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Article

Quantum Holism, Superluminality, and Einstein Causality

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Abstract: Within the framework of the EPR Gedankenexperiment we show that quantum mechanics is not incomplete but nonlocal and holistic. The attempt to use this quantum non-locality for the transmission of superluminal signals is exposed to serious objections which are based on very general theorems. However, we show that one of the theorems is equivalent to the impossibility of superluminal signals. Hence, the question whether superluminal EPR signals are possible can not be decided in this way.

Keywords: EPR Gedankenexperiment, holism, separability, realism, Einstein causality

1 Introduction

The present paper is concerned with three quite general problems of quantum systems consisting of two subsystems that are not separable (cf. Peres 1996). First, we investigate the question, whether the entanglement of the subsystems provides a new kind of quantum holism (Esfeld 1999) which is unknown in classical physics. We show that in the EPR-Gedankenexperiment (Einstein/Podolsky/Rosen, 1935) in the version of *Bohm and Aharonov* (Bohm/Aharonov, 1957) the entanglement of the subsystems does not imply incompleteness – as Einstein conjectured – but non-locality and a new kind of holism which is based on the objective un-decidedness of properties of the subsystems (Busch/Lahti/Mittelstaedt 1992, Mittelstaedt 1998b). Second, we investigate the problem whether the non-local correlations, which can be tested by experiment, allow for the transmission of signals. Non-locality implies in this case that the signals, if they exist, are instantaneous and hence superluminal. However, there are strong arguments against superluminal signals which would violate Einstein causality (d'Espagnat 1984; d'Espagnat 1989, especially p. 94; d'Espagnat 1994, pp. 117, 145, 352; de Muynck 1984; Mittelstaedt/Stachow 1983; Schlieder 1969). – Finally, we analyze these arguments and show that the micro-causality condition of relativistic quantum field theory excludes entanglement induced superluminal signals but that this condition is justified by the exclusion of superluminal signals. Hence, we are confronted here with a vicious circle, and the question whether there are superluminal EPR-signals cannot be answered in this way.

2 Entanglement and quantum holism

2.1 The EPR-BA-Gedankenexperiment

In the EPR-Gedankenexperiment in the version of *Bohm and Aharonov* (Bohm/Aharonov, 1957) we consider two different spin $1/2$ systems S_1 and S_2 (e.g. proton and neutron) with Hilbert spaces H_1 and H_2, respectively, and assume that the compound system $S = S_1 + S_2$ is in the pure state

$$\Psi(S) = \frac{1}{\sqrt{2}} \left(\varphi^{(1)}_{\mathbf{n}} \otimes \varphi^{(2)}_{-\mathbf{n}} - \varphi^{(1)}_{-\mathbf{n}} \otimes \varphi^{(2)}_{\mathbf{n}} \right) . \tag{1}$$

Here we denote the spin observables with respect to the systems S_1 and S_2 by $\sigma_1(\mathbf{n})$ and $\sigma_2(\mathbf{n})$, respectively, where the spin direction is described by a unit vector $\mathbf{n}$ (in the Poincaré sphere, Mittelstaedt 1998b). The eigenstates are then given by $\varphi^{(1)}_{\pm\mathbf{n}}$ and $\varphi^{(2)}_{\pm\mathbf{n}}$ and fulfil the eigenvalue equations (with eigenvalues $s_k = \pm 1$)

$$\sigma_k(\mathbf{n})\varphi^{(k)}_{\pm\mathbf{n}} = \pm\varphi^{(k)}_{\pm\mathbf{n}} . \tag{2}$$

for $k = 1, 2$. If S is in the pure state $\Psi(S)$ given by Eq. (2), the subsystems S_1 and S_2 are said to be in the "reduced" mixed states W_1 and W_2, respectively (cf. appendix A "Reduced mixed states").

The observables $\sigma_1(\mathbf{n})$ and $\sigma_2(\mathbf{n})$ can also be written as observables $A_1(\mathbf{n})$ and $A_2(\mathbf{n})$ of the compound system $S = S_1 + S_2$

$$A_1(\mathbf{n}) := \sigma_1(\mathbf{n}) \otimes \mathbb{1}_2 \quad \text{and} \quad A_2(\mathbf{n}) := \mathbb{1}_1 \otimes \sigma_2(\mathbf{n}) . \tag{3}$$

A measurement of the observable $A_1(\mathbf{n})$, say, transforms in a first step (pre-measurement) the state operator $W = P[\Psi]$ of the compound system S - where $P[\Psi]$ denotes the projection operator onto Ψ - into a mixed state $W'(A_1(\mathbf{n}), W)$ and leads in a second step (reading) to the measurement results $\mu\{\sigma_1(\mathbf{n})\}$ and $\mu\{\sigma_2(\mathbf{n})\}$ of the two spin observables.

There is a strong correlation between the measurement results of the spin observables such that

$$\mu\{\sigma_1(\mathbf{n})\} = \pm 1 \Leftrightarrow \mu\{\sigma_2(\mathbf{n})\} = - \pm 1 \tag{4}$$

holds. This means that if $A_1(\mathbf{n})$ was measured with the result $s_1 = +1$, say, then a measurement of $A_2(\mathbf{n})$ will lead with certainty to the result $s_2 = -1$. If the second measurement refers to an observable $A_2(\mathbf{n}')$ with a different spin direction $\mathbf{n}' \neq \mathbf{n}$, then one can no longer predict the result $s_2' = -1$ with certainty. In this case quantum mechanics provides the probability $p_{12}(\mathbf{n}, -\mathbf{n}') = \frac{1}{4}(1 + \mathbf{n} \cdot \mathbf{n}')$

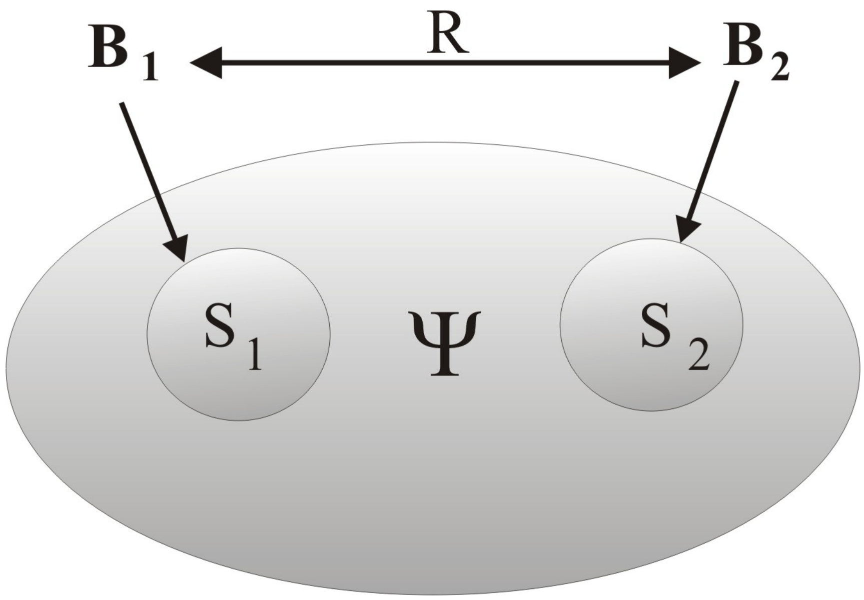

Figure 1: Compound system S with state Ψ and subsystems S_1 and S_2. The observers B_1 and B_2 can measure observables of the subsystems S_1 and S_2, respectively. R is the distance between B_1 and B_2.

for measuring the $\mathbf{n}$-spin on system S_1 and the $(-\mathbf{n})$-spin on S_2, and hence the conditional probability $p(\mathbf{n}, -\mathbf{n}') = \frac{1}{12}(1 + \mathbf{n} \cdot \mathbf{n}')$ for obtaining the $(-\mathbf{n}')$-spin on system S_2 if the $\mathbf{n}$-spin was measured on system S_1.

Generally, quantum measurements are performed by observers who are equipped with measurement apparatuses. Here we consider two observers B_1 and B_2 and apparatuses M_1 and M_2 for measurements of the observables $\sigma_1(\mathbf{n})$ and $\sigma_2(\mathbf{n})$ respectively. We will assume here, that the compound system has a large extension and that the subsystems S_1 and S_2 as well as the observers B_1 and B_2 have a macroscopic distance R. In the experiments of Aspect/Gangier/Rogier (1982) the distance R is about 14m. More recent experiments of Gisin et al. (2000) work with distances of about 10 kilometers (Fig. 1).

The state $\Psi(S)$ of the compound system $S_1 + S_2$ is an *entangled* state. This means that $\Psi(S)$ is not *separable*. Generally, a pure state $\Psi(S)$ of a system S consisting of two subsystems S_1 and S_2 is called *separable*, if $\Psi(S)$ can be written as a tensor product $\Psi(S) = \Psi_1(S_1) \otimes \Psi_2(S_2)$ with pure states $\Psi_1(S_1) \in H_1$ and $\Psi_2(S_2) \in H_2$. If the compound system S is in a mixed state W, $(\mathrm{tr}(W) = 1)$, then the state W is called *separable* if it can be written as $W = \sum_i p_i W_i^1 \otimes W_i^2$ where W_i^1 and W_i^2 are mixed states of the subsystems S_1 and S_2, respectively (cf., e.g., Peres 1996). The special case that $W = P[\varphi]$ is a pure state is contained in this more general definition of separability.

2.2 The EPR-incompleteness argument

On the basis of this Gedankenexperiment, which can also be realized by photons, the EPR argument can be derived if the following two principles are taken for granted.

1. The principle of reality $R := R_1 \rightarrow R_2$, i.e. "if R_1 then R_2", where
   (R_1) *"the value A_i of an observable A can be determined without changing the system S"*
   (R_2) *"a property $P(A_i)$ which corresponds to this value of A pertains to the system S".*
2. The principle of locality $L := L_1 \rightarrow L_2$, i. e. 'if L_1 then L_2", where
   (L_1) *"two systems cannot interact with each other"*
   (L_2) *"a measurement with respect to one system cannot change the other system".*

If after the preparation of the state $\Psi(S)$, the systems S_1 and S_2 are separated into distant regions of space, then the systems cannot interact with each other. Hence the premise L_1 of the locality principle is fulfilled, and thus the conclusion L_2 is valid. This means that the measurement of $\sigma_1(\mathbf{n})$ cannot change S_2 in any way. However, since the result s_1 of the $\sigma_1(\mathbf{n})$ measurement determines the value $s_2 = -s_1$ of the observable $\sigma_2(\mathbf{n})$, the premise R_1 of the reality principle is fulfilled. Hence we obtain the conclusion R_2 which means that the value s_2 of $\sigma_2(\mathbf{n})$ pertains to the system S_2 after the preparation.
Since these arguments can be applied to spin observables $\sigma_1(\mathbf{n})$ and $\sigma_2(\mathbf{n})$ with arbitrary directions $\mathbf{n}$, it follows that for any direction $\mathbf{n}$ the value s_2 of $\sigma_2(\mathbf{n})$ pertains to system S_2 after the preparation of $\Psi(S)$. This means that the observable $\sigma_2(\mathbf{n})$ can be *weakly objectified* with respect to the mixed state $W_2(S_2)$ of the subsystem S_2, or that the mixed state $W_2(S_2)$ admits an *ignorance interpretation* with respect to the states $\varphi_{\mathbf{n}}^{(2)}$ and $\varphi_{-\mathbf{n}}^{(2)}$ (cf. Busch/Lahti/Mittelstaedt 1992). *Einstein* was convinced that on the basis of this result he could demonstrate at least the *incompleteness* of quantum theory, even if he could not question its *validity* (Bohr, 1949). The argument reads as follows: On the one hand, the value of $\sigma_2(\mathbf{n})$ is objectively determined even if the observer does not know it. On the other hand, quantum mechanics does not allow to determine this value and provides only probabilities for the values of $\sigma_2(\mathbf{n})$. Hence, quantum mechanics is *incomplete* since it does not describe the full *reality* (Einstein/Podolsky/Rosen, 1935). However, this latter conclusion is not correct.

2.3 The EPR contradiction

Ignorance interpretation of the mixed state $W_2(S_2)$ is not in accordance with quantum mechanics for the following reason. Assume that $\sigma_2(\mathbf{n})$ is weakly objectified with respect to the system S_2 in the state W_2. Then we can attribute

a value $s_2 \in \{1, -1\}$ of $\sigma_2(\mathbf{n})$ to the system S_2 in the mixed state W_2 such that this value s_2 pertains to the system with probability $p = \frac{1}{2}$.
This conclusion means that the observable $A_2(\mathbf{n}) = \mathbb{1}_1 \otimes \sigma_2(\mathbf{n})$ is weakly objectified with respect to the compound system S in the pure state Ψ. This implies that probabilities for values of an observable B of the compound system S must be calculated by means of the mixed state

$$W_\Psi = \frac{1}{2} P[\varphi_{\mathbf{n}}^{(1)} \otimes \varphi_{-\mathbf{n}}^{(2)}] + \frac{1}{2} P[\varphi_{-\mathbf{n}}^{(1)} \otimes \varphi_{\mathbf{n}}^{(2)}] \ . \tag{5}$$

Hence, for the test observable

$$B\left(\mathbf{n}', \mathbf{n}''\right) := \sigma_1\left(\mathbf{n}'\right) \otimes \sigma\left(\mathbf{n}''\right) = \sum_{i,k} B_{ik} P_{ik}\left(\mathbf{n}', \mathbf{n}''\right) \tag{6}$$

with the notation

$$P_{ik}\left(\mathbf{n}', \mathbf{n}''\right) := P\left[\varphi_{i\mathbf{n}'}^{(1)} \otimes \varphi_{k\mathbf{n}''}^{(2)}\right] \quad \text{with} \quad i, k \in \{1, 2\} \ , \tag{7}$$

the probabilities of the eigenvalues $B_{++} = B_{--} = 1$ and $B_{+-} = B_{-+} = -1$ read

$$p_\Psi\left(B_{ik}\right) = \mathrm{tr}\{W_\Psi P_{ik}\left(\mathbf{n}', \mathbf{n}''\right) \ . \tag{8}$$

For the special choices of Ψ, A, W_Ψ and B it follows

$$p_\Psi\left(B_{ik}\right) = \mathrm{tr}\{P[\Psi] P_{ik}\left(\mathbf{n}', \mathbf{n}''\right)\} = \frac{1}{4}\left(1 - B_{ik}\left(\mathbf{n}' \cdot \mathbf{n}''\right)\right) \tag{9}$$

$$= \mathrm{tr}\{W_\Psi \cdot P_{ik}\left(\mathbf{n}', \mathbf{n}''\right)\} = \frac{1}{4}\left(1 - B_{ik}\left(\mathbf{n} \cdot \mathbf{n}'\right)\left(\mathbf{n} \cdot \mathbf{n}''\right)\right) \ , \tag{10}$$

Hence the condition Eq. (8) of weak objectification (value attribution) assumes for all values B_{ik} the special form

$$\mathbf{n}' \cdot \mathbf{n}'' - \left(\mathbf{n} \cdot \mathbf{n}'\right)\left(\mathbf{n} \cdot \mathbf{n}''\right) = 0 \ . \tag{11}$$

Since this equation is violated in quantum mechanics except for a few special triples $(\mathbf{n}, \mathbf{n}', \mathbf{n}'')$, it follows that weak objectification of A and hence ignorance interpretation of W_2 is in general not compatible with quantum mechanics. This contradiction between the consequences of the principles R and L and quantum mechanics is the content of the EPR-paradox. It should be mentioned that from condition (11) one can easily derive the inequalities

$$\begin{aligned} |\mathbf{n}' \cdot \left(\mathbf{n} - \mathbf{n}''\right)| &\leq \mathbf{n} \cdot \left(\mathbf{n} - \mathbf{n}''\right) \ , \\ |\mathbf{n}' \cdot \left(\mathbf{n} + \mathbf{n}''\right)| &\leq \mathbf{n} \cdot \left(\mathbf{n} + \mathbf{n}''\right) \ . \end{aligned} \tag{12}$$

Triples of vectors $(\mathbf{n}, \mathbf{n}', \mathbf{n}'')$ that fulfil these inequalities satisfy Bell's inequalities in accordance with quantum mechanics. However, for arbitrary triples of vectors $(\mathbf{n}, \mathbf{n}', \mathbf{n}'')$ Bell's inequalities contradict quantum mechanics. For more details about the validity domain of Bell's inequalities in quantum mechanics we refer to the literature (Mittelstaedt 1998b, pp. 101-102 and Busch et al. 1992).

2.4 Resolution of the EPR contradiction – non-locality

A measurement of $\sigma_1(\mathbf{n})$ on the system S_1 corresponds to a measurement of $A_1(\mathbf{n}) = \sigma_1(\mathbf{n}) \otimes \mathbb{1}_2$ on the compound system S. According to the theory of quantum measurements in the sense of von Neumann (1932), in the first step of the measuring process the state $W = P[\Psi]$ of S is transformed into a mixture of states $\Gamma(W_L) = \{\varphi_{\mathbf{n}}^{(1)} \otimes \varphi_{-\mathbf{n}}^{(2)}, \varphi_{-\mathbf{n}}^{(1)} \otimes \varphi_{\mathbf{n}}^{(2)}\}$, and in a second step the result s_1 is read by the observer. Hence a measurement of $\sigma_1(\mathbf{n})$ on S_1, i.e. of $A_1(\mathbf{n})$ on S, (with or without reading) induces a change of the initial state W_2 of S_2 into a mixture of states $\Gamma(W_2) = \{\varphi_{\mathbf{n}}^{(2)}, \varphi_{-\mathbf{n}}^{(2)}\}$, irrespective of the spatial distance of the systems S_1 and S_2.

One could think again of two observers B_1 and B_2 separated by a large distance R, where B_1 measures $\sigma_1(\mathbf{n})$, and B_2 measures $\sigma_2(\mathbf{n})$. It is obvious that under these conditions the locality principle L is untenable. Even if "*two systems cannot interact with each other*" and thus L_1 is fulfilled, a measurement with respect to one system can change the other one in such a way that some observable is objectified, in disagreement with L_2. A relaxation of the locality principle which is in accordance with the measuring process is then given if L *is* weakened into the relaxed principle of locality

(L') $L_1 \to L_2'$

with

(L_2') *"a measurement with respect to one system can change the other one at most such that some observable is objectified on this system".*

If we make use only of the relaxed locality principle $L' = L_1 \to L_2'$, then the paradox mentioned disappears since from L_2' we can no longer derive the premise R_1 of the reality principle. Hence we can neither deduce R_2 nor the weak objectification relation (11) nor Bell's inequalities (12). However, the price for this consistency is very high: The weak locality principle allows for some *"objectification at a distance"*, a nonlocal influence on a physical system, the dynamics of which is completely unknown.

The EPR paradox and its resolution lead to two important consequences for the interpretation of quantum mechanics, *non-locality* and *holism*. Since the locality principle, which was conceived by *Einstein*, turned out to be untenable in quantum mechanics, non-locality must be considered as a basic structure of this theory. In the EPR-Gedankenexperiment the compound system S which is composed of the subsystems S_1 and S_2 is in the entangled state $\Psi(S_1 + S_2)$. Even if the interaction between the systems is turned off and if their spatial distance becomes very large and macroscopic the compound system will remain in the entangled state and will never become *separable*. There is no element in the dynamics of the system S which leads to a decoherence of the entangled state $\Psi(S)$.

On the basis of this non-locality argument it becomes obvious that the state $\Psi(S)$ can never be decomposed into a tensor product $\Psi(S) = \Psi_1(S_1) \otimes \Psi_2(S_2)$

of states $\Psi_1 \in H_1$ and $\Psi_2 \in H_2$, and that the subsystems S_1 and S_2 can only be described by the reduced mixed states $W_1(S_1)$ and $W_2(S_2)$, respectively. Moreover, the entanglement of $\Psi(S)$ implies that the mixed states $W_1(S_1)$ and $W_2(S_2)$ do not admit ignorance interpretation. This means that it is not possible to attribute values of the observables $\sigma_1(\mathbf{n})$ and $\sigma_2(\mathbf{n})$ to the subsystems S_1 and S_2 respectively, even if the compound system is in a pure state $\Psi(S)$ which provides maximal information about the system S and in particular about the value of the S-observable $\sigma = A_1 + A_2$.

Hence, we are confronted here with a rather strange situation: The observer possesses a maximal knowledge of the total system $S = S_1 + S_2$ but the subsystems S_1 and S_2 can only be described incompletely by the mixed states W_1 and W_2. In particular, the knowledge of the reduced mixed states W_1 and W_2 does not allow to determine the state $\Psi(S)$ of the compound system. This situation in which the knowledge about the compound system is more than the sum of all information about the subsystems is usually called *holism.* Note, however, that in quantum mechanics the holism is further strengthened, since for an observer who knows the state $\Psi(S)$ the properties of the subsystems S_1 and S_2 are not only subjectively unknown but objectively undecided.

3 Non-locality and superluminal signals

3.1 EPR - communication

Prior to the measurement of the observables $A_1(\mathbf{n})$ and $A_2(\mathbf{n})$ the compound system S is in the pure state $W = P[\Psi]$, and the subsystems S_1 and S_2 are in the mixed states

$$W_1 = \frac{1}{2}P\left[\varphi_{\mathbf{n}}^{(1)}\right] + \frac{1}{2}P\left[\varphi_{-\mathbf{n}}^{(1)}\right] , W_2 = \frac{1}{2}P\left[\varphi_{\mathbf{n}}^{(2)}\right] + \frac{1}{2}P\left[\varphi_{-\mathbf{n}}^{(2)}\right] . \tag{13}$$

A *Lüders measurement* (cf. Appendix B) of $A_1(\mathbf{n}) = \sigma_1(\mathbf{n}) \otimes \mathbb{1}_2$ transforms W into $W_L(A_1(\mathbf{n}), W)$ and the mixed states W_1 and W_2 into the mixtures of states $\Gamma(W_1(\mathbf{n}))$ and $\Gamma(W_2(\mathbf{n}))$, respectively, which admit an ignorance interpretation. Hence a measurement of $\sigma_1(\mathbf{n})$ at the subsystem S_1 by the observer B_1 induces the objectification of the observable $\sigma_2(\mathbf{n})$ of the subsystem S_2 (in distance R) and hence the transition from *mixed state* W_2 to the *mixture of states* $\Gamma(W_2(\mathbf{n}))$. Therefore, the question arises whether this "objectification at a distance" can be used by observer B_1 for transmitting a signal to the other observer B_2. The sender B_1 (*Alice*) of the one-bit signal would use the alternative (measurement of σ_1-no measurement) and the receiver B_2 (*Bob*)[1] would receive the message by the alternative mentioned (Fig 2).

The observer B_1 (*Alice*) cannot send a signal to B_2 (*Bob*) by performing a *single* measurement of $\sigma_1(\mathbf{n})$. If B_1 obtains the measurement result $\mu(\sigma_1(\mathbf{n})) = 1$, say, then B_2 will obtain with certainty the result $\mu(\sigma_2(\mathbf{n})) = -1$. However, this result does not contain any useful information about B_1. If B_2 (*Bob*) measures

[1] This terminology is adopted from communication theory.

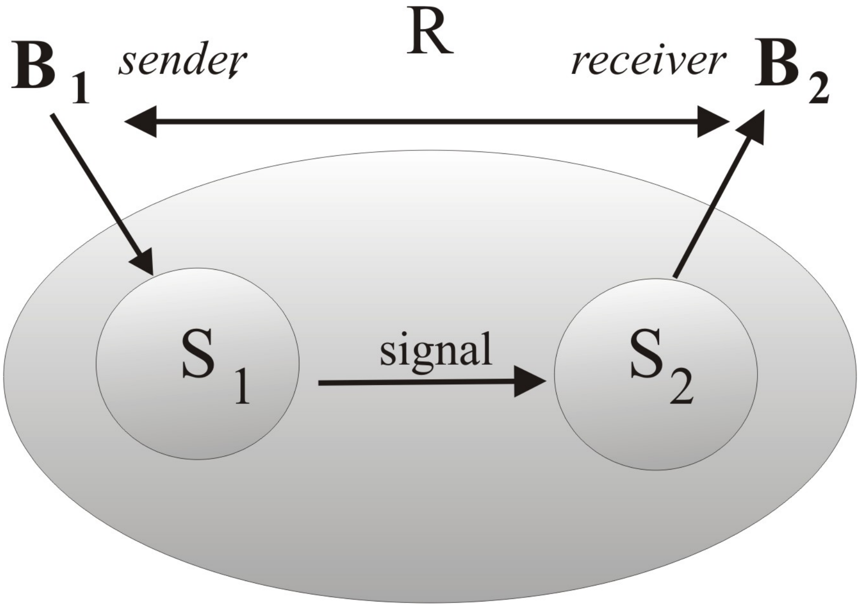

Figure 2: The observer B_1 (sender) tries to send a signal to B_2 (receiver). A $\sigma_1(\mathbf{n})$-measurement by B_1 induces a $\sigma_2(\mathbf{n})$-objectification which must be recognized by B_2.

$\sigma_2(\mathbf{n})$ then he will obtain in any case one of the values ± 1 and it does not matter whether or not B_1 has performed a $\sigma_1(\mathbf{n})$-measurement. It would matter if B_2 were in the position to measure the probabilities $p(+1)$ and $p(-1)$ of the two values $+1$ and -1, but this is not possible by means of a single measurement. It would be possible if, after the $\sigma_1(\mathbf{n})$-measurement, the state of S_2 could be cloned such that an ensemble of identically prepared states could be measured. However, according to an important theorem (Wootters/Zurek, 1982) in quantum mechanics a single state cannot be cloned.

If B_1 performs a series of $\sigma_1(\mathbf{n})$-measurements then she will obtain a sequence of measurement results $\mu\{\sigma_1(\mathbf{n})\} = \pm 1$ with probabilities $p(\pm 1) = \frac{1}{2}$. However, irrespective of the special result ± 1, any $\sigma_1(\mathbf{n})$-measurement (without reading) transforms the state $W = P[\Psi]$ of $S_1 + S_2$ into the *Lüders* mixture $W_L\left(A_1(\mathbf{n}), W\right)$ and the *mixed state* W_2 of S_2 into the *mixture of states* $\Gamma\left(W_2(\mathbf{n})\right)$, which admits an ignorance interpretation. B_1 could try to use this objectification at a distance for sending a signal (of one bit) to B_2. There are two possibilities for B_1: She can either perform a series of $\sigma_1(\mathbf{n})$-measurements or she does not perform any measurement at all. The other observer B_2 (*Bob*) has then to find out whether or not B_1 has made a series of measurements by measuring an observable $A_2\left(\mathbf{n}'\right) = \mathbb{1}_1 \otimes \sigma_2(\mathbf{n}')$ (with $\mathbf{n}' \neq \mathbf{n}$) many times. On the basis of the measurement results obtained in this way *Bob* can calculate expectation values. If the expectation values of $A_2\left(\mathbf{n}'\right)$ with respect to W and $W_L\left(A_1(\mathbf{n}), W\right)$ are different then B_2 can decide whether or not B_1 has made a series of $\sigma_1(\mathbf{n})$-measurements, – and in this way receive a one bit signal from B_1.

3.2 Locality condition and quantum causality

There is an important argument which shows that the two expectation values cannot be distinguished and that consequently quantum correlations cannot be used for the transmission of superluminal signals. The argument is very general and holds in quantum logic (Mittelstaedt, 1983) as well as in quantum field theory (Schlieder, 1969). However, it is based on assumptions which are partly hypothetical and not completely settled. – In the Minkowskian space-time M we consider a quantum system S with state W which refers to the entire space-time (Heisenberg state), and two finite regions $R_1 \subset M$ and $R_2 \subset M$ with a space-like distance, denoted here by $R_1 \approx R_2$. Furthermore, we consider two local observable algebras $\mathcal{A}_1 = \mathcal{A}_1(R_1)$ and $\mathcal{A}_2 = \mathcal{A}_2(R_2)$, the elements of which are measurable in R_1 and R_2, respectively, by two local observers B_1 and B_2 whose world lines are shown in Fig. 3. The causal future of R_1 and R_2 is denoted here by $J^+(R_1)$ and $J^+(R_2)$, respectively.
For two local observables $A_1 \in \mathcal{A}_1$ and $A_2 \in \mathcal{A}_2$ in quantum field theory one usually assumes as an axiom the locality condition ($\mathcal{L}$)

$$\text{If } A_1 \in \mathcal{A}_1(R_1) \text{ and } A_2 \in \mathcal{A}_2(R_2) \text{ and } R_1 \approx R_2, \text{ then } [A_1, A_2] = 0\,. \quad (\mathcal{L})$$

This means that two observables which are measurable in space-time regions with space-like distance commute in quantum mechanics. The justification of this locality condition will be discussed later.
Next, we consider a quantum system S with the state operator W and two observables A_1 and A_2. If the spectral decomposition[2] of A_1 reads $A_1 = \sum_k A_1^k P(A_1^k)$, then a *Lüders measurement* (cf. Appendix B) of A_1 (without reading) transforms the state W of S into the *Lüders mixture*

$$W_L(A_1, W) = \sum_k P(A_1^k)\, W P(A_1^k)\ . \tag{14}$$

The expectation values of the observable A_2 before and after the pre-measurement of A_1 is then given by the expressions

$$\langle A_2, W\rangle = \mathrm{tr}\{A_2 \cdot W\}\ , \quad \langle A_2, W_L\rangle = \mathrm{tr}\{A_2 \cdot W_L\}\ , \tag{15}$$

which are, in general, different. However, if the observables A_1 and A_2 commute, then the expectation values would be equal and vice versa. This is the content of *Lüders' theorem*

$$[A_1, A_2] = 0 \Leftrightarrow \forall W : \langle A_2, W\rangle = \langle A_2, W_L\rangle\ . \quad (T_L)$$

In other words, for commuting observables A_1 and A_2, the expectation value of A_2 does not depend on whether or not A_1 was measured before.
We will now combine the locality condition L with the *Lüders' Theorem* (T_L). If we are given a system S with state W and local observables A_1 and A_2 which

[2] For the sake of simplicity we restrict the considerations here to (sharp) observables with a discrete spectrum.

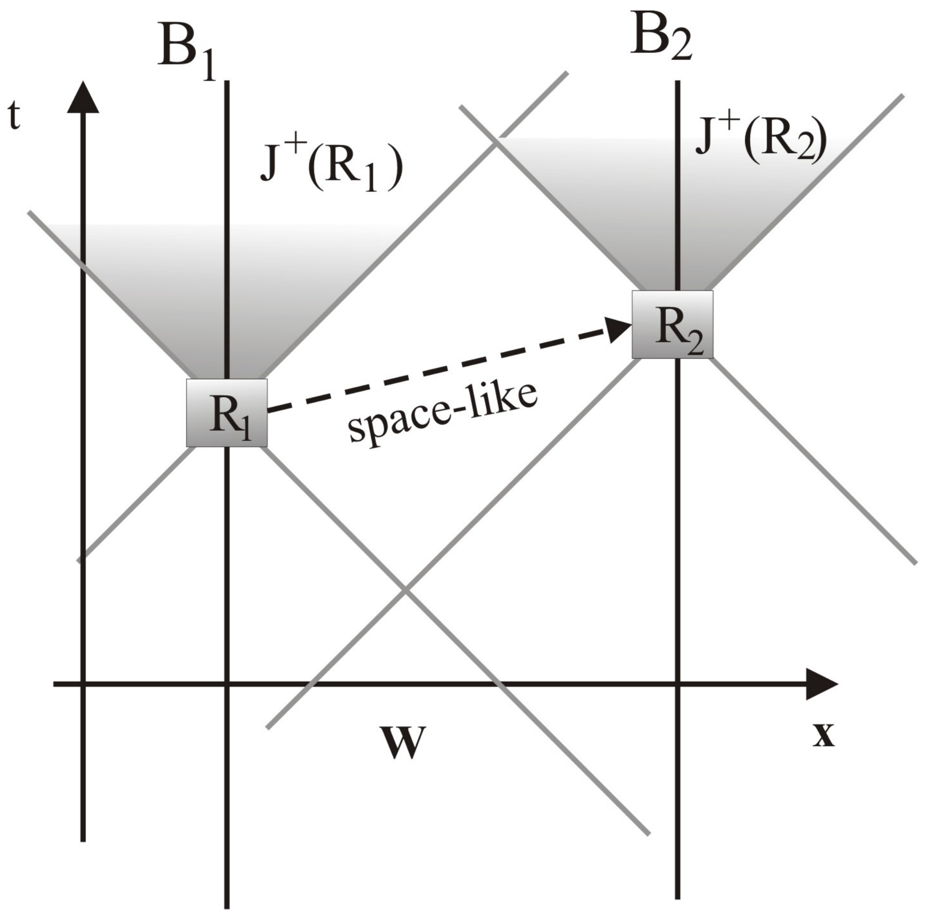

Figure 3: Space-time regions R_1 and R_2 in the Minkowski space with space-like distance, observers B_1 and B_2 and their world lines. $J^+(R_1)$ and $J^+(R_2)$ are the causal futures of R_1 and R_2, respectively.

are measurable in space-time regions R_1 and R_2, respectively, with space-like distance, then the locality condition ($\mathcal{L}$) implies the commutativity of A_1 and A_2. Furthermore, by means of *Lüders' Theorem* (T_L), we find

$$\forall W : \langle A_2, W \rangle = \langle A_2, W_L(A_1, W) \rangle \ . \tag{16}$$

Hence, for regions R_1 and R_2 with space-like distance, an observer B_2 cannot distinguish by measurements of A_2 in R_2 whether or not another observer B_1 has made a *Lüders measurement* of A_1 in R_1. Consequently, for regions R_1 and R_2 with space-like distance an observer B_1 in R_1 cannot send a superluminal signal (with velocity $v_S > c$) to another observer B_2 in R_2 by measuring or not measuring the observable $A_1 \in \mathcal{A}_1(R_1)$. This result is expressed by the *quantum causality condition* (C_Q)

$$R_1 \approx R_2 \Rightarrow \forall W : (\langle A_2, W \rangle = \langle A_2, W_L(A_1, W) \rangle) \ . \tag{C_Q}$$

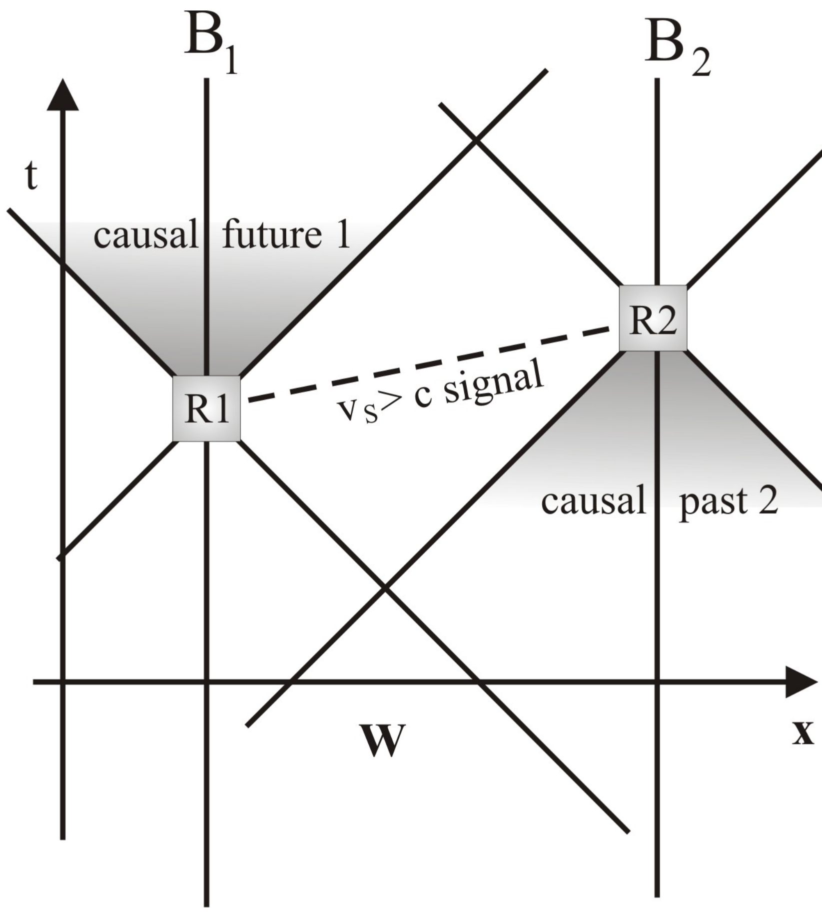

Figure 4: The quantum causality condition prevents the transmission of $(v_S > c)$ signals from R_1 to R_2.

Applied to the EPR- communication problem we find that B_1 (*Alice*) cannot send a superluminal signal to B_2 (*Bob*) by measuring (or not) the spin-observable $\sigma_1(\mathbf{n})$ (Fig. 4).
This no-go theorem for superluminal signals that are based on EPR correlations is often called the "no-signalling theorem" and considered as a fundamental principle that excludes superluminal signals of the kind mentioned (Redhead/Riviere 1997 and Redhead 1999). However, there are still some problems of the justification of this "theorem". We will discuss these questions in the following subsection 3.3.

3.3 Locality and superluminality

The described way of reasoning against superluminal signals is based on the locality condition

($\mathcal{L}$) If $A_1 \in \mathcal{A}_1\,(R_1)$ and $A_2 \in \mathcal{A}_2\,(R_2)$ and $R_1 \approx R_2$, then $[A_1, A_2] = 0$

There are two kinds of justifications for this condition ($\mathcal{L}$). The first one is based on quantum field theory, the second one, which will not be considered here, on

relativistic quantum logic (Mittelstaedt/Stachow, 1983). The two ways of reasoning are, however, not independent from each other and, what is more important, not independent of the problem discussed here. The first argument, which was formulated by Schlieder (1969), justifies the condition ($\mathcal{L}$) by its consequences for quantum measurements, in particular by the exclusion of superluminal signals: Since without the locality condition ($\mathcal{L}$) the two observables were – in general – incommensurable, *Lüders' theorem* could not be applied to the EPR situation. Hence, the two expectation values $\langle A_2, W\rangle$ and $\langle A_2, W_L(A_1, W)\rangle$ would be different and superluminal signals between regions R_1 and R_2 with space-like distances would become possible. Since signals of this kind are not in accordance with special relativity, the locality condition ($\mathcal{L}$) must be presupposed in order to avoid a violation of relativity and quantum causality (C_Q).
For *Lüders measurements* M_L, which fulfil the *Lüders theorem* (T_L), the locality condition ($\mathcal{L}$) implies quantum causality (C_Q).

$$(T_L) \Rightarrow ((\mathcal{L}) \Rightarrow (\mathrm{C_Q})) \ . \tag{17}$$

Indeed, from ($\mathcal{L}$), (T_L), and (C_Q), we get

$$R_1 \approx R_2 \overset{(\mathcal{L})}{\Rightarrow} [A_1, A_2] = 0 \overset{(T_L)}{\Rightarrow} \forall W: \ (\langle A_2, W\rangle = \langle A_2, W_L(A_1, W)\rangle) \ . \tag{18}$$

If there were an independent proof of ($\mathcal{L}$) from first principles, then the implication (17) would indeed lead to a justification of (C_Q) – and thus to an exclusion of ($v_S > c$) quantum signals. Here, however, the locality condition is not derived from first principles but justified by its consequence to exclude superluminal quantum signals, i e. by the implication

$$(T_L) \Rightarrow ((\mathrm{C_Q}) \Rightarrow (\mathcal{L})) \ . \tag{19}$$

Using again Eqs. ($\mathcal{L}$), (T_L) and (C_Q), the implication (19) can easily be obtained by

$$R_1 \approx R_2 \overset{(C_Q)}{\Rightarrow} \forall W: \ (\langle A_2, W\rangle = \langle A_2, W_L(A_1, W)\rangle) \overset{(T_L)}{\Rightarrow} [A_1, A_2] = 0 \, . \tag{20}$$

Hence, taking together Eqs. (17) and (19), it follows that under the assumption of *Lüders measurements* the principles ($\mathcal{L}$) and (C_Q) are equivalent (cf. also de Muynck 1984). This means that the present way of reasoning for quantum causality (C_Q) must not be considered as a proof of this principle but rather as a *petitio principii*: On the one hand locality implies quantum causality but on the other hand quantum causality is used as an argument for justifying the locality condition. Consequently, entanglement-induced superluminal quantum signals cannot be excluded in this way (Mittelstaedt, 1998a). – It is a very remarkable result that the equivalence of ($\mathcal{L}$) and (C_Q), which is demonstrated here for observables corresponding to self-adjoint operators (or projection valued measures), is not restricted to this kind of (sharp) observables. Indeed, in a recent paper (Busch, 1999) the same equivalence was found to be also valid for unsharp

observables represented by positive operator valued (POV) measures. Hence, our argument holds very general.
Summarizing these results we find that the locality principle ($\mathcal{L}$) is not suited to exclude superluminal EPR-signals and to justify quantum causality (C_Q), since the locality principle itself is justified by quantum causality. For this reason, the question of whether EPR-correlations can be used for superluminal signals, cannot be answered in this way.

4 Conclusion

A careful analysis of the EPR-Gedankenexperiment shows that quantum mechanics is not incomplete but nonlocal in a very specific weak sense. This non-locality is based on the quantum mechanical entanglement of the subsystems which are not strictly separable. The weak non-locality allows for some kind of *objectification* at a distance but presumably not for an *action* at a distance in the classical sense. Hence, entanglement-induced instantaneous and thus superluminal signals, which would violate Einstein causality, are not possible at first glance.
However, a more detailed investigation of this no-go argument for superluminal signals – the "no - signalling theorem" – shows that it is based on the micro-causality axiom of relativistic quantum field theory which is usually justified by the exclusion of superluminal signals. Hence, the no-signalling theorem and the axiom of micro-causality are equivalent. Consequently, we are confronted here with a *petitio principii*, and the question whether there are entanglement-induced superluminal signals cannot be answered in this way. It is important to note that this argument holds for the most general kind of observables in quantum mechanics.

A Reduced mixed states

If a compound system $S = S_1 + S_2$ is prepared in a pure state

$$\Psi(S) = \frac{1}{\sqrt{2}}\left(\varphi_{\mathbf{n}}^{(1)} \otimes \varphi_{-\mathbf{n}}^{(2)} - \varphi_{-\mathbf{n}}^{(1)} \otimes \varphi_{\mathbf{n}}^{(2)}\right) \tag{21}$$

with orthonormal eigenstates $\varphi_{\mathbf{n}}^{(1)}$ and $\varphi_{-\mathbf{n}}^{(2)}$, then the *reduced mixed states* of the subsystems S_1 and S_2 are

$$W_1 = \frac{1}{2}P\left[\varphi_{\mathbf{n}}^{(1)}\right] + \frac{1}{2}P\left[\varphi_{-\mathbf{n}}^{(1)}\right] = \frac{1}{2}\mathbb{1}_1 \, , \tag{22}$$

$$W_2 = \frac{1}{2}P\left[\varphi_{\mathbf{n}}^{(2)}\right] + \frac{1}{2}P\left[\varphi_{-\mathbf{n}}^{(2)}\right] = \frac{1}{2}\mathbb{1}_2 \, , \tag{23}$$

where by $P[\varphi]$ we denote the projection operator onto φ.

B Lüders measurement

Let A be a discrete and non-degenerate observable $A = \sum_i A_i \varphi_i$ with eigenvalues A_i and eigenstates φ_i such that we have the eigenvalue equation $A\varphi_i = A_i \varphi_i$. The simplest kind of measurement of the observable A in a given state φ that is not an eigenstate, transforms the preparation φ into the mixture

$$W(\varphi, A) = \sum_i |(\varphi_i, \varphi)|^2 P[\varphi_i] \ . \tag{24}$$

If A is degenerate, and can be expressed as $A = \sum_i A_i P(A_i) = \sum_i A_i \sum_k P\left[\varphi_i^k\right]$, where $P(A_i)$ is the projection operator that projects onto the subspace which belongs to the eigenvalue A_i , then the measurement of A in the state φ transforms the preparation φ into the mixture

$$W_L(\varphi, A) = \sum_i (\varphi, P(A_i)\varphi) P(A_i) \ . \tag{25}$$

This mixed state is called "*Lüders mixture*" and the measurement that leads to a Lüders mixture is called *"Lüders measurement"*.

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