Capital Budgeting: A Comprehensive Guide

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Foreword

This booklet, originally published in German[1], has its origins in lectures, seminars, and consulting cases that I conducted during my time as a university lecturer in Germany. Over the years, it has served as a foundational text for an introductory course on investment calculation methods. The objective of the booklet is to present the most important investment calculation methods in a concise and accessible format, allowing readers to quickly acquire basic knowledge.

An exercise section complements the theoretical part, providing opportunities to apply the concepts learned. Key financial figures can easily be calculated using Excel's financial mathematics functions, which are detailed in a separate booklet dedicated to Excel formulas[2].

This revised English edition has been machine-translated and carefully edited with the help of ChatGPT.

I hope the booklet proves helpful to international students seeking to master capital budgeting methods.

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Kempten, January 2025

[1] Pflaumer, P.: *Grundwissen Investitionsrechnung, 4th Edition, Kempten 2018.* `https://www.researchgate.net/publication/351587409`

[2] Pflaumer, P.: *Excel Formulas for Financial Mathematics and Investment Analysis, Studies of the Competence Center for Business Development and Consulting e. V.*, Kempten, 2024. `https://www.researchgate.net/publication/377178441`

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Chapter 1

Introduction

1.1 Concept of Investment

The term "investment" is defined differently across the literature. In its narrowest sense, investment refers to the production and acquisition of tangible assets that are part of fixed capital. A broader, balance sheet-oriented definition of investment involves the allocation of company capital (liabilities) to company assets (assets). This definition encompasses both payments for tangible assets and financial assets. In its broadest sense, investment refers to a series of payments for acquiring an asset that generates returns or reduces future obligations. This broader concept includes converting cash into both tangible goods (e.g., land, buildings, materials, securities) and intangible goods (e.g., patents, human capital). The cash flow-based definition of investment is particularly well-suited for assessing profitability through financial mathematical methods, as it allows for a comprehensive comparison of all cash inflows and outflows associated with the investment.

1.2 Types of Investments

Investments can be categorized based on the type of asset into real and financial investments. Real investments involve payments for tangible assets (e.g., land, machinery, raw materials) and intangible assets (e.g., employee training, research and development). Financial investments involve placing capital in nominal assets such as receivables, equity shares, or securities.

Based on the investment's purpose, we distinguish between replacement, rationalization, and expansion investments. Replacement investments involve substituting an existing asset with a new one of the same type and quality. Rationalization investments, however, are made with the expectation that the new asset will be more cost-efficient than the old one. When an additional asset increases technical capacity, this is referred to as an expansion investment. Investments are further classified by their frequency: they can either be one-time (e.g., land purchases) or recurring (e.g., replacement investments).

1.3 Cash Flows and Investment Decisions

Cash flows refer to the timing, amounts, and risks associated with all financial inflows and outflows related to an investment. The net difference between these financial inflows and outflows is termed cash flow. Cash flows quantify the financial impacts of an investment decision and serve as the basis for calculating profitability across various investment opportunities. The results of these calculations provide a critical foundation for informed investment decisions.

1.4 Investment Appraisal Methods

Investment appraisal is a quantitative method used to assess the profitability of a project. It includes all associated inflows and outflows, the timing of payments, and information regarding payment risk. The following overview summarizes the primary methods used in investment appraisals.

1.4.1 Overview of Methods

I. **Deterministic Methods**

- A. Static Methods
  1. Cost comparison
  2. Profit comparison
  3. Return comparison
  4. Payback period
- B. Dynamic Methods
  1. Net present value (NPV)
  2. Annuity method
  3. Internal rate of return (IRR)
  4. Other interest rate methods

II. **Stochastic Methods**

- Sensitivity analysis
- Risk analysis
  - Analytical methods
  - Simulation methods

1.5 Cost of Capital

The cost of capital represents the minimum return that an investor requires, given the risk associated with the investment. It reflects a realistic target return. Typically, the cost of capital is aligned with the normal rate of return expected within the company or industry. The lower bound for the cost of capital is usually the long-term market interest rate.

1.6 Terminology

It is assumed that cash flows occur at the end of each year and that there is initially complete certainty about future payments. The following terms are used:

- e_t: Receipt at time t
- a_t: Payment at time t
- c_t: Net cash flow at time t ($c_t = e_t - a_t$)
- I_0: Initial investment outlay at time 0
- n: Investment duration in years
- C_0: Net present value
- p: Nominal interest rate (e.g., 5%)
- i: Decimal interest rate $i = \frac{p}{100}$ (e.g., 0.05)
- r: Internal rate of return
- r_B: Baldwin rate of return (capital return)
- a: Equivalent annuity
- n_A: Dynamic payback period
- R: Static return on average tied-up capital
- ROI: Return on investment
- t_A: Static payback period
- A: Depreciation

1.7 Investment Project Planning, Execution, and Control

1.7.1 Investment Idea

a) Internal

An investment idea can originate within the company, based on its strategic goals, operational needs, or desire to expand. It may stem from identifying opportunities to improve efficiency, reduce costs, or develop new products.

b) External

Alternatively, external factors like market conditions, customer demand, technological advancements, or competition can inspire the investment. These ideas are crucial in responding to external opportunities and threats in a dynamic market environment.

1.7.2 Determination of the Cost of Capital

The cost of capital is a critical benchmark in investment analysis, representing the minimum return required for a project to be considered viable. It reflects the opportunity cost of using company funds, taking into account risk, inflation, and alternative investment opportunities. Proper determination ensures that only projects with expected returns above this threshold are undertaken.

1.7.3 Establishment of the Cash Flow Series for the Investment

a) Duration

The project's duration must be clearly defined, including the time horizon over which the investment will generate returns. A well-planned time frame helps in forecasting cash flows accurately.

b) Discounting Date
The point in time when cash flows are discounted back to present value should be established. Typically, this is the time when the investment begins, but it could vary depending on the project structure.

c) Investment Outflows
This step involves identifying all upfront and ongoing costs related to the investment. These include not only initial purchase or construction costs but also maintenance, upgrades, and any operational expenses.

d) Cash Flows Before and After Taxes
Forecasting the cash flows generated by the investment is essential, considering both pre-tax and post-tax scenarios. Tax considerations can significantly impact the project's profitability and must be integrated into the analysis.

e) Financing Alternatives
Different financing options (debt, equity, or hybrid) can influence the project's risk and return profile. Choosing the appropriate mix of financing is vital for optimizing capital structure and reducing overall project risk.

1.7.4 Evaluation

a) Net Present Value (NPV)
NPV is a fundamental method of evaluating an investment's profitability, representing the difference between the present value of cash inflows and outflows. A positive NPV indicates a profitable investment.

b) Annuity Method
The annuity method converts the NPV into equal annual amounts, making it easier to compare investments with different durations.

c) Internal Rate of Return (IRR)
IRR is the discount rate at which NPV equals zero, providing a measure of the investment's return. It helps compare the project's profitability against the cost of capital.

d) Baldwin Rate of Return

This variation of IRR adjusts for varying capital requirements over time, offering a more refined profitability analysis.

e) Payback Period

The payback period measures how long it takes for the investment to recover its initial costs. Although simple, this method doesn't account for the time value of money or cash flows after the payback period.

f) Static Methods

While dynamic methods are preferred, static methods (e.g., cost comparison) provide a quick, albeit less comprehensive, way to assess investment efficiency. They focus on a single period rather than the entire project lifecycle.

1.7.5 Risk Analysis

a) Model

Risk analysis starts with selecting an appropriate model to assess uncertainty in the investment's cash flows, whether deterministic or stochastic.

b) Deterministic and Stochastic Factors

Deterministic factors assume certain outcomes, while stochastic factors incorporate randomness. Both must be considered to understand the range of possible results.

c) Risk Profiles

A risk profile outlines the distribution of possible outcomes, helping investors understand the likelihood of achieving specific returns.

d) Dependencies

It is important to account for interdependencies between various factors that affect the project, such as market conditions, operational variables, and economic factors.

e) Simulations

Monte Carlo simulations and other techniques can model different scenarios to predict how risk factors impact the project's profitability.

f) Interpretation
Key statistics, such as the mean, standard deviation, and probability of achieving certain results, help in understanding the risk-return trade-off. These insights allow for more informed decision-making.

1.7.6 Decision-Making

a) Cash Flow Series
Decisions should be based on the entire cash flow series, ensuring that both inflows and outflows are accounted for in the context of the project's risk profile.

b) Objectives
The investment's alignment with company objectives (profitability, market expansion, sustainability) must be considered in the decision-making process.

c) Other Factors (Personnel, Technical, Environmental, etc.)
Aside from financial considerations, personnel needs, technical feasibility, and environmental impact should be factored into the decision.

d) Risk
The project's overall risk must be assessed, and only those investments where the expected return justifies the risk should be selected.

1.7.7 Investment Control

a) Deviations Between Planned and Actual Values
Regular monitoring is essential to identify deviations between forecast and actual cash flows. This step helps in identifying discrepancies early and allows for corrective action.

b) Analysis of Deviations and Identification of Causes
Understanding the reasons for deviations is key to improving the ac-

curacy of future forecasts and ensuring that the project remains on track.

c) Implementation of Corrections

When significant deviations occur, corrective measures (such as cost-cutting, reallocation of resources, or strategic pivots) may be necessary to bring the project back in line with its goals.

Chapter 2

Static Methods of Investment Appraisal

2.1 Preliminary Remarks

Dynamic methods (such as the net present value method and the internal rate of return method) are based on cash inflows and outflows and account for the timing of payments through discounting. In contrast, static methods are based on the costs and revenues of a single period. They disregard the different timing of relevant figures used for the calculation. Static methods, which rely on figures from **cost accounting**, are also referred to as one-period methods. Accordingly, dynamic methods are sometimes referred to as multi-period methods.

In static methods, **imputed interest** is often calculated to represent the **opportunity cost** of the capital employed by the entrepreneur. These static methods, commonly used in small and medium-sized enterprises, are characterized by easily interpretable metrics. The calculations are straightforward, and the required data is typically available from the company's cost accounting system.

However, the primary criticism of static methods is their single-period perspective. The present value, and therefore the profitability of an investment, depends not only on the size of influencing factors but also on their timing. Two investment alternatives may differ in profitability even if their static returns are identical. Consequently, the metrics from static methods can only approximate those of dynamic methods. The following example will illustrate the application of **cost accounting methods** in static investment appraisal:

Example:

An investor plans to purchase equipment for producing organic cheese wedges. Two machines, Type A and Type B, are available. The purchase prices, variable costs, and fixed costs are listed in the following table. The equipment will be used for 4 years, and straight-line depreciation will be applied to the residual value. The imputed interest rate is 10%.

Machine Type	**A**	**B**
Purchase Price	28,000	32,000
Resale Value (after 4 years)	8,000	10,000
Fixed Costs (excluding depreciation and imputed interest per year)	21,400	21,900
Variable Costs per Unit	0.36	0.33

Table 2.1: Comparison of two machines for investment decision

2.2 Cost Comparison Method

The starting point of this method is the calculation of the total costs or the average costs per year. If the output quantity of the various investment alternatives is the same, the alternative with the lowest total cost should be chosen. If the output quantity of the investment alternatives differs, the alternative with the lowest average or unit costs is preferable.

With a linear relationship between total costs K and output x, the linear cost function is:

$$K = K_f + k_v \cdot x$$

where:

- K = Total costs,
- K_f = Fixed costs,
- k_v = Variable cost per unit,
- x = Output quantity.

The average or unit cost function is derived by dividing the total cost by the output quantity, which yields:

$$k = \frac{K}{x} = \frac{K_f}{x} + k_v$$

In static methods, imputed interest is calculated for the capital employed by using the average capital as the calculation basis. Assuming a straight-line depreciation of the residual value R_n over n years, the annual depreciation is:

$$A = \frac{I_0 - R_n}{n}$$

where:

- I_0 = Initial investment,
- R_n = Residual value.

Thus, the residual value after n years is:

$$R_n = I_0 - n \cdot A$$

The average capital employed is:

$$\bar{K} = \frac{I_0 + R_n}{2}$$

This formula assumes that the capital employed decreases uniformly from I_0 to R_n over the duration n.

The formula for $\bar{K}$ arises from the geometric property of a linear decrease. The function $C(t)$, representing capital employed at time t, forms a straight line from I_0 to R_n. The average is equivalent to the height of the rectangle with the same area as the trapezoid under the $C(t)$ curve:

$$\bar{K} = \frac{1}{n} \int_0^n C(t)\, dt = \frac{I_0 + R_n}{2}$$

When capital employed decreases exponentially, such as with declining balance depreciation, the average capital employed is calculated using *Keyfitz's formula*:

$$\bar{K} = \frac{I_0 - R_n}{\ln(I_0) - \ln(R_n)}$$

This formula accounts for the dominance of larger capital values in earlier periods due to the exponential decay.

For imputed interest, the following formula is used as an estimate:

$$Z = \frac{p}{100} \cdot \bar{K} = \frac{p}{100} \cdot \frac{I_0 + R_n}{2}$$

For two investment alternatives, the **critical output quantity** x_{crit} is the quantity at which the advantage of one alternative shifts to the other. If the cost functions for investments A and B are given by:

$$K_A = K_{fA} + k_{vA} \cdot x$$

and

$$K_B = K_{fB} + k_{vB} \cdot x$$

the total costs are equal at the critical output quantity x_{crit}, where:

$$K_A = K_B$$

or equivalently:

$$K_{fA} + k_{vA} \cdot x_{\text{crit}} = K_{fB} + k_{vB} \cdot x_{\text{crit}}$$

Solving for x_{crit} gives:

$$x_{\text{crit}} = \frac{K_{fB} - K_{fA}}{k_{vA} - k_{vB}}$$

The critical output quantity can only be calculated if the two linear cost functions intersect, meaning:

$$K_{fB} > K_{fA} \quad \text{and} \quad k_{vB} < k_{vA}$$

Example:

1. Which alternative should be chosen for an annual production of 25,000 units?
2. What is the critical output quantity?

3. Which alternative should be chosen if the production estimate for Machine A is 20,000 units and for Machine B is 25,000 units?

Solution:

The imputed interest and depreciation are:

Machine Type	**A**	**B**
Depreciation	5,000	5,500
Imputed Interest	1,800	2,100

The cost functions for Machines A and B are:

$$K_A = 28,200 + 0.36 \cdot x$$
$$K_B = 29,500 + 0.33 \cdot x$$

1. For a production of 25,000 units, the total costs are:

$$K_A = 28,200 + 0.36 \cdot 25,000 = 37,200$$

$$K_B = 29,500 + 0.33 \cdot 25,000 = 37,750$$

2. The critical output quantity is:

$$x_{\text{crit}} = \frac{29,500 - 28,200}{0.36 - 0.33} = 43,333 \text{ units}$$

3. The unit costs for Machine A and B are:

$$k_A = \frac{28,200}{20,000} + 0.36 = 1.77$$

$$k_B = \frac{29,500}{25,000} + 0.33 = 1.51$$

Thus, Machine B is more cost-effective for a production quantity of 25,000 units.

2.3 Profit Comparison Method

In addition to costs, the **Profit Comparison Method** also includes revenues in the investment calculation. Instead of comparing costs, profits are compared. The investment alternative with the highest profit should be selected. One point of criticism, however, is that differing levels of investment expenditures are not directly considered. The higher profit of the more favorable investment may result from higher investment outlays. Therefore, the profit comparison method is only appropriate without further checks when the investment outlays are the same. If they differ, the profit from the difference in investment outlays between the higher and lower investment must be taken into account.

Example:

Profit Comparison with Differing Investment Outlays

	Investment X	**Investment Y**
Investment Outlay	100,000	60,000
Annual Profit	15,000	12,500
Duration	4 years	4 years

Which alternative should be chosen?

Solution:

Alternative X is only more favorable if the additional investment of 40,000 for Alternative Y generates an annual profit that is less than 2,500.

2.4 Break-Even Analysis

The break-even analysis is used to determine the output quantity $x_{\text{B.E.}}$ (break-even quantity), from which a positive profit is generated. Assuming a linear cost function, the profit G is:

$$G = p \cdot x - K = p \cdot x - (K_f + k_v \cdot x) = (p - k_v) \cdot x - K_f$$

or

$$G = d \cdot x - K_f$$

where:

- p = price ($p \cdot x$ = revenue),
- $d = (p - k_v)$ = contribution margin per unit.

For the break-even quantity $x_{\text{B.E.}}$, the condition is:

$$G = 0 = d \cdot x_{\text{B.E.}} - K_f$$

which gives:

$$x_{\text{B.E.}} = \frac{K_f}{d}$$

If the output quantity exceeds the break-even quantity, the company generates profits.

Example:

1. Calculate the profits for an annual production quantity of 25,000 units. The selling price is 1.70 per cheese wedge.
2. At what annual production quantity do the two machines break even?

Solution:

a)

$$G_A = 1.70 \cdot 25,000 - 37,200 = 5,300$$

and

$$G_B = 1.70 \cdot 25,000 - 37,750 = 4,750$$

b)

$$x^A_{\text{B.E.}} = \frac{28,200}{1.70 - 0.36} = 21,045$$

and

$$x^B_{\text{B.E.}} = \frac{29,500}{1.70 - 0.33} = 21,533$$

2.5 Profitability Comparison Method

In this method, the profitability of the average invested capital is calculated:

$$R = \frac{\bar{G}}{\bar{K}} \cdot 100$$

where:

- $\bar{G}$ = Average profit,
- $\bar{K} = \frac{I_0 + R_n}{2}$ = Approximation of the average invested capital.

The investment alternative with the highest profitability should be chosen. For investment projects with different outlays I_0, the profitability of the additional investment must also be evaluated.

Example:

Profitability Comparison with Different Investment Outlays

	Investment X	**Investment Y**
Investment Outlay	100,000	60,000
Annual Profit	15,000	12,500
Duration	4 years	4 years
Residual Value	0	0

Solution:

$$R_X = \frac{15,000}{50,000} \cdot 100 = 30\%$$

$$R_Y = \frac{12,500}{30,000} \cdot 100 = 41.67\%$$

Investment Y requires an average capital binding that is 20,000 lower. Given that Investment Y already has a higher rate of return (41.67% compared to 30% for Investment X), **Investment Y is more favorable than Investment X**.

When the average profit is related to the initial invested capital I_0, the measure of profitability is known as the "Return on Investment" (ROI):

$$ROI = \frac{\bar{G}}{I_0} \cdot 100$$

If the residual value is $R_n = 0$, then:

$$R = \frac{\bar{G}}{\frac{I_0}{2}} \cdot 100 = \frac{2 \cdot \bar{G}}{I_0} \cdot 100$$

From this, it follows that:

$$R = 2 \cdot ROI$$

Example:

Calculate the profitabilities R and ROI for the initial example with an annual production quantity of 25,000 units. The selling price is 1.70 per cheese wedge.

Solution:

$$R_A = \frac{5,300}{0.5 \cdot (28,000 + 8,000)} \cdot 100 = 29.4\%$$

and

$$ROI_A = \frac{5,300}{28,000} \cdot 100 = 18.9\%$$

$$R_B = \frac{4,750}{0.5 \cdot (32,000 + 10,000)} \cdot 100 = 22.6\%$$

and

$$ROI_B = \frac{4,750}{32,000} \cdot 100 = 14.8\%$$

2.6 Amortization Comparison Method

Liquidity considerations take precedence when a company uses the **Amortization Comparison Method** (Pay-Off or Pay-Back Method) for investment accounting. The goal is to recover the invested capital as quickly as possible. Reasons for this may include limiting risk (e.g., in investments in developing countries) or taking the opportunity to invest in a new, potentially more lucrative project after a short time.

The amortization period t_A (in years) is calculated as the quotient of the investment outlay and the average cash flow:

$$t_A = \frac{I_0}{\bar{c}}$$

The annual cash flow can be determined either directly, as the difference between inflows and outflows, or indirectly, as the sum of profit, depreciation, and imputed costs. Since the profit is:

$$G = p \cdot x - K_a - K_{na}$$

the calculation of the cash flow yields:

$$G + K_{na} \quad \text{(indirect)} = p \cdot x - K_a \quad \text{(direct)}$$

where:

- $p \cdot x$ = Revenue (cash flow),
- K_a = Cash-effective costs (e.g., wages, rent, energy costs),
- K_{na} = Non-cash-effective costs (e.g., depreciation and imputed costs).

The investment that has the shortest amortization period is considered favorable.

Example:

Calculate the Amortization Periods Considering Imputed Interest.

Solution:

With:

$$\text{Cash Flow} = \text{Profit} + \text{Depreciation} + \text{Imputed Costs}$$

we get:

$$\bar{c}_A = 5,300 + 5,000 + 1,800 = 12,100$$

and

$$\bar{c}_B = 4,750 + 5,500 + 2,100 = 12,350$$

Thus,

$$t_A^A = \frac{28,000}{12,100} = 2.31 \text{ years}$$

and

$$t_A^B = \frac{32,000}{12,350} = 2.59 \text{ years}$$

Chapter 3

Dynamic Methods of Capital Budgeting

3.1 Key Metrics for Constant Cash Flows

The following example will demonstrate how the key metrics of dynamic investment appraisal are derived.

Example:

An investment in a silver mine of 100,000 leads to annual inflows of 50,000 and annual outflows of 30,000 for 10 years. After 10 years, the mine is depleted, so no residual value is obtained. Is the investment worthwhile with a 5% discount rate?

If the mine is not purchased, the investor could instead invest their capital at the discount rate using compound interest (base alternative).

The final wealth (final or terminal value) from the base alternative after 10 years is calculated as:

$$K_n^B = I_0 \cdot q^n = 100,000 \cdot 1.05^{10} = 162,889.46$$

If the silver mine is purchased, it is assumed that the annual returns can be reinvested at the discount rate. Therefore, the final wealth is calculated as the future value of an annuity of 20,000.

The final wealth (future value of an annuity) for the investment alternative after 10 years is:

$$K_n^I = c \cdot \frac{q^n - 1}{q - 1} = 20,000 \cdot \frac{1.05^{10} - 1}{1.05 - 1} = 251,557.85$$

The "purchase of the silver mine" alternative leads to a higher final wealth than the base alternative. Therefore, from a financial perspective, the investment in the silver mine should be made. The difference in final wealth is:

$$251,557.85 - 162,889.46 = 88,668.39$$

In general, an investment is worthwhile if the following holds:

$$-I_0 + \frac{c}{q^n} \cdot \frac{q^n - 1}{q - 1} > 0$$

The value:

$$C_0 = -I_0 + \frac{c}{q^n} \cdot \frac{q^n - 1}{q - 1} = -I_0 + c \cdot RBF_p^n$$

is called the net present value (NPV) in investment appraisal (for constant cash flows), where $RBF_p^n = \frac{1}{q^n} \cdot \frac{q^n-1}{q-1}$ refers to the present value annuity factor, as provided in financial tables.

For large time horizons n, the NPV for constant cash flows can be approximated by:

$$C_0 = -I_0 + \frac{c}{q-1}$$

since:

$$\lim_{n\to\infty} \frac{c}{q^n} \cdot \frac{q^n - 1}{q-1} = \frac{c}{q-1}$$

Formally, the NPV is the difference between the discounted cash flows and the initial investment outlay. In this case, the NPV is:

$$C_0 = -100,000 + \frac{20,000}{1.05^{10}} \cdot \frac{1.05^{10} - 1}{1.05 - 1} = 54,434.70$$

The NPV can also be seen as the discounted difference between the final wealth values:

$$C_0 = \frac{K_n^I - K_n^B}{q^n} = \frac{88,668.39}{1.05^{10}} = 54,434.70$$

The NPV indicates the increase in wealth at time $t = 0$ that the investor achieves compared to investing at the discount rate. An investment should be made if the NPV is positive, and it should be rejected if it is negative. The following relationship exists between the final wealth of the investment alternative and the NPV:

$$K_n^I = (C_0 + I_0) \cdot q^n$$

$$c \cdot \frac{q^n - 1}{q-1} = \left(-I_0 + \frac{c}{q^n} \cdot \frac{q^n - 1}{q-1} + I_0\right) \cdot q^n$$

If the NPV is distributed evenly over n equal annuities, the **equivalent annuity** for the investment is:

$$a = C_0 \cdot q^n \frac{q-1}{q^n - 1} = c - I_0 \cdot q^n \frac{q-1}{q^n - 1}$$

In this case, the equivalent annuity, or the annual wealth increase over the investment period, is:

$$a = 54,434.70 \cdot 1.05^{10} \frac{0.05}{1.05^{10} - 1}$$

$$= 20,000 - 100,000 \cdot 1.05^{10} \frac{0.05}{1.05^{10} - 1} = 7,049.54$$

The duration n_A, which satisfies the condition:

$$C_0 = -I_0 + \frac{c}{q^{n_A}} \frac{q^{n_A} - 1}{q - 1} = 0$$

is referred to as the **dynamic payback** period. Solving for n_A gives:

$$n_A = \frac{\ln\left(\frac{c}{c - I_0 \cdot (q-1)}\right)}{\ln q} = \frac{\ln\left(\frac{20{,}000}{20{,}000 - 100{,}000 \cdot 0.05}\right)}{\ln 1.05} = 5.89 \text{ years}$$

The limit of the dynamic payback period as $q \to 1$ is the static payback period, given by $t_A = \frac{I_0}{c}$. The dynamic payback period is longer than the static one because, in addition to recovering the invested capital, the investment must also generate a return (interest) on the capital used.

3.2 Baldwin Interest Rate

The interest rate at which the initial capital I_0 grows to the final wealth K_n^I is called the Baldwin interest rate or the rate of return on capital (cf. Baldwin, 1959). The Baldwin interest rate indicates the return on the invested capital:

$$
\begin{aligned}
r_B &= \left(\sqrt[n]{\frac{K_n^I}{I_0}} - 1\right) \cdot 100 \\
&= \left(\sqrt[n]{\frac{c \cdot \frac{q^n-1}{q-1}}{I_0}} - 1\right) \cdot 100 \\
&= \left(\sqrt[n]{\frac{(C_0 + I_0) \cdot q^n}{I_0}} - 1\right) \cdot 100
\end{aligned}
$$

For the silver mine example, the Baldwin interest rate is calculated as:

$$r_B = \left(\sqrt[10]{\frac{251,557.85}{100,000}} - 1\right) \cdot 100 = 9.66$$

The investment is worthwhile because the Baldwin interest rate is higher than the discount rate.

3.3 Internal Rate of Return (IRR)

The internal rate of return r differs from the Baldwin interest rate. The IRR indicates the return on the capital employed at each moment. With an initial investment of 100,000, the IRR corresponds to the interest rate at which an annuity of 20,000 can be paid for 10 years. It is the interest rate at which the NPV of the investment becomes zero:

$$C_0 = 0 = -I_0 + \frac{c}{q_r^n} \cdot \frac{q_r^n - 1}{q_r - 1}$$

where $q_r = 1 + \frac{r}{100}$.

Mathematically, the IRR can be calculated using the financial Excel function `RATE`. For the above example, the IRR is 15.1% (`RATE(10, 20000, -100000)`).

Thus, if 100,000 is invested at 15.1%, a annuity of 20,000 can be financed for 10 years. The Excel formula is `PMT(15.1%, 10, -100000) = 20001.21` (see Pflaumer, 2024, p. 29).

The investment is worthwhile because the IRR is higher than the discount rate.

Note: If the NPV is positive, both the Baldwin interest rate and the IRR are higher than the discount rate. When evaluating a single investment project, calculating just one of these metrics is sufficient to assess the profitability of the investment.

3.4 Key Metrics for Variable Cash Flows

In many practical investments, the cash flows will vary in size. The net present value (NPV) is still defined as the difference between the discounted cash flows and the investment outlay, as follows:

$$C_0 = -I_0 + \frac{c_1}{q} + \frac{c_2}{q^2} + \ldots + \frac{c_n}{q^n}$$

In the cash flow c_n, the ongoing surplus from operational activities is included, along with the residual value of the investment asset.

To calculate the Baldwin interest rate, the formula remains the same as before:

$$r_B = \left(\sqrt[n]{\frac{K_n^I}{I_0}} - 1 \right) \cdot 100 = \left(\sqrt[n]{\frac{(C_0 + I_0) \cdot q^n}{I_0}} - 1 \right) \cdot 100$$

To determine the internal rate of return (IRR), the NPV equation is set to zero. The equation for the internal rate of return in general is:

$$C_0 = 0 = -I_0 + \frac{c_1}{q_r} + \frac{c_2}{q_r^2} + \ldots + \frac{c_n}{q_r^n}$$

The internal rate of return can be determined using the Excel function IRR (see Pflaumer, 2024, p. 53).

Example:

An investment project with an outlay of 1,000,000 and a duration of four years generates the following cash flows: - Year 1: 200,000 - Year 2: 400,000 - Year 3: 400,000 - Year 4: 308,000. The discount rate is 8%.

The net present value is calculated as follows:

$$\begin{aligned} C_0 &= -1,000,000 + \frac{200,000}{1.08} + \frac{400,000}{1.08^2} + \frac{400,000}{1.08^3} + \frac{308,000}{1.08^4} \\ &= 72,042.80 \end{aligned}$$

The Baldwin interest rate is calculated as:

$$r_B = \left(\sqrt[4]{\frac{(72,042.80 + 1,000,000) \cdot 1.08^4}{1,000,000}} - 1 \right) \cdot 100 = 9.89$$

The equation for the internal rate of return is:

$$C_0 = 0 = -1,000,000 + \frac{200,000}{q_r} + \frac{400,000}{q_r^2} + \frac{400,000}{q_r^3} + \frac{308,000}{q_r^4}$$

To estimate the internal rate of return without suitable software, it is advisable to create a small table calculating the NPV based on different discount rates:

p	NPV	p	NPV
0%	308,000.00	8%	72,042.80
1%	274,356.23	9%	47,226.59
2%	242,019.27	10%	23,290.76
3%	210,923.80	11%	194.85
4%	181,008.41	12%	-22,099.35
5%	152,215.38	13%	-43,627.94
6%	124,490.38	14%	-64,425.06
7%	97,782.26	15%	-84,523.00

Table 3.1: NPV values at different discount rates

From the table (the tabular representation of the so-called NPV function), it can be seen that the internal rate of return is slightly above 11%.

The exact calculation using the financial Excel function `IRR`, with the cash flows located in column C, rows 3 to 7, yields: `=IRR(C3:C7)` = 11.01%.

t	Cash Flows
0	$-1,000,000$
1	$200,000$
2	$400,000$
3	$400,000$
4	$308,000$

Note: At a discount rate of 0%, the NPV corresponds to the sum of all cash flows minus the investment outlay.

The following figure shows the Net Present Value (NPV) function depending on the interest rate. The intersection of the x-axis with the vertical dotted line provides a method to graphically determine the internal rate of return (IRR). It can be observed that it is approximately 11%.

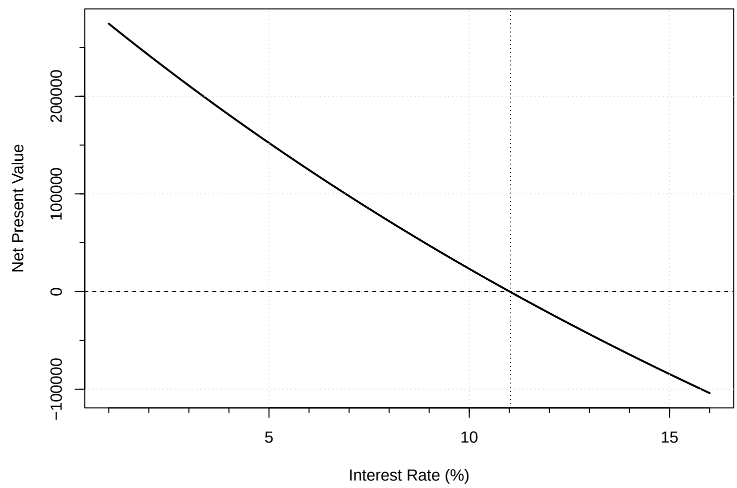

Figure 3.1: Net Present Value Function depending on Interest Rate

3.5 Analytical Determination of the Internal Rate of Return

For certain cash flows, the internal rate of return can be analytically determined by setting the NPV equation to zero and solving for q_r or r.

Examples:

1. The investment outlay is followed only by a cash flow in year 1:

$$c_1 > I_0.$$

The equation for the internal rate of return is:

$$C_0 = -I_0 + \frac{c_1}{q_r} = 0.$$

From this, the internal rate of return is calculated as:

$$r = \left(\frac{c_1}{I_0} - 1\right) \cdot 100,$$

which also corresponds to the Baldwin interest rate.

2. The investment outlay is followed by positive cash flows c_1 and c_2 in years 1 and 2:

$$c_1 + c_2 > I_0.$$

The positive solution of the resulting quadratic equation

$$C_0 = -I_0 + \frac{c_1}{q_r} + \frac{c_2}{q_r^2} = 0$$

gives the internal rate of return.

3. If the investment outlay is followed only by a cash flow at the end of the investment period, with $c_n > I_0$:

$$C_0 = -I_0 + \frac{c_n}{q_r^n} = 0$$

results in:

$$r = \left(\sqrt[n]{\frac{c_n}{I_0}} - 1\right) \cdot 100,$$

which corresponds to the Baldwin interest rate.

3.6 Conventional and Irregular Investments

So far, we have only considered investments with so-called conventional cash flow patterns, where an initial investment outlay is followed only by positive cash flows, and the total sum of flows exceeds the outlay. Conventional investments are a special case of normal investments, where (multiple) outflows occur first, followed by inflows, with the total inflows exceeding the total outflows. Both conventional and normal investments fall under the category of regular investments, and there exists exactly one real, positive internal rate of return (IRR). For conventional investments, the net present value (NPV) function is also continuously decreasing as a function of the discount rate.

When the cash flows do not follow these patterns, there may be zero, one, or even multiple IRRs. Norström (1972) provided a sufficient condition for the existence of a unique nonnegative internal rate of return (IRR) for certain investment projects. This condition ensures that the IRR is unique and nonnegative, simplifying the decision-making process in capital budgeting by avoiding the complications of multiple or non-existent IRRs.

These investments, referred to as irregular investments, make the internal rate of return less suitable as a measure of investment profitability compared to NPV. Examples and formal identification criteria for irregular investments can be found, for instance, in Bernhard (1979) and Durand (1974). However, the simplest method to identify an irregular investment is by graphically plotting the NPV function, from which the number of IRRs can immediately be identified (see, e.g., Pflaumer, 2024, pp. 23 f.) Therefore, before calculating any investment metrics, it is always advisable to sketch the NPV function. Investments with irregular cash flows often arise when the investment is financed by a loan.

When evaluating a single project, it does not matter which metric is used for the investment decision. Either all metrics will lead to the acceptance of the project, or all will reject it. However, when comparing mutually exclusive investments, the NPV method and the IRR method can lead to different decisions. Mutually exclusive projects are investments that cannot be carried out simultaneously. For example, an investor cannot build both an office building and an apartment building on the same plot of land. In such cases, the investor must use the NPV method as the basis for the investment decision to ensure that the highest wealth increase is achieved. Using the IRR method can lead to suboptimal investment decisions if the NPV functions intersect.

3.7 Static and Dynamic Profitability

In practice, static profitability metrics (such as ROI and R) are often used, even though dynamic profitability (i.e., the internal rate of return, IRR) can be just as easily calculated. It is important to note that static profitability values are only approximations of the IRR and that their use can lead to incorrect investment decisions.

Example:

A machine costs 27,450. It is depreciated linearly over 5 years down to a residual value of zero. The annual cash flow is 10,000. Calculate the IRR and the static profitability metrics.

Solution:

Internal Rate of Return (IRR):

$$\text{IRR} = \text{RATE}(5, -10000, 27450) = 24.01\%$$

Static Profitability Metrics:

$$\begin{aligned} ROI &= \frac{G}{I_0} \cdot 100 = \frac{c - A}{I_0} \cdot 100 = \frac{c - \frac{I_0}{n}}{I_0} \cdot 100 \\ &= \left(\frac{c}{I_0} - \frac{1}{n} \right) \cdot 100 = \left(\frac{10,000}{27,450} - \frac{1}{5} \right) \cdot 100 = 16.43\% \end{aligned}$$

$$R = 2 \cdot ROI = 32.86\%$$

since the residual value $R_5 = 0$.

It is evident that in this case, the IRR is significantly overestimated by R and underestimated by the ROI.

The relationship between static profitability metrics and the IRR can be derived easily for an investment with constant cash flows and a residual value of zero.

For this case, the IRR is determined by the following equation:

$$0 = -I_0 + c \cdot \frac{1}{q^n} \cdot \frac{q^n - 1}{q - 1}$$

with

$$q = 1 + \frac{r}{100}$$

or

$$\frac{c}{I_0} = q^n \cdot \frac{q-1}{q^n - 1}$$

Since in this special case

$$ROI = \frac{G}{I_0} \cdot 100 = \left(\frac{c - \frac{I_0}{n}}{I_0} \right) \cdot 100 = \left(\frac{c}{I_0} - \frac{1}{n} \right) \cdot 100,$$

we get

$$ROI = \left(q^n \cdot \frac{q-1}{q^n - 1} - \frac{1}{n} \right) \cdot 100$$

and with

$$q = 1 + \frac{r}{100},$$

after some transformations (see Levy/Sarnat, 1986),

$$ROI = \frac{r}{1 - \frac{1}{\left(1 + \frac{r}{100}\right)^n}} - \frac{100}{n}$$

and

$$R = 2 \cdot ROI = 2 \cdot \left(\frac{r}{1 - \frac{1}{\left(1 + \frac{r}{100}\right)^n}} - \frac{100}{n} \right).$$

For $n = 1$ and for an infinite time horizon:

$$ROI = r$$

and

$$R = 2 \cdot r.$$

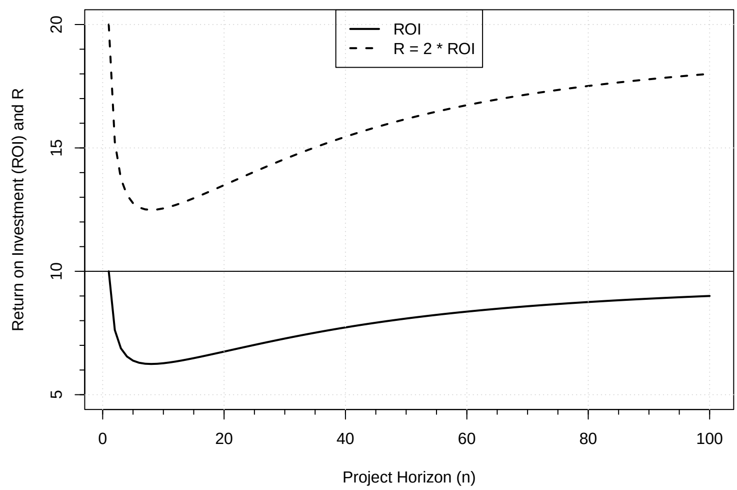

Figure 3.2: ROI and R as a function of project horizon

Figure 3.2 shows the relationship between static and dynamic profitability as a function of time when the IRR is 10%.

The internal rate of return (IRR) is underestimated by the ROI in this case (with the exception of $n = 1$). The underestimation is greatest at a time horizon of 8 years (ROI = 6.24). After that, the underestimation decreases as the time horizon increases. As the time horizon lengthens, the ROI asymptotically approaches the IRR r. In contrast, the capital profitability R overestimates the IRR, and this overestimation increases with the lengthening time horizon.

Note: Static profitability values can deviate significantly from the IRR, and their use can lead to incorrect investment decisions. Therefore, calculating the IRR is always recommended as an alternative, as it is just as simple to compute as static profitability values.

3.8 Static and Dynamic Cost Comparisons

In practice, cost comparisons of investment projects are often conducted statically. A better alternative is the dynamic cost comparison, which explicitly considers the timing of cost outflows. The absolute values of the negative net present values (NPVs) are interpreted as "dynamic total costs" for the investment projects. The alternative with the least negative NPV is selected. The absolute values of the equivalent annuities represent the annual "dynamic costs."

Example:

An entrepreneur plans to purchase a company car. Two brands, A and B, are available for consideration. The purchase price and costs are detailed in the following table. The depreciation is calculated linearly to the respective residual value. The imputed interest rate is 10%. The car is intended to be sold after four years. For which alternative should the entrepreneur choose from a financial perspective, assuming an annual mileage of 35,000 km? What is the critical mileage per year at which the costs of the two alternatives are equal?

Brand	**A**	**B**
Purchase Price (USD)	30,000	40,000
Resale Value (USD)	10,000	15,000
Fixed Cash Costs per Year (USD)	8,000	9,000
Variable Costs per km (USD)	0.40	0.30

Table 3.2: Cost Comparison for Company Car Purchase

Solution:

A) **Static Analysis**:

The calculation of the imputed interest is:

$$Z = \frac{p}{100} \cdot \frac{I_0 + R_n}{2}$$

Brand	**A**	**B**
Depreciation (USD)	5,000	6,250
Imputed Interest (USD)	2,000	2,750
Fixed Cash Costs (USD)	8,000	9,000
Total Fixed Costs (USD)	15,000	18,000

Table 3.3: Static Cost Analysis

$$K_A = 15,000 + 0.4 \cdot x$$
$$K_B = 18,000 + 0.3 \cdot x$$

The total annual costs are: For A: 29,000. For B: 28,500.

The critical mileage is given by:

$$x_{crit} = \frac{18,000 - 15,000}{0.40 - 0.30} = 30,000 \text{ km.}$$

B) **Dynamic Analysis**:

The net present value functions are:

$$C_{0A} = -30,000 - (8,000 + 0.4 \cdot 35,000) \cdot \frac{1}{1.1^4} \cdot \frac{1.1^4 - 1}{0.1} + \frac{10,000}{1.1^4}$$

$$= -30,000 - 22,000 \cdot \frac{1}{1.1^4} \cdot \frac{1.1^4 - 1}{0.1} + \frac{10,000}{1.1^4} = -92,907$$

$$C_{0B} = -40,000 - (9,000 + 0.3 \cdot 35,000) \cdot \frac{1}{1.1^4} \cdot \frac{1.1^4 - 1}{0.1} + \frac{15,000}{1.1^4}$$

$$= -40,000 - 19,500 \cdot \frac{1}{1.1^4} \cdot \frac{1.1^4 - 1}{0.1} + \frac{15,000}{1.1^4} = -91,567$$

The equivalent annuities are:

$$a_A = -92,907 \cdot 1.1^4 \cdot \frac{0.1}{1.1^4 - 1} = -29,309$$

$$a_B = -91,567 \cdot 1.1^4 \cdot \frac{0.1}{1.1^4 - 1} = -28,887.$$

To calculate the dynamic critical mileage, set the NPVs equal:

$$-30,000 - (8,000 + 0.4 \cdot x) \cdot \frac{1}{1.1^4} \cdot \frac{1.1^4 - 1}{0.1} + \frac{10,000}{1.1^4} =$$
$$-40,000 - (9,000 + 0.3 \cdot x) \cdot \frac{1}{1.1^4} \cdot \frac{1.1^4 - 1}{0.1} + \frac{15,000}{1.1^4}$$

Solving for x gives:

$$x_{crit} = 30,774 \text{ km}.$$

In this example, both static and dynamic methods lead to the same financial benefit. For financial and accounting reasons, one should choose brand B.

Chapter 4

Metrics Considering Taxes and Financing

4.1 Net Present Value After Taxes

Taxes must be considered as actual cash outflows when making investment decisions. In principle, all taxes can be included in investment accounting, but the many complexities of tax laws make the calculations highly complex. Furthermore, tax laws change frequently. Therefore, this section will not delve into the specifics of tax systems but will instead present a fundamental approach to incorporating taxes in investment decisions. This approach can be easily modified to address specific tax issues on a case-by-case basis.

For simplicity, a constant income tax rate T is assumed, where $0 < T < 1$. Tax liabilities are treated as cash outflows, and tax refunds are treated as cash inflows, which occur at the end of the period in which they arise.

For proportional taxes, T can be interpreted as the marginal tax rate. The marginal tax rate indicates how much additional tax must be paid if income increases by 1 unit. For example, if income rises by 1,000 units and the marginal tax rate is 40% (T=0.4), an additional

400 units in taxes must be paid.

Since taxes are levied on profits rather than cash flows, depreciation must be subtracted from pre-tax cash flows to determine the taxable income.

Example:

A bus company has calculated the following pre-tax cash flows. The cash flow in year 4 includes the resale value (equal to the book value) of the bus, which is 330,000. The interest rate is 10%.

t	0	1	2	3	4
c_t	$-660,000$	$124,600$	$149,000$	$173,400$	$503,400$

Calculate the after-tax cash flows assuming a yearly straight-line depreciation of 82,500 and a tax rate $T = 0.3$ (i.e., 30%).

The after-tax cash flows can be determined using the following scheme:

	0	1	2	3	4
Pre-tax cash flows	$-660,000$	$124,600$	$149,000$	$173,400$	$503,400$
- Resale value					$330,000$
- Depreciation		$82,500$	$82,500$	$82,500$	$82,500$
= Pre-tax profit		$42,100$	$66,500$	$90,900$	$90,900$
- Taxes		$12,630$	$19,950$	$27,270$	$27,270$
= Post-tax profit		$29,470$	$46,550$	$63,630$	$63,630$
+ Depreciation		$82,500$	$82,500$	$82,500$	$82,500$
+ Resale value					$330,000$
= Post-tax cash flow	$-660,000$	$111,970$	$129,050$	$146,130$	$476,130$

After calculating post-tax profit, depreciation is added back to arrive at the post-tax cash flows. The resale value of 330,000 is not subject to income tax, as it matches the book value, and is considered separately when determining the post-tax cash flows.

Since the interest income of the base alternative (e.g., purchasing a bond with a pre-tax interest rate of 10%) is also subject to income tax, the post-tax discount rate for the investment alternative is reduced to:

$$p^* = (1 - T) \cdot p = (1 - 0.3) \cdot 10 = 7$$

The net present value after taxes can be defined as follows:

$$C_0^* = -I_0 + \sum_{t=1}^{n} \frac{(1 - T) \cdot (c_t - A_t) + A_t}{(q^*)^t}$$

or

$$C_0^* = -I_0 + \sum_{t=1}^{n} \frac{(1 - T) \cdot c_t}{(q^*)^t} + \sum_{t=1}^{n} \frac{T \cdot A_t}{(q^*)^t}$$

where:

- T = constant tax rate
- A_t = depreciation in year t
- $q^* = 1 + \frac{p^*}{100} = 1 + \frac{(1-T) \cdot p}{100} = (1 - T) \cdot q + T$
- p^* = post-tax discount rate

The net present value after taxes consists of two components. The first component reflects the market-determined cash flows, which are subject to uncertainties. The second component represents the present value of tax savings $T \cdot A_t$, which are considered guaranteed cash flows. This present value can be viewed as a subsidy from the government to the investment.

When calculating the internal rate of return after taxes, the post-tax cash flows should be used.

Example:

Calculate the net present value and internal rate of return after taxes for the bus company example. The pre-tax discount rate is 10%.

$$C_0^* = -660,000 + \frac{111,970}{1.07} + \frac{129,050}{1.07^2} + \frac{146,130}{1.07^3} + \frac{476,130}{1.07^4} = 39,885.03$$

The internal rate of return after taxes is approximately 9.1%. This value is calculated using the Excel IRR function with the post-tax cash flows.

The Baldwin rate after taxes is calculated as:

$$r_B^* = 100 \cdot \left(\sqrt[4]{\frac{(660,000 + 39,885.03) \cdot 1.07^4}{660,000}} - 1 \right) = 8.58\%$$

If the resale value exceeds (or falls below) the book value, an extraordinary profit (or loss) arises and must be considered for tax purposes.

Example:

For the bus investment, let's assume a resale value of 360,000 under otherwise identical conditions. The cash flow in year 4 can be calculated as follows:

	$t = 4$
Pre-tax cash flow	$173,400$
Resale value	$(360,000)$
Book value	$(-330,000)$
+Extraordinary income	$30,000$
-Depreciation	$82,500$
=Pre-tax profit	$120,900$
-Taxes	$36,270$
=Post-tax profit	$84,630$
+Depreciation	$82,500$
+Book value	$330,000$
=Post-tax cash flow	$497,130$

The net present value in this case is:

$$C_0^* = -660,000 + \frac{111,970}{1.07} + \frac{129,050}{1.07^2} + \frac{146,130}{1.07^3} + \frac{497,130}{1.07^4} = 55,905.03$$

4.2 Tax Paradox

In general, the net present value after taxes decreases as the income or corporate tax rate increases. The phenomenon where the net present value after taxes increases with a rising tax rate is known as the "tax paradox" in investment accounting (see Reinhardt, 1994). This section will examine the tax paradox in more detail, focusing on constant annual cash flows and linear depreciation.

Example:

An entrepreneur is considering whether to invest in Country A, which has a 20% corporate tax rate, or Country B, where the tax rate is 40%. Both countries offer the same other advantages. The pre-tax discount rate is 10%. The initial investment is 20,000, and the pre-tax annual cash flow is 11,530, with an investment period of 2 years.

The investment will be depreciated linearly to a zero residual value. In which country should the investment be made?

The results after taxes are:

	$Country A$	$Country B$
$Net Present Value$	15.36	16.98
IRR	8.06%	6.06%
$Baldwin Rate$	8.04%	6.06%
$Final Wealth$	23,345.92	22,491.08

The analysis incorrectly suggests that the investor should consider investing in country B, which has a higher tax rate, as the net present value (NPV) is higher compared to country A.

In this case, the net present value function increases up to a tax rate of approximately 40%, after which it decreases.

When the tax paradox occurs, the net present value method is not a suitable criterion for decision-making. In cases where mutually exclusive investment projects are being compared, the final wealth values should be used as a decision-making tool.

For constant cash flows, the net present value after taxes can be expressed as:

$$C_0^* = -I_0 + (1-T) \cdot c \cdot \frac{1}{q^{*n}} \cdot \frac{q^{*n}-1}{q^*-1} + T \cdot \frac{I_0}{n} \cdot \frac{1}{q^{*n}} \cdot \frac{q^{*n}-1}{q^*-1}$$

By rearranging:

$$C_0^* = -I_0 + c \cdot \frac{1}{q^{*n}} \cdot \frac{q^{*n}-1}{q^*-1} - T \cdot \left(c - \frac{I_0}{n}\right) \cdot \frac{1}{q^{*n}} \cdot \frac{q^{*n}-1}{q^*-1}$$

The cause of the paradox becomes apparent. A tax increase leads to additional cash outflows ($\rightarrow$ reduction in the net present value after taxes) but also reduces the post-tax discount rate ($\rightarrow$ increase in the net present value after taxes), provided $c > \frac{I_0}{n}$. If the positive interest effect is larger than the negative cash flow effect, the paradox occurs. The paradox is more likely to arise when the annual cash flow c is small, the pre-tax discount rate p is large, and the tax rate T is small. There is no tax paradox in the final wealth value, provided it is a conventional investment, i.e., $n \cdot c > I_0$.

4.3 Capital Value After Financing

When an investment is financed through a loan, the cash flow series must be adjusted to account for the financing. While the investment's cash flow series (CFI) starts with an outflow followed by inflows, financing has a cash flow series (CFF) that begins with an inflow followed by outflows.

The cash flow series after accounting for financing (Net-CF) is formed by adding the investment cash flow series (CFI) and the financing cash flow series (CFF).

Example

The cash flow series for a bus investment is as follows:

t	**0**	**1**	**2**	**3**	**4**
$\mathbf{c}_t$	-660,000	124,600	149,000	173,400	503,400

Table 4.1: Cash Flow Series for the Bus Investment

30% of this investment must be financed externally. The interest rate on the loan p_S is 10%, which is the same as the discount rate. The loan and accrued interest will be repaid at the end of the investment's term. Determine the net present value (NPV).

t	CFI	CFF	Net-CF
0	-660,000	198,000	-462,000
1	124,600	0	124,600
2	149,000	0	149,000
3	173,400	0	173,400
4	503,400	-289,891.80	213,508.20

Table 4.2: Cash Flow Series after Financing

Solution

The NPV for the cash flow series after financing is:

$$NPV = 50,520.18$$

The Baldwin rate is:

$$12.89\%$$

At time 0, a loan of 198,000 (30% of 660,000) is taken. The loan and accrued interest are repaid at time 4, amounting to:

$$198,000 \cdot 1.1^4 = 289,891.80$$

The NPV of the cash flow series after accounting for financing does not change because the NPV of the financing cash flow series is zero.

	CFI	CFF	Net-CF
NPV	50,520.18	0.00	50,520.18
$\mathbf{r_B}$	12.05%	—	12.89%

Table 4.3: Net Present Value and Baldwin Rate Comparison

Note

When the loan interest rate p_S and the discount rate p are the same, the type and amount of financing have no effect on the NPV. However, financing does affect the NPV when the discount rate p and loan interest rate p_S differ, as is common in practice.

Example: Case 1 - $p_S > p$

Assume the loan interest rate p_S is 12%. How does this affect the cash flow series and the NPV after financing? The discount rate is $p = 10$.

t	CFI	CFF	Net-CF
0	-660,000	198,000	-462,000
1	124,600	0	124,600
2	149,000	0	149,000
3	173,400	0	173,400
4	503,400	-311,557.83	191,843.17

Table 4.4: Cash Flow Series After Financing with $p_S > p$

The NPV of the cash flow series is now:

$$NPV = 35,722.67$$

The Baldwin rate is:

12.07%

	CFI	CFF	Net-CF
NPV	50,520.18	-14,797.51	35,722,67
$\mathbf{r_B}$	12.05%	—	12.07%

Table 4.5: Net Present Value and Baldwin Rate Comparison

Example: Case 2 - $p_S < p$

Now assume the loan interest rate p_S is 8%. Determine the NPV after financing. The discount rate is $p = 10$.

t	CFI	CFF	Net-CF
0	-660,000	198,000	-462,000
1	124,600	0	124,600
2	149,000	0	149,000
3	173,400	0	173,400
4	503,400	-269,376.81	234,023.19

Table 4.6: Cash Flow Series After Financing with $p_S < p$

	CFI	CFF	Net-CF
NPV	50,520.18	14,012.01	64,532.19
$\mathbf{r_B}$	12.05%	—	13.65%

Table 4.7: Net Present Value and Baldwin Rate Comparison

The NPV of the cash flow series is now:

$$NPV = 64,532.19$$

The Baldwin rate is:

$$13.65\%$$

Leverage Effect:

If the discount rate is higher than the loan interest rate, it is advantageous to increase the proportion of external financing. The NPV and Baldwin rate increase as the proportion of external financing increases. This is called the leverage effect. However, in practice, the level of debt will be limited (optimal debt ratio). Banks will not grant an unlimited amount of credit, and as debt increases, the risk of loss due to fluctuating interest rates and uncertain cash flows also increases.

The loan repayment can be made either in a lump sum or in regular installments. There are two main types of repayment: **installment repayment** and **annuity repayment**.

In **installment repayment**, equal repayment amounts (Repayment = Loan/Term) are made over the loan period. As the outstanding debt decreases, interest payments also reduce, leading to a decrease in total payments over time (Total Payment = Repayment + Interest).

In **annuity repayment**, fixed payments are made throughout the loan period. As the debt decreases, the interest portion of each payment decreases, while the repayment portion increases correspondingly.

Example: A bus investment of 660,000 is to be financed with 30% debt. The loan interest rate is 8%. The loan repayment occurs: a) In a lump sum at the end of the term, b) In four equal installment repayments, c) In four equal annuities. Which financing option should be chosen if the discount rate is $p = 10$?

The repayment plan for the installment loan is as follows:

Year	Outstanding Debt	Repayment	Interest	Total Payment
1	198,000	49,500	15,840	65,340
2	148,500	49,500	11,880	61,380
3	99,000	49,500	7,920	57,420
4	49,500	49,500	3,960	53,460
Sum	—	198,000	39,600	237,600

Table 4.8: Repayment plan for the installment loan

The annual repayment amount is calculated as $198,000/4 = 49,500$. The interest is calculated based on the outstanding debt.

The annuity for the annuity repayment is 59,780, calculated using the Excel PMT function: `=PMT(8%, 4, -198000)` (see Pflaumer, 2024, p. 30).

The repayment plan for the annuity loan is as follows:

After calculating the annuity, the interest is determined based on

Year	Outstanding Debt	Repayment	Interest	Total Payment
1	198,000	43,940	15,840	59,780
2	154,060	47,455	12,325	59,780
3	106,605	51,252	8,528	59,780
4	55,353	55,352	4,428	59,780
Sum	—	197,999	41,121	239,120

Table 4.9: Repayment plan for the annuity loan

the outstanding debt. The repayment amount is the difference between the annuity and the interest. The new outstanding debt is obtained by subtracting the repayment from the previous outstanding debt. Small discrepancies may occur in the repayment plan due to rounding.

When combining the cash flows of the investment and financing, the following results are obtained:

t	CFI	LS	IR	AR
0	-660,000	-462,000	-462,000	-462,000
1	124,600	124,600	59,260	64,820
2	149,000	149,000	87,620	89,220
3	173,400	173,400	115,980	113,620
4	503,400	234,023	449,940	443,620

Table 4.10: Comparison of cash flows for investment and different financing options

CFI = Cash Flow of Investment, LS = Lump Sum Repayment, IR = Installment Repayment, AR = Annuity Repayment

Metric	CFI	LS	IR	AR
NPV	50,520	64,532	58,739	59,026
IRR	12.76%	15.66%	14.16%	14.22%
Baldwin Rate	12.05%	13.65%	13.34%	13.36%

Table 4.11: Net present value and rates for different repayment methods

The results show that delaying loan repayment as long as possible is advantageous when the loan interest rate p_S is lower than the discount rate p; external funds are cheaper than internal funds. Conversely, if the loan interest rate exceeds the discount rate, the loan should be repaid as quickly as possible.

Cash Flows after Accounting for Taxes and Financing

To calculate cash flows after accounting for taxes and financing, the following schema should be followed, considering that loan interest directly related to operating income is tax-deductible. If the residual values are higher (or lower) than the corresponding book values, extraordinary gains (or losses) are incurred, which must be considered for tax purposes:

Description	**Value**
Cash flow	
- Residual value	
- Depreciation	
- Loan interest	
= Project profit before taxes	
- Taxes	
= Project profit after taxes	
+ Depreciation	
+ Financing cash flow without loan interest	
+ Residual value	
= Net cash flow (after taxes and financing)	

Table 4.12: Cash Flow Analysis Accounting for Taxes and Financing

4.4 Leasing

Leasing is a specific form of financing where assets are rented or leased by a leasing company or the manufacturer. Whether leasing or purchasing (with or without credit financing) is more advantageous can only be determined through investment calculations. For this purpose, the net present values (NPV) of the leasing and purchasing alternatives, considering tax aspects (leasing rates are tax-deductible for businesses), are calculated, and the alternative with the higher NPV is selected. Generally, leasing becomes more advantageous with higher discount rates, while purchasing is more favorable with lower rates. The point at which both leasing and purchasing are equally advantageous is known as the critical discount rate or the critical leasing rate.

Example: A hotel owner is considering whether to buy 50 oriental carpets at 10,000 each or lease them for an annual payment of 1,650 per carpet. After a basic rental period of 7 years, the owner has the option to purchase the carpets for a residual value of 30%. The pre-tax discount rate is 10%, and the corporate tax rate is 40%. The carpets are depreciated linearly to their residual value. Should the owner lease or buy?

The NPV method is applied here in the context of a dynamic cost comparison: no direct cash flows from the hotel's business operations are assigned to the carpets. Therefore, a negative NPV results. The depreciation reduces profits and leads to annual tax savings of 400.

Year (t)	**0**	**1**	**2-6**	**7**
Cash Flow	-10,000	0	0	3,000
- Depreciation		1,000	1,000	1,000
= Profit Before Tax		-1,000	-1,000	-1,000
- Tax		-400	-400	-400
= Profit After Tax		-600	-600	-600
+ Depreciation		1,000	1,000	1,000
= Cash Flow After Tax	-10,000	400	400	3,400

Table 4.13: Cash Flows: Purchasing an Oriental Carpet

The NPV (Net Present Value) of purchasing a carpet is calculated as:

$$C_{0K} = -5,771.88$$

using a post-tax discount rate of 6%.

Leasing results in annual tax savings of 660, so the after-tax leasing costs are 990 per year. The NPV of leasing is:

$$C_{0L} = -5,526.56$$

Year (t)	**0**	**1**	**2-6**	**7**
Cash Flow	0	-1,650	-1,650	-1,650
- Tax		-660	-660	-660
= Cash Flow After Tax	0	-990	-990	-990

Table 4.14: Cash Flows: Leasing an Oriental Carpet

Since the NPV of leasing is higher than the NPV of purchasing, leasing is more economically advantageous in this case.

Additionally, we calculate the difference in NPVs:

$$C_{0D} = C_{0K} - C_{0L} = -5,771.88 + 5,526.56 = -245.32$$

If the NPV of the differential investment is positive, purchasing is more advantageous; if it is negative, leasing is preferable. The pre-tax discount rate at which both alternatives are equally advantageous is 9.01%. For lower rates, purchasing is better, and for higher rates, leasing offers a greater NPV.

To calculate the critical leasing rate L_{crit}, we use the Net Present Value (NPV) formula for leasing as a function of the leasing rate L:

$$C_{0L} = -(1 - T) \cdot L_{crit} \cdot \frac{1}{q^{*n}} \cdot \frac{q^{*n} - 1}{q^* - 1} = -0.7 \cdot (1 - 0.4) \cdot L_{crit} \cdot 5.5824$$

This is then equated to the NPV of purchasing:

$$C_{0K} = -5,771.88$$

Solving for L_{crit}, we find:

$$L_{crit} = 1,723.24$$

Thus, if the leasing rate exceeds 1,723.24, purchasing the asset becomes more economically advantageous.

Chapter 5

Sensitivity Analysis in Capital Budgeting

Sensitivity analysis is a crucial technique used in capital budgeting to evaluate how changes in key assumptions and variables impact the overall financial outcomes of an investment project. This method allows decision-makers to assess the robustness of their projections and identify which variables significantly influence the project's net present value (NPV) or internal rate of return (IRR).

By systematically varying one parameter at a time while holding others constant, sensitivity analysis provides insights into the potential risks and uncertainties associated with capital investments. Sensitivity analysis is an essential tool in capital budgeting that enhances decision-making by evaluating the effects of variable changes on project outcomes. By identifying critical factors influencing financial projections and utilizing different analysis methods, companies can better understand risks, prepare for uncertainties, and optimize their investment strategies.

When used alongside other analytical methods, sensitivity analysis contributes to more robust capital budgeting processes, ultimately leading to more informed investment decisions.

Several methods of sensitivity analysis offer different insights into the potential variability of project outcomes, as detailed in Section 5.1.

5.1 Methods of Sensitivity Analysis

5.1.1 One-way Sensitivity Analysis

This straightforward approach involves altering one variable while keeping all other variables constant. For example, a company might evaluate how changes in sales volume affect NPV. This method quickly identifies which variables have the most significant impact on the investment's success.

5.1.2 Two-way Sensitivity Analysis

This method varies two input variables simultaneously to observe the interaction effects on NPV or IRR. For instance, a company could assess how changes in both sales volume and operating costs impact project outcomes. This approach provides a more comprehensive view of how different factors interact, helping managers understand combined effects on financial projections.

5.1.3 Scenario Analysis

Unlike one-way and two-way sensitivity analyses, scenario analysis examines multiple variables simultaneously by creating specific scenarios (e.g., best-case, worst-case, and most likely scenarios). This method is particularly useful for understanding the impact of different combinations of factors, allowing decision-makers to explore a range of potential outcomes based on varying assumptions.

5.2 Applications of Sensitivity Analysis

5.2.1 Critical Value Method

The critical value method is another useful approach in sensitivity analysis, particularly in capital budgeting. This method focuses on identifying the threshold values of key variables that determine whether a project is financially viable. A critical aspect of this analysis is determining the minimum duration n required for a project with constant cash flows to achieve a positive net present value (NPV).

Understanding the Critical Value Method

In capital budgeting, NPV is calculated as the difference between the present value of cash inflows and the present value of cash outflows. The formula for NPV can be expressed as follows:

$$\text{NPV} = \sum_{t=1}^{n} \frac{c_t}{(1+i)^t} - I_0$$

Where:

- c_t = Cash flow at time t
- i = Discount rate
- I_0 = Initial investment
- n = Duration of the project

For projects with constant cash flows, the cash flow c remains the same for each period. The NPV equation can be simplified to:

$$\text{NPV} = c \cdot \frac{1}{q^n} \cdot \frac{q^n - 1}{q - 1} - I_0 = c \cdot \left(\frac{1 - (1+i)^{-n}}{i} \right) - I_0$$

To determine the minimum duration n that results in a positive NPV, we need to set the NPV equation greater than or equal to zero:

$$c \cdot \left(\frac{1 - (1+i)^{-n}}{i} \right) - I_0 \geq 0$$

Rearranging the equation gives:

$$n \geq \frac{-\ln\left(1 - \frac{iI_0}{c}\right)}{\ln(1+i)}$$

Example: Given the following parameters:

- Initial investment $I_0 = 100,000$
- Annual cash flow $c = 20,000$
- Discount rate $i = 0.15$

This formula determines the minimum duration n required for the project to achieve a positive net present value (NPV). For this example, the project must run for at least 10 years with annual cash flows of 20,000 and a discount rate of 15% to ensure a positive NPV. Using the given parameters:

$$n \geq \frac{-\ln\left(1 - \frac{0.15 \cdot 100{,}000}{20{,}000}\right)}{\ln(1+0.15)} = \frac{-\ln(0.25)}{\ln(1.15)} \approx 9.92$$

Thus, the project duration must satisfy $n \geq 10$.

5.2.2 Best Case, Medium Case, and Worst Case Analysis

In capital budgeting, conducting best case, medium case, and worst case analyses is an effective method to evaluate the sensitivity of a project's NPV to variations in cash flows. This approach enhances decision-making by understanding the range of potential outcomes based on different assumptions.

Step 1: Define the Scenarios

- **Best Case Scenario:** Estimate the maximum expected cash flow for each period, assuming everything goes better than expected.
- **Medium Case Scenario:** Estimate the most likely or average cash flow for each period, serving as the baseline scenario.
- **Worst Case Scenario:** Estimate the minimum expected cash flow for each period, accounting for the possibility of adverse outcomes.

For the earlier example, assume the following cash flows:

- Best Case Cash Flows: 30,000 annually
- Medium Case Cash Flows: 20,000 annually
- Worst Case Cash Flows: 10,000 annually

Calculate NPV for Each Scenario

Using the NPV formula:

$$\text{NPV} = \sum_{t=1}^{n} \frac{c_t}{(1+i)^t} - I_0$$

Assuming the initial investment I_0 is 100,000 and the discount rate i is 15%, we calculate the NPVs for the best, medium, and worst case scenarios.

5.2.3 Sensitivity Analysis of Influencing Factors (NPV Sensitivity Chart)

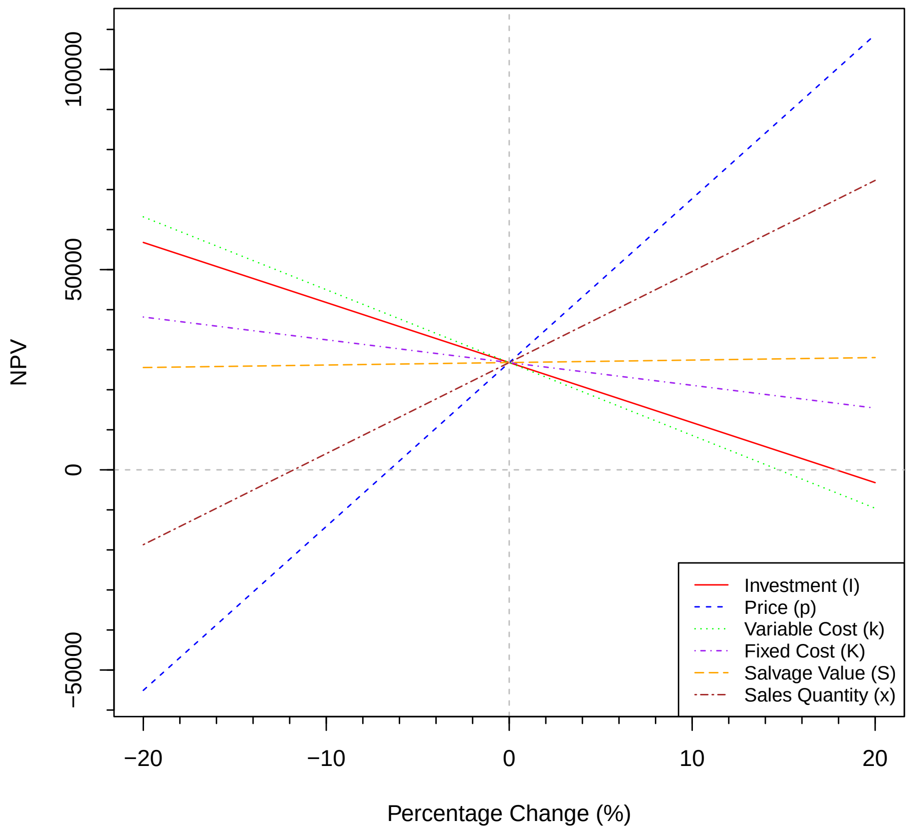

Figure 5.1: Sensitivity Analysis of NPV with respect to different parameters

This method involves varying key influencing factors—such as cash flow, investment duration, salvage value, and discount rate—by a specified percentage (e.g., 10%). The absolute change in NPV resulting from these variations is then calculated and visualized in an NPV sensitivity chart. In this chart, the x-axis represents the percentage change in the influencing factors, while the y-axis displays the corresponding change in NPV.

This visualization facilitates the quick identification of significant influencing factors, with steeper curves indicating greater importance. The intersection of the curve with the x-axis shows how much each factor can increase before the NPV becomes negative.

Scenario

A local business, EcoTech Solutions, is considering an investment in a new production line to manufacture environmentally friendly gadgets. The initial investment required for this project is $150,000, which includes machinery, setup costs, and other capital expenditures. EcoTech expects to sell these gadgets at a price of $180 per unit, with a variable production cost of $80 per unit.

The company forecasts a fixed annual operating cost of $15,000, which includes maintenance, salaries, and utilities. The project has an estimated lifespan of 5 years, after which the machinery can be sold for a salvage value of $10,000. EcoTech expects to sell 600 units of the gadget each year, and it uses an interest rate of 10% (represented as an interest factor of $q = 1.1$) to discount future cash flows.

Planning

Here are the specific parameters that EcoTech is using to evaluate the project:

- Initial Investment: $150,000
- Price per Unit (p): $180

- Variable Cost per Unit (k): \$80
- Fixed Cash Outflow per Year (K): \$15,000
- Salvage Value (S): \$10,000
- Interest Factor (q): 1.1 (Interest Rate of 10%)
- Investment Horizon (n): 5 years
- Sales Quantity (x): 600 units per year

Method

EcoTech applies the following Net Present Value (NPV) formula to calculate the profitability of the project:

$$\text{NPV} = I_0 + ((p-k)\cdot x - K)\cdot\left(\frac{1/q^n \cdot (q^n - 1)}{q-1}\right) + \frac{S}{q^n}$$

Where:

- I_0 is the initial investment
- p is the price per unit
- k is the variable cost per unit
- K is the fixed annual outflow
- S is the salvage value
- q is the interest rate factor
- n is the project duration (in years)
- x is the number of units sold per year

Calculation

By plugging in the values into the NPV formula:

$$\text{NPV} = -150,000 + ((180 - 80) \cdot 600 - 15,000) \cdot \left(\frac{1/1.1^5 \cdot (1.1^5 - 1)}{1.1 - 1} \right) + \frac{10,000}{1.1^5}$$

After evaluating the formula, EcoTech finds that the Net Present Value (NPV) of the project is $26,794.62.

Result

The NPV of $26,794 suggests that the project is expected to generate more cash inflows than outflows, making it financially viable. This NPV figure serves as the basis for constructing the NPV sensitivity chart, which will help the company assess how changes in various assumptions—such as sales quantity, price per unit, and costs—affect the overall profitability of the project.

NPV Sensitivity Chart

To further assess potential risks and uncertainties, EcoTech conducts a sensitivity analysis, presenting the results in the NPV Sensitivity Chart shown in the following figure. By varying one parameter (e.g., price per unit, variable cost, sales quantity) at a time while keeping the others constant, the company can identify which factors have the most significant impact on the project's NPV. This will allow them to make informed decisions about how to mitigate potential risks and ensure the long-term success of the investment.

This example not only illustrates how the NPV is calculated but also serves as a starting point for a deeper analysis of the project's sensitivity to changes in key variables.

The sensitivity analysis revealed several critical values that are important for EcoTech Solutions to monitor closely. These critical values are the thresholds at which the NPV becomes zero, meaning that exceeding these values would result in a negative NPV and render the project financially unfeasible, assuming all other variables remain constant.

The steepness of the curves in the NPV sensitivity chart, which is reflected in Figure 5.1, provides a clear indication of which variables are most crucial to the project's financial outcome. The steeper a curve is, the more sensitive the NPV is to changes in that particular variable. This means that even slight deviations from the initial assumptions for these variables can result in significant changes to the NPV, making them critical points of focus for monitoring and control.

For example, in the case of price (p) and variable costs (k), their curves are among the steepest, indicating that small changes in either of these can greatly impact the NPV. This highlights the need for closely monitoring market prices and ensuring efficient cost control mechanisms are in place. On the other hand, variables like fixed cash outflows (K) and salvage value (S) have less steep curves, indicating that the project's NPV is less sensitive to fluctuations in these parameters. While still important, they do not require the same level of attention and monitoring as price or variable costs. Overall, the steeper the curve in the chart, the more critical the variable is to the project's success, and greater attention should be paid to minimizing risks and uncertainties associated with these highly sensitive factors.

Critical Values

- The critical price (p) of the product is \$168.22. If the selling price drops below this level, the project's NPV becomes negative. This highlights the importance of maintaining a price above this threshold or identifying strategies to cut costs and enhance profitability.

- The critical variable cost (k) is \$91.78. If the per-unit cost of production increases beyond this value, the investment would lose its financial viability. Keeping production costs under control is therefore vital to ensure continued profitability.
- The critical sales quantity (x) is 529.3 units per year. Falling below this sales volume would result in a negative NPV. Ensuring sufficient demand and sales is crucial, as underperformance in this area directly threatens the project's financial success.
- The critical investment amount (I) is \$176,795. If the initial investment required exceeds this threshold, the project will no longer generate positive returns. Proper budgeting and cost control during the initial setup phase are important to prevent overspending.

These critical values help decision-makers identify the most sensitive aspects of the project and set benchmarks that must be met to maintain profitability. They serve as key indicators for what needs to be closely managed and monitored throughout the investment's life cycle, ensuring that the project remains financially sustainable even in the face of changing conditions.

While the project appears financially viable under the base assumptions, the sensitivity analysis reveals that it is highly sensitive to changes in price, variable costs, and sales volume. These factors significantly impact the Net Present Value (NPV), highlighting the need for careful monitoring and strategic adjustments. Moving forward, it's important to manage these variables closely to ensure the project's long-term success and profitability. By addressing these key risks, decision-makers can be better equipped to make informed choices and optimize the investment's performance.

Chapter 6

Stochastic Capital Budgeting and Risk Analysis

In practical applications of capital budgeting, the importance of decision-making under uncertainty has increasingly gained significance, particularly in large-scale projects. This is because, when evaluating an investment, key future variables that influence the outcome are often uncertain at the time of the decision. Uncertainties regarding future cash flows may stem from developments in the sales and procurement markets, as well as internal factors within the company. Stochastic investment analysis helps quantify these uncertainties by assigning probability distributions to influencing factors, which are derived either objectively or subjectively. In this context, stochastic investment analysis refers to a calculation method in which the profitability of an investment is assessed by incorporating statistical distributions for the key influencing factors. As a result, the metric used to evaluate profitability, typically net present value (NPV) or internal rate of return (IRR), also becomes stochastic. By analyzing the distribution of these metrics, the associated risks can be assessed, enabling the implementation of risk management strategies.

In practice, there are two main approaches to stochastic investment analysis: analytical methods, based on mathematical-statistical formulas, and simulation methods, typically carried out using specialized software. When referring to risk analysis for investment decisions, it is often the stochastic investment analysis through simulation that is meant.

6.1 Analytical Methods

Through assumptions about distribution behavior, mathematical methods are used to derive the probability distribution of NPV or IRR. This approach was first described by Hillier (1963) and Wagle (1967). However, their assumptions are highly restrictive, and the method itself is complex. A simpler approach, based on subjective estimations, was proposed by Jöckel and Pflaumer (1981), which allows risk quantification through basic calculations. They use NPV as the metric for assessing investment profitability. Key determinants of cash flow, such as prices, variable costs, sales volumes, and research and marketing costs, are treated as stochastic. For each influencing factor, a probability distribution or risk profile is specified. This includes an upper and lower bound—values that will not be exceeded or fall below. Additionally, a median value is specified, representing the point at which there is an equal likelihood of under- or overestimation. This distribution is sometimes referred to as a "double rectangle distribution" in statistical literature. Assuming stochastic independence of influencing factors, provide a simple formula for calculating the expected value and variance of NPV. The square root of the variance (standard deviation) serves as a measure of investment risk. When comparing multiple investment options, the expected values and standard deviations can be plotted in a risk-return portfolio to identify optimal investments.

The method described here assumes stochastic independence of the influencing factors. Jöckel and Pflaumer expand their analytical model by allowing for dependencies, which can be described using regression models for correlations between variables (Jöckel and Pflaumer, 1980) or time series models for temporal dependencies (Jöckel and Pflaumer, 2024). A key finding from their research is that failing to account for positive correlations can lead to a significant underestimation of investment risk.

6.2 Simulation Methods

As with analytical methods, the influencing factors of NPV are modeled stochastically. However, the distribution of NPV or IRR is determined through simulations (e.g., see Savvides, 1994). The first risk simulation for investment decisions was conducted by Hess and Quigley (1963) for decisions in the chemical industry. The method gained widespread recognition through Hertz (1964), who applied and refined risk analysis at McKinsey & Co.

Simulation methods have two major advantages over analytical methods: First, the distribution of NPV and IRR can be determined even with various forms of dependencies and any type of distribution for influencing factors. Second, risk simulation does not require extensive knowledge of mathematical statistics, making it easier for practitioners to understand compared to analytical methods. The author's extensive experience in seminars and consultancy has shown the challenges in conveying the quantitative aspects of risk to decision-makers, despite most managers being fully aware of the risks associated with their decisions. If risk quantification were limited to analytical models, it would often be neglected. However, the basic principles of simulation methods can be taught in just a few hours, increasing the willingness to incorporate risk into investment decisions, especially for large projects. Stochastic investment analysis not only aids in risk evaluation but also encourages systematic discussion of possible influencing factors, leading to insights that simple deterministic investment calculations would never uncover.

6.3 A Quick and Simple Stochastic Method

In the context of investment analysis and firm valuation, understanding the inherent risks associated with cash flows is paramount. Traditional deterministic models, while providing a straightforward means of estimating a firm's value, often fall short in capturing the uncertainty and variability of real-world cash flows. Such models typically rely on the assumption of stable growth rates, which may not hold true, especially in dynamic and unpredictable market environments.

Given the limitations of deterministic approaches—particularly when historical data is scarce or insufficient—an alternative method becomes necessary. The random walk (RW) hypothesis offers a pragmatic solution for assessing investment risk in these scenarios. This approach assumes that cash flows evolve over time as a stochastic process, incorporating randomness and reflecting the uncertainty inherent in financial markets.

Utilizing the random walk framework allows for a more nuanced understanding of risk, enabling analysts to derive confidence intervals—such as the commonly used 2-sigma bounds—that quantify the potential variability in future cash flows. While it may be perceived as a "quick and dirty" method, particularly in the absence of comprehensive information on trends or seasonal effects, the random walk provides a critical baseline for evaluating risk. It offers a means to make informed decisions, even in the face of incomplete data, thus ensuring that uncertainty is accounted for in the investment valuation process.

The following section introduces the random walk model, demonstrating its use in evaluating investment risk while examining its strengths and limitations in firm valuation through three key formulas. Notably, the formula for long-term investments does not require estimation of the time horizon, making it a particularly valuable tool for assessing uncertainty.

6.3.1 Random Walk Overview

A random walk is a mathematical model that describes a sequence of steps in which the direction of each step is determined randomly. Typically represented as a stochastic process, a random walk can be visualized as a path taken by an object that moves in discrete steps, where each step's size and direction are influenced by random variables. This concept is widely used in various fields, including finance, physics, and statistics, to model phenomena that exhibit unpredictable behavior over time. In finance, for example, the random walk hypothesis suggests that stock prices follow a random path, making it challenging to predict future movements based solely on historical data.

A random walk is a stochastic process where each period's value depends on the previous period's value, plus a random fluctuation. In financial terms, this models cash flows where each period's value is the sum of the previous cash flow and a random shock. Mathematically, it is expressed as:

$$u_t = u_{t-1} + \epsilon_t$$

Where:

- u_t is the cash flow at time t,
- ϵ_t is an independent and identically distributed (i.i.d.) random shock with mean $E[\epsilon_t] = 0$ and variance σ^2.

This process reflects the inherent unpredictability in financial projections, with cash flows fluctuating randomly over time without following a clear trend.

6.3.2 Key Formulas for Present Value and Risk Assessment

The variability in cash flows modeled as a random walk directly impacts the present value distribution, introducing uncertainty. Below are three important formulas that account for the stochastic nature of cash flows and their effect on firm valuation, as discussed in *Data Analysis for Firm Valuation* (Jöckel and Pflaumer, 2024):

1. **Variance of Discounted Cash Flows**
   The variance of the present value when cash flows are discounted over n periods at a rate $q = 1 + i$ (where i is the interest rate) is

given by:

$$\text{VAR(discounted)} = \frac{n + \frac{q^{2n+2}-q^2}{q^2-1} - \frac{2q(q^n-1)}{q-1}}{q^{2n}(1-q)^2} \cdot \sigma^2$$

This formula demonstrates how the uncertainty grows over time, with the variability of future cash flows increasing as the number of periods rises.

2. **Variance of Undiscounted Cash Flows**
When the discount rate approaches zero (i.e., the cash flows are not discounted), the variance of the total cash flows over n periods is:

$$\text{VAR(undiscounted)} = \frac{n(n+1)(2n+1)}{6} \cdot \sigma^2$$

This shows how, without discounting, the cumulative uncertainty grows rapidly as the time horizon extends.

3. **Variance for an Infinite Time Horizon**
For an infinite time horizon, the variance of the present value of discounted cash flows is:

$$\text{VAR(infinite)} = \frac{q^2}{(q+1)(q-1)^3} \cdot \sigma^2$$

This formula simplifies the risk assessment over an extended period, showing that while discounting reduces variance, the inherent randomness of cash flows still leads to increasing uncertainty.

6.3.3 Estimating the Variance of the Random Walk

To estimate the variance σ^2 from cash flow differences, we can calculate the variance of the differences directly, as these represent the random shocks (increments) in the random walk. Here's a simple approach to estimating σ^2:

1. Compute the first differences of the cash flow series:

$$\Delta CF = CF_t - CF_{t-1}$$

2. Calculate the sample variance of these differences:

$$\hat{\sigma}^2 = \frac{1}{n-1}\sum_{t=1}^{n}(\Delta CF_t - \Delta\bar{C}F)^2$$

Where $\Delta\bar{C}F$ is the mean of the differences, and n is the number of observations.

This estimation provides an approximation of σ^2, the variance of the random shocks driving the random walk.

6.3.4 Example

A firm has generated the following cash flows (in millions) over the last 10 years:

8; 6.9; 6.4; 9.5; 9.7; 9.9; 13.4; 14; 11.8; 10.4

The firm operates in a competitive industry and has a discount rate of 20% due to the high risk associated with future cash flows.

a) Using the perpetuity formula, estimate the firm's value under the assumption that the average cash flow from the past 10 years will continue indefinitely into the future.

b) Assuming that the cash flows follow a random walk, estimate the mean and variance of the increments.

c) Calculate the standard deviation and a 2-sigma interval of the firm value.

Solution:

a) Average Cash Flow and Firm Value:

The average cash flow over the past 10 years is calculated as:

$$\text{Average cash flow} = \frac{CF_1 + CF_2 + \cdots + CF_{10}}{10} = 10\,\text{million}$$

Using the perpetuity formula, the firm's value is estimated as:

$$V = \frac{\text{Average cash flow}}{\text{Discount rate}} = \frac{10}{0.2} = 50\,\text{million}$$

b) Random Walk – Mean and Variance of the Increments:

From the cash flow increments:

$$\Delta CF = -1.1; -0.5; 3.1; 0.2; 0.2; 3.5; 0.6; -2.2; -1.4$$

We calculate:

- Mean of the increments:

$$\bar{\epsilon} = \frac{\sum_{i=1}^{n} \Delta CF_i}{9} = 0.2667$$

- Variance of the increments:

$$\sigma^2 = \frac{\sum_{i=1}^{n} (\Delta CF_i - \bar{\epsilon})^2}{8} = 3.74$$

c) Standard Deviation and 2-Sigma Interval of Firm Value:

Using the previously derived formula for an infinite time horizon, we calculate the variance of the firm value:

$$\text{Firm value variance} = \frac{1.2^2}{(1.2+1)\cdot(1.2-1)^3} \cdot 3.74 = 306$$

This yields a standard deviation of:

$$\sigma = \sqrt{306} = 17.49$$

The 2-sigma interval for the firm value, expressing the stochastic nature of the firm's valuation, is approximately:

$$\text{2-sigma interval} = \text{Firm value} \pm 2\sigma = 50 \pm 2 \cdot 17.49$$

$$\text{2-sigma interval} = (15.02\,\text{million}, 84.98\,\text{million})$$

This wide range highlights the high level of uncertainty in the firm's valuation, underscoring the importance of accounting for variability in cash flows when assessing firm value.

6.4 Further Stochastic Investment Models

While the random walk model offers valuable insights, we now present three alternative stochastic investment models that we have developed and utilized. These models provide distinct approaches to investment modeling, independent of the random walk framework.

1. **A Quick and Easy Tool to Assess the Risk of a Capital Investment Project:** This model, based on the work of Jöckel and Pflaumer (1981), provides a straightforward procedure for evaluating investment risk without the need for complex simulations. By employing a normal approximation, it assesses the net present value (NPV) based on stochastic and deterministic factors such as prices, sales, and variable costs. This approach facilitates clear investment planning and allows for quick

calculations within an Excel framework. *(Available at: https://www.researchgate.net/publication/356906257)*

2. **Risk Analysis in Capital Investment Appraisal with Correlated Cash Flows:** This approach focuses on risk analysis in capital budgeting by introducing simple analytical methods that accurately measure risk for projects with correlated cash flows. It serves as a faster alternative to traditional Monte Carlo simulations, demonstrating reliability and practicality in evaluating investment risks. *(Available at: https://www.researchgate.net/publication/319645061)*

3. **Using ARMA Models in Stochastic Enterprise Valuation:** This model integrates Autoregressive Moving Average (ARMA) models to capture temporal dependencies in cash flows. By accounting for autocorrelation, it provides a more accurate estimation of variance in a firm's value distribution, enhancing risk assessment and leading to more informed investment decisions. *(Available at: https://www.researchgate.net/publication/379120854)*

Each of these models offers unique strengths, serving as valuable tools for assessing investment risk and supporting informed decision-making alongside the random walk approach.

Chapter 7

Problems and Exercises

Detailed solutions are found in Pflaumer (2018).

1. **Investment Project**
   An investment project with an initial outlay of €1 million results in the following cash flows:

   - Year 1: €200,000
   - Year 2: €400,000
   - Year 3: €400,000
   - Year 4: €308,000

   The discount rate is 8%.

   (a) Calculate:
      i. The net present value (NPV)
      ii. The terminal or final value
      iii. The Baldwin return rate
      iv. The internal rate of return.

   (b) Sketch the NPV function.

2. **Investment Alternatives**
   A company with a discount rate of 10% must choose between two investment alternatives:

- **Machine A:**
  - Investment outlay: €1,000,000
  - Annual cash flow: €200,000
  - Useful life: 10 years
  - Residual value: €0
- **Machine B:**
  - Investment outlay: €500,000
  - Annual cash flow: €100,000
  - Useful life: 10 years
  - Residual value: €270,000

(a) Calculate the NPVs.

(b) Determine the IRRs.

(c) Which alternative is preferable?

3. **Purchase of a Rental Property**
   For the purchase of a rental property, with a resale planned after 30 years, the following information is given:

   - Purchase price: €1,000,000
   - Average annual cash flow for the first decade: €50,000
   - Average annual cash flow for the second and third decades: €80,000
   - Estimated resale value after 30 years: €1,000,000

   (a) Should the property be purchased at a discount rate of 5%? Calculate the NPV, terminal values for the baseline and investment alternatives, and the equivalent annuity.

   (b) What are the Baldwin return rate and the IRR?

4. **Investment Alternatives A and B**
   Two investment alternatives A and B are available. The investment outlays are €70,000 for A and €80,000 for B. The discount rate is 14%. The following cash flows (in €) are projected:

Year	1	2	3	4	5
A	10,000	20,000	30,000	45,000	60,000
B	25,000	25,000	25,000	25,000	25,000

(a) Calculate:
   i. The NPVs.
   ii. The Baldwin return rates.
(b) Which investment is preferable?

5. **Financed Machine Purchase**
A machine with a useful life of 4 years costs €80,000. It is depreciated linearly to zero, and no residual value is expected. The annual cash flows are €30,000. The acquisition costs will be 50% financed through a loan with an interest rate of 20%. The loan will be repaid as quickly as possible from the machine's cash flows. The discount rate is 10%.
   (a) Calculate the NPV.
   (b) Calculate the NPV after accounting for taxes, assuming a constant profit tax rate of 60%.

6. **Investment Project with Taxes**
An investment project with an outlay of €1 million results in the following cash flows:
   - Year 1: €300,000
   - Year 2: €400,000
   - Year 3: €400,000
   - Year 4: €500,000

   The discount rate is 10% and the profit tax rate is 60%. Depreciation is done linearly to a residual value of €0.
   (a) Calculate the NPV and terminal value after taxes.
   (b) Determine the Baldwin return rate after taxes.
   (c) Calculate the IRR after taxes.

7. **Investment Alternatives A and B**
A company with a discount rate of 10% must choose between two investment alternatives A and B, both with a useful life of 6 years. Investment A requires an initial outlay of €80,000 and results in the following: NPV = €12,434, IRR = 20%. For investment B, the following data is available:

- Initial outlay: €100,000
- Annual cash flow: €25,000
- Residual value: €12,500

(a) Calculate the NPV and IRR for B.

(b) Determine the Baldwin return rates for A and B.

(c) Which alternative is preferable (justify your answer)?

8. **Investment Alternatives**
Two investment alternatives A and B are characterized by the following cash flows:

Year	0	1	2	3	4
A	-20,000	0	0	0	48,830
B	-12,000	5,540	5,540	5,540	5,540

The discount rate is 20%.

(a) Calculate the IRRs.

(b) Calculate the Baldwin return rates.

(c) Which investment is preferable (justify your answer)?

(d) At what discount rate do the NPV functions intersect?

9. **Machinery Purchase**
An entrepreneur is considering the purchase of equipment for producing cheese wedges, with a useful life of 4 years. The initial outlay is €200,000, and the equipment would save €80,000 annually by reducing staff costs. It is depreciated linearly to zero, but the equipment can be sold for €50,000 at the end of year 4.

(a) Calculate the NPV at a discount rate of 10%, assuming a loan of €100,000 is taken out, which will be repaid in equal installments over four years. The loan interest rate is 8%.

(b) Calculate the terminal value.

(c) Calculate the NPV after taxes, assuming a constant tax rate of 50%. Extraordinary gains are taxable, and loan interest reduces taxes. Any losses can be offset internally.

10. **Buy or Lease**
An entrepreneur must decide whether to purchase or lease a machine worth €100,000, with a useful life of 8 years. The lease payment is €12,000 in the first year and increases by 10% annually. If purchased, the entrepreneur would depreciate the machine by €10,000 per year. The machine can be sold for €20,000 at the end of its useful life. At a pre-tax discount rate of 8% and an income tax rate of 50%:

    (a) Is purchasing or leasing preferable?
    (b) Calculate the critical leasing rate before taxes for the first year.
    (c) The machine generates an annual cash flow of €20,000. Calculate the NPV of the more favorable alternative.

11. **Financing a Vehicle**
A sales representative is considering a vehicle that costs €50,000. The resale value after three years is €20,000. With a discount rate of 10% (pre-tax), he is evaluating the following financing alternatives:

    (a) **A**: Pay €20,000 in equity and finance the rest with a loan over three years at 15% p.a.
    (b) **B**: Lease the vehicle with an annual rate of €15,000.

    Depreciation is linear to the resale value, and the income tax rate is 40%.

    (a) Which financing option should the representative choose? Justify your decision using the NPV method.
    (b) At what annual leasing rate would both financing alternatives be equally favorable?
    (c) At what discount rate would both alternatives be equally favorable?

12. **Gift or Loan**
A student must choose between a gift of €10,000 or a €70,000 interest-free loan, repaid in seven annual installments of €10,000.

    (a) Evaluate the advantages of each option using the NPV method.

(b) At what interest rate does the loan become more favorable?

13. **Zero Baldwin Rate**
An investment of €100,000 with a 5-year duration results in a Baldwin rate of 0% at a discount rate of 10%.
    - What is the NPV of this investment?

14. **Flight School Investment**
A flight school plans to purchase an airplane for €150,000. Fixed annual costs amount to €15,000, and the school charges €180 per flight hour, with variable costs of €80 per hour. The airplane is expected to be sold after 5 years for €100,000. The discount rate is 10%.
    (a) How many flight hours must the airplane be rented annually for the Baldwin rate of return to exceed 12%?

15. **Equal Cash flows**
A project with a 2-year duration requires an initial investment of €100,000.
    (a) What must the equal annual cash flows be to achieve an internal rate of return of 10%?
    (b) What must the flows be to achieve a Baldwin rate of 10%?

    The discount rate is 8%.

16. **Silver Mine Investment**
An investor is considering an €80,000 stake in a silver mine, operational for four years, with expected annual cash flows of €22,000. A loan of €40,000 is taken at 10% interest p.a., with repayment at the end of four years. Interest is paid annually. The investment is tax-advantaged, with a 70% depreciation in the first year and linear depreciation in the following years. The post-tax discount rate is 6.5%, and the investor's tax rate is 50%.
    (a) Calculate NPV, IRR, and Baldwin rate after tax if the investment is sold for €20,000 after four years.
    (b) Repeat the calculation for two scenarios: first, with a resale value of €8,000, and second, with a resale value of €20,000.
    (c) Sketch the NPV function for a resale value of €8,000.

17. **Buy or Rent**
A couple is deciding whether to buy or rent a home. The purchase price is €200,000, with 80% financed through a 20-year loan at 8% interest, repaid in equal annual annuities. Insurance costs €500 in the first year, and maintenance costs €1,000, expected to rise by 5% and 3%, respectively, each year. The property is expected to increase in value by 3% annually, while rent starts at €10,000 with a 3% annual increase. The couple has €40,000 in savings, earning 5% interest annually.

    (a) Use the NPV method to evaluate whether buying or renting is more advantageous (n=20).
    (b) At what annual increase in property value are both options equally favorable?
    (c) Determine the discount rate at which both alternatives are equally favorable.
    (d) How does the decision change if the loan is repaid in equal installments?

18. **Car Replacement**
The purchase price of a car is €30,000, with a resale value of 75% after one year. The resale value then decreases by 10% annually. Repair and maintenance costs start at €500 in the first year, increasing by €500 each year.

    - Determine the optimal replacement time, assuming the new car price remains unchanged and the discount rate is 5%.

19. **Investment Alternatives**
Two investment alternatives, A (5-year duration) and B (4-year duration), each require an initial outlay of €75,000. The discount rate is 10%, and the tax rate is 60%. One-third of the investment is financed through a loan repaid in equal annuities. Depreciation is linear to zero over time. The cash flows are as follows:

Year	1	2	3	4	5
A	20,000	20,000	30,000	45,000	60,000
B	40,000	40,000	40,000	40,000	–

    - Which alternative is preferable? Justify your answer using an appropriate capital budgeting metric.

20. **Country Comparison**
An entrepreneur must decide whether to invest in Country A (with a tax rate of 20%) or Country B (with a tax rate of 30%). Both countries offer similar advantages, and the pre-tax discount rate is 15%. Depreciation is linear to zero over time. The expected cash flows are:

- Initial investment: €3,000,000
- Annual pre-tax cash flows: €800,000
- Investment duration: 6 years

- In which country should the entrepreneur invest? Justify your answer using an appropriate capital budgeting metric.

21. **Investment with Constant Cash Flow**
An investment with an outlay of Io generates a perpetual annual cash flow of c. What is the Baldwin return rate for this investment at a discount rate of p?

22. **Investment Decision: Projects X and Y**
An entrepreneur must choose between Project X with an outlay of €20,000 or Project Y with an outlay of €12,000. The cash flows for years 1 to 4 are as follows:

Year	1	2	3	4
Cash Flows from X	3,000	4,000	8,000	8,000
Cash Flows from Y	5,000	5,000	5,000	3,286

The discount rate is 6% before taxes, and the profit tax rate is 30%. The annual depreciation is €4,000 for Project X and €2,000 for Project Y.

(a) Calculate the Baldwin return rates after taxes, assuming the residual values of Projects X and Y equal their book values. Which investment is preferable?

(b) Determine the internal return rates after taxes, assuming the residual value of X is €6,000 and that of Y is €0. Which investment is preferable?

23. **Car Purchase Decision**
An entrepreneur is considering whether to buy a diesel car for €26,500 or a gasoline car for €25,000. The diesel car incurs an annual tax €245 higher than the gasoline car. The useful life is assumed to be 5 years, and the diesel car has a resale value €400 higher than the gasoline car. The following table provides the fuel consumption and price:

	Diesel Car	Gasoline Car
Consumption (liters per 100 km)	6.4	9.0
Price per liter (Jan. 2003)	€0.924	€1.094

Determine the annual mileage at which purchasing the diesel car is more advantageous than purchasing the gasoline car, assuming a discount rate of 4%.

24. **Commercial Real Estate Investment**
An investment in commercial real estate costing €1,000,000 will yield annual cash flows of €100,000 for 5 years. The property can be resold for €1,000,000 after 5 years. A portion of the investment will be financed through a loan that will be repaid in a lump sum at the end of the term, with annual interest payments. Sketch the internal return rate IRR as a function of the debt-to-equity ratio V (leverage function) for two scenarios: (a) 8% p.a. interest and (b) 12% p.a. interest.

25. **Machine Acquisition**
An entrepreneur plans to acquire a machine costing €300,000. The useful life is projected to be four years, with no residual value. The expected annual cash flows are €100,000. The machine must be paid for immediately, with €150,000 of this financed through an 8% p.a. loan. The discount rate is 6% per year. The entrepreneur is presented with three repayment options, with interest paid annually.

    (a) The loan is repaid in four equal installments.
    (b) The loan is repaid in four equal annuities.
    (c) The loan is repaid as quickly as possible from the cash flows from the machine.

Should the entrepreneur proceed with the acquisition? Which repayment option should he choose? Calculate the respective NPVs for each scenario.

26. **Cheese Cutter Investment**
An entrepreneur considers investing in a cheese cutter with a planned life of 6 years. The purchase price is €180,000, and it is expected to save €40,000 per year in labor costs. Depreciation is linear to a residual value of €0, with a projected resale value of €30,000 at the end of 6 years. The profit tax rate is 50% and the pre-tax discount rate is 8%.

    (a) Assess the profitability of this investment using the NPV and internal rate of return methods.

    (b) Calculate the NPV after taxes if the entire investment is financed at an interest rate of 8%. The loan will be repaid at the end of the period, with annual interest payments. Possible losses can be offset within the company. Sketch the NPV function after taxes and financing.

27. **Cheese Production Investment**
An investment of €100,000 in a cheese production plant will yield €20,000 annually for 10 years. The discount rate is 5%. The useful life is projected to be 10 years, and the plant will be depreciated linearly to a residual value of €0.

    (a) Calculate the NPV, Baldwin return rate, and IRR.

    (b) Calculate the NPV after taxes at a profit tax rate of 40%.

    (c) Calculate the NPV after financing and after taxes if a loan of €50,000 is taken out, to be repaid in 10 equal annuities at an interest rate of 8% per year.

28. **Investment Analysis**
An investment of €400,000 yields cash flows of €206,000 in the first year and €300,000 in the second year. The discount rate is 8%.

    (a) Determine the NPV, Baldwin return rate, and IRR.

    (b) Determine the IRR after taxes with a profit tax rate of 40% if the investment is depreciated linearly to a residual value of €0 over 2 years.

(c) Calculate the NPV and Baldwin return rate after financing and taxes if a loan of €100,000 is taken out, to be repaid in 2 equal installments with an interest rate of 6% per year.

29. **Cheese Production Equipment Comparison**
An entrepreneur must decide between two cheese production machines, both priced at €80,000. Machine A incurs annual fixed costs of €20,000, while Machine B incurs annual fixed costs of €25,000. The variable costs per unit are €0.30 for Machine A and €0.25 for Machine B. The selling price for cheese is €0.80 per unit.

    (a) Calculate the total costs, average costs, and profit functions for both machines.

    (b) Determine the production level at which total costs are equal for both alternatives.

    (c) Calculate the contribution margins and the break-even quantities for both machines.

    (d) How many units must be sold with Machine B to achieve a return on investment (ROI) of 20%?

    (e) Calculate the payback period for Machine B at a production level of 50,000 units per year, assuming that 20% of the fixed costs and all variable costs are cash-effective.

30. **Ski Lift Investment**
An investment in a ski lift of €1,000,000 generates annual cash flows of €200,000 for 10 years. Depreciation is linear to zero over time. The discount rate is 12%.

    (a) Calculate the NPV and determine the risk associated with this investment.

    (b) Assume the cash flows of €200,000 represent an average estimate, with a possible deviation of up to 40% from the mean. If the deviations are uniformly distributed and cash flows are stochastically independent, calculate the standard deviation and the probability of a negative NPV.

31. **Internal Rate of Return Approximation**
How is the approximate formula for the internal rate of return derived for a long-term investment with constant cash flows?

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