuable chance for researchers and educators to gain insights into students' understanding, particularly of students with mathematical difficulties who may have issues expressing their strategies. It helps to uncover the diverse strategies used when students work on mathematical tasks, e.g., quantity recognition (Schindler & Lilienthal, 2018).

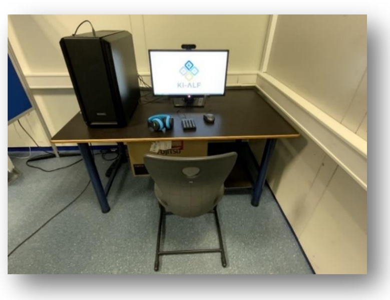


Eye tracker

Method

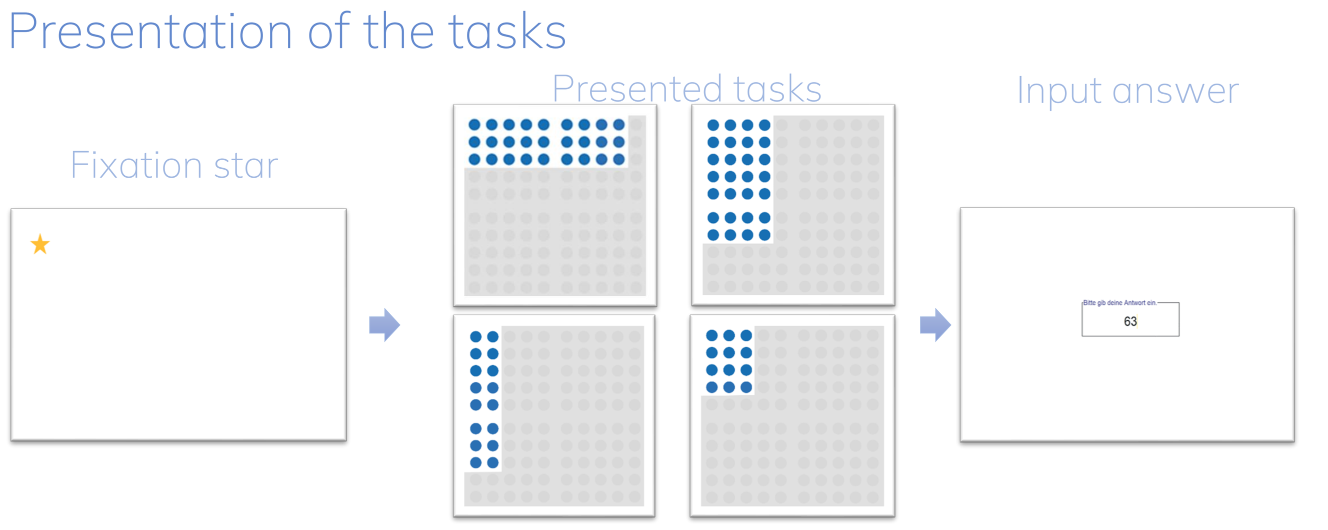
Data collection

- Participants
  - 163 grade 5 students aged from 9.8-12.8 years (M = 10.6, SD = 0.5) of a German inclusive comprehensive school
- Devices
  - Monitor (24" HD monitor, 60 Hz refresh rate)
  - Tobii Pro X3-120 (infrared, binocular, 120 Hz)



Desk setting

Presentation of the tasks



Eye-tracking data

- Average accuracy: 1.1°, SD: 0.9°, corresponding to error about 1.2 – 1.3 cm on the 24" monitor
- Gaze-overlaid videos (gazes displayed as a semi-transparent moving dot)

Qualitative data analysis

Following three stages, we identified students' strategies when working on multiplication tasks with AR (Mayring, 2014; similar to Schindler et al., 2019).

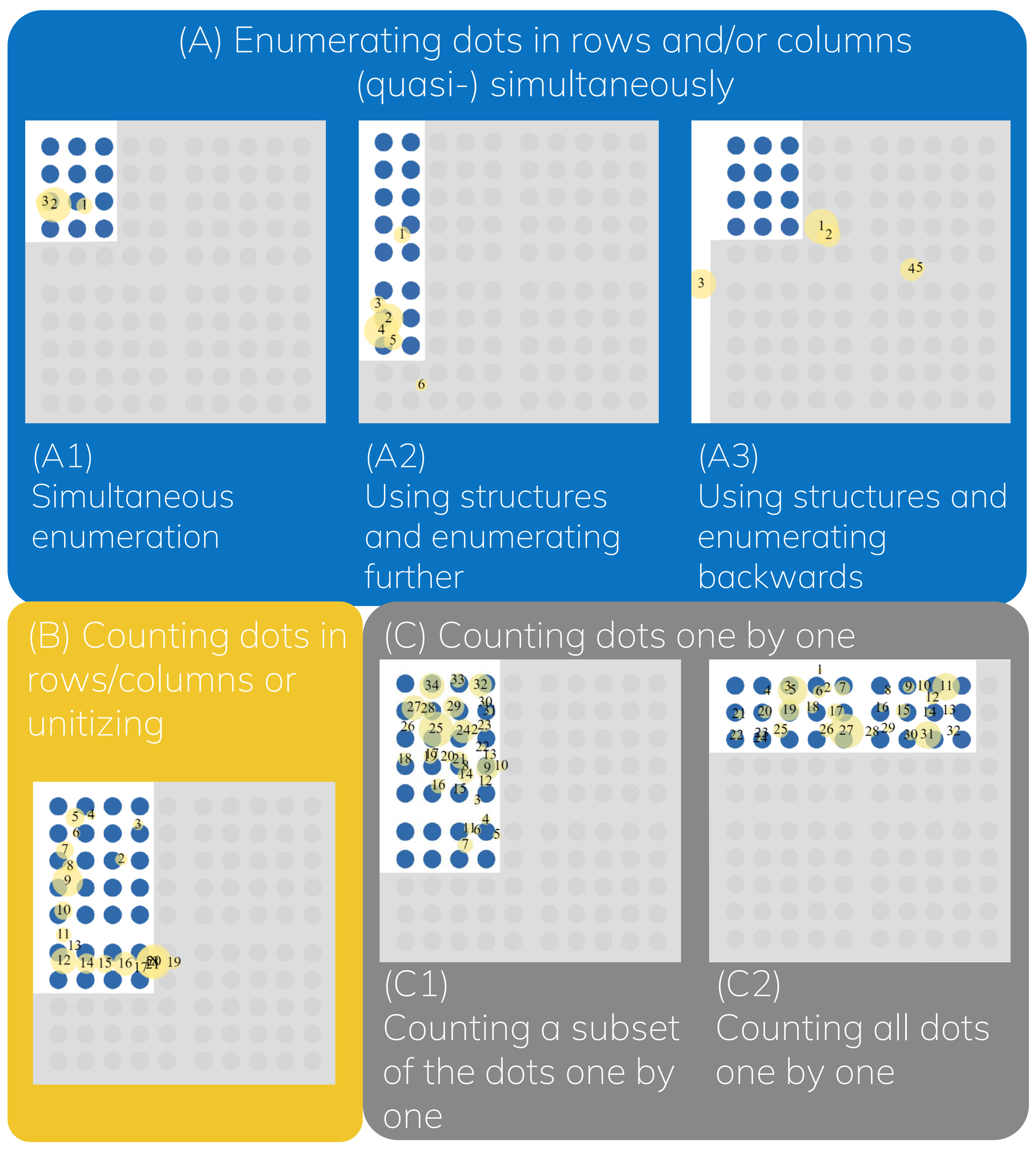
Stage 1 Coding based on an initial categorization developed by watching videos, describing the gazes, and grouping the similar descriptions

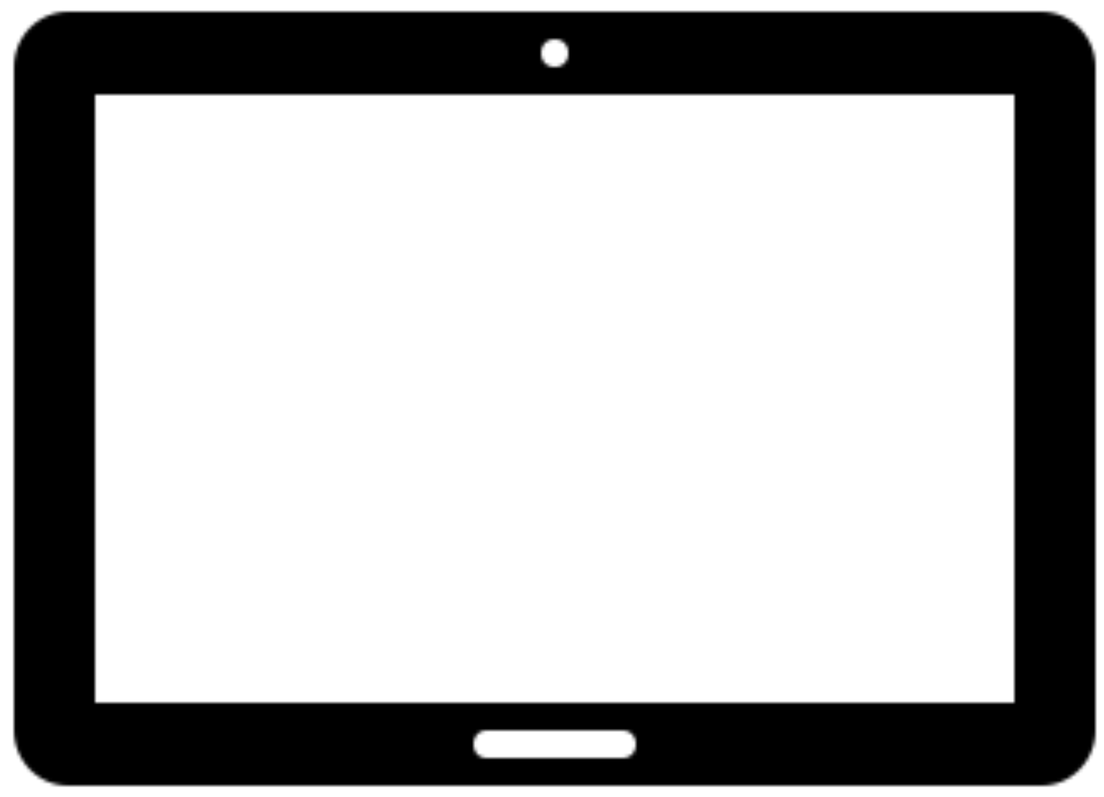
Stage 2 Refinement of the categorization by thoroughly discussing dissimilarities in code assignments

Stage 3 Final coding with a high interrater agreement indicated by Cohen's Kappa (κ = 0.874)

Results

1. Multiplication strategies of fifth-graders

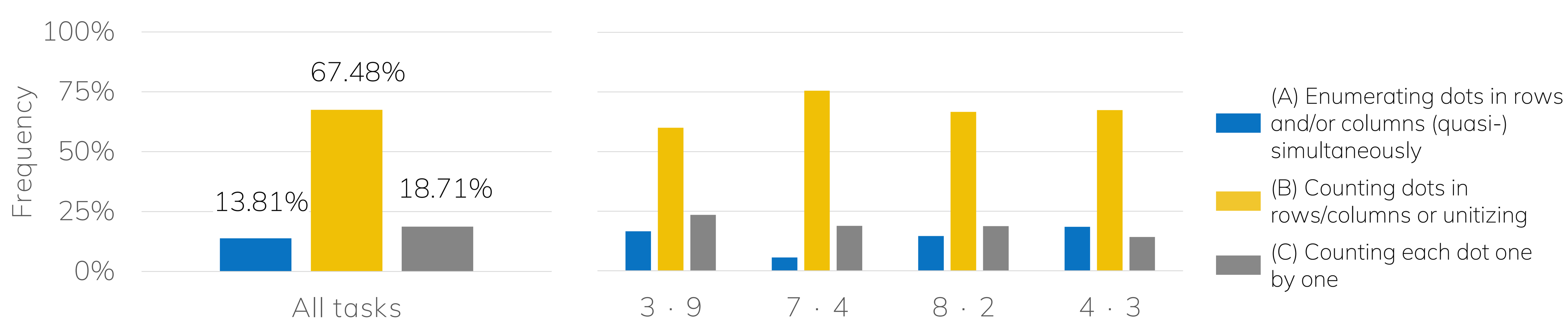




(Also available online, see Lai et al., 2024)

- Gazes of strategies (A2) and (A3) indicate that students enumerated the number of dots in rows and columns using structural hints of the 100-dot field (i.e., the structure of 5 and structure of 10).
- The name of strategy (B) reflects the uncertainty in distinguishing if it involves counting dots in rows/columns or the process of unitizing.

2. Distribution of strategy use



- Almost a fifth of the students (18.71%) used strategy (C) Counting each dot one by one.
- Strategy (B) was used most frequently across four tasks. Strategy (A), which was the least used overall, was used more often than strategy (C) in task 4 · 3. Conversely, for the other three tasks, strategy (C) was used more often than strategy (A).
- Notably, strategy (A) was used less in task 7 · 4.

Discussion

- By analyzing eye-tracking videos, three kinds of strategies were identified used by fifth graders when working on multiplication tasks with AR, echoing the findings of Barmby et al. (2009) and Bolden et al. (2015).
- Almost a fifth of the multiplication tasks were solved using counting strategy (C) by fifth graders, which indicates an insufficient understanding of multiplication. This resonated with the findings of Mulligan (2002).
- Eye tracking allowed us to analyze students' strategies and their use for a large number of students.
- This study highlights the need to support students' understanding of multiplication, even at the beginning of secondary school.

SPONSORED BY THE Federal Ministry of Education and Research

RAHMENPROGRAMM EMPIRISCHE BILDUNGSFORSCHUNG

Acknowledgement

KI-ALF coordinated by Prof. Dr. Maike Schindler has received funding from the Federal Ministry of Education and Research as a part of the program KI-ALF (01NV2123). The responsibility for the content of this publication remains with the authors.

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