On Interdependent Preferences and Sequential Structures in Rent-Seeking Contests

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Preface

This dissertation draws on research I undertook during the time in which I held a scholarship of the Deutsche Forschungsgemeinschaft (DFG) at the Graduiertenkolleg "Allokationstheorie, Wirtschaftspolitik und kollektive Entscheidungen", and later while I was a teaching and research assistant at the chair "Mikroökonomik" at the Technische Universität Dortmund. The present thesis has strongly been influenced by and profited from discussions with professors and fellow students of the Graduiertenkolleg, as well as presentations as various national and international conferences. I am very grateful to all supported my work in that way. Financial support by the Deutsche Forschungsgemeinschaft is gratefully acknowledged.

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Contents

List of Symbols

a	Effort level of players with independent preferences
b	Effort level of players with interdependent preferences
c	Relative effort level of players with independent preferences
d	Marginal cost of effort
$f_i(x)$	Impact function
g	Welfare gain of a subgame with the favorite moving early in contrast to a simultaneous move game
i, j, k	Index for a player
m	Size of a subgroup of all players, especially the number of players with independent preferences
n	Number of players
p_i	Winning probability of player i
$q := \frac{b-a}{a}$	Relative additional effort of players with interdependent preferences
q_1, q_2, q_3	Auxiliary functions
r	Technology parameter
$\overline{r}_I$	Upper limit for the value of r for which equilibrium type I can be observed
$\underline{r}_{III}$	Lower limit for the value of r for which equilibrium type III can be observed
s_1, s_2, s_3	Parameter of a special preference function
t	Point in time
u	Utility function, depending on λ
x_i	Effort of player i
$\hat{x}$	Effort of an underdog
$\overline{x}$	Highest effort chosen in a contest
$\mathbf{x}$	Vector of efforts
$\mathbf{x}_{-i}$	Vector of the efforts of the opponents of player i
y	Effort level of a mutant player
A	Total aggregate effort of all players
A_{-1}	Total aggregate effort of all players but player 1
A_{-i}	Total aggregate effort of all players but player 1 and player i, $i \neq 1$
B	$B(Z, \lambda) := b^r(Z, \lambda)$
C	$C := a^r$
iE	Subgame where player i moves earlier than her opponents (who are assumed to choose simultaneously)
F_i	Preference function of player i
I	Set of players with independent preferences
J	Set of players with interdependent preferences
iL	Subgame where player i moves later than her opponents (who are assumed to choose simultaneously)

M	Set of a subgroup of all contestants
$N_{sym}(\Gamma)$	Set of symmetric Nash-equilibria of the game Γ
Pr	Subgame where the early mover preeempts all of her opponents
R_I	Set of all technology parameters with increasing returns to scale, for which equilibrium type I can be observed
S	Subgame with simultaneous moves
T	$T := (n-2)^2V^2 + 4(n-1)x_1V$
V; V_i	Rent, prize to win in the contest; personal prize for player i
W^i	Welfare level of subgame i
Z	$Z := ma^r + (n-m-1)b^r$
α_i	Weight that player i puts on relative payoff compared to material payoff
β_1, β_2	Parameter of the linear preference function.
γ	$\gamma := \frac{\beta_2}{\beta_1+\beta_2}$
θ	Relative valuation of the prize for the favorite
κ	Number of rounds in a tournament
λ_i	Preference parameter of player i
ν	Constant factor for adjusting winning probabilities
ξ	Parameter of a special discontinuous preference function
π_i	Material payoff of player i
ρ_i	Relative payoff function of player i
σ_i	Parameter of the impact function of player i
τ_i	Probability that player i moves early in a mixed strategy equilibrium
ϕ	$\phi(n) := \frac{4n-7+\sqrt{16n^2-40n+33}}{2(n-1)}$ Upper limit to which an underdog prefers not to move late
φ	$\varphi(z) := \frac{z^r}{A}V - z$ Payoff function given a fixed aggregate effort
ψ	Parameter representing valuations with identical payoffs in different subgames
$\omega(y)$	Loss function of a mutant, depending on her effort y
Δ	Difference between payoff levels of players with independent and interdependent preferences
Ξ	Additional payoff of a favorite when switching from $1L$ to $1E$
Ω	$\Omega := \sum_{i=2}^{n} \frac{1}{V_i}$
$\mathcal{T}$	Set of feasible timing choices
$\mathcal{V}$	Set of feasible valuations
$\mathcal{X}$	Set of feasible effort choices

Chapter 1

Introduction

> ”In many market-oriented economies, government restrictions upon economic activity are pervasive facts of life. These restrictions give rise to rents in a variety of forms, and people often compete for these rents.” Anne O. Krueger, 1974

1.1 Motivation

In many situations, we can observe that several individuals or institutions compete for an exogenously given asset. In such situations they spend money or other valuable resources to increase their winning probability, yet without exerting any influence on the value of the asset. This asset can take many different forms. While the obvious example of an asset is a fixed amount of money, we can also think of other examples: a job position, an office (e.g. in a presidential election race), a patent, a license, a territorium (in a war), a right (in some lawsuits), or a title (e.g. in sport contests). We call such situations rent-seeking contests.

As already addressed, election campaigns are a standard example of contests. We will now use one very prominent campaign as an example to demonstrate the relevance of contests in economic and political reality. In November 2008, the election for the president of the United States, presumably the most powerful office in the world, was held. Of course, this office is a very valuable asset, and candidates are willing to pay in order to increase their probability of winning this election. In such an election, it would be far more efficient to simply inform voters about the political stance of each candidate, and then elect the one with values matching those of the majority of the electorate. However, each candidate has an interest in convincing voters to vote for him. Consequently, a huge amount of resources is dissipated in the race between the candidates for those votes that finally decide the election.

In the election campaign 2008, this race started long before the election itself. The final decision about which candidates will be admitted to the actual election in November has been made in the end of August for Barack Obama (by the Democratic Party) and in the beginning of September for John McCain (by the Republican Party). Yet, in advance, this attractive position initiated another contest, namely the contest for the candidacy, the so-called primary elections. Every candidate in these primary elections entered the race with the one goal: to become the next president of the United States.

While, by January 31 2008, there were still more than nine months to go to the actual election, the top fifteen different candidates of the two major parties had spent a total amount of not less than 546 millions US-$ (CNN 2008 a,b).[1] The chairman of the Federal Election Commission, Michael Toner, announced at the beginning of 2007 that he expected the total expenditures on this year's presidential election to exceed one billion US-$.[2] This example demonstrates how important a contest for a single position can be. However, it is questionable whether or not these expenditures and the campaign itself improve the candidates' abilities and qualifications for the office of the US president. Millions of dollars are spent with the sole aim of influencing the allocation of the asset "presidential office".

In this work, we focus on two different aspects of rent-seeking contests, namely interdependent preferences and sequential structures. With respect to interdependent preferences, we will restrict parts of our analysis to considering the impact of spite. From a theoretical point of view, this is the most interesting form of interdependent preferences. While it is hard to verify the presence of spite in an election campaign, some aspects of the present presidential campaign indicate that candidates gain advantages from spiteful behavior. In 2008, the most intensive duel between the candidates was fought between Barack Obama, who became the designated candidate of the Democratic Party and finally was elected for the President of the United Stated, and Hillary Clinton. Although both candidates represent the same party and thus represent similar ideas, there were frequent news stories about how one of the candidates (or their staff members) had attacked the other in order to damage his or her credibility. In this intra-party election campaign, we find several examples that show how the candidates try to enhance their winning chances through spiteful behavior. For example, Barack Obama reproached Hillary Clinton that she would not care about social policy (FOX News, 2007, Washington Post, 2007) and lied concerning her attitude to the NAFTA trade agreement (Die Welt, 2008). Clinton accused Obama of being unreliable in energy policy (NY Daily News, 2008) or foreign affairs (Focus, 2007). Often, it was not the political issues themselves that attracted attention, but rather, the sharpness of their personal attacks (Focus, 2007, Die Zeit, 2008). Here, one could see that in the pre-election campaign it was not only the aim of a candidate to enhance one's own standing, but also to damage the image of the opponent. We will allude to the problem of interdependent preferences in contests in a more general framework in Chapters 3 to 5.

The second focus of this work lies on sequential structures. This part analyzes how the timing of decisions changes far the incentives of a player to choose a certain strategy. Think of a contest, like an election campaign, where two candidates present their

[1] This number does not include the cost incurred by candidates of smaller parties and independent candidates.

[2] As a comparison: total expenditures from the three recent US-president election campaigns are given by approximately 575 million US$ in 1996, 650 million US-$ in 2000 and a billion US-$ in 2004. (CNN, 2008c)

manifesto one after another, in such a way that the second candidate can react to the strategy of the first. Since the first player has already committed to her manifesto, she cannot strategically react to the second candidates' program, while the second candidate of course can react to her opponent's manifesto. This decision situation is different from one, where both candidates have to publish their manifesto simultaneously. For this reason, it is advantageous for the opponents to consider the effects of this situation, when they choose the date to publish their manifesto. For the addressed example of the U.S. presidential election, when looking at the dates when the two most promising candidates presented their manifesto, it is striking that in the last six decades, beginning with the first post-war election campaign in 1948 up to this year's presidential race, it was always the candidate from the opposition who chose the earlier presentation date. The candidate who represented the same party as the president in office always chose the later timing (Morgan, 2003). As we will see, such a sequential structure is a rational result for a contest between two candidates of different strength (cf. Leininger, 1993), but not necessarily for a contest with more than two candidates. Situations where more than two players join a sequential contest are the main issue of the Chapters 6 to 8.

1.2 The History of Contest Theory

Adam Smith (1776) was the first person to scientifically study economic problems. He came to the result that in markets the individually rational behavior of every participant colludes to a welfare maximum, driven by the famous invisible hand. This result, however, was only valid for perfect price competition. It was Cournot (1838) who first showed that in a market competition where competitors do not decide about the price but about the quantity, profit maximization by duopolists will induce a social loss. Applying the equilibrium concept proposed by Nash (1951), that also drives Cournots result, we find a large variety of games, where individual rationality maximizes neither welfare nor joint output. In some games - especially the famous prisoners' dilemma put forward by Luce and Raiffa (1957) - individual rationality even fails to reach Pareto-efficiency, i.e. it would be possible in the equilibrium outcome to increase one agent's utility without decreasing her opponents' utility. In several papers, Tullock (1967, 1971, 1980) introduced the idea that also competition for a fixed amount of money leads to an equilibrium outcome that is not pareto-efficient if it induces wasteful efforts. The latter game is well-known as "rent-seeking contest".

The term "rent-seeking" has first been put forward by Krueger (1974), but the idea of rent-seeking as a welfare problem is some years older. In 1967, Gordon Tullock explored that the possibility of tariff policy unintentionally creates incentives for firms in the affected industry to use up resources for political lobbying. The scenario he examined was the introduction of a tariff policy that is being discussed in the political process. The tariff policy in question would be a mere redistribution without any productive contribution. However, Tullock showed that such a situation creates incentives for firms in the concerned industry to use up resources for political lobbying in order to either foster or prevent the tariff depending on whether the firms gains or loses from its presence. To sum up, in a rent-seeking contest, rational agents use up valuable resources for a contest that cannot generate welfare.

Technically speaking, rent-seeking characterizes a situation, where two or more persons can perform a certain action in order to receive a positive expected share of a **fixed** amount of an asset that is distributed. The actions that are performed with the aim to receive the rent are not productive.[3] Yet, the problem is that, despite of the lack of productivity, rent-seeking games, in general, create incentives to dissipate scarce resources that could have been better spent in (more) productive ways.

As already mentioned, Tullock first addressed rent-seeking as a problem, still without using the term "rent-seeking". In his early work where he discusses the effects of a potential implementation of a tariff policy, he introduces two further similar examples, namely a competition for a monopoly position and the prevention of theft.[4] He showed that, also in these situations, there are incentives for different parties to invest in a contest for a payoff of fixed size.[5] Tullock (1967, 1971) argues that whenever some kind of welfare transfer is achievable that valuable resources are spent by several parties in order to either obtain or prevent the transfer. In a further step, Anne Krueger (1974) puts forward a formal model for the problem of competition for a tariff policy, applying the problem to the specific situations of India and Turkey. With her model, she is the first to analytically establish the welfare loss of rent-seeking. Besides, she also introduced the term "rent-seeking" into the literature.[6]

The scientific analysis of the problem of rent-seeking entered a new era when Tullock (1980) developed a handy and simply-structured model of a rent-seeking contest, the so-called Tullock-contest. This model is an important basis for a broad range of literature (see Lockard and Tullock, 2001, for a compendium). This model will be the basis of all the objective functions looked upon in this work. His major intention when introducing this function is to demonstrate how rent-seeking generates dissipation. He shows the first-order-conditions for some numerical examples to show that the amount of efforts spent in a rent-seeking contest can be quite substantial. Besides the discussion of existence of equilibria, his main issue is that resources are shown to be wasted in this setup. Obviously, waste occurs whenever the chosen effort of some player is positive.

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[3]In some examples like e.g. R&D-competitions effort can indeed be productive. Another example for productive contests are sports contests, where the effort increases the utility of the spectators. However, we will restrict our analysis to the assumption of non-productive efforts.

[4]Tullock analyzes a game between a thief and a potential victim, where the one invests in burgling tools and the other one in respective security devices.

[5]Of course, Tullock pays tribute to the fact that the mere existence of a monopoly is detrimental from a social point of view. However, he reckons that the size of the so-called Harberger triangle is neglectable in contrast to the loss created by the rent-seeking activity.

[6]Krueger (1974), p. 291.

1.3 An Introduction to Interdependent Preferences

For decades, it has been standard in economic literature to assume that agents are *homo oeconomicus*. This means that the players perfectly rationally maximize their own payoff, without caring about the payoffs of other players. But looking into real life, we notice that people indeed do care about what other people receive, in an altrusitic as well as in a spiteful way. We call preferences that do not only depend on the own payoff of the respective player but also on the payoffs of others *interdependent preferences*.

The role of interdependent preferences, notably altruism, in economic problems was first addressed by Mill (1887): "On the happiness of others, on the improvement of mankind [...] followed not as a means, but as itself an ideal [...] they find happiness by the way."[7] Younger approaches in analyzing interdependent preferences go back to Hamilton (1971), an evolutionary biologist, whose work found first economic attention by Schaffer (1988), and Gary Becker (1974, 1976), who contributed the concepts of social interactions (1974) and altruism (1976) to economic literature. Both Hamilton and Becker emphasized the role, evolution played in revealing interdependent preferences as stable. The branch of economic literature, where interdependent preferences have found most application, is indeed evolutionary game theory. Further fields of economics where the role of interdependent preferences has been analyzed are educational subsidies (Lommerud, 1989), wealth taxes (Konrad, 1990) or risk decision (Konrad and Lommerud, 1993). Hopkins and Kornienko (2004) provide an analysis of a game between status-oriented preferences to show that welfare is lower in the presence of negatively interdependent preferences. Finally, Sobel (2005) gives an comprehensive overview over the role of interdependent preferences in economic contexts and literature.

The first empirical evidence for interdependent preferences was provided in Duesenberry (1949). He explored the role of status-seeking for the theory of savings. In this work, he showed "that the interdependence of consumer preferences affects the choice between consumption and saving [...]". Besides Duesenberry's empirical basis, the papers by Bush (1994a,b) and Kapteyn et al. (1997) provide empirical support for the economic relevance of interdependent preferences that is based on field data. Moreover, experimental evidence has continuously been questioning the plausibility of independent pay-off maximization over the past decade. For instance, Levine (1998) takes results from ultimatum experiments and the final round of a centipede experiment in order to pin down the distribution of altruistic and spiteful preferences. This distribution then allows him to explain results, e.g. in public good contribution games or in early rounds of the centipede game. Other examples for the presence of interdependent preferences can be found in Davis and Holt (1993). Introspection commonly yields the same result.

A very important issue in the analysis of interdependent preferences is the question of whether interdependent preferences yield a higher payoff than independent preferences. The works of Bester and Güth (1998) and Possajennikov (2000) show that in a certain class of games altruism can be evolutionarily stable. Beyond that, Kockesen et al. (2000a) showed that spiteful behavior can also be advantageous in the sense that

[7] Mill (1887), p. 142, quoted via Seckler (1975).

maximization of these preferences provides the player a relatively higher payoff than her opponents. While, in a biological context, this means a higher evolutionary fitness, in an economic context, this represents a higher market share. Both interpretations demonstrate that the spiteful player has a better chance to survive than her opponents, and thus this behavior is evolutionarily more advantageous. For contests the evolutionary stability of spite was first addressed by Hehenkamp et al. (2004) and Leininger (2003). Kockesen et al. (2000b) interpret this evolutionary advantage as a strategic advantage such that a player who consciously decides to choose a negative interdependent preference (e.g. by employing a spiteful manager) yields a higher payoff than her opponents with independent preferences. The work at hand shows that this also holds for rent-seeking contests. Chapter 3 of our analysis establishes this strategic advantage for spiteful preferences, while Chapter 4 sheds some light on the limits of this advantage and specifies the parameter ranges for successful interdependent preferences. Overly spiteful behavior can also be disadvantageous. Finally, we take a short glance at the question, which preferences prevail in an evolutionary process.

1.4 Sequential Structures

In this section, we want to give a short introduction to the literature on sequential structures in contests. In a sequential contest, players decide upon their effort at different times, such that players moving late already know the choice of the opponents that move early. This creates the opportunity for the early mover to commit to a certain effort level and for the late mover to react to this commitment, which introduces an additional strategic component into the game. In contrast to repeated contests, in a sequential game, there is only one prize to be distributed while a repeated contest offers a prize in every single round.[8]

The idea of sequential structures in a market game stems from the analysis of a duopoly by Stackelberg (1934). He analyzed sequential games in a Cournot and a Bertrand duopoly. In a sequential Cournot duopoly, the early moving player receives a higher payoff than in a simultaneous move game while the late mover receives a lower payoff.

In contest theory, sequential structures have been analyzed since the 1980's. In a seminal work, Dixit (1987) analyzed the impact of an exogenous order of moves on the strategic effects a sequential structure introduces to a contest between two players. He denotes the player who has the higher winning probability in Nash-equilibrium as the favorite. Dixit now finds that the favorite's equilibrium effort, when she is the first mover, is higher than her equilibrium effort in a simultaneous move game. Yet, Baik and Shogren (1992) showed that, in a subgame-perfect equilibrium of a model with an endogenized order of moves, the stronger player will wait until the weaker player has chosen her action in the early stage. If the sequential order is determined endogenously, both players will

[8]For literature on repeated contests see e.g. Amegashie (2005), Linster (1994), Münster (2004), Netzer and Wiermann (2006).

expend less effort than in the simultaneous game. Leininger (1993) derived this result for rent-seeking contests with a Tullock-payoff function, explicitly formulating effort choices and equilibrium payoffs both in the simultaneous and the sequential game. In how far the analyses of Baik and Shogren and Leininger can be converted from one to another, has been shown by Nitzan (1994). Since Leininger's work focuses on the special case of rent-seeking contests while Baik and Shogren keep their results general, we will use the article by Leininger as a basis for our analysis of the n-player-case in rent-seeking contests with constant returns to scale. Unfortunately, we have to state that the analysis of a fully endogenized order of moves is tractable only for the case of three players. However, we find that even with this simple modification from the two-player- to the three-player-case, results concerning equilibria and welfare change. Leininger (1993) found that, for **any** combination of heterogenous players, the stronger one will always choose late, and this will be welfare maximizing among the feasible subgames and thus an improvement over the result of Tullock's simultaneous move game. As we will see, these results cannot be extended to the three-player-case to their full extent.

Further extensions of the analysis of sequential rent-seeking for the two-player-case have been put forward by Morgan (2003) and Yildirim (2005). In a sequential contest, Morgan allows for uncertainty about whether a player will have a high or a low valuation at the moment when she decides upon her decision timing. He shows that, in equilibrium, players always move sequentially, although this need not necessary mean that the stronger player will move late. Morgan can replicate the result that both players have a higher expected payoff from the sequential game compared to the simultaneous decision. However, the total expected amount of effort spent in the contest will be higher than in the simultaneous move game.[9] Yildirim (2005) concentrates on the question of how equilibria change if players are allowed to increase their effort bids after their first bid. He establishes the existence of multiple equilibria, among them one equilibrium in the simple simultaneous game and another one that is consistent with a Stackelberg-equilibrium. But since our research aims at an extension of the player set, let us now throw a glance at works that deal with more than two players.

The first approach to analyze contests with more than two players in sequential structure has been proposed by Gradstein and Konrad (1999) who seek for an effort-maximizing contest structure under the assumption of homogenous players. They find that a contest designer can maximize effort with a simultaneous move contest for decreasing returns to scale, while, for increasing returns to scale, the contests (with n players) should be played in κ rounds of two-player contests, where only the winner enters the next round and $\kappa = \log_2 n$. Yet, Gradstein and Konrad assume simultaneous choice in each subcontest. A first attempt to analyze three-player-contests with truly sequential structures is proposed by Glazer and Hassin (2000) who analyzed the game where three homogenous players compete for a rent while deciding upon effort in three different points of time, one after another. The aggregate effort is, in this case, higher

[9] At the first glance this might seem contradictory. However, the higher costs of the contest are outweighed by a higher probability of assigning the rent to the player with higher valuation, which means a higher expected return of the contest.

than in the simultaneous game, and thus welfare is lower.

Finally, Konrad and Leininger (2007) proposed a sequential analysis of an n-player all-pay-auction.[10] They find that in a sequential all-pay-auction with endogenous moves there exists an equilibrium with a pareto-efficient outcome. No wasteful effort will be exerted. In this equilibrium each but the strongest player will decide in the first stage not to make any positive effort in the subgame-perfect equilibrium. The favorite will then get the rent for free in the second stage, reducing the dissipation to zero, which is surprisingly good news from a welfare point of view.

1.5 Outlook

Let us now sketch our proceeding. Before starting the main part of our work, we will first present the basic model and its most important features. The main part of this work can be split in two parts. The first is concerned with interdependent preferences, while the second concentrates on sequential structures.

In the first part, we deal with interdependent preferences. We begin by putting up a model where players of two different types are faced with a rent-seeking contest. While the first type (*individualists*) is only interested in maximizing absolute payoff, the second type (*status-seekers*) also cares about the relative payoff. In this scenario, we analyze participation conditions for both types and put up the question of which combinations of parameters provide the status-seekers a higher material payoff. In a further step, we relax the assumptions on the set of preference parameter and allow for altruistic and a much broader range of spiteful preferences, distinguishing between weakly spiteful preferences that are less spiteful than relative payoff maximization, and strongly spiteful preferences. We analyze the incentives of these players to spend positive effort. The main focus of this chapter of our work is to outline the conditions under which more spiteful players experience a strategic advantage. Finally, the question arises, whether there is an optimal preference level. It was Schaffer (1989) who first draws the relation between evolutionarily stable preferences and relative payoff maximization. Later, Eaton and Eswaran (2003) showed for rent-seeking contests with constant returns to scale that maximizing the relative payoff is an evolutionarily stable preference. We will generalize the latter for a much broader range of rent-seeking contests, including those with slightly increasing returns to scale.

The second part concentrates on multi-player-contests with an endogenous order of moves. The articles by Baik and Shogren (1992) and Leininger (1993) show that for two-player-contests, the stronger player will always decide to choose late, while the weaker one will decide to choose early. Leininger, furthermore, shows that this is the optimal order of moves if the aim is to reduce the effort spent by the contestants. The second part of our work seeks to generalize these results to a larger number of players. In the first chapter on sequential structures, we assume a two-stage-game in which **three**

[10] To see the relationship between all-pay-auctions and Tullock-contests with constant returns to scale, see Section 2.2.

players can decide at the first stage whether to decide upon their effort early or late, and, in the second stage, they play a sequential (or simultaneous) rent-seeking contest depending on the chosen timings of the first stage. We provide efforts and payoffs of every feasible subgame and compare them. In the next step, we analyze a game where one favorite faces $(n-1)$ equal opponents. These opponents are assumed to be weaker, in the sense that they have a lower valuation for the prize. In this setting, we look for the endogenous order of moves for two different cases: one with a first-mover-right for the strongest player and one without. This enables us to evaluate, from a welfare point of view, whether such a first-mover-right should be granted. Finally, we analyze a general n-player-contest and ask under which conditions players have an incentive to deviate from a contest with simultaneous moves. As a result we will be able to state a general condition under which players will decide upon a simultaneous move contest in a subgame-perfect equilibrium.

Chapter 2

The Theory of Contests

This chapter gives an introduction to the technical framework of this work. In the prior chapter, we mentioned that Tullock (1980) has advanced the theory of rent-seeking by introducing a handy model. In this model, he described a contest for a rent of fixed size V, where a player i exerts a non-negative effort x_i in order to receive a non-negative payoff. The payoff function has the following form:[1]

$$\pi_i(\mathbf{x}) = \begin{cases} \frac{x_i^r}{\sum_{j=1}^n x_j^r} V - x_i & \text{if } \exists j = 1, ..., n : x_j > 0 \\ \frac{1}{n} & \text{if } x_j = 0, \forall j = 1, ..., n, \end{cases} \tag{2.1}$$

with $\mathbf{x} = (x_1, ..., x_n)$. The parameter r represents the returns to scale of the underlying contest technology and n denotes the number of contestants. Typically, we assume that the term $\frac{x_i^r}{\sum_{j=1}^n x_j^r}$ depicts the probability that player i wins the total amount of the prize V. Therewith, we implicitly assume that a player maximizing (2.1) is risk-neutral. This term can also be interpreted as displaying the share of the prize that player i receives depending on her effort. Yet, although there is no difference from an analytical point of view, we will restrict the discussion of our results to the interpretation of this term as a winning probability.

Tullock calculates the equilibrium and finds that the first-order-condition is given by: $x_i = r\frac{n-1}{n^2}V$. This effort will be chosen in the Nash-equilibrium of a contest between homogenous players if the second-order-condition is fulfilled, which holds for $r < \frac{n}{n-1}$ (cf. Perez-Castrillo and Verdier, 1992). The aggregate effort is given by $r\frac{n-1}{n}V$ and, from a welfare point of view, these efforts are unproductive waste. In the remainder of this section, we will allude to some features that can occur in rent-seeking contest in different settings.

We should, furthermore, point out that our analysis uses the playing-the-field assumption, which is common in contest theory and has implications mostly for evolutionary questions. "Playing-the-field" means that, if a population of players is observed playing

[1] We extended the original model such that the payoff is also defined, if no player exerts a positive. While it is conceivable, that if no player applies for the rent, no player will receive it, we assume that in this case the winning probability is equally distributed among the players.

a game, then all players participate in the game together. In contrast to analyses where repeatedly e.g. two players are chosen out of a population to play a game, under the playing-the-field assumption evolutionary stability yields equivalent outcomes regardless of whether evolution works at strategy or at preference level for a large variety of games (see Eaton and Eswaran, 2003). We will show in Chapter 5 that this also holds for all rent-seeking contests with decreasing and for some with increasing returns to scale.

2.1 Axiomatic Foundation

While this work concentrates on the Tullock-contest, for which the winning probability has a logit form and the impact function is a power function, we should, nevertheless, stress that the theory of contests goes beyond this easily tractable model. An extensive overview of the field of contests in general is given by Konrad (2007). Let us start our discussion by referring to models where not only the assumption that the contest success function is a power function is dropped, but also the assumption of the logit form, namely the works of Dixit (1987) and Malueg and Yates (2004).[2] Both these analyses look only at contests between two players. Dixit's work focuses on a game with an exogenous structure where one player moves earlier than the other. His result for the general case is that, for a positive cross derivative of her winning probability, a player will exert higher effort than in a simultaneous-move Nash-equilibrium.[3]

Malueg and Yates assumed the existence of an interior solution of the simultaneous move game and found some additional results concerning the comparative statics. In particular, they pointed out how the influence of valuations on the effort levels, the winning probabilities and the final payoff depend on the cross derivative of the winning probabilities. All of these results have been previously adapted to Tullock-contests (see e.g. Baik, 1994, Nti, 1999, Stein, 2002).

Yet, most of the literature concentrates on contests with a logit form. On the first glance this seems to be arbitrary. However, Skaperdas (1996) has proposed a system of axioms that are reasonable for the winning probability in a contest with homogenous players and immediately imply a logit contest success function. His axioms are:

1. The winning probabilities are true probabilities, i.e., they lie in the interval $[0, 1]$ for every player, and sum up to one for all players.[4]

2. Raising one player's effort strictly increases this player's winning probability, while it decreases the other players' winning probabilities.

[2]Baik and Shogren (1992) analyze their sequential game not only for a logit but also for a probit form. Yet their analysis does not cover contests as general as Dixit and Malueg and Yates.

[3]In the case of a Tullock-contests, this is for player i: $\frac{\partial^2 p_i}{\partial x_i \partial x_j} = \frac{\partial^2}{\partial x_i \partial x_j}\left(\frac{x_i^r}{x_i^r + x_j^r}\right) = \frac{\partial}{\partial x_j}\left(\frac{r x_i^{r-1} x_j^r}{\left(x_i^r + x_j^r\right)^2}\right)$ $= \frac{r(x_i x_j)^{r-1}}{(x_i + x_j)^3}\left(x_i^r - x_j^r\right)$. This is positive if player i exerts the higher effort in equilibrium.

[4]For an example violating this axiom, see Nti (1997).

3. Skaperdas called for anonymity. This means that each player can be exchanged with any other player without harming the results.

4. Let $p_i(\mathbf{x})$ denote the winning probability of player i in the contest among all players, and $p_i^M(\mathbf{x}^M)$ denote the winning probability of player i in a contest among a subgroup M of the initial group of players, say of size m. Then, the winning probabilities of the players in this subgroup will not depend on M, if effort levels do not change, i.e. $\frac{p_i^M(\mathbf{x}^M)}{p_j^M(\mathbf{x}^M)} = \frac{p_i(\mathbf{x})}{p_j(\mathbf{x})}$.[5]

5. Whenever the contest is played only among a subgroup of the initial group of players, the winning probabilities of the participating players do not depend on non-participating players.

Skaperdas showed that any probability function that satisfies these axioms is a logit function, i.e., $p_i(\mathbf{x}) = \frac{f(x_i)}{\sum_{j=1}^{n} f(x_j)}$. In a further step, he introduced a sixth axiom, namely the one of zero-homogeneity.

6. Increasing the effort of all players by the same factor does not alter the winning probability of any player. Specifically, this means that the contest success function does not depend on the unit of efforts, which can be interpreted as the currency if prize and efforts are monetary variables.

Skaperdas established that the probability function used in the Tullock-contest is the only one that fulfills the Axioms 1 through 6. This means that, if one accepts the axioms of Skaperdas, it is sufficient to analyze a Tullock-contest instead of a general contest.

Note that Skaperdas' axiom 3 requires homogenous players. This axiom has been regarded critically in economic literature. Thus, Kooreman and Schoonbeek (1997) and Clark and Riis (1998) extended the analysis of Skaperdas by dropping Axiom 3 and came to the result that the axioms 1, 2, 4, 5 and 6 are equivalent to:[6]

$$p_i(\mathbf{x}) = \frac{\sigma_i x_i^r}{\sum_{j=1}^{n} \sigma_j x_j^r}.$$

However, for our analysis, the results of Skaperdas are sufficient since we do not assume players to be heterogenous with respect to their contest success function but only with respect to their preferences or to their valuations. The functional form of the winning probability is not affected by this heterogeneity.

[5] See appendix for a short discussion.

[6] Kooreman and Schoonbeek (1997) analyzed this case for two players, while Clark and Riis (1998) looked at the n-player-case.

2.2 On the Returns to Scale

In this section, we want to focus on the role that the technology parameter r plays in Tullock's payoff function. While we will restrict the analysis in the main part of this work to small values of r and, mostly, to $r = 1$, it is, nevertheless, worthwhile discussing what will happen if r becomes large.

The parameter in the power function represents how the marginal effectiveness of the efforts develops. While, for small values of $r < 1$, the first marginal effort is infinitively effective for an effort level of zero and marginal effectiveness decreases in effort, higher values of $r > 1$ represent a contest function with increasing returns to scale.

Let us first take a look at the limit cases. On the one hand, if r takes the value 0, the impact function x^r is reduced to 1 for every positive effort level x. We thus have a fair lottery where every player has an equal chance of winning, regardless of her effort. Since a player receives the prize with probability $p_i = \frac{1}{n}$ even if she chooses zero effort, an optimal reaction of a player cannot be to choose a positive effort level $\hat{x}$. In equilibrium $\mathbf{x} = (0, ..., 0)$.

The converse limit case, $r = \infty$, has a far more important interpretation. In this case, every player pays the costs for her effort, but the player with the highest effort choice, if unique, wins the prize with certainty. This form of contest is better known as an all-pay-auction. Under the assumption of complete information and simultaneous decisions, Hillman and Riley (1989) showed that, in an all-pay-auction with heterogenous players, only the two players with highest valuations are active, in the sense that they choose a positive effort with a positive probability. In equilibrium, the player with the highest valuation (V_1) plays a mixed strategy, where she chooses her effort level from an equal distribution between zero and the second highest valuation (V_2). The player with the second highest chooses a strictly positive effort with probability $\frac{V_2}{V_1}$, and with the probability $\frac{V_1 - V_2}{V_1}$ she chooses an effort of zero. In the case that she exerts a positive effort, she also chooses her effort from an equal distribution on $[0, V_2]$. Players with a lower valuation remain inactive. As a result, the player with the highest valuation receives an expected payoff of $V_1 - V_2$, while all the other players receive an expected payoff of zero. Baye et al. (1996) provide a complete analysis of all equilibria in the all-pay-auction, where they relax the implicit assumption that the highest and the second highest valuation are unambiguous. Especially if the two highest valuations take equal value, no player receives a positive payoff. In this case, we have full dissipation. Overdissipation will never occur. Konrad and Leininger (2007) propose a model of an all-pay-auction, where they find an equilibrium in which no dissipation exists, as we already discussed in detail in Section 1.4.

Let us now concentrate on positive but finite values of r, which we will focus on for the rest of this work. In general, this class of games can be divided into four different cases each with different properties. The first and most prominent is the one with $r = 1$ that represents a contest technology with constant returns to scale. Many works on the field of contest theory (e.g., Leininger, 1993, Linster, 1994, Stein, 2002, Schoonbeek and Winkel, 2006) concentrate on this special case, among others, because of its easy tractability.[7] Apart from its prominence with concern to manageability, this case also takes a special role from the point of view of results. As we will see, it depicts the border betweeen two further areas of interest with different properties.

For the case of decreasing returns to scale where $r < 1$, we find an interesting contrast to constant returns in the sense that the reaction function is always positive. While, in a setting with $r = 1$ and heterogenous players, as we will see in Chapter 4, some players might abstain from participation, every player has an incentive to make a positive investment if returns to scale are decreasing. The reason for this is that the marginal return on invested effort converges to infinity as the effort itself converges to zero (cf. Cornes and Hartley, 2005). This means that, for the case of non-participation (i.e. an effort level of zero), the marginal gain of the first bit of effort is infinitely high, while the marginal cost is a finite (and, in general, constant) number. Thus, positive investment always pays.

Next, let us focus on the case of slightly increasing returns to scale meaning that $r \in \left(1, \frac{n}{n-1}\right]$. In contrast to the case where $r = 1$ for increasing returns to scale ($r > 1$), the general properties of the contest, such as existence and uniqueness of (pure-strategy) equilibria, depend on the number of players n in the contest. Hence, we split the analysis for increasing returns to scale in two subcases where the threshold $\frac{n}{n-1}$ is a function of n. For a contest with homogenous players, this is the threshold up to which maximizing payoff leads to non-negative payoff in an interior Nash-equilibrium. As a consequence, in the presence of increasing returns to scale, the number of players must be known in order to determine the type of equilibrium that can occur. As we will see in section 3.6, the uniqueness cannot even be ensured for $r \in \left(1, \frac{n}{n-1}\right]$ if the assumptions of payoff maximizing and homogeneity are relaxed (see the discussion below).

In his seminal paper, Tullock (1980) showed that, if returns to scale are high enough, the first-order-conditions for a number of numerical examples hint at effort levels with a dissipation level higher than the prize (overdissipation).[8] Indeed, these numerical examples are those characterized by $r > \frac{n}{n-1}$. But since payoff maximization is assumed to be the major concern of players in the standard rent-seeking contest, overdissipation cannot occur in equilibrium, because every player could, by reducing her effort to zero, ensure that she receives zero payoff. As a consequence, the equilibrium analysis becomes more complex for $r > \frac{n}{n-1}$,[9] since the second-order-condition is no longer fulfilled. However, this challenging case has hardly been dealt with in literature. The first important contribution in this field was made by Perez-Castrillo and Verdier (1992). They characterize pure-strategy equilibria for $r > \frac{n}{n-1}$, and show that, for $2 \leq \frac{r}{r-1}$ (which is equivalent to $r \leq 2$), a Nash-equilibrium is given when m players compete actively in the contest (where $2 \leq m \leq \frac{r}{r-1} < n$), while the remaining players stay outside the contest. Note that this means that $r \leq \frac{m}{m-1}$.

The only work that analyzes equilibria for the case of $r > 2$ is the one of Baye

[7]E.g., while the first derivative of the impact function in general is rx^{r-1}, for $r = 1$ it is 1!

[8]Yet, for these effort levels, the second-order-conditions of a payoff maximization are not fulfilled.

[9]As we will see later, overdissipation can indeed occur, if the standard assumptions are relaxed.

et al. (1994) who restricted their work to the two-player-case.[10] They calculate some Nash-equilibria for parameter constellations, where continuous choice of effort is not feasible, but effort choice must take integer values. They do not give a general solution to the problem. Yet, they find that, in their setting, overdissipation will never occur in Nash-equilibrium.

2.3 The Role of Heterogeneity

In his standard model, Tullock (1980) had assumed that all players were identical. In real life, we observe that, in many contests, players differ in several aspects such as valuation for rent (Chapter 6 to 8), costs in exerting efforts, effectiveness of efforts or objective functions (Chapter 3 to 5). Throughout this work, we will assume that players can be heterogenous in at least one of these points.

For the introductory motivation, let us focus on heterogeneity that can arise from the payoff function. The further chapters will then address the questions of how results change if other variables than the own material payoff function, especially the payoff of the opponents, play a role for the decision of the contestant or if we introduce different timing aspects into the decision structure.

To illustrate the three possible levels of heterogeneity in the material payoff function let us write a general payoff function as:[11]

$$\pi_i(\mathbf{x}) = \frac{\sigma_i x_i^r}{\sum_{j=1}^n \sigma_j x_j^r} \cdot V_i - d_i x_i. \tag{2.2}$$

Different values of V_i represent heterogeneity in the valuation of the rent, a difference in d_i between the players signals a heterogenous cost structure for the effort and different values of σ_i represent differences in the effectiveness of effort.

The first-order-condition for the maximization of (2.2) is given by:

$$\frac{\partial \pi_i(\mathbf{x})}{\partial x_i} \overset{!}{=} 0 \Leftrightarrow r \cdot \sigma_i \cdot x_i^{r-1} \left(\sum_{k \neq i} \sigma_k x_k^r \right) \cdot \frac{V_i}{d_i} = \left(\sum_{j=1}^n \sigma_j x_j^r \right)^2 . \tag{2.3}$$

The only way that valuation and cost of effort influence this reaction function is via the factor $\frac{V_i}{d_i}$. Thus, an increase (decrease) of the valuation and a decrease (increase) of the effort costs by the same factor induce the same change in the reaction function of player i.

With respect to effectiveness, however, results are no longer equivalent. Comparative statics of both heterogeneity in valuation and in effectiveness have been put forward by

[10] In a comment, Tullock (1995) claims, that Perez-Castrillo and Verdier (1992) have already looked upon this case. However, Perez-Castrillo and Verdier restrict their analysis to $r \leq 2$.

[11] Although heterogeneity with respect to the technology parameter r would be also conceivable, this issue has not been addressed in the literature so far. Except for the huge reduction in calculatory complexity, a motivation for assuming homogenous technology scales is given by the axiomatic theory of heterogenous players in contests, cf. Section 2.1.

Stein (2002). He shows that, while an increase in valuation will always increase individual and total effort, a change in effectiveness has ambiguous effects. An increase in σ_i only increases total effort if σ_i is relatively small. In this case, the players become more similar, which intensifies competition in the contest. If a strong player becomes more effective, total effort decreases. In a similar way, changes in valuation and effectiveness have very different effects on the player's own effort. While an increase in valuation always incites her to choose a higher effort level, increasing one's effectiveness eases up the player's situation, such that she reduces her effort in most situations. The effect of changes in valuation and effectiveness on winning probabilities, payoffs and the opponents' effort levels are, however, comparable.

Part I

Interdependent Preferences

Chapter 3

Strategic Advantage of Negatively Interdependent Preferences[1]

3.1 Introduction

In this chapter, we explore the implications of negatively interdependent preferences in the strategic context of rent-seeking contests. The focus is on heterogeneous populations where part of the players maximize material payoff while others are additionally concerned with their relative position. To this aim, we will introduce the notion of relative payoff, that represents the difference between one's own material payoff and the average material payoff of all players.

In this setup, we will allude to a number important issues that help to characterize the equilibria of this game. We start with discussing the conditions that have to be fulfilled such that players actively participate in this game. Beyond this, we present some results that illustrate how the relative size of effort can determine the relative size in material payoff.

In the core of this chapter, we seek for conditions that are necessary and sufficient to obtain the result that players with interdependent preferences receive the higher material payoff in equilibrium. Here, we first concentrate on the two-player-contest, an ubiquitous example for which we find that the spiteful player will always receive a higher material equilibrium payoff. Thereafter, we show that this result generalizes to multi-player-contests if returns to scale are non-increasing. Finally, we turn to increasing returns to scale. Here, the results are not as straightforward if there are more than two players. We distinguish four types of equilibria that are characterized by (a) whether or not interdependent preferences lead to a higher effort level and (b) whether both types of players choose a positive effort level or only one type. We find that in general all four types of equilibria can occur for certain parameter ranges. Even more, we find that at least three types of these equilibria can occur for the same parameter constellation, which immediately implies that non-uniqueness of equilibrium can occur for games with more than two players. Examples are provided.

[1]This chapter represents joint work with Burkhard Hehenkamp, cf. Guse and Hehenkamp (2006).

A related analysis to ours has been presented by Kockesen et al. (2000a, 2000b). They establish the strategic advantage of negatively interdependent preferences for certain classes of supermodular and submodular games, which do not include general rent-seeking contests. Individuals, who have an additional concern for their relative position, act more aggressively than players who just care about their own material pay-off. This gives them higher pay-off in equilibrium. However, Kockesen et al's results do not cover rent-seeking contest. First, this is because they focus on cases where either strategic complements or strategic substitutes are present. In rent-seeking contests, by contrast, payoff functions have both a range of strategic complementarity and a range of strategic substitutability. Second, Kockesen et al. restrict their analysis to non-negative payoff functions, a premise that, in the context of rent-seeking contests, would simply assume the issue of dissipation away. It thus remains an open question whether the strategic advantage of negatively interdependent preferences and the phenomenon of overdissipation prevail in the economically important context of rent-seeking contests.

Whether status-seeking preferences have a strategic advantage over individualistic ones or not, has a number of interesting applications, two of which we will introduce here.

For the first one, imagine a situation, where a decision-maker is about to delegate a task that involves play of a rent-seeking contest to one out of a group of possible delegates - which one should he select? Then - provided the type of rent-seeking contest gives a strategic advantage to status-seeking preferences - he should pick the person that will presumably be most concerned about his relative performance in the contest.

The second application is related to the first. Consider the decision whether to delegate or not and, in the case of delegation, which incentives to provide the delegate with. This framework was originally proposed and developed by Vickers (1985) and Fershtman and Judd (1987) in the context of oligopoly. There is an increasing amount of literature on strategic delegation in rent-seeking contests (see e.g. Baik and Kim, 1997, Kräkel and Sliwka, 2002, or Baik, 2003), all of which, among other restrictions, restrict to the rather special case of a constant returns to scale technology. Notice that we deal with a much more general class of payoff functions, covering the whole range of decreasing, constant and increasing returns to scale. In the conclusion, we will comment on the implications of our results.

3.2 The model

In our model, we examine rent-seeking contests, where the material payoff is characterized by (2.1). The effort of each player is non-negative. With slight abuse of notation, we write $x = (x_h, x_{-h})$ when it is convenient to single out player h's strategy.

We consider a population where the set of players is divided into two groups. Each member of the first group, $i \in I \equiv \{1, ..., m\}$, picks her effort in order to maximize material payoff, $\pi_i(\mathbf{x})$, where $\mathbf{x}$ is the vector of effort levels of all n players. Members of the first group have *independent preferences*. We call them *individualists*.

Members of the other group, $j \in J \equiv \{m+1, ..., n\}$, additionally care about their own material performance relative to that of the other players. We call them *status-seekers.*

The function F below represents their (negatively) interdependent preferences with respect to absolute and relative material payoff. More precisely, they choose their effort level in order to maximize

$$F_j(\mathbf{x}) = F\left(\pi_j\left(\mathbf{x}\right), \rho_j\left(\mathbf{x}\right)\right), \tag{3.1}$$

where we assume F to be strictly increasing in both arguments and where the relative payoff, $\rho_j\left(\mathbf{x}\right)$, is given by

$$\rho_j\left(\mathbf{x}\right) := \pi_j\left(\mathbf{x}\right) - \frac{1}{n}\sum_{h=1}^{n}\pi_h\left(\mathbf{x}\right). \tag{3.2}$$

Since these types of preferences do not only depend on absolute material payoff, but also on relative material payoff, we call these preferences *(negatively) interdependent.*

Alternative specifications of relative payoff are also possible. For example, a player might compare her own payoff with the average payoff of the other players:

$$\tilde{\rho}_j\left(\mathbf{x}\right) := \pi_j\left(\mathbf{x}\right) - \frac{1}{n-1}\sum_{h\neq n}\pi_h\left(\mathbf{x}\right). \tag{3.3}$$

However, although the equilibrium slightly changes, the class of solutions does not. Thus, the general results we establish with (3.2) could also be implemented with (3.3).

Notice that, is important to distinguish between *material payoff*, which determines the material success of players and their strategies, and *utility*, which represents the players' preferences that can be independent or interdependent. While, for individualists, the notions of material payoff and utility coincide, they differ for status-seekers. The following example introduces the class of linear interdependent preferences.

Example 1 *Consider the following interdependent utility function, which assumes preferences to be linear in absolute and relative material payoff,*

$$\begin{aligned} F\left(\pi_j(\mathbf{x}), \rho_j\left(\mathbf{x}\right)\right) &= \beta_1\pi_j(\mathbf{x}) + \beta_2\rho_j\left(\mathbf{x}\right) = \left(\beta_1 + \beta_2\frac{n-1}{n}\right)\pi_j(\mathbf{x}) - \frac{\beta_2}{n}\sum_{h\neq n}\pi_h(\mathbf{x}) \\ &= (\beta_1+\beta_2)\pi_j(\mathbf{x}) - \frac{\beta_2}{n}\left[V - \sum_{h=1}^{n}x_h\right], \end{aligned} \tag{3.4}$$

where $\beta_1, \beta_2 > 0$ guarantee the mentioned monotonicity properties. Notice that for fixed $\beta_1 > 0$ and $\beta_2 = 0$ maximizing (3.4) is equivalent to maximizing (2.1). Similarly, given any fixed $\beta_2 > 0$, maximizing (3.4) is equivalent to maximizing relative material payoff if $\beta_1 = 0$.

As is standard in game theory, we assume that, in equilibrium, each player takes the other players' efforts as given, i.e., we search for a Nash-equilibrium of the game that is induced by the independent and interdependent preferences. As in Kockesen et al., our focus is on intragroup symmetric equilibria, where all players with identical preferences choose the same effort level, i.e., where $x_i = x_1, \forall i \in I$, and $x_j = x_n, \forall j \in J$. We denote these equilibria by $\hat{\mathbf{x}} = ([a]_m, [b]_{n-m}) \in N_{sym}(\Gamma_F(m))$.

For our further analysis, we need to define the notions of strategic advantage and dissipation:

Definition 1 *Consider a game $\Gamma_F(m)$ for some $k \in \{1, ..., n-1\}$. Then, (negatively) interdependent preferences yield a (strict) strategic advantage over independent preferences, if and only if*

$$\pi_j(\hat{\mathbf{x}}) \overset{(>)}{\geq} \pi_i(\hat{\mathbf{x}}), \forall i \in I \; \forall j \in J \; \forall \hat{\mathbf{x}} \in N_{sym}(\Gamma_F(m)).$$

Definition 2 *An equilibrium $\hat{\mathbf{x}}$ displays overdissipation (full or underdissipation), if and only if $\sum_{h=1}^{n} x_h >$ ($=$ or $<$) V, respectively.*

It follows that in situations where status-seekers have a strategic advantage over individualists there will be no overdissipation. If the strategic advantage is strict, this implies underdissipation. To see this, notice that an individualist will only exert positive effort if her material payoff would be non-negative, i.e., $\pi_i(\hat{\mathbf{x}}) \geq 0$. Otherwise, she could do better with $x_i = 0$. In case of a strategic advantage, it follows that $\pi_j(\hat{\mathbf{x}}) \geq \pi_i(\hat{\mathbf{x}}) \geq 0$. Consequently, the material payoff of these players is non-negative. This implies that the sum over all material payoffs is also non-negative, $\sum_{h=1}^{n} \pi_h(\hat{\mathbf{x}}) \geq 0$. Because of $\sum_{h=1}^{n} \pi_h(\hat{\mathbf{x}}) = V - \sum_{h=1}^{n} x_h$, this is equivalent to $V \geq \sum_{h=1}^{n} x_h$. Thus, the sum of all effort levels is lower than the prize at stake. There will be no overdissipation.

In case of a strict strategic advantage, we have $\pi_j(\hat{\mathbf{x}}) > \pi_i(\hat{\mathbf{x}}) \geq 0$. Therefore, $\sum_{h=1}^{n} \pi_h(\hat{\mathbf{x}}) > 0$ and $V > \sum_{h=1}^{n} x_h$, which excludes the case of full dissipation. Accordingly, we must have underdissipation whenever a strict strategic advantage is present. We have shown:

Proposition 1 *Suppose interdependent preferences yield a strategic advantage over independent preferences. Then, dissipation is, at most, full, i.e., $\sum_{h=1}^{n} x_h \leq V$. If the strategic advantage is strict, we have underdissipation, i.e., $\sum_{h=1}^{n} x_h < V$.*

3.3 Preliminary results

We start with deriving general conditions under which individualists and status-seekers optimally participate in the contest. Subsequently, we derive first order conditions that characterize equilibrium behavior. We conclude this section with a preliminary lemma that helps to analyze the strategic advantage of negatively interdependent preferences.

3.3.1 Participation constraints

We, first, give the conditions under which individualists optimally participate and under which they optimally stay out of the contest. Subsequently, we turn to the respective conditions for status-seekers.

Individualists

On the one hand, individualists $i \in I$ optimally participate in the contest (i.e. $a > 0$) whenever doing so yields non-negative payoff in equilibrium (zero payoff is what they would earn leaving the contest):

$$\pi_i(\hat{\mathbf{x}}) \geq \pi_i(0, \hat{\mathbf{x}}_{-i}) \quad \Leftrightarrow \quad \frac{a^r}{ma^r + (n-m)b^r} V - a \geq 0. \tag{PCI}$$

On the other hand, non-participation is optimal to individualists when no positive effort level gives positive material payoff, i.e., when $\pi_i(\hat{\mathbf{x}}) = 0 \geq \pi_i(x_i, \hat{\mathbf{x}}_{-i})$, for all effort levels $x_i \geq 0$. The following lemma provides an equivalent characterization of non-participation:

Lemma 1 *Fix $\hat{\mathbf{x}} = ([a]_m, [b]_{n-m}) \in N_{sym}(\Gamma_F(m))$. If $a = 0$ then $x_i = 0$ is optimal to an individualist $i \in I$ if and only if $r > 1$ and*

$$\frac{(r-1)^{r-1}}{r^r} V^r \leq (n-m)b^r. \tag{NPCI}$$

Proof. To prove this lemma, we simply refer to Perez-Castrillo and Verdier (1992). They show in their first proposition that non-participation in a contest is optimal whenever $\frac{(r-1)^{r-1}V^r}{r^r}$ is lower than the aggregate effort of the other contestants, which is here given here by $(n-m)b^r$. ■

Status-seekers

Status-seekers $j \in J$ participate in the contest whenever this gives higher utility with respect to $F(\cdot)$ than non-participation would. Contrary to individualists, status-seekers additionally evaluate their *relative* payoff that would result from non-participation. Correspondingly, a status-seeker's participation constraint reads:

$$F(\pi_j(\hat{\mathbf{x}}), \rho_j(\hat{\mathbf{x}})) \geq F(\pi_j(0, \hat{\mathbf{x}}_{-j}), \rho_j(0, \hat{\mathbf{x}}_{-j})). \tag{PCJ}$$

Without knowing the concrete utility function $F(\cdot)$, we can hardly compare the two expressions in (PCJ). Therefore, it is not possible to give a general characterization of a status-seeker's participation constraint. To get a grip on this problem, we first investigate the relationship between absolute and relative payoff. Lemma 2 summarizes our findings.

Lemma 2 *Let $\mathbf{x}', \mathbf{x}'' \in X^n$ be two arbitrary effort profiles that only differ in their j:th component, i.e., $\mathbf{x}' = (x_1, .., x'_j, .., x_n)$, $\mathbf{x}'' = (x_1, .., x''_j, .., x_n)$ and $x'_j < x''_j$. Then, the following implications hold:*

$$\begin{array}{llll} (i) & \pi_j(\mathbf{x}') \leq \pi_j(\mathbf{x}'') & \Rightarrow & \rho_j(\mathbf{x}') < \rho_j(\mathbf{x}''), \\ (ii) & \rho_j(\mathbf{x}') \geq \rho_j(\mathbf{x}'') & \Rightarrow & \pi_j(\mathbf{x}') > \pi_j(\mathbf{x}''). \end{array}$$

Proof. Inserting the definitions of $\rho_j(\cdot)$ and $\pi_j(\cdot)$, one obtains

$$\begin{aligned}\rho_j(\mathbf{x}') - \rho_j(\mathbf{x}'') &= \left[\pi_j(\mathbf{x}') - \tfrac{1}{n}\textstyle\sum_h \pi_h(\mathbf{x}')\right] - \left[\pi_j(\mathbf{x}'') - \tfrac{1}{n}\textstyle\sum_h \pi_h(\mathbf{x}'')\right] \\ &= [\pi_j(\mathbf{x}') - \pi_j(\mathbf{x}'')] - \left[\tfrac{1}{n}\textstyle\sum_h \pi_h(\mathbf{x}') - \tfrac{1}{n}\textstyle\sum_h \pi_h(\mathbf{x}'')\right] \\ &= [\pi_j(\mathbf{x}') - \pi_j(\mathbf{x}'')] - \tfrac{1}{n}[x''_j - x'_j],\end{aligned}$$

where the last equality follows from $\sum_h \pi_h(\mathbf{x}) = V - \sum_h x_h$. Then, the two claims hold because of $x''_j > x'_j$. ■

Notice that Lemma 2 does not only cover switches between participation and non-participation, but, more generally, analyzes the effects of an increase in effort (or a decrease). Part (i) says that *any* increase in effort that increases absolute payoff will also increase relative payoff, while part (ii) states the converse: *Any* decrease in effort that increases relative payoff leads to an increase in absolute payoff.

It, thus, follows from Lemma 2 that an effort profile $\hat{\mathbf{x}} = ([a]_m, [b]_{n-m})$ can only represent an intragroup symmetric equilibrium if, for any status-seeker $j \in J$, (i) no higher effort level $x_j > b$ increases her absolute payoff π_j and (ii) no lower effort level $x_j < b$ increases her relative payoff ρ_j. So, if one of the two conditions applied, i.e., if (i) $x_j > b$ increased absolute payoff π_j or if (ii) $x_j < b$ increased relative payoff ρ_j, then Lemma 2 would imply that both absolute and relative payoff increase, which would lead to a higher utility level with respect to $F(\cdot)$ – the former choice of $\hat{\mathbf{x}}_j = b$ would not have been optimal. We have thus established the following corollary:

Corollary 1 *Fix $\hat{\mathbf{x}} = ([a]_m, [b]_{n-m}) \in N_{sym}(\Gamma_F(k))$. Then, for all status-seekers $j \in J$, we have (i) $\pi_j(\hat{\mathbf{x}}) \geq \pi_j(x_j, \hat{\mathbf{x}}_{-j})$, for all $x_j > b$ and (ii) $\rho_j(\hat{\mathbf{x}}) \geq \rho_j(x_j, \hat{\mathbf{x}}_{-j})$ for all $x_j < b$.*

Returning to the problem of characterizing the status-seekers' participation constraint, we can now derive a necessary and a sufficient condition for the optimal participation of status-seekers. The necessary one says that non-participation may not lead to higher relative payoff $\rho_j(0, \hat{\mathbf{x}}_{-j})$, since by Lemma 2, this would also imply higher absolute material payoff $\pi_j(0, \hat{\mathbf{x}}_{-j})$ and, hence, higher utility with respect to $F(\cdot)$. Thus, non-participation would be optimal in this case. On the other hand, the sufficient condition says that status-seekers will optimally exert positive effort when participation results in higher absolute payoff than non-participation. By Lemma 2, this also implies higher relative payoff and thus higher utility with respect to $F(\cdot)$. The corollary below sums up our findings:

Corollary 2 *(Optimality of status-seekers' participation)*
Fix $\hat{\mathbf{x}} = ([a]_m, [b]_{n-m}) \in N_{sym}(\Gamma_F(m))$. Then, for all status-seekers $j \in J$, (i) $b > 0$ implies $\rho_j(\hat{\mathbf{x}}) \geq \rho_j(0, \hat{\mathbf{x}}_{-j})$ and (ii) $\pi_j(\hat{\mathbf{x}}) \geq \pi_j(0, \hat{\mathbf{x}}_{-j})$ implies $b > 0$.

In contrast, non-participation is optimal to a status-seeker when any positive effort level does not increase her utility with respect to $F(\cdot)$, i.e., when

$$F(\pi_j(\hat{\mathbf{x}}), \rho_j(\hat{\mathbf{x}})) \geq F(\pi_j(x_j, \hat{\mathbf{x}}_{-j}), \rho_j(x_j, \hat{\mathbf{x}}_{-j})) \qquad \text{(NPCJ)}$$

for $\hat{\mathbf{x}} = (0, \hat{\mathbf{x}}_{-j})$ and for all $x_j \geq 0$. Again, without knowing the functional form of $F(\cdot)$, we can only give a necessary and a sufficient condition.

Non-participation can only be optimal for a status-seeker j when participation with an arbitrary effort level $x_j > 0$ does not increase her absolute material payoff above the zero payoff margin (which results from non-participation). Otherwise, a positive effort level does not only increase j's material payoff, but by Lemma 2, also decreases all other players' material payoff. Again, both absolute and relative payoff increase so that utility increases with respect to $F(\cdot)$ – non-participation would not have been optimal.

Similar to the above, non-participation is, in fact, optimal when any positive effort level $x_j > 0$ reduces relative payoff. By Lemma 2, this also implies a reduction in absolute payoff and hence in utility. We have established:

Corollary 3 *(Optimality of status-seekers' non-participation)*
Fix $\hat{\mathbf{x}} = ([a]_m, [b]_{n-m}) \in N_{sym}(\Gamma_F(m))$. *Then, (i)* $b = 0$ *implies* $\pi_j(\hat{\mathbf{x}}) \geq \pi_j(x_j, \hat{\mathbf{x}}_{-j})$, *for all* $x_j > 0$ *and (ii)* $\rho_j(\hat{\mathbf{x}}) \geq \rho_j(x_j, \hat{\mathbf{x}}_{-j})$ *implies* $b = 0$, *for all* $x_j > 0$.

We conclude the analysis of participation constraints with the example of linear interdependent utility functions. For this class of status-seeking preferences, we can determine an equivalent characterization.

Example 1 (continued) *Let* $\hat{\mathbf{x}} = ([a]_m, [b]_{n-m})$ *denote an intragroup symmetric Nash equilibrium and* $\check{\mathbf{x}} = (0, \hat{\mathbf{x}}_{-j})$ *the alternative strategy profile where status-seeker j opts out of the contest. Recall that*

$$F\left(\pi_j(\mathbf{x}), \rho_j\left(\mathbf{x}\right)\right) = \beta_1 \pi_j(\mathbf{x}) + \beta_2 \rho_j\left(\mathbf{x}\right) = (\beta_1 + \beta_2)\pi_j(\mathbf{x}) - \frac{\beta_2}{n}\left[V - \sum_{h=1}^{n} x_h\right]. \tag{3.5}$$

Consequently, the participation constraint of status-seekers, (PCJ), is equivalent to

$$\begin{aligned}
F_j(\hat{\mathbf{x}}) &\geq F_j(\check{\mathbf{x}}), \\
(\beta_1 + \beta_2)\pi_j(\hat{\mathbf{x}}) - \frac{\beta_2}{n}\left[V - \sum_{h=1}^{n} \hat{x}_h\right] &\geq (\beta_1 + \beta_2)\pi_j(\check{\mathbf{x}}) - \frac{\beta_2}{n}\left[V - \sum_{h=1}^{n} \check{x}_h\right], \\
(\beta_1 + \beta_2)\left[\frac{b^r}{ma^r + (n-m)b^r}V - b\right] - \frac{\beta_2}{n} b &\geq 0, \\
\frac{b^r}{ma^r + (n-m)b^r} \cdot \frac{n(\beta_1 + \beta_2)}{n\beta_1 + (n-1)\beta_2} V - b &\geq 0.
\end{aligned} \tag{3.6}$$

Here, the third row follows from $\pi_j(\check{\mathbf{x}}) = 0$ *and* $\hat{x}_h = \check{x}_h$ *for* $h \neq j$, *while the fourth row is a multiple of the third one. Setting*

$$\lambda = \frac{n(\beta_1 + \beta_2)}{n\beta_1 + (n-1)\beta_2}, \tag{3.7}$$

we can interpret λV as the implicit personal value a status-seeker ascribes to the material prize V. (A status-seeker with personal value λV will participate in the contest with material value V if and only if an individualist would participate in a contest with material value λV.) Notice that $\beta_1, \beta_2 > 0$ implies $\lambda \in (1, \frac{n}{n-1})$. Put verbally, the monotonicity assumptions put an upper boundary on the implicit personal value λV. We will allude to this issue in more detail in the following chapter.

3.3.2 Characterizing equilibrium behavior

For individualists $i \in I$, the first order condition of a local absolute payoff maximum at $x_i = a > 0$ can be derived from (2.1) as

$$\frac{\partial \pi_i}{\partial x_i}(\hat{\mathbf{x}}) = 0 \quad \Leftrightarrow \quad ra^{r-1}[(m-1)a^r + (n-m)b^r]V = [ma^r + (n-m)b^r]^2. \qquad \text{(FOCI)}$$

As is well known in the rent-seeking literature (cf. Pérez-Castrillo and Verdier, 1992), any solution to the first order condition that satisfies the participation constraint (PCI) also satisfies the second order condition for a local maximum. Moreover, the only other candidate for a global maximum is $x_i = 0$. Hence, for any individualist $i \in I$, an effort level $x_i > 0$ is optimal and leads to non-negative payoff $\pi_i(\hat{\mathbf{x}}) \geq 0$ if and only if $x_i = a$ satisfies (FOCI) and (PCI).

A status-seeker j's effort level $\hat{x}_j = b > 0$ is optimal when it maximizes utility with respect to $F(\cdot)$, i.e., when

$$F(\pi_j(\hat{\mathbf{x}}), \rho_j(\hat{\mathbf{x}})) \geq F(\pi_j(x_j, \hat{\mathbf{x}}_{-j}), \rho_j(x_j, \hat{\mathbf{x}}_{-j})),$$

for all $x_j \geq 0$. Unfortunately, we cannot derive general conditions without knowing the particular functional form of $F(\cdot)$. However, we can build on the insight of Lemma 2 in order to derive a lower and an upper boundary on the derivative of j's material payoff function. To this end, let us first determine the first derivative of j's absolute and relative payoff function, respectively:

$$\frac{\partial \pi_j}{\partial x_j}(\hat{\mathbf{x}}) = \frac{r\hat{x}_j^{r-1}\sum_{h\neq j}\hat{x}_h^r}{\left[\sum_h \hat{x}_h^r\right]^2}V - 1, \qquad (3.8)$$

$$\frac{\partial \rho_j}{\partial x_j}(\hat{\mathbf{x}}) = \frac{r\hat{x}_j^{r-1}\sum_{h\neq j}\hat{x}_h^r}{\left[\sum_h \hat{x}_h^r\right]^2}V - 1 + \frac{1}{n} = \frac{\partial \pi_j}{\partial x_j}(\hat{x}) + \frac{1}{n}. \qquad (3.9)$$

Obviously, $\frac{\partial \pi_j}{\partial x_j}(\hat{\mathbf{x}}) \geq 0$ implies $\frac{\partial \rho_j}{\partial x_j}(\hat{\mathbf{x}}) > 0$ and, conversely, $\frac{\partial \rho_j}{\partial x_j}(\hat{\mathbf{x}}) \leq 0$ implies $\frac{\partial \pi_j}{\partial x_j}(\hat{\mathbf{x}}) < 0$. These implications represent the marginal analogue to Lemma 2. Similar to the corresponding corollary, (i) $\frac{\partial \pi_j}{\partial x_j}(\hat{\mathbf{x}}) > 0$ and (ii) $\frac{\partial \rho_j}{\partial x_j}(\hat{\mathbf{x}}) < 0$ cannot hold in any intragroup symmetric Nash equilibrium. Therefore, in the former case, any status-seeker $j \in J$ could strictly increase her interdependent utility $F(\cdot)$ exerting slightly higher effort $b+\varepsilon$, while, in the latter case, he could gain from a slightly lower effort level $b-\varepsilon$ (for $\varepsilon > 0$ sufficiently small). Therefore, in any intragroup symmetric Nash equilibrium $\hat{x} = ([a]_m, [b]_{n-m}) \in N_{sym}(\Gamma_F(m))$, we have

$$\frac{\partial \pi_j}{\partial x_j}(\hat{\mathbf{x}}) \leq 0 \quad \text{ and } \quad \frac{\partial \rho_j}{\partial x_j}(\hat{\mathbf{x}}) = \frac{\partial \pi_j}{\partial x_j}(\hat{\mathbf{x}}) + \frac{1}{n} \geq 0.$$

Inserting $\hat{\mathbf{x}} = ([a]_m, [b]_{n-m})$ in (3.8) and (3.9), the inequalities imply

$$\frac{n-1}{n} < \frac{rb^{r-1}[ma^r + (n-m-1)b^r]}{[ma^r + (n-m)b^r]^2} V < 1,$$

which can be written as

$$rb^{r-1}[ma^r + (n-m-1)b^r]\, \lambda V = [ma^r + (n-m)b^r]^2, \tag{FOCJ}$$

for some $\lambda \in (1, \frac{n}{n-1})$. Comparing (FOCI) and (FOCJ), we obtain the same intuitive interpretation of λ that we introduced in Example 1: Given λ, the expression λV represents the implicit personal value a status-seeker ascribes to the material prize V. From $\lambda > 1$, we have $\lambda V > V$, that is, a status-seeker associates a higher personal value with the prize V than does an individualist (whose personal value coincides with the material value). Just as in the example, the feasible range of values, $\lambda \in (1, \frac{n}{n-1})$, is bounded from above for the analysis in this chapter. However, as our analysis will later show, this does not always prevent status-seekers from wasteful competition.

Example 1 (continued) *Consider the example of linear interdependent utility functions defined by (3.4). Calculating the first order condition of status-seekers, we obtain*

$$\frac{n(\beta_1 + \beta_2)}{n\beta_1 + (n-1)\beta_2} r\hat{x}_j^{r-1} \left(\sum_{h \neq j} \hat{x}_h^r \right) V = \left[\sum_h \hat{x}_h^r \right]^2, \tag{3.10}$$

for any intra-group symmetric Nash equilibrium $\hat{\mathbf{x}} = ([a]_k, [b]_{n-k})$. Not surprisingly, equation (3.10) reduces to (FOCJ) for $\lambda = \frac{n(\beta_1+\beta_2)}{n\beta_1+(n-1)\beta_2}$.

Notice that the class of linear interdependent utility functions spans the range of implicit personal values of $\lambda V \in (V, \frac{n}{n-1}V)$. Moreover, for utility functions from this class, the interpretation as implicit personal value λV does not only apply in equilibrium (such as in the first order condition, (3.10)), but also describes optimal behavior out of equilibrium (such as seen in the participation constraint, (3.6)). In contrast, the out-of-equilibrium value of λ will depend on the absolute and relative payoff for alternative specifications of interdependent utility.

In case both groups take active roles in equilibrium, i.e., $a, b > 0$, we can combine the equations (FOCI) and (FOCJ) to obtain

$$a^{r-1}[(m-1)a^r + (n-m)b^r] = \lambda b^{r-1}[ma^r + (n-m-1)b^r]\,.$$

We divide both sides of this equation by b^{2r-1} and get

$$c^{r-1}[(m-1)c^r + (n-m)] = \lambda[mc^r + (n-m-1)] \tag{FOCIJ}$$

where we have substituted $c := \frac{a}{b}$. Together with the participation constraints (PCI) and (PCJ), this combined first order condition (FOCIJ) completely characterizes the intragroup symmetric equilibrium. Having determined c, we can then use the first order conditions (FOCI) and (FOCJ) to find the equilibrium solutions of a and b. These are given by

$$a = \frac{c^r[(m-1)c^r + n - m]}{[kc^r + n - m]^2} rV \qquad \text{and} \tag{SOLA}$$

$$b = \frac{[mc^r + n - m - 1]}{[mc^r + n - m]^2} r\lambda V = \frac{c^{r-1}[(m-1)c^r + n - m]}{[mc^r + n - m]^2} rV. \tag{SOLB}$$

Unfortunately, the combined first order condition (FOCIJ) does not allow the calculation of a general solution (which, of course, would depend on m, n, λ and r). Therefore, we have to fall back on numerical methods to find an equilibrium value of c. Fortunately, however, we are able to establish the strategic advantage of interdependent preferences without relying on numerical solutions.

3.3.3 Analyzing the strategic advantage

The following Lemma helps us to characterize the set of intragroup symmetric Nash equilibria that display the strategic advantage of interdependent preferences.

Lemma 3 *(i) For any* $m \in \{1, \ldots, n-1\}$, *any technology parameter* $r > 0$, *and any intragroup symmetric Nash equilibrium* $x \in N_{sym}(\Gamma_F(m))$, *we have*

$$\pi_j(\mathbf{x}) \overset{(>)}{\geq} \pi_i(\mathbf{x}) \qquad \Rightarrow \qquad x_j \overset{(>)}{\geq} x_i$$

for all $i = 1, \ldots, m$ *and all* $j = m+1, \ldots, n$.

(ii) For $m = n-1$, *for any technology parameter* $r > 0$, *and any intragroup symmetric Nash equilibrium* $\mathbf{x} \in N_{sym}(\Gamma_F(n-1))$, *we have*

$$\pi_n(\mathbf{x}) \overset{(>)}{\geq} \pi_i(\mathbf{x}) \qquad \Leftrightarrow \qquad x_n \overset{(>)}{\geq} x_i.$$

for all $i = 1, \ldots, n-1$.

Proof. See appendix. ■

Part (i) provides a necessary condition for the presence of a strategic advantage of interdependent preferences. Status-seekers (or interdependent preferences) can only be better off than individualists (or independent preferences) if they choose a higher effort level $x_j \geq x_i$. Part (ii) shows that a higher effort level is also sufficient for the strategic advantage of interdependent preferences in cases where only one player has interdependent preferences. Notice that part (ii) also covers the ubiquitous case of two-player contests.

The proof of Lemma 3 reveals that the individualists play the crucial role in deriving the necessary condition for a strategic advantage of interdependent preferences. Only when the individualists choose lower effort levels, $x_1 \leq x_n$, their choice x_1 can be optimal against x_n if there is a strategic advantage. Otherwise (i.e. if $x_1 > x_n$), the individualist could do strictly better with choosing the lower effort level x_n instead of x_1, which contradicts $\mathbf{x}$ representing an equilibrium.

In contrast, in part (ii), it is the single status-seeker who takes the key role in establishing the result. Even if there were no strategic advantage, the status-seeker nevertheless would choose the higher effort level $x_n > x_1$. Without a strategic advantage, the status-seeker has negative relative payoff. If she mimicked the individualists, she would realize zero relative payoff. This means, in order to be optimal in terms of her interdependent preferences, the effort level x_n must yield higher material payoff than effort level x_1, in contradiction to the non-existence of the strategic advantage. Thus, a higher effort level in equilibrium implies status-seekers have a strategic advantage.

3.4 Two-player contests

In this chapter, we focus on the ubiquitous special case of two-player contests. Typical examples include legal conflicts such as lawsuits or most contests in political lobbying.

One general result for two-player contests is the following theorem, which identifies two-player contests as the case where the status-seeker always experiences a strategic advantage over the individualist.

Theorem 1 *For any two-player contest,* $n = 2$, *any technology parameter* $r > 0$, *and any Nash equilibrium* $\mathbf{x} \in N_{sym}(\Gamma_F(1))$, *we have* $\pi_2(\mathbf{x}) \geq \pi_1(\mathbf{x})$.

The proof builds on part (ii) of Lemma 3. We establish that, in any equilibrium $\mathbf{x}$ where the status-seeker exerts lower effort than the individualist, she could increase both her relative and her material payoff by slightly increasing her effort level. Consequently, $\mathbf{x}$ cannot represent an equilibrium.

Proof. Let $r > 0$, fix $\mathbf{x} = (x_1, x_2) \in N_{sym}(\Gamma_F(1))$, and suppose to the contrary that $\pi_2(\mathbf{x}) < \pi_1(\mathbf{x})$. From Lemma 3, it follows that $x_2 < x_1$. Moreover, we have $x_2 > 0$, because otherwise x_1 cannot be a best response for player 1 against x_2. Since $\pi_i(x_1, x_2)$, $i = 1, 2$, is differentiable for $x_1, x_2 > 0$, it follows that $\frac{\partial \pi_1(x_1,x_2)}{\partial x_1} = 0$, which is equivalent to

$$r x_1^{r-1} x_2^r V = (x_1^r + x_2^r)^2. \tag{3.11}$$

But then $x_1 > x_2$ implies that

$$\frac{\partial \pi_2(x_1, x_2)}{\partial x_2} = \frac{r x_1^r x_2^{r-1} V}{(x_1^r + x_2^r)^2} - 1 = \frac{x_1}{x_2} - 1 > 0, \tag{3.12}$$

where the last equality follows from (3.11). Because of $\rho_2(x_1, x_2) = \frac{1}{2}[\pi_2(x_1, x_2) - \pi_1(x_1, x_2)]$ and

$$\frac{\partial \pi_1(x_1, x_2)}{\partial x_2} = -\frac{r x_1^r x_2^{r-1} V}{(x_1^r + x_2^r)^2} < 0,$$

inequality (3.12) implies $\frac{\partial \rho_2(x_1,x_2)}{\partial x_2} > 0$. Hence, there exist some $\hat{x}_2 \in (x_2, x_2+\varepsilon)$ such that

$$\pi_2(x_1, \hat{x}_2) > \pi_2(x_1, x_2) \qquad \text{and} \qquad \rho_2(x_1, \hat{x}_2) > \rho_2(x_1, x_2),$$

in contradiction to x_2 being optimal for player 2 against x_1 (in terms of player 2's interdependent preferences). Thus, $\pi_2(\mathbf{x}) \geq \pi_1(\mathbf{x})$. ■

As a consequence of Theorem 1, notice that no overdissipation can ever occur in two-player contests where an individualist and a status-seeker interact.

We conclude by investigating the two-player case for the class of linear interdependent preferences. In this case, we can solve for the equilibrium explicitly.

Example 1 (continued) *Combining the first order conditions of the two players, we obtain $b = a\lambda$ or, equivalently, $c = \frac{a}{b} = \frac{1}{\lambda}$. Replacing c in (SOLA) and (SOLB) by $\frac{1}{\lambda}$, one can determine the equilibrium values of a and b, respectively. Building on these values, it is possible to calculate material equilibrium payoffs and to derive participation constraints that hinge upon the parameter values of λ and r only.*

3.5 Non-increasing marginal efficiency – The case $r \leq 1$

After looking at the case of two-player contests, we now extend our analysis to general n-player contests. Throughout the rest of this chapter, we assume $n > 2$. For these general n-player contests, the case of increasing marginal efficiency ($r > 1$) can result in non-participation by one of the two groups. Therefore, we split our analysis into two parts: the case of non-increasing marginal efficiency, presented in this section, and the case of increasing marginal efficiency, which we postpone to the next section.

The following theorem establishes the strategic advantage of status-seeking preferences for the case of non-increasing marginal efficiency. More precisely, we show (i) that at most one intragroup symmetric equilibrium exists, (ii) that status-seekers exert higher effort in equilibrium, and (iii) that this implies a strategic advantage of negatively interdependent preferences.

Theorem 2 *Fix $r \leq 1$ and any $m \in \{1, \dots, n-1\}$ and let $\hat{\mathbf{x}} = ([a]_m, [b]_{n-m}) \in N_{sym}(\Gamma_F(m))$. Then, (a) $\hat{x}$ is the only intragroup symmetric equilibrium; (b) interdependent preferences yield a strategic advantage over independent preferences, i.e. $\pi_n(\mathbf{x}) \geq \pi_1(x)$; (c) all players exert positive effort levels, status-seekers a higher one, i.e., $b \geq a > 0$; and (d) if $b > a$ then the strategic advantage is strict, $\pi_n(\mathbf{x}) > \pi_1(\mathbf{x})$. Finally, (e) if F is differentiable, then $b > a$, in fact, holds so that the strategic advantage is strict, i.e., $\pi_n(\mathbf{x}) > \pi_1(\mathbf{x})$.*

Proof. See appendix. ■

Although, in general, it is not possible to determine the equilibrium strategies for arbitrary values of $r \leq 1$, one can solve the first order conditions for the special case of linear contest technologies, i.e., where $r = 1$.

3.5.1 The special case of constant marginal efficiency $(r = 1)$

In order to illustrate Theorem 2, we determine the equilibrium for the class of preferences introduced in Example 1:

Example 1 (continued) *Consider again the example of linear interdependent preferences. The first order condition of the maximization problem* $\max_{x_j} F_j(x)$ *is:*

$$\left(\sum_{h=1}^{n} x_h\right)^2 = \frac{V\left(\beta_1+\beta_2\right)}{\beta_2\left(n-1\right)+\beta_1 n}\sum_{h\neq n} x_h, \tag{3.13}$$

as derived in the appendix. Assuming intragroup symmetry, the first order condition of status-seekers becomes

$$\left(ma+\left(n-m\right)b\right)^2 = \frac{V\left(\beta_1+\beta_2\right)}{\beta_2\left(n-1\right)+\beta_1 n}\left(ma+\left(n-m-1\right)b\right), \tag{3.14}$$

while the first-order condition for individualists reads

$$\left(ma+\left(n-m\right)b\right)^2 = \frac{V}{n}\left(\left(m-1\right)a+\left(n-m\right)b\right). \tag{3.15}$$

In equilibrium, both (3.14) and (3.15) hold, such that the respective right hand sides have to be equal to each other, which implies

$$c = \frac{a}{b} = \frac{\beta_1 n+\beta_2 m}{\beta_2\left(n+m-1\right)+\beta_1 n} \tag{3.16}$$

as derived in the appendix. By Theorem 2, the strict strategic advantage is strict if $a < b$ *or* $\frac{\beta_1 n+\beta_2 m}{\beta_1 n+\beta_2 m(n+m-1)+\beta_1 n} < 1$*, which is equivalent to*

$$\beta_1 n+\beta_2 m < \beta_2\left(n+m-1\right)+\beta_1 n \Leftrightarrow 0 < \beta_2\left(n-1\right) \Leftrightarrow n > 1.$$

Thus, in an intragroup symmetric linear rent-seeking contest with linear interdependent preferences, a status-seeker always has a strict strategic advantage over an individualist, since the last condition, $n > 1$*, is trivially satisfied. The equilibrium values of* a *and* b *can be explicitly calculated by inserting* c *into (SOLA) and (SOLB), respectively.*

Finally, we illustrate that we cannot skip the restriction of differentiability of F in Theorem 2. To this end, we present a discontinuous and hence non-differentiable but increasing utility function F, where a status-seeker cannot gain by increasing her effort b above a. Suppose the equilibrium is $([a]_m, [a]_{n-m})$, where both groups of players choose strategy a.

Proposition 2 $\exists\xi = \frac{V^2}{\varepsilon n}$*, with* $\varepsilon \in]0;a[$*, such that for the utility function*

$$F_j\left(\pi_j(\mathbf{x});\rho_j(\mathbf{x})\right) = \begin{cases} \xi+\rho_j(\mathbf{x}) & \text{if } \pi_j(\mathbf{x}) \geq \frac{V}{n}-a \\ \rho_j(\mathbf{x}) & \text{if } \pi_j(\mathbf{x}) < \frac{V}{n}-a \end{cases} \tag{3.17}$$

in an intragroup symmetric equilibrium, the status-seekers will choose $b = a$.

Proof. Notice that $\frac{V}{n} - a$ is the maximum value that absolute payoff takes in Nash-equilibrium. Hence, with higher effort $x_j = b$, the absolute payoff shrinks so that the lower line of equation (3.17) is the relevant one to any player j. Furthermore, for $a = b$, we have $\rho_j(\mathbf{x}) = 0$, since all players choose the same strategy. The value of (3.17) in this situation is $F_j = \xi$.

The question, we address next, is how much is the maximum that player j can get by increasing her effort. To this aim, we maximize ρ subject to $b > a$. The maximization problem $\frac{\partial \rho_j(\mathbf{x})}{\partial b} \stackrel{!}{=} 0$ implies

$$b = \sqrt{Van} - a\,(n-1)\,, \tag{3.18}$$

which is larger than a in the case of underdissipation (see appendix). The second-order condition is satisfied. Inserting (3.18) in (2.1) and (3.2) yields

$$\begin{aligned} \rho_j(\mathbf{x}) \quad &= \tfrac{n-1}{n}\pi_j(\mathbf{x}) - \tfrac{1}{n}\textstyle\sum_{h\neq j}\pi_h(\mathbf{x}) \\ &= \tfrac{n-1}{n}\left(\tfrac{V}{\sqrt{Van}} - 1\right)\left(\sqrt{Van} - an\right) \\ &< \left(\tfrac{\sqrt{Van}}{an} - 1\right)(V - an) \\ &\leq \left(\tfrac{V}{\varepsilon n} - 1\right)V < \tfrac{V^2}{\varepsilon n} = \xi, \end{aligned} \tag{3.19}$$

where the second equality is established in the appendix. Thus, there exists a ξ, such that no player j with utility function (3.17) can reach a higher utility level when increasing her effort b above the effort level a of her opponents. ■

3.6 Increasing marginal efficiency - The case $r > 1$

We finally turn to investigating the case of increasing marginal efficiency, i.e. $r > 1$. In this case, we make out four different types of equilibria depending on (a) which group exerts higher effort and on (b) whether both groups or just one group take active roles in equilibrium. Having characterized each equilibrium type separately, we conclude with illustrating that different equilibrium types can, in fact, result from the same parameter constellation.

More specifically, for $r > 1$ sufficiently small, it turns out that both groups always participate actively in equilibrium. Depending on the interdependent utility function F there might now exist an additional equilibrium, where individualists choose higher expenditure levels than status-seekers. In this equilibrium, individualists will earn higher payoff. Moreover, we illustrate that the existence of this equilibrium requires that status-seekers put a higher weight on relative payoff when they opt out of the contest than when they participate actively in equilibrium. Put intuitively, when status-seekers' preferences display a strong aversion to leaving a contest, this will be costly to them in equilibrium.

However, if status-seekers' relative weight between absolute and relative payoff is constant, then there is only one equilibrium where both groups take active roles. Moreover, for $r > 1$ sufficiently small, there is no other equilibrium type. In that case, interdependent preferences unambiguously yield a strategic advantage over independent preferences.

For larger values of r, we observe two additional types of equilibria, where only one group of players – either status-seekers or individualists – takes an active role in equilibrium. Here, the relative size of the two groups, $\frac{m}{n-m}$, determines the size of the parameter regions associated with these two additional types of equilibria.

In the equilibrium where status-seekers drive individualists out of the contest, the strategic advantage prevails, provided that $r > 1$ is relatively small. However, for higher values of r, the competition within the group of status-seekers intensifies, resulting in a negative material payoff. Notice that opting out is not an option for these players (provided r is not too large). Opting out by one status-seeker would raise the other status-seekers' absolute payoff above the zero payoff level, while it fixes her own payoff and the payoff of individualists to zero. In this type of equilibrium, since opting out will reduce a status-seeker's own relative payoff, the status-seeker would reject this option. Finally, if r is too large, opting out becomes profitable (and there is no longer any such equilibrium in pure strategies).

In the equilibrium where only individualists choose positive effort levels and status-seekers refrain from competition, individualists realize non-negative payoff, while status-seekers end up with zero payoff. Basically, this parameter region is characterized by the fact that all active players realize very small payoffs: Any status-seeker that entered with some arbitrary positive effort level would earn negative material payoff.

Finally, if $r > 1$ is too large, then no intragroup symmetric equilibrium in pure strategies exists and (presumably) we are "back to the bog" of mixed strategy equilibria.

Altogether, we have to deal with the four above-mentioned types of equilibria. A fifth type of effort profile, where both groups remain inactive, does not represent an equilibrium, since, in that case, any player has an incentive to deviate. By exerting slightly positive effort she could gain the full prize.

We call the four possible equilibrium types

Type I: $b \geq a > 0$ – both groups are active in equilibrium, status-seekers choose higher effort levels;

Type II: $a > b > 0$ – both groups are active in equilibrium, individualists choose higher effort levels;

Type III: $b > a = 0$ – only status-seekers engage in the contest;

Type IV: $a > b = 0$ – only individualists are active.

Below, we establish these results and address whether status-seekers experience a strategic advantage over individualists. Subsequently, we relate the equilibrium types to the contest parameter $r > 1$. We find the following four equilibrium ranges:

$$\begin{array}{lll}
\text{Type I:} & R_I := (1, \overline{r}_I) & \overline{r}_I \leq \frac{n}{n-1} \leq \frac{3}{2}, \\
\text{Type II:} & R_{II} := (1, \overline{r}_{II}) & \overline{r}_{II} \leq \min\{\frac{m}{m-1}, \frac{1}{\lambda}\frac{n^2}{(n-1)^2}\} \leq \frac{9}{4}, \\
\text{Type III:} & R_{III} := (\underline{r}_{III}, \overline{r}_{III}) & \underline{r}_{III} > 1 \text{ and } \overline{r}_{III} \leq \frac{1}{\lambda}\frac{n}{n-1}\frac{n-m}{n-m-1} \leq 3, \\
\text{Type IV:} & R_{IV} := (\underline{r}_{IV}, \overline{r}_{IV}) & \underline{r}_{IV} > 1 \text{ and } \overline{r}_{IV} \leq \frac{m}{m-1} \leq 2.
\end{array}$$

We will show that $n - m > 1$ is necessary for equilibria of type III and $m > 1$ for equilibria of type IV so that the corresponding expressions above are well-defined. For equilibria of type II and in case of $m = 1$, one has to replace the inequality with $\overline{r}_{II} \leq \frac{n^2}{\lambda(n-1)^2}$.

Finally, for $r > \max\{\overline{r}_I, \overline{r}_{II}, \overline{r}_{III}, \overline{r}_{IV}\}$, there is no equilibrium in pure strategies. In this work, we refrain from working out the equilibria in mixed strategies that presumably occur in this parameter region.

3.6.1 Characterization of equilibrium types

We start with characterizing equilibrium types I and II, where both groups of players choose positive expenditure levels. Moreover, as the subsequent characterization of the other equilibrium types III and IV will show, equilibria of type I and II are the only equilibrium candidates for $r > 1$ sufficiently small.

Equilibrium Type I

Let $\hat{\mathbf{x}} = ([a]_m, [b]_{n-m})$ denote an intragroup symmetric equilibrium with $b \geq a > 0$. Equilibria of type I are present whenever $r > 1$ and r is sufficiently close to one.

Before we state the theorem, we rewrite the participation constraints as functions of $c = \frac{a}{b}$. By assumption, we have $c \leq 1$. Individualists $i \in I$ participate if and only if $\pi_i(\hat{\mathbf{x}}) \geq 0$, i.e., if

$$\frac{a^r}{ma^r + (n-m)b^r}V - a = \frac{c^r}{mc^r + n - m}V - \frac{c^r[(m-1)c^r + n - m]}{[mc^r + n - m]^2}rV \geq 0,$$

which is equivalent to

$$c^r[m - r(m-1)] \geq (r-1)(n-m), \tag{3.20}$$

or (given that $m > 1$) to

$$r < \frac{m}{m-1} \qquad \text{and} \qquad c \geq \underline{c} := \left[\frac{(r-1)(n-m)}{m - r(m-1)}\right]^{\frac{1}{r}}. \tag{3.21}$$

Notice that the restriction $c \leq 1$ implies $\underline{c} \leq 1$ or $(r-1)(n-m) \leq m - r(m-1)$, which is equivalent to $r \leq \frac{n}{n-1}$. Since $m < n$ implies $\frac{n}{n-1} < \frac{m}{m-1}$, it follows that individualists participate if and only if $r \leq \frac{n}{n-1}$ and $c \geq \underline{c}$. Moreover, we have that $\overline{r}_I \leq \frac{n}{n-1}$.

Similarly, status-seekers participate only if $\rho_j(0, \hat{\mathbf{x}}_{-j}) \leq \rho_j(\hat{\mathbf{x}})$, which is equivalent to $\pi_j(\hat{\mathbf{x}}) + \frac{b}{n} = \frac{V}{mc^r + n - m} - \frac{n-1}{n}b \geq 0$. Inserting (SOLB), we obtain

$$n\left[mc^r + n - m\right] \geq r\lambda(n-1)\left[mc^r + n - m - 1\right], \tag{3.22}$$

which provides another upper boundary on r (if $m < n - 1$), because $c > 0$ implies

$$r \leq \frac{n\left[mc^r + n - m\right]}{\lambda(n-1)\left[mc^r + n - m - 1\right]} < \frac{n(n-m)}{\lambda(n-1)(n-m-1)}. \tag{3.23}$$

Combining the two upper boundaries on r, we conclude that

$$\bar{r}_I \leq \min\left\{\frac{n(n-m)}{\lambda(n-1)(n-m-1)}, \frac{n}{n-1}\right\} = \frac{n}{n-1}$$

if $m < n-1$. The equality holds because $\lambda < \frac{n}{n-1}$ implies $\frac{n(n-m)}{\lambda(n-1)(n-m-1)} > \frac{n-m}{n-m-1} > \frac{n}{n-1}$.

Finally, since in any equilibrium of type I both groups participate, the first order condition is given by (FOCIJ).

Theorem 3 *Fix $r > 1$, any $m \in \{1, \ldots, n-1\}$ and let $\hat{\mathbf{x}} = ([a]_m, [b]_{n-m}) \in N_{sym}(\Gamma_F(m))$ such that $b \geq a > 0$. Then, status-seekers have a strategic advantage over individualists.*

Proof. See appendix. ■

While equilibria of type I represent the type of equilibrium that one would expect after the analysis in the preceding sections, it turns out that other types of equilibria can be present, depending on the contest parameter r and, as in the following subsection, on the type of status-seeking preferences.

Equilibrium type II

Under equilibrium type II, both groups are active in equilibrium and individualists choose higher expenditure levels than status-seekers. Accordingly, let $\hat{\mathbf{x}} = ([a]_m, [b]_{n-m})$ denote any intragroup symmetric equilibrium such that $a > b > 0$. From part (i) of Lemma 3, it directly follows that individualists will always earn higher payoff in equilibrium. The question remains whether there exists such an equilibrium. Our first result in this paragraph shows that equilibria of type II do not exist if the interdependent utility function of status-seekers, F, is linear in both absolute and relative material payoff.

Theorem 4 *Fix $r > 1$ and $m \in \{1, \ldots, n-1\}$. Suppose $F(\pi, \rho) = \beta_1\pi + \beta_2\rho$, where $\alpha, \beta > 0$. Then, no intragroup symmetric equilibrium of type II exists.*

To establish the Theorem, the following Lemma is helpful. We replace the aggregate behavior of other players in the first order condition of status-seekers (FOCJ), that is $ma^r + (n-k-1)b^r$, with Z and obtain

$$rb^{r-1}Z\lambda V = [Z + b^r]^2. \tag{3.24}$$

Then, Lemma 4 derives properties of the function $b(Z, \lambda)$, which is implicitly defined by (3.24). Similarly, we define $B(Z, \lambda)$ by setting $B(Z, \lambda) := [b(Z, \lambda)]^r = b^r(Z, \lambda)$.

Lemma 4 *Fix $r > 1$ and any $m \in \{1, \ldots, n-1\}$ and let $\hat{\mathbf{x}} = ([a]_m, [b]_{n-m}) \in N_{sym}(\Gamma_F(m))$ with $a, b > 0$. Then, we have*
(i) $\frac{\partial B(Z,\lambda)}{\partial Z} \geq -1 \quad \Leftrightarrow \quad Z \leq \frac{r}{r-1}B$,
(ii) $\frac{\partial b(Z,\lambda)}{\partial \lambda} > 0 \quad \Leftrightarrow \quad Z < \frac{r+1}{r-1}B$,
(iii) $Z \leq \frac{r}{r-1}B \quad \Rightarrow \quad a \leq b$.

Proof of the Lemma. See appendix. ■

Parts (i) and (ii) derive conditions that characterize partial derivatives of $B(Z,\lambda)$ and $b(Z,\lambda)$, respectively, which then, in part (iii), are shown to be sufficient for the non-existence of equilibrium type II. Therefore, to prove Theorem 4, it is sufficient to establish that, for any intragroup symmetric equilibrium, the sufficient condition in part (iii) is satisfied.

Proof of Theorem 4. Let $\hat{\mathbf{x}} = ([a]_m, [b]_{n-m}) \in N_{sym}(\Gamma_F(m))$, where $F(\pi,\rho) = \beta_1\pi + \beta_2\pi$ and $\beta_1, \beta_2 > 0$. Without loss of generality, let player n be a representative status-seeker and set $\check{\mathbf{x}} = ([a]_m, [b]_{n-m-1}, 0)$. Set $\gamma = \frac{\beta_2}{\beta_1+\beta_2}$. Then, player n will participate in equilibrium, i.e., $b > 0$, if and only if

$$\begin{aligned} & F_n(\hat{\mathbf{x}}) \geq F_n(\check{\mathbf{x}}) \\ \Leftrightarrow\ & (1-\gamma)\pi_n(\hat{\mathbf{x}}) + \gamma\rho_n(\hat{\mathbf{x}}) \geq (1-\gamma)\pi_n(\check{\mathbf{x}}) + \gamma\rho_n(\check{\mathbf{x}}) \\ \Leftrightarrow\ & \pi_n(\hat{\mathbf{x}}) - \frac{\gamma}{n}\left[V - \sum_{h=1}^{n}\hat{x}_h\right] \geq 0 - \frac{\gamma}{n}\left[V - \sum_{h=1}^{n}\check{x}_h\right] \\ \Leftrightarrow\ & \frac{b^r V}{ma^r + (n-m)b^r} - \frac{n-\gamma}{n} b \geq 0, \end{aligned}$$

where the last equivalence follows from $\check{x}_h = \hat{x}_h$ for all $h \neq n$. Substituting (SOLB), $c = \frac{a}{b}$, and $\lambda = \frac{n}{n-\gamma}$, we can equivalently express this as

$$\begin{aligned} & \frac{V}{mc^r + n - m} \geq \frac{1}{\lambda}\frac{mc^r + n - m - 1}{(mc^r + n - m)^2} r\lambda V \\ & \Leftrightarrow mc^r + n - m \geq r\left[mc^r + n - m - 1\right]. \end{aligned}$$

Finally, inserting $c = \frac{a}{b}$ gives us

$$\begin{aligned} ma^r + (n-m)b^r &\geq r\left[ma^r + (n-m-1)b^r\right] \\ \Leftrightarrow Z + B \geq rZ, & \end{aligned}$$

which implies $Z \leq \frac{1}{r-1}B < \frac{r}{r-1}B$ and, by Lemma 4 (iii), $a \leq b$. Thus, no equilibrium of type II exists. ■

Notice that the Proof of Theorem 4 does not fully exploit the power of Lemma 4, since the lemma also implies non-existence of type II equilibria for larger values of Z (namely for $Z \in [\frac{1}{r-1}B, \frac{r}{r-1}B]$).

Theorem 4 confirms that, for interdependent utility functions with constant relative weight between absolute and relative material payoff, there is only one equilibrium type with both groups taking active roles, namely, equilibrium type I. Thus, in that case, status-seekers unambiguously experience a strategic advantage over individualists. However, the following two examples illustrate that an equilibrium of type II can, in fact, result if we allow for utility functions that put higher relative weight on relative material payoff under opting out than under participation in equilibrium.

Example 2 *Consider* $(r,m,n,V) = (1.41, 2, 4, 1)$ *and solve (FOCIJ) for* $\lambda = 1.01$ *numerically. This gives* $c = 2.9884$ *and the corresponding equilibrium values of* a *and* b

as $(a,b) = (0.3416, 0.1143)$. *The absolute material payoffs of individualists and status-seekers amount to* $(\pi_i, \pi_j) = (0.0704, -0.0263)$, *respectively. The relative (material) payoff in equilibrium* $\hat{\mathbf{x}}$ *is* $\rho_j(\hat{\mathbf{x}}) = -0.0484$ *and under "opting out",* $\check{\mathbf{x}}$*, it amounts to* $\rho_j(\check{\mathbf{x}}) = -0.0506$. *Consequently, the necessary condition for participation of status-seekers is satisfied (i.e.* $\rho_j(\hat{\mathbf{x}}) \geq \rho_j(\check{\mathbf{x}})$*). Then, it turns out that*

$$F(\pi(\mathbf{x}), \rho(\mathbf{x})) = s_1 \pi(\mathbf{x})^{2s_3+1} + s_2 \pi(\mathbf{x}) + \rho(\mathbf{x}) \tag{3.25}$$

for $s_1 = 2.378 \cdot 10^{441}, s_2 = 0.000001$ *and* $s_3 = 140$*, represents an interdependent utility function that*
(a) gives rise to a first order condition (FOCJ) that, for $\lambda = 1.01$*, has the same solution* $(a,b) = (0.3416, 0.1143)$ *as the first order condition (FOCIJ) above and that*
(b) satisfies the necessary and sufficient participation constraint of status-seekers, (PCJ), $F_j(\hat{\mathbf{x}}) - F_j(\check{\mathbf{x}}) = 0.00001240 \geq 0$.

Example 3 *Set* $(r, m, n, V) = (1.01, 2, 4, 1)$ *and solve (FOCIJ) for* $\lambda = 1.000001$*. This results in* $c = 1016.5738$*. The corresponding equilibrium values of* a *and* b *can be calculated using (SOLA) and (SOLB). An interdependent utility function implying* $\lambda = 1.000001$ *and satisfying the participation constraint of status-seekers is then given by the above example, (3.25), for* $s_1 = 9.576 \cdot 10^{67244}, s_2 = 0.000001$ *and* $s_3 = 7293$.

Example 2 shows that equilibria of type II might exist even when equilibria of type I do not (notice that $r = 1.41 > \frac{n}{n-1} = \frac{4}{3}$). In contrast, Example 3 illustrates that type II equilibria also exist if $r > 1$ is close to one (possibly for some different utility function of status-seekers). From each example separately, it follows that Theorem 4 cannot be generalized to interdependent utility functions that are additively separable in absolute and relative payoff.

Let us finally comment on the conditions that characterize equilibria of type II. First, participation by individualists requires that inequality (3.20) is satisfied. Since c now exceeds one, we can no longer conclude that $\underline{c} \leq 1$. This is actually the case in Example 2, where $\underline{c} \simeq (1.3898)^{\frac{1}{1.41}} \simeq 1.2630$ and $c \simeq 2.9884$. It is hence compatible with equilibria of type II that $r > \frac{n}{n-1}$ (like in Example 2). Accordingly, it is now the conditions $r < \frac{m}{m-1}$ (for $m > 1$) and $c \geq \underline{c}$ that characterize participation by individualists (for $m = 1$, this participation constraint reduces to $c \geq \underline{c}$).

While the participation constraint of individualists has changed, the necessary condition for participation of status-seekers (3.22) and the first order condition characterizing equilibrium behavior (FOCIJ) remain the same as under equilibrium type I. Similarly, the necessary participation constraint of status-seekers provides an upper boundary on r:

$$r \leq \frac{n\,[mc^r + n - m]}{\lambda(n-1)\,[mc^r + n - m - 1]} \leq \frac{n^2}{\lambda(n-1)^2},$$

where the second inequality follows now from $c \geq 1$. We can thus conclude that $\overline{r}_{II} \leq \min\{\frac{m}{m-1}, \frac{1}{\lambda}\frac{n^2}{(n-1)^2}\}$.

Equilibrium Type III

We continue with equilibrium type III, where status-seekers choose such high effort levels that they drive individualists out of the contest. Let $\hat{\mathbf{x}} = ([a]_m, [b]_{n-m})$ denote a corresponding intragroup symmetric equilibrium with $b > a = 0$.

On the one hand, if there is more than one status-seeker, i.e., if $n - m > 1$, then the first order condition of status-seekers FOCJ gives us the equilibrium value of b,

$$b = \frac{(n-m-1)}{(n-m)^2} r\lambda V. \tag{3.26}$$

Non-participation is optimal to individualists if

$$\frac{(r-1)^{r-1}}{r^{2r}} \leq \lambda^r \frac{(n-m-1)^r}{(n-m)^{2r-1}}. \tag{3.27}$$

Participation of status-seekers can only be optimal if $\rho_j(\hat{\mathbf{x}}) \geq \rho_j(0, \hat{\mathbf{x}}_{-j})$, which is equivalent to $\pi_j(\hat{\mathbf{x}}) + \frac{b}{n} \geq 0$. Inserting (3.26) we obtain

$$r \leq \frac{1}{\lambda} \frac{n}{n-1} \frac{n-m}{n-m-1}. \tag{3.28}$$

It follows that $\overline{r}_{III} \leq \frac{1}{\lambda}\frac{n}{n-1}\frac{n-m}{n-m-1}$. Interdependent preferences yield a strategic advantage if and only if $\pi_j(\hat{x}) \geq 0 = \pi_i(\hat{x})$ or, equivalently, if

$$r \leq \frac{1}{\lambda} \frac{n-m}{n-m-1}. \tag{3.29}$$

Notice that

$$\frac{1}{\lambda}\frac{n-m}{n-m-1} < \frac{1}{\lambda}\frac{n}{n-1}\frac{n-m}{n-m-1},$$

so that a negative payoff to status-seekers is compatible with their participation in equilibrium.

On the other hand, for $n - m = 1$, there is no equilibrium of type III. In this case, for instance, $\check{x}_n = \frac{b}{2} > 0$ would imply both higher absolute payoff, $\pi_n(\hat{\mathbf{x}}) < \pi_n([0]_{n-1}, \frac{b}{2})$, and, by Lemma 2, higher relative payoff. Consequently, $\hat{\mathbf{x}} = ([0]_{n-1}, b)$ cannot represent an intra-group symmetric Nash equilibrium.

Theorem 5 summarizes our findings:

Theorem 5 *Fix $r > 1$ and any $k \in \{1, \ldots, n-2\}$ and let $\hat{x} = ([0]_m, [b]_{n-m}) \in N_{sym}(\Gamma_F(m))$ with $b > 0$. Then, conditions (3.26), (3.27) and (3.28) hold. Interdependent preferences yield a strategic advantage if and only if condition (3.29) applies. For $m = n-1$ no equilibrium of type III exists.*

Observe that constraint (3.27) provides a lower bound on r, denoted by $\underline{r}_{III}$, that strictly exceeds one. To see $\underline{r}_{III} > 1$, take the limit $r \to 1$ on both sides of (3.27). While the left hand side goes to one, the right hand side converges to $\lambda\frac{n-m-1}{n-m}$, which is strictly lower than one because of

$$\lambda\frac{n-m-1}{n-m} \leq \frac{n}{n-1}\frac{n-2}{n-1} < 1.$$

Hence, in the limit, constraint (3.27) is not satisfied. On the other hand, for $r = \frac{n-m}{n-m-1} \leq \overline{r}_{III}$ this constraint holds. Therefore, it follows from continuity of both sides in (3.27) that there exists an $\underline{r}_{III} \in (1, \frac{n-m}{n-m-1})$ such that there is no equilibrium of type III in the interval $(1, \underline{r}_{III})$.

Equilibrium type IV

Finally, consider the remaining type of intragroup symmetric equilibria $\hat{\mathbf{x}} = ([a]_m, [b]_{n-m})$, where $a > b = 0$. Similar to the above, there is no equilibrium of type IV if $m = 1$. However, for $m > 1$, it follows from the first order condition of individualists, (FOCI), that

$$a = \frac{m-1}{m^2} rV, \tag{3.30}$$

where individualists participate if and only if $\pi_i(\mathbf{x}) \geq 0$. Participation is equivalent to

$$r \leq \frac{m}{m-1}, \tag{3.31}$$

which implies $\overline{r}_{IV} \leq \frac{m}{m-1}$.

Obviously, given this type of equilibrium, status-seekers never experience a strict strategic advantage. They optimally remain out of the contest if $\pi_j(x_j, \hat{\mathbf{x}}_{-j}) < \pi_j(\hat{\mathbf{x}}) = 0$, for all positive effort levels where $x_j > 0$. Inserting the equilibrium value of a, we obtain that non-participation of status-seekers can only be optimal if

$$\frac{(r-1)^{r-1}}{r^{2r}} \leq \frac{(m-1)^r}{m^{2r-1}}. \tag{3.32}$$

Theorem 6 *Let $r > 1$, $m \in \{2, \dots, n-1\}$ and $\hat{\mathbf{x}} = ([a]_m, [0]_{n-m}) \in N_{sym}(\Gamma_F(m))$ with $a > 0$. Then, we have $r \leq \frac{m}{m-1}$ and the conditions (3.30) and (3.32) apply. Individualists realize higher payoff. For $m = 1$, no equilibrium of type IV exists.*

Observe that each type IV equilibrium, such as characterized in the above theorem, exactly corresponds to a Nash equilibrium of the standard Tullock contest between m players, where all of them have standard (i.e. individualistic) preferences. Since these contests have a unique equilibrium for any $r \leq \frac{m}{m-1}$, the same holds for equilibria of type IV.

Corollary 4 *For any $r > 1$, there exists at most one equilibrium of type IV.*

Similar to the remark in the end of the discussion of Equilibrirm Type III, it can be shown that constraint (3.32) provides a lower bound $\underline{r}_{IV} < \overline{r}_{IV}$ such that no equilibrium of type IV exists in the interval $(1, \underline{r}_{IV})$. Thus, for $r > 1$ sufficiently small, namely for $r \in (1, \min\{\underline{r}_{III}, \underline{r}_{IV}\})$, we can only have equilibria of types I and II.

3.6.2 Equilibrium multiplicity

In order to illustrate that the equilibrium types I, III and IV can occur for the same constellation of parameter values, we refer to the example characterized by the parameter values $\lambda = 1.001$, $n = 10$; $m = 5$ and $r = 1.1$. Without loss of generalit, we set $V = 1$.

First let us investigate equilibrium type I, where members of both groups are active and $b > a$. Solving (FOCIJ) numerically for c yields a ratio of expenditure levels of $c \approx 0.9560$. Inserting c in (SOLA) and (SOLB) gives us the effort level of each group, $a = 0.0968$ and $b = 0.1013$. Then, we have to ensure participation of each group. Individualists optimally participate because they receive positive absolute payoff under participation, $\pi_i = 0.0007 > 0$. Status-seekers' absolute payoff is $\pi_j = 0.0012 > \pi_i$. Since opting out would lead to zero absolute payoff, it follows from Corollary 2 that status-seekers also optimally participate in this contest.

Equilibrium type III can occur for this parameter constellation, provided inequalities (3.26) to (3.28) hold. Inequality (3.26) applies for an effort of $b = 0.176$. Inserting the above parameter values in (3.27) yields $0.644 \leq 0.667$, which tells us that there is no incentive for the individualists to join the game. Participation of the status-seekers is guaranteed by (3.28) because this inequality is satisfied as well: $1.1 \leq 1.388$. Finally, inequality (3.29) holds with $1.1 \leq 1.249$, which implies the strategic advantage of the status-seekers. Each status-seeker experiences an absolute material payoff of $\pi_j = 0.024$.

For an equilibrium of type IV, three similar conditions have to apply. First, inequality $r \leq \frac{m}{m-1}$ becomes $1.1 \leq 1.25$ for our set of parameter values and hence is satisfied. Second, the optimality condition of individualists (3.30) is fulfilled if they choose $a = 0.176$. Third, inequality (3.32) holds because of $0.644 \leq 0.666$, ensuring that status-seekers do not participate. As mentioned earlier, individualists earn higher payoff, since status-seekers opt out of the contest, while individualists experience non-negative payoff of $\pi_i = 0.024$.

Having illustrated that different equilibria can occur for the same parameter constellation, we content ourselves with noting that the instance of equilibrium multiplicity creates an equilibrium selection problem. This has to be taken into account when applying our results.

3.7 Concluding remarks

In this chapter, we have examined symmetric rent-seeking contests where two groups of players are heterogenous with respect to preferences. One group has independent preferences, while the other group acts according to interdependent preferences. The focus of our analysis was whether status-seekers experience a strategic advantage over individualists. We found that such a strategic advantage has two major implications in rent-seeking contests. First, whenever status-seekers experience a strategic advantage, aggregate effort never exceeds the value of the rent at stake, i.e., there is no overdissipation. Second, a strategic advantage of status-seekers is not compatible with them exerting lower effort than individualists. In cases where one status-seeker only interacts with individualists, the converse holds.

Subsequently, we investigated for which classes of rent-seeking contests the strategic advantage is present in equilibrium. It turned out that the ubiquitous case of two-player contests and the class of n-player contests with non-increasing marginal efficiency ($r \leq 1$) display this property. This result extends to n-player contests with increasing marginal efficiency ($r > 1$), if status-seekers maximize some convex combination of absolute and relative material payoff and if $r > 1$ and r is sufficiently small. However, for general interdependent utility functions, there can exist another type of equilibrium, where individualists exert higher effort and earn higher material payoff. (This type can exist for $r > 1$ sufficiently small as well as for larger values of r). For larger values of r, we also observe two additional types of equilibria, where only one group of players are active in equilibrium and the other group of players exert no effort at all. Thus, for general contests with increasing marginal efficiency, we can no longer conclude that status-seekers (or negatively interdependent preferences) have a strategic advantage over individualists (or independent preferences).

Next, we comment on the two suggested applications to the theory of strategic delegation. Without further investigations, we can only address delegation problems, where the delegating decision-maker is mainly concerned with relative performance (e.g. market share). Due to the payoff structure of rent-seeking contests, it seems obvious, however, that our conclusions (at least locally) extend to delegation problems where absolute payoff objectives are in the foreground.

Let us first consider the problem whether to delegate or not (and, in case of delegation, which incentives to provide the delegate with). From our results, it follows that, for rent-seeking contests with two players ($n = 2$) or with non-increasing marginal efficiency ($r \leq 1$), any subgame-perfect equilibrium will involve a positive delegation decision. A delegating firm can perform better than its opponents if she compensates her delegate according to relative rather than to absolute performance. For contests with increasing marginal efficiency ($r > 1$), there is no definitive answer. One first has to solve the equilibrium selection problem.

Given a positive delegation decision in the above delegation problem, the second delegation problem arises. Which individual should one select from a pool of delegates that are heterogeneous with respect to the degree of interdependency in utility. In this second application, we refer to intrinsic rather than to extrinsic incentives in delegation. Again, we find a positive answer for the case of two players or for contests with non-increasing marginal efficiency. In these two cases, the decision-maker should always pick the status-seeker, when she can choose between a status-seeker on one hand and an individualist on the other hand.

However, in the end, we find a type of prisoner's dilemma for both rent-seeking delegation games. Comparing the equilibrium, where all other firms choose to compensate according to relative performance rather than absolute performance (or to pick status-seekers rather than individualists), material payoff will be lower than in the situation where all firms refrain from doing so.

The theory shows us that, in a two-player-contest, the player that maximizes relative payoff and, therefore, shows spiteful behavior will be more successful. The presence of the strategic advantage we pointed out can, therefore, explain why two candidates for the same position have a **strategic** incentive to behave spitefully against each other in an election campaign. If one candidate acts spitefully while the other one does not, the candidate with the spiteful behavior is more likely to win the race. This also held for the race for between Barack Obama and Hillary Clinton. Although both these candidates show respect for each other, as they say after the race, and even consider a cooperation (Süddeutsche 2008), they did not avoid an aggressive battle against each other. Since the only aim of the campaign was to prove to be more capable than the opponent, the candidates did not only gain from presenting themselves as especially qualified, but also from discrediting the opponent and worsening his or her position. In the next chapters we will allude to the limits of this result and show that overly spiteful behavior will not benefit a candidate.

Chapter 4

On Heterogeneity in Interdependent Preferences

4.1 Introduction

In Chapter 3, we established that, in a broad range of situations, players with spiteful preferences achieve higher material payoff in Tullock-contests (Tullock, 1980) than players with standard independent preferences, especially for non-increasing returns to scale. However, we focused above on a model with only two different types of players. In this chapter, we restrict our analysis to additively separable preferences and non-increasing returns to scale. For this case, we generalize the model in so far as allowing for complete heterogeneity among the players rather than confining our analyses to two types of characters. Furthermore, we allow for all levels of altruism and non self-destructive spite. However, this strategic advantage no longer prevails when too strong spiteful preferences induce overdissipation and thus negative material payoff for all active players. We find that, for constant returns to scale, overdissipation can occur and induce a higher material payoff for less spiteful players. Furthermore, we look at the question which incentives players have to join the contest at all. In the presence of overdissipation, only spiteful players will choose a strictly positive effort.

Another important result of this chapter is that interdependent preferences with different levels are equivalent to different valuations of the players. Therefore, we will also refer to the literature that deals with asymmetric valuations. Asymmetric valuations have been looked upon in rent-seeking literature in different contexts (e.g. Hillman and Riley 1989, Leininger, 1993, Baik, 1994). Nti (1999) was the first to analyze the game with asymmetric valuations (and independent preferences) in detail. However, his results are still restricted to the two-player-case. They show that the player with higher valuation will spend more effort and the ratio of their expenditure to their valuation is the same for both players. He also finds that the effort of the player with higher valuation increases in both valuations, while the effort of the player with lower valuation increases only in her own valuation but decreases in her opponents' valuation. Note that, in contrast to our analysis, Nti's results do not only hold for non-increasing but also for increasing returns to scale. Stein (2002) analyzed asymmetric valuations in an

n-player context with constant returns to scale. His result that the own effort is higher if the own valuation is higher is equivalent to our results that more spiteful players exert higher efforts, and thus our results generalize his result to the case of decreasing returns to scale. However, it holds for both analyses that the equivalence to our work ends when looking at the material payoff, since the prize players compete for in our setting is not the prize they can receive in the end. This leads us to the result that, in contrast to the models of Nti (1999) and Stein (2002), more spiteful players do not necessarily gain the higher expected payoff. Indeed, if the players in the game are sufficiently spiteful, we are able to find that the less spiteful players receive the higher material payoff. For the specific case of constant returns to scale, this condition reduces to overdissipation. We show that overdissipation can occur in the presence of spiteful preferences. From a technical point of view, we exploit the fact that rent-seeking contests are aggregative games. This is in line with the findings of Possajennikov (2003), Alos-Ferrer and Ania (2005) and Leininger (2006).

4.2 Effort Choice

The model we present here builds on the analysis in Chapter 3. The material payoff function considered here is also given by (2.1). Let us now point out the differences in the assumptions between the models in this chapter and the previous one. At first, we restrict the values for r considered here to $r \in]0,1]$. Next, we concentrate on a linear objective function of the form

$$F_i(\mathbf{x}) = \pi_i(\mathbf{x}) + \alpha_i \sum_{h=1}^{n} \frac{1}{n} \pi_h(\mathbf{x}), \tag{4.1}$$

which can be seen as a specification of (3.1).[1] Both these restrictions are made to avoid technical inconvenience. Chapter 3 captures an analysis with objective functions of the form (4.1) in so far as $\alpha_i \in (-1,0)$ for the status-seekers and $\alpha_i = 0$ for the individualists. We take this as a starting point to show how an extension of the set of players to an arbitrary number of different players can generalize or respectively change the results of Chapter 3, when allowing for any values of $\alpha_i > -n$. The latter restriction ensures that the utility function of each player positively depends on her own payoff. If player i is characterized by $\alpha_i > 0$, we call her an *altruist*. In contrast, if player i is characterized by $\alpha_i < 0$, we stick to the notation *status-seeker*. For the case where $\alpha_i = 0$, equation (4.1) reduces to equation (2.1).

To simplify the analysis we introduce the following parameter:

$$\lambda_i := \frac{n}{n + \alpha_i}.$$

From $\alpha_i \in]-n, \infty[$, it follows that $\lambda_i \in \mathbb{R}^+$. In the next sections, we will distinguish the players according to the preference parameter λ_i. Note that $\alpha_i < 0$ implies $\lambda_i > 1$

[1]The proposed function is equivalent to $(\alpha + 1)\pi_i - \alpha\rho_i$.

and $\alpha_i > 0$ leads to $\lambda_i < 1$. So $\lambda_i > (<)1$ represents the preference of a status-seeker (altruist).

From Fang (2002), we know that a rent-seeking game with non-increasing returns to scale has a unique Nash-equilibrium for any positive prize V. This result also holds true for asymmetric valuations. Therefore, another problem with the same optimality condition will have the same solution. We exploit this to state

Lemma 5 *In an n-player rent-seeking contest with material payoff function (2.1) and $r \leq 1$, a player maximizing an interdependent utility function in the form of (4.1) with $\alpha_i > -n$ acts according to the same reaction function as if she would maximize material payoff competing for a prize of $\lambda_i V$. Uniqueness of the Nash-equilibrium implies that in equilibrium the same effort is chosen as if competing for $\lambda_i V$.*

Proof. See appendix. ■

As already adressed, the game where two players compete for prizes of different size was already analyzed by Nti (1999). Instead of deriving Nti's results again in our framework, we will renounce a specific analysis of two-player-contests, and remark that, except for the final proposition concerning the expected payoff, Nti's results can be transferred to our analysis as the special case of two-player-contest, even for increasing returns to scale.

Next, we will compare the properties of the behavior of active players with different levels of preference parameters. We define:

Definition 3 *A player i is called* active *in equilibrium $\hat{\mathbf{x}}$, if and only if the ith component of $\hat{\mathbf{x}}$ is given by $x_i > 0$.*

Before analyzing active players in detail, we shall discuss up front the incentives for a player to be active in a rent-seeking contest with non-increasing returns to scale. For rent-seeking contests with decreasing returns of scale ($r < 1$), Cornes und Hartley (2005) already established that every player in the game has an incentive to exert strictly positive effort. On the contrary, for constant returns to scale, relatively altruistic players may have an incentive not to invest in the contest. We find:

Lemma 6 *Let n players decide whether to join a rent-seeking contest with material payoff function (2.1) and $r = 1$. Let each player i be characterized by $\alpha_i > -n$ such that she acts like competing for $\lambda_i V$. In equilibrium, player i will only choose a positive effort $x_i > 0$, if in equilibrium it holds that*

$$\lambda_i V > \sum_{j=1}^{n} x_j.$$

Proof. See appendix. ■

Now consider a game where $\lambda_i V$ can be interpreted as the personal valuation of player i in a game with heterogenous valuations and independent preferences. In this case one can interpret the result of Lemma 6 in the following way: a player only chooses positive effort in the contest if she considers the aggregate effort spent in the game to represent underdissipation in the game. Put differently, a player in a contest with heterogenous valuations will only exert positive effort in equilibrium if the aggregate effort does not exceed her own valuation.

Let us now assume for the rest of this chapter that all n players are spiteful enough to participate in equilibrium. Now, we analyze which effect the preference parameter λ_i has on the level of effort a player exerts. Every player i maximizes a utility function that can be written as a function of $\lambda_i \in \mathbb{R}^+$, $\forall i = 1, ..., n$, since equation (4.1) can be transformed to

$$u_i \left((\pi_j(\mathbf{x}))_{j=1,\ldots,n} | \lambda_i \right) = \pi_i(\mathbf{x}) + \left(\frac{1}{\lambda_i} - 1 \right) \sum_{j=1}^{n} \pi_j(\mathbf{x}).$$

Given this individual preference parameter λ_i, we can rewrite the first-order-condition of player i as

$$r x_i^{r-1} \sum_{h \neq i} x_h^r \lambda_i V = \left(\sum_{h=1}^{n} x_h^r \right)^2 =: A^2, \tag{4.2}$$

where A represents the aggregate effort of all n players. Note that in every situation all players face the same aggregate effort. So for any pair of players i and j, $i, j = 1, ..., n$, we can conclude from (4.2) that

$$r x_i^{r-1} \sum_{h \neq i} x_h^r \lambda_i V = r x_j^{r-1} \sum_{h \neq j} x_h^r \lambda_j V. \tag{4.3}$$

From (4.3), we deduce the following proposition, which generalizes the result of Theorem 2 (c) in Chapter 3.

Proposition 3 *Consider a rent-seeking contest with material payoff function (2.1) and $r \leq 1$. Then for efforts x_i and x_j of two players i and j, characterized by λ_i and λ_j respectively, we find that in equilibrium $\lambda_i > \lambda_j \Leftrightarrow x_i > x_j$.*

Proof. See appendix. ■

Obviously, these results are in line with Stein (2002) and those of Chapter 3. However, they do not imply that a higher value of λ_i also leads to higher material payoff. We will show in the next section that this need not necessarily hold in our general setting.

4.3 Effects on the Level of Material Payoff

In order to more clearly distinguish the cases where spiteful preferences induce a higher payoff, let us introduce the following notions:

Definition 4 *Define a preference function of form (4.1) as "weakly spiteful" if $0 > \alpha_i \geq -1$, or equivalently if $1 < \lambda_i \leq \frac{n}{n-1}$. In contrast, define a preference function of form (4.1) as "strongly spiteful" if $\alpha_i < -1$, i.e. if $\lambda_i > \frac{n}{n-1}$.*

Chapter 3 showed that a preference function is weakly spiteful if it increases in both absolute and relative payoff $\rho_i(\mathbf{x}) = \pi_i(\mathbf{x}) - \overline{\pi}(\mathbf{x})$. We found for certain classes of contests with weakly spiteful preferences that more spiteful players experience the higher absolute payoff in equilibrium. Like in Chapter 3, the aim of this analysis is to show under which conditions more spiteful preferences can receive a higher material payoff. We will see that the above result cannot be generalized to the entire set of interdependent preferences, since the absence of strongly spiteful players played an important role in Chapter 3. For games with a player set that either contain no or only strongly spiteful players, we are able to find:

Theorem 7 *Assume that n players play a rent-seeking contest with non-increasing returns to scale, where it holds that $\lambda_i \leq (\geq)\frac{n}{n-1}$, $\forall i = 1, ..., n$. Then players with a more spiteful preference realize a higher (lower) material payoff in equilibrium, i.e.*

$$\lambda_i > \lambda_j \Rightarrow \pi_i > (<)\pi_j, \quad \forall i, j = 1, ..., n, i \neq j.$$

Proof. The proof follows the line of Theorem 2. It is given in the appendix. ■

Without loss of generality, say that $\lambda_1 \leq ... \leq \lambda_n$. Then, we can conclude from Theorem 7:

Corollary 5 *Assume that n players play a rent-seeking contest with non-increasing returns to scale. Then, it holds for the material payoff in equilibrium that*

$$\pi_1(\mathbf{x}) \leq (\geq)\pi_2(\mathbf{x}) ... \leq (\geq)\pi_n(\mathbf{x}), \text{ if } \lambda_n \leq \frac{n}{n-1}\left(\lambda_1 \geq \frac{n}{n-1}\right)$$

with $\pi_i(\mathbf{x}) < (>)\pi_{i+1}(\mathbf{x})$, *whenever* $\lambda_i < \lambda_{i+1}$, $\forall i \in 1, ..., n-1$.

Corollary 6 *If players with the same preference parameter participate in a rent-seeking contest, they will choose the same effort level and thus receive the same material payoff.*

Corollary 6 supports the assumption of Chapter 3 where we restrict the analysis to intragroup symmetric equilibria. The corollary now tells us that these equilibria are the only one to arise in a rent-seeking game with utility functions of the type (4.1). Let us next state a lemma that gives a more extensive characterization of a strategic advantage.

Lemma 7 *Assume a rent-seeking game with payoff function (2.1) and utility function (4.1). For two active players i and j, we find that, in equilibrium, $\lambda_i > \lambda_j$ results in $\pi_i(\mathbf{x}) \overset{(>)}{\geq} \pi_j(\mathbf{x})$, if and only if*

$$\frac{x_i^r - x_j^r}{x_i - x_j} \overset{(>)}{\geq} \frac{\sum_{h=1}^n x_h^r}{V}.$$

Proof. From Proposition 3, we know that $x_i > x_j$ has to hold. Then, the condition follows from equivalence to $\pi_i(\mathbf{x}) \geq (>)\pi_j(\mathbf{x})$. ■

Let us now focus on the case of constant returns to scale. By inserting $r = 1$ in the analysis of Lemma 7, we find

Proposition 4 *Consider a rent-seeking game with $r = 1$. Then in equilibrium it holds:*

$$\{\lambda_i > \lambda_j \Leftrightarrow \pi_i(\mathbf{x}) > (<)\, \pi_j(\mathbf{x})\} \Leftrightarrow \sum_{h=1}^{n} x_h < (>)\, V \Leftrightarrow \pi_i(\mathbf{x}) > (<)\, 0 \; \forall i = 1, ..., n$$

Proof. The first part follows directly from Lemma 7. The latter equivalence sign follows directly from rearranging $\pi_i > (<)\, 0$ for $r = 1$. ■

Proposition 4 says that the less spiteful players can indeed earn a higher material payoff in certain settings. Note that Stein (2002) finds that the player with higher valuation will always perform better in terms of material payoff in a simultaneous rent-seeking game with constant returns to scale. The difference from our analysis is the following: while Stein assumes the higher value of the prize to be actually based on a difference in the material payoff function, in our setting, the higher valuation is only felt by the player and is not reflected in the actual material payoff.

Proposition 4 shows that there are limits to the strategic advantage of negatively interdependent preferences even in contests with non-increasing returns to scale. The reason for this is that players with strongly spiteful preferences concentrate more on reducing their opponents' payoff than on increasing their own. We will use the following example to illustrate that the strategic advantage can indeed be on the side of the less spiteful player.

Example 4 *Assume a contest with $n = 2$ and $r = V = 1$, where the players are characterized by $\lambda_1 = 2$ and $\lambda_2 = 3$. In equilibrium, they will choose $x_1 = \frac{12}{25}$ and $x_2 = \frac{18}{25}$, resulting in aggregated effort of $1.2V$, which indicates overdissipation. The material payoffs are given by $\pi_1 = -\frac{2}{25}$ and $\pi_2 = -\frac{3}{25}$, which means $\pi_1 > \pi_2$.*

Of course, the result of Example 4 depends on player 2 being a status-seeker. At first glance, it is not obvious that player 1 also has to be a status-seeker. However, we find:

Corollary 7 *Consider an equilibrium of an n-player rent-seeking contest with constant returns to scale, i.e. $r = 1$, where overdissipation - and thus a strategic advantage for the less spiteful player - occurs. Then all active players will be status-seekers.*

Proof. From Proposition 4, we know that every active player will receive a negative material payoff if overdissipation occurs in equilibrium. To show that in this case a positive effort cannot be the best response of a player i characterized by $\lambda_i \leq 1$, we refer to the appendix. ■

Note that this does not necessarily imply full or overdissipation for cases where two status-seekers meet in a rent-seeking contest with constant returns to scale. A counterexample can be directly derived from Theorem 7 and Proposition 4.

4.4 Conclusion

We present an analysis that captures the range of all levels of interdependent preferences that attach an overall positive utility to own material payoff. Self-destructive preferences are excluded. We find that, for Tullock-contests with non-increasing returns to scale for each pair of players, the player with the more spiteful, preference exerts higher effort in equilibrium. As an implication, the results that Chapter 3 points out for contests with non-increasing returns to scale do not depend on the assumption of intragroup symmetric equilibrium. Furthermore, we find that, in an equilibrium with underdissipation, the player with the most spiteful preference will earn the highest material payoff. For the case of two-player-contests, we can refer to Nti (1999) to show that this result is not restricted to non-increasing returns to scale.

As another important insight, we saw that results change with a too high level of spite. The results that spiteful preferences yield a strategic advantage in a broad range of games have to be qualified with respect to the assumptions behind analyses of Kockesen et al. (2000) and Chapter 3. As we saw, relaxing these assumptions can reverse the result.

The results of this chapter also have implications for contests with asymmetric valuations. Mathematically, this problem is equivalent to ours for the most part. Results concerning effort choice or winning probability in equilibrium can be smoothly transferred from the one framework to the other. Of course, this means that the additional findings we obtain for the effort choice in multi-player games with decreasing returns to scale can also be applied to games with asymmetric valuations. On the other hand, this analogy is restricted only to some of our results. Since players in games with interdependent preferences and with asymmetric valuations interpret the role of the heterogeneous prize competed for in a different way, results concerning the material payoff cannot be transferred. Hence, we can find that spiteful preferences can induce overdissipation while we know that asymmetric valuations cannot.

Chapter 5

Evolutionarily Stable Preferences

The analysis of heterogenous preferences in rent-seeking contests put forward in the previous chapter brings about the question which preference parameter is evolutionarily stable. A preference is evolutionarily stable if a population of players maximizing this preference cannot be successfully invaded by a mutant maximizing a different preference function. This means that a mutant will never receive a strictly higher payoff than the players with evolutionarily stable preferences. This chapter shows that, for a game where players receive a material payoff determined by (2.1), players with a preference parameter of $\lambda_i = \frac{n}{n-1}$, or respectively $\alpha_i = -1$, will receive an at least weakly higher payoff against any mutant in a playing-the-field contest with complete information, as long as r is not too large. As we know from Chapter 4, this preference parameter constitutes the border between weakly and strongly spiteful preferences. For this case, (4.1) reduces to $\rho_i(\mathbf{x}) := \pi_i(\mathbf{x}) - \frac{1}{n}\sum_{h=1}^{n} \pi_h(\mathbf{x})$. The maximization of this equation is equivalent to the maximization of the relative payoff function

$$\rho_i(\mathbf{x}) := \pi_i(\mathbf{x}) - \frac{1}{n-1}\sum_{h \neq i} \pi_h(\mathbf{x}). \tag{5.1}$$

With respect to the first-order-condition, it has already been shown by Schaffer (1989) that relative payoff maximizers, i.e. players maximizing (5.1), have evolutionarily stable preferences. Eaton and Eswaran (2003) showed that, for constant returns to scale, relative payoff is indeed an evolutionarily stable preference function. Furthermore, they show that, under the playing-the-field assumption, the evolutionarily stable preferences leads to playing the evolutionarily stable strategy. Hehenkamp et al. (2004) showed that the strategies played by relative payoff maximizers are evolutionarily stable, if evolution works on the level of strategies. Let us now show that relative payoff maximization is evolutionarily stable in rent-seeking contests for a much broader set of parameters than only for constant returns to scale, including every level of decreasing returns to scale. This means we want to focus on the question under which circumstances such a population can be successfully invaded by a mutant j with the preference function

$$F_j(\mathbf{x}) \neq \rho_j(\mathbf{x}), \tag{5.2}$$

in the sense that the mutant's preference function brings about a higher material payoff. Here, we stick to the assumption of playing-the-field. An analysis of evolutionary stability in Tullock-contests that relaxes the playing-the-field assumption is given by Leininger (2008).

To establish evolutionary stability in our setup, we calculate the reaction function of a player with objection function (5.1) if she reacts rationally and optimally, to the behavior of her opponents, and we show that the mutant cannot receive a higher material payoff.

Without loss of generality, we denote the mutant by n, and her equilibrium effort by x_n. The first-order-condition of each incumbent player i is given by

$$\frac{\partial \rho_i(\mathbf{x})}{\partial x_i} \overset{!}{=} 0$$

$$\Leftrightarrow \left(\sum_{j \neq i} x_j^r\right) x_i^{r-1} r V \frac{n}{n-1} = \left(\sum_{h=1}^{n} x_h^r\right)^2, \tag{5.3}$$

as we can see from $\lambda_i = \frac{n}{n-1}$ and equation (4.2). Since we know that (5.3) displays the equilibrium behavior of all players $i = 1, ..., n-1$, we can conclude that, for each pair of players i and j with $i, j = 1, ..., n-1$, it has to hold that

$$\left(\sum_{h \neq i} x_h^r\right) x_i^{r-1} r V \frac{n}{n-1} = \left(\sum_{h \neq j} x_h^r\right) x_j^{r-1} r V \frac{n}{n-1}$$

or equivalently

$$\frac{\sum_{h=1}^{n} x_h^r - x_i^r}{\sum_{h=1}^{n} x_h^r - x_j^r} = \frac{x_j^{r-1}}{x_i^{r-1}}. \tag{5.4}$$

As we can see from the proof of Proposition 3, which can be generalized to $r > 1$, we conclude from (5.4) that

$$x_i = x_j = x_1, \forall i, j = 1, ..., n-1. \tag{5.5}$$

Now inserting (5.5) in (5.3) and denoting $x_1 =: x$ and $x_n =: y$ yields

$$((n-2)x^r + y^r)\, x^{r-1} r V \frac{n}{n-1} = ((n-1)x^r + y^r)^2$$

$$\Leftrightarrow \frac{V}{(n-1)x^r + y^r} = \frac{(n-1)x^r + y^r}{(n-2)x^r + y^r} \frac{n-1}{x^{r-1} r n}. \tag{5.6}$$

The population is stable against invading mutants if, for every mutant strategy y generated by preference of type (5.2), it holds that

$$\pi_1(\mathbf{x}) \geq \pi_n(\mathbf{x})$$

$$\Leftrightarrow \frac{x^r}{(n-1)x^r + y^r} V - x \geq \frac{y^r}{(n-1)x^r + y^r} V - y$$

$$\Leftrightarrow \frac{x^r - y^r}{(n-1)x^r + y^r} V \geq x - y. \tag{5.7}$$

This condition ensures that the incumbents receive the (at least weakly) higher material payoff. Inserting (5.6) we can transform (5.7) to

$$\omega(y) := (n-1)\frac{x^r - y^r}{x^{r-1}rn} \cdot \frac{(n-1)x^r + y^r}{(n-2)x^r + y^r} - (x-y) \geq 0 \tag{5.8}$$

Whenever (5.8) holds, the mutant does not obtain a strictly higher payoff than the incumbents. We can now show for a certain range of r that (5.8) always holds, meaning that incumbents will receive the higher material payoff regardless of which particular strategy y maximizes (5.2). Note that from a mutant's point of view ω displays the relative loss she has to bear in contrast to the incumbents. We will now show that even when minimizing this difference, or equivalently maximizing her advantage against the incumbents, she will not end up with a strictly higher payoff in equilibrium. This becomes intuitively clear, since the incumbents also maximize the difference of their material payoff to their opponents. However, formally we still have to show that $\omega \geq 0$ holds. For this, we derive the first and the second derivative of ω, given by

$$\omega'(y) = 1 - \left(\frac{y}{x}\right)^{r-1} \frac{n-1}{n\left((n-2)x^r + y^r\right)} \left[(n-1)x^r + y^r + \frac{x^2 r - x^r y^r}{(n-2)x^r + y^r}\right]$$

and

$$\begin{aligned}\omega''(y) = &-\frac{n-1}{x^{r-1}n} \cdot \frac{y^{r-2}}{\left((n-2)x^r + y^r\right)^3} \Big\{(r-1)\Big[\Big(n^3 - 5n^2 + 9n - 6\Big)x^{3r} \\ &+ \Big(3n^2 - 11n + 11\Big)x^{2r}y^r + (3n-6)x^r y^{2r} + y^{3r}\Big] - 2r(n-1)x^{2r}y^r\Big\}.\end{aligned} \tag{5.9}$$

Next we continue by showing that

1. $\omega(x) = 0$,
2. $\omega'(x) = 0$ to show us that $y = x$ is a candidate for a local minimum of $\omega(y)$,
3. $\omega''(y) > 0, \forall y > 0$ to ensure that $y = x$ is both local and global minimum of $\omega(y)$. Since we then have shown that $\omega(x) = 0$, we can conclude that ω is weakly positive for all positive values of y.

At first, inserting $y = x$ into $\omega(y)$ yields

$$(n-1)\frac{x^r - x^r}{x^{r-1}rn} \cdot \frac{(n-1)x^r + x^r}{(n-2)x^r + x^r} - (x-x) = 0.$$

Next inserting $y = x$ into $f'(y)$ yields

$$\begin{aligned}&1 - \left(\frac{x}{x}\right)^{r-1} \frac{n-1}{n\left((n-2)x^r + x^r\right)} \left[(n-1)x^r + x^r + \frac{x^2 r - x^r x^r}{(n-2)x^r + x^r}\right] \\ &= 1 - 1\frac{n-1}{n(n-1)x^r}\left[nx^r + \frac{0}{(n-1)x^r}\right] = 1 - 1 = 0.\end{aligned}$$

The third step is not that straightforward. However, it is easy to see that $q_1 := \frac{n-1}{x^{r-1}n} \cdot \frac{y^{r-2}}{((n-2)x^r+y^r)^3}$ is positive. Furthermore it can be shown that for $n \geq 2$ the term

$$q_2 := \left[\left(n^3 - 5n^2 + 9n - 6\right) x^{3r} + \left(3n^2 - 11n + 11\right) x^{2r} y^r + (3n-6) x^r y^{2r} + y^{3r}\right]$$

is positive. From this, we conclude

Lemma 8 *Assume a rent-seeking contests with n players. Then there exists a technology parameter $r^* > 1$, such that for every technology with $r < r^*$ it holds that $f''(y) > 0$.*

Proof. See appendix. ■

Now, since we have shown that the difference between the material payoff of an incumbent and a mutant is minimized by $y = x$, and is not negative for $r < r^*$, we can conclude the following theorem:

Theorem 8 *Assume a rent-seeking contest with n players. Then there exists an $r^* > 1$ such that, for every rent-seeking contest with a technology $r < r^*$, a population of status-seekers maximizing (5.1) cannot be successfully invaded by any other preference type.*

With this, we could show that relative payoff maximization is an evolutioanrily stable preference for all rent-seeking contests with non-increasing returns to scale and also for some contests with increasing returns to scale, provided the scale effects are not too large.

Our analyses of Chapters 3 to 5 provides strong support for the anaylsis put forward by Hehenkamp et al. (2004). We find that, as long as all players are not or only weakly spiteful, then a more spiteful preference yields a higher payoff for the respective player if the technology is characterized by non-increasing returns to scale. For increasing returns to scale, we point out that several equilibria with different properties may occur, if only two types of players are present, where one type is characterized by independent and the other one by weakly interdependent preferences.

Furthermore, we showed that preferences that maximize relative payoff are stable against other intruding preferences and, thus, evolutionarily stable. In a world where different types repeatedly interact with each other, it will be the relative payoff maximizers who survive in the long run. However, while spiteful players that act according to relative payoff maximization are more successful compared to other preferences, we have shown that the absolute payoff reduces to zero if all players act according to relative payoff maximization. In homogenous populations with less spiteful preferences, players can receive positive absolute payoff though. This creates a *preference dilemma.* From a welfare perspective, the aggregate absolute payoff that the players will get in equilibrium is larger if the players are less spiteful. For a homogenous population, this means that every player gains in absolute terms without losing relative payoff if all players behave less spitefully. Yet, there are individual incentives to deviate from this behavior, so in a preference equilibrium all players will behave as relative payoff maximizers.

Part II

Sequential Structures

Chapter 6

Sequential Structures in Rent-Seeking Contests: The Three-Player-Case

This part of our work now concentrates on sequential structures in contests with more than two players. Each of the analyses presented in this part have a common structure. In the first stage of the game, all contestants decide (simultaneously) about a point in time when they will choose their effort. At the end of this stage, the results of the timing decision are publicly announced and are common knowledge for the rest of the game. Starting in the second stage, the players will commit to their effort sequentially, depending on which stages were chosen in the timing decisions of the first stage.[1] Once a player has committed to her effort choice, it is common knowledge among all players. Finally, in the last stage, the payoffs of the players are determined by Tullock-contest with a payoff function of the form:

$$\pi_i(\mathbf{x}) = \begin{cases} \frac{x_i}{\sum_{j=1}^{n} x_j} V_i - x_i & \text{if } \exists j = 1, ..., n : x_j > 0 \\ \frac{1}{n} & \text{if } x_j = 0, \forall j = 1, ..., n. \end{cases} \tag{6.1}$$

This payoff function is derived from the Tullock function (2.1), but with constant returns to scale ($r = 1$) and the possibility of heterogenous valuations (i.e. V_i instead of V). For two players, Leininger (1993) presented such an analysis with two timings, finding that in the subgame-perfect equilibrium the player with the higher valuation V_i will always decide to move late, while the player with the lower valuation will choose to move early. As a result, aggregate effort in this equilibrium will be lower than in a simultaneous move Nash-equilibrium, and each player's payoff increases, which means that, from a welfare point of view, the equilibrium in the sequential game is a Pareto-improvement to the equilibrium in the simultaneous game.[2]

[1]Note that, if all players have chosen the same stage before, the game of effort choice will be a simultaneous one.

[2]The game we present here requires a rather complex notion of strategies. We present in the appendix, how a strategy is defined here.

Concerning the relevance of sequential structures for contests in practical application, we already refered to Morgan's (2003) statement about the parties' National Conventions in U.S. presidential election campaigns. He had found that the two big parties always chose their timing such that the party to which the current president belongs schedules its National Convention later. If one interprets the governing party as the one with the higher valuation (which can be supported e.g. by referring to switching costs that both parties have to bear if the opposing party wins), this observation runs completely in line with the results of Leininger (1993).

An example for a system with more than two (major) parties can be found in the parliamentary election in Saxony-Anhalt 2006. The Social Democratic Party (SPD) and the Left Party (PDS) decided upon their manifestos on the 14th and the 16th of January, 2006, respectively, while the incumbent, the Christian Democratic Union (CDU), did not present their manifesto until the 31st of January, 2006. Final decision about this manifesto was on 25th of February. A manifesto can be interpreted as committing oneself to a strategy. This can be interpreted as a simultaneous decision of SPD and PDS, while the CDU, the party providing the president, chose a timing late enough to be able to react to the manifestoes of the SPD and the PDS. As we will show later, this timing choice is exactly the order that our model would predict.

6.1 Introduction

In this chapter, we want to extend the analysis put forward by Leininger (1993) to a contest in which three players decide whether they will choose their effort in an early or in a late stage. To this aim, we analyze the equilibria of the feasible subgames, which enables us to determine the subgame-perfect equilibrium. We will find that a player only has an incentive to choose late if she exerts a higher effort in a simultaneous move game than both her opponents taken together. Otherwise, all players will decide simultaneously on the early stage. Mixed strategy equilibria can occur for a small subset of the regarded cases when a second player also has an incentive to move late. Parameter constellations where only two players are active are moved to the appendix.

In addition, we compare the aggregate payoff levels of the different subgames in order to be able to see whether the sequential structures are still able to reduce dissipation. This is not necessarily the case. Especially for rather homogenous populations, we find that simultaneous moves are not only played on the equilibrium path, but also yield the highest aggregate payoff among all feasible subgames.

6.2 The model

We model a rent-seeking contests with three players $i = 1, 2, 3$. The payoff of each player, dependent on the effort vector, is given by (6.1). Similarly to Leininger (1993), we examine a sequential game where every player commits herself to one of two points in time ($t = 1$ or $t = 2$) before choosing her effort. Let us assume that, in the case of a mixed strategy over the timing choice, the actual decision time is drawn *before*

$t = 1$. This justifies the assumption that players have complete information concerning the subgame they play.

Assume throughout this paper, that $V_1 \geq V_2 \geq V_3$. To ensure participation of all three players in equilibrium, let us assume that

$$V_3 \geq \frac{V_1}{2}. \tag{6.2}$$

For any player i with a valuation of V_i, the valid combinations of V_j and V_k are illustrated in Figure B.1. The inequality (6.2) might be considered a restrictive assumption. However, it allows us to focus on true three-player-cases, whereas here we skip cases where one of the players would prefer negative effort on the equilibrium path and discuss them in Chapter A.2.

There are three generally feasible subgames: the first one, where all players move simultaneous; the second one, where two players move early and one player moves late; and the third one, where one player moves first while the others move later. In the first step of our analysis, we will investigate the equilibria of these subgames one after another. This allows us later on to derive the subgame-perfect equilibrium of the timing game.

6.2.1 The simultaneous game

If all three players end up with the same choice of timing decision, they play a standard rent-seeking contest with heterogenous valuations as put forward by Stein (2002). The first-order-condition of the payoff maximization is given by:

$$\frac{\partial \pi_i(\mathbf{x})}{\partial x_i} = \frac{x_j + x_k}{(x_i + x_j + x_k)^2} V_i - 1 \stackrel{!}{=} 0, \; i, j, k = 1, 2, 3, \; j \neq i \neq k. \tag{6.3}$$

Equating this equation for every player, we can deduce the equilibrium behavior that brings about $(x_j + x_k) V_i = (x_i + x_k) V_j = (x_i + x_j) V_k$, which yields

$$x_i = x_j \frac{V_i}{V_j} + x_k \frac{V_i - V_j}{V_j}. \tag{6.4}$$

Then

$$x_j = x_k \frac{((V_i - V_j)V_k - V_i V_j)}{V_i V_j - (V_i + V_j)V_k}. \tag{6.5}$$

Inserting (6.5) in (6.4) yields

$$x_i = x_k \left(\frac{V_j V_k - V_i (V_j + V_k)}{V_i V_j - (V_i + V_j) V_k} \right). \tag{6.6}$$

Now inserting (6.5) and (6.6) in (6.3) gives us the combined first-order-condition:

$$x_k = 2 V_i V_j V_k \frac{V_i V_k + V_j V_k - V_i V_j}{(V_i V_j + V_i V_k + V_j V_k)^2},$$

for any player $k = 1, 2, 3$. Now, non-negative effort requires that, for any combination of players $i, j, k = 1, 2, 3$ with $i \neq j \neq k$, it holds that

$$V_i \geq \frac{V_j V_k}{V_j + V_k}. \tag{6.7}$$

If (6.7) does not hold, the aggregate effort in this game is higher than player i's valuation, $\frac{V_j V_k}{V_j + V_k}$, thus she has no incentive to spend positive effort. Players j and k play a standard two-player simultaneous move game, as analyzed by Leininger (1993). It can be shown that players j and k will also choose an aggregative effort of $\frac{V_j V_k}{V_j + V_k}$ in this situation.

Yet, if (6.7) holds for all three players calculating equilibrium payoffs results in

$$\pi_i^S = \frac{V_i \left(V_i V_j + V_i V_k - V_j V_k\right)^2}{\left(V_i V_j + V_i V_k + V_j V_k\right)^2}, \tag{6.8}$$

for any $i = 1, 2, 3$, where the superscript S denotes the simultaneous move game. Now having derived the payoff a player can gain from a simultaneous move game, we need to know the payoffs of the sequential games to derive the subgame-perfect equilibrium of the total game. These are derived in the next two subsections.

6.2.2 Two players move early

In this subsection, we focus on the subgame where player k has chosen to move late while the players i and j decided to move first. Then player k takes the actions of i and j in $t = 1$ as given when deciding in $t = 2$. She maximizes

$$\pi_k(\mathbf{x}) = \frac{x_k}{x_i + x_j + x_k} V_k - x_k$$

with the first-order condition $\frac{\partial \pi_k}{\partial x_k} \stackrel{!}{=} 0 \Leftrightarrow (x_i + x_j) V_k = (x_i + x_j + x_k)^2$. From this, we can conclude

$$x_k = \sqrt{(x_i + x_j) V_k} - (x_i + x_j). \tag{6.9}$$

Note that, for player k, only the aggregate effort of her opponents matters, not the individual choices. In $t = 1$, players i and j know that (6.9) characterizes the reaction of player k and take this into consideration. This transforms the payoff function of an early mover i to

$$\pi_i(x_i, x_j, x_k(x_i, x_j)) = \frac{x_i}{\sqrt{(x_i + x_j) V_k}} V_i - x_i.$$

The first-order-condition can be transformed to

$$\frac{\partial \pi_i(x_i, x_j, x_k(x_i, x_j))}{\partial x_i} = \frac{\sqrt{(x_i + x_j) V_k} - \frac{x_i V_k}{2\sqrt{(x_i + x_j) V_k}}}{(x_i + x_j) V_k} V_i - 1 \stackrel{!}{=} 0. \tag{6.10}$$

Since (6.10) holds for both early movers i and j, we can conclude (see appendix for a derivation)

$$x_j = x_i \frac{2V_j - V_i}{2V_i - V_j}. \tag{6.11}$$

This means that $x_i + x_j = x_i \frac{V_i+V_j}{2V_i-V_j}$. Inserting this in (6.10) and solving for x_i yields:

$$x_i^{kL} = \frac{9V_i^2 V_j^2 (2V_i - V_j)}{4 (V_i + V_j)^3 V_k}, \tag{6.12}$$

where the superscript kL denotes the subgame in which player k is the only one to move late (L). Thus, the participation constraint for an early moving player i in this subgame is given by $V_i \geq \frac{V_j}{2}$.

Returning to the most interesting case of three active players, we find that player k's effort can be derived by inserting (6.12) for both early movers in (6.9):

$$x_k^{kL} = \frac{3V_i V_j}{4 (V_i + V_j)^2 V_k} (2V_k (V_i + V_j) - 3V_i V_j). \tag{6.13}$$

Player k is willing to choose a non-negative effort in this subgame, if and only if

$$V_k \geq \frac{3V_i V_j}{2 (V_i + V_j)}. \tag{6.14}$$

If (6.14) does not hold, player k cannot gain a positive payoff from this subgame, and only the players i and j will be active in the equilibrium of this subgame.

From (6.12) and (6.13), we can calculate the aggregate effort spent in this subgame. We will need this value to derive the levels of payoff each player receives in equilibrium. We find:

$$\sum_{h=i,j,k} x_h = \frac{3V_i V_j}{2 (V_i + V_j)}. \tag{6.15}$$

Remark that the critical value for the participation of player k in (6.14) takes the same value. Furthermore, we see from (6.15):

Lemma 9 *Consider a rent-seeking contest where two players bid in the first period and one player bids in the second period. If all players exert positive effort, then total dissipation only depends on the valuations of the two players bidding in the first stage.*

Now, we can derive the equilibrium payoffs in this subgame from the above calculated effort:

$$\pi_i^{kL} = \frac{3V_i^2 V_j (2V_i - V_j)^2}{4 (V_i + V_j)^3 V_k}, \tag{6.16}$$

$$\pi_j^{kL} = \frac{3V_i V_j^2 (2V_j - V_i)^2}{4 (V_i + V_j)^3 V_k} \tag{6.17}$$

and

$$\pi_k^{kL} = \frac{(2V_k (V_i + V_j) - 3V_i V_j)^2}{4 (V_i + V_j)^2 V_k}. \tag{6.18}$$

Let us now look at the question: what happens if at least one participation constraint is binding? Note that, if a participation constraint is binding, it is always the weakest

	$V_3 > \frac{2}{3}V_1$	$V_3 \leq \frac{2}{3}V_1$
$V_3 > \frac{2}{3}V_2$	$x_1 \geq V_3 - 2V_3\frac{V_2-V_3}{V_2}$ $x_1 \leq 2V_3\frac{V_1-V_3}{V_1}$	$x_1 \geq V_3 - 2V_3\frac{V_2-V_3}{V_2}$ $x_1 \leq \frac{V_3^2}{V_1}$
$V_3 \leq \frac{2}{3}V_2$	*does not occur for* $V_1 \geq V_2$	$x_1 \geq V_3 - \frac{V_3^2}{V_2}$ $x_1 \leq \frac{V_3^2}{V_1}$

Table 6.1: Feasible equilibrium efforts of player 1, if $V_3 \in \left(\frac{V_1V_2}{V_1+V_2}, \frac{3}{2}\frac{V_1V_2}{V_1+V_2}\right)$.

player, namely player 3, who is affected. Since (6.2) holds for player 3, her participation is never critical if she moves early. Now presume she moves late. If her opponents choose $x_1 + x_2 \geq \frac{3}{2}\frac{V_1V_2}{V_1+V_2}$ in the first stage, she will choose zero effort (see (6.14)). Yet, if the early movers play a two-player game in $t = 1$ without regard to player 3, the aggregate effort takes the value of $\frac{V_1V_2}{V_1+V_2}$. Hence a plain two-player-contest is only played in $t = 1$ if $V_3 \leq \frac{V_1V_2}{V_1+V_2}$. Yet, this can be shown to contradict (6.2). For $V_3 \in \left(\frac{V_1V_2}{V_1+V_2}, \frac{3}{2}\frac{V_1V_2}{V_1+V_2}\right)$, however, a three-player-contest is played, where players 1 and 2 strategically react to player 3's presence in order to preempt her. Unfortunately, this game does not have a unique equilibrium, but rather a continuum of equilibria. We present the equilibrium effort of player 1 in these equilibria in Table 6.1. Table 6.1 gives the upper and lower limits for the effort of player 1. For each single feasible value of x_1, there exists an equilibrium, in which player 1 chooses x_1 and player 2 chooses $x_2 = V_3 - x_1$ on the first stage, while player 3 chooses $x_3 = 0$. For a derivation of the values given in Table 6.1, we refer to the appendix.

Now we have determined the payoff structure of a contest where two players move early, while a third one chooses her effort after her opponents. To be able to analyze the total game of timing decision, however, we still need to look at the situation, where only one player decides in the early stage. In the next subsection, we take a closer look at this subgame.

6.2.3 One player moves early

So let us now assume that only player i decides on her effort in the first round while the players j and k move in the second round. The late movers simultaneously decide upon their effort. So, the first-order-condition of a late mover is given by

$$\frac{\partial \pi_k(\mathbf{x})}{\partial x_k} \stackrel{!}{=} 0 \Leftrightarrow (x_i + x_j)\, V_k = (x_i + x_j + x_k)^2 . \tag{6.19}$$

In equilibrium, equating the first-order-condition of both late movers leads to $(x_i + x_k)\, V_j = (x_i + x_j)\, V_k$, which is equivalent to $x_k = \frac{V_k}{V_j}(x_i + x_j) - x_i$. Inserting this in (6.19) yields:

$$(x_i + x_j)\, V_k = \left((x_i + x_j)\,\frac{V_k}{V_j} + x_j\right)^2 \tag{6.20}$$

As shown in the appendix, we can conclude from this that the optimal behavior of a late moving player is given by

$$x_j^{iE} = \frac{V_j^2 V_k - 2V_j V_k x_i - 2V_k^2 x_i + \sqrt{V_j^3 V_k \left(V_j V_k + 4V_k x_i + 4V_j x_i\right)}}{2\left(V_j + V_k\right)^2}, \tag{6.21}$$

where the superscript iE denotes the subgame in which player i is the only one who moves early (E). This is anticipated by player i in the first period and she maximizes

$$\pi_i\left(x_i\right) = \frac{x_i}{x_i + x_j^{iE}\left(x_i\right) + x_k^{iE}\left(x_i\right)} V_i - x_i.$$

This leads her to a first-order-condition of

$$\left(x_j^{iE}\left(x_i\right) + x_k^{iE}\left(x_i\right) - x_i\left(\frac{\partial x_j^{iE}\left(x_i\right)}{\partial x_i} + \frac{\partial x_k^{iE}\left(x_i\right)}{\partial x_i}\right)\right) V_i = \left(x_i + x_j^{iE}\left(x_i\right) + x_k^{iE}\left(x_i\right)\right)^2. \tag{6.22}$$

Inserting (6.21) in (6.22) yields the maximization problem of the early mover, which is solved by

$$x_i^{iE} = \frac{V_i^2\left(V_j + V_k\right)^2 - V_j^2 V_k^2}{4V_j V_k\left(V_j + V_k\right)}, \tag{6.23}$$

as shown in the appendix. This is a non-negative term for $V_i \geq \frac{V_j V_k}{V_j + V_k}$. As mentioned above, this threshold value is exactly the aggregate effort that will be spent on the late stage between players j and k, if i chooses zero effort. Now by inserting (6.23) in (6.21), we find the effort of the players j and k (For a derivation, see appendix):

$$x_j^{iE} = \frac{V_j V_k\left(2V_j + V_k\right) - \frac{V_i^2}{V_j}\left(V_j + V_k\right)^2 + 2V_i V_j\left(V_j + V_k\right)}{4\left(V_j + V_k\right)^2}. \tag{6.24}$$

For a late-moving player j, a non-negative effort only fulfills the first-order-condition for

$$V_j \geq \tilde{V}_j := \frac{V_i - V_k + \sqrt{V_i^2 + 6V_i V_k + V_k^2}}{4}, \tag{6.25}$$

with player i moving early and and player k moving late. Again, if this condition is not fulfilled, player j cannot gain any positive payoff in this subgame.

Given (6.25), the aggregate effort in this subgame is thus given by

$$x_i^{iE} + x_j^{iE} + x_k^{iE} = \frac{V_i V_j + V_i V_k + V_j V_k}{2\left(V_j + V_k\right)}, \tag{6.26}$$

and equilibrium payoffs are given by

$$\pi_i^{iE} = \frac{\left(V_i V_j + V_i V_k - V_j V_k\right)^2}{4V_j V_k\left(V_j + V_k\right)}, \tag{6.27}$$

$$\pi_j^{iE} = \frac{\left(2V_j^2 + V_jV_k - V_iV_j - V_iV_k\right)^2}{4V_j\left(V_j+V_k\right)^2} \text{ and} \tag{6.28}$$
$$\pi_k^{iE} = \frac{\left(2V_k^2 + V_jV_k - V_iV_j - V_iV_k\right)^2}{4V_k\left(V_j+V_k\right)^2},$$

as derived in the appendix.

Let us now take a closer look at the case where (6.25) is not fulfilled. Remark that $\tilde{V}_j$ can be shown to be strictly larger than $\frac{V_i}{2}$. For $V_j \in \left(\frac{V_i}{2}; \tilde{V}_j\right)$, player j quits participation. Then the early moving player i cannot gain from increasing her effort beyond the point where player j quits active participation, contrary to the implicit assumption behind (6.22) that the solution is interior. Thus, the best response of player j will be to choose an optimal reaction of zero in (6.21). The equilibrium effort of player i is given by $x_i = \frac{V_j^2}{V_k}$. Player k will choose an effort of $x_k = V_j - \frac{V_j^2}{V_k}$. Obviously, $x_j = 0$. Denoting this game by "Pr" for Preemption, we can write down the payoffs as:

$$\pi_i^{Pr} = \frac{(V_i - V_j)V_j}{V_k} \wedge \pi_j^{Pr} = \frac{(V_k - V_j)^2}{V_k} \wedge \pi_k^{Pr} = 0.$$

6.3 Comparing the subgame payoffs

6.3.1 Decision Structure

So far, we have analyzed the equilibria of the subgames. This enables us to reduce the total game to a game of a timing decision. Equilibrium behavior in the subgames is now presumed and the normal form of the timing game is given by:

		Player j	
		$t=1$	$t=2$
Pl. i	$t=1$	$\pi_i^S, \pi_j^S, \pi_k^S$	$\pi_i^{jL}, \pi_j^{jL}, \pi_k^{jL}$
	$t=2$	$\pi_i^{iL}, \pi_j^{iL}, \pi_k^{iL}$	$\pi_i^{kE}, \pi_j^{kE}, \pi_k^{kE}$

Player k moves in $t=1$.

		Player j	
		$t=1$	$t=2$
Pl. i	$t=1$	$\pi_i^{kL}, \pi_j^{kL}, \pi_k^{kL}$	$\pi_i^{iE}, \pi_j^{iE}, \pi_k^{iE}$
	$t=2$	$\pi_i^{jE}, \pi_j^{jE}, \pi_k^{jE}$	$\pi_i^S, \pi_j^S, \pi_k^S$

Player k moves in $t=2$.

Each player can be faced with four different problems for the timing decision, two of which can be solved equivalently, since it does not matter for the analysis whether e.g. player i faces player j moving early and player k moving late, or the other way round. So, we can formulate the decision situation in three problems.

- **Problem I:** What is a player's best choice if her opponents decide late?
- **Problem II:** What is a player's best choice if her opponents decide early?
- **Problem III:** What is a player's best choice if one of her opponents decides early, while the other one decides late?

Note that these problems are in a way hypothetical since they assume that the player facing the problem decides last about his choosing time, whereas we regard choice of timing as a simultaneous game in our analysis. Yet, solving these problems is inevitable for finding the Nash-equilibrium of the timing game.

Problem I

Problem I can be solved by comparing the payoff player i receives from the simultaneous game with the one she gets from moving first. If the considered player is active in both subgames, the payoff for moving first is larger if $\pi_i^{iE} > \pi_i^S$ which is equivalent to

$$\left(V_i\left(V_j + V_k\right) - V_jV_k\right)^2 > 0 \tag{6.29}$$

for $V_i > \frac{V_jV_k}{V_j+V_k}$, as shown in the appendix. For $V_i \leq \frac{V_jV_k}{V_j+V_k}$, player i will stay outside the contest and thus receive zero payoff in both cases. In this case, she will be indifferent between both timings then.

Thus, in a subgame where all players move late simultaneously, every player has an incentive to deviate to an early move. This result is not too surprising, since it can already be established in the two-player-case analyzed by Leininger (1993).

Problem II

Problem II can be solved by comparing the payoff player k receives from a simultaneous game with the one in the subgame where she is the only late mover. For an interior solution in both subgames, she will prefer moving early if $\pi_k^S \geq \pi_k^{kL}$. Comparing the respective variants of (6.8) and (6.13), we find that $\pi_k^S \geq \pi_k^{kL}$ is fulfilled with equality for

$$V_k = \frac{\psi V_iV_j}{V_i + V_j}$$

with $\psi \in \left\{\frac{3-\sqrt{57}}{8}, \frac{3+\sqrt{57}}{8}, 3\right\}$. However, it is sufficient to concentrate on cases where the player is willing to generate a non-negative effort as the only late mover. This happens if $V_k \geq \frac{3}{2}\frac{V_iV_j}{V_i+V_j}$. Since $\frac{3+\sqrt{57}}{8} < \frac{3}{2}$, the only situation where $\pi_k^S = \pi_k^{kL}$ holds and player k chooses a positive effort is then given by $V_k = 3\frac{V_iV_j}{V_i+V_j}$. Now it is easy to show that player k would only choose to move late if $V_k > 3\frac{V_iV_j}{V_i+V_j}$. Given assumption (6.2), this holds for at most one player. We will present further intuition for this result in subsection 6.3.3.

Problem III

Problem III can be solved by comparing the payoff of player j when joining the one player moving early with the payoff when joining the one player moving late. She prefers moving late if:

$$\pi_j^{kL} > \pi_j^{iE}. \tag{6.30}$$

From (6.17) and (6.28) in (6.30) leads to:

$$\frac{3V_iV_j^2\left(2V_j-V_i\right)^2}{4\left(V_i+V_j\right)^3V_k} > \frac{\left(2V_j^2+V_jV_k-V_iV_j-V_iV_k\right)^2}{4V_j\left(V_j+V_k\right)^2}.$$

For most of the constellations captured by (6.2), this can be shown to hold for all three players. Yet, if player 1 is the player moving late, while player 3 is the player moving early, and V_3 lies only slightly above the threshold $\frac{V_1}{2}$, player 2 can have an incentive to join the late mover. Table B.1 in the appendix gives an overview of the range of constellations for which this might happen.[3] It contains about 3 % of the entire constellations captured by (6.2). Note that Table B.1 also provides an upper limit for the valuation of a player, up to which she prefers to join the late moving opponent instead of joining the early mover.

Let us now subsume what we learn from Table B.1. A player only has an incentive to join the late mover if one player is (at least weakly) stronger and the other player is strictly weaker. There is only one case where a player with highest valuation has a weak incentive to join the late mover, and that is, if she has the same valuation as the late mover ($V_1 = V_2$), while $V_3 = \frac{V_1}{2}$. Note that this is a border case. The strongest player never has a strict incentive to move late if facing this situation; the weakest player never has any incentive at all to move late.

6.3.2 Equilibria

The previous analysis has revealed the conditions under which players prefer early commitment to their effort and when they would rather wait, given their opponents' decisions. From these results, we can deduce which of the subgames lies on the equilibrium path of a subgame-perfect Nash-equilibrium.

We find that, in a situation where all players choose late, there is always an incentive to deviate and move early. Thus, all players moving late cannot be an equilibrium. In a situation where all players move early, however, no player has an incentive to deviate to a later move if $V_1 \leq \frac{3V_2V_3}{V_2+V_3}$ holds for the strongest player. Thus, this situation characterizes an equilibrium. Furthermore, we find that a constellation where one of the weaker player moves late on her own can never be equilibrium. In a similar way, there is no equilibrium with two players moving late, since there is always at least one player who has an incentive to deviate to moving early. The strongest, as well as the weakest, player will always prefer an earlier move to a late move, if there is a second late mover.

Now, if $V_1 > \frac{3V_2V_3}{V_2+V_3}$, player 1 has an incentive to move late. Unless we are in a constellation captured by Table B.1, both weaker opponents have no incentive to move late in any situation, such that we end up in an equilibrium where the strongest player moves late, while both the others move early. Yet, for the cases captured by Table B.1, we find that player 2 also has an incentive to join the late mover. However, if player 2

[3] Note that Table B.1 also presents constellations where this problem occurs for the case of $V_3 > \frac{V_1}{2}$, that is discussed in Chapter A.2.

chooses late, player 1 no longer has an incentive to move late herself. Hence there is no equilibrium in pure strategies, but only in mixed strategies. To derive these strategies, let us denote the probability that player i will assign to an early move in her mixed strategy by τ_i, $i = 1, 2$, whereas she chooses $t = 2$ with probability (1-τ_i). In a mixed strategy equilibrium, each player must be indifferent between the alternatives she can choose. For player 2, this means that her expected payoff from moving early equals her expected payoff from moving late. The expected payoffs are given by $E(\pi_2|_E) = \tau_1\pi_2^S + (1-\tau_1)\pi_2^{1L}$ for an early move and $E(\pi_2|_L) = \tau_1\pi_2^{2L} + (1-\tau_1)\pi_2^{3E}$ for a late move. Equating these equations yields

$$\tau_1 = \frac{\pi_2^{3E} - \pi_2^{1L}}{\pi_2^{3E} + \pi_2^S - \pi_2^{1L} - \pi_2^{2L}}.$$

The derivation of ρ_2 runs analogously and it shows that:

$$\tau_2 = \frac{\pi_1^{2L} - \pi_1^{3E}}{\pi_1^{2L} + \pi_1^{1L} - \pi_1^S - \pi_1^{3E}}.$$

In the appendix, we show that the values for τ_i, $i = 1, 2$ indeed lie in the interval $[0, 1]$ for the relevant parameters. The case that (6.30) is violated for some player j and $V_1 < \frac{3V_2V_3}{V_2+V_3}$ never occurs. We summarize:

Theorem 9 *In a game where three players decide whether they will choose effort for a rent-seeking contest early or late, the following holds in a subgame-perfect equilibrium:*

- *If the three players are relatively homogenous, i.e.,* $V_1 \leq \frac{3V_2V_3}{V_2+V_3}$*, all players move simultaneously in the early stage.*
- *If* $V_1 > \frac{3V_2V_3}{V_2+V_3}$ *and (6.30) holds for* $j = 2$ *(with* $i = 3$ *and* $k = 1$*), player 1 moves late while the two weaker players move early.*
- *If* $V_1 > \frac{3V_2V_3}{V_2+V_3}$ *and (6.30) does not hold for* $j = 2$ *(*$i = 3$ *and* $k = 1$*), player 3 moves in* $t = 1$*, while both her opponents choose a mixed strategy concerning their timing decision. The probability with which player* i*,* $i = 1, 2$*, moves early is given by* ρ_i*.*

6.3.3 Comparison to the two-player-case

For a sequential contest with two players, Dixit (1987) has shown that a player who receives a winning probability of more than $\frac{1}{2}$ in the simultaneous move game will choose a higher effort if she moves alone early than she will in a simultaneous move game. Furthermore, he found that a player with a winning probability lower than $\frac{1}{2}$ will exert a lower effort level moving alone early than in the simultaneous move game. Hence, as analyzed by Baik and Shogren (1992) and Leininger (1993), the weaker player always has an incentive to prevent the stronger player's early move, while the strong player is better off waiting, since the weak player behaves less aggressively as the first mover. The weaker player will thus move first in a subgame-perfect equilibrium, while the stronger player will move last. In our three-player-model, we find that the strongest player will only move late if $V_1 \geq 3\frac{V_2V_3}{V_2+V_3}$. This is the condition which induces that, in a simultaneous move equilibrium, player 1's effort is higher than the aggregate effort of both her opponents. This directly induces a winning probability of more than $\frac{1}{2}$ in the model at hand. Here we find how the results of the two-player-case can be generalized to the three-player-case.

Corollary 8 *If the effort of the strongest player in the simultaneous move game is (strictly) higher than the aggregate effort of her opponents, i.e., her winning probability is higher than $\frac{1}{2}$, then she has a (strict) incentive to move late. The reverse also holds.*

Proof. As we can conclude from the previous analysis:

$$x_1^S = 2V_1V_2V_3\frac{V_1(V_2+V_3)-V_2V_3}{(V_1V_2+V_1V_3+V_2V_3)^2} \wedge x_2^S + x_3^S = 4\frac{V_1V_2^2V_3^2}{(V_1V_2+V_1V_3+V_2V_3)^2}.$$

Then:

$$x_1^S > x_2^S + x_3^S \Leftrightarrow 2V_1V_2V_3\frac{V_1(V_2+V_3)-V_2V_3}{(V_1V_2+V_1V_3+V_2V_3)^2} > 4\frac{V_1V_2^2V_3^2}{(V_1V_2+V_1V_3+V_2V_3)^2}$$

$$\Leftrightarrow V_1(V_2+V_3)-V_2V_3 > 2V_2V_3 \Leftrightarrow V_1 > \frac{3V_2V_3}{V_2+V_3}.$$

■

The analysis of this section revealed how the players behave in equilibrium when they choose their decision time. The question of which decision time would be socially desirable is addressed in the next section.

6.4 Welfare Aspects

As already known in contest literature, the distribution of a rent via a contest is in general not optimal but generates dissipation. If one assumes the welfare level to be given by

$$W = \sum_{i=1,2,3} \pi_i, \tag{6.31}$$

then it is obvious that welfare is maximized when the total prize is allocated to the player with highest valuation for free. Of course, this allocation cannot be achieved in a contest. Therefore, another interesting question arises, namely, what is the least inefficient order of moves. We now address the question which order of moves generates the highest welfare level. In order to compare in which subgame welfare is highest, we first calculate the level of welfare in each subgame:

- In the simultaneous move subgame, welfare is given by

$$W^S = \frac{V_i(V_j-V_k)^2 + V_j(V_i-V_k)^2 + V_k(V_j-V_i)^2 + V_iV_jV_k}{V_iV_j+V_iV_k+V_jV_k}.$$

- In the subgame where the players i and j move early, welfare is given by:

$$W^{kL} = \frac{3V_iV_j\left(V_i^2 - V_iV_j + V_j^2\right) + V_k^2\left(V_i + V_j\right)^2 - 3\left(V_i + V_j\right)V_iV_jV_k}{\left(V_i + V_j\right)^2 V_k}.$$

- In the subgame where only player i moves early, welfare is given by:

$$W^{iE} = \frac{V_jV_k\left(2V_j^2 + V_jV_k + 2V_k^2\right) + V_i^2\left(V_j + V_k\right)^2 - 4V_iV_jV_k\left(V_j + V_k\right)}{2V_jV_k\left(V_j + V_k\right)},$$

  if (6.25) holds for both late players. Otherwise, if, for player j, (6.25) is violated, the welfare level is given by[4]

$$W^{Pr} = \frac{V_j(V_i - V_k) + V_k(V_k - V_j)}{V_k}.$$

We find that the simultaneous move game is indeed the subgame with the highest welfare level, if the strongest player 1 is rather weak, i.e., $V_1 \leq \frac{3V_2V_3}{V_2+V_3}$. Unlike in the two-player-case, the introduction of a sequential structure cannot reduce the dissipation level. For $V_1 \geq \frac{3V_2V_3}{V_2+V_3}$, W^{1E} takes the highest value if (6.25) holds. If not, it can be shown that the highest welfare is indeed reached by the game where the strongest player moves early and the weakest player is preempted, as long as $V_1 < \frac{V_2\left(V_2^2 - V_2V_3 + V_3^2\right)}{V_2^2 - V_3^2}$. For $V_1 > \frac{V_2\left(V_2^2 - V_2V_3 + V_3^2\right)}{V_2^2 - V_3^2}$, we find that the strongest player moving late on her own creates the highest welfare. We summarize:

Proposition 5 *If a contest is played between three players in two points in time, the order of moves that generates the highest welfare level in the equilibrium of the subgame is given:*

- *by all three players choosing simultaneously if the three players are relatively homogenous, i.e.,* $V_1 < \frac{3V_2V_3}{V_2+V_3}$*,*
- *by simultaneous moving or the strongest moving early on her own or the strongest player moving late on her own, if* $V_1 = \frac{3V_2V_3}{V_2+V_3}$*.*
- *by the player with the highest valuation moving early and her opponents moving late for* $V_1 \in \left(\frac{3V_2V_3}{V_2+V_3}; \frac{V_2\left(V_2^2 - V_2V_3 + V_3^2\right)}{V_2^2 - V_3^2}\right)$*. Note that the weakest player will not choose positive effort in this case.*
- *by the strongest player moving late, and her opponents moving early, if* $V_1 \geq \frac{V_2\left(V_2^2 - V_2V_3 + V_3^2\right)}{V_2^2 - V_3^2}$*.*

[4] The case that (6.25) is violated for both players j and k cannot occur, as long as (6.2) holds.

Together with Theorem 9 we can conclude:

Proposition 6 *For* $V_1 \leq \frac{3V_2V_3}{V_2+V_3}$ *or* $V_1 \geq \frac{V_2\left(V_2^2-V_2V_3+V_3^2\right)}{V_2^2-V_3^2}$*, the order of moves that creates the highest welfare level is the same one that the subgame-perfect equilibrium brings about. Otherwise, welfare can be improved over the subgame-perfect equilibrium by choosing another order of moves than the one played on the equilibrium path.*

Another interesting result can be found by comparing the payoffs from the simultaneous game with the payoffs received when the strongest player moves late. If we make this comparison for every single player, we find:

Lemma 10 *Let* V_1*,* V_2 *and* V_3 *be such that, in the equilibria of the subgames* S *and* $1L$*, every player is active. Then for* $V_1 < \frac{3V_2V_3}{V_2+V_3}$*, the simultaneous move game pareto-dominates subgame* $1L$*. Yet, for* $V_1 > \frac{3V_2V_3}{V_2+V_3}$*, subgame* $1L$ *pareto-dominates the simultaneous move game.*

6.5 Conclusion

We have shown that, in a rent-seeking contest where three players can decide endogenously about the sequential structure, different timing strategies are only played on the equilibrium path if the strongest player is stronger than her opponents together. We measure this in terms of efforts chosen in the simultaneous move game. Put differently, a player only has an incentive to move late if her winning probability exceeds $\frac{1}{2}$. Note that, for the two-player-contest analyzed by Leininger (1993), this always holds. Yet, this si different in a three-player-contest. If, in such a contest, the strongest player is too weak, the three players will move simultaneously in the subgame-perfect equilibrium. For some cases where one player is strong enough to be stronger than her opponents, it may occur that the secodn strongest player also has an incentive to choose late. In this situation, the timing decisions of these two players will not be in pure strategies. Concerning welfare aspects, we found that the order of moves that evolves in a subgame-perfect equilibrium only creates the highest aggregate payoff in a subset of all possible constellations, e.g. if the simultaneous move game is played on the equilibrium path. For the case of the strongest player moving late, for Leininger (1993) showed is welfare improving in the two-player-case, we found that in the three-player-case it can be more favorable if the strongest player moves early as the only player. In contrast, whenever the simultaneous move game is played on the equilibrium path, the welfare can not be increased by changing the order of moves. Compared to Leininger's result that a welfare improvement by sequential choice is always possible for two heterogenous players, our result is rather bad news. We have to state that, in the presence of a third player, a welfare improvement may not necessarily be possible. Furthermore, where it is possible, it will not be exhausted to its full extent any more.

Chapter 7

Incentives in the Strategic Choice of Decision Timings

7.1 Introduction

In the previous chapter, we found that, as soon as a third player is introduced, sequential moves only occur in a subgame-perfect equilibrium for a rather heterogenous population. If players are rather homogenous, a simultaneous move game is played in subgame-perfect equilibrium. In this chapter, we take a further step to look at sequential multi-player-contests, and relax the assumption of only three players. In order to keep the results manageable, we have to make some other restrictions; we assume that one player has a higher valuation than her opponents, while we assume the opponents to be equal.

As an application, one might think of a contest for a market license that grants a monopoly rent (which can be reasonable in the case of a natural monopoly). For such a case Harris and Vickers (1985) argue that the incumbent firm has a higher profit from winning the rent, which we can interpret as having a higher valuation. The reason for the higher profit is that an incumbent firm does not have to bear additional market entry costs such as for the acquisition of machinery and personnel. For such a setup, we will examine whether such an incumbent firm has an incentive to move earlier or later than her opponents.

In order to keep the analysis more general, from now on, we will use a different terminology. Following Dixit (1987) we call the firm with the higher valuation - and thus with a stronger position - the "favorite", while we call those players with the lower valuation "underdogs". With respect to the underdogs, we restrict our analysis to show that, in equilibrium, a single underdog does not have an incentive to choose a late move when the remaining underdogs choose early. This holds irrespectively of the timing decision of the favorite.

With respect to the favorite, we will see that she will always opt for an earlier move if allowed. If not, we can find a threshold value for her valuation, above which she will choose to decide later than her opponents. For a lower valuation simultaneous move is played in the subgame-perfect equilibrium. For $n = 3$, the threshold value should take the same value as the one in Chapter 6 for two equal players, since the involved subgames are identical. This indeed happens.

For the case where the favorite has a first-mover right, we find that she will always exert this right. If she faces at least two other players and her valuation is sufficiently higher than the one of her opponents, it pays for the favorite to exert such a high effort that her opponents will not have any incentive to exert a positive effort. The welfare analysis will show that, whenever this situation occurs, this is the best feasible sequential order from a welfare point of view. Note that then the stronger player is the only active player in the game. Other games where only one player is active in equilibrium have been analyzed by Konrad and Leininger (2007) for all-pay-auction and by Schoonbeek and Winkel (2006) for incomplete information.

When relaxing the assumption of the first-mover-right for the favorite, there is no equilibrium in which she chooses her effort before her opponents. Yet, the results of Leininger (1993) that the favorite will always choose to move after her opponent in a two-player contest can again be generalized to the multi-player case in so far that she moves late if her winning probability in the simultaneous movbe game exceeds $\frac{1}{2}$, similar to the results in Chapter 6. If the underdogs taken together exert a higher effort in equilibrium, it does not pay for the favorite to separate in timing strategy and the simultaneous game will be played.

From the view of mechanism design, we are also able to make a statement about whether or not a first-mover-right should be granted to the player with the highest valuation. We will provide a threshold value for the relative valuation, from which point on the first-mover-right, and thus the game structure in which the favorite moves first, will bring about a higher social welfare than the game where her opponents decide in the first round.

7.2 Setup of the game

7.2.1 The model

The payoff structure of the game we analyze is described by a Tullock-contest (Tullock, 1980). We assume that the contest technology has constant returns to scale. Let the number of players be fixed to n, denote the non-negative effort of player i by x_i. The expected payoff of player i is then determined by (6.1). We assume

$$V_1 = \theta V_i, \quad \theta > 1 \wedge i = 2, ..., n,$$

where θ represents the relative valuation of the favorite. For simplification, we write $V_i =: V$, $\forall i = 2, ..., n$. With this, we assume the players i, $i = 2, ..., n$ to have identical valuations.

7.2.2 Timing decision

The aim of this chapter is to focus on the incentives for the favorite to choose her timing decision. We will consider two variants of the meta-game:

1. Like the underdogs, the favorite chooses a timing $t = 1$ or $t = 2$. She cannot secure herself an earlier choice than her opponents.

2. The favorite chooses a timing $t = 0$, $t = 1$ or $t = 2$. This means that the favorite can exert the exclusive right of the first move.

For the case of a mixed strategy concerning the timing decision, let the realization of the strategy be drawn before $t = 0$. Before the first effort choice is made, the subgame played is common knowledge. The strategy set of the favorite's timing decision consists of two alternative timings in variant 1 and three alternatives in variant 2. The timing decision of the favorite determines in which subgame the contest will be played. For now, we will simply assume that the underdogs choose in $t = 1$. Later on, we will point out that an underdog does not have any incentive to deviate from $t = 1$ to $t = 2$. Thus, the equilibria presented here are subgame-perfect equilibria of a game in which the underdogs can decide whether to choose their effort in $t = 1$ or $t = 2$.

Considering the subgames where the underdogs move simultaneously, we call the subgame where all players choose simultaneously "Game S", the subgame where the favorite moves last is called "Game L", and the one where she moves first is called "Game E". As we will see, the latter subgame will be only relevant for the analysis of variant 2. Let us next analyze these subgames

7.3 The Subgames

7.3.1 Equilibrium Behavior in the Simultaneous Game

Consider a rent-seeking contest with constant returns to scale, where all players decide simultaneously about their effort. Then the favorite maximizes $\pi_1(\mathbf{x}) = \frac{x_1}{\sum_{j=1}^{n} x_j}\theta V - x_1$,while each of the underdogs maximizes $\pi_i(\mathbf{x}) = \frac{x_i}{\sum_{j=1}^{n} x_j}V - x_i$, $2 \leq i \leq n$. The first-order-condition for the favorite is given by:

$$\frac{\partial \pi_1(\mathbf{x})}{\partial x_1} = 0 \Leftrightarrow \theta \sum_{j=2}^{n} x_j V = \left(\sum_{j=1}^{n} x_i \right)^2, \tag{7.1}$$

while the underdogs decide according to

$$\frac{\partial \pi_i(\mathbf{x})}{\partial x_i} = 0 \Leftrightarrow \sum_{j=1, j\neq i}^{n} x_j V = \left(\sum_{j=1}^{n} x_i \right)^2, \quad 2 \leq i \leq n. \tag{7.2}$$

From Proposition 3, we can see that, from (7.2), we can conclude that $x_i = x_j$, $2 \leq i, j \leq n$. Equating (7.1) and (7.2) and inserting $x_i = x_j$ yields:

$$x_1 = \left[(n-1)\theta - (n-2) \right] x_i. \tag{7.3}$$

Equilibrium efforts are then given by (see appendix for derivations):

$$x_1^S = \frac{(n-1)\theta}{(1+(n-1)\theta)^2}\left((n-1)\theta-(n-2)\right)V \tag{7.4}$$

and

$$x_i^S = \frac{(n-1)\theta}{(1+(n-1)\theta)^2}V, \quad 2 \leq i \leq n. \tag{7.5}$$

Equilibrium payoffs are thus given by

$$\pi_1^S = \frac{\left((n-1)\theta-(n-2)\right)^2}{(1+(n-1)\theta)^2}\theta V \tag{7.6}$$

and

$$\pi_i^S = \frac{V}{(1+(n-1)\theta)^2}, \quad 2 \leq i \leq n. \tag{7.7}$$

These results build on the assumption that all players move simultaneously. But from the literature (e.g. Dixit, 1987, Baik and Shogren, 1992, Leininger, 1993) and the previous chapter, we know that it can be a stable equilibrium, if players do not decide at the same time but sequentially.

7.3.2 Equilibrium Behavior in Subgame L

For our analysis, let us next assume the favorite chooses to decide upon his effort level after her opponents do. She takes the effort level of the underdogs as given. Again, we can conclude from the results of Proposition 3 that all underdogs will choose the same effort in the first round. Denoting this effort level by $\hat{x}$, the first-order-condition of the favorite is given by

$$\frac{\partial \pi_1(x_1,[\hat{x}]_{n-1})}{\partial x_1} = \frac{(n-1)\hat{x}}{(x_1+(n-1)\hat{x})^2}\theta V - 1 = 0$$

$$\Leftrightarrow x_1 = \sqrt{(n-1)\hat{x}\theta V} - (n-1)\hat{x}. \tag{7.8}$$

Her opponents anticipate this in the first stage and are also able to see, in how far this action depends on their own effort x_i. As mentioned above, in equilibrium, $x_i = x_j, \quad i,j = 2,...,n$. But in the decision process, note that an underdog i will interpret "$(n-1)\hat{x}$" as "$(n-2)\hat{x}+x_i$", taking the other underdogs' reactions as given, but not so her own. Each underdog maximizes her payoff π_i over her own effort x_i. The reduced payoff function is given by $\pi_i(x_1,[\hat{x}]_{n-1}) = \frac{x_i V}{\sqrt{\theta[(n-2)\hat{x}+x_i]V}} - x_i$. The equilibrium behavior of an underdog is thus characterized by

$$\frac{\partial \pi_i(x_1,[\hat{x}]_{n-1})}{\partial x_i} = \frac{\sqrt{\theta\left[(n-2)\hat{x}+x_i\right]V} - \frac{x_i}{2}\frac{\theta V}{\sqrt{\theta[(n-2)\hat{x}+x_i]V}}}{\theta\left[(n-2)\hat{x}+x_i\right]V}V - 1 = 0. \tag{7.9}$$

Since in equilibrium all underdogs behave the same, we can set $\hat{x} = x_i$ and find that (7.9) is equivalent to

$$x_i^L = \hat{x} = \frac{(2n-3)^2}{4(n-1)^3\theta}V. \tag{7.10}$$

Now inserting (7.10) in (7.8) gives us the strategy of the favorite:

$$x_1^L = \frac{(2n-3)\left(2(n-1)\theta - (2n-3)\right)}{4(n-1)^2\theta}V. \tag{7.11}$$

From the strategy, we can calculate the favorite's level of payoff from moving last:

$$\pi_1^L = \frac{(2(n-1)\theta - 2n+3)^2}{4(n-1)^2\theta}V. \tag{7.12}$$

Her opponents' payoff is given by

$$\pi_i^L = \frac{(2n-3)}{4(n-1)^3\theta}V. \tag{7.13}$$

7.3.3 Equilibrium Behavior in Subgame E

In this section, we assume that the favorite will choose her effort x_1 in the first stage and the underdogs will follow in the second stage. Each underdog takes the effort of the first player x_1 as exogenously given and maximizes in the second stage $\pi_i(\mathbf{x}) = \frac{x_i}{x_1+\sum_{j=2}^n x_j}V - x_i, \quad 2 \leq i \leq n$. The first-order-condition is given by

$$\frac{\partial \pi_i(\mathbf{x})}{\partial x_i} \overset{!}{=} 0 \Leftrightarrow \left(x_1 + \sum_{j=2, j\neq i}^{n} x_j\right) V = \left(x_1 + \sum_{j=2}^{n} x_j\right)^2. \tag{7.14}$$

For a symmetric equilibrium, which will come into place due to Proposition 3, we can turn (7.14) into

$$\left(x_1 + (n-2)x_i\right) V = \left(x_1 + (n-1)x_i\right)^2. \tag{7.15}$$

However, we cannot directly conclude from this player i's reaction function without regarding the participation constraint. From Lemma 6 we know that a player i will only choose positive effort in equilibrium, if, for her valuation V_i, it holds that $V_i > \sum_{j=1}^n x_j$. This means, that a player will only exert a positive effort if her valuation for the prize exceeds the level of dissipation in the game. A player will only take part in the contest as long as the total effort spent in the game does not exceed her own valuation. If we pay attention to this restriction, we can derive the reaction function of an underdog to the favorite's behavior from (7.15), while taking into account the optimal behavior of the other underdogs.

Lemma 11 *In an intragroup symmetric equilibrium where the favorite moves first, the optimal reaction function of an underdog i with $2 \leq i \leq n$ is given by*

$$x_i = \hat{x}(x_1) = \begin{cases} \frac{(n-2)V+\sqrt{(n-2)^2V^2+4(n-1)x_1V}}{2(n-1)^2} - \frac{x_1}{n-1}, & \text{if } x_1 \leq V \\ 0 & \text{, if } x_1 \geq V. \end{cases} \tag{7.16}$$

Proof. See appendix. ∎

Like above, let us in the following denote the effort of an underdog, that is only viewed upon in the aggregate, by $\hat{x}$. The favorite anticipates the reaction of the underdogs. Assuming that the favorite's effort will not exceed V, her payoff function turns to

$$\pi_1(x_1, [\hat{x}(x_1)]_{n-1}) = \frac{x_1}{x_1 + (n-1)\hat{x}}\theta V - x_1$$

$$\Leftrightarrow \pi_1(x_1, [\hat{x}(x_1)]_{n-1}) = x_1 \left(\frac{2(n-1)\theta V}{(n-2)V + \sqrt{(n-2)^2V^2 + 4(n-1)x_1V}} - 1 \right). \tag{7.17}$$

Since for $x_1 \geq V$ the underdogs always react with a zero effort, it is obvious that any choice of $x_1 > V$ leads to a lower payoff for the favorite than $x_1 = V$, and thus cannot maximize her payoff. So, we can restrict our analysis to $x_1 \leq V$. One can show that the payoff function in (7.17) is concave, so we can conclude that the optimal strategy of the favorite is characterized by $x_1 < V$, if for $x_1 = V$ it holds that $\frac{\partial \pi_1}{\partial x_1} < 0$. On the other hand, for $\frac{\partial \pi_1(V,[\hat{x}(V)]_{n-1})}{\partial x_1} \geq 0$, we find that she will choose $x_1 = V$. Solving this equation, we find

Proposition 7 *The favorite will choose to invest $x_1 = V$ and thus exclude her opponents from the contest if and only if, for her relative valuation θ, holds that*

$$\theta \geq \frac{n}{n-1}.$$

Her payoff will then be given by $\pi_1 = (\theta - 1)V$.

Proof. See appendix. ∎

Beyond the results of Proposition 7, let us also calculate how the players behave in this subgame, if $\theta < \frac{n}{n-1}$. In this case, the equilibrium effort of the favorite is the solution to the first derivative of (7.17) being zero. The solution is given by

$$x_1^E = \frac{(n-1)^2\theta^2 - (n-2)^2}{4(n-1)}V. \tag{7.18}$$

Inserting $\theta = \frac{n}{n-1}$ in (7.18) yields $x_1^E = V$. Therefore, the optimal effort of the favorite is continuous in θ. Reinserting (7.18) in (7.16) yields the effort an underdog exerts if $\theta < \frac{n}{n-1}$:

$$x_i^E = \frac{n(n-2) + 2(n-1)\theta - (n-1)^2\theta^2}{4(n-1)^2}V. \tag{7.19}$$

Equilibrium payoffs in this subgame are thus given by

$$\pi_1^E = \begin{cases} \frac{((n-1)\theta-(n-2))^2}{4(n-1)}V, & \text{if } \theta < \frac{n}{n-1} \\ (\theta-1)V, & \text{if } \theta \geq \frac{n}{n-1}, \end{cases} \tag{7.20}$$

and

$$\pi_i^E = \begin{cases} \frac{(n-(n-1)\theta)^2}{4(n-1)^2}V, & \text{if } \theta < \frac{n}{n-1} \\ 0, & \text{if } \theta \geq \frac{n}{n-1}. \end{cases} \tag{7.21}$$

So far, this section has concentrated on the analysis of subgames that differ in the timing decision of the favorite. Yet it is also conceivable that an underdog could have an incentive to choose a different timing from $t = 1$. We address this issue by presenting:

Lemma 12 *Given that all $(n-2)$ players j with $j = 2, ..., n$, $j \neq i$ choose their effort in $t = 1$, player i, $i = 2, ..., n$, has an incentive also to choose her effort in $t = 1$. This result holds regardless of the timing decision of the favorite.*

Proof. See appendix. ■

7.4 Subgame-perfect Equilibria

This section analyzes the subgame-perfect equilibria of the games introduced in Subsection 7.2.2. From Lemma 12, we can conclude that a combination of timing decisions, where the underdogs move in $t = 1$ and the favorite does not have a strict incentive to choose another timing choice, is the timing decision of a subgame-perfect equilibrium. Therefore, to be able to find such equilibria it is sufficient to compare the payoffs, which the subgames analyzed in Section 7.3 would deliver to the favorite. We continue our analysis by determining, which subgame produces the highest payoff for the favorite given θ. The payoffs the favorite gains in each subgame are given by equation (7.6) for the simultaneous game, by equation (7.20) if she chooses first and by equation (7.12) if she chooses after her opponents. Now by equating these equations pairwise, we find

Proposition 8 *Let all underdogs decide about their effort in $t = 1$. Then*

1. *for any valuation θV with $\theta > 1$, the favorite gains a higher equilibrium payoff from moving early, i.e., $t = 0$, than from moving late, i.e., $t = 2$. This means: $\pi_1^E > \pi_1^L, \quad \forall \theta > 1, \quad n \geq 3$.*

2. *For any valuation θV with $\theta > 1$, the favorite gains a higher equilibrium payoff from moving early, i.e., $t = 0$, than from simultaneous moves, i.e., $t = 1$. This means: $\pi_1^E > \pi_1^S, \quad \forall \theta > 1, \quad n \geq 3$.*

3. *The equilibrium payoff of the favorite is (strictly) higher from moving late ($t = 2$) than from a simultaneous move ($t = 1$) if and only if θ is (strictly) higher than the threshold value $\hat{\theta} = \frac{2n-3}{n-1}$. This means $\pi_1^L \gtrless \pi_1^S$, iff $\theta \gtrless \hat{\theta}, \quad n \geq 3$.*

Proof. See appendix. ■

Remark that, whenever $\theta < \frac{2n-3}{n-1}$ holds, the favorite has a strict preference for moving earlier. Now having analyzed the relation of the favorite's payoff in different subgames, we can take a closer look at the game in total. We comprise

Theorem 10

1. *For variant 1, the subgame-perfect equilibria are characterized by*
   (a) *the favorite choosing (7.4) and each underdog choosing (7.5), both in $t = 1$, if $\theta < \frac{2n-3}{n-1}$, and by*
   (b) *the favorite choosing (7.11) in $t = 2$ and each underdog choosing (7.10) in $t = 1$, if $\theta > \frac{2n-3}{n-1}$.*
2. *For variant 2, the subgame-perfect equilibrium is given by the favorite choosing (7.18) in $t = 0$ and each underdog choosing (7.19) (in $t = 1$), for all values of $\theta > 1$.*

Proof. The theorem follows from Proposition 8. ■

For variant 1, Theorem 10 also holds for the two-player-case as one can see from Leiniger (1993). In this case, the threshold value $\frac{2n-3}{n-1}$ reduces to 1. This means that the favorite will always choose late. This is the same result that was shown by Leininger. Furthermore, this result runs in line with the major result of Chapter 6. If we assume for the analysis of Chapter 6 that both weaker players have the same valuation, then this is the same game as the one presented here with $n = 3$. Both Theorems 9 and 10 state that, if the favorite's valuation is higher than $\frac{3}{2}$ of the underdog's valuation, she will move later, while if her valutaion is lower than $\frac{3}{2}$ of the underdog's valuation, she will prefer a simultaneous move game.

Similar to Corollary 8, where we exposed that the favorite will prefer to move later if her effort in the simultaneous move game is higher than the aggregate effort of her opponents we find here that:

Corollary 9 *In subgame-perfect equilibrium, the favorite will prefer a late move to a simultaneous move if her equilibrium effort in the simultaneous move game exceeds the aggregate effort of her opponents, i.e., if her winning probability from the simultaneous move game exceeds $\frac{1}{2}$.*

Proof. From (7.4) and (7.5), we can see that:

$$x_1^S > (<)(n-1)x_i^S \Leftrightarrow (n-1)\theta - (n-2) > (<)n - 1 \Leftrightarrow \theta > (<)\frac{2n-3}{n-1}.$$

■

Furthermore, we can see from Theorem 10 that whenever the favorite has the opportunity to choose earlier, she will do so. This implies directly that the equilibrium path of the game changes depending on whether the favorite is granted a first-mover-right.

7.5 Welfare Implications

In this section, we analyze which type of sequential structure would maximize the social welfare given the valuation structure. In particular, this section answers the question: which timing a contest designer who maximizes welfare would assign to the favorite? Let us therefore define welfare as the sum of payoffs of all players, i.e.,

$$W := \sum_{j=1}^{n} \pi_j. \tag{7.22}$$

When looking at the welfare level achieved in game E, we have to split up the analysis into two cases. For $\theta \geq \frac{n}{n-1}$, it is obvious that the welfare level is given by

$$W^E = (\theta - 1)V. \tag{7.23}$$

Now looking at $\theta < \frac{n}{n-1}$, we find from (7.20) and (7.21) that for this case

$$W^E = \frac{1 + (n-1)^2 (\theta - 1)^2}{2(n-1)} V. \tag{7.24}$$

Now for the simultaneous game the welfare level is given by

$$W^S = \frac{((n-1)\theta - (n-2))^2 \theta + (n-1)}{[1 + (n-1)\theta]^2} V. \tag{7.25}$$

Finally, in the situation where the favorite moves after her opponents, we have

$$W^L = \frac{2(n-1)\theta^2 + (2n-3)(1-2\theta)}{2(n-1)\theta} V. \tag{7.26}$$

From the levels of welfare, we can derive which sequence of moves reaches the highest welfare level in which parameter constellation.

Proposition 9

1. *Welfare is higher in the equilibrium of the subgame where the favorite moves early than in the case of simultaneous choice, i.e., $W^E > W^S$, if and only if $\theta > \frac{n}{n-1}$.*

2. *Welfare is higher in the equilibrium of the subgame where the favorite moves early than when she moves late, i.e., $W^E > W^L$, if and only if $\theta > \frac{n+1+\sqrt{n^2-6n+13}}{2(n-1)}$.*

3. *The welfare level in the equilibrium of the simultaneous move game is (strictly) higher than in the game with the favorite moving late, i.e., $W^S > W^L$ if and only if $\theta < \frac{2n-3}{n-1}$.*

Proof. See appendix. ■

Now from Proposition 9, we can conclude that

Theorem 11 *Let $n \geq 3$. Then, subgame S brings about the highest welfare in equilibrium, if $\theta \in \left(1, \frac{n}{n-1}\right)$ and subgame E brings about the highest welfare in equilibrium, if $\theta > \frac{n}{n-1}$.*

By comparing these results with Theorem 10, we find under which circumstances equilibrium behavior leads to the subgame with highest welfare:

Corollary 10 *Consider in both variants the subgame-perfect equilibrium to be played. Then in variant 1, the subgame with the highest welfare is played if $\theta < \frac{n}{n-1}$, while in variant 2, the subgame with the highest welfare is played if $\theta > \frac{n}{n-1}$.*

Proof. The proof follows directly from Theorem 10 and Theorem 11. ■

This is especially interesting for the design of contest rules. Consider an institution that has to constitute the rules of a rent-seeking contest with the above described set of players. Let the appropriate social welfare function be given by (7.22) and the institution has to decide whether the favorite should be allowed to make an early move in $t = 0$. Now Corollary 10 tells us that the institution should allow the favorite to move early, if and only if $\theta \geq \frac{n}{n-1}$. Note that in this scenario, the favorite will be the only player choosing a poitive effort.

7.6 Conclusions

This chapter analyzes a game where a favorite faces a number of identical opponents in a sequential rent-seeking contest. First, we focus on the incentives of the favorite given that the underdogs choose a fixed timing. We find that the favorite will always prefer an early move to both simultaneous and late move, if there are at least two underdogs. She will only prefer the late move to simultaneous move, if her valuation is so high that her effort in the simultaneous game would be higher than the sum of the effort levels chosen by her opponents. Furthermore, we show that the underdogs do not have an incentive to deviate from an early move. Therefore the presented results also represent a subgame-perfect equilibrium if the underdogs are also allowed to choose their timing, too. So far, the model generalizes parts of the results by Leininger (1993) and Chapter 6.

Furthermore, we discuss the role of a first-mover-right. We find that such a right will be exerted by the favorite if offered. From a welfare point of view, it should be granted that, whenever the favorite chooses such a high effort level if she moves early, she will be the only player who chooses a strictly positive effort. This result tells us that only one firm should actively participate in the contest. However, since this player preempts her opponents, she does not exert zero effort (as she would do if she was the only player at all, i.e. $n = 1$), but a not negligible positive effort level. This is because her passive opponents still play an important role in the contest, since they threaten to choose a positive effort once the favorite reduces her own effort level. In this situation, the favorite wins with probability 1. If the valuation of the favorite is so low that it does not pay for her to preempt her opponents, then a simultaneous move game creates a higher welfare. For this case, it would be optimal not to grant a first-mover-right. This would result in simultaneous moves on the equilibrium path, since a late move would not pay for the favorite.

Chapter 8

When is the Assumption of Simultaneous Moves in Contests Justfiable?

8.1 Introduction

The previous chapters have analyzed the sequential structure of rent-seeking contests with (a) three heterogenous players and (b) n players but only two different types of players. In the next step, we will look at the incentives that a sequential structure creates in a contest with n different types of players. Due to complexity, we will focus on the question whether or not sequential structure will occur in a subgame-perfect equilibrium at all. Many analyses in contest theory assume *ex ante* that players choose to move simultaneously without knowing their opponent's choice (e.g. Tullock, 1980, Stein, 2002). But the previous analysis as well as earlier literature (e.g. by Dixit, 1987, Baik and Shogren, 1992, Leininger, 1993) show that simultaneous moves are not necessarily played on the equilibrium path. These results especially of Baik and Shogren and Leininger undermine the *ad-hoc* assumption of simultaneous moves since we now know that, under full information, two players will always arrange themselves in an order of moves different from the simultaneous move game. Yet, these results are restricted to two players. The results of the Chapters 6 and 7 indicate that the simultaneous move game can indeed be part of a subgame-perfect equilibrium without incentives to deviate. Our analysis will now analyze under which general conditions a subgame with simultaneous moves can be reached on the subgame-perfect equilibrium path.

8.2 When will Late Move be Preferred?

Since this chapter builds up on the previous chapters, the payoff structure of the game we analyze here is also described by a Tullock-contest (Tullock, 1980) with constant returns to scale. As above, let the number of active players be given by n and each player i's payoff be given by (6.1). Now, let every player have the opportunity to either commit to their effort in an early point of time, $t = 1$, or to delay the decision to the late timing, $t = 2$. Let us assume that every player knows who decides in $t = 1$ and who decides in $t = 2$, when she chooses her effort.

As the aim of analysis is to determine the set of constellations where the simultaneous move game occurs on the equilibrium path we should make clear upfront what this means:

A simultaneous move game is played on the equilibrium path if every player chooses her effort at the same point of time and no single player has an incentive to deviate from her strategy. This can occur in two possible ways. The first one is that every player moves in $t = 1$ and no one has an incentive to deviate to $t = 2$. The other one is that every player moves in $t = 2$ and no one has an incentive to deviate to $t = 1$. To be able to identify the presence of such incentives, we first have to know the payoffs that players can gain from the simultaneous move game. The Nash-equilibrium of this game has already been studied by Stein (2002). By denoting the number of active players by n, we can write the effort that an active player i spends in the equilibrium of a simultaneous move game as:

$$x_i^S = \frac{n-1}{\sum_{j=1}^n \frac{1}{V_j}} - \left(\frac{n-1}{\sum_{j=1}^n \frac{1}{V_j}} \right)^2 \frac{1}{V_i}. \tag{8.1}$$

From this, we can calculate the aggregate effort as $\sum_{j=1}^n x_j = \frac{n-1}{\sum_{j=1}^n \frac{1}{V_j}}$ and the payoff of an individual player i as

$$\pi_i^S = V_i \left(1 - \frac{n-1}{V_i \sum_{j=1}^n \frac{1}{V_j}} \right)^2, \ \forall i = 1, .., n. \tag{8.2}$$

Now assume that all players would choose $t = 1$. If, without loss of generality, player 1 would deviate, she would be the only player to move late. In this case, player 1 will take the aggregate effort of her opponents' as given. We denote it by $A_{-1} := \sum_{i=2}^n x_i$. Her decision will then be

$$\frac{\partial \pi_1(\mathbf{x})}{\partial x_1} = \frac{A_{-1}}{(x_1 + A_{-1})^2} V_1 - 1 = 0$$

$$\Leftrightarrow x_1 = \sqrt{A_{-1} V_1} - A_{-1}. \tag{8.3}$$

Her opponents, of course, anticipate this in the first stage and are also able to see how this action depends on their own effort x_i. Let us now look at how a player $i = 2, ..., n$ will behave in this situation, reacting to the aggregate effort of the $(n-2)$ early movers $A_{-i} := \sum_{j=2, j \neq i}^n x_j$. Note that $A_{-i} = A_{-1} - x_i$.

So, the payoff of player i moving early can be written as

$$\pi_i(\mathbf{x}) = \frac{x_i V_i}{\sqrt{(A_{-i} + x_i) V_1}} - x_i.$$

This payoff function is maximized by

$$\Leftrightarrow x_i = 2A_{-1} - 2\frac{\sqrt{V_1}}{V_i} (A_{-1})^{\frac{3}{2}}, \tag{8.4}$$

as shown in the appendix.

Similarly to Stein (2002), we can now determine the aggregate effort in the early period by summing up (8.4) for every player $i = 2, ..., n$. Denoting $\Omega = \sum_{i=2}^{n} \frac{1}{V_i}$ we receive:

$$A_{-1} := \sum_{i=2}^{n} x_i = 2(n-1)A_{-1} - 2A_{-1}^{\frac{3}{2}}\sqrt{V_1}\Omega \Leftrightarrow 2\sqrt{A_{-1}V_1}\Omega = 2n-3$$

$$\Leftrightarrow \sqrt{A_{-1}} = \frac{2n-3}{2\sqrt{V_1}\Omega}.$$

Thus,

$$A_{-1} = \frac{(2n-3)^2}{4V_1\Omega^2}. \tag{8.5}$$

Then, the payoff of an early mover i is given by:

$$\pi_i^{1L} = \frac{1}{V_i}\frac{2n-3}{\Omega V_1}\left(V_i - \frac{2n-3}{2\Omega}\right)^2.$$

On the other hand, this enables us to calculate the effort of the late mover by:

$$x_1^{1L} = \sqrt{A_{-1}V_1} - A_{-1} = \frac{2n-3}{2\Omega}\left(1 - \frac{2n-3}{2\Omega V_1}\right).$$

It is only non-negative for

$$V_1 \geq \frac{2n-3}{2\left(\sum_{j=1,j\neq i}^{n} \frac{1}{V_j}\right)}.$$

The payoff player 1 recieves in this situation is given by:

$$\pi_1^{1L} = \frac{\sqrt{A_{-1}V_1} - A_{-1}}{\sqrt{A_{-1}V_1}}V_1 - \sqrt{A_{-1}V_1} + A_{-1}$$

$$= V_1 - 2\sqrt{A_{-1}V_1} + A_{-1} = \left(\sqrt{V_1} - \sqrt{A_{-1}}\right)^2,$$

which holds for any aggregate effort A_{-1} that players spend on the early stage. Inserting (8.5), we receive player 1's payoff in equilibrium:

$$\pi_1^{1L} = \left(\sqrt{V_1} - \sqrt{\frac{(2n-3)^2}{4V_1\Omega^2}}\right)^2 = V_1\left(1 - \frac{2n-3}{2V_1\Omega}\right)^2.$$

This enables us to compare the payoffs from the simultaneous move game to the one, where one player moves later.

Lemma 13 *Let n players participate in a rent-seeking contest with constant returns to scale. Then, we find that player 1 moving late alone brings about a (strictly) higher equilibrium payoff than the simultaneous move game, iff*

$$V_1 \geq (>)\frac{2n-3}{\sum_{i=2}^{n} \frac{1}{V_i}}.$$

Proof. See appendix. ■

Note that this threshold value reduces for $n = 2$ to $V_1 \geq V_2$, which underpins the result of Leininger (1993). Yet, for $n \geq 3$, we find that the threshold value is strictly above the harmonic mean of the opponent's valuations such that there is a substantial set of combination, where no player exceeds the threshold value.

From this result, we are able to conclude:

Proposition 10 *Let n players participate in a rent-seeking contest with simultaneous moves. If there is no player with $V_1 > \frac{2n-3}{\sum_{j=2}^{n} \frac{1}{V_j}}$, then no player has an incentive to deviate to a later move.*

In the previous chapters, we found that a player will only choose a late move instead of a simultaneous move game if she spends a higher effort in the equilibrium of the simultaneous move game than all her opponents together. We are also able to find a corresponding result in this model:

Corollary 11 *A player will prefer a late move to a simultaneous move, if her equilibrium effort in the simultaneous move game exceeds the aggregate effort of her opponents, i.e., if her winning probability in the simultaneous move game exceeds $\frac{1}{2}$.*

Proof. See appendix. ■

Now let us shed some light on the question whether a deviation to an earlier point in time can be profitable.

8.3 When will Early Move be Preferred?

In this section, we will check whether a situation where all players move late can also be an equilibrium. If a player would deviate from this equilibrium, we would end up in a subgame, where one player moves early and $n-1$ players move late. Now assume, without loss of generality, that player 1 moves early on her own. Let $A = \sum_{i=1}^{n} x_i$ and remember that $\Omega := \sum_{i=2}^{n} \frac{1}{V_i}$ and $A_{-1} = \sum_{i=2}^{n} x_i$. Then the first-order-condition of a late mover i maximizing (6.1) is:

$$(A - x_i)V_i = A^2 \Leftrightarrow x_i = A - \frac{A^2}{V_i}, i = 2, ..., n.$$

Summing this up for $i = 2, ..., n$ yields:

$$A_{-1} = (n-1)A - A^2\Omega.$$

From the definitions, it is obvious that $A := A_{-1} + x_1$. Then, it follows that

$$A_{-1} = \frac{n-2}{2\Omega} - x_1 + \sqrt{\frac{x_1}{\Omega} + \frac{(n-2)^2}{4\Omega^2}}.$$

The early mover's payoff function is then given by

$$\pi_1 = \frac{x_1}{A_{-1} + x_1} V_1 - x_1 = \frac{2\Omega x_1}{(n-2) + \sqrt{4\Omega x_1 + (n-2)^2}} V_1 - x_1.$$

The first-order-condition is then given by $\frac{\partial \pi_1}{\partial x_1} = 0$, which can be transformed to

$$x_1 = \frac{\Omega^2 V_1^2 - (n-2)^2}{4\Omega}.$$

The resulting payoff is given by:

$$\pi_1^{1E} = \frac{(\Omega V_1 - (n-2))^2}{4\Omega}.$$

The payoff from the simultaneous move game is given by (8.2), which for player 1 can be written as: $\pi_1^S = V_1 \left(1 - \frac{n-1}{\Omega V_1 + 1}\right)^2$. One can show:

Lemma 14 *Let n players participate in a rent-seeking contest. We will find that an early move always brings about a higher equilibrium payoff than the simultaneous game. Unless $n = 2$ and $V_1 = V_2$, the difference is strictly positive.*

Proof.

$$\pi_1^{1E} > \pi_1^S \Leftrightarrow \frac{(\Omega V_1 - (n-2))^2}{4\Omega} > V_1 \left(\frac{\Omega V_1 + 1 - n + 1}{\Omega V_1 + 1}\right)^2 \Leftrightarrow (\Omega V_1 + 1)^2 > 4\Omega V_1$$

$$\Leftrightarrow \Omega^2 V_1^2 + 2\Omega V_1 + 1 > 4\Omega V_1 \Leftrightarrow \Omega^2 V_1^2 - 2\Omega V_1 + 1 > 0 \Leftrightarrow (\Omega V_1 - 1)^2 > 0,$$

which is strictly fulfilled unless $\Omega V_1 = 1$. The latter is equivalent to $\sum_{i=2}^{n} \frac{1}{V_i} = \frac{1}{V_1}$, which necessarily holds for all but one player, if $n \geq 3$. ■

We learnt that, whenever there are more than two players, there is always a player who has a strict incentive to move early in the considered situation. Even for $n = 2$, Leininger (1993) showed that simultaneous moves in $t = 2$ can only be equilibrium if $V_1 = V_2$. Yet, this is not a unique equilibrium.

Theorem 12 *A simultaneous move in $t = 1$ is only part of a subgame-perfect equilibrium if all players are rather homogenous, i.e. no player with $V_1 > \frac{2n-3}{\sum_{i=2}^{n} \frac{1}{V_i}}$ exists. A simultaneous move in $t = 2$ is never part of a subgame-perfect equilibrium.*

In line with Corollary 8 and 9, we are able to find that this result is also driven by the strongest player exerting more than half of the effort spent in a simultaneous move game.

Corollary 12 *In a sequential rent-seeking contest, players will coordinate their timing to a simultaneous move game if, in such an equilibrium, no player spends a higher effort than all of her opponents together, i.e., no player has a winning probability of more than $\frac{1}{2}$.*

Proof. See appendix. ■

This enables us to conclude state that, for rather homogenous populations of at least three players, the assumption of simultaneous move can indeed be justified.

8.4 Conclusion

In this chapter, we looked at a sequential contest with an arbitrary number of heterogenous players and focused on the question: under which conditions are simultaneous moves played on the equilibrium path of a subgame-perfect equilibrium? To this aim, we looked at the incentives a player has to deviate from a simultaneous move game.

We found that, if deviation to an earlier point of time is possible, there is always a player that has a strict incentive to move earlier than her opponents if the number of players exceeds two. Even for two players, Leininger had shown, that such a situation cannot be a prominent equilibrium, but only one of many possible equilibria with equal payoff structure if players are homogenous.

Yet, we found that, if there is no earlier point in time simultaneous moves can indeed be played in the subgame-perfect equilibrium if the players are sufficiently homogenous. As we already observed in Chapters 6 and 7, we find that also in this setting a player is only willing to choose a later timing than her opponents if her effort in the simultaneous move game is higher than the aggregate effort of her oponents. Otherwise, the *ad-hoc* assumption of simultaneous moves can indeed be justified by an endogenously chosen order of moves.

Chapter 9

Conclusion

This work contributes to two different branches of literature in contest theory. As a common feature, both parts focus on rent-seeking contests with a Tullock-contest-function, assuming that the expended effort is wasted.

In the first part, we focus on interdependent preferences. At first, we build up a model where individuals of two homogenous groups compete for a rent. One group has independent preferences, while the other one has negatively interdependent preferences. The aim of this analysis is to point out under which conditions the spiteful players attain a higher material payoff. We find that there is a broad range of situations where spiteful preferences experience this strategic advantage, especially for two-player-contests and contests with non-increasing returns to scale. For increasing returns to scale, we discuss all possible forms of intragroup symmetric Nash-equilibria (concerning the order of efforts) and point out under which conditions they can or cannot occur. Furthermore, we discuss possible applications of this result, which can be interpreted as a strategic advantage to negatively interdependent preferences, such as the implications that the possibility of *choosing* spiteful preferences has on decisions concerning delegation.

In the next chapter, we relax the assumptions of only two different types and the restriction to weakly spiteful preferences for a contest with linear preference functions. We find that, among participating players, more spiteful players receive a higher material payoff, as long as underdissipation is observed. This happens especially in the absence of strongly spiteful players. In contrast, if the latter dominate the game, the less spiteful players will receive a higher payoff, although it is negative. Furthermore, we show that, for each player with an interdependent preference, there exists a valuation (different from the true valuation) such that a player with independent preferences and this valuation behaves like the corresponding player with interdependent preferences. This interdependency equivalent valuation is higher than the true valuation if the player is spiteful and lower if the player is altruistic. Concerning the participation of players, we find that for constant returns to scale positive effort choice cannot be taken for granted. We show that a player quits active participation if the dissipation level is higher than the interdependency equivalent valuation.

The last section of the first part addresses the relevance of interdependent preferences for evolutionary questions. We find that relative payoff maximization constitutes an evolutionarily stable preference for a large scale of contests, including also contests with increasing returns to scale. Note that relative payoff maximization results from exactly that preference that separates weakly and strongly preference. In so far, this result goes in line with the result that, in a population with only weakly spiteful players, a more spiteful preference yields a higher equilibrium payoff, while in a population with only strongly spiteful preferences, the reverse holds.

The second part of the work analyzes what happens if the order of moves is endogenized into a contest with more than two players. We find that, for more than two players, there is always a range of parameter constellations in the n-player-case where players will choose to move simultaneously in the subgame-perfect equilibrium, namely if players are rather homogenous in the sense that the valuations do not differ too much.

For the two special cases for which we provide a welfare analysis (in Chapter 6 for three players, and in Chapter 7 for n players, where $n-1$ players are equal), we find that a simultaneous move game indeed represents the order of moves that leads to the highest aggregate paoyff if it played on the equilibrium path.

The first analysis we present in this part focuses on a contest with three players that can choose their timing between an early and a late move. We find that the strongest player has an incentive to choose late if she is stronger than her opponents together (in terms of effort chosen in a simultaneous move game). If, however, no player dominates her opponents, all players will choose to move early in a subgame-perfect equilibrium. For a small range of parameter constellations, we find that not only the strongest but also the second strongest can have an incentive to move late. For this case, an equilibrium comprises mixed strategies cocerning the timing decision. We also compare the welfare in terms of aggregate payoff of the feasible subgames and find that, whenever the simultaneous move game is played, this is the best subgame, while for more heterogenous players, the subgame played on the equilibrium path need not be the one with the highest welfare.

In the next chapter, we look at the incentive of a strong player facing $(n-1)$ equal opponents. As in the case of three players, she will only move late in equilibrium if she is stronger than her opponents together. In addition, we analyze the effect of a first-mover-right of the stronger player. We find that the favorite will always exert the first-mover-right if granted. If the favorite is so strong that an early move leads to preemption of her opponents than this situation even raises the welfare level above the level of the equilibrium without first-mover-right. In this case, the first-mover-right should be granted to the strongest player.

Finally, in Chapter 8, we look at a game between n heterogenous players with an endogenized order of moves. We analyze under which condition it can be an equilibrium that all players coordinate to the same timing of effort choice, which is a standard assumption in many models of rent-seeking contests. In line with the preceding chapters, we come to the result that simultaneous move can only occur on the equilibrium path, if there is no player who chooses an equilibrium effort higher than the aggregate effort of her opponents. We provide a general condition telling us when a simultaneous game will be played on the subgame-perfect equilibrium path. This condition generalizes both the conditions for three players and for $(n-1)$ equal players. Furthermore, we find that simultaneous moves can only occur on the earliest possible point of time, since there is always an incentive for any player to deviate to an earlier timing. It is remarkable, that for a population with only homogenous players, simultaneous moves will always be played on the subgame-perfect equilibrium path.

In the introduction, we already alluded to some features of election campaigns, that we regard in our contest models. For examples, we adduced that, despite the political closeness, the Democratic candidates for the presidential election, Barack Obama and Hillary Clinton, do not shrink back from worsening the opponent's position. As our theoretical analysis showed, it can indeed be reasonable for a contestant to intentionally harm one's opponents and not only to focus on one's own payoff. Spiteful players, if they are not too spiteful, will be more successful in winning contests than players with independent preferences, and the same seems to hold for politicians. The evolutionary analysis of Chapter 5 shows that spiteful preferences, that induce a behavior equivalent to relative payoff maximization are an evolutionarily stable preference, if an evolutionary process selects players according to their preferences.

Applying this to the example of presidential candidates, it is obvious that a candidate for presidency will have to endure quite a number of contests to get into the position where they are able to enter the contest for the candidacy of the party (e.g. to be an esteemed senator). These contests weed out the less successful candidates, meaning Obama and Clinton have already gone through an evolutionary process. Such a process filters out politicians that behave like relative payoff maximizers as the most successful ones.

For the second part of our work, the theory cannot be applied to the US-presidential election campaign, because the pre-election campaign has a more complex structure than our model, while in the major presidential campaign that started in August 2008 there were only two candidates left, while our findings only regard at least three players. But, the theory is applicable to three-party-systems. A typical example for a three-party-system can be found in Eastern Germany. If we measure the strength of a party in a state by its number of seats in the parliament, then CDU was the strongest party in Saxony-Anhalt before the last parliament election with 48 seats, while the SPD and the PDS were of equal sterngth with 25 seats. For this combination, our theory predicts, that both smaller parties will move simultaneously while the CDU moves later. This indeed happened. Beyond the scope of election campaigns, there are far more fields where contest theory plays a role. So, Chapter 7 can make a contribution on whether a market incumbent should be granted a first-mover-right if a contest for the market license is played. Yet, the most general result of our analysis of sequential structures is that whenever there is no player who exerts a higher effort than the aggregate effort of her opponents, simultaneous moves will be played on the equilibrium path. This can serve as justification for the *ad-hoc*-assumption of simultaneous moves that is often made in the literature on contest theory.

The work at hand delivers an extended understanding of the role that interdependent preferences and endogenized order of moves play in rent-seeking contests, especially if more than two players are involved. However, we restricted our analysis of the sequential structure to a discrete set of two points in time. As an extension, it would be conceivable to extend the analysis of Chapter 6 to three timings. Furthermore, it is not clear what will happen in equilibrium if a first point in time is not clearly defined. As an example, one might fall back on the order of moves in election campaigns. In general, there is not a real first point of time where an election campaign can be started earliest. Yet, it is not clear what this means to the equilibrium behavior in our model, which says that, for more than two players, there are always at least two players who want to choose as early as possible.

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Chapter A

Appendix

A.1 Definition of a strategy

In our context, a strategy (t, x) is a combination of a timing decision t and an effort choice x. The timing decision t can be viewed upon as a function $t : \mathcal{V} \to \mathcal{T}$. $\mathcal{V}$ is the set of vectors of valuation that are feasible. Most generally, it is given by $\mathbb{R}^n_+$ for a contest with n players, but we will restrict our analysis for the main parts of our work to the most interesting cases, where players do not differ in valuation too much, such that all players still have an incentive to choose a positive effort. Since, in this part of our work, we assume players to differ only with respect to their valuation, $\mathcal{V}$ can also be interpreted as the set of players.

$\mathcal{T}$ represents the set of timings from which players can choose their actual timing decision. In most cases, we restrict our analysis to $\mathcal{T} = \{1, 2\}$, yet in Chapter 7, we will also use $\mathcal{T} = \{0, 1, 2\}$.

The effort choice x is a function that maps valuations and timing choices of all players and already executed effort choices of opponents who decided for an earlier move to the non-negative real numbers. Depending on the chosen timing decision $\hat{t}$, it is characterized by

$$x(\hat{t}) : \mathcal{V} \times \mathcal{T} \times \mathcal{X}_{\hat{t}} \to \mathbb{R}_{\geq 0},$$

where $\mathcal{X}_{\hat{t}}$ represents the set of feasible effort vectors of players who chose $t < \hat{t}$.

A.2 Equilibria where $V_1 > 2V_3$

A.2.1 The subgames

The aim of this section is to derive the equilibria of the game where three players play a sequential rent-seeking contest with endogenous timing and $V_1 > 2V_3$. The regarded subgames are the following cases:

At first, let us analyze the equilibria for every single subgame. Whenever helpful, we will fall back on wht learned from Chapter 6.

		Player 2	
		$t=1$	$t=2$
Pl.	$t=1$	S_E	$2L$
1	$t=2$	$1L$	$3E$

Player 3 moves in $t=1$.

		Player 2	
		$t=1$	$t=2$
Pl.	$t=1$	$3L$	$1E$
1	$t=2$	$2E$	S_L

Player 3 moves in $t=2$.

The subgames S_E and S_L In the simultaneous move game, player 3 quits, if $V_3 \leq \frac{V_1V_2}{V_1+V_2}$. This can be shown to be strictly smaller than $\frac{V_1}{2}$, if $V_1 > V_2$.

This means that inactivity of player 3 does not follow from $V_1 > 2V_3$. For $V_3 > \frac{V_1V_2}{V_1+V_2}$, the results from Chapter 6 can be apllied, while for the case where it is indeed optimal for her to be inactive, we find, that players 1 and 2 play a standard two-player-contest. The aggregate effort will be $x_1 + x_2 = \frac{V_1V_2}{V_1+V_2}$, and thus player 3, who has a lower valuation, is not willing to interfere in the two-player-contest.

The subgame $3L$ In the inner solution that fulfills the first-order-conditions for an equilibrium in the three-player-game, player 3 is active, if and only if

$$V_3 \geq \frac{3}{2}\frac{V_1V_2}{V_1+V_2}$$

$$\frac{3}{2}\frac{V_1V_2}{V_1+V_2} < \frac{V_1}{2} \Leftrightarrow V_1 > 2V_2.$$

This means for $V_1 > 2V_3$ player 3 will only be active if player 2 will be inactive. This can be shown to lead to a contradiction whenever $V_3 \leq V_2$. In this subgame, we will not observe a game with three, but only with two, active players. Yet, this does not mean that these two players act as if in a true two-player-contest. If players 1 and 2 ignore 3, they choose $x_1 + x_2 = \frac{V_1V_2}{V_1+V_2}$, as analyzed by Leininger, 1993. Yet for $V_3 \in \left(\frac{V_1V_2}{V_1+V_2}, \frac{3}{2}\frac{V_1V_2}{V_1+V_2}\right)$, we find that player 3 has an incentive to choose a positive effort if the aggregate effort of players 1 and 2 is $\frac{V_1V_2}{V_1+V_2}$. As a result, players 1 and 2 increase their effort above the level in the two-player-game in order to preempt player 3. An aggregate effort of $x_1 + x_2 = V_3$ is needed to ensure the inactivity of player 3. Yet, this does not tell us how the aggregate effort is divided between the players. To characterize an equilibrium, we should find for both players 1 and 2 that their payoffs decrease if they a) increase or b) decrease their effort, given the effort of her active opponent and the reaction function of player 3.

a) An increase of the effort would not induce a reaction from player 3. Thus, the marginal change in payoff is given in this situation by $\frac{\partial \pi_i}{\partial x_i} = \frac{x_j}{(x_i+x_j)^2}V_i - 1$, which should become non-positive in order to make an increase in effort non-profitable. This means:

$$\frac{\partial \pi_1}{\partial x_1} = \frac{x_2}{(x_1+x_2)^2}V_1 \leq 1 \Leftrightarrow x_2V_1 \leq (x_1+x_2)^2$$

$$\Leftrightarrow x_2 \geq \frac{(x_1+x_2)^2}{V_1}, \tag{A.1}$$

and by symmetry:

$$x_1 \leq \frac{(x_1+x_2)^2}{V_2}. \tag{A.2}$$

b) If a player decreases her effort, player 3 has an incentive to enter the game, and to choose $x_3 = \sqrt{(x_1+x_2)V_3} - (x_1+x_2)$, such that the aggregate effort will be $\sum_{i=1}^{3} x_i = \sqrt{(x_1+x_2)V_3}$. The marginal payoff of a change in effort by player 1 is thus given by:

$$\frac{\partial \pi_1(x_1,x_2,x_3(x_1,x_2))}{\partial x_1} = \frac{\partial}{\partial x_1}\frac{x_1}{\sqrt{V_3(x_1+x_2)}}V_1 - x_1 = \frac{V_1\left(\sqrt{x_1+x_2} - \frac{x_1}{2\sqrt{x_1+x_2}}\right)}{\sqrt{V_3}(x_1+x_2)} - 1.$$

This should be positive, such that a decreasing effort reduces the payoff of the player:

$$\frac{V_1\left(\sqrt{x_1+x_2} - \frac{x_1}{2\sqrt{x_1+x_2}}\right)}{\sqrt{V_3}(x_1+x_2)} \geq 1 \Leftrightarrow V_1 - \frac{x_1 V_1}{2(x_1+x_2)} \geq \sqrt{V_3(x_1+x_2)}$$

$$\Leftrightarrow \frac{x_1 V_1}{2(x_1+x_2)} \leq V_1 - \sqrt{V_3(x_1+x_2)}$$

$$\Leftrightarrow x_1 \leq 2(x_1+x_2) - \frac{2(x_1+x_2)}{V_1}\sqrt{V_3(x_1+x_2)}, \tag{A.3}$$

and by symmetry:

$$x_2 \leq 2(x_1+x_2) - \frac{2(x_1+x_2)}{V_2}\sqrt{V_3(x_1+x_2)}. \tag{A.4}$$

Now, in an equilibrium, it has to hold that $x_1 + x_2 = V_3$, such as (A.1), (A.2), (A.3) and (A.4). One can show that, from (A.1), (A.2) and $x_2 = V_3 - x_1$, it follows that

$$V_3 - \frac{V_3^2}{V_1} \leq x_1 \leq \frac{V_3^2}{V_2}, \tag{A.5}$$

while from (A.3), (A.4) and $x_2 = V_3 - x_1$, it follows that

$$V_3 - 2V_3\frac{V_2 - V_3}{V_2} \leq x_1 \leq 2V_3\frac{V_1 - V_3}{V_1}. \tag{A.6}$$

Therefore, it is necessary for the existence of an equilibrium that

$$V_3 - \frac{V_3^2}{V_1} \leq \frac{V_3^2}{V_2} \wedge V_3 - 2V_3\frac{V_2 - V_3}{V_2} \leq 2V_3\frac{V_1 - V_3}{V_1}.$$

$$\Leftrightarrow V_3 \leq V_3^2\frac{V_1 + V_2}{V_1 V_2} \wedge V_1 V_2 - 2V_1(V_2 - V_3) \leq 2V_1V_2 - 2V_2V_3$$

$$\Leftrightarrow V_1V_2 \leq V_3V_1 + V_3V_2 \wedge (2V_1 + 2V_2)V_3 \leq 3V_1V_2$$

$$\Leftrightarrow V_3 \geq \frac{V_1 V_2}{V_1 + V_2} \wedge V_3 \leq \frac{3}{2} \frac{V_1 V_2}{V_1 + V_2},$$

which is exactly the interval where the problem occurs.

Now, we have to show that both intervals defined by (A.5) and (A.6) intersect. This holds since $V_3 - \frac{V_3^2}{V_1} = V_3 \frac{V_1 - V_3}{V_1} \leq 2V_3 \frac{V_1 - V_3}{V_1}$, which holds trivially, and

$$V_3 - 2V_3 \frac{V_2 - V_3}{V_2} \leq \frac{V_3^2}{V_2} \Leftrightarrow V_2 V_3 - 2V_2 V_3 + 2V_3^2 \leq V_3^2 \Leftrightarrow V_3^2 \leq V_3 V_2 \Leftrightarrow V_3 \leq V_2,$$

which holds per definition. Note that $\frac{V_3^2}{V_2} < 2V_3 \frac{V_1 - V_3}{V_1} \Leftrightarrow V_3 < 2\frac{V_1 V_2}{V_1 + 2V_2}$, which always holds if $V_3 < V_2$ and $V_3 < \frac{V_1}{2}$. Then, we can summarize that, if $V_3 \in \left(\frac{V_1 V_2}{V_1 + V_2}, \frac{3}{2}\frac{V_1 V_2}{V_1 + V_2}\right)$, a Nash-equilibrium has the following properties: player 1 will choose some $x_1 \in \left(\max\left(V_3 \frac{V_1 - V_3}{V_1}, V_3 - 2V_3 \frac{V_2 - V_3}{V_2}\right),\right.$ while player 2 will choose $x_2 = V_3 - x_1$. Note that this implies non-uniqueness of the Nash-equilibrium.

For $V_3 \leq \frac{V_1 V_2}{V_1 + V_2}$, players 1 and 2 play a standard two-player-contest.

The subgame $2L$ In this subgame, player 3 quits if $V_3 < \frac{V_1}{2}$. Thus, we find that player 3 is inactive. Player 1 and 2 play a sequential game where they need not expect interference by player 3. Aggregate effort is $\frac{V_1}{2}$ (see Leininger, 1993), the payoff of player 2 is given by $\frac{(2V_2 - V_1)^2}{4V_2}$ if $V_2 > \frac{V_1}{2}$, elseif 0.

The subgame $1L$ For $V_3 > \frac{V_2}{2}$ every player is active in this subgame's Nash-equilibrium (cf. Chapter 6). For $V_3 \leq \frac{V_2}{2}$, player 3 quits. Player 2 and 1 play a two-player sequential game, with an aggregate effort of $\frac{V_2}{2} > V_3$. Hence, player 3 has no incentive to join the contest.

The subgame $1E$ Player 3 quits if $V_3 \leq \frac{V_1 - V_2 + \sqrt{V_1^2 + 6V_1 V_2 + V_2^2}}{4}$. This holds since

$$\frac{V_1 - V_2 + \sqrt{V_1^2 + 6V_1 V_2 + V_2^2}}{4} > \frac{V_1}{2} \Leftrightarrow V_1 - V_2 + \sqrt{V_1^2 + 6V_1 V_2 + V_2^2} > 2V_1$$

$$\Leftrightarrow \sqrt{V_1^2 + 6V_1 V_2 + V_2^2} > V_1 + V_2 \Leftrightarrow V_1^2 + 6V_1 V_2 + V_2^2 > V_1^2 + 2V_1 V_2 + V_2^2.$$

which holds for strict positive valuations. Player 3 quits and has no incentive to interfere in the two-player-contest equilibrium

The subgame $2E$ Player 3 quits active participation if:

$$V_3 \leq \frac{V_2 - V_1 + \sqrt{V_1^2 + 6V_1 V_2 + V_2^2}}{4}$$

$$\frac{V_2 - V_1 + \sqrt{V_1^2 + 6V_1 V_2 + V_2^2}}{4} > \frac{V_1}{2} \Leftrightarrow V_2 - V_1 + \sqrt{V_1^2 + 6V_1 V_2 + V_2^2} > 2V_1$$

$$\Leftrightarrow \sqrt{V_1^2+6V_1V_2+V_2^2} > 3V_1 - V_2 (\Leftrightarrow) V_1^2+6V_1V_2+V_2^2 > 9V_1^2-6V_1V_2+V_2^2 \Leftrightarrow V_2 > \frac{2}{3}V_1.$$

Thus, if V_2 is rather small compared to V_1, player 3 has an incentive not to quit participation.

For $V_2 > \frac{2}{3}V_1$, if player 3 who is absent is ignored, players 1 and 2 play a standard two-player-contest where the aggregate effort will be $\frac{V_2}{2}$, which is smaller than $\frac{V_2-V_1+\sqrt{V_1^2+6V_1V_2+V_2^2}}{4}$. Thus, for $V_3 \in \left(\frac{V_2}{2}, \frac{V_2-V_1+\sqrt{V_1^2+6V_1V_2+V_2^2}}{4}\right)$, player 3 does not have an incentive to choose a positive effort in the three-player-case, but to interfere in the two-player-contest.

Thus, in equilibrium, players 1 and 2 will choose their efforts, such that the sum of efforts will be V_3. To this aim, player 2 chooses her effort, such that the reaction of her opponents in $t=2$ leads to:

$$x_2 + x_1(x_2) + x_3(x_2) = V_3,$$

which leads to $x_3(x_2) = 0$.

$$x_2 + x_1(x_2) + x_3(x_2) = V_3 \Leftrightarrow$$

$$\frac{V_1^2V_3 + V_1V_3^2 - 4V_1V_3x - 2V_1^2x_2 - 2V_3^2x + (V_1+V_3)\sqrt{V_1V_3\left(V_1V_3+4(V_1+V_3)x_2\right)}}{2(V_1+V_3)^2} + x_2$$

$$= \frac{V_1V_3(V_1+V_3)}{2(V_1+V_3)^2} - 2x_2\frac{V_1^2+2V_1V_3+V_3^2}{2(V_1+V_3)^2} + x_2 + \frac{\sqrt{V_1V_3\left(V_1V_3+4(V_1+V_3)x_2\right)}}{2(V_1+V_3)} = V_3$$

$$\Leftrightarrow V_1V_3 + \sqrt{V_1V_3\left(V_1V_3+4(V_1+V_3)x_2\right)} = V_3\left(2V_1+2V_3\right)$$

$$\Leftrightarrow \sqrt{V_1V_3\left(V_1V_3+4(V_1+V_3)x_2\right)} = V_3\left(V_1+2V_3\right)$$

$$\Leftrightarrow V_1^2V_3^2 + 4V_1V_3x_2(V_1+V_3) = V_3^2\left(4V_3^2+4V_3V_1+V_1^2\right)$$

$$\Leftrightarrow 4V_1V_3(V_1+V_3)x_2 = 4V_3^4+4V_3^3V_1 \Leftrightarrow x_2V_1(V_1+V_3) = V_3^2(V_1+V_3) \Leftrightarrow x_2 = \frac{V_3^2}{V_1}.$$

Inserting this into the reaction function of players 1 and 3 yields $x_3 = 0$ and $x_1 = \frac{V_3(V_1-V_3)}{V_1}$.

Now, we have to show that this is optimal from the point of view of player 2 to choose $x_2 = \frac{V_3^2}{V_1}$. Her payoff is then given by $\pi_2 = \frac{V_3(V_2-V_3)}{V_1}$. For a deviation from this effort to a lower effort, player 3 will choose positive effort, and the payoff of player 2, depending on her own effort, and her opponent's reaction function is given by:

$$\pi_2 = \frac{x_2 \cdot 2(V_1+V_3)}{V_1V_3+\sqrt{V_1V_3+4(V_1+V_3)x_2}}V_2 - x_2$$

$$\frac{\partial \pi_2}{\partial x_2} = 2(V_1+V_3)V_2\frac{V_1V_3+\sqrt{V_1V_3+4(V_1+V_3)x_2} - \frac{V_1V_3 4(V_1+V_3)x_2}{2\sqrt{V_1V_3+4(V_1+V_3)x_2}}}{\left(V_1V_3+\sqrt{V_1V_3+4(V_1+V_3)x_2}\right)^2} - 1.$$

For $x_2 = \frac{V_3^2}{V_1}$, this should be (at least weakly) positive, such that deviation for player 2 is not profitable.

$$\frac{\partial \pi_2}{\partial x_2}\left(\frac{V_3^2}{V_1}\right) \geq 0$$

$$\Leftrightarrow 2\left(V_1+V_3\right)V_2\left(V_1V_3+\sqrt{V_1V_3+4(V_1+V_3)\frac{V_3^2}{V_1}}-\frac{V_1V_34(V_1+V_3)\frac{V_3^2}{V_1}}{2\sqrt{V_1V_3+4(V_1+V_3)\frac{V_3^2}{V_1}}}\right)$$

$$\geq \left(V_1V_3+\sqrt{V_1V_3+4(V_1+V_3)\frac{V_3^2}{V_1}}\right)^2$$

$$\Leftrightarrow 2\left(V_1+V_3\right)V_2\left(V_1V_3+V_3(V_1+2V_3)-2V_3^2\frac{V_1+V_3}{V_1+2V_3}\right) \geq 4V_3^2\left(V_1+V_3\right)^2$$

$$\Leftrightarrow V_2\left(1-\frac{V_3}{V_1+2V_3}\right) \geq V_3$$

$$\Leftrightarrow V_3^2+(V_1-V_2)\frac{V_3}{2}-\frac{V_1V_2}{2} \leq 0$$

$$\Rightarrow V_3 \leq \frac{V_2-V_1+\sqrt{V_1^2+6V_1V_2+V_2^2}}{4}.$$

Deviation to a higher payoff cannot be profitable as well, since player 3 would not have an influence on players 1 and 2 any more. Yet, if it was optimal despite the absence of player 3, it should also occur in the two-player-case. This does not happen, and thus, player 2 has no incentive to deviate from $x_2 = \frac{V_3^2}{V_1}$.

For $V_3 < \frac{V_2}{2}$, a standard two-player sequential game is played by players 1 and 2, while for $V_3 > \frac{V_2-V_1+\sqrt{V_1^2+6V_1V_2+V_2^2}}{4}$, a standard three-player sequential game is played.

The subgame $3E$ Player 3 chooses a positive effort if $V_3 > \frac{V_1V_2}{V_1+V_2}$. According to the main findings of subgame S_E, we find here that, for this case, the three-player-case (with three *active* players) is played, while for $V_3 \leq \frac{V_1V_2}{V_1+V_2}$, the players 1 and 2 play a two-player-contest.

A.2.2 The equilibrium path

Let us now focus on which subgame is stable on the equilibrium path and in which subgame deviation pays for at least one player. To this aim, we will check for every subgame whether any player has an incentive to deviate.

Is S_E stable? Subgame S_E is a three-player-game, if $V_3 > \frac{V_1V_2}{V_1+V_2}$. If not, only player 1 and 2 participate actively.

A deviation of player 1 would lead to subgame $1L$. For the latter case, i.e., $V_3 \leq \frac{V_1V_2}{V_1+V_2}$, we find that, for $V_3 \leq \frac{V_2}{2}$ this would also be a two-player-game, and we can apply Leininger's (1993) result. Then S_E is not stable.

Now for $V_3 \in \left[\frac{V_2}{2}, \frac{V_1V_2}{V_1+V_2}\right]$, player 1 decides between a simultaneous two-player-game, where she receives $\frac{V_1^3}{(V_1+V_2)^2}$ and moving late in a three-player-game, receiving $\frac{(2V_1(V_2+V_3)-3V_2V_3)^2}{4V_1(V_2+V_3)^2} = V_1\left(1-\frac{3}{4}\frac{V_2V_3}{V_1(V_2+V_3)}\right)$. Deviation pays if

$$\frac{V_1^3}{(V_1+V_2)^2} < V_1\left(1-\frac{3}{4}\frac{V_2V_3}{V_1(V_2+V_3)}\right)$$

$$\Rightarrow V_1^2(V_2+V_3) < V_1(V_1+V_2)(V_2+V_3) - \frac{3}{4}V_1V_2V_3 - \frac{3}{4}V_2^2V_3$$

$$\Leftrightarrow 0 < V_1V_2^2 + V_1V_2V_3 - \frac{3}{4}V_1V_2V_3 - \frac{3}{4}V_2^2V_3$$

$$\frac{V_1V_2V_3}{4} + V_2^2\left(V_1 - \frac{3}{4}V_3\right) > 0,$$

which holds since $V_1 > V_3$.

For $V_3 > \frac{V_1V_2}{V_1+V_2}$ player 3 is active in subgame S_E as well as in $1L$. The analysis of Chapter6 can thus be applied to show that S_E **is stable if** $V_1 \leq 3\frac{V_2V_3}{V_2+V_3}$.

Is $3L$ stable? For $V_3 \in \left[\frac{V_1V_2}{V_1+V_2}, \frac{3}{2}\frac{V_1V_2}{V_1+V_2}\right]$, player 3 is active in S_E, but not in $3L$, thus she will prefer moving early. For $V_3 > \frac{3}{2}\frac{V_1V_2}{V_1+V_2}$, all three players will be active in S_E as well as in $3L$, but only the strongest player has an incentive to move later. Thus, $3L$ cannot be stable.

For $V_3 < \frac{V_1V_2}{V_1+V_2}$ we find that

$$\frac{V_2 - V_1 + \sqrt{V_1^2 + 6V_1V_2 + V_2^2}}{4} > \frac{V_1V_2}{V_1+V_2}$$

$$\Leftrightarrow (V_1+V_2)\sqrt{V_1^2+6V_1V_2+V_2^2} > V_1^2 + 4V_1V - 2 - V_2^2$$

$$\Leftrightarrow 00(V_1^2 + 2V_1V_2^2 + V_2^2)(V_1^2 + 6V_1V_2 + V_2^2) > (V_1^2 + 4V_1V - 2 - V_2^2)^2$$

$$= V_1^4 + 6V_1^3V_2 + V - 1^2V_2^2 + 2V_1^3V_2 + 12V_1^2V_2^2 + 2V_1V_2^3 + V_1^2V_2^2 + 6V_1V_2^3 + V_2^4$$

$$> V_1^4 + 8V_1^3V_2 + 14V_1^2V_2^2 - 8V_1V_2^3 + V_2^4$$

$$\Leftrightarrow 8V_1^3V_2 + 14V_1^2V_2^2 + 8V_1V_2^3 > 8V_1^3V_2 + 14V_1^2V_2^2 - 8V_1V_2^3.$$

In this case, player 3 will neither have an incentive to interact in $3L$ nor in $2E$. Thus, both subgames will be true two-player-games. From Leininger (1993), we know that in this situation the stronger player will prefer late move. Thus, player 1 will deviate from $3L$ to game $2E$. $3L$ will never be stable.

Is $2L$ stable? For $V_2 \leq \frac{V_1}{2}$, player 2 will be inactive in $2L$, and thus she has an incentive to deviate to S_E. For $V_2 > \frac{V_1}{2}$ we have to distinguish for different valuations of player 3. Let us first regard the case, where player 3 is active. Then, player 2 will prefer $2L$ to S_E, if $\frac{(2V_2-V_1)^2}{4V_2} > \frac{(V_2(V_1+V_3)-V_1V_3)^2}{(V_2(V_1+V_3)+V_1V_3)^2}V_2$. Yet this never holds, if $V_2 \in \left[\frac{V_1}{2}, V_1\right]$ and $V_3 \geq \frac{V_1V_2}{V_1+V_2}$.

Now if player 3 is inactive in S_E, then player 2 would receive $\frac{V_2^3}{(V_1+V_2)^2}$ in S_E. Deviation to S_E will thus pay if: $\frac{(2V_2-V_1)^2}{4V_2} < \frac{V_2^3}{(V_1+V_2)^2}$. The latter is equivalent to

$$V_1^3 - 2V_1^2V_2 - 3V_1^2V_2^2 + 4V_2^3 < 0,$$

which holds for $V_2 \in \left(\frac{V_1}{2}, V_1\right)$.

Thus subgame $2L$ is never stable.

Is $1L$ stable? For $V_3 \leq \frac{V_2}{2}$, player 1 and 2 play a plain two-player-game, regardless of when player 3 does (not) move. For this situation, the **subgame $1L$ is stable**!

For $V_3 > \frac{V_2}{2}$, $1L$ is a three-player-game. Player 3 has no incentive to deviate, either because her payoff is zero from deviation or from the analysis in the standard three-player-case. Player 1 only has an incentive to deviate to subgame S_E, if $V_3 \geq \frac{V_1V_2}{V_1+V_2}$ (see above) and $V_1 \leq 3\frac{V_2V_3}{V_2+V_3}$. A deviation of player 2 would lead to subgame $3E$. This pays only if $\pi_2^{3E} > \pi_2^{1L}$. Unfortunately, this can happen for some parameter ranges that are too complex to be captured formally. Table B.1 gives an overview over the cases where player 2 deviates from $1L$ to $3E$, if $V_3 > \frac{V_1V_2}{V_1+V_2}$. For $V_3 \in \left(\frac{V_2}{2}, \frac{V_1V_2}{V_1+V_2}\right)$, player 2 has to decide between a three-player-game where she moves early and a simultaneous two-player-game at the late stage. This is also represented in the Table (as Maximum) in appendix B of this work. Player 3 never has an incentive to deviate to $2E$.

Is $1E$ stable? In subgame $1E$, player 2 receives $\pi_2^{1E} = \frac{(2V_2-V_1)^2}{4V_2}$, if $V_2 \geq \frac{V_1}{2}$, and zero elseif. The latter, of course, creates a strict incentive for her to deviate to $3L$. But also for $V_2 \geq \frac{V_1}{2}$, deviation to $3L$ would pay her:

- Regard $V_3 \in (\frac{V_1V_2}{V_1+V_2}, \frac{3V_1V_2}{2(V_1+V_2)})$ first.[1] Then, the minimal payoff she would get here from a deviation, i.e., $\pi_2^{3L}(x_1 = \frac{V_3^2}{V_2}, x_2 = V_3 - \frac{V_3^2}{V_2})$, is larger than π_2^{1E}:

$$\pi_2^{3L}(min) = \frac{V_3(V_2-V_3)}{V_2}\left(\frac{V_2}{V_3} - 1\right) = \frac{(V_2-V_3)^2}{V_2} = \frac{(2V_2-2V_3)^2}{4V_2}$$

$$\frac{(2V_2-2V_3)^2}{4V_2} \geq \frac{(2V_2-V_1)^2}{4V_2}$$

$$\Leftrightarrow 2V_2 - 2V_3 \geq 2V_2 - V_1 \Leftrightarrow V_1 \geq 2V_3.$$

[1] $\frac{3V_1V_2}{2(V_1+V_2)}$ is larger than $\frac{V_1}{2}$, if V_2 is larger than $\frac{V_1}{2}$.

- Now, consider $V_3 \leq \frac{V_1 V_2}{V_1+V_2}$. Players 1 and 2 play a true two-player-game in $3L$, where player 2 receives: $\frac{V_2^3}{(V_1+V_2)^2}$. This is larger than $\pi_2^{1E} = \frac{(2V_2-V_1)^2}{4V_2}$ for $\frac{V_2}{V_1} \in \left[\frac{1}{2}, 1\right]$, as we show here:

$$\frac{V_2^3}{(V_1+V_2)^2} \geq \frac{(2V_2-V_1)^2}{4V_2}$$

$$\Leftrightarrow V_1\left(V_1^3 - 2V_1V_2^2 - 3V_1V_2^2 + 4V_2^3\right) \leq 0$$

$$\Leftrightarrow 4V_1^4\left(\frac{V_2}{V_1} + \frac{\sqrt{17}+1}{8}\right)\left(\frac{V_2}{V_1} - \frac{\sqrt{17}-1}{8}\right)\left(\frac{V_2}{V_1} - 1\right) \leq 0.$$

  This holds for values from the regarded interval.

As a consequence, $1E$ is never stable.

Is $2E$ stable? For $V_3 > \frac{V_2}{2}$, player 3 has a strict incentive to deviate to $1L$. For the case that player 3 is active in $2E$ it stems from the analysis of Chapter 6 that she will prefer an early move. If on the other hand she chooses zero effort in $2E$ and $V_3 > \frac{V_2}{2}$ holds, then player 3 can receive a positive payoff by deviating and, thus, $2E$ cannot be stable. For $V_3 \leq \frac{V_2}{2}$, she is inactive in both subgames $2E$ and $1L$ and, thus, indifferent. Players 1 and 2 play a true two-player-game, regardless of when player 3 does (not) move. For this situation, the **subgame $2E$ is stable**!

Is $3E$ stable? Assume first that $V_3 < \frac{V_1V_2}{V_1+V_2}$. In this case, player 3 will be inactive in this subgame, as well as in subgame $2L$. Since player 1 has an incentive in the two-player-game to move early, given that player 2 moves late, $3E$ is not stable in this area. For larger values of V_3, one can show that the payoff player 1 would gain from moving early in two-player-game is larger than from subgame $3E$ with 3 active players. Subgame $3E$ is thus never stable.

Is S_L stable? For $V_3 > \frac{V_1V_2}{V_1+V_2}$ we find that player 3 is active in S_L as well as in $3E$. Thus, we can rely on the calculation of Chapter 6, which shows that she will deviate to an earlier move.
For $V_3 \leq \frac{V_1V_2}{V_1+V_2}$, player 3 is inactive in S_L. Now $\frac{V_1V_2}{V_1+V_2} < \frac{V_2-V_1+\sqrt{V_1^2+6V_1V_2+V_2^2}}{4}$:

$$\frac{V_1V_2}{V_1+V_2} < \frac{V_2 - V_1 + \sqrt{V_1^2+6V_1V_2+V_2^2}}{4}$$

$$\Leftrightarrow 4V_1V_2 < V_2^2 - V_1^2 + (V_1+V_2)\sqrt{V_1^2+6V_1V_2+V_2^2}$$

$$\Leftrightarrow \frac{V_1^2+4V_1V_2-V_2^2}{V_1+V_2} < \sqrt{V_1^2+6V_1V_2+V_2^2}$$

$$V_1^4 + 4V_1^3V_2 - V_1^2V_2^2 + 4V_1^3V_2 + 16V_1^2V_2^2 - 4V_1V_2^3 - V_1^2V_2^2 - 4V_1V_2^3 + V_2^4$$
$$> V_1^4 + 2V_1^3V_2 + V_1^2V_2^2 + 6V_1^3V_2 + 12V_1^2V_2^2 + 6V_1V_2^3 + V_1^2V_2^2 + 2V_1V_2^3 + V_2^4$$

$$\Leftrightarrow 8V_1^3V_2 + 14V_1^2V_2^2 - 8V_1V_2^3 < 8V_1^3V_2 + 14V_1^2V_2^2 + 8V_1V_2^3.$$

Thus player 3 will also be inactive in subgame $2E$. This means that, for this constellation, the subgames $2E$ and S_L represent two-player-contests, for which Leininger (1993) showed that player 2 will move early. Thus player 2 will deviate, and subgame S_L is never stable.

A.2.3 Equilibria

In the subgame-perfect equilibrium, we find the following behavior:

For combinations of valuations captured by Table B.1, players 1 and 2 will choose a mixed strategy with respect to their timing decision. Player 3 will move early.

For the remaining combinations of valuations, we find:

Whenever $V_3 \geq \frac{V_1V_2}{V_1+V_2} \wedge V_1 \leq 3\frac{V_2V_3}{V_2+V_3}$ we find that a subgame-perfect equilibrium is characterized by all players moving early. (Subgame S_E) This is a unique equilibrium, if at least one condition holds with strict inequality.

Whenever $V_3 \leq \frac{V_1V_2}{V_1+V_2} \vee V_1 \geq 3\frac{V_2V_3}{V_2+V_3}$ there is a subgame-perfect equilibrium in which the strongest player moves late, while both weaker players move early, unless the valuations lie in the parameter range characterized by Table B.1. (Subgame $1L$) It is unique, if $V_3 > \frac{V_2}{2}$ and at least one of the above conditions holds with strict inequality.

Whenever $V_3 \leq \frac{V_2}{2}$, there exists an equilibrium in which both the strongest and the weakest player move late while the intermediate player chooses early. (Subgame $2E$) This equilibrium is never unique and player 3 will not be active.

Chapter B

Proofs

B.1 Comments on the Introduction

Annotation to Axiom 4 in Skaperdas (1996) While we present the Axiom 4 as

$$\frac{p_i^M(\mathbf{x})}{p_j^M(\mathbf{x})} = \frac{p_i(\mathbf{x})}{p_j(\mathbf{x})}, \forall i, j \in M \, \forall x_i, x_j \in S, \tag{B.1}$$

Skaperdas writes:

$$p_i^M(\mathbf{x}) = \frac{p_i(\mathbf{x})}{\sum_{j=1}^m p_j(\mathbf{x})}, \forall i \in M. \tag{B.2}$$

Let us now show that both postulations are equivalent, as long as Axiom 1 holds. At first, we can rewrite (B.1) as

$$\frac{p_i^M(\mathbf{x})}{p_i(\mathbf{x})} = \frac{p_j^M(\mathbf{x})}{p_j(\mathbf{x})},$$

which obviously only holds for **every** $x_i \in S$ if the function $\frac{p_i^M(\mathbf{x})}{p_i(\mathbf{x})}$ is a constant function, or put differently, $\exists \nu \in \mathbb{R} : p_i^M(\mathbf{x}) = \nu \cdot p_i(\mathbf{x})$, $\forall x_i \in S$. In (B.2), Skaperdas presents this functional form, choosing $\nu = \frac{1}{\sum_{j=1}^m p_j(\mathbf{x})}$. This choice is due to Axiom 1, which assures that the sum of winning probabilities yields one. ■

B.2 Proofs of Chapter 3

B.2.1 Preliminary Results

Proof of Lemma 3. **Part (i):** Fix any $x = ([a]_k, [b]_{n-m}) \in N_{sym}(\Gamma_F(m))$ for some $a, b \in [0, \infty)$ with $a > b$. It follows from the hypothesis, from negative spillovers, and from symmetry of the material game that

$$\pi_1([a]_m, [b]_{n-m}) \leq \pi_n([a]_m, [b]_{n-m}) < \pi_n(b, [a]_{m-1}, [b]_{n-m}) = \pi_1(b, [a]_{m-1}, [b]_{n-m}),$$

which contradicts a representing an optimal response of player 1 against $([a]_{m-1}, [b]_{n-m})$. Thus, $b \geq a$.

Part (ii): Let $m = n-1$ and $x = ([a]_{n-1}, b)$. All we have to show is that $b > a$ implies $\pi_n(x) > \pi_1(x)$. Suppose, to the end of contradiction, that $b > a$ and $\pi_n(x) \leq \pi_1(x)$. Observe that $\pi_n(x) \leq \pi_1(x)$ implies $\rho_n(x) = \pi_n(x)(1 - \frac{1}{n}) - \frac{1}{n}\sum \pi_1(x) \leq 0$. Since b is a best response of player n against $[a]_{n-1}$ (with regard to player n's interdependent preferences $F(\pi_n, \rho_n)$), it follows from $\rho([a]_n) = 0 \geq \rho_n(x)$ that $\pi_n([a]_{n-1}, b) \geq \pi_n([a]_n)$. Thus, negative spillovers and symmetry of the material game imply

$$\pi_1([a]_{n-1}, b) < \pi_1([a]_n) = \pi_n([a]_n) \leq \pi_n([a]_{n-1}, b),$$

in contradiction to $\pi_n(x) \leq \pi_1(x)$. ■

B.2.2 Non-increasing marginal efficiency

Proof of Theorem 2. **Part 1** *(Properties)* Fix $r \leq 1$ and $m \in \{1, \ldots, n-1\}$ and let $\hat{x} = ([a]_m, [b]_{n-m}) \in N_{sym}(\Gamma_F(m))$. The following expressions will turn out useful in establishing the proof:

$$\rho_j(\mathbf{x}) = \left[\frac{x_j^r}{\sum_h x_h^r} - \frac{1}{n}\right] V - \left[x_j - \frac{1}{n}\sum_h x_h\right], \tag{B.3}$$

$$\frac{\partial \rho_j}{\partial x_j}(\mathbf{x}) = \frac{r x_j^{r-1} \sum_{h \neq j} x_h^r}{\left(\sum_h x_h^r\right)^2} V - 1 + \frac{1}{n}, \tag{B.4}$$

and

$$\frac{\partial \pi_h}{\partial x_h}(\mathbf{x}) = \frac{r x_h^{r-1} \sum_{i=1, i\neq h}^n x_i^r}{\left(\sum_i x_i^r\right)^2} V - 1. \tag{B.5}$$

These hold for any $\mathbf{x}$, any $j \in J \equiv \{m+1, \ldots, n\}$, and any $h \in \{1, \ldots, n\}$. Notice that, for all $\mathbf{x}$, $\frac{\partial \rho_j(\mathbf{x})}{\partial x_j} < 0$ implies $\frac{\partial \pi_j(\mathbf{x})}{\partial x_j} < 0$ and, inversely, $\frac{\partial \pi_j(\mathbf{x})}{\partial x_j} > 0$ implies $\frac{\partial \rho_j(\mathbf{x})}{\partial x_j} > 0$. Moreover, neither of the two cases can be part of an intragroup symmetric Nash equilibrium. Therefore, in the former case, any status-seeker would be strictly better off with $b - \varepsilon$, while, in the latter case, she could do strictly better expending effort $b + \varepsilon$, where $\varepsilon > 0$ has to be chosen sufficiently small in each case. Therefore, in any intragroup symmetric equilibrium $\hat{\mathbf{x}}$, it has to be, for any status-seeker $j \in J$, that $\frac{\partial \rho_j(\mathbf{x})}{\partial x_j} \geq 0$ and $\frac{\partial \pi_j(\hat{\mathbf{x}})}{\partial x_j} \leq 0$. From (B.4) and (B.5), it thus follows that $0 \leq \frac{\partial \pi_j}{\partial x_j}(\hat{\mathbf{x}}) + \frac{1}{n} \leq \frac{1}{n}$, which is equivalent to

$$\frac{r b^{r-1}[(m-1)a^r + (n-m)b^r]}{[m a^r + (n-m)b^r]^2} V = \gamma, \tag{B.6}$$

for some $\gamma \in [\frac{n-1}{n}, 1]$.

Claim (i): $a > 0$. Suppose to the end of contradiction that $a = 0$. According to Proposition 1 in Pérez-Castrillo and Verdier (1992), for non-increasing contest technologies ($r \leq 1$) an effort level $a = 0$ can only maximize player 1's material pay-off if $r = 1$. Therefore, it suffices to derive a contradiction for $r = 1$.

On the one hand, $a = 0$ implies $b < \frac{V}{n-m}$ since, otherwise, $\sum_h \hat{x}_h = (n-m)b \geq V$ entails

$$\frac{\partial \rho_n}{\partial x_n}(\hat{\mathbf{x}}) = \frac{(n-m-1)V}{(n-m)^2 b} - 1 + \frac{1}{n}$$
$$\leq \frac{n-m-1}{n-m} - \frac{n-1}{n} = \frac{-m}{(n-m)n} < 0$$

and hence $\frac{\partial \pi_n(\hat{\mathbf{x}})}{\partial x_n} < 0$. By the argument mentioned above, player n would do better in terms of his interdependent preferences if he expended $t = b - \varepsilon$ with $\varepsilon > 0$ sufficiently small. However, this contradicts $\hat{x}_n = b$ being optimal for player n against $\hat{x}_{-n} = ([a]_m, [b]_{n-m-1})$. Thus, $b < \frac{V}{n-m}$.

On the other hand, $a = 0$ implies $b \geq \frac{V}{n-m}$ since, otherwise, $a = 0$ could not maximize player 1's material pay-off. This holds since, for $r = 1$, the best response function of player 1 is given by $x_1(Z) = \max\{0, \sqrt{VZ} - Z\}$, $Z > 0$, (where $Z = \sum_{h \neq 1} x_h$) and because $x_1(Z) > 0$ if and only if $Z < V$. (Notice again that $b \geq \frac{V}{n-m}$ implies $\sum_h \hat{x}_h = (n-m)b \geq V$.)

Taken together, $b < \frac{V}{n-m}$ and $b \geq \frac{V}{n-m}$ yield a contradiction so that we have established $a > 0$.

Claim (ii): $b \geq a$. Suppose to the contrary that $b < a$. It suffices to show $\frac{\partial \pi_n(\hat{\mathbf{x}})}{\partial x_n} > 0$ for this implies $\frac{\partial \rho_n(\mathbf{x})}{\partial x_n} > 0$, which yields a contradiction to $\hat{x}_n = b$ being optimal for player n against $\hat{x}_{-n} = ([a]_m, [b]_{n-m-1})$. As mentioned above, in this case, player n would be better off with $t = b + \varepsilon$.

To see $\frac{\partial \pi_n(\hat{\mathbf{x}})}{\partial x_n} > 0$, insert the first order condition of player 1,

$$\frac{\partial \pi_1}{\partial x_1}(\hat{x}) = \frac{ra^{r-1}[(m-1)a^r + (n-m)b^r]}{[ma^r + (n-m)b^r]}V - 1 = 0,$$

into $\frac{\partial \pi_n(\hat{\mathbf{x}})}{\partial x_n}$. This results in

$$\frac{\partial \pi_n}{\partial x_n}(\hat{x}) = \frac{rb^{r-1}[ma^r + (n-m-1)b^r]}{ra^{r-1}[(m-1)a^r + (n-m)b^r]} - 1,$$

which is strictly positive because of $b < a$, $\left(\frac{b}{a}\right)^{r-1} > 1$, and $r \leq 1$. Thus, $b \geq a$.

Claim (iii): $\pi_n(\hat{\mathbf{x}}) \geq \pi_1(\hat{\mathbf{x}})$, which holds strictly if $b > a$. Obviously, $b = a$ implies $\pi_n(\hat{x}) = \pi_1(\hat{x})$, so we only have to deal with $b > a$.

Let us consider the case $r = 1$ first. In this case, $\pi_h(\hat{\mathbf{x}})$ reduces to $x_h[\frac{V}{ma+(n-m)b} - 1]$ and $\pi_n(\hat{\mathbf{x}}) > \pi_1(\hat{\mathbf{x}})$ is equivalent to $(b-a)[V - ma - (n-m)b] > 0$. Because of $b > a$, it suffices to show $ma + (n-m)b < V$.

Suppose to the contrary that $ma + (n-m)b \geq V$. If $ma + (n-m)b > V$, then $\pi_h(\hat{\mathbf{x}}) < 0$, for all players h. However, expending $x_1 = 0$, player 1 would be strictly better off because of $\pi_1(0, \hat{\mathbf{x}}_{-1}) = 0$. Hence, a would not have been optimal. On the other hand, if $ma + (n-m)b = V$, then $\pi_h(\hat{\mathbf{x}}) = 0$, for all players h. In this case, $x_1 = a - \varepsilon$, for $\varepsilon > 0$ sufficiently small, gave player 1 strictly positive pay-off, in contradiction to a being optimal. Thus, it must be that $ma + (n-m)b < V$ so that the claim follows.

Now consider $r < 1$. Define $\varphi : [a, b] \to \mathbb{R}$ by

$$\varphi(z) \equiv \frac{z^r}{ma^r + (n-m)b^r}V - z,$$

and notice that $\varphi(a) = \pi_1(\hat{x})$ and $\varphi(b) = \pi_n(\hat{x})$. Therefore, we have to show that $\varphi(b) > \varphi(a)$. At first, we determine the first and second derivative of $\varphi(\cdot)$ as

$$\varphi'(z) = \frac{rz^{r-1}}{ma^r + (n-m)b^r}V - 1 \qquad \text{and} \qquad \varphi''(z) = \frac{r(r-1)z^{r-2}}{ma^r + (n-m)b^r}V < 0.$$

Since $\varphi(\cdot)$ is strictly concave on $[a,b]$, it suffices to show that $\varphi'(b)>0$. Then, $\varphi'(z)>0$, for all $z\in[a,b]$, implies that $\varphi(a)<\varphi(b)$.

To see $\varphi'(b)>0$, we insert the first order condition for status-seekers (B.6) into $\varphi'(b)$, which yields

$$\begin{aligned}\varphi'(b) &= \tfrac{rb^{r-1}}{ma^r+(n-m)b^r}V-1=\gamma\tfrac{ma^r+(n-m)b^r}{ma^r+(n-m-1)b^r}-1\\ &\geq \tfrac{n-1}{n}\tfrac{ma^r+(n-m)b^r}{ma^r+(n-m-1)b^r}-1,\end{aligned}$$

which can be shown to be strictly positive, using $b>a$.

Thus, $\pi_n(\hat{\mathbf{x}})>\pi_1(\hat{\mathbf{x}})$, for any $\hat{\mathbf{x}}\in N_{sym}(\Gamma_F(m))$ with $b>a$ and any $r\leq 1$.

Claim (iv): $\pi_n(\hat{\mathbf{x}})>\pi_1(\hat{\mathbf{x}})$ if $F(\cdot,\cdot)$ is differentiable. By Claim (iii), we only have to show that in this case $b\neq a$, which implies $b>a$ and hence $\pi_n(\hat{\mathbf{x}})>\pi_1(\hat{\mathbf{x}})$.

Suppose to the contrary that $b=a$. By symmetry of the material game, we have that $\frac{\partial\pi_n}{\partial x_n}(\hat{\mathbf{x}})=\frac{\partial\pi_1}{\partial x_1}(\hat{\mathbf{x}})=0$ since player 1 maximizes his material pay-off. Moreover, $\frac{\partial\pi_n}{\partial x_n}(\hat{\mathbf{x}})=0$ implies $\frac{\partial\rho_n}{\partial x_n}(\hat{\mathbf{x}})=\frac{\partial\pi_n}{\partial x_n}(\hat{\mathbf{x}})+\frac{1}{n}=\frac{1}{n}>0$. Hence, it follows from $\partial_2F>0$ that

$$\frac{\partial F(\hat{\mathbf{x}})}{\partial x_n}=\partial_1F\ \underbrace{\frac{\partial\pi_n(\hat{\mathbf{x}})}{\partial x_n}}_{=0}+\partial_2F\ \frac{\partial\rho_n(\hat{\mathbf{x}})}{\partial x_n}=\partial_2F\ \frac{1}{n}>0,$$

so that there exists $\varepsilon>0$ such that

$$F(\pi_n(\hat{\mathbf{x}}_{-n},b+\varepsilon),\rho_n(\hat{\mathbf{x}}_{-n},b+\varepsilon))>F(\pi_n(\hat{\mathbf{x}}),\rho_n(\hat{\mathbf{x}})).$$

Similar to the above arguments, this yields a contradiction to $\hat{\mathbf{x}}$ being optimal for player n.

Part 2 *(Uniqueness)* Fix $r\leq 1$, any $k\in\{1,\ldots,n-1\}$ and any $\hat{x}=([a]_m,[b]_{n-m})\in N_{sym}(\Gamma_F(m))$. By Part 1, we have $0<a\leq b$. Set $c:=\frac{a}{b}$. Obviously, $c\in(0,1]$. For any status-seeker, $i\in\{1,\ldots,k\}$, positive effort $a>0$ implies $\frac{\partial\pi_i}{\partial x_i}(\hat{\mathbf{x}})=0$, which is equivalent to

$$a^{r-1}[(m-1)a^r+(n-m)b^r]rV=[ma^r+(n-m)b^r]^2. \tag{B.7}$$

Similarly, for any status-seeker, $j=\{m+1,\ldots,n\}$, we have $\frac{\partial\pi_j}{\partial x_j}(\hat{\mathbf{x}})\leq 0$ and $\frac{\partial\rho_j}{\partial x_j}(\hat{\mathbf{x}})\geq 0$, which implies

$$b^{r-1}[ma^r+(n-m-1)b^r]rV=\gamma[ma^r+(n-m)b^r]^2, \tag{B.8}$$

for some $\gamma\in[\frac{n-1}{n},1]$. Combining equations (B.7) and (B.8) and replacing $\frac{a}{b}$ by c, it follows that

$$c^{1-r}[mc^r+(n-m-1)]=\gamma[(m-1)c^r+(n-m)]. \tag{B.9}$$

Define $\varphi:(0,1]\to\mathbb{R}$ by $\tilde{c}\mapsto\varphi(\tilde{c})=m\tilde{c}-\gamma(m-1)\tilde{c}^r+(n-m-1)\tilde{c}^{1-r}-\gamma(n-m)$. Obviously, c solves (B.9) if and only if $\varphi(c)=0$. Therefore, it suffices to show that $\varphi(\tilde{c})=0$ implies $\varphi'(\tilde{c})>0$, which implies that the equation $\varphi(\tilde{c})=0$ and hence equation (B.9) has a unique solution on $(0,1]$.

Inserting $\varphi(\widetilde{c})=0$ into the derivative $\varphi'(\widetilde{c})$ yields

$$\begin{aligned}\varphi'(\widetilde{c}) &= m-\gamma(m-1)r\widetilde{c}^{r-1}+(1-r)(n-m-1)\widetilde{c}^{-r}\\ &= \tfrac{1}{\widetilde{c}}[\underbrace{m\widetilde{c}-\gamma(m-1)\widetilde{c}^{r}+(n-m-1)\widetilde{c}^{1-r}}_{=\gamma(n-m)}\\ &\quad +(1-r)\gamma(m-1)\widetilde{c}^{r}-r(n-m-1)\widetilde{c}^{1-r}]\\ &= \tfrac{1}{\widetilde{c}}\left[\gamma(n-m)+(1-r)\gamma(m-1)\widetilde{c}^{r}-r(n-m-1)\widetilde{c}^{1-r}\right].\end{aligned}$$

To establish $\varphi'(\widetilde{c})>0$, we show

$$\gamma(n-m)>r(n-m-1)\widetilde{c}^{1-r}. \tag{B.10}$$

For $m=n-1$, this is obvious. For $1\leq m<n-1$, observe that $\gamma\geq\frac{n-1}{n}$, $\widetilde{c}\in(0,1]$, and $r\leq 1$ imply

$$\frac{\gamma(n-m)}{n-m-1}\geq\frac{(n-1)(n-m)}{n(n-m-1)}=\frac{n^2-mn-n+m}{n^2-mn-n}>1\geq r\widetilde{c}^{1-r}. \tag{B.11}$$

Comparing the left and the right hand side, claim (B.10) can be seen as true. ∎

Constant marginal efficiency

Proof of (3.13). Equation (3.13) represents the first-order condition of the maximum of (3.4). Thus, we differentiate:

$$\frac{\partial F_j(\mathbf{x})}{\partial x_j}=\frac{\partial}{\partial x_j}\left(\left(\beta_1+\beta_2\frac{n-1}{n}\right)\pi_j(\mathbf{x})-\frac{\beta_2}{n}\sum_{h\neq j}\pi_h(\mathbf{x})\right), \tag{B.12}$$

where $\frac{\partial\pi_j(\mathbf{x})}{\partial x_j}=\frac{Vrx_j^{r-1}}{\left(\sum_{h=1}^n x_h^r\right)^2}\left(\sum_{h=1}^n x_h^r-x_j^r\right)-1$ and $\frac{\partial\pi_i(\mathbf{x})}{\partial x_j}=-\frac{Vrx_i^r x_j^{r-1}}{\left(\sum_{h=1}^n x_h^r\right)^2}$. Therefore, (B.12) becomes

$$\left(\beta_1+\beta_2\frac{n-1}{n}\right)\left(\frac{Vrx_j^{r-1}}{\left(\sum_{h=1}^n x_h^r\right)^2}\left(\sum_{h=1}^n x_h^r-x_j^r\right)-1\right)+\frac{\beta_2}{n}\sum_{h\neq j}\frac{Vrx_h^r x_j^{r-1}}{\left(\sum_{h=1}^n x_h^r\right)^2}.$$

Since we assume that $r=1$, this expression reduces to

$$\left(\beta_1+\beta_2\frac{n-1}{n}\right)\left(\frac{V}{\left(\sum_{h=1}^n x_h\right)^2}\left(\sum_{h=1}^n x_h-x_j\right)-1\right)+\frac{\beta_2}{n}\sum_{h\neq j}\frac{Vx_h}{\left(\sum_{h=1}^n x_h\right)^2}.$$

We set $\frac{\partial F_j(\mathbf{x})}{\partial x_j}=0$ and get

$$\frac{V}{\left(\sum_{h=1}^n x_h^r\right)^2 n}\left[\left(\beta_1 n+\beta_2\left(n-1\right)\right)\sum_{h\neq n}x_h+\beta_2\sum_{h\neq n}x_h\right]=\beta_1 n+\beta_2\left(n-1\right)$$

$$\Leftrightarrow \frac{V}{\left(\sum_{h=1}^{n} x_h^r\right)^2} (\beta_1 + \beta_2) \sum_{h \neq n} x_h = \beta_1 n + \beta_2 (n-1)$$

$$\Leftrightarrow \frac{V(\beta_1 + \beta_2)}{\beta_2 (n-1) + \beta_1 n} \sum_{h \neq n} x_h = \left(\sum_{h=1}^{n} x_h^r\right)^2 .$$

■

Proof of (3.16). To show that (3.16) holds, we take (3.14) and (3.15) and equate them. We receive:

$$\frac{V}{n}((m-1)a + (m-k)b) = \frac{V(\beta_1 + \beta_2)}{\beta_1 + \beta_2 (n-1) n} (ma + (n-m-1)b)$$

$$\Leftrightarrow (\beta_1 + \beta_2 (n-1) n)((m-1)a + (n-m)b) = n(\beta_1 + \beta_2)(ma + (n-m-1)b)$$

$$\begin{aligned} \Leftrightarrow \quad & nm\beta_2 a - n\beta_2 a - m\beta_2 a + \beta_2 a + nm\beta_1 a - n\beta_1 a \\ & + n^2\beta_2 b - nm\beta_2 b - n\beta_2 b + m\beta_2 b + n^2\beta_1 b - nm\beta_1 b \\ = \quad & nm\beta_2 a + nm\beta_1 a + n^2\beta_2 b - nm\beta_2 b - n\beta_2 b + n^2\beta_1 b - nm\beta_1 b - n\beta_1 b \end{aligned}$$

$$\Leftrightarrow a(\beta_2(1-(n+m)) - \beta_1 n) = b(-\beta_2 m - \beta_1 n)$$

$$\Leftrightarrow a = b\frac{\beta_2 m + \beta_1 n}{\beta_2 (n+m-1) + \beta_1 n}.$$

■

Proof of Proposition 2. First, recall equation (3.18), $b = \sqrt{Van} - a(n-1)$. Then, $b > a$ follows from $V > an \Leftrightarrow Van > (an)^2 \Leftrightarrow \sqrt{Van} > an \Leftrightarrow \sqrt{Van} - a(n-1) > a$.

To see the equality in (3.19), notice that $\pi_h([a]_{n-1}, b) = \frac{a}{(n-1)a+b} V - a$, for all $h \neq j$. Using (3.18), we obtain

$$\begin{aligned} \rho_j([a]_{n-1}, b) &= \tfrac{n-1}{n} \pi_j([a]_{n-1}, b) - \tfrac{1}{n} \textstyle\sum_{h \neq j} \pi_h([a]_{n-1}, b) \\ &= \tfrac{n-1}{n} \left(\tfrac{b-a}{(n-1)a+b} V - (b-a) \right) \\ &= \tfrac{n-1}{n} \left(\tfrac{\sqrt{Van}-an}{\sqrt{Van}} V - (\sqrt{Van} - an) \right) \\ &= \tfrac{n-1}{n} \left(\tfrac{V}{\sqrt{Van}} - 1 \right) \left(\sqrt{Van} - an \right). \end{aligned}$$

■

B.2.3 The case $r > 1$

Type I equilibria ($b \geq a > 0$)

Proof of Theorem 3. To establish the strategic advantage for the case where $b \geq a > 0$ and $r > 1$, we look upon the difference Δ between the (material) pay-off of status-seekers and the one of individualists. We define

$$\Delta := \pi_j - \pi_i. \tag{B.13}$$

In case of a (strict) strategic advantage, Δ is non-negative (positive). Inserting (2.1) in (B.13) for an intragroup symmetric equilibrium yields us:

$$\Delta = \frac{b^r - a^r}{ma^r + (n-m)b^r} V - b + a. \tag{B.14}$$

For $b = a$, this yields $\Delta = 0$; status-seekers experience a weak strategic advantage.
Let us now look at the less plain case where $b > a$. We will see that the strategic advantage will be strict, i.e., $\Delta > 0$. After solving (FOCIJ), we can take c as given and, thus, replace b by $\frac{a}{c}$ with $c \in (0,1)$. Inserting this in (B.14) yields:

$$\Delta = \frac{\left(\frac{a}{c}\right)^r - a^r}{ma^r + (n-m)\left(\frac{a}{c}\right)^r} V - \frac{a}{c} + a = \frac{\frac{1}{c^r} - 1}{k + \frac{n-k}{c^r}} V - a\left(\frac{1}{c} - 1\right). \tag{B.15}$$

From (FOCI), it follows that the best response of an individualists as a function of c is given as

$$a = \frac{m + \frac{n-m}{c^r} - 1}{\left(m + \frac{n-m}{c^r}\right)^2} rV. \tag{B.16}$$

Inserting (B.16) in (B.15) results in

$$\begin{aligned}
\Delta = V & \left(\frac{\frac{1}{c^r} - 1}{m + \frac{n-m}{c^r}} - \left(\frac{1}{c} - 1\right) \frac{m + \frac{n-m}{c^r} - 1}{\left(m + \frac{n-m}{c^r}\right)^2} r \right) \\
= & \frac{V}{\left(m + \frac{n-m}{c^r}\right)^2} \left[\left(\frac{1}{c^r} - 1\right)\left(m + \frac{n-m}{c^r}\right) \right. \\
& \left. - \left(\frac{1}{c} - 1\right)\left(m + \frac{n-m}{c^r}\right) r + \left(\frac{1}{c} - 1\right) r \right] \\
= & \frac{V}{\left(m + \frac{n-m}{c^r}\right)^2} \left[\left(\frac{n-m}{c^r} + m\right)\left(\frac{1}{c^r} - 1 - \frac{r}{c} + r\right) + r\left(\frac{1}{c} - 1\right) \right] \\
= & \underbrace{\frac{V}{\left(m + \frac{n-m}{c^r}\right)^2}}_{>0} \left\{ \underbrace{\left(\frac{n-m}{c^r} + m\right)}_{>0} \left[\left(\frac{1}{c}\right)^r - 1 - r\left(\frac{1}{c} - 1\right) \right] + \underbrace{r\left(\frac{1}{c} - 1\right)}_{>0} \right\}.
\end{aligned} \tag{B.17}$$

From this, we can see that

$$\left(\frac{1}{c}\right)^r - 1 - r\left(\frac{1}{c} - 1\right) \geq 0 \tag{B.18}$$

is sufficient for a strict strategic advantage. Writing $\frac{1}{c} := 1 + q$, we can turn (B.18) into

$$(1+q)^r \geq r(1 + q - 1) + 1 = 1 + rq.$$

Since $q > 0$ and $r > 1$, we can apply the Generalized Bernoulli Inequality, which tells us that

$$(1+q)^r > 1 + rq, \quad \forall q > -1, r > 1.$$

Applying this knowledge to (B.17) implies

$$\Delta > 0,$$

that is, higher effort $b > a$ results in a strict strategic advantage of interdependent preferences. ■

Type II equilibria ($a > b > 0$)

Proof of Lemma 4. Fix $r > 1$ and $m \in \{1, \ldots, n-1\}$ and let $\hat{\mathbf{x}} = ([a]_m, [b]_{n-m}) \in N_{sym}(\Gamma_F(m))$ such that $a, b > 0$. Recall $Z = ma^r + (n-m-1)b^r$, $B(Z,\lambda) = [b(Z,\lambda)]^r = b^r(Z,\lambda)$, and that $b(Z,\lambda)$ solves

$$[Z + b^r]^2 - rb^{r-1}Z\lambda V = 0. \tag{B.19}$$

Then, (B.19) is equivalent to

$$B^{\frac{1}{r}}[Z+B]^2 - rBZ\lambda V = 0. \tag{B.20}$$

Part (i): We implicitly differentiate (B.20) to obtain

$$\begin{aligned} \frac{\partial B}{\partial Z} &= -\frac{2B^{\frac{1}{r}}[Z+B] - rB\lambda V}{\frac{1}{r}B^{\frac{1}{r}-1}[Z+B]^2 + 2B^{\frac{1}{r}}[Z+B] - rZ\lambda V} \\ &= -\frac{2B^{\frac{1}{r}}[Z+B] - B^{\frac{1}{r}}[Z+B]^2 Z^{-1}}{\frac{1}{r}B^{\frac{1}{r}-1}[Z+B]^2 + 2B^{\frac{1}{r}}[Z+B] - B^{\frac{1}{r}}[Z+B]^2 B^{-1}}, \end{aligned} \tag{B.21}$$

where the equality follows from (B.20). The numerator is non-negative if and only if $Z \geq B$. Similarly, the denominator is positive if and only if

$$\begin{aligned} \frac{1}{r}[Z+B] + 2B - [Z+B] &> 0 \\ \Leftrightarrow \qquad \frac{r+1}{r-1}B &> Z. \end{aligned}$$

Now, consider $Z \leq B$ first. Since this implies $Z < \frac{r+1}{r-1}B$, we have $\frac{\partial B}{\partial Z} \geq 0$ for all $Z \leq B$. On the other hand, if $B < Z < \frac{r+1}{r-1}B$, then we have $\frac{\partial B}{\partial Z} < 0$. Since both the numerator and the denominator of (B.21) are positive, $\frac{\partial B}{\partial Z} \geq -1$ is equivalent to

$$\begin{aligned} \text{Numerator (B.21)} &\leq \text{Denominator (B.21)} \\ \Leftrightarrow \quad -B^{\frac{1}{r}}[Z+B]^2 Z^{-1} &\leq \frac{1}{r}B^{\frac{1}{r}-1}[Z+B]^2 - B^{\frac{1}{r}}[Z+B]^2 B^{-1} \\ \Leftrightarrow \qquad -B &\leq \frac{1-r}{r}Z \\ \Leftrightarrow \qquad \frac{r}{r-1}B &\geq Z. \end{aligned}$$

Since $Z \leq \frac{r}{r-1}B$ implies $Z < \frac{r+1}{r-1}B$, this completes the proof of part (i).

Part (ii): Notice first that $\frac{\partial B}{\partial \lambda}(Z,\lambda) = r[b(Z,\lambda)]^{r-1}\frac{\partial b}{\partial \lambda}(Z,\lambda)$ and that $Z > 0$ implies $b(Z,\lambda) > 0$. Therefore, it is sufficient to show that $\frac{\partial B}{\partial \lambda}(Z,\lambda) > 0$ if and only if $Z < \frac{r+1}{r-1}B$. Implicitly differentiating (B.20), we obtain

$$\frac{\partial B}{\partial \lambda} = -\frac{-rBZV}{\frac{1}{r}B^{\frac{1}{r}-1}\left[Z+B\right]^2 + B^{\frac{1}{r}}2\left[Z+B\right] - rZ\lambda V}.$$

Obviously, $\frac{\partial B}{\partial \lambda}$ is strictly positive if and only if the denominator is strictly positive, which by part (i) is equivalent to $\frac{r+1}{r-1}B > Z$.

Part (iii): Suppose $a > b$ and $Z \leq \frac{r}{r-1}B$. Then, part (ii) implies that $\frac{\partial B}{\partial \lambda}(Z',\lambda) > 0$, for all $0 < Z' \leq Z$. Derive a condition from the first order condition of an individualist that is similar to (B.20), namely,

$$C^{\frac{1}{r}}\left[Y+C\right]^2 - rCYV = 0, \tag{B.22}$$

where $C = a^r$ and $Y = (m-1)a^r + (n-m)b^r$. Comparing (B.22) with (B.20), it follows that $C(Y) = B(Y,1)$. Since $a > b$ implies $Y < Z$, we can thus apply part (ii) and $\lambda > 1$ to conclude that

$$C(Y) = B(Y,1) < B(Y,\lambda). \tag{B.23}$$

Moreover, it follows from part (i) that $\frac{\partial B}{\partial Z}(Z',\lambda) \geq -1$, for all $Z' \leq \frac{r}{r-1}B$, and hence

$$\begin{aligned} \frac{B(Z,\lambda) - B(Y,\lambda)}{Z - Y} &\geq -1 \\ \Leftrightarrow \qquad B(Y,\lambda) - B(Z,\lambda) &\leq Z - Y. \end{aligned} \tag{B.24}$$

Combining the two inequalities (B.23) and (B.24), we obtain

$$\begin{aligned} C(Y) - B(Z,\lambda) &< B(Y,\lambda) - B(Z,\lambda) \\ &\leq Z - Y = a^r - b^r = C - B, \end{aligned}$$

which yields a contradiction because $\hat{\mathbf{x}} = ([a]_m, [b]_{n-m}) \in N_{sym}(\Gamma_F(k))$ implies $C(Y) = C$ and $B(Z,\lambda) = B$. Thus, $a \leq b$. ■

B.3 Proofs of Chapter 4

Proof of Lemma 5. The first order condition of the maximization problem of player i is given by $\frac{\partial F_i(\pi_i(\mathbf{x}),\rho_i(\mathbf{x}))}{\partial x_i} \overset{!}{=} 0$, which is equivalent to

$$rx_i^{r-1}\sum_{j\neq i} x_j^r \frac{n}{n+\alpha_i} V = \left(\sum_{h=1}^{n} x_h^r\right)^2. \tag{B.25}$$

Now assume, on the other hand, that player i maximizes the utility function (2.1) in a rent-seeking contest with a fixed prize of $\tilde{V} := \lambda_i V$. His first-order-condition is then given by

$$\frac{\partial \pi_i(\mathbf{x})}{\partial x_i} = \frac{rx_i^{r-1}\sum_{j\neq i} x_j^r}{\left(\sum_{h=1}^{n} x_h^r\right)^2}\tilde{V} - 1 \overset{!}{=} 0$$

$$\Leftrightarrow r x_i^{r-1} \sum_{j \neq i} x_j^r \lambda_i V = \left(\sum_{h=1}^{n} x_h^r \right)^2. \tag{B.26}$$

Obviously, since we assumed $\lambda_i := \frac{n}{n+\alpha_i}$, the decision situation in (B.25) and (B.26) is the same. Since we know from Fang (2002) that maximization of the material payoff (2.1) for a fixed prize V has a unique solution, we can apply this result also to (B.26). We find that there is one unique solution for the optimization of the material payoff competing for $\lambda_i V$, and from (B.25) and (B.26), we can conclude that this is also the unique solution in maximizing (4.1) when competing for a prize of V. ■

Proof of Lemma 6. The first-order-condition of a player i is given by

$$\sum_{j \neq i} x_j \lambda_i V - \left(\sum_{j \neq i} x_j + x_i \right)^2 = 0. \tag{B.27}$$

A choice of $x_i = 0$ is endogenously optimal if

$$\sum_{j \neq i} x_j \lambda_i V = \left(\sum_{j \neq i} x_j \right)^2 \Leftrightarrow \lambda_i V = \sum_{j=1}^{n} x_j.$$

Note that $\sum_{j=1}^{n} x_j$ and $\sum_{j \neq i} x_j$ can be set equal, since $x_i = 0$. Applying the implicit function theorem to equation (B.27) results in

$$\frac{\partial x_i}{\partial \lambda_i} = -\frac{\sum_{j \neq i} x_j}{-2 \sum_{j=1}^{n} x_j} > 0, \quad \forall x_i > -\sum_{j \neq 1} x_j,$$

telling us that a player i that is characterized by $\lambda_i V < \sum_{j=1}^{n} x_j$ would then prefer negative effort. Thus, she stays outside the contest. ■

Proof of Proposition 3. At first we show how (4.3) turns into the ratio of the preference parameters:

$$(4.3) \Leftrightarrow x_i^{r-1} \sum_{h \neq i} x_h^r \lambda_i = x_j^{r-1} \sum_{h \neq j} x_h^r \lambda_j$$

$$\Leftrightarrow \frac{\lambda_i}{\lambda_j} = \frac{A - x_j^r}{A - x_i^r} \left(\frac{x_i}{x_j} \right)^{1-r}, \tag{B.28}$$

where $A := \sum i = 1^n x_i^r$.

Part (i): Assume $\lambda_i > \lambda_j$. This means that the left-hand-side of equation (B.28) is larger than 1. For equating (B.28), at least one of the factors on the right-hand-side also has to be larger than 1. This means:

$$\frac{A - x_j^r}{A - x_i^r} > 1 \vee \left(\frac{x_i}{x_j} \right)^{1-r} > 1$$

$$\Rightarrow x_i > x_j \vee (x_i > x_j \wedge r < 1).$$

Thus from $\lambda_i > \lambda_j$ follows $x_i > x_j$. Due to reciprocity, we can also conclude that $\lambda_i < \lambda_j \Rightarrow x_i < x_j$

Part (ii): Now if $\lambda_i = \lambda_j$, the left-hand-side of (B.28) equates 1. Then for the two factors on the right-hand-side, it has to hold that either one has to be smaller and one larger than 1, or both have to equate 1. One can show that the first of these possibilities induces a contradiction. Thus the latter must hold, which means that $\frac{x_i}{x_j} = 1$ or equivalently $x_i = x_j$ has to hold. Since these results cover the whole range of possibilities for the ratio of x_i and x_j, we can conclude that the equivalency has to hold, too. ■

Proof of Theorem 7. We will restrict the proof to the case, where $\lambda_i \leq \frac{n}{n-1}$, $\forall i$. The proof for $\lambda_i \geq \frac{n}{n-1}$, $\forall i$ runs analogously. Assume, like in Chapter 3, the aggregate to be spent and, in consequence, A to be fixed. Then, we can define a function $\varphi(z) : \mathbb{R}^+ \rightarrow \mathbb{R}$, such that:

$$\varphi(z) = \frac{z^r}{A}V - x. \tag{B.29}$$

Note that, applying Proposition 3, it is sufficient to show that a higher effort induces higher material payoff. For this purpose, we show that the first derivative of (B.29), $\varphi'(z) = \frac{rz^{r-1}}{A}V - 1$, is positive over the whole range of efforts chosen in the contest. Denoting the highest effort chosen in equilibrium by $\overline{x}$, we do this by showing, that $\varphi(x)$ is concave for any effort level and $\varphi'(\overline{x}) > 0$, from which we can conclude that material payoff is strictly increasing in λ_i, whenever effort is strictly increasing.

The second derivative of (B.29) is given by $\varphi''(z) = \frac{r(r-1)z^{r-2}}{A}V \leq 0, \quad r \leq 1$, which already proves concavity. Now let us look $\varphi'(\overline{x})$.

$$\varphi'(\overline{x}) = \frac{r\overline{x}^{r-1}}{A}V - 1 \tag{B.30}$$

Denoting λ_i of the player i choosing $\overline{x}$ by $\overline{\lambda}$, we can transform (4.2) to:

$$\overline{x}^{r-1} = \frac{A^2}{r(A - \overline{x}^r)\overline{\lambda}V}. \tag{B.31}$$

Inserting (B.31) in (B.30) yields:

$$\varphi'(\overline{x}) = \frac{A}{(A - \overline{x}^r)\overline{\lambda}} - 1 \geq \frac{n-1}{n}\frac{A}{A - \overline{x}^r} - 1 = \frac{\frac{n-1}{n}A - (A - \overline{x}^r)}{A - \overline{x}^r} = \frac{\overline{x}^r - \frac{A}{n}}{A - \overline{x}^r} > 0.$$

The last expression is larger than zero since $\overline{x}^r$ is per definition the largest summand of A. Since we assumed a heterogenous population, the largest summand of A is larger than the average summand, $\frac{A}{n}$. The numerator is strictly positive. Since A is a sum of several positive summands, where $\overline{x}^r$ is only one of them, the denominator is also strictly positive, and thus the player with higher preference parameter realizes the higher material payoff. ■

Proof of Corollary 7. We have to show here that, for any player i with $\lambda_i \leq 1$, an effort level of $x_i > 0$ cannot be the best response to an aggregate effort where $\sum_{j \neq i} x_j > V$. To this aim, remember that $\lambda_i \leq 1$ implies $\alpha_i \geq 0$. Denote with $\mathbf{x}_{i0} =$

$(x_1, ..., x_{i-1}, 0, x_{i+1}, ..., x_N)$. Then, if i remains passive, her objective function will take the value

$$F(\mathbf{x}_{i0}) = \frac{\alpha_i}{N} \sum_{j \neq i} \pi_j(\mathbf{x}_{i0}) = \frac{\alpha_i}{N} \sum_{j \neq i} x_j \frac{V - \sum_{j \neq i} x_j}{\sum_{j \neq i} x_j}.$$

Now suppose to the contrary that she would choose $x_i > 0$ as best response to some $\sum_{j \neq i} x_j > V$. Writing $\mathbf{x}_i = (x_1, ..., x_{i-1}, x_i > 0, x_{i+1}, ..., x_N)$ we have:

$$F(\mathbf{x}_i) = \left(1 + \frac{\alpha_i}{N}\right)\left(\frac{x_i}{\sum_{j \neq i} x_j + x_i} V - x_i\right) + \frac{\alpha_i}{N} \sum_{j \neq i} \left(\frac{x_j}{\sum_{k \neq i} x_k + x_i} V - x_j\right)$$

$$= \left(1 + \frac{\alpha_i}{N}\right) x_i \underbrace{\frac{V - \sum_{j \neq i} x_j - x_i}{\sum_{j \neq i} x_j + x_i}}_{<0} + \frac{\alpha_i}{N} \sum_{j \neq i} x_j \underbrace{\frac{V - \sum_{j \neq i} x_j - x_i}{\sum_{j \neq i} x_j + x_i}}_{< \frac{V - \sum_{j \neq i} x_j}{\sum_{j \neq i} x_j}} < F(\mathbf{x}_{i0}).$$

■

B.4 Proofs of Chapter 5

Proof of Lemma 8. As we said, q_1 and q_2 are positive. Furthermore, define $q_3 := 2(n-1)x^{2r}y^r$. This turns $\omega''(y) > 0$ to

$$-q_1((r-1)q_2 - q_3 r) > 0,$$

with $q_1, q_2, q_3 > 0$. From this, we can conclude to

$$r < 1 + \frac{q_3}{q_2 - q_3} =: r^* > 1, \text{ if } q_2 > q_3.$$

If $q_2 \leq q_3$, we end up in $q_2 > r(q_2 - q_3)$, which is then always fulfilled. ■

B.5 Proofs of Chapter 6

Derivation of (6.11): Equating (6.10) for player i and player j yields:

$$\frac{\sqrt{(x_i + x_j) V_k} - \frac{x_i V_k}{2\sqrt{(x_i + x_j) V_k}}}{(x_i + x_j) V_k} V_i = \frac{\sqrt{(x_i + x_j) V_k} - \frac{x_j V_k}{2\sqrt{(x_i + x_j) V_k}}}{(x_i + x_j) V_k} V_j$$

$$\Leftrightarrow \sqrt{(x_i + x_j) V_k} V_i - \frac{x_i V_k}{2\sqrt{(x_i + x_j) V_k}} V_i = \sqrt{(x_i + x_j) V_k} V_j - \frac{x_j V_k}{2\sqrt{(x_i + x_j) V_k}} V_j$$

$$\Leftrightarrow 2(x_i + x_j) V_i V_k - x_i V_i V_k = 2(x_i + x_j) V_j V_k - x_j V_j V_k$$

$$\Leftrightarrow 2V_i x_j + V_i x_i = 2V_j x_i + V_j x_j \Leftrightarrow x_j = x_i \frac{2V_j - V_i}{2V_i - V_j}.$$

■

Derivation of Table 6.1:

We look at a rent-seeking contest, where player 3 decides on her effort in $t = 2$, while players 1 and 2 choose their effort levels in $t = 1$. Assume that $\frac{V_1 V_2}{V_1+V_2} < V_3 < \frac{3}{2}\frac{V_1 V_2}{V_1+V_2}$. The participation constraint of player 3 says, that she will be inactive in the standard three-player-model. The standard model implicitly assumes that a player who is strictly worse off from participating chooses a negative effort, resulting in a negative winning probability. Since this is not feasible by assumption, the reaction functions of players 1 and 2 cannot be adapted in this case. Let us now focus on what will actually be the equilibrium effort of player 1 and 2.

Presume, first, that players 1 and 2 completely ignore the presence of player 3 and play a mere two-player-game. According to Leininger (1993), their efforts would be $x_1 = \frac{V_1^2 V_2}{(V_1+V_2)^2} \wedge x_2 = \frac{V_1 V_2^2}{(V_1+V_2)^2}$, which means that $x_1 + x_2 = \frac{V_1 V_2}{V_1+V_2}$. The latter is smaller than V_3 per assumption. Yet, (6.9) shows that for $x_1 + x_2 < V_3$, player 3 will choose a positive effort. Hence, if players 1 and 2 would behave as if in a two-player-contest, player 3 would choose a positive effort. This cannot constitute an equilibrium.

From the standard analysis of the three-player-case, we can conclude that $x_3 > 0$ cannot be part of an equilibrium in this setting since this equilibrium would also be feasible without the restriction on $x_3 \geq 0$.

In order to keep player 3 out of the contest, the aggregate effort of players 1 and 2 has to be at least V_3. Furthermore it has to hold that each active player has a positive return of the last marginal effort. Hence, the first derivative of the payoff must be weakly positive for the case that $x_1 + x_2 = V_3$. We illustrate this for player 1:

$$\frac{\partial \pi_1 (x_1, x_2, x_3^*(x_1, x_2))}{\partial x_1} \geq 0|_{x_1+x_2=V_3}$$

$$\Leftrightarrow \frac{\partial}{\partial x_1} \frac{x_1}{\sqrt{(x_1+x_2)V_3}} V_1 - x_1 \geq 0|_{x_1+x_2=V_3}$$

$$\Leftrightarrow \frac{\sqrt{(x_1+x_2)V_3} - \frac{x_1 V_3}{2\sqrt{(x_1+x_2)V_3}}}{(x_1+x_2)V_3} V_1 \geq 1|_{x_1+x_2=V_3}$$

$$\Leftrightarrow V_1 V_3 - \frac{1}{2} x_1 V_1 \geq V_3^2 \Leftrightarrow x_1 \leq 2V_3 \frac{V_1 - V_3}{V_1}.$$

For player 2, it analogously has to hold that

$$x_2 \leq 2V_3 \frac{V_2 - V_3}{V_2}.$$

The upper limit for aggregate effort $x_1 + x_2 = 4V_3 - 2\frac{V_1+V_2}{V_1 V_2} V_3^2$ for these calculations should lie above V_3. This holds, if and only if $V_3 < \frac{3}{2}\frac{V_1 V_2}{V_1+V_2}$.

Furthermore, we have to check whether a player has an incentive to deviate to a higher effort. If one of the players does so, she only has to face one opponent, since player 3 was already preempted without an increase of her opponents efforts. An aggregate effort of V_3 is thus stable, if

$$\frac{\partial \pi_1^{n=2}(x_1,x_2)}{\partial x_1} \leq 0|_{x_1+x_2=V_3} \Leftrightarrow \frac{\partial}{\partial x_1}\left(\frac{x_1}{x_1+x_2}V_1 - x_1\right) \leq 0|_{x_1+x_2=V_3}$$

$$\Leftrightarrow \frac{x_2}{(x_1+x_2)^2}V_1 - 1 \leq 0|_{x_1+x_2=V_3} \Leftrightarrow x_2 \leq \frac{V_3^2}{V_1},$$

and respectively player 2 has no incentive to increase her effort if $x_1 \leq \frac{V_3^2}{V_2}$.

Here, we can see that this analysis provides an additional upper limit for effort vectors (x_1, x_2) that is given by $\frac{V_3^2}{V_1} + \frac{V_3^2}{V_2} = \frac{V_1+V_2}{V_1V_2}V_3^2$. This is larger than V_3 if and only if $V_3 > \frac{V_1V_2}{V_1+V_2}$. ■

Derivation of (6.21): From (6.20), we can conclude

$$(x_i + x_j)\,V_k = \left((x_i + x_j)\,\frac{V_k}{V_j} + x_j\right)^2$$

$$\Leftrightarrow x_j^2 + x_jV_j^2\frac{2\left(\frac{V_k}{V_j}\right)^2 x_i + 2\frac{V_k}{V_j}x_i - V_k}{(V_k+V_j)^2} + \frac{V_kx_i}{(V_k+V_j)^2}\left(V_kx_i - V_j^2\right) = 0$$

$$\Rightarrow x_j = \frac{V_kV_j^2 - 2V_k^2x_i - 2V_kV_jx_i}{2\,(V_k+V_j)^2} \pm$$

$$\sqrt{\frac{\left(V_kV_j^2 - 2V_k^2x_i - 2V_kV_jx_i\right)^2}{4\,(V_k+V_j)^4} + \frac{V_kx_i\left(V_j^2 - V_kx_i\right)}{4\,(V_k+V_j)^4}4\,(V_k+V_j)^2}$$

$$= \frac{V_kV_j^2 - 2V_k^2x_i - 2V_kV_jx_i}{2\,(V_k+V_j)^2} \pm \frac{1}{2\,(V_k+V_j)^2} \cdot V_j^3V_k\left(V_jV_k + 4x_i\,(V_k+V_j)\right)^{\frac{1}{2}}.$$

■

Derivation of (6.23):

Let us first take a look at the aggregate effort the late moving players will exert in reaction to player i's choice:

$$x_j^E + x_k^E = \frac{V_jV_k + \sqrt{V_jV_k\left(V_jV_k + 4\,(V_j+V_k)\,x_i\right)}}{2\,(V_j+V_k)} - x_i. \tag{B.32}$$

Furthermore, the first-order-condition of player i depends on the derivation $\left(\frac{\partial x_j^*(x_i)}{\partial x_i} + \frac{\partial x_k^*(x_i)}{\partial x_i}\right)$, which equals the derivation of (B.32) for x_i. This is given by

$$\frac{\partial\left(x_j^*(x_i) + x_k^*(x_i)\right)}{\partial x_i} = \sqrt{\frac{V_jV_k}{V_jV_k + 4\,(V_j+V_k)\,x_i}} - 1. \tag{B.33}$$

Now, inserting (B.32) and (B.33) into (6.22) yields

$$\left[\frac{V_jV_k+\sqrt{V_jV_k\left(V_jV_k+4\left(V_j+V_k\right)x_i\right)}}{2\left(V_j+V_k\right)}-\sqrt{\frac{V_jV_k}{V_jV_k+4\left(V_j+V_k\right)x_i}x_i}\right]V_i$$
$$=\left[\frac{V_jV_k+\sqrt{V_jV_k\left(V_jV_k+4\left(V_j+V_k\right)x_i\right)}}{2\left(V_j+V_k\right)}\right]^2.$$

One can show that this equation is solved by

$$x_i=\frac{V_i^2\left(V_j+V_k\right)^2-V_j^2V_k^2}{4V_jV_k\left(V_j+V_k\right)}. \tag{B.34}$$

■

Derivation of (6.24):
Inserting (6.23) in (6.21) yields:

$$x_j=\frac{V_j^2V_k-2V_k\left(V_j+V_k\right)\frac{V_i^2\left(V_j+V_k\right)^2-V_j^2V_k^2}{4V_jV_k\left(V_j+V_k\right)}}{2\left(V_j+V_k\right)^2}$$
$$+\frac{V_j\sqrt{V_jV_k\left(V_jV_k+4\left(V_j+V_k\right)\frac{V_i^2\left(V_j+V_k\right)^2-V_j^2V_k^2}{4V_jV_k\left(V_j+V_k\right)}\right)}}{2\left(V_j+V_k\right)^2}$$
$$\Leftrightarrow x_j^*=\frac{V_jV_k\left(2V_j+V_k\right)-\frac{V_i^2}{V_j}\left(V_j+V_k\right)^2+2V_iV_j\left(V_j+V_k\right)}{4\left(V_j+V_k\right)^2}.$$

■

Derivation of (6.26):
From (6.24), we can conclude:

$$x_j^*+x_k^*=\frac{V_jV_k\left(2V_k+V_j+2V_j+V_k\right)-\left(V_j+V_k\right)^2\left(\frac{V_i^2}{V_j}+\frac{V_i^2}{V_k}\right)+2V_i\left(V_j+V_k\right)^2}{4\left(V_j+V_k\right)^2}$$
$$=\frac{3V_j^2V_k^2+2V_iV_jV_k\left(V_j+V_k\right)-V_i^2\left(V_j+V_k\right)^2}{4V_jV_k\left(V_j+V_k\right)}.$$

Then

$$x_i^*+x_j^*+x_k^*=\frac{V_i^2\left(V_j+V_k\right)^2-V_j^2V_k^2}{4V_jV_k\left(V_j+V_k\right)}+\frac{3V_j^2V_k^2+2V_iV_jV_k\left(V_j+V_k\right)-V_i^2\left(V_j+V_k\right)^2}{4V_jV_k\left(V_j+V_k\right)}$$
$$=\frac{V_iV_j+V_iV_k+V_jV_k}{2\left(V_j+V_k\right)}.$$

■

Derivation of (6.27) and (6.28):

$$\pi_i = \frac{V_i^2 (V_j + V_k)^2 - V_j^2 V_k^2}{4V_j V_k (V_j + V_k)} \cdot \frac{2V_iV_j + 2V_iV_k - V_iV_j - V_iV_k - V_jV_k}{V_iV_j + V_iV_k + V_jV_k}$$
$$= \frac{(V_iV_j + V_iV_k - V_jV_k)^2}{4V_jV_k (V_j + V_k)}.$$
$$\pi_j = \frac{V_jV_k (2V_j + V_k) - \frac{V_i^2}{V_j} (V_j + V_k)^2 + 2V_iV_j (V_j + V_k)}{4 (V_j + V_k)^2}$$
$$\cdot \frac{2V_j^2 + 2V_jV_k - V_iV_j - V_iV_k - V_jV_k}{V_iV_j + V_iV_k + V_jV_k}$$
$$= \frac{\left(2V_j^2 + V_jV_k - V_iV_j - V_iV_k\right)^2}{4V_j (V_j + V_k)^2}.$$

π_k can be derived analogously. ■

Derivation of (6.29):

By inserting (6.8) and (6.27), we get:

$$\pi_i^{iE} > \pi_i^S \Leftrightarrow \frac{(V_iV_j + V_iV_k - V_jV_k)^2}{4V_jV_k (V_j + V_k)} > \frac{V_i (V_iV_j + V_iV_k - V_jV_k)^2}{(V_iV_j + V_iV_k + V_jV_k)^2}$$
$$\Leftrightarrow (V_iV_j + V_iV_k + V_jV_k)^2 > 4V_iV_jV_k(V_jV_k)$$
$$\Leftrightarrow V_i^2(V_j + V_k)^2 + 2V_iV_jV_k(V_jV_k) + V_j^2V_k^2 > 4V_iV_jV_k(V_jV_k)$$
$$\Leftrightarrow V_i^2(V_j + V_k)^2 - 2V_iV_jV_k(V_jV_k) + V_j^2V_k^2 > 0$$
$$\Leftrightarrow (V_i (V_j + V_k) - V_jV_k)^2 > 0.$$

■

Annotations to τ_i:

$\tau_1 \in [0, 1] \Leftrightarrow \pi_2^{3E} \geq \pi_2^{1L} \wedge \pi_2^S > \pi_2^{2L}$.

$\pi_2^{3E} \geq \pi_2^{1L}$ specifies the case we are interested in, since this has to hold in order for player 2 to have an incentive to deviate from joining the weak player moving early to joining the strong player moving late.

$\pi_2^S > \pi_2^{2L}$: From the analysis of problem II, we know that this is equivalent to $V_2 \leq \frac{3V_1V_3}{V_1+V_3}$. Let us now show that this has to hold if $V_3 \geq \frac{V_1}{2}$, as assumed in (6.2). Note that $\frac{3V_1V_3}{V_1+V_3}$ increases in V_3. Then, we find:

$$\frac{3V_1V_3}{V_1 + V_3} \geq \frac{3V_1\frac{V_1}{2}}{V_1 + \frac{V_1}{2}} = V_1.$$

Yet, $V_2 \leq V_1$ holds by assumption, and thus $V_2 \leq \frac{3V_1V_3}{V_1+V_3}$.

$\tau_2 \in [0, 1] \Leftrightarrow \pi_1^{2L} \geq \pi_1^{3E} \wedge \pi_1^L > \pi_1^S$

$\pi_1^{2L} \geq \pi_1^{3E}$ always holds, as we can see from the analysis of Problem III.

Since we only regard mixed strategies for the case $V_1 \geq \frac{3V_2}{V_3}V_2 + V_3$, $\pi_1^L > \pi_1^S$ holds, as we have shown when analyzing Problem II.

B.6 Proofs of Chapter 7

Derivations of Equilibrium Strategies and Payoffs

Inserting $x_i = x_j$ and (7.3) in (7.1) yields:

$$\theta(n-1)x_iV = \left[\left((n-1)+(n-1)\theta-(n-2)\right)x_i\right]^2$$
$$\Leftrightarrow \theta(n-1)Vx_i = \left(1+(n-1)\theta\right)^2 x_i^2$$
$$\Leftrightarrow x_i = \frac{(n-1)\theta}{\left(1+(n-1)\theta\right)^2}V \quad (\vee x_i = 0). \tag{B.35}$$

Remark that $x_i = 0$ will not be chosen by any player i, since we know from Stein (2002) that at least two players will choose positive effort in the simultaneous game, while Chapter 4 tells us that players with identical valuation choose the same effort level. (7.4) is derived by inserting (B.35) in (7.3). It follows:

$$\pi_1 = \frac{(n-1)\theta}{\left(1+(n-1)\theta\right)^2}\left((n-1)\theta-(n-2)\right)V\left(\frac{\theta V}{\sum_{j=1}^n x_j}-1\right)$$

$$= \frac{(n-1)\left((n-1)\theta-(n-2)\right)\theta}{\left(1+(n-1)\theta\right)^2}V\cdot\left[\frac{\theta V}{\frac{(n-1)\theta}{(1+(n-1)\theta)^2}\left[\left((n-1)\theta-(n-2)\right)V+(n-1)V\right]}-1\right]$$

$$= \frac{(n-1)\left((n-1)\theta-(n-2)\right)\theta}{\left(1+(n-1)\theta\right)^2}\frac{\left(1+(n-1)\theta\right)^2-(n-1)\left((n-1)\theta+1\right)}{(n-1)\left((n-1)\theta+1\right)}V$$

$$= \frac{\left((n-1)\theta-(n-2)\right)^2}{\left(1+(n-1)\theta\right)^2}\theta V$$

$$\pi_i = \frac{(n-1)\theta}{\left(1+(n-1)\theta\right)^2}V\left(\frac{\left(1+(n-1)\theta\right)V}{(n-1)\theta V}-1\right)$$
$$= \frac{V}{\left(1+(n-1)\theta\right)^2}.$$

Proof of Lemma 11. The optimal reaction function on all opponents is given by

$$x_i = \begin{cases} \sqrt{V\left(\sum_{j\neq i}x_j\right)}-\sum_{j\neq i}x_j, & \text{if } \sum_{j\neq i}x_j \leq V \\ 0 & \text{, if } \sum_{j\neq i}x_j \geq V, \end{cases}$$

which generalizes the equilibrium in the sequential rent-seeking game introduced by Leininger (1993). Now since all underdogs behave the same, this yields

$$x_i = \sqrt{\left(x_1+(n-2)x_i\right)V}-x_1-(n-2)x_i, \text{ for } x_1+(n-2)x_i \leq V.$$

Due to symmetric behavior of the underdogs, the non-activity of one underdog in equilibrium implies non-activity of all underdogs, and thus $x_i = 0$, if $\sum_{j\neq i}x_j \geq V$.

On the second stage, the game between the underdogs behaves like a simultaneous game, resulting in:

$$(n-1)x_i = \sqrt{(x_1 + (n-2)x_i)\, V} - x_1$$
$$\Leftrightarrow (n-1)^2 x_i^2 + (2(n-1)x_1 - (n-2)V)\, x_i + x_1(x_1 - V) = 0.$$

After applying the quadratic formula, we get a reaction function that describes each underdog's optimal reaction to the favorite's and the other underdogs' actions. It is given by:

$$x_i = \frac{(n-2)V - 2(n-1)x_1}{2(n-1)^2} +$$
$$\sqrt{\frac{(n-2)^2V^2 - 4(n-1)(n-2)x_1V + 4(n-1)^2x_1^2}{4(n-1)^4} + \frac{x_1(V-x_1)}{(n-1)^2}}$$
$$= \frac{1}{2(n-1)^2}\left[(n-2)V + \sqrt{(n-2)^2V^2 + 4(n-1)x_1V}\right] - \frac{x_1}{n-1}, \tag{B.36}$$

which is the optimal behavior of an underdog if she actively takes part in the game. Note that, for $x_1 = V$, (B.36) delivers an optimal interior solution of $x_i = 0$, such that the reaction function of the underdogs is continuous. ■

Proof of Proposition 7. Denoting $T := (n-2)^2V^2 + 4(n-1)x_1V$ we find:

$$\frac{\partial \pi_1}{\partial x_1} = \frac{2(n-1)\theta V}{(n-2)V + \sqrt{T}} - 1 - x_1 \frac{2(n-1)\theta V \frac{1}{\sqrt{T}}}{\left[(n-2)V + \sqrt{T}\right]^2} \frac{1}{2} 4(n-1)V$$

$$= \frac{2(n-1)\theta V}{(n-2)V + \sqrt{T}} \cdot \left[1 - \frac{2x_1(n-1)V}{\left((n-2)V + \sqrt{T}\right)\sqrt{T}}\right] - 1.$$

We will now insert $x_1 = V$ into $\frac{\partial \pi_1}{\partial x_1}$ to show that $\frac{\partial \pi_1(x_1=V)}{\partial x_1} \geq 0$ is equivalent to $\theta \geq \frac{n}{n-1}$. Beforehand, let us mention that T transforms to

$$T = (n^2 - 4n + 4)V^2 + (4n-4)V^2 = n^2V^2.$$

The condition for the favorite excluding her opponents is thus given by

$$\frac{2(n-1)\theta V}{(n-2)V + nV} \cdot \left[1 - \frac{2(n-1)V^2}{((n-2)V + nV)\, nV}\right] \geq 1$$
$$\Leftrightarrow \theta \geq \frac{n}{n-1}.$$

■

Proof of Lemma 12.

To prove Lemma 12, we have to show that an underdog (at least weakly) prefers moving in $t = 1$ to moving in $t = 2$ if all the other underdogs move in $t = 1$ and

1. the favorite also moves in $t = 1$

2. the favorite moves later, i.e., in $t = 2$

3. the favorite moves earlier, i.e., in $t = 0$.

We will now allude to these three different cases. However, we will only give an extensive analysis to only the first case:

1. Let us first assume that the favorite and $(n-2)$ underdogs move early, while one underdog denoted by n chooses late. Player n will then maximize her payoff with the first-order-condition

$$\frac{\partial \pi_n(\mathbf{x})}{\partial x_n} = 0 \Leftrightarrow \sum_{h=1}^{n-1} x_h V = \left(\sum_{h=1}^{n-1} x_h\right)^2.$$

The reaction function of player n is thus given by

$$x_n = \sqrt{\left(\sum_{h=1}^{n-1} x_h\right) V} - \sum_{h=1}^{n-1} x_h.$$

Each player i, $i = 1, , n-1$, anticipates this in the early stage, and thus maximizes

$$\pi_i(\mathbf{x}_{-n}, x_n(\mathbf{x}_{-n})) = \frac{x_i}{\sqrt{\left(\sum_{j=1}^{n-1} x_j\right) V}} \theta_i V - x_i,$$

where $\theta_i = \theta$ for $i = 1$ and $\theta_i = 1$ for $i \neq 1$. The first-order-condition of payoff-maximization by player i is given by

$$\frac{\partial \pi_i(\mathbf{x}_{-n}, x_n(\mathbf{x}_{-n}))}{\partial x_i} = \frac{\sqrt{\left(\sum_{h=1}^{n-1} x_h\right) V} - \frac{1}{2} x_i \sqrt{\frac{V}{\sum_{h=1}^{n-1} x_h}}}{\left(\sum_{h=1}^{n-1} x_h\right) V} \theta_i V - 1 \stackrel{!}{=} 0$$

$$\Leftrightarrow \sqrt{\left(\sum_{h=1}^{n-1} x_h\right) V} - \frac{1}{2} x_i \sqrt{\frac{V}{\sum_{h=1}^{n-1} x_h}} = \frac{\sum_{h=1}^{n-1} x_h}{\theta_i}$$

$$\Leftrightarrow x_i = 2 \sum_{h=1}^{n-1} x_h - 2 \left(\sum_{h=1}^{n-1} x_h\right)^{\frac{3}{2}} \frac{1}{\sqrt{V} \theta_i}.$$

Summing this up for all early moving players results in:

$$\sum_{h=1}^{n-1} x_h = 2(n-1) \sum_{h=1}^{n-1} x_h - 2 \left(\sum_{h=1}^{n-1} x_h\right)^{\frac{3}{2}} \frac{1}{\sqrt{V}} \cdot \sum_{h=1}^{n-1} \frac{1}{\theta_h}$$

$$\Leftrightarrow \sum_{h=1}^{n-1} x_h = 2(n-1) \sum_{h=1}^{n-1} x_h - 2 \left(\sum_{h=1}^{n-1} x_h\right)^{\frac{3}{2}} \frac{1}{V} \cdot \frac{(n-2)\theta + 1}{\theta}$$

$$\Leftrightarrow 2\frac{(n-2)\theta+1}{\theta\sqrt{V}}\sqrt{\sum_{h=1}^{n-1} x_h} = 2n-3$$

$$\Leftrightarrow \sum_{h=1}^{n-1} x_h = \frac{(2n-3)^2\theta^2}{4((n-2)\theta+1)^2}V.$$

From this, we are able to conclude that

$$x_n = \sqrt{\sum_{h=1}^{n-1} x_h V} - \sum_{h=1}^{n-1} x_h = \frac{(2n-3)\theta}{2((n-2)\theta+1)}V - \frac{(2n-3)^2\theta^2}{4((n-2)\theta+1)^2}V$$

$$= \frac{(2n-3)\theta\,(2(n-2)\theta+2-(2n-3)\theta)}{4((n-2)\theta+1)^2}V$$

$$\Leftrightarrow x_n = \frac{(2n-3)(2-\theta)\theta}{4((n-2)\theta+1)^2}V. \tag{B.37}$$

The aggregate effort of all players, including player n, is then given by:

$$\sum_{h=1}^{n} x_h = \frac{(2n-3)^2\theta^2}{4((n-2)\theta+1)^2}V + \frac{(2n-3)(2-\theta)\theta}{4((n-2)\theta+1)^2}V$$

$$= \frac{(2n-3)\theta V}{4((n-2)\theta+1)^2}\left((2n-3)\theta+2-\theta\right)$$

$$= \frac{2(n-2)\theta+2}{\left(2(n-2)\theta+2\right)^2}(2n-3)\theta V$$

$$= \frac{(2n-3)\theta V}{2(n-2)\theta+2}.$$

Then, the payoff of player n is given by

$$\pi_n(\mathbf{x}) = x_n\frac{V-\sum_{h=1}^{n} x_h}{\sum_{h=1}^{n} x_h} = \frac{(2n-3)(2-\theta)\theta}{4((n-2)\theta+1)^2}V\cdot\left(V - \frac{(2n-3)\theta}{2(n-2)\theta+2}V\right)\frac{2(n-2)\theta+2}{(2n-3)\theta V}$$

$$= \frac{(2-\theta)}{2(n-2)\theta+2}\cdot\frac{(2n-4)\theta+2-(2n-3)\theta}{2(n-2)\theta+2}V$$

$$\Leftrightarrow \pi_n^{nL} = \frac{(2-\theta)^2}{4((n-2)\theta+1)^2}V \tag{B.38}$$

If we now consider under which parameter constellation this payoff is larger than the payoff from the simultaneous move game, we find that the payoff function in (7.7) is larger than in (B.38), if $\theta \in \left[\frac{4n-7-\sqrt{16n^2-40n+33}}{2(n-1)}, 0\right] \cup \left[\frac{1}{n-1}, \frac{4n-7+\sqrt{16n^2-40n+33}}{2(n-1)}\right]$. We regard only values of $\theta \geq 1$ and it can be be seen from (B.37) that the late moving player will only choose a positive effort, if $\theta < 2$. For $n \geq 2$, it holds that $1 \geq \frac{1}{n-1}$ and $2 \leq \frac{4n-7+\sqrt{16n^2-40n+33}}{2(n-1)} := \phi(n)$.

The latter can be shown by inserting $n = 2$ into $\phi(n)$ (which yields $\theta \approx 2.56155$) and then showing that ϕ increases in n. To this aim, we can show that $\frac{\partial \phi}{\partial n} > 0$ is equivalent to $(n-1)^2 > 0$, for $n \neq 1$. Therefore, the relevant values of θ lie in $\left[\frac{1}{n-1}, \frac{4n-7+\sqrt{16n^2-40n+33}}{2(n-1)}\right]$, and we can conclude that an underdog has always an at least weakly higher payoff from moving in $t = 1$ than from moving in $t = 2$, if all of her opponents move in $t = 1$.

2. Next, we show that an underdog will stick to her choice of $t = 1$, if the favorite moves in $t = 2$ (at least for $\theta > \frac{2n-3}{n-1}$). To this aim, we consider that $(n-2)$ players stick to $t = 1$, while the players 1 and n move in $t = 2$. In analogy to the analysis behind (B.34), one can show that the aggregate effort by the $(n-2)$ early movers is given by

$$\sum_{h=2}^{n-1} x_h = \left(\left(\frac{\theta(n-3) + (2n-5)}{4\sqrt{\theta(\theta+1)}(n-2)} + \frac{\sqrt{\theta^2(n-1)^2 + 4\theta(n-2)^2 + (2n-5)^2 - 2\theta(n-3)}}{4\sqrt{\theta(\theta+1)}(n-2)}\right)^2 - \frac{\theta}{4(\theta+1)}\right) V.$$

Following the analysis from Chapter 6 for the case of late movers, we can compute the equilibrium effort of both late moving players and their respective payoffs. This enables us to calculate player n's payoff from deviating to a late move, if the favorite moves late, while the remaining players stick to $t = 1$. We skip the formula here, since it is too complex, but simulations show that it never pays for an underdog to join the late moving favorite.

3. Now, consider the case where we allow the favorite to move earlier than the underdogs, i.e., in $t = 0$. Note that, in this case, in subgame-perfect equilibrium the underdogs choose an effort of $x_i = 0$ if $\theta \geq \frac{n}{n-1}$. Since all underdogs have the same valuations, they will make the same decision concerning the question whether or not to choose a strictly positive effort in any equilibrium. This means that, for these levels of θ, it is optimal for all underdogs to choose $x_i = 0$, regardless of whether they choose in $t = 1$ or $t = 2$. Since the function of the favorite's reaction to the aggregate effort of the underdogs in $t = 0$ does not depend on whether the underdogs move simultaneously or sequentially, it is given by (7.16). This allows us to conclude from the analysis of Section 7.3.3 that the underdogs are inactive, if and only if $\theta \geq \frac{n}{n-1}$. For lower values of θ, the comparison becomes rather complex, but simulations show that underdogs will prefer an earlier move to a later move.

■

Proof of Proposition 8. (i) We will compare the payoffs given by the equations (7.20) and (7.12) and find that the latter is smaller for every $\theta > 1$ and every $n \geq 3$.

The case $\theta < \frac{n}{n-1}$:

$$\pi_1^E > \pi_1^L$$

$$\Leftrightarrow \frac{((n-1)\theta - (n-2))^2}{4(n-1)} V > \frac{(2(n-1)\theta - 2n + 3)^2}{4(n-1)^2\theta} V$$

$$\Leftrightarrow \Xi(\theta) := (n-1)^3\theta^3 - 2n(n-1)^2\theta^2 + \left(n^2+4n-8\right)(n-1)\theta - (2n-3)^2 > 0.$$

To prove $\Xi(\theta)$ being positive for $\theta > 1$, one can show that $\Xi(1) > 0$ and $\Xi'(\theta) > 0$ for $\theta > 1$ and $n \geq 3$.

The case $\theta \geq \frac{n}{n-1}$:

We now similarly insert:

$$(\theta - 1)V > \frac{\left(2(n-1)\theta - 2n + 3\right)^2}{4(n-1)^2\theta}V$$

$$\Leftrightarrow 4(n-1)^2\theta^2 - 4(n-1)^2\theta > 4(n-1)^2\theta^2 - 4(n-1)(2n-3)\theta + (2n-3)^2$$
$$\Leftrightarrow 4(n-1)\theta(2n-3-n+1) > (2n-3)^2.$$

Since we deal with the case $\theta \geq \frac{n}{n-1}$, we can write

$$4(n-1)(n-2)\theta \geq 4n^2 - 8n > 4n^2 - 12n + 9 \Leftrightarrow 4n > 9.$$

The latter holds for any $n \geq 3$. So we can conclude that $\pi_1^E > \pi_1^L$ for every $\theta > 1$ and $n \geq 3$.

(ii) **The case $\theta < \frac{n}{n-1}$:** To this aim, we compare the equations (7.20) and (7.6):

$$\pi_1^E > \pi_1^S \Leftrightarrow \frac{\left((n-1)\theta - (n-2)\right)^2}{4(n-1)}V > \frac{\left((n-1)\theta - (n-2)\right)^2}{\left((n-1)\theta + 1\right)^2}\theta V$$

$$\Leftrightarrow (n-1)^2\theta^2 + 2(n-1)\theta + 1 > 4(n-1)\theta \Leftrightarrow \left((n-1)\theta - 1\right)^2 > 0.$$

This is obviously fulfilled for every $\theta > 1$ and $n \geq 3$.

The case $\theta \geq \frac{n}{n-1}$:

$$\pi_1^E > \pi_1^S$$
$$\Leftrightarrow (\theta - 1)V > \frac{\left((n-1)\theta - (n-2)\right)^2}{\left(1 + (n-1)\theta\right)^2}\theta V$$
$$\Leftrightarrow (n-1)^2\theta(\theta - 1) > 1.$$

For $\theta \geq \frac{n}{n-1}$, we can write

$$(n-1)^2\theta(\theta - 1) \geq (n-1)^2\theta\left(\frac{n}{n-1} - \frac{n-1}{n-1}\right) = (n-1)\theta > 1.$$

(iii) Let us now take a look at the condition, under which the payoffs of moving simultaneously and moving later are equal for player 1, i.e., $\pi_1^S = \pi_1^L$. This is equivalent to

$$\frac{\left((n-1)\theta - (n-2)\right)^2}{\left(1 + (n-1)\theta\right)^2}\theta V - \frac{\left(2(n-1)\theta - 2n + 3\right)^2}{4(n-1)^2\theta}V = 0. \tag{B.39}$$

Calculating the roots of (B.39) yields:

$$\theta = \frac{2n-3}{n-1} \vee \theta = \frac{\frac{n}{2} - \frac{9}{8} + \frac{\sqrt{16n^2-40n+33}}{8}}{n-1} \vee \theta = \frac{\frac{n}{2} - \frac{9}{8} - \frac{\sqrt{16n^2-40n+33}}{8}}{n-1}.$$

For $n \geq 3$, the last two roots can be shown to be smaller than 1, such that we find player 1 indifferent between the outcome of subgame L and that of subgame S, only if $\theta = \frac{2n-3}{n-1}$. Due to continuity, it is sufficient to notice that, for $\theta = 1$, it holds that $\pi_1^L < \pi_1^S$ and, for $\theta = 2$ on the other hand, it holds that $\pi_1^L > \pi_1^S$, which proves the result of Proposition 8 (iii). ■

Proof of Proposition 9.

(i) Let us first look at the case where $\theta < \frac{n}{n-1}$. Then $W^E > W^S$ implies

$$\frac{1 + (n-1)^2(\theta - 1)^2}{2(n-1)} > \frac{\left((n-1)\theta - (n-2)\right)^2 \theta + (n-1)}{\left(1 + (n-1)\theta\right)^2},$$

which can be shown to be equivalent to

$$\Leftrightarrow g(\theta) = (n-1)^4\theta^4 - 2(n-1)^4\theta^3 + (n-1)^4\theta^2 - 2(n-1)^2\theta^2 + 2(n-1)^2\theta - n(n-2) > 0.$$

Note that $g(1) < 0$, while $g\left(\frac{n}{n-1}\right) = 0$. Next, let us show that $\theta = \frac{n}{n-1}$ is the only the root of $g(\theta)$ for $\theta \in \left]1, \frac{n}{n-1}\right]$. To this aim, we analyze the first derivative of h, given by

$$\begin{aligned} g'(\theta) &= 4(n-1)^4\theta^3 - 6(n-1)^4\theta^2 + 2(n-1)^4\theta - 4(n-1)^2\theta + 2(n-1)^2 \\ &= (n-1)^4\left(4\theta^3 - 6\theta^2 + 2\theta\right) + (n-1)^2(2 - 4\theta). \end{aligned}$$

Since $g(\theta)$ is a polynomial of the third degree, we can conclude that it has, at most, one local minimum and one local maximum. From $g(1) < 0$ and $g\left(\frac{n}{n-1}\right) = 0$, we can conclude that there can only be a root in $\left]1, \frac{n}{n-1}\right[$, if the only local maximum lies in this interval. But since $g'(0) = 2(n-1)^2 > 0$ and $g'(1) = -2(n-1)^2 < 0$, we can conclude that the local maximum is given by some $\theta \in]0, 1[$. Thus, there is no root in $\left]1, \frac{n}{n-1}\right[$, or put differently, both subgames only reach the same level of welfare if and only if $\theta = \frac{n}{n-1}$. Since $g(1) < 0$, it is obvious that, for $1 < \theta < \frac{n}{n-1}$, it holds that $W^E < W^S$.

Now looking at $\theta \geq \frac{n}{n-1}$ we have that

$$W^E > W^S \Leftrightarrow (\theta - 1) > \frac{\left(n-1)\theta - (n-2)\right)^2 \theta + (n-1)}{\left(1 + (n-1)\theta\right)^2},$$

which can be transformed to

$$\Leftrightarrow \theta^2 - \theta - \frac{n}{(n-1)^2} > 0.$$

Applying the quadratic formula, we find that the latter condition is fulfilled whenever $\theta > 1$ exceeds the threshold value

$$\theta_1 = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{n}{(n-1)^2}} = \frac{1}{2} + \sqrt{\frac{n^2 + 2n + 1}{4(n-1)^2}} = \frac{(n-1) + (n+1)}{2(n-1)} = \frac{n}{n-1}.$$

(ii) As above, we have to split the proof. For $\theta < \frac{n}{n-1}$, we find

$$W^E > W^L$$

$$\Leftrightarrow \frac{1+(n-1)^2(\theta-1)^2}{2(n-1)}V > \frac{2(n-1)\theta^2+(2n-3)(1-2\theta)}{2(n-1)\theta}V$$
$$\Leftrightarrow (n-1)^2\theta^3 - 2n(n-1)\theta^2 + \left(n^2+2n-4\right)\theta - (2n-3) > 0.$$

Since the left-hand-side of the last inequality is a polynomial of the third degree, it has at most three roots. These can be shown to be given by $\theta = \frac{n+1-\sqrt{n^2-6n+13}}{2(n-1)}$, $\theta = 1$ and $\theta = \frac{n+1+\sqrt{n^2-6n+13}}{2(n-1)}$. Only for the latter root does it hold that $\theta > 1$. From the sign of the first coefficient, we can conclude that $W^E > W^L$ holds for $\theta \in \left]\frac{n+1+\sqrt{n^2-6n+13}}{2(n-1)}, \frac{n}{n-1}\right]$, while for $\theta < \frac{n+1+\sqrt{n^2-6n+13}}{2(n-1)}$ we find $W^E < W^L$.

Now for $\theta \geq \frac{n}{n-1}$ we have:

$$\theta - 1 > \frac{2(n-1)\theta^2+(2n-3)(1-2\theta)}{2(n-1)\theta}$$
$$\Leftrightarrow 2(2n-3-n+1)\theta - 2n + 3 > 0$$
$$\Leftrightarrow 2(n-2)\theta - (2n-3) > 0.$$

Since we assumed that $\theta > \frac{n}{n-1}$, we can write

$$2(n-2)\theta - (2n-3) > 2\frac{n^2-2n}{n-1} - (2n-3) = \frac{n-3}{n-1} \geq 0.$$

Thus, we can conclude that, for $\theta > \frac{n}{n-1}$, it always holds that $W^E > W^L$.

(iii)

$$W^S > W^L$$
$$\Leftrightarrow \frac{\left((n-1)\theta-(n-2)\right)^2\theta + (n-1)}{\left(1+(n-1)\theta\right)^2} > \frac{2(n-1)\theta^2+(2n-3)(1-2\theta)}{2(n-1)\theta}$$
$$\Leftrightarrow -2(n-1)^2\theta^3 + (n-1)(5n-9)\theta^2 - 2\left(n^2-5n+5\right)\theta - (2n-3) > 0.$$

The only root with $\theta > 1$ to the function on the left-hand-side of the last inequality is given by $\theta = \frac{2n-3}{n-1}$. Because the left-hand-side of that inequality is positive for $\theta = 1$, we can conclude that $W^S > W^L$ for $\theta < \frac{2n-3}{n-1}$, while, for $\theta > \frac{2n-3}{n-1}$, it has to hold that $W^S < W^L$. ■

B.7 Proofs of Chapter 8

Derivation of (8.4)

$$\frac{\sqrt{(A_{-i}+x_i)V_1}V_i - \frac{x_iV_iV_1}{2\sqrt{(A_{-i}+x_i)V_1}}}{(A_{-i}+x_i)V_1} - 1 = 0$$

$$\Leftrightarrow \frac{\sqrt{A_{-1}V_1}V_i - \frac{x_iV_iV_1}{2\sqrt{A_{-1}V_1}}}{A_{-1}V_1} = 1$$

$$(A_{-1}V_1)^{\frac{3}{2}} = A_{-1}V_1V_i - \frac{1}{2}x_iV_iV_1$$

$$\Leftrightarrow A_{-1}^{\frac{3}{2}}\sqrt{V_1} - A_{-1}V_i = -\frac{1}{2}x_iV_i$$

$$\Leftrightarrow x_i = 2A_{-1} - 2\frac{\sqrt{V_1}}{V_i}(A_{-1})^{\frac{3}{2}}.$$

Proof of Lemma 13. Player i would prefer moving late if

$$\pi_1^{1L} \geq \pi_1^S \Leftrightarrow V_1\left(1 - \frac{2n-3}{2V_1\Omega}\right)^2 \geq V_1\left(1 - \frac{n-1}{V_1\left(\frac{1}{V_1}+\Omega\right)}\right)^2$$

$$\left(V_1 - \frac{2n-3}{2\Omega}\right)^2 \geq \left(V_1 - \frac{n-1}{\left(\frac{1}{V_1}+\Omega\right)}\right)^2$$

$$\Leftrightarrow V_1^2 - (2n-3)\frac{V_1}{\Omega} + \frac{(2n-3)^2}{4\Omega^2} \geq V_1^2 - \frac{2(n-1)V_1}{\left(\frac{1}{V_1}+\Omega\right)} + \frac{(n-1)^2}{\left(\frac{1}{V_1}+\Omega\right)^2}$$

$$\Leftrightarrow -(2n-3)\frac{V_1}{\Omega} + \frac{(2n-3)^2}{4\Omega^2} \geq -\frac{(2n-2)V_1}{\frac{1}{V_1}+\Omega} + \frac{(n-1)^2}{\left(\frac{1}{V_1}+\Omega\right)^2}$$

$$\Leftrightarrow -(2n-3)V_1\Omega\left(\frac{1}{V_1}+\Omega\right)^2 + \frac{(2n-3)^2}{4}\left(\frac{1}{V_1}+\Omega\right)^2$$

$$\geq -(2n-2)V_1\Omega^2\left(\frac{1}{V_1}+\Omega\right) + (n-1)\Omega^2$$

$$\Leftrightarrow -(2n-3)V_1\Omega - 2(2n-3)V_1^2\Omega^2 - (2n-3)V_1^3\Omega^3$$

$$+\frac{(2n-3)^2}{4} + \frac{(2n-3)^2}{2}V_1\Omega + \frac{(2n-3)^2}{4}V_1^2\Omega^2$$

$$\geq -(2n-2)V_1^3\Omega^3 + (n^2-2n+1-2n+2)V_i^2\Omega^2$$

$$\Leftrightarrow V_1^3\Omega^3 + V_1^2\Omega^2\left(\frac{21}{4} - 3n\right) + V_1\Omega\left(2n^2 - 3n + \frac{15}{2}\right) + \left(n^2 - 3n + \frac{9}{4}\right) \geq 0$$

$$\Leftrightarrow (V_1\Omega - (2n-3))\left(V_1\Omega + \frac{9}{8} - \frac{n}{2} + \frac{\sqrt{16n^2-40n+33}}{8}\right)$$

$$\cdot\left(V_1\Omega + \frac{9}{8} - \frac{n}{2} - \frac{\sqrt{16n^2-40n+33}}{8}\right) \geq 0.$$

Now it is easy to see, for which values of $V_1\Omega$ this fulfills with equality. For the roots of this inequality, we find that

$$\frac{n}{2} - \frac{9+\sqrt{16n^2-40n+33}}{8} < 0 < \frac{n}{2} - \frac{9-\sqrt{16n^2-40n+33}}{8} < \frac{2n-3}{2} < 2n-3,$$

whenever $n \geq 2$. For positive values of V_1, player 1 will thus be willing to move late, choosing a non-negative effort if

$$V_1 \geq \frac{2n-3}{\sum_{i=2}^{n} \frac{1}{V_i}}.$$

■

Proof of Corollary 11. We want to show that:

$$x_1^S \geq \sum_{h=2}^{n} x_i^S \Leftrightarrow V_1 \geq \frac{2n-3}{\sum_{i=2}^{n} \frac{1}{V_i}}.$$

To this aim, we start by inserting the respective values for each player in (8.1), and receive:

$$x_1^S \geq \sum_{h=2}^{n} x_i^S \Leftrightarrow \frac{n-1}{\sum_{i=1}^{n} \frac{1}{V_i}} - \frac{(n-1)^2}{\left(\sum_{i=1}^{n} \frac{1}{V_i}\right)^2} \frac{1}{V_1} \geq \frac{(n-1)^2}{\sum_{i=1}^{n} \frac{1}{V_i}} - \frac{(n-1)^2}{\left(\sum_{i=1}^{n} \frac{1}{V_i}\right)^2} \Omega$$

$$\Leftrightarrow 1 - \frac{n-1}{\sum_{h=1}^{n} x_i^S} \frac{1}{V_1} \geq (n-1) - \frac{n-1}{\sum_{i=1}^{n} \frac{1}{V_i}} \Omega$$

$$\frac{1}{V_1} \leq \sum_{i=2}^{n} \frac{1}{V_i} - \frac{n-2}{n-1} \sum_{i=1}^{n} \frac{1}{V_i} = \left(1 - \frac{n-2}{n-1}\right) \sum_{i=2}^{n} \frac{1}{V_i} - \frac{n-2}{n-1} \frac{1}{V_1}$$

$$\Leftrightarrow \frac{2n-3}{n-1} \frac{1}{V_1} \leq \frac{1}{n-1} \sum_{i=2}^{n} \frac{1}{V_i}$$

$$\Leftrightarrow V_1 \geq \frac{2n-3}{\sum_{i=2}^{n} \frac{1}{V_i}}.$$

■

Proof. For the proof, we show that, in the simultaneous move game, $x_i < A_{-1} \Leftrightarrow V_1 < \frac{2n-3}{\sum_{i=2}^{n} \frac{1}{V_i}}$:

$$\frac{n-1}{\Omega + \frac{1}{V_1}} - \left(\frac{n-1}{\Omega + \frac{1}{V_1}}\right)^2 \frac{1}{V_1} < \sum_{i=2}^{n} x_i^S = \frac{(n-1)^2}{\Omega + \frac{1}{V_1}} - \left(\frac{n-1}{\Omega + \frac{1}{V_1}}\right)^2 \Omega$$

$$\Leftrightarrow \left(\frac{n-1}{\Omega + \frac{1}{V_1}}\right)^2 \left(\Omega - \frac{1}{V_1}\right) < (n-2) \frac{n-1}{\Omega + \frac{1}{V_1}}$$

$$\Leftrightarrow (n-1)\left(\Omega - \frac{1}{V_1}\right) < (n-2)\left(\Omega + \frac{1}{V_1}\right) \Leftrightarrow V_1 < \frac{2n-3}{\Omega}.$$

■

B.8 Tables

Early Mover V_3/V_1	Minimum V_2/V_1	Maximum V_2/V_1	Early Mover V_3/V_1	Minimum V_2/V_1	Maximum V_2/V_1
0.164431	0.196790	0.196790	0.35	0.444018	0.658962
0.17	0.203812	0.207567	0.36	0.458668	0.684530
0.18	0.216466	0.227668	0.37	0.473566	0.709698
0.19	0.229172	0.248724	0.38	0.488741	0.734449
0.2	0.241934	0.270212	0.39	0.504232	0.758775
0.21	0.254755	0.293591	0.4	0.520081	0.782673
0.22	0.267638	0.317308	0.41	0.536343	0.806146
0.23	0.280589	0.341793	0.42	0.553083	0.829201
0.24	0.293613	0.366962	0.43	0.570385	0.851847
0.25	0.306715	0.392717	0.44	0.588357	0.874095
0.26	0.319902	0.418952	0.45	0.607143	0.895959
0.27	0.333182	0.445553	0.46	0.626942	0.917452
0.28	0.346563	0.472406	0.47	0.648037	0.938588
0.29	0.360054	0.499396	0.48	0.670871	0.959382
0.3	0.373667	0.526418	0.49	0.696183	0.987984
0.31	0.387412	0.553371	0.5	0.72541	1.00000
0.32	0.401303	0.580170	0.51	0.76215	0.95630
0.33	0.415356	0.606740	0.52	0.82810	0.88522
0.34	0.429588	0.633020	0.52099	0.85650	0.85650

Table B.1: Range when joining the late mover 1 is preferable for 2

B.9 Figures

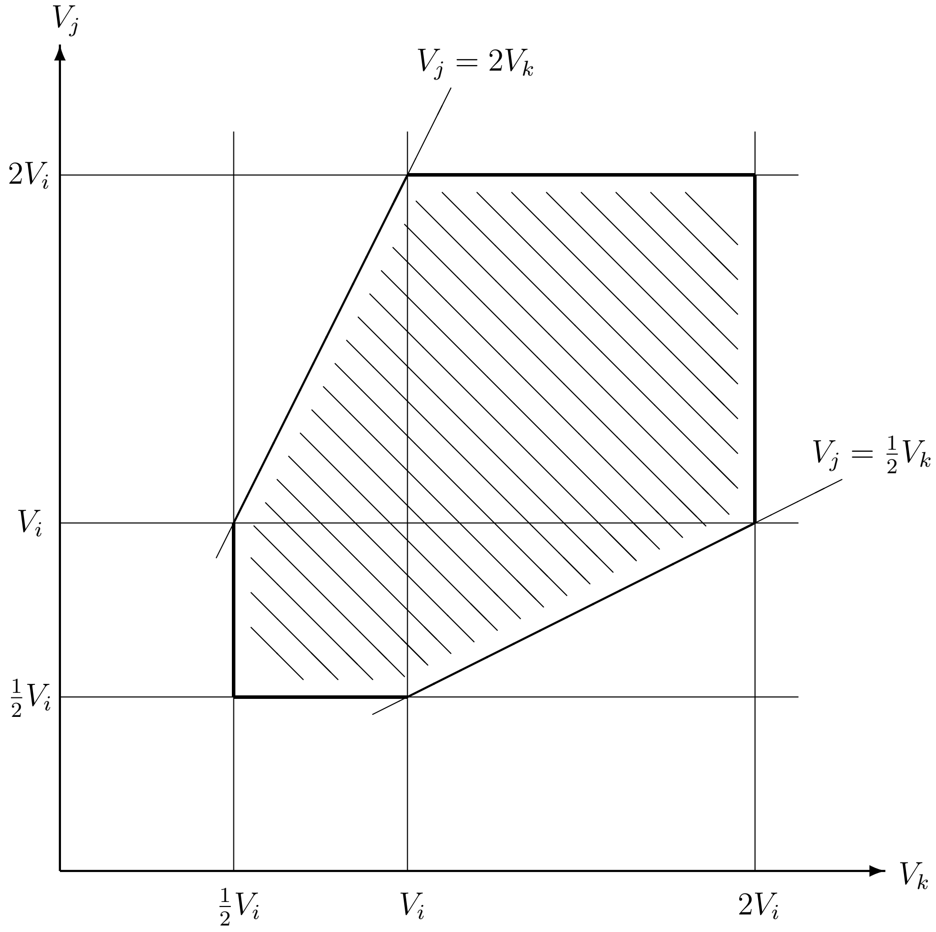

Figure B.1: The figure illustrates the constellations of valuations that are allowed by (6.2). Other constellations than the ones in the hatched area are only considered in the appendix.

Note that (6.2) can be split up into $V_i \geq \frac{V_j}{2}$ and $V_i \geq \frac{V_k}{2}$, which can be transformed for player 1, 2 and 3, respectively, to:

$$\frac{V_i}{2} \leq V_j \leq 2V_i,$$

$$\frac{V_i}{2} \leq V_k \leq 2V_i \text{ and}$$

$$\frac{V_k}{2} \leq V_j \leq 2V_k.$$