EXPERIMENTAL DESIGN FOR QUALITY IMPROVEMENT IN THE PRESENCE OF TIME-TRENDS

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Thesis

Submitted in fulfillment of the

Requirement for the Degree of

"Doktor der Naturwissenschaften"

at University of Dortmund

Dortmund, Germany

Dortmund, 2004

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Date of Oral Examination: 13. October 2004

Abstract

This thesis concentrates on an aspect of experimental design for quality improvement. It is often observed that there is a time trend that influences the experimental data for a given design. For instance, this might be due to a machine part that deteriorates during experimentation. This always leads to data with different distribution under the same experimental setting when observed at a later period of time.

In this dissertation we want to deal with the problems of time trends for a given process. The theoretical and practical aspects of such time trends were taken into consideration. In the theoretical aspect we try to improve methods to get rid of time trends influence by determination of trend resistance using factorial design. Systematic run order in which the estimates for factorial effects of interest are time trends resistant are considered. Here, time trends are modelled as linear and quadratic functions. Several approaches for constructing systematic run order of two levels fractional factorial designs are reviewed. All the reviewed approaches give the same possible number of linear trend resistant contrasts for two levels fractional factorial designs. An attempt to construct trend resistant Plakett Burman designs is also presented.

In the practical aspect, a funnel experiment was used to demonstrate the time trend problem and we also tried to identify the principal determiners of the time trend problem established in the funnel experiment. The results of our experiment show that (i) the run times get considerably larger when the ball bearing has run several times (presence of time trend), (ii) two independent funnels of same type behaved differently, and (iii) the funnel is responsible for the trend in the exemplified experiment.
As another part of the practical work, comparison of the systematic run order with the randomized and standard run orders of a 2^{k-p} fractional factorial designs were studied. The comparison study is divided into two parts. In the first part, half normal plots was used to compare the standard run order with the systematic run

orders to determine which of them is more sensitive to presence of active contrasts. The sensitivity analysis shows that the systematic run order is more sensitive to presence of active contrast than the randomized and standard run orders.
In the second part, a simulation study was used to compare the performance of the run orders under consideration and to compute the critical values needed for determination of the performance criteria. The performance of the standard, randomized, and systematic run orders was measured by taking the probabilities of false rejection and the probabilities of effect detection of active contrasts. Our results show that the randomized run order managed to keep the nominal level, while the systematic run order did not. Additionally, when there were active factors, the systematic run order did not achieve more power than the randomized run order.

In general, when factorial/fractional factorial experiments are conducted over sequence of time for quality improvement, randomizing the run order of the design is an appropriate proceeding. However, when randomization is expensive or not feasible, then systematic run order that are time trends resistant should be used.

Dedication

To

my wife: Dupe,

my son: K.S. Adekeye (jr.),

my daughter: Stella, and

Prof. Dr. J. Kunert for all his help!.

Where there is no wisdom there is no reverence, and where there is no reverence there is no wisdom; where there is no understanding there is no knowledge, and where there is no knowledge there is no understanding.——Ethics of the fathers. Chapter 3

Acknowledgment

"The race is not to the swift neither is the battle to the strong, nor does food come to the wise or wealth to the brillant or favor to the learned: but time and chance happen to them all"— The preacher.

My foremost appreciation goes to my advisor Prof Joachim Kunert, who without him my coming to Dortmund to undergo this training will not have been possible. Also the successful completion of this dissertation will not be possible without his words of courage and untired efforts of guiding me at all the stages of writing this thesis– I have learned a lot from him.

My appreciation also goes to the Graduate School of Production Engineering and Logistics, University of Dortmund for funding this programme and for all support during the course of this programme. Special appreciation to Frau Meni Syrou, the Coordinator of Graduate School of Production Engineering and Logistics, University of Dortmund.

I will like to thank the Dean and the entire members of staff of the faculty of Statistics, University of Dortmund for their assistance during the whole period of this programme.

To my wife and children, I say a very big thank you for their moral, physical and emotional support during the painstaking period of compiling this thesis. Without them I wouldn't be able to complete at the time I had completed.

My appreciation to Dr. Theis for going through the manuscript several times, his useful comments and suggestions has helped to shape the final compilation of this Thesis. To Dr. Martina Erdbrügge, for introducing me to S-Plus language.

To my Friends: Bola Olayiwola, Mr and Mrs Emma Okoruwa, Mr and Mrs Abraham

Abbas, Mr and Mrs Segun Robort, Femi Odutuyo, Sanjay Jadhov, Pastor Sarpong, The Olusanya (Mannheim), Solomon Ige, Amor Messauodi and Ramzi Telmoudi. I say a very big thank you for helping me one time or the other during my study in Dortmund.

My acknowledgment will not be complete if I failed to mention the name of a wonderful friend, Dr. Samson Adebayo, I really appreciate his contribution towards my life.

This list will not be exhaustive if I failed to acknowledge the following: Prof. and Mrs Osanaiye, Prof. Jolayemi, Prof Oyejola, Mr. Yusuf, Pastor Salami, Rev. and Mrs Adetunji, Engineer and Mrs Abolude, Chief M.I. Odeniyi and many others that space will not allow me to mention for their support and words of encouragement when I decided to come for this programme.

I appreciate the prayer and concern of my parents, the entire E.O.Adekeye family, the entire D.B. Bamigboye family, members of De-Motivator, and my youthful friend Mr and Mrs Oyetoyan.

Above all, I thank the almighty God for giving me the grace, courage, strength, and for seeing me through up-till this stage of my life. To him be all glory, honor and adoration.

University of Dortmund, Germany K. S. Adekeye
October 2004

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Chapter 1

Introduction

The competitiveness in today market place has caused many companies to examine how they can improve the quality of their products in order to maintain or increase their market share. Also, the focus on quality improvement for some past decades is shifting into the design and development phase of a product. Therefore, frequently heard expressions such as "Quality by design" and "Do it right the first time" express the changing philosophy that quality should be built into the product at the design stage.

An important application of statistical methods to industrial research is the design and analysis of experiment in connection with the improvement of manufacturing processes. The objective of any statistical investigation is to improve the quality of a product or to produce the product more economically without losing its desired quality. Design of experiments can often speed the improvement or optimization process in major steps. This applies to most types of industrial research where the research may involve the examination of many different factors and the problem is how best to design the experiments in order to estimate the effects of these factors. In many of the physical systems that engineers work on there are usually many factors potentially affecting the response variable. Therefore, highly fractionated two level factorial designs are employed in the industry as screening designs to identify which of the many experimental factors are important to the response variable. Screening experiments are used to sift through a set of factors to identify those that have impacts on the response. Effects that are large enough to be of practical importance will be called active effects, and factors that are involved in one or more active effects will be called active factors. The primary goals of a screening experiment are to (i) identify the active factors and (ii) to provide a simple model that captures the essential features of the relationship between these active factors and the response. Three empirical principles for most screening applications as given by Miller and Sitter (2001) are effect sparsity, effect hierarchy and effect heredity. These three principles justify the use of small fractions of factorial designs to determine how many factors are apt to be active. Assuming that only a relatively small portion of the factors in an experiment will be active is synonymous with the factor sparsity principle. In the factor hierarchy principle, the assumption made is based on the claim that main effects are more likely to be active than two factor interactions, and two factor interactions in turn are more likely to be active than three or more factor interactions. For the effect heredity, the assumption is based on the statement that an interaction effect can only be active if at least one of the factors involved has an active main effect.

1.1 Motivation and Coverage of Study

When engineers perform factorial experiments, they usually have one machine or a pilot plant and they are therefore compelled to conduct their experimental runs in sequence. However, when runs are made in time sequence, each observation may be affected by a trend which is a function of time or position. Engineers are therefore faced with the problems of time-trends.

The response from a factorial experiment carried out in a time sequence may be affected by uncontrollable variables that are highly correlated with the time in which they occur (Bailey, Cheng, and Kipnis;1992). The usual advice given to experimenters is that the order of runs should be randomized before the experiment is performed. However, any particular random run order may or may not be adequate

and hence randomization may lead to a run order whereby the estimates of factor effects of interest are adversely affected by the presence of trend. Therefore, a systematic run order in the presence of time-trend may improve the efficiency with which factor effects are estimated. It is therefore pertinent to consider systematic run orders in which the estimates for factor effects of interest are trend resistant.
On the other hand, there are authors who do not even accept that randomizing the run order of a factorial design is a useful precaution against time trends, see e.g. Grima, Tort-Martorell and de León (2003). It is not clear that the randomization argument really works for saturated fractional factorial designs: each design with n runs has $n-1$ contrasts that may become influenced by the time trend. Note that there are only $(n-1)n/2$ possible run orders for each column. So there must always be some columns of the design that are heavily influenced by the time trend.

The coverage of this study on one part is to choose those sets of ordered contrast that provide efficient estimation of all desired effects and interactions (that is, a trend resistant design) and on another part is to compare the performance of the trend resistant run order with the randomized run orders of a fractional factorial design. Hence, we will focus on construction of systematic fractional factorial run order in which the estimates of the main effects of interest are time-trend resistant for linear and quadratic (that is, first and second order trend resistant design). Extension of the time-trend resistant design for the two factor interactions (designs that are time-trend resistant to both the main effects and the two factor interactions) will also be considered. Run orders with small number of factor level changes that at the same time provide good protection against biased estimates of the main and two factor interaction effects resulting from a polynomial time-trend will also be studied. Further, the time-trend problem will be demonstrated using the funnel experiment and factors that produce the time-trend in the experiment will be identified. The results obtained from the exemplified experiment will be used to compare the performance of the constructed systematic run order with the randomized and standard run order of a fractional factorial design via simulation studies.
In Chapter 2, different approaches for constructing a time-trend resistant design

are reviewed. In addition to the two levels fractional factorial time-trend resistant design, a time-trend resistant Plackett Burman design is also presented. The exemplified experiment is discussed in Chapter 3 as a practical case to demonstrate the time-trend problem. Chapter 4 deals with comparison of the constructed time-trend resistant design with the standard and randomized run orders of an unreplicated two-levels fractional factorial design. Finally, in Chapter 5 the conclusion and discussion of results are presented.

1.2 Review of literature

The basic idea of trend resistant designs is that certain of the ordered contrasts appearing in the system are orthogonal to linear and to quadratic trends. For an experimental design with k factors and n runs, the time-trend resistance model for the response of interest for the experiment can be represented in the form

$$y_i = \mu + f'(x_i)\alpha + h'(t_i)\beta + \varepsilon_i, \tag{1.1}$$

where, y_i, $1 \leq i \leq n$, represent the observed response from an experiment, μ is the grand mean, $f(x_i)$ is a $k \times 1$ vector representing the settings of main effects and interaction effects in the model at design point x_i, $h(t_i)$ is a $q \times 1$ vector of the polynomial expansion for the time-trend expressed as a function of time t, α is a $k \times 1$ vector of parameters of interest, β is a $q \times 1$ vector of parameters of the polynomial time-trend, and ε_i, $1 \leq i \leq n$, are the random errors which are assumed to be independently normal with mean zero and constant variance σ^2. Equation (1.1) can be re-written in a matrix form as:

$$Y = 1_n\mu + F\alpha + H\beta + \varepsilon, \tag{1.2}$$

where μ is as earlier defined, F is the $n \times k$ design matrix with i^{th} row consisting of the $f'(x_i)$, H is the $n \times q$ matrix with i^{th} row consisting of the $h'(t_i)$, Y is a column vector of n observations, and ε is the vector of errors. A design is said to be time-trend resistant if the contrast effects are orthogonal to the polynomial trend components. In Equation (1.2), if $F'H = 0$, then the design matrix F will be said to be time-trend resistant.

There has been a steady interest in the design of experiments in the presence of time-trends. Cox (1951, 1952) initiated the study of systematic design for the efficient estimation of treatment effects in the presence of a smooth polynomial trend. Hill (1960) presented experimental designs which allow adjustment for time trends. Draper and Stoneman (1968) gave good run orders for 2^3 factorial and 2^{k-p}, $k-p=3$, fractional factorial designs with eight runs when only the main effects are of interest. They also considered the number of factor level changes for each of the designs and in order to measure the correlation between a given effect contrasts and the row numbers in the design matrix, they used the statistic "time count". This statistic is employed later in this study to determine the resistant property of run orders of two levels designs. Phillips (1964, 1968) considered the use of magic squares, magic rectangles and similar concepts to construct some factorial and other designs orthogonal to linear and occasionally quadratic trends. Dickinson (1974) found sequences with minimal factor level changes for 2^4 and 2^5 experiments. He used the run order with the maximum correlation of the main effects with a linear time-trend as an evaluation criterion for finding run orders with minimum number of factor level changes. Dickinson showed that the statistic time count is simply the numerator in the ordinary Pearson product moment correlation between the given effect and the row number, and he presented the denominator as the quantity $\sqrt{N\left[(N^2-1)/12\right]}$. Therefore, the Pearson product moment correlation coefficient between a given effect contrasts and time is $\rho_d = \frac{TC}{\sqrt{N[(N^2-1)/12]}}$, where N is the number of runs in the design and TC is the statistic time count. Therefore, a time count of zero implies a zero value of ρ_d. Joiner and Campbell (1976) gave some specific examples as to when trend effects can occur in sequential experiments. Cheng and Jacroux (1988) constructed run orders of 2^k and 2^{k-p} fractional factorial designs in which the estimates of the main effects and the two-factor interaction effects are orthogonal to some polynomial trends. John (1990) treated 2^k and 3^k factorial designs by using the foldover principle. Cheng (1990) concentrated on the ordering of the treatment combination of 2^k factorial designs by using the foldover method. Cheng and Steinberg (1991) considered the problem of finding trend robust run orders when the time effects are modelled via time series model. Bailey, Cheng, and Kipnis (1992) extended and unified the work of Cheng and Jacroux (1988), Coster and Cheng (1988), and Cheng (1990) to general symmetric and assymmetrical factorial designs. Atkinson and Donev (1996) provided a general solution to the design of experiment in the presence of time-trends. Tack and Vandebroek (2001) proposed an optimality criterion that strikes a balance between cost-efficiency and trend resistant designs. Some of the aforementioned works are reviewed extensively later in this study.

Chapter 2

Construction of Trend Resistant Run Orders of Two-Level Designs: A Review

In this chapter, various methods of constructing some contrast sequences in which the runs are orthogonal to at least a linear trend are reviewed. The chapter is divided into two major sections. The first section deals with the construction of two levels fractional factorial designs that are time-trend resistant to the main effects and all or some two factor interaction effects. The second section deals with Plackett Burman saturated designs that are time-trend resistant. For the two level designs under consideration, we used the statistic time count to measure the degree of time-trend for an effect column of the model design matrix of a given design. This is taken as the inner product of the effect column and the row number, and it measures the correlation between a given effect and time. The definition of time count as used in this study is given below.

Definition: Time-count

For an experimental design with N runs and k factors each at two levels, let $\mathrm{X} = [\ \mathrm{x}_1, \mathrm{x}_2, ...,\mathrm{x}_N]'$, where $\mathrm{x}_1, \mathrm{x}_2, ...,\mathrm{x}_N$ represent the row vectors with x_{ji} representing the vector of the standardized setting of factor i in the j^{th} observation of the design matrix, X. Here, $x_{ji} \in \{1, -1\}$ depending on whether in the j^{th} observation, factor i is at high or low level, respectively. Then we define time count as

$$TC(r)_i = \sum_{j}^{N} \mathbf{x_{ji}} \bullet j^r, \tag{2.1}$$

$i = 1, 2, ..., 2^k - 1$; $j = 1, 2, ..., N$; r is the degree of resistance and it can take values from 1 to $k - 1$.

If $TC(r)_i$ in Equation (2.1) equals zero, it implies that contrast i is r time-trend resistant. A design with $TC(r)_1 = TC(r)_2 = ... = TC(r)_{2^k-1} = 0$ is therefore, a trend resistant design of order r.

Another feature of interest which is taken into consideration in constructing time-trend resistant designs in the literature is the number of times the factor level changes in the setting. This is because some factors may be more difficult to change than others, more costly, or may require more time to return to a controlled state. It should be noted, that when all the factor levels are equally expensive to change, minimizing the cost of level changes is the same as minimizing the total number of level changes (Cheng 1985). Therefore, a time-trend resistant design with minimum total number of factor level changes is desirable.

2.1 Two level fractional factorial designs

For any number of variables in a full factorial experiment performed at two setting levels, say high and low, the number of trials required equals the number of setting levels raised to the power of the number of variables investigated. For instance, a four variables orthogonal design at high and low levels setting will require 2^4 trials in which each factor will have 8 trials at high setting and 8 trials at low setting. It is possible that the experimenter will only need four trials at both settings in order to determine if a specific variable has a major effect on the response. Fractional factorial designs allow experimenters to remove some of the trials required by the full factorial designs while the orthogonality of the designs are maintained.

Before discussing the construction methods for trend resistant factorial designs, we give a brief description of some notations used in this study. Suppose there are k factors with each of the factors at two levels, the runs comprising the experimental design are conveniently set out in either of two notations. In the first notation, the factors are identified by capital letters and their two levels by the presence or absence of the corresponding lower case letters. When all the factors are at their "low" level a ϕ is used. In the second notation, the factors are identified also by capital letters but the two levels of each factor are denoted by either a minus (-) or by a plus (+) sign, or by minus and plus one (± 1). The list of experimental runs is called the design matrix. For example, consider a two factors experiment with each of the factor at two levels. The four experimental runs of a 2^2 factorial design using the notations described above can be represented as follows:

$$\begin{bmatrix} run & 1 \\ run & 2 \\ run & 3 \\ run & 4 \end{bmatrix} = \begin{bmatrix} - & - \\ + & - \\ - & + \\ + & + \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ +1 & -1 \\ -1 & +1 \\ +1 & +1 \end{bmatrix} = \begin{bmatrix} \phi \\ a \\ b \\ ab \end{bmatrix}$$

In general, let the two levels be identify with the elements of $GF(2)$, the Galois field of order 2. Then the set of all the 2^k level combinations can conveniently be represented by the k- dimensional linear space $\{x : x = (x_1, x_2, ..., x_k)$, $x_i = \{-1, +1\}\}$ over $GF(2)$, denoted by V^k. A 2^{k-p} fractional factorial designs can be defined as a $(k-p)$ dimensional subspace of V^k, or equivalently as the solution set of p simultaneous linear equations $\mathbf{xa_i} = \mathbf{0}$, where $\mathbf{a_i}$, $1 \leq i \leq p$, are linearly independent vectors in V^k, written as column vector. A Galois field usually denoted as GF (s), is a finite field with s^n elements where s is a prime integer and n is the degree of polynomial. The s factor levels are usually denoted by $0, 1, ..., s-1$, with 0 the additive identity and 1 the multiplicative identity in $GF(s)$.

The relationship that generates a 2^{k-p} fractional factorial design can be written as: *name of the last(k) factor= product of names of the first (k-p) factors.*

This relationship is called the *generator* of the design. It specifies how the column of signs are made up for the last factor. For each defining relation, the number of letters in the right hand side of the expression that describes the relationship is called *length* of the relation. If there is one or more defining relation (s) for a design, then the length of the shortest defining relation is called *resolution* of the design. For example, the *generator* for a 2^{4-1} fractional factorial design is $D = ABC$, the *defining relation* is $I = ABCD$, the length of the shortest defining relation is 3. Therefore, the resolution of the design is III. There is a lot of literature in this area (see e.g., Box and Hunter, part I & II (1961), Vardeman and Jobe (1999)).

The construction procedure for a 2^{k-p} fractional factorial design used in this study is hereby presented. Let $N = 2^{k-p}$ be the number of runs in the design, k the number of factors in the experiment, and p the fractionation sought. The procedure follows the stepwise sequence below:

(i) Define the generating equation (Defining relation)

(ii) Let the entries of the first column follow the sequence $(N/2)\{+\}$, $(N/2)\{-\}$.

(iii) Let the entries of the second column follow the sequence $(N/2^2)\{+\}, (N/2^2)\{-\}, (N/2^2)\{+\}, (N/2^2)\{-\}$.

(iv) Let the entries of the third column follow the sequence $(N/2^3)\{+\}, (N/2^3)\{-\}, N/2^3\{+\}, ..., N/2^3\{-\}$.

(v) Continue until the $(k-p)^{th}$ column is obtained. This column entries should follow the sequence $(N/2^{k-p})\{+\}, (N/2^{k-p})\{-\}, (N/2^{k-p})\{+\}, ..., (N/2^{k-p})\{-\}$.

(vi) Obtain the entries for the k^{th} column by taking the product of the first $k - p$ columns as stated in the defining relation in step (i).

(vii) Obtain the entries for two or more factor interaction $(2^{k-p}-1-(k-p))$ columns using the columnwise multiplicative rule (Finney 1945) on the columns obtained in steps (ii) to (vi).

It should be noted that for each of the 1, 2, ..., $(k-p)$ columns and the columns obtained in step (vi), there should be N entries. The entries for the two-factor interaction columns are found by pairwise multiplication of the entries for the main effect contrasts, while the higher (more than two) order factor interaction columns are obtained by using the columnwise multiplication of the entries for the main effect contrasts.

2.1.1 Trend resistant two level fractional factorial designs

There are various approaches that have been proposed for the construction of trend resistant two levels factorial designs. This section reviews methods for constructing two-levels run order fractional factorial designs that are robust against time trend given by Daniel and Wilcoxon (1966), Coster and Cheng (1988), Cheng and Jacroux (1988), John (1990), Cheng and Steinberg (1991), and Jacroux and Ray (1991).
Daniel and Wilcoxon (1966) used the linear Chebychev polynomial coefficient to develop plans that are trend resistant for two-levels factorial experiments, Coster and Cheng (1988) proposed a generalized fold over method from a sequence of generators, John (1990) generalized the results of Daniel and Wilcoxon and connected it with class of foldover designs. An algorithm for the reverse foldover approach was presented by Cheng and Steinberg (1991), and the Kronecker product was proposed by Jacroux and Ray (1991). The Daniel and Wilcoxon approach is hereafter referred to as DW approach, KP for Kronecker product, while we retain the term foldover, reverse foldover, and generalized foldover as originally used.
Following the reverse foldover algorithm, we will present a modified version of the reverse foldover algorithm to achieve a factorial design that is robust against linear trend with minimum cost. Furthermore, an easy to implement algorithm will be presented for each of the reviewed approaches.

Daniel and Wilcoxon (DW) approach

Daniel and Wilcoxon approach is based on reassigning the contrast settings of higher (more than two) order interactions of the standard run order that are orthogonal to time-trend coefficient of the order sought to represent the main effect contrasts (columns of the model design matrix). They used the Chebyshev polynomial coefficient to represent the j in Equation (2.1). Their procedure for constructing time-trend resistant fractional factorial designs can be summarized in the following steps:

(i) Write down the main effects entries to get the main effects design matrix.

(ii) Complete the model design matrix for all the contrasts in a canonical order using the pairwise multiplicative rule for the two factor interactions and the columnwise multiplicative rule for more than two factor interactions.

(iii) Determine the time count for each contrast using Equation (2.1).

(iv) Remove the columns in the model design matrix with non-zero time count.

(v) If the time count for the columns with the main effects are non-zero, then reassign the columns with zero time count to represent the main effect columns.

(vi) Use the new assignment for the main effect columns in (v) to complete the new model design matrix as in step (ii).

The model design matrix obtained in (vi) will be trend resistant for at least the main effect contrasts and probably some two factor interaction contrasts. It should be noted that there might be situations when the number of columns with zero time count will be less than the number of main effect contrasts in the experiment. In such situation, use of prior knowledge about the factors by the engineers (experimenter) should be employed so that assignment will be done according to order of importance of the main effects. It should be noted that there is a great risk that the resulting design on assigning as above might lead to designs whereby the main effects are aliased with two factor interaction effects (resolution III design). Cheng and Jacroux (1988) approach improve on Daniel and Wilcoxon's work as described below.

Cheng and Jacroux approach

Cheng and Jacroux's (1988) idea is based on designating some high-order interaction contrasts in the standard ordering as the main effects. This is the same as Daniel and Wilcoxon's (1966) approach. They show that in the standard order of a two level complete k factors design, any h- factor interactions is orthogonal to a $h-1$ degree polynomial trend, where h represents two or more factor interactions. Therefore, designating the setting of some high order interaction contrasts of the standard order as the main effects and some two factor interaction contrasts can give a run order of factorial or fractional factorial experiment in which the main effects and some two factor interaction contrasts are orthogonal to high degree polynomial trends. Cheng and Jacroux give a number of theorems to summarize their results with accomplish algorithms to each theorem for construction of both the main effects and two-factor interactions trend resistant factorial/fractional factorial designs. In this study, we focus on Cheng and Jacroux theorems for constructing trend resistant 2^{k-p} fractional factorial designs. These are described below.
Consider a 2^{k-p} fractional factorial two levels design where k is the number of factors in the experiment and p the fractionation sought. Let $q = k - p$ and $d = k - 3p$, then

(i) If $p = 1$, $k \geq 7$ is odd, there exists a run order of a 2^{k-p} fractional factorial design defined by $I = -A_1A_2....A_k$ such that all the main effect contrasts are at least $(k-5)$-trend resistant and all two factor interaction contrasts are at least linear trend resistant.

(ii) If $p = 1$ and $k \geq 8$ is even, there exists a run order of a 2^{k-p} fractional factorial design defined by $I = -A_1A_2....A_{k-3}$ such that all the main effect contrasts are at least $(k-5)$-trend resistant and all two factor interaction contrasts are at least linear trend resistant.

(iii) If $p = 2$, $q \geq 8$ is even, and $d \geq 4$, then there exists a 2^{k-p} fractional factorial design whose runs can be ordered such that all the main effect contrasts are at least $(d-1)$-trend resistant and all the two factor interaction contrasts are at least linear trend resistant.

(iv) If $p \geq 3$, $q \geq 8$ is even, and $d \geq 2$, then there exists a 2^{k-p} fractional factorial design whose runs can be ordered such that all the main effect contrasts are at least $(d+1)$-trend resistant and all the two factor interaction contrasts are at least linear trend resistant.

(iv) If $p \geq 3$, $q \geq 7$ is odd, and $d \geq 3$, then there exists a 2^{k-p} fractional factorial design whose runs can be ordered such that all the main effect contrasts are at least (d)-trend resistant and all the two factor interaction contrasts are at least linear trend resistant.

It should be noted that the defining relation $I = A_1A_2....A_k$ will give the same results as $I = -A_1A_2....A_k$. That is, if the defining relation is $I = A_1A_2....A_k$ or $I = -A_1A_2....A_k$, the results in (i) and (ii) which depend on the generating equation used will still give all the main effect contrasts to be at least $(k-5)$-trend resistant and all the two factor interaction contrasts to be at least linear trend free as claimed by Cheng and Jacroux (1988).

Foldover approach

The procedure proposed by John (1990) depends on the foldover principle which in turn depends on the sequence of generators chosen and the order in which they appear. A fold over design is a design with run order in which the complementary points are run in the same order as the original points. Two design points are said to be complementary if one of the design points is obtained by changing the levels of all the factors in the second design point. For example, in a 2^4 factorial design with factors designated as A, B, C, and D, the design points a and bcd are complementary to each other and are, therefore, a foldover pair. The foldover approach for a 2^k design, say H, with Q runs, say, involve the addition of a new 2^k design, say H', which also has Q runs in which the signs of some or all the factor columns of H

are reversed. Therefore, if H^* is the foldover design for design H with the sign of all the factors in H reversed, then

$$H^* = \left[\begin{array}{c} H \\ \hline H' \end{array}\right] = \left[\begin{array}{c} H \\ \hline -H \end{array}\right] = \left[\begin{array}{cccc} H_{11}, & H_{12}, & \ldots, & H_{1k} \\ \vdots & \vdots & & \vdots \\ H_{Q1}, & H_{Q2}, & \ldots, & H_{Qk} \\ \hline -H_{11}, & -H_{12}, & \ldots, & -H_{1k} \\ \vdots & \vdots & & \vdots \\ -H_{Q1} & ,-H_{Q2} & ,\ldots, & -H_{Qk} \end{array}\right].$$

From above, it is seen that a foldover design H^* with $2Q$ runs is a design that consists of Q foldover pairs. In a foldover approach, if we start with a design with Q runs and fold it over, we get a design with $2Q$ runs in which all the main effect factors are at least linear trend resistant. The procedure for constructing a trend resistant design using the foldover approach is given as follows:

Let H^* be a sequence of 2Q points from a 2^k factorial design, where Q is the number of runs in the design. Further, let H^* be a partition into two sequences as above, where the first subsequence of the $2Q$ points is denoted by H and the second subsequence of the $2Q$ points is denoted by H'. In H^*, each factor in both H and H' appears at its high level (+1) and low level (-1) exactly $Q/2$ times. It should be noted that the partition of H^* depends on the sequence of generators used to generate the initial design H. Consider the first point (run) of H to be ϕ (that is, all the factors are at their low level). For a particular factor say A, of the design H, let $n_A(-1)$ and $n_A(+1)$ be the vectors of the run numbers in H when factor A is at its low and high levels, respectively. Further, let $S_A(-1)$ be the sum of the vector of the run numbers in H in which A is at its low level (that is, $(n_A(-1))^T1$) and $S_A(+1)$ be the sum of the run numbers in H in which A is at its high level (that is $(n_A(+1))^T1$). Also let $n'_A(-1)$ and $n'_A(+1)$ denote the vector of the run numbers of factor A in H' when factor A is at its low and high levels, respectively. We define $S'_A(-1)$ and $S'_A(+1)$ to be the sum of the vectors of the run numbers in H' when factor A is at its low and high levels, respectively. That is, $S'_A(-1) = (n'_A(-1))^T1$

and $S'_A(+1) = (n'_A(+1))^T 1$. Then the main effect of factor A will be linear trend resistant if $S_A(-1) + S'_A(-1) = S_A(+1) + S'_A(+1)$. This holds if the level of factor A in the $(Q+i)^{th}$ run is the opposite of its level in the i^{th} run.

For example, using the foldover approach to construct a 2^3 factorial design such that all the three main effect factors are at least linear trend resistant. We start by constructing a half fraction of the desired design (that is, a 2^{3-1} design). This design consist of 4 runs with defining relation $I = ABC$. Thus H is given by

$$H = \begin{bmatrix} -1 & -1 & +1 \\ +1 & -1 & -1 \\ -1 & +1 & -1 \\ +1 & +1 & +1 \end{bmatrix}.$$

Folding over of H gives another design H'. If we combine the two designs H and H' as described earlier, then we have $H^* = 2^3$ design. That is,

$$H^* = \begin{bmatrix} H \\ H' \end{bmatrix} = \begin{bmatrix} -1 & -1 & +1 \\ +1 & -1 & -1 \\ -1 & +1 & -1 \\ +1 & +1 & +1 \\ +1 & +1 & -1 \\ -1 & +1 & +1 \\ +1 & -1 & +1 \\ -1 & -1 & -1 \end{bmatrix}.$$

To show that the three main effect factors (columns) in H^* are linear trend resistant, we proceed as follows: For factor A (first column in H^*), $n_A(-1) = 1, 3$; $n_A(+1) = 2, 4$; $n'_A(-1) = 6, 8$; and $n'_A(+1) = 5, 7$. These imply $S_A(-1) = 4$; $S_A(+1) = 6$; $S'_A(+1) = 12$; and $S'_A(-1) = 14$. Thus, $S_A(-1) + S'_A(-1) = S_A(+1) + S'_A(+1) = 18$. Therefore, factor A in H^* is linear trend resistant. Similarly, for factor B (second column in H^*), $n_B(-1) = 1, 2$; $n_B(+1) = 3, 4$; $n'_B(-1) = 7, 8$; and $n'_B(+1) = 5, 6$. These imply $S_B(-1) = 3$, $S_B(+1) = 7$, $S'_B(+1) = 11$, and $S'_B(-1) = 15$. Thus, $S_B(-1) + S'_B(-1) = S_B(+1) + S'_B(+1) = 18$. Therefore, factor

B in H^* is linear trend resistant. For factor C (third column in H^*), $n_C(-1) = 2, 3$; $n_C(+1) = 1, 4$; $n'_C(-1) = 5, 8$; and $n_C(+1) = 6, 7$. These imply $S_C(-1) = S_C(+1) = 5$, and $S'_C(+1) = S'_C(-1) = 13$. Thus, $S_C(-1) + S'_C(-1) = S_C(+1) + S'_C(+1) = 18$. Therefore, factor C in H^* is also linear trend resistant.
Suppose we want to have the first run in H^* to be ϕ, then we have to use another defining relation $I = -ABC$ to construct H in order to have a foldover design that will also be linear trend resistant for all the three main effect columns.

In what follows, we present an algorithm for the foldover approach for constructing trend resistant designs. This algorithm is based on the principle of the reverse foldover algorithm given by Cheng and Steinberg (1991). The steps for the algorithm are as follows:

(i) Assign the longest linear sequence of letters in the design as the first generator. This should contain k letters.

(ii) Select the remaining generators by choosing at each step the longest linear sequence of letters in the design that preserves a generator set. These can be any of the $\binom{k}{k-1}$ $(k-1)$-factor interactions. Ties may be broken arbitrarily.

(iii) start with ϕ as the first run in the design.

(iv) After writing down the first 2^v runs, generate the next 2^v runs by writing down the first 2^v runs and thereafter multiply each of the new runs by the $(v+1)^{st}$ generator, where $0 \leq v < k$.

(v) Repeat step (iv) until the entire design has been generated.

To illustrate the foldover algorithm, we consider a 2^4 design. Let the factors for the design be represented as A, B, C, and D. Following the steps of the algorithm above we have the following:

→ Assign *abcd* as the first generator.

→ The remaining three generators should be any of the $\binom{4}{3}$ 3-factor interactions. If we choose *abc*, *abd*, and *acd* as the remaining generators, then we proceed as follows:

→ Let the first run be ϕ.

→ The first two runs will be

$$\phi,\ abcd \tag{2.2}$$

→ Multiplying (2.2) by the second generator (that is, *abc*) gives the run order

$$\phi,\ abcd,\ abc,\ d \tag{2.3}$$

→ Multiplying (2.3) by the third generator (that is, *abd*) gives the run order

$$\phi,\ abcd,\ abc,\ d,\ abd,\ c,\ cd,\ ab. \tag{2.4}$$

→ Multiplying (2.4) by the fourth generator (that is, *acd*) gives the run order

$$\phi,\ abcd,\ abc,\ d,\ abd,\ c,\ cd,\ ab,\ acd,\ b,\ bd,\ ac,\ bc,\ ad,\ a,\ bcd. \tag{2.5}$$

The design generated by run order (2.5) has the following properties. For factor A (first column), $S_A(-1) = S_A(+1) = 18$, and $S'_A(+1) = S'_A(-1) = 50$. Thus, $S_A(-1) + S'_A(-1) = S_A(+1) + S'_A(+1) = 68$. Similarly for factor B (second column), $S_B(-1) = S_B(+1) = 18$, and $S'_B(+1) = S'_B(-1) = 50$. Thus, $S_B(-1) + S'_B(-1) = S_B(+1) + S'_B(+1) = 68$. Also for the third and fourth columns of the design generated by run order (2.5), that is, factors C and factor D, $S_C(-1)+S'_C(-1) = S_C(+1)+S'_C(+1) = S_D(-1)+S'_D(-1) = S_D(+1)+S'_D(+1) = 68$. Therefore, all the main effect factors of the design generated by run order (2.5) are linear trend resistant. If we use Cheng and Steinberg (1991) trend resistant criteria, the generator sequence $\{abcd, abc, abd, acd\}$ that produce run order (2.5) have the letters a, b, c, and d appearing at least 3 times. Also the linear and quadratic time counts (r=1 and r= 2 in Equation (2.1)) for each of the factors in the design equals zero. Thus, all the main effect factors in the design generated by run order (2.5) are both linear and quadratic trend resistant. Therefore, our foldover algorithm yields a design that is at least linear trend resistant for all the main effects.

Reverse foldover approach

The reverse foldover approach is another equivalent way to apply the foldover approach. It also depends heavily on the generator sequence used. The procedure is described as follows:
Given a generator set, $t_1, t_2, ..., t_{k-p}$ for a 2^{k-p} fractional factorial design, where a set of $k-p$ runs is a generator set if no product of some or all the runs in the set equals ϕ. Then the runs in the 2^{k-p} fractional factorial design will be generated by starting with ϕ, followed by t_1. After 2^s factor combinations, $s < (k-p)$, have been generated, they are followed by their products with t_{s+1} in reverse order. That is, the run order generated by the reverse foldover approach involve taking the products of all the points in the generator set sequentially in a reverse order. For example, suppose the generator set for a complete 2^2 design is $\{a, b\}$. Then, the run order generated by reverse foldover approach will be $\{\phi, a, ab, b\}$. This run order is linear trend resistant for the first factor in the design.
Cheng and Steinberg (1991) presented an algorithm for the reverse foldover approach for constructing two level factorial design in which almost all the factors are robust against both linear and quadratic trend with maximum level changes. Following the reverse foldover algorithm of Cheng and Steinberg (1991), we present a modified version of the reverse foldover algorithm for constructing trend resistant 2^k designs. A modified version of the algorithm is presented because the run orders generated using Cheng and Steinberg's version give designs with maximum number of factor level changes but not linear trend resistant for all the main effects (see Cheng and Steinberg 1991). For instance, for a 2^4 design using the reverse foldover algorithm, the sequence of generators will be $\{abcd, abc, abd, acd\}$. The design obtained with this generator sequence has a linear Time-Count (r=1 in Equation 2.1) of 8,0,0,0 for the main effect factors A, B, C, and D, respectively. Hence, the obtained design is not linear trend resistant for all the main effect factors. In addition, the number of factor level changes for the design obtained with the reverse foldover algorithm equals 53. This implies very high cost for achieving a nearly linear trend resistant design.

The modified reverse foldover algorithm and the reverse foldover algorithm of Cheng and Steinberg (1991) both depend heavily on the sequence of generators used. Also steps (iv), (v) and (vi) of the modified reverse foldover presented in this study are the same as steps (iii), (iv) and (v) of the Cheng and Steinberg's version. The only differences in the two versions is the principle for selecting the generator sequence.

The modified version of the reverse foldover algorithm involve the following steps:

(i) Assign as the first generator any of the letters for the main effect factors or any linear sequence of two letters representing the two factor interactions in the desired design.

(ii) Select the second generator to be any linear sequence of the two letters representing the two factor interactions in the desired design. This should be chosen such that the product of the first two generators will be exhaustive.

(iii) Select the remaining generators by choosing at each step any other letter in the design representing the main effects or the factor interactions such that the generator set are preserved. This should not be any of the $\binom{k}{k-1}$ $(k-1)$-factor interactions.

(iv) Choose ϕ as the first run in the design.

(v) After writing down the first 2^v runs, $0 \leq v < k$, generate the next 2^v runs by writing down the first 2^v runs in reverse order and multiply each of the new runs by the $(v+1)^{st}$ generator.

(vi) Repeat step (v) until the entire design has been generated.

Caution should be taken in selecting the sequence of generators such that letters from the main effect factors do not follow each other in an alphabetical order. In the above algorithm, if we replace k by $k - p$, then we have an algorithm for constructing a trend resistant 2^{k-p} design.

In order to illustrate the modified reverse foldover algorithm, we consider a 2^4 design. Let the factors for the design be represented as A, B, C, and D as before. Following the steps of the algorithm above we have:

→ Assign c as the first generator.

→ Assign ab as the second generator.

→ The remaining two generators should not be any of the $\binom{4}{3}$ 3-factor interactions. Therefore, if we choose d and b as the remaining generators, then we proceed as follows:

→ Let the first run be ϕ.

→ The first two runs will be

$$\phi,\ c \tag{2.6}$$

→ Reversing the order in (2.6) and multiplying by the second generator (that is, ab), we have the run order

$$\phi,\ c,\ abc,\ ab \tag{2.7}$$

→ Reversing the order in (2.7) and multiplying by the third generator (d), we have the run order

$$\phi,\ c,\ abc,\ ab,\ abd,\ abcd,\ cd,\ d \tag{2.8}$$

→ Reversing the order in (2.8) and multiplying by the fourth generator (b), we have the run order

$$\phi,\ c,\ abc,\ ab,\ abd,\ abcd,\ cd,\ d,\ bd,\ bcd,\ acd,\ ad,\ a,\ ac,\ bc,\ b \tag{2.9}$$

The generator sequence that produces run order (2.9) is $\{c, ab, d, b\}$. The design generated by run order (2.9) has the following properties. For factor A (first column) of the design, $S_A(-1) = S_A(+1) = 18$, and $S'_A(+1) = S'_A(-1) = 50$. Thus, $S_A(-1) + S'_A(-1) = S_A(+1) + S'_A(+1) = 68$. Similarly for factor B (second column), $S_B(-1) = S_B(+1) = 18$, and $S'_B(+1) = S'_B(-1) = 50$. Thus, $S_B(-1) + S'_B(-1) = S_B(+1) + S'_B(+1) = 68$. These scenarios are the same for factor C (third column). For factor D(fourth column), $S_D(-1) = 10$, $S_D(+1) = 26$, $S'_D(+1) = 42$, and $S'_D(-1) = 58$. Thus, $S_D(-1)+S'_D(-1) = S_D(+1)+S'_D(+1) = 68$. This implies a linear time count (r=1 in Equation (2.1)) of zero for each of the main factors in the design. Therefore, all the main effect factors of the design generated by run order (2.9) are linear trend resistant.

From the above results, it suffices to say that the modified reverse foldover algorithm produces designs that are at least linear trend resistant. In addition, the number of factor level changes for the design given in run order (2.9) equals 19. This is far less than the number of factor level changes of 53 obtained with Cheng and Steinberg's (1991) reverse foldover algorithm. Thus, with the modified version presented in this study, it is possible to have a linear trend resistant factorial design with minimum number of factor level changes and hence minimum cost!.

Generalized foldover approach

The generalized foldover approach (GFA) is another equivalent way to apply the foldover approach for constructing a trend resistant design. It was first proposed by Coster and Cheng (1988). They made use of the technique based on generalized foldover scheme to construct systematic run order of fractional factorial designs with minimum number of factor level changes which simultaneously have all main effect factors being orthogonal to a polynomial time trend. Coster (1993) presents a modification to the generalized foldover method of Coster and Cheng (1988). The modified method involves the specification of sufficient conditions on the appearance of factors at high levels in sequence of generators of a fractional factorial design in such a way that two and higher order interactions together with the main effects are orthogonal to trend.

The generalized foldover technique for a factorial design, say H, with k factors each and s (s is a prime number) levels, is defined as a design technique which combines all the possible foldovers of $1/s^p$ fractional run sequence of the factorial design H to make another $1/s^{p+1}$ fractional run sequence of the factorial design H, $1 \leq p < k$. For instance, consider an s levels complete factorial design H say, with k factors. Let $Z = (z_1, z_2, ..., z_k)$, $z_i \in GF(s), i = 1, 2, .., k$, denote the treatments or the treatments combination which consists of level combinations, one level from each factor, where GF(s) is as earlier defined. Further, let U be any $1/s^p$ fractional run sequence of the design H, denoted by, $U = [z_1, z_2, ..., z_t]'$, where $z_j \in Z, j = 1, 2, ...t,\ t = s^{k-p}$ represent the $(1 \times k)$ row vector one level from each factor of the design matrix. Then, for any given foldover generator $g = g_1, g_2, ..., g_k, g_i \in GF(s), i = 1, 2, ..., k$, the run sequence U(g) is another run sequence of H obtained by folding over of U by g. Therefore, the generalized foldover of the run sequence U by g, for any given run sequence for the $1/s^p$ fraction of a factorial design H, say U, and foldover generator $g \notin U \cup \{0'\}$, is a run sequence for $1/s^{p+1}$ fraction of the factorial design H and is define as

$$U^* = \begin{bmatrix} U \\ U(g) \\ U(2g) \\ \vdots \\ U((s-1)g) \end{bmatrix}.$$

The construction of linear trend resistant run order using the GFA depends on certain conditions. These are presented as follows. Let $A_1, A_2, ..., A_k$ be k factors of a two levels factorial design, $H = (2^{k-p})$, and $g, = [g_1, ..., g_{k-p}]'$, be a foldover generator matrix for constructing trend resistant design. Then, (i) Any of the factors A_j, $1 \leq j \leq k$ is linear trend resistant if there are at least two high level elements of factors A_j in g. (ii) For any two factors say, A_b and A_c, $1 \leq b \neq c \leq k$, if there are at least two pairs $(z_{ib}, z_{ic}) \in g,\ 1 \leq i \leq k - p$, such that one element is at low-level and the other element is at high-level, then the A_b x A_c interactions is linear trend resistant. In other words, to achieve main effects r-trend resistant designs, we need a foldover generator sequence that has each factor appearing at a high (+) level and its foldover level in $(r + 1)$ generators. Interactions involving two factors are r-trend resistant if one factor is at high (+) level while the other is at low (-) level in $(r + 1)$ generators with the appropriate foldover levels. In the generalized foldover approach, a design is said to be r trend resistant if each factor of the design appears at least $(r+1)$ times in the sequence of generators used to generate the design. The above description reflects that the construction of linear trend resistant design by GFA is decided by the choice of a generator matrix.

In order to illustrate the GFA, we consider the construction of a 2^{4-1} linear trend resistant fractional factorial design for the main effect factors. Following the GFA procedure as stated above, we start by choosing $k - p = 3$ foldover generators. Let the three independent foldover generators be bc, ac, and abc that is,

$$g = \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} bc \\ ac \\ abc \end{bmatrix}. \tag{2.10}$$

Further, let $U_0 = \phi$ be the first run. Then the systematic run order using the GFA as described earlier is determine as follows:

For the first generator $g_1 = bc$, let the generalized foldover of U_0 by g_1 be denoted by U_1, then, we have

$$U_1 = \begin{bmatrix} U_0 \\ U_0(g_1) \end{bmatrix} = \begin{bmatrix} \phi \\ bc \end{bmatrix}.$$

Similarly, the generalized foldover of U_1 by $g_2 = ac$ denoted by U_2 is

$$U_2 = \begin{bmatrix} U_1 \\ U_1(g_2) \end{bmatrix}, U_1(g_2) = \begin{bmatrix} ac \\ ab \end{bmatrix}.$$

Thus,

$$U_2 = \begin{bmatrix} U_0 \\ U_0(g_1) \\ U_1(g_2) \end{bmatrix} = \begin{bmatrix} \phi \\ bc \\ ac \\ ab \end{bmatrix}.$$

Also the generalized foldover of U_2 by g_3 denoted by U_3 is given as

$$U_3 = \begin{bmatrix} U_2 \\ U_2(g_3) \end{bmatrix}, U_2(g_3) = \begin{bmatrix} abc \\ a \\ b \\ c \end{bmatrix}.$$

Thus, the run sequence combining U_0, U_1, U_2 denoted by U^* is

$$U^* = U_3 = \begin{bmatrix} U_0 \\ U_0(g_1) \\ U_1(g_2) \\ U_2(g_3) \end{bmatrix} = \begin{bmatrix} \phi & bc & ac & ab & abc & a & b & c \end{bmatrix}'.$$

Therefore, the half fraction of 2^4 design using the generalized foldover approach with the generator matrix in (2.10) is the run order

$$\phi,\ bc,\ ac,\ ab,\ abc,\ a,\ b,\ c \tag{2.11}$$

In the generator matrix in (2.10), all the three letters a, b and c appear at least twice, hence, factors A, B and C are linear trend resistant. Also, the linear time count (TC(1)) equals zero for all the factors in the design generated by run order (2.11). This is a confirmation that the design is linear trend resistant for all the main effect factors. It should be noted that the generalized foldover approach will give the same run order with the foldover approach if the same sequence of generators is used.

The result from the example above (run order 2.11) can be obtained using the modified reverse foldover algorithm with $\{bc, ab, c\}$ as the sequence of generators. Thus, the foldover, reverse foldover and the generalized foldover approaches can yield the same result depending on the sequence of generators used.

Kronecker product of matrices

The Kronecker product between matrices can be used to construct trend resistant contrasts for two level factorial designs. Jacroux and Ray (1991) show that for two vectors that are trend resistant of certain degrees, the Kronecker product of the two vectors will be trend resistant. The procedure for constructing trend resistant design is summarized in lemma 1 of Jacroux and Ray (1991). The first part of their lemma is emphasized here for constructing time-trend resistant fractional factorial run orders. The procedure is described as follows:
Let L_1 be an $m \times 1$ vector of +1's and -1's of a design matrix such that L_1 is not orthogonal to time-trend effects (that is, $TC \neq 0$). Further, let L_2 be an $n \times 1$ vector of +1's and -1's of the same design matrix which is k trend resistant. Then the Kronecker product between L_1 and L_2, that is, $L_1 \otimes L_2$ will be k trend resistant, where $\otimes$ is the Kronecker product between the two vectors.
As an example, if $L_1 = [a_1, a_2, ..., a_m]'$ and $L_2 = [b_1, b_2, ..., b_m]'$, then
$L_1 \otimes L_2 = [a_1b_1, a_1b_2, .., a_1b_n, a_2b_1, a_2b_2, ..., a_2b_n, ..., a_mb_1, a_mb_2.....a_mb_m]'$.
With this approach, the highest degree polynomial trend that a contrast in a full 2^k design can be orthogonal to is $k-1$, where k is the number of contrasts in the design.

Jacroux and Ray Algorithm

An algorithm given by Jacroux and Ray (1991) for constructing r-trend resistant designs is presented as follows:
For a 2^k factorial design, let $s_1, s_2, ..., s_k$ represent the main effect contrasts derived from the standard ordering. Then, the steps in the algorithm for constructing r-trend resistant two levels k factors factorial designs are given below.

(i) Select a set of k generators from any 2 or more factor interaction letters generated in the standard ordering. This should contain at least $r + 1$ letters out of $s_1, s_2, ..., s_k$.

(ii) Assign each of the main effects to each of the generators in (i). That is, A_1 to say $s_1 * s_2$, A_2 to say $s_2 * s_3$, and so on.

(iii) Obtain the run order of the design indicated by the assignment in (ii).

The run order in step (iii) will be linear trend resistant for at least all the main effect contrasts in the full 2^k design. It should be noted that the above algorithm is also applicable for 2^{k-p} design.

2.1.2 Example: DW approach

In this example, the approach of Daniel and Wilcoxon described earlier in Section 2.1.1 is used to construct a linear trend resistant design. Suppose we are interested in a 16 runs ($N = 16$) resolution V design, then we will construct a time-trend resistant design of the form of a 2_V^{5-1} fractional factorial design. A resolution V fractional factorial design is used because with this resolution type design, it is possible to get estimates for all the main and two factor interaction effects separately without "confusing" them. Let the five factors for a 2^{5-1} experiment be represented by A, B, C, D and E with each of them at two levels. Following the steps presented in Section 2.1 for constructing factorial design and the algorithm for the DW approach in Section 2.1.1, we have $N = 2^{5-1} = 16$ for a 2_V^{5-1} design. Suppose the defining relation is given by $E = ABCD$. Then, the entries of the first four columns will be: $[(16/2)\{+\}, (16/2)\{-\}]'$,

$[(16/4)\{+\}, (16/4)\{-\}, (16/4)\{+\}, (16/4)\{-\}]'$,
$[(16/8)\{+\}, (16/8)\{-\}, (16/8)\{+\}, (16/8)\{-\}, (16/8)\{+\}, (16/8)\{-\}, (16/8)\{+\}, (16/8)\{-\}]'$,
$[+, -, +, -, +, -, +, -, +, -, +, -, +, -, +, -]'$.

The entries for the k^{th} column, that is, the fifth column is obtained by taking the product of the first $(k-1)$ columns as stated in the defining relation. Thus, the five columns representing the settings of the five main factors are presented in a matrix form below.

$$\begin{bmatrix} + & + & + & + & + & + & + & + & - & - & - & - & - & - & - & - \\ + & + & + & + & - & - & - & - & + & + & + & + & - & - & - & - \\ + & + & - & - & + & + & - & - & + & + & - & - & + & + & - & - \\ + & - & + & - & + & - & + & - & + & - & + & - & + & - & + & - \\ + & - & - & + & - & + & + & - & - & + & + & - & + & - & - & + \end{bmatrix}'$$

Using the five column vectors in the matrix above, we complete the construction by obtaining all the possible pairs for the five columns using the pairwise multiplicative rule. The structure for the obtained design for all the 2^{k-1} contrasts is presented in Table 2.1.

Table 2.1: Standard run orders for 2^{5-1} fractional factorial design

	Contrast														
	Main factor					Two factor interactions and aliased factor									
Run No.	A	B	C	D	E	AB (CDE)	AC (BDE)	BC (ADE)	DE (ABC)	AD (BCE)	BD (ACE)	CE (ABD)	CD (ABE)	BE (ACD)	AE (BCD)
1	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
2	+	+	+	-	-	+	+	+	+	-	-	-	-	-	-
3	+	+	-	+	-	+	-	-	-	+	+	+	-	-	-
4	+	+	-	-	+	+	-	-	-	-	-	-	+	+	+
5	+	-	+	+	-	-	+	-	-	+	-	-	+	+	-
6	+	-	+	-	+	-	+	-	-	-	+	+	-	-	+
7	+	-	-	+	+	-	-	+	+	+	-	-	-	-	+
8	+	-	-	-	-	-	-	+	+	-	+	+	+	+	-
9	-	+	+	+	-	-	-	+	-	-	+	-	+	-	+
10	-	+	+	-	+	-	-	+	-	+	-	+	-	+	-
11	-	+	-	+	+	-	+	-	+	-	+	-	-	+	-
12	-	+	-	-	-	-	+	-	+	+	-	+	+	-	+
13	-	-	+	+	+	+	-	-	+	-	-	+	+	-	-
14	-	-	+	-	-	+	-	-	+	+	+	-	-	+	+
15	-	-	-	+	-	+	+	+	-	-	-	+	-	+	+
16	-	-	-	-	+	+	+	+	-	+	+	-	+	-	-
LTC	-64	-32	-16	-8	0	0	0	0	0	0	0	0	0	0	0
QTC	-1088	-544	-272	-136	0	256	128	64	0	64	32	0	16	0	0

() Represent the aliased structure for the two factor interaction contrasts. The aliased structure for the main effect contrasts are: $A \leftrightarrow BCDE, B \leftrightarrow ACDE, C \leftrightarrow ABDE, D \leftrightarrow ABCE, E \leftrightarrow ABCD$. Where $\leftrightarrow$ represent "Aliased to". LTC = Linear time count (r=1 in Equation (2.1)), QTC = Quadratic time count (r=2 in Equation (2.1).

It is observed from Table 2.1 that the two factor interaction contrasts with both linear and quadratic time count of zero are the contrasts that are aliased with interactions ABC, ACD, ABD, BCD, and $ABCD$. Therefore, to get a design that is both linear and quadratic time-trend resistant of resolution V with the defining relation $E = ABCD$, we used the four two-factor interaction contrasts in Table 2.1 that have a zero value of linear and quadratic time-counts. On reassigning as follows: $A \longrightarrow AE(BCD)$, $B \longrightarrow DE(ABC)$, $C \longrightarrow CE(ABD)$, $D \longrightarrow BE(ACD)$, and E as the product of all the four factors as stated in the defining relation, we get a design that is robust against second order time-trend for all the main factors A, B, C, D, and E. On taking the pairwise multiplication of the five main factors, we get the contrasts for the two-factor interactions. Table 2.2 presents the structure of the obtained design.

Table 2.2: Time-trends resistant 2^{5-1} fractional factorial design

	Contrast														
	Main factor					Two-factor interactions and aliased factor									
Run No.	A	B	C	D	E	AB (CDE)	AC (BDE)	AD (BCE)	AE (BCD)	BC (ADE)	BD (ACE)	BE (ACD)	CD (ABE)	CE (ABD)	DE (ABC)
1	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
2	-	+	-	-	-	-	+	+	+	-	-	-	+	+	+
3	-	-	+	-	-	+	-	+	+	-	+	+	-	-	+
4	+	-	-	+	+	-	-	+	+	+	-	-	-	-	+
5	-	-	-	+	-	+	+	-	+	+	-	+	-	+	-
6	+	-	+	-	+	-	+	-	+	-	+	-	-	+	-
7	+	+	-	-	+	+	-	-	+	-	-	+	+	-	-
8	-	+	+	+	-	-	-	-	+	+	+	-	+	-	-
9	-	-	-	-	+	+	+	+	-	+	+	-	+	-	-
10	+	-	+	+	-	-	+	+	-	-	-	+	+	-	-
11	+	+	-	+	-	+	-	+	-	-	+	-	-	+	-
12	-	+	+	-	+	-	-	+	-	+	-	+	-	+	-
13	+	+	+	-	-	+	+	-	-	+	-	-	-	-	+
14	-	+	-	+	+	-	+	-	-	-	+	+	-	-	+
15	-	-	+	+	+	+	-	-	-	-	-	-	+	+	+
16	+	-	-	-	-	-	-	-	-	+	+	+	+	+	+
LTC	0	0	0	0	0	-8	-16	-32	-64	0	0	0	0	0	0
QTC	0	0	0	0	0	-136	-272	-544	-1088	16	32	64	64	128	256

() Represent the aliased structure for the two factor interaction contrasts. LTC and QTC are as earlier define.

Removing all the columns with non-zero LTC in the designs in Table 2.2, we have a design that is linear time-trend resistant with eleven contrasts in 16 runs. The design structure is presented in Table 2.3. It satisfies the orthogonal property of saturated fractional factorial designs and has the same defining relation as the design in Table 2.1 (standard order).

Table 2.3: Linear time-trend resistant 2^{5-1} fractional factorial design

Run No.	Contrast										
	Main factor					Two-factor interactions and aliased factor					
	A	B	C	D	E	BC (ADE)	BD (ACE)	BE (ACD)	CD (ABE)	CE (ABD)	DE (ABC)
1	+	+	+	+	+	+	+	+	+	+	+
2	-	+	-	-	-	-	-	-	+	+	+
3	-	-	+	-	-	-	+	+	-	-	+
4	+	-	-	+	+	+	-	-	-	-	+
5	-	-	-	+	-	+	-	+	-	+	-
6	+	-	+	-	+	-	+	-	-	+	-
7	+	+	-	-	+	-	-	+	+	-	-
8	-	+	+	+	-	+	+	-	+	-	-
9	-	-	-	-	+	+	+	-	+	-	-
10	+	-	+	+	-	-	-	+	+	-	-
11	+	+	-	+	-	-	+	-	-	+	-
12	-	+	+	-	+	+	-	+	-	+	-
13	+	+	+	-	-	+	-	-	-	-	+
14	-	+	-	+	+	-	+	+	-	-	+
15	-	-	+	+	+	-	-	-	+	+	+
16	+	-	-	-	-	+	+	+	+	+	+
LTC	0	0	0	0	0	0	0	0	0	0	0

() Represent the aliased structure for two factor interaction contrasts and LTC is as earlier define.

The design in Table 2.3 is linear time-trend resistant with eleven contrasts in 16 runs, this will be referred to as the linear time-trend resistant design in Chapter 4. The general result obtained from above is presented in the Theorem below.

Theorem

For any 2^k factorial designs or 2^{k-p} fractional factorial designs, there are $2^k - 1 - k$ columns of the model design matrix for a full factorial and $2^{k-p} - 1 - (k-p)$ columns of the model design matrix for a fractional factorial design that are linear trend resistant.

Proof

We give a proof for a 2^k design, the proof for a 2^{k-p} design follows the same pattern. Let k be the number of factors in a design arranged in a standard order, and let $N = 2^k$. Then there will be

$\begin{pmatrix} k \\ 2 \end{pmatrix}$ two- factor interactions, $\begin{pmatrix} k \\ 3 \end{pmatrix}$ three- factor interactions, ..., $\begin{pmatrix} k \\ h \end{pmatrix}$ h- factor interactions, and $\begin{pmatrix} k \\ k \end{pmatrix}$ k-factor interactions.

It is a known fact that any h-factor interactions is $(h-1)$-trend resistant (see Cheng and Jacroux, 1988). That is, any two or more factor interactions have a linear time count of zero(LTC=0). Hence taking the sum of all the possible interaction contrast of a factorial design, that is

$$\begin{pmatrix} k \\ 2 \end{pmatrix} + \begin{pmatrix} k \\ 3 \end{pmatrix} + \ldots + \begin{pmatrix} k \\ h \end{pmatrix} + \ldots + 1$$

will give the number of possible linear trend resistant contrasts in a factorial design. Now to show that

$$\begin{pmatrix} k \\ 2 \end{pmatrix} + \begin{pmatrix} k \\ 3 \end{pmatrix} + \ldots + \begin{pmatrix} k \\ h \end{pmatrix} + \ldots + 1 = 2^k - 1 - k. \tag{2.12}$$

Solving the LHS of Equation (2.12), we have

$$\frac{k(k-1)}{2!} + \frac{k(k-1)(k-2)}{3!} + \ldots + \frac{k(k-1)(k-2)\ldots(k-h-1)}{h!} + \ldots + 1 \tag{2.12.1}$$

$$\Longrightarrow \frac{k(k-1)}{2}\left\{1+\frac{(k-2)}{3}+\ldots+\frac{(k-2)(k-3)\ldots(k-h-1)}{h(h-1)(h-2)\ldots 3}+\ldots\right\}+1 \qquad (2.12.2)$$

It can easily be shown by mathematical induction that the expression in (2.12.2) equals $2^k - 1 - k$.

An alternative proof which we found to be more straight forward than the one above is presented as follows: In a standard run order of a 2^k design, there are $2^k - 1$ contrasts (columns) out of which $\binom{k}{1}$ are main effects. These main effects are not linear trend resistant (see Table 2.1 on page 28). If we take away the k main effects from the possible number of contrasts in a 2^k design, then we have $2^k - 1 - \binom{k}{1}$ contrasts left for two and higher factor interactions. But the two and higher order factor interactions of the standard run order of a 2^k design are linear trend resistant (see Table 2.1 on page 28). Therefore, it suffices to say that there are $2^k - 1 - k$ contrasts of a 2^k design that are linear trend resistant.

To determine how many factor levels need to be changed in a fractional factorial design, we use the 2_v^{5-1} fractional factorial design in Table 2.1 for illustration. This principle of determining the number of factor level changes is the same for all other designs in this chapter. The first five columns of the design in Table 2.1 represent the main effect contrasts under study. In order to ascertain the number of factor changes in a design, the number of changes in level is determined by counting the number of differences in the plus and minus in a pair of rows in the design matrix. For example, from run number 1 to run number 2, we have 2 differences (main effect factor D and E). From run number 2 to run number 3, we have 2 differences (main effect factor C and D); 3 to 4: 2 differences; 4 to 5: 4 differences; 5 to 6: 2 differences; 6 to 7: 2 differences; 7 to 8: 2 differences; 8 to 9: 4 differences; 9 to 10: 2 differences; 10 to 11: 2 differences; 11 to 12: 2 differences; 12 to 13: 4 differences; 13 to 14: 2 differences; 14 to 15: 2 differences; 15 to 16: 2 differences. Therefore, the design in Table 2.1 has a total of 36 factor level changes. This is equivalent to the number of level changes for the standard run order for a five factors resolution V fractional factorial design. Hence, the design in Table 2.1 will henceforth in this study be referred to as the standard run order. In a similar way, the number of factor level changes for the main effect contrasts in the constructed time-trend resistant design of second order degree presented in Table 2.3 equals 48. This is higher than that of the standard run order in Table 2.1. The number of factor level changes for the main effect contrasts in the constructed time-trend resistant design of second order degree obtained by using the other approaches reviewed were less than 48. For example, using the foldover approach with generator sequence $\{abcd, abc, abd, acd\}$, the total number of factor level changes is 43. Also, the generalized foldover approach and the Jacroux and Ray algorithm with generator sequence $\{abc, abd, acd, bcd\}$ give the total number of factor level changes of 38. The Modified reverse foldover algorithm with generator sequence $\{c, ab, d, b\}$ produce a total number of factor level changes of 19 as earlier mentioned.

The approach to be used when constructing trend resistant full/fractional factorial designs depends on the aim and nature of the experiment. If all the experimental factors under consideration are equally expensive, then the total number of factor level changes will not be an issue to be considered and hence focus could be on having a systematic run order that is polynomial trend resistant of certain degree. It should be noted that the result obtained above using the DW approach will also be obtained if we used the foldover scheme with generator sequence $\{abcd, abc, abd, acd\}$ or the GFA and Jacroux and Ray's (1991) algorithm with generator sequence $\{abc, abd, acd, bcd\}$. Also, the modified version of reverse foldover with generator sequence $\{c, ab, d, b\}$ produce the same result with the other approaches. Using the KP approach, a design that is robust against first and second order trend with the same property as the design in Table 2.3 is plausible.

2.2 Trend resistant Plackett Burman designs

Plackett Burman design is suitable for studying k factors, each with L levels, and N runs. It is constructed by using a generating vector which is first written down as a column/row. A second column/row is obtained by moving down the elements of the previous columns/rows once and placing the last element in the first/last position. The procedure is repeated until all the k columns/rows are obtained. Finally a row of elements all representing the first factor level is added to complete the design. When $L = 2$ (two levels), a final row of -1's/+1's is to be added (see Plackett and Burman, 1946). In this work, we are interested in a 12 runs Plackett Burman design which will henceforth be referred to as PB_{12}. Plackett Burman designs are identical to fractional orthogonal designs when N is a power of two, hence a trend resistant Plackett Burman design (TRPBd) is also identical to its fractional counterpart.

For $r = 1$ in Equation (2.1), none of the contrasts of a Plackett Burman design gives a TC of zero. Thus, none of the contrast in PB_{12} is even linear trend resistant. On taking the pairwise contrast, that is, $\binom{11}{2}$ two factor interactions, 5 of these are linear trend resistant but are not orthogonal to each other. Hence they can not be taken to be any of the columns of a Plackett Burman design.
Following the approach suggested by Jacroux and Ray (1991) for constructing main effects only fractional factorial designs when the number of available experimental units is a multiple of 4, we construct a trend resistant Plackett Burman design (TRPBd). We start with a Plackett Burman matrix of order 12 (PB_{12}). By rearranging the rows within the PB_{12} design, we developed another set of columns that are at least linear trend resistant using computer search approach. The obtained result is presented in Table 2.4. To determine the degree of resistance for the obtained design in Table 2.4, we compute the statistic time count. This is presented in the last row of Table 2.4.

Table 2.4: Linear trend resistant 12-runs Plackett Burman design

Run No.	Contrast										
	A	B	C	D	E	F	G	H	I	J	K
1	-	+	+	+	-	+	-	-	+	-	-
2	-	-	+	+	+	-	+	-	-	+	-
3	+	+	+	-	+	-	-	+	-	-	-
4	+	-	-	-	+	+	+	-	+	-	-
5	-	+	-	-	-	+	+	+	-	+	-
6	+	-	+	-	-	+	-	-	-	+	+
7	+	-	-	+	-	-	-	+	+	+	-
8	+	+	+	+	+	+	+	+	+	+	+
9	-	-	+	-	-	-	+	+	+	-	+
10	-	-	-	+	+	+	-	+	-	-	+
11	+	+	-	+	-	-	+	-	-	-	+
12	-	+	-	-	+	-	-	-	+	+	+
LTC	0	2	-20	0	0	-10	0	6	4	2	34

The obtained design presented in Table 2.4 has only four linear trend resistant contrasts. Our search in this study for constructing Plackett Burman trend resistant design cannot achieve more than four linear trend resistant columns for a 12 runs Plackett Burman design. Therefore, a twelve runs Plackett Burman design with four columns being linear trend resistant is possible. Although, the number of linear trend resistant columns for a 12 runs Plackett Burman design achieved here is a slight improvement on the result by Jacroux and Ray (1991) which has only three linear trend resistant columns, it is too small to be an acceptable design. Therefore, an alternative approach is necessary.

The construction of orthogonal main effects with more than four linear trend resistant contrast for a PB_{12} seems to be a very complex one. However, if the orthogonality assumption is relaxed, it is possible using semifolding principle to have a PB_{12} design with at least 10 out of the eleven contrasts being robust against linear trend. Semifolding involves using half of a foldover design to fold the points that are at the high /low level of a factor . For an overview of semifolding, see Mee and Peralta (2000), John (2000) and Kowalski (2002).

On applying the semifolding principle, the first half of the rows of the PB_{12} design were folded. The obtained design is presented in Table 2.5. The last row of Table 2.5 represent the computed TC(1) for each of the factors. Thus, we have a PB_{12} design with ten contrasts being linear trend resistant. It should be noted that a naive version of the semifolding principle is employed in this study.

Table 2.5: Non-orthogonal linear trend resistant PB_{12} design

Run No.	Contrast										
	A	B	C	D	E	F	G	H	I	J	K
1	+	+	+	+	+	+	+	+	+	+	+
2	+	+	-	+	-	-	+	-	-	-	+
3	+	+	+	-	+	-	-	+	-	-	-
4	+	-	-	-	+	+	+	-	+	-	-
5	+	-	+	-	-	+	-	-	-	+	+
6	+	-	-	+	-	-	-	+	+	+	-
7	-	-	-	-	-	-	-	-	-	-	-
8	-	-	+	-	+	+	-	+	+	+	-
9	-	-	-	+	-	+	+	-	+	+	+
10	-	+	+	+	-	-	-	+	-	+	+
11	-	+	-	+	+	-	+	+	+	-	-
12	-	+	+	-	+	+	+	-	-	-	+
LTC	-36	0	0	0	0	0	0	0	0	0	0

The information matrix for the design in Table 2.4 is $12I_{11}$, where I_{11} represents an (11×11) identity matrix. Thus, the design in Table 2.4 is orthogonal. The information matrix for the design in Table 2.5 given below shows that only factor A is orthogonal to all the other main effect factors and all the other main effect factors are in turn orthogonal to factor A. Each factor pair of the remaining 10 main effect factors (B through K) has correlation $\pm 1/3$. Thus, the design in Table 2.5 is a non orthogonal design.

$$\begin{bmatrix} 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 12 & 4 & 4 & 4 & -4 & 4 & 4 & -4 & -4 & 4 \\ 0 & 4 & 12 & -4 & 4 & 4 & -4 & 4 & -4 & 4 & 4 \\ 0 & 4 & -4 & 12 & -4 & -4 & 4 & 4 & 4 & 4 & 4 \\ 0 & 4 & 4 & -4 & 12 & 4 & 4 & 4 & 4 & -4 & -4 \\ 0 & -4 & 4 & -4 & 4 & 12 & 4 & -4 & 4 & 4 & 4 \\ 0 & 4 & -4 & 4 & 4 & 4 & 12 & -4 & 4 & -4 & 4 \\ 0 & 4 & 4 & 4 & 4 & -4 & -4 & 12 & 4 & 4 & -4 \\ 0 & -4 & -4 & 4 & 4 & 4 & 4 & 4 & 12 & 4 & -4 \\ 0 & -4 & 4 & 4 & -4 & 4 & -4 & 4 & 4 & 12 & 4 \\ 0 & 4 & 4 & 4 & -4 & 4 & 4 & -4 & -4 & 4 & 12 \end{bmatrix}.$$

Information matrix for the design in Table 2.5

Another possibility we consider is the use of the foldover approach described earlier in Section 2.1.1. This approach is used here to construct linear trend resistant Plackkett Burman design. On using the foldover principle on PB_{12}, a 24 runs PB design with 12 contrasts was obtained. Using Miller and Sitter (2001) notation, this is designated as PB_{12+12} design. Using the foldover principle, we start by writing down the first 12 rows (runs) for the PB_{12} design. This is follow by folding over all the first 12 rows. The design obtained has all the eleven main-effect columns to be linear trend resistant (that is, LTC of zero). The design is presented in Table 2.6. For this design, each main effect column is orthogonal to all other main effect columns and to all two factor interaction columns. In order to search for more linear trend resistant columns for the PB_{12+12} design, we employ the pairwise multiplicative rule on the columns of the design presented in Table 2.6. Out of the 55 two factor interaction columns, only four of them are found to be linear trend resistant. These four two factor interactions are however, correlated with each other and hence non-orthogonal.

Table 2.6: Orthogonal linear trend resistant PB_{12+12} design

	Contrast										
Run No.	A	B	C	D	E	F	G	H	I	J	K
1	-	-	+	-	-	-	+	+	+	-	+
2	-	+	-	-	-	+	+	+	-	+	-
3	+	-	-	-	+	+	+	-	+	-	-
4	-	-	-	+	+	+	-	+	-	-	+
5	-	-	+	+	+	-	+	-	-	+	-
6	-	+	+	+	-	+	-	-	+	-	-
7	+	+	+	-	+	-	-	+	-	-	-
8	+	+	-	+	-	-	+	-	-	-	+
9	+	-	+	-	-	+	-	-	-	+	+
10	-	+	-	-	+	-	-	-	+	+	+
11	+	-	-	+	-	-	-	+	+	+	-
12	+	+	+	+	+	+	+	+	+	+	+
13	+	+	-	+	+	+	-	-	-	+	-
14	+	-	+	+	+	-	-	-	+	-	+
15	-	+	+	+	-	-	-	+	-	+	+
16	+	+	+	-	-	-	+	-	+	+	-
17	+	+	-	-	-	+	-	+	+	-	+
18	+	-	-	-	+	-	+	+	-	+	+
19	-	-	-	+	-	+	+	-	+	+	+
20	-	-	+	-	+	+	-	+	+	+	-
21	-	+	-	+	+	-	+	+	+	-	-
22	+	-	+	+	-	+	+	+	-	-	-
23	-	+	+	-	+	+	+	-	-	-	+
24	-	-	-	-	-	-	-	-	-	-	-
LTC	0	0	0	0	0	0	0	0	0	0	0

The information matrix for the design in Table 2.6 is $24I_{11}$, where I_{11} is as earlier defined. For this design, each of the two factor interaction columns is orthogonal to all the other two factor interaction columns with which it shares a common factor. While the columns for the pairs of the two factor interactions that do not share a common factor has correlation $\pm 1/3$. Hence the design in Table 2.6 is an orthogonal main effects linear trend Plackett Burman design.

Three different possibilities of constructing linear trend resistant Plackett Burman design have been presented. Each of these possibilities gives different result.

The first one shows that we can have **4** linear trend resistant contrasts (columns) in 12 runs with the orthogonality property being satisfied. The second one shows that we can have **10** linear trend resistant contrasts (columns) in 12 runs with the orthogonality property being violated. The third one shows that we can have **11** linear trend resistant contrasts (columns) in 24 runs with the orthogonality property being satisfied. Therefore, if the experimenter is interested in high number of linear trend resistant PB_{12} design columns, he has to be prepared to pay the price of losing the orthogonality property. However, to retain the orthogonality property with high number of linear trend resistant columns, more number of experimental runs is the price to pay.

Chapter 3

The Funnel Experiment: A Practical Case Study of DoE

The funnel experiment is a simple physical experiment that is used as a teaching aid for statistical design of experiment (Gunter B. 1993). It was first proposed by Deming (1986) and later with a different view by MacGregor (1990) for teaching the importance of design of experiment to quality improvement. In this study, the funnel experiment is used as a practical case study to illustrate time-trends problem in a fractional factorial setting. Our device for the funnel experiment which is based on an idea of Joachim Kunert (Universität Dortmund) is an improvement on the funnel experiment of Bert Gunter (1993) device. For our device, only one person can handle the machine to conduct experiments, whereas according to Gunter (1993) at least three people are needed to operate the machine in order to conduct an experiment.

The practical aspect of this study is to show that there is a time-trend in the exemplified (funnel) experiment and to identify and exclude the factors that are responsible for the proclaimed time-trend in the process that we use as a practical case study. Figure 3.1 presents the picture of the machine used in the experiment. The funnel experiment is made up of the following materials: A funnel made of hard plastic/ aluminium with very wide opening of about 25cm wide and a narrow long tip of about 8.5cm long/ without tip. A rod of fixed length (100 cm long) with a small opening at the rear end where the ball stopper is fixed. A pole made of steel which stand as a supporting tool where the rear part of the rod rests on. A pole basement which holds the pole standing erect. A clapper used as a supporting tool for the rod connected to the pole at the rear end. A device holding the rod at the front. A box (funnel basement) made of wood with a wide opening at the upper end on which the funnel is sited and a narrow opening at the lower end housing the funnel tip. A ball bearing of fixed size of about 18mm in diameter. A digi-timer stop watch which is used to measure the response variable. The machine is set on a flat table for smooth conduct of the experiment as shown in Figure 3.1

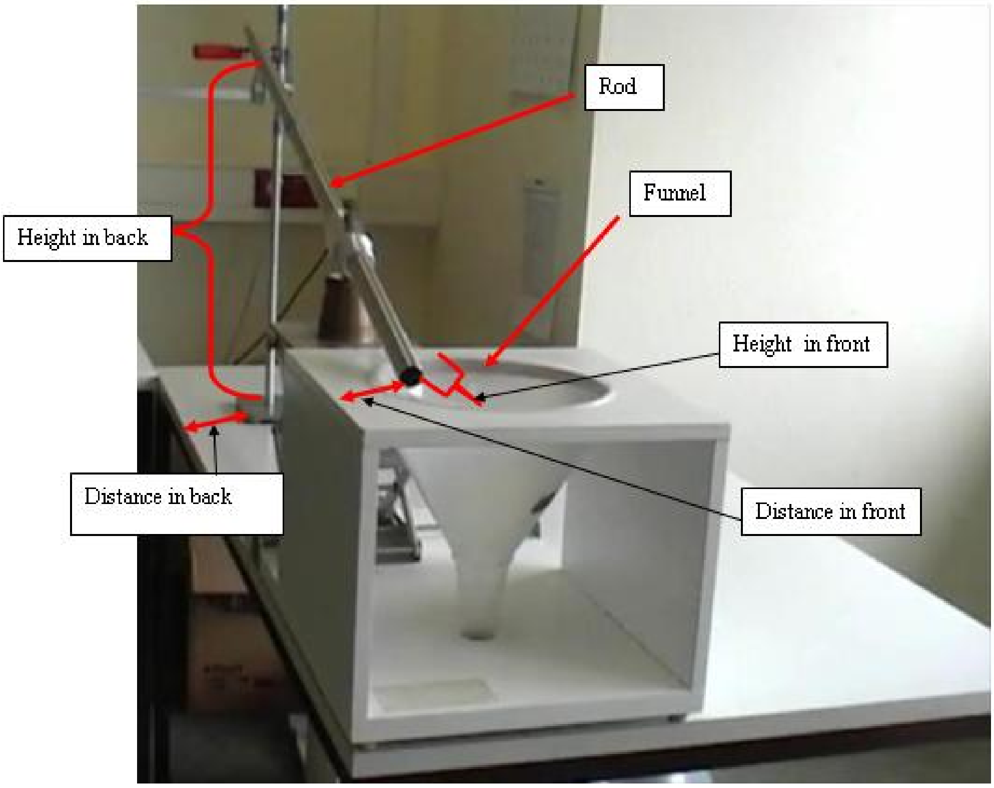


Figure 3.1: *Experimental settings*

The pictures of the two types of funnels (plastic and aluminium) used in the conduct of the experiments are presented in Figure 3.2.

Figure 3.2: *Funnel with tips(LHS) and American type Funnel(RHS)*

The experiment is designed to evaluate the running time of the ball bearing through a rod of fixed length spinning within the funnel in a spiral path and finally exiting at the basement of the funnel. The response variable of the experiment is the running time of the ball bearing. This is measured using the stop watch by taking the time the ball bearing is released at the edge of the rod to the time it exits at the basement of the funnel.

The following experimental settings are taken to be of importance in order to measure the response from the experiment.

- The length between the point on the pole where the clapper holds the rod to the pole and the pole basement, this is designated as height in back.
- The distance between the pole basement and the edge of the table where the machine is placed, this is designated as distance in back.
- The length between the rod tip and the funnel opening, this is called the height in front.
- The distance between the rod tip and the edge of the funnel basement top, this is designated as distance in front.

To achieve the set objective, we plan two major factorial experiments. The first one is to show that there is a time-trend in the funnel experiment and the second is to determine the factor(s) responsible for the time-trend.

3.1 Experiment to show time-trends in the funnel

In this section, all the experiments were conducted using the plastic type funnel (funnel with tip). The aim of the first experiment is to demonstrate the time-trend problem. Five factors each at two levels were selected to be the important design factors of interest. These factors and their levels are presented in Table 3.1.

Table 3.1: Experimental Factors and Their Levels

Factors	Description	Level 1	level 2
A	Distance in front	7.0cm	10.0cm
B	Height in front	0.5cm	1.0cm
C	Distance in back	5.0cm	10.0cm
D	Height in back	40.0cm	50.0cm
E	Size of ball	Small(10mm)	Big(18mm)

A pilot experiment was conducted using a resolution V fractional factorial design by varying each of the five experimental factors in Table 3.1 at two levels in sixteen experimental runs. For each setting of level combinations, the experiment was conducted by taking four consecutive runs in a row before resetting the level of the factors. That is, there are four replicates per factor setting. This is to have a preliminary knowledge of the funnel experiment. On ranking the outcomes it was discovered that only the setting A (low– 7.0 cm), B (high– 1.0 c m), C (low– 5.0 cm), D (high– 50.0 cm), and E (high– Big ball) shows evidence of presence of a time-trend. The main experiment was then conducted by keeping the settings that showed evidence of trend obtained from the pilot experiment fixed. For the main experiment, two different machines were used to run the experiments. The experiment with Machine 1 was conducted at a particular time on a day while the experiment with Machine 2 was conducted around the same time on another day. To run a complete set of experiments (16 runs) approximately 10 minutes were needed. This implies that the time lag between consecutive runs in the experiment is approximately 38 seconds. The following observations were made during the experiments:

(a) The running time from when the ball is released at the edge of the rod length to when it exits from the rod is approximately constant for all runs.

(b) The ball bearing on exit from the rod length usually hits the same spot on the funnel before spinning within the funnel.

Because of the second observation, we carried out another experiment by turning the funnel approximately 180^o in order to change the point which the ball hits on the funnel. The experiment was repeated by keeping all other settings as before. The observed running time for the experiment from the two machines are presented in Table 3.2.

Table 3.2: Experimental Results for Machines 1 and 2

	Observed running time(in sec.)							
Machine 1	22.13	23.49	23.32	24.26	23.70	23.92	24.07	24.09
	25.06	25.36	24.32	24.97	25.03	26.09	25.40	26.02
Machine 2	21.35	21.36	22.31	21.98	23.07	23.29	22.89	23.71
	23.18	23.73	24.30	23.30	23.68	23.49	23.51	24.19

To determine whether there is a time-trend in the experimental results presented in Table 3.2, a trend analysis was conducted. The nonparametric Spearman's criterion was used to detect the existence of trend in the series of the experimental data. The Spearman rank-correlation is described as

$$R_{sp} = 1 - \frac{6\sum_i^n D_i^2}{n^3 - n}, \tag{3.1}$$

where n is the total number of observation in the series, i is the chronological order number and D_i is the difference between the rankings of the measured variable in chronological order and the series of measurements. The test statistic for the hypothesis of no trend, that is, $H_0 : R_{sp} = 0$ (there is no trend), against the alternative hypothesis, $H_1 : (R_{sp} < or > 0)$ (there is a trend) is given by:

$$t_{sp} = R_{sp}\sqrt{\frac{n-2}{1-R_{sp}^2}}, \tag{3.2}$$

where the statistic t_{sp} is assumed to have a student's t-distribution, with $v = n - 2$ degrees of freedom. A series is said to have no trend if at a given α level of significance t_{sp} lies in the interval:

$$t(v, \alpha/2) < t_{sp} < t(v, 1 - \alpha/2), \tag{3.3}$$

otherwise the series is adjudged to have trend.

The two experimental results presented in Table 3.2 were tested for existence of trend using Equations (3.1), (3.2), and (3.3). The obtained trend analysis results give t_{sp} values of 8.469 and 5.3140 for machines 1 and 2 data, respectively. Using $\alpha = 0.05$ level of significance, the interval given by Equation (3.3) for $v = 14$ is obtained to be [-2.1448, 2.1448]. The p values for the obtained t_{sp} values are $p < 0.0001$ and $p = 0.0001$ for machines 1 and 2 data, respectively. Since the computed t_{sp} values for the results from the two machines lies outside the computed interval, and their respective p values are less than 0.05, it suffices to conclude that there is trend in the two experimental data and hence the process that produces them. The plot of the experimental results in Figure 3.3 also shows that the running time increases with time (run number). Therefore, we can conclude that there is a time-trend in the funnel experiment.

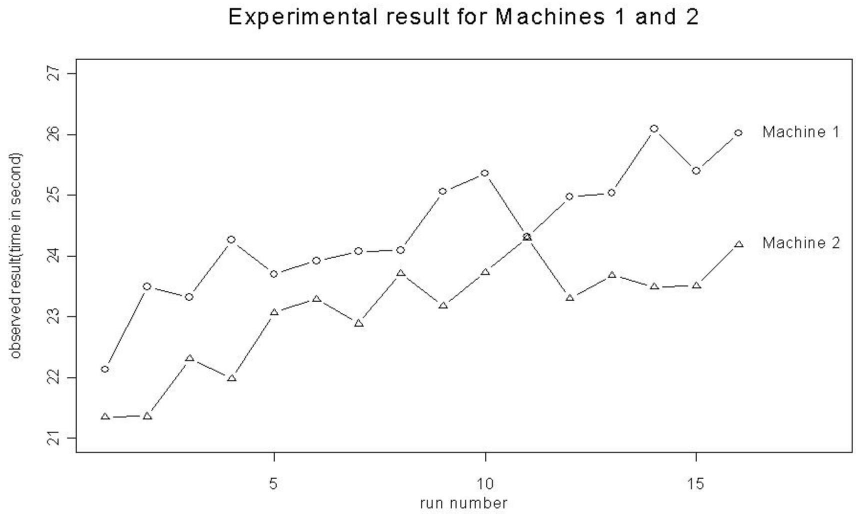


Figure 3.3: *Experimental results showing time-trends*

3.2 Experiment to determine factors responsible for the time-trend

To determine the factor(s) that are responsible for the observed time-trend in the funnel experiment discovered in Section 3.1, factorial and cross over experiments were used. The experimental factors considered are the ball bearing, the funnel, and the rod. These three factors are the movable (that is, assemble and disassemble) factors in the experimental setting. In order to achieve the objective in this section, we planned two different settings for each of the factors. The two settings for each of the experimental factors used are of the same magnitude, size, shape, and are made of the same materials.

To investigate which of the three experimental factors mentioned above has a significant effect on the observed time-trend, different experiments were conducted varying each of the three factors in different settings with 20 experimental runs. The 20 experimental runs were partitioned into two sequences each containing 10 runs. These experiments are factorial experiments with two factors each at two levels and cross-over experiment with a factor at two levels conducted over two periods of time. In the first experiment, a machine was used with the other two factors being varied. The aim is to see which of the two varied factors has a significant effect on the time-trend in the experiment. The second experiment was planned with two machines (funnels), one rod and one ball. The aim of the second experiment is to check for the influence of the funnel on the observed time-trend. A factor is adjudged to have a significant effect on the observed time-trend in the experiment if the observed running time keep on increasing irrespective of changes in sequence. That is, $|t_{sp}| \geq t(v, 1 - \alpha/2)$ for the two sequences.

3.2.1 Factorial experiment for ball and rod influence

Two levels factorial experiment was conducted to determine the ball and rod influence on the observed time-trend in the funnel experiment. We started the experiment by excogitating the ball and the rod with the funnel fixed. That is, the experiment was conducted by varying the ball and the rod using a machine (fixed funnel). Hence, we have a case with n = 2 factors each at two levels. The experiment was conducted on a standard order setting with the sequence

$$\{\phi, 1, 2, 12\}, \tag{3.4}$$

where the entries in the sequence represent no changes, changes in factor 1, changes in factor 2, and changes in both factors 1 and 2 (interaction), respectively.

To conduct the factorial experiment with n = 2, the first run with the setting ϕ was used, that is keeping all the factors at (-) level. 10 runs were first conducted and we waited for a time lag of 30 minutes before conducting the second set of 10 runs. Hence we have a total number of 20 runs for the setting where all the factors are at (-) level. Experiment with factor 1 was conducted on the second day. Here the experiment involved changing the ball bearing. We did the first 10 runs by using ball 1 and we waited for the same time lag as before, thereafter we used ball 2 to do the second 10 runs. On the third day, the experiment with changes in factor 2 was conducted. Here the first 10 runs were done by using ball 1 with rod 1. After waiting for the period of 30 minutes, we changed to the second rod (rod 2), and used the same ball as in the first part of the experiment to do another set of 10 runs. On the fourth day, the experiment involved changing of the rod and the ball. This is to determine the influence of the interaction between the ball and the rod on the time trend. The first 10 runs were done by using ball 1 and rod 1. After the waiting time, the second 10 runs were done by using ball 2 and rod 2. Thus, a period of four days was used to conduct a complete two factors experiment. The experimental setting and the observed responses are presented in Table 3.3.

Table 3.3: First Experiment: Observed Results For Factorial Experiment

	No changes in both		Changes in ball		Changes in rod		Changes in both	
run	r1b1	r1b1	r1b1	r1b2	b1r1	b1r2	r1b1	r2b2
1	22.56	24.85	24.55	29.87	26.82	26.63	28.60	28.88
2	22.89	24.32	25.53	26.04	25.78	25.56	28.34	28.62
3	24.32	25.33	25.89	26.29	26.96	26.63	27.84	27.60
4	24.63	25.69	26.00	26.28	26.56	21.82	28.63	28.77
5	25.59	25.40	26.43	27.11	27.17	27.09	29.77	28.77
6	25.06	25.07	27.16	26.96	26.63	27.29	29.25	29.57
7	24.42	25.65	26.99	27.19	26.99	28.29	28.93	29.25
8	25.50	25.23	26.69	28.08	27.07	27.86	28.86	29.70
9	24.55	25.59	27.38	27.47	26.46	27.83	28.41	28.39
10	25.73	25.87	27.73	26.97	27.27	27.26	28.89	28.26
Mean	24.52	25.3	26.44	27.23	26.77	26.63	28.75	28.78
stand. dev	1.08	0.46	0.96	1.11	0.44	1.86	0.53	0.63
Coff.of var.	4.39	1.82	3.64	4.08	1.64	6.97	1.83	2.18
t_{sp}	2.94	2.26	8.75	0.84	1.3	3.07	1.26	0.07

An examination of the observed running time in Table 3.3 and the graphical display of the results in Figure 3.4 shows that the running times increase with days (see the mean for each factor sequence in Table 3.3) and with the run numbers for some of the factor sequence.

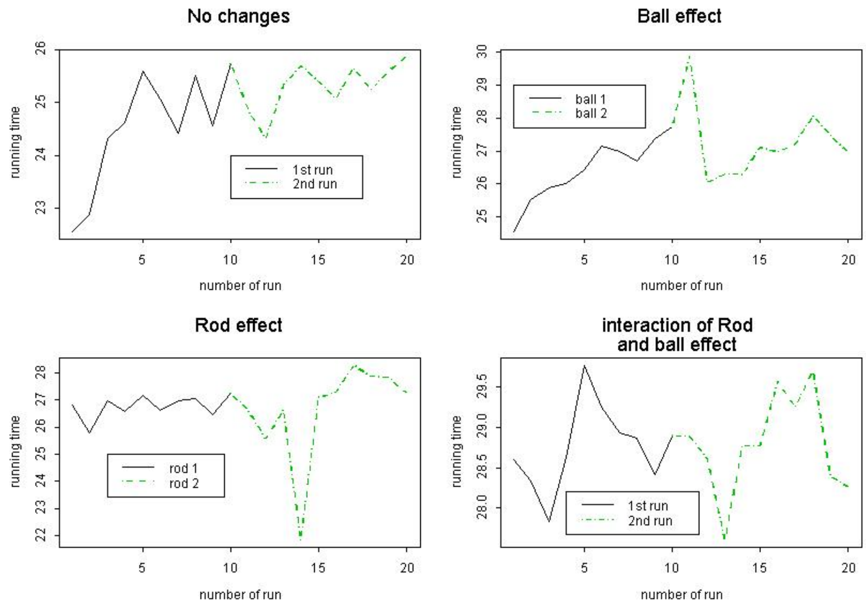


Figure 3.4: *Influence of the ball and rod effects on the time-trend in the funnel experiment*

Examination of the graphs in Figure 3.4 reveals the following:

(i) The running time increases with the run numbers for the case where no changes in experimental setting was made. That is, when the factors setting is fixed, there is trend in the experiment.

(ii) The result with ball effect shows increase in the running time with the run numbers for the first 10 runs (that is experiment with ball 1), but thereafter there is a slight fall and then a stable increase in the running time with ball 2.

(iii) The behavior of the observed running time with rod 1 is totally different from that of rod 2. The variation in the running time with rod 1 is approximately constant, while there is a large variation in the observed running time with rod 2. Also an outlier point is observed in the result for rod 2.

(iv) The plot for the interaction between the rod and the ball shows not much difference when the ball and the rod were changed. In this plot, no trend is observed.

The degree of variations of the observed running time for each of the factor sequence was computed using the standard deviation and the coefficient of variation. The absolute differences in the standard deviations and the coefficient of variation equal 0.62(2.57), 0.15(0.44), 1.42(5.33) and 0.1(0.35) for the factor settings: no changes (ϕ), changes in ball, changes in rod, and interaction in both factors, respectively. Here, the values in the parenthesis above represent the absolute difference for coefficient of variation of each setting. The high difference in both the standard variation and the coefficient of variation for the rod effect might be due to the outlier point at the fourth run in the second sequence.
A trend analysis was conducted on the results using Equation (3.1) through (3.3). The computed t_{sp} values for each sequence of factor settings are presented in the last row of Table 3.3. Using $\alpha = 0.05$, the decision interval for $n = 10$ is $[-2.306,\ 2.306]$. Therefore, a sequence (level) for each of the factor settings is adjudged to have a trend if its t_{sp} lies outside the interval

$$-2.306 < t_{sp} < 2.306. \tag{3.5}$$

That is, if $|t_{sp}| \geq 2.306$ for a factor sequence, then we say that there is presence of trend in that factor sequence. Using the interval in (3.5) to analyze the computed t_{sp} in the last row of Table 3.3, the following are inferred:

(a) For settings with no changes and changes in ball, there is trend in Sequence 1 whereas there is no trend in Sequence 2.

(b) For the setting with changes in rod, there is no trend in Sequence 1 (Rod 1) whereas there is existence of trend in Sequence 2 (Rod 2).

(c) There was no existence of trend for the setting where both factors were changed.

Generally speaking, in Figure 3.4, the observed running time can be said to increase with time for the plots for no changes, ball effect, and rod effect (if the outlier point is removed). However, using the combination of the trend analysis results and eye inspection of Figure 3.4, we can say that neither the ball nor the rod have a significant effect on the time-trend in the experiment. Further, a one-one comparison of the running time (response) for the changes in ball show that the responses for ball 2 are greater than those of ball 1 in 80% of the cases considered. While for the changes in rod, the responses for rod 2 are greater than those of rod 1 in 40% of the cases. In 60% of the cases considered, the responses from the interaction between rod 2 and ball 2 are greater than those from rod 1 and ball 1. This could be interpreted to mean that the ball is likely to have a higher effect ("red X") and the interaction as the "pink X".

The above results do not give a clear picture of the possible factor that is responsible for the observed time-trend in the funnel experiment. In order to achieve the set aim, more experiments were conducted using the sequence in (3.4) in a randomized manner. The mode of operation was the same as above but the waiting time was reduced from 30 minutes to 5 minutes. This was done to remove the influence of long time lag from the experiment. All together eight experiments were conducted. The observed results for the remaining seven different factorial experiments along with the experimental settings are presented in Tables 3.4 through 3.10.

Table 3.4: Second Experiment: Observed Results For Factorial Experiment

	No changes in both		Changes in rod		Changes in ball		Changes in both	
run	r1b1	r1b1	r1b1	r2b1	b1r1	b2r1	r1b1	r2b2
1	35.59	36.67	34.73	36.70	37.01	35.67	36.95	37.26
2	35.99	36.17	34.60	36.89	37.58	37.91	36.99	35.07
3	35.99	35.86	34.64	36.94	35.57	37.49	36.99	39.04
4	36.60	36.97	36.63	37.00	36.97	35.99	38.31	36.76
5	36.18	36.78	34.78	36.24	37.90	37.38	37.06	36.77
6	36.55	36.37	34.53	36.46	36.97	37.29	37.47	35.27
7	36.93	36.27	34.65	34.74	37.03	37.53	37.73	37.47
8	36.53	36.07	33.87	36.87	38.49	37.63	37.27	36.33
9	36.23	35.98	36.14	37.61	36.85	37.77	35.83	36.83
10	36.40	35.94	36.68	37.59	37.30	37.09	36.87	36.99
mean	36.3	36.31	35.12	36.7	37.17	37.17	37.15	36.78
stand.dev	0.38	0.38	0.98	0.81	0.76	0.75	0.64	1.12
coeff.of var.	1.06	1.05	2.79	2.21	2.05	2.02	1.73	3.04
t_{sp}	1.91	-1.26	0.69	0.76	0.4	0.61	-0.38	0.05

Table 3.5: Third Experiment: Observed Results For Factorial Experiment

	Changes in ball		No Changes in both		Changes in both		Changes in rod	
run	b1r1	b2r1	r1b1	r1b1	b1r1	b2r2	r1b1	r2b1
1	34.82	34.90	36.82	38.03	33.43	35.20	36.37	34.67
2	37.03	36.83	36.60	37.49	36.06	35.12	33.65	36.49
3	37.59	37.50	37.19	36.94	33.90	37.02	35.13	36.70
4	36.98	36.31	37.31	34.83	34.75	38.19	34.93	37.62
5	37.94	36.69	36.54	36.12	36.09	37.87	36.53	37.00
6	36.32	34.88	36.72	34.58	36.51	37.42	38.19	35.54
7	37.05	36.53	37.84	35.53	36.41	38.36	35.78	36.80
8	37.13	34.90	37.13	35.03	34.35	37.83	35.83	37.27
9	37.11	35.63	37.14	35.78	36.51	37.23	34.43	34.70
10	36.40	38.39	37.55	34.74	34.91	36.43	36.32	36.84
mean	36.84	36.26	37.08	35.91	35.29	37.07	35.72	36.36
stand.dev	0.85	1.19	0.42	1.22	1.16	1.15	1.27	1.04
coeff. of var.	2.32	3.27	1.13	3.39	3.3	3.11	3.55	2.85
t_{sp}	0.43	0.07	1.35	-2.66	1.57	0.96	0.26	0.76

Table 3.6: Fourth Experiment: Observed Results For Factorial Experiment

	No changes in both		Changes in ball		Changes in rod		Changes in both	
run	b1r1	b1r1	b1r1	b2r1	r1b1	r2b1	r1b1	r2b2
1	22.97	24.13	24.51	24.57	24.49	26.69	26.46	27.25
2	23.21	24.03	24.60	25.69	25.49	26.67	25.87	26.24
3	23.45	24.38	24.54	26.33	25.91	26.99	25.37	26.98
4	23.72	24.38	24.35	26.27	26.23	26.97	26.49	26.03
5	23.23	24.90	25.09	26.31	26.43	27.11	26.38	26.15
6	23.38	24.53	25.29	26.14	26.18	27.26	26.40	26.72
7	23.89	24.54	25.33	26.07	26.77	26.93	26.86	26.83
8	24.01	24.13	24.54	26.20	27.19	27.20	26.49	26.79
9	24.19	24.55	24.75	26.59	26.87	26.52	26.68	27.03
10	24.12	24.69	25.03	26.43	26.83	27.31	26.58	26.59
mean	23.62	24.43	24.8	26.06	26.24	26.96	26.34	26.66
stand.dev	0.43	0.27	0.35	0.58	0.8	0.27	0.43	0.4
coeff. of var.	1.81	1.12	1.42	2.21	3.03	0.99	1.63	1.52
t_{sp}	5.55	2.21	1.57	2.19	6.42	1.21	3.89	-0.02

Table 3.7: Fifth Experiment: Observed Results For Factorial Experiment

	Changes in both		No Changes in both		Changes in rod		Changes in ball	
run	b1r1	b2r2	b1r1	b1r1	r1b1	r2b1	r1b1	r1b2
1	32.77	35.96	33.27	32.72	32.90	35.50	33.74	34.64
2	32.92	34.66	22.65	33.42	33.79	36.02	33.52	34.24
3	34.16	34.84	33.78	33.53	32.95	35.38	33.58	33.47
4	33.43	35.75	34.07	33.63	36.52	33.63	35.57	33.33
5	36.59	34.68	33.91	33.94	34.09	36.19	33.87	36.19
6	34.77	36.12	32.81	33.13	33.03	36.21	35.29	34.86
7	35.21	37.27	32.91	33.14	34.23	35.98	35.35	35.66
8	36.89	36.01	33.53	32.97	35.55	33.94	33.83	35.17
9	37.66	34.08	33.73	33.52	33.49	36.07	36.02	35.38
10	34.99	35.89	33.30	32.99	33.88	35.63	36.77	34.53
mean	34.94	35.53	32.4	33.3	34.04	35.46	34.75	34.75
stand.dev	1.69	0.94	3.45	0.37	1.17	0.93	1.18	0.91
Coeff. of var.	4.84	2.66	10.65	1.11	3.44	2.62	3.4	2.62
t_{sp}	4.02	0.43	0.22	-0.4	1.17	0.47	3.41	1.35

Table 3.8: Sixth Experiment: Observed Results For Factorial Experiment

	Changes in ball		No Changes in both		Changes in rod		Changes in both	
run	b1r1	b2r2	b1r1	b1r1	r1b1	r2b1	r1b1	r1b2
1	26.98	27.89	28.67	28.56	29.09	28.41	27.88	28.51
2	27.57	27.95	28.93	28.83	30.74	29.00	28.22	30.71
3	27.33	28.23	28.63	29.35	30.27	30.08	28.10	30.96
4	27.34	27.89	29.17	28.42	29.67	29.97	28.23	29.72
5	28.43	29.03	28.96	30.06	31.13	30.29	28.47	31.46
6	27.87	28.37	29.15	30.00	30.82	30.76	28.16	31.15
7	27.41	28.50	30.39	29.81	30.58	29.87	28.98	31.00
8	28.31	28.73	28.75	30.44	30.19	30.79	28.97	31.46
9	28.23	28.80	29.29	30.51	30.57	30.77	29.08	31.53
10	28.21	28.50	29.92	30.35	30.34	31.18	28.74	31.42
mean	27.77	28.39	29.19	29.63	30.34	30.11	28.48	30.79
stand.dev	0.51	0.4	0.57	0.79	0.59	0.86	0.43	0.97
coeff. of var.	1.83	1.41	1.94	2.67	1.95	2.86	1.51	3.14
t_{sp}	2.49	2.88	2.57	4.42	0.54	4.65	4.02	3.8

Table 3.9: Seventh Experiment: Observed Results For Factorial Experiment

	Changes in rod		Changes in ball		No Changes in both		Changes in both	
run	b1r1	b2r2	b1r1	b1r1	r1b1	r2b1	r1b1	r1b2
1	30.56	30.43	28.83	30.27	31.10	32.01	31.28	31.90
2	30.71	31.25	28.95	31.19	31.73	31.25	31.98	33.03
3	31.27	30.83	29.76	30.93	31.63	31.69	30.84	31.74
4	31.67	30.54	29.37	32.00	31.93	31.86	32.76	32.61
5	31.51	30.67	29.91	30.52	31.53	31.95	32.29	33.33
6	31.24	30.98	30.87	30.81	31.76	31.60	32.29	33.13
7	32.29	31.16	31.57	32.07	30.48	31.08	31.95	32.70
8	31.53	31.48	30.47	31.10	32.11	32.28	31.22	33.02
9	31.21	31.33	30.93	31.46	31.20	32.53	32.96	33.12
10	30.69	31.67	31.23	31.62	32.10	31.43	32.87	33.52
mean	31.27	31.03	30.19	31.2	31.56	31.77	32.04	32.81
stand.dev	0.53	0.41	0.97	0.6	0.51	0.45	0.74	0.59
coeff. of var.	1.68	1.33	3.2	1.91	1.61	1.42	2.3	1.79
t_{sp}	0.47	3.41	5.55	1.54	0.88	0.22	1.63	2.19

Table 3.10: Eight Experiment: Observed Results For Factorial Experiment

	Changes in rod		No Changes in both		Changes in Ball		Changes in both	
run	b1r1	b1r2	b1r1	b1r1	r1b1	r1b2	r1b1	r1b2
1	32.18	32.28	32.13	31.83	32.81	35.50	32.25	34.89
2	31.36	33.09	31.52	32.67	34.73	34.75	32.60	33.39
3	30.71	32.48	31.84	34.31	34.64	34.73	34.69	34.83
4	31.25	31.93	31.78	34.97	32.53	35.59	33.49	35.22
5	32.42	32.45	31.83	31.89	35.03	35.10	35.14	35.88
6	32.50	32.00	31.92	34.34	34.40	35.26	32.84	36.21
7	31.61	32.88	31.57	34.28	34.41	34.49	34.97	34.36
8	32.09	32.27	34.59	32.85	34.27	34.85	35.30	33.41
9	31.88	32.33	32.11	34.41	34.59	35.47	33.13	34.58
10	31.98	32.33	31.73	34.27	34.91	35.69	35.47	35.60
mean	31.8	32.4	32.1	33.58	34.23	35.14	33.99	34.84
stand.dev	0.56	0.36	0.9	1.15	0.86	0.42	1.25	0.95
coeff. of var.	1.77	1.1	2.79	3.44	2.51	1.19	3.66	2.73
t_{sp}	0.58	-0.42	0.19	1.04	0.61	0.58	2.57	0.22

The summary of results of the preliminary analysis for the seven experiments in Table 3.4 through 3.10 are presented as follows:

- ↦ From Table 3.4, the absolute differences of the standard deviation and the coefficient of variation for each of the factor settings are 0(0.01), 0.17(0.58), 0.01(0.03), 0.48(1.31) for the ϕ, rod , ball, and interaction effects, respectively. The values in the parenthesis above and thereafter in this section represent the absolute difference in the coefficient of variation. For this experiment, none of the computed t_{sp} fall outside the interval given in (3.5). Further, the mean of the second sequence is greater than the mean of the first sequence only for the rod effect. This suggests that there is no trend in the experiment, and hence no conclusion is visible from the experiment.

- ↦ In the results for the third experiment (see Table 3.5), the absolute differences of the standard deviation and the coefficient of variation for each of the factor settings are 0.34(0.95), 0.8(2.26), 0.01(0.19), 0.23(0.7) for the ball, ϕ, interaction, and rod effects, respectively. In this experiment, only the t_{sp} for the ϕ effect fall outside the interval in (3.5), this suggests that there is no trend in the sequence for both the rod and the ball. Using the mean value, the mean of the second sequence is greater than the first sequence for both the interaction and rod effects. Therefore, no conclusion is visible from the experiment.

- ↦ From the fourth experimental results (see Table 3.6), the absolute differences of the standard deviation and the coefficient of variation for each of the factor settings are 0.16(0.69), 0.23(0.79), 0.53(2.04), 0.03(0.11) for the ϕ, ball, rod, and interaction effects, respectively. The computed t_{sp} for the ϕ, rod, and interaction effects are greater than the critical value (that is, they lie outside the interval given in (3.5)). Hence there is presence of trend in the sequences for the said factors. Also the mean of the second sequence is greater than the mean of the first sequence for all the factors in this experiment. Since the absolute difference in the coefficient of variation for the rod effect is relatively high compared to the other factors with trend, then the rod might be a possible candidate responsible for the trend in the experiment.

- ↦ In the results for the fifth experiment (see Table 3.7), the absolute differences of the standard deviation and the coefficient of variation for each of the factor settings are 0.75(2.18), 3.08(9.54), 0.24(0.82), 0.27(0.78) for the interaction, ϕ, rod, and ball effects, respectively. Also, the mean of the second sequence is greater than the mean of the first sequence for all the factors except for the ball effect that has equal mean value. Further, the computed t_{sp} implies that trend is present only in the interaction and ball effects sequence and since the absolute differences in the computed degree of variation for the ball effect is not so high in comparison to effects of other factors, then the interaction effect is taken to be the only candidate that can be considered.

- ↦ Table 3.8 shows the results for the sixth experiment. For this experiment, the absolute differences of the standard deviation and the coefficient of variation for each of the factor settings for the ball, ϕ, rod, and interaction effects are 0.11(0.42), 0.22(0.73), 0.27(0.91), and 0.54(1.63), respectively. The computed t_{sp} for all the factor settings lies outside the interval in (3.5). This implies that there is presence of trend in the experiment. Examination of the mean shows that the mean of the second sequence is greater than the mean of the first sequence for all the factors except for only the rod effect. The interaction effect which has an outstanding absolute different coefficient of variation is a likely candidate responsible for the observed trend in the experiment.

- ↦ The results of the seventh experiment (see Table 3.9) reflect that there is presence of trend in the experiment since the mean of the second sequence is greater than the mean of the first sequence for all the factors. However, an examination of the t_{sp} reflect that only the ball effect could be adjudged to be a possible candidate responsible for the trend in the experiment.

- ↦ In Table 3.10 (the eight experimental result), the absolute differences in the standard deviation and the coefficient of variation for each of the factor settings are 0.2(0.67), 0.25(0.65), 0.44(1.32), 0.3(0.93) for the rod, ϕ, ball, and interaction effects, respectively. There is presence of trend in the experiment since the mean of the second sequence is greater than the mean of the first sequence for all the factors. However, only the computed t_{sp} value for one of the sequence for interaction effect implies presence of trend. Therefore, for this experiment we cannot make any viable conclusion.

The experimental results in Tables 3.3 to 3.10 are summarized according to the experimental factors and are presented graphically in Figure 3.5.

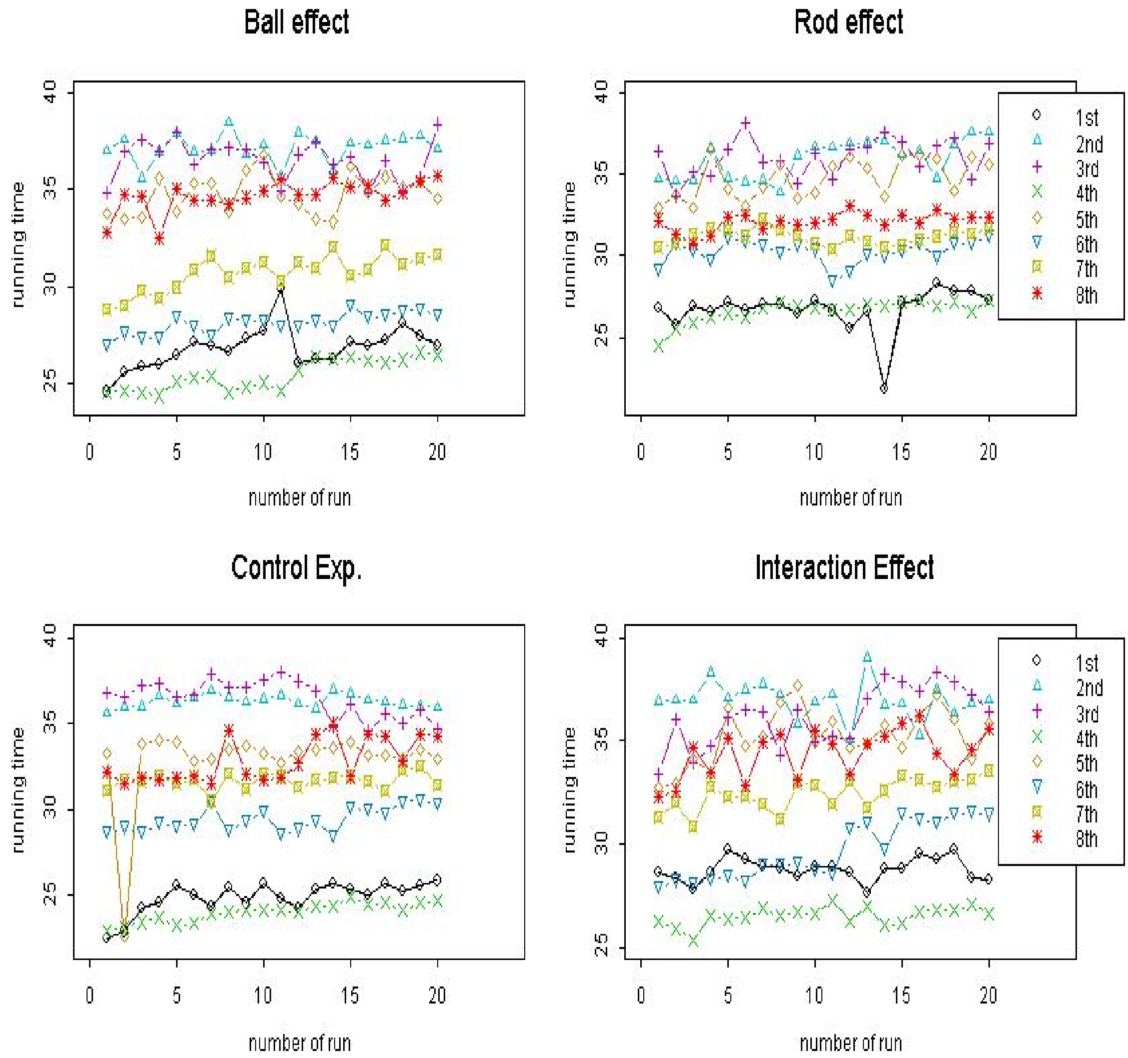


Figure 3.5: *Summary plot for factorial experimental result for the ball and rod effects*

From the graphs in Figure 3.5, the following are inferred: (a) the ball has an effect on the time-trend in five of the experiments, (b) presence of trend due to the rod

effect are seen in two of the experiments, and (c) the interaction between the ball and the rod has an effect on the time-trend in four of the experiments.
Since only one-quarter of the eight experiments shows the rod to be a possible candidate, then the rod can be said to have no or very little effect on the trend in the experiment. Therefore, we are left with only two possible factors that could be responsible for the time-trend in the experiment. These two possibilities are the ball and the interaction between the ball and the rod. It should however, be noted that the above submission is based on the preliminary analysis of the experimental data.

In order to make a comprehensive analysis of the experimental data in Tables 3.3 to 3.10, we summarized the data into a two factorial experiment according to the experimental factors. The response is taken to be the sum of differences for each experimental columns in Tables 3.3 to 3.10. Thus we have thirty-two runs with two factors. The design matrix and the summarized response are presented in Table 3.11.

Table 3.11: Summary of Factorial Experimental Results

	Factors		
Run	Ball	Rod	Response
1	-1	-1	7.75
2	1	-1	7.91
3	-1	1	-1.45
4	1	1	0.29
5	-1	-1	0.09
6	1	-1	0.08
7	-1	1	15.79
8	1	1	-3.68
9	-1	-1	-11.77
10	1	-1	-5.81
11	-1	1	6.47
12	1	1	17.75
13	-1	-1	8.09
14	1	-1	12.57
15	-1	1	7.26
16	1	1	3.23
17	-1	-1	9.03
18	1	-1	-0.07
19	-1	1	14.12
20	1	1	5.87
21	-1	-1	4.47
22	1	-1	6.21
23	-1	1	-2.28
24	1	1	23.09
25	-1	-1	2.11
26	1	-1	10.08
27	-1	1	-2.34
28	1	1	7.66
29	-1	-1	14.80
30	1	-1	9.11
31	-1	1	6.06
32	1	1	8.49

To analyze the data in Table 3.11, we used the two levels factorial model given in matrix form by

$$y = \beta X + \varepsilon \tag{3.6}$$

where y is an $(n \times 1)$ vector of response, X is an $(n \times p)$ matrix of the regressor factors,

$\beta = [\beta_0, \beta_1, \beta_2, \beta_{12}]'$ is a $(p \times 1)$ vector of unknown parameters with β_0 representing the overall mean effect, β_1 representing the ball effect, β_2 representing the rod effect, and β_{12} representing the interaction between ball and rod effect. ε is an $(n \times 1)$ vector of random errors, with $\varepsilon \sim NID(0, \sigma^2)$. On analyzing the data in Table 3.11, the parameter estimates $(\hat{\beta})$ for the model given in (3.6) are found to be 5.6556, 0.7681, 0.99, and 0.4237 for the overall mean, ball, rod, and interaction effects, respectively. That is, $\hat{\beta} = [0.6556, 0.7681, 0.99, 0.4237]'$. The analysis of variance table to test for the significance of the estimated parameter effects is presented in Table 3.12 below.

Table 3.12: Analysis of Variance Table For Factorial Experimental Results

Source	DF	Sum of Square	Mean Square	F-value	p-value
ball	1	18.881	18.88051	0.3268800	0.5720637
rod	1	31.363	31.36320	0.5429939	0.4673245
ball& rod	1	5.746	5.74605	0.0994819	0.7547911
Residuals	28	1617.273	57.75977		
Total	31	1673.263			

The estimated p-values on the above ANOVA table imply that the ball, rod, and interaction between ball and rod effects are not significantly different from zero. Thus we can conclude that neither the ball, the rod nor the interaction between ball and rod have a significant effect on the running time of the ball bearing. Furthermore, using the parameter estimates and the information on the ANOVA table, we computed the confidence interval for the mean response of the running time. A 100(1-α) percent confidence interval on the mean response at the point say, $X_0 = [1, x_{01}, ..., x_{0k}]'$ given by

$$\hat{y_0} - t_{\alpha/2,n-p}\sqrt{\hat{\sigma}^2 X_0'(X'X)^{-1}X_0} \leq y \leq \hat{y_0} + t_{\alpha/2,n-p}\sqrt{\hat{\sigma}^2 X_0'(X'X)^{-1}X_0}$$

was used. Here $\hat{y_0} = \hat{\beta}X_0'$ is the mean response at point X_0, $x_{0i} \in \{1, -1\}, i = 1, 2, ..., k$, n is the number of run in the model design matrix X, and p the number of estimated parameter. Therefore, the 95% confidence interval on the mean for the sum of differences in the running time for the two sequence experiments conducted with ball 1 rod 1 is [-0.2497, 8.8922], and for ball 1 rod 2 is [0.8828, 10.0247]. Similarly for ball 2 rod 1 we have [0.4391, 9.5809] and for ball 2 rod 2, we have [3.2666, 12.4084]. Hence, the smallest mean for the sum of differences in the running time for the two sequences in the experiments should be approximately -0.25, and the maximum sum of differences of the running time should be 12.41.

An alternative way in which the experimental results in Tables 3.3 to 3.10 can be analyzed is to arrange the different experiments as block factorial experiment. This is done by using each of the eight factorial experiments as block. The differences in the sum of the experimental results of the two sequences for each factor in Table 3.3 to 3.10 are taken to be the response variable. This is obtained by taking the sum of the first sequence of a factor away from the sum of the second sequence of that factor. Table 3.13 presents the summary for the differences in the sum of the experimental results of the two sequences for each factor arranged as a blocked factorial experiment.

Table 3.13: Two Factorial Experiment With Block Effect

	Factors				
Block	ϕ	Rod	Ball	interaction	Total
1	7.75	7.91	-1.45	0.29	14.5
2	0.09	0.08	15.79	-3.68	12.28
3	-11.77	-5.81	6.47	17.75	6.64
4	8.09	12.57	7.26	3.23	31.15
5	9.03	-0.07	14.12	5.87	28.95
6	4.47	6.21	-2.28	23.09	31.49
7	2.11	10.08	-2.34	7.66	17.51
8	14.80	9.11	6.06	8.49	38.46
Total	34.57	40.08	43.63	62.7	180.98

The model for a two factorial experiment with block effect is

$$y_{ijkl} = \mu + \xi_i + \beta_j + \xi\beta_{ij} + \gamma_k + \varepsilon_{ijkl} \tag{3.7}$$

where μ represents the overall mean effect, ξ represents the ball effect, β represents the rod effect, $\xi\beta$ represents the interaction between ball and rod effect, γ represents the block effect, and ε represents the random error term. Using the model in Equation (3.7) to analyze the data in Table 3.13, the analysis of variance table is similar to that of Table 3.12. The sum of square for the rod, ball and interaction between ball and rod effects are the same as that of Table 3.12. The sum of square for the block and error effects equal 224.1837 and 1393.0895, respectively. Hence, the p-values for the rod, ball, interaction, and block effects are 0.50, 0.60, 0.77, and 0.84, respectively. Thus, none of the ball, the rod, nor the interaction between ball and rod has a significant influence on the trend in the experiment.

Examinations with the eye during the conduct of the experiments show that the time it takes the ball to roll through the rod is approximately constant at 1.25 seconds for all the experiments conducted irrespective of changes in factor. However, the time it takes the ball to spiral through the funnel to the uppermost part of the funnel tip and within the funnel tip varies (does not remain constant) from one experiment to another. Therefore, the funnel is suspected to have an influence on the time trend in the experiment.

3.2.2 Experiment for funnel influence

In order to determine the influence of the funnel and the interaction between ball and funnel on the observed time-trend in the experiment, we planned another factorial experiment. The experiment involves two similar balls of the same size (as before), a rod and two similar funnels. For this experiment one machine was used. We started by conducting the first 10 runs with Ball 1 and Funnel 1. The second 10 runs were obtained by using the same funnel with the second ball (Ball 2) after a time lag of 5 minutes. The process was repeated with another funnel (Funnel 2). Hence the experiment is a two factors experiment conducted over a day. The obtained results are presented in Figure 3.6.

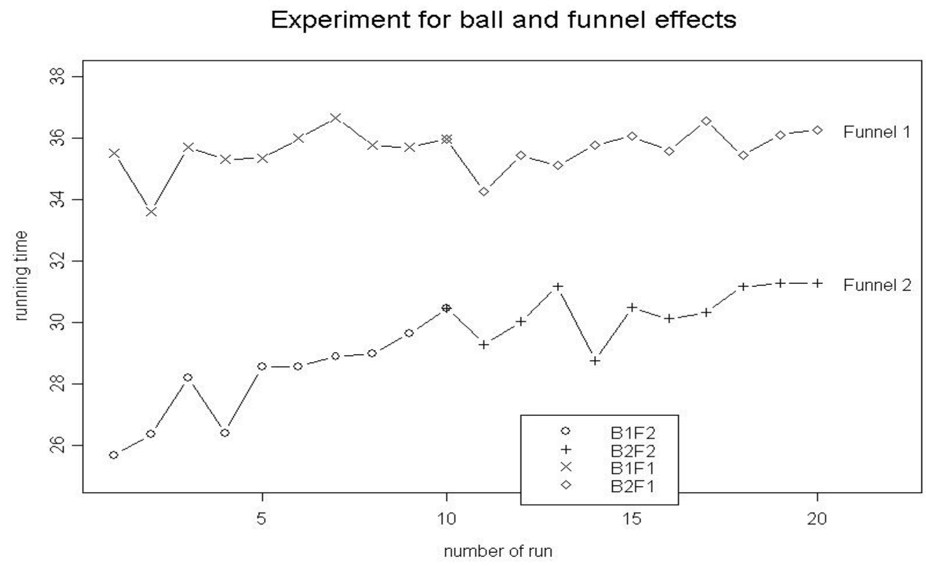


Figure 3.6: *Influence of the ball and funnel effects on the time-trend in the experiment. B1F1 = result from Ball 1 with Funnel 1, B2F1 = result from Ball 2 with Funnel 1, B1F2 = result from Ball 1 with Funnel 2, B2F2 = result from Ball 2 with Funnel 2*

The computed $|t_{sp}|$ values for the experimental data plotted on Figure 3.6 are 1.85 for Funnel 1, and 9.16 for Funnel 2. For these experimental data sets, the interval given in Equation (3.3) equals [-2.100, 2.100]. Therefore, there is no trend in the data obtained with Funnel 1 while there is trend in the data obtained with Funnel 2. Although the above result may have happened by chance, it is an indication that the process behaved differently in the two funnels. This result leads us to the next section.

Cross-over experiment for funnel influence

In this section, because of the differences in the behavior of the funnel, we planned a cross-over experiment to determine the influence of the funnel on the observed time-trend in the experiment. This is to take care of the possibility of any carry over effect on the experimental response. The experiment was conducted with a ball, a rod, and two funnels over a period of two days. We made use of cross-over experimental design setting with two funnels conducted over two periods in two sequences. Hence, we have a 2×2 cross-over experiment. It should be noted that the period effect in the carry over setting represents the time-trend effect in this case. The experiment is conducted as follows: On day 1, the first 10 runs were done by using Funnel 1 along with a ball and a rod. Thereafter we changed to Funnel 2. Using the same rod and ball, we conducted another 10 runs. A transition period of five minutes is between the two partition of runs, this is the period we used to change the funnel. On day 2, the first 10 runs were conducted with Funnel 2 along with the same rod and ball used on day 1. The funnel was changed to Funnel 1 and we conducted the remaining 10 runs using the same ball and rod as before. The setting of the experiments and the observed results are presented in Table 3.14.

Table 3.14: Cross-Over Experimental Setting and Observed Results

	Sequence 1					Sequence 2				
	(Funnel 1)					(Funnel 2)				
Day 1	25.55	26.40	26.70	27.00	26.38	14.89	15.68	16.24	16.60	16.70
	27.18	27.52	27.20	28.04	26.34	16.87	17.51	17.55	17.13	17.47
	(Funnel 2)					(Funnel 1)				
Day 2	17.23	16.81	17.05	16.76	17.19	25.15	25.57	25.99	25.89	26.77
	16.65	17.69	17.20	17.99	17.33	26.52	27.77	26.93	28.18	27.13

The averages of the observed running time for the four cells in Table 3.14 are 26.831, 16.664, 17.19, and 26.59 for (Funnel 1 day 1), (Funnel 2 day 1), (Funnel 2 day 2), and (Funnel 1 day 2) respectively. The average running time in the first sequence decreases with days but reverse is the case for the second sequence. A close examination shows that the average running time for Funnel 1 is higher than that of Funnel 2, hence the pattern for the sequence. To determine existence of trend in each cell of Table 3.14, a trend test was carried out using Equations (3.1) through (3.3). The obtained t_{sp} and its corresponding p- values giving in parenthesis thereafter for day 1 are 1.5425(0.0808) and 5.9457(0.0002) for funnels 1 and 2, respectively. Similarly for day 2, we have 6.4212 (0.0001) and 1.7(0.0638) for funnels 1 and 2, respectively. If we compare the obtained t_{sp} values with the interval [-2.1, 2.1], it implies none existence of trend, existence of trend, no existence of trend, and existence of trend in the order (Funnel 1 day 1), (Funnel 2 day 1), (Funnel 2 day 2), and (Funnel 1 day 2), respectively. A striking behavior in the above results is the consistency of the trend pattern for the funnels with days. That is, irrespective of the changes in days, there seems to be trend in the results obtained with Funnel 2. The plot of the observed results for the two days are presented in Figure 3.7.

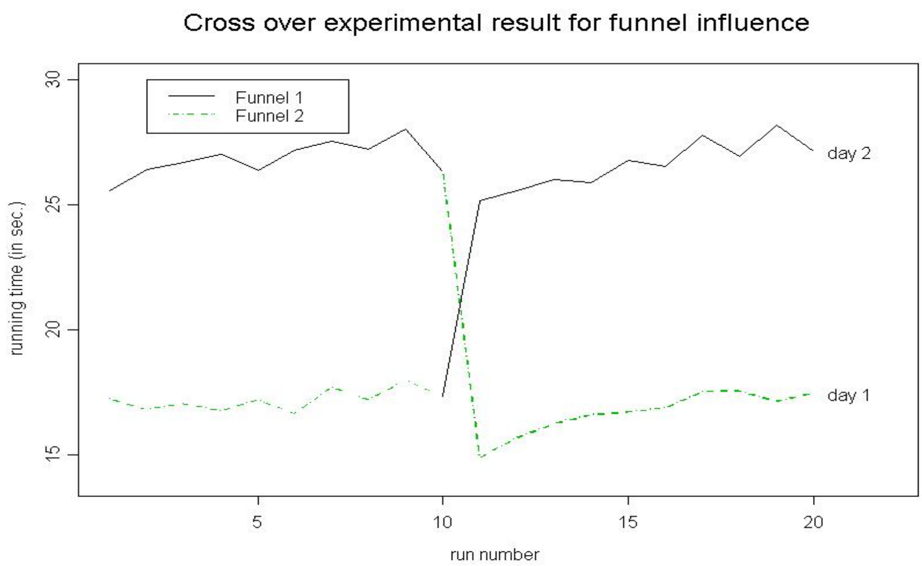


Figure 3.7: *Influence of the funnel on the time-trend in the experiment. The experiment was conducted over a period of two days*

The pattern of the plotted results in Figure 3.7 and the large difference in the averages of the observed response for the two funnels depicts differences in the performance of the funnels. On comparing the plot for the two days (see Figure 3.7), it is observed that the response increases with time on the second day despite changes in funnel, but on the first day, the scenario is totally different. The running time in Funnel 1 has a similar pattern on both days, which is similar for Funnel 2. Thus, the pattern of the observed running time (experimental response) for the funnels on both days are almost alike. In addition, the observed running time for Funnel 2 is far less than that of Funnel 1 irrespective of changes in day and sequence. From the above results and inference, it is clear that the funnel behaved differently and there is no possibility that day has some influence on the observed trend in the experiment.

To confirm that periods (days) have a significant influence on the trend in the experiment, we conducted another cross over experiment on a single day using the "abba" design setting. Two identical funnels, a ball and a rod were used. We started with Funnel 1 and conducted 10 runs. The funnel was changed and we waited for five minutes before conducting another 10 plus 10 runs using the same ball and rod. Therefore 20 runs were conducted on Funnel 2 before we changed back to Funnel 1 and conducted another 10 runs. The observed results are display in Figure 3.8. For more on the "abba" design (see for example, Toutenburg ;1995 and Ratkowsky, Evans, & Alldredge; 1993).

The computed t_{sp} and p-values (in parenthesis) for the data that generated Figure 3.8 are 7.75(0.0000), 3,28(0.0056), 5.21(0.0004), and -0.26(0.4007) for Sequence 1 Funnel 1 (S1F1), Sequence 1 Funnel 2 (S1F2), Sequence 2 Funnel 1 (S2F1), and Sequence 2 Funnel 2 (S2F2), respectively. Similarly the computed t_{sp} and p-values for the data for funnels 1 and 2 are 19.64(0.0000) and 3.88(0.0005), respectively. Therefore, the trend analysis on the data that generated Figure 3.8 confirmed presence of trend in the two funnels. Hence, there is time trend in the two funnels and therefore changes in days have little or no influence on the trend in the experiment.

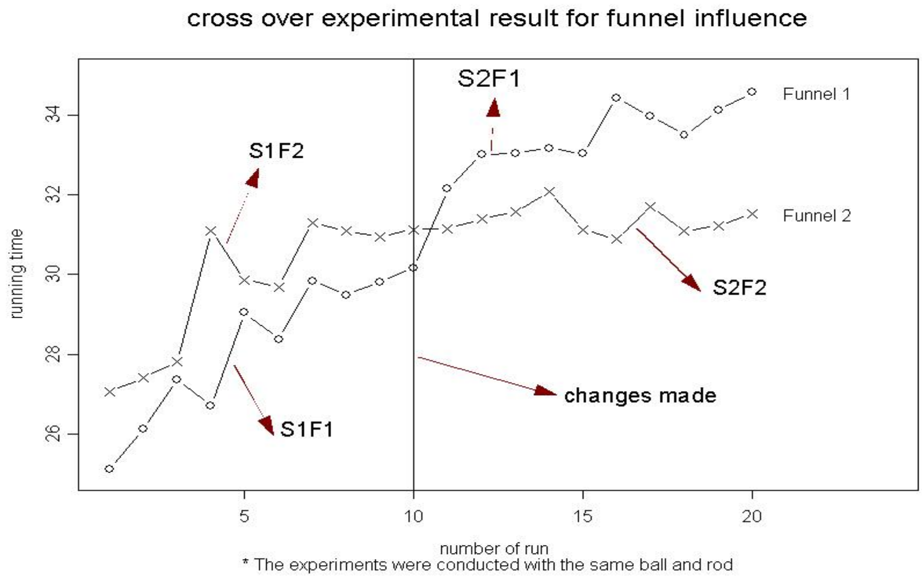


Figure 3.8: *Influence of the funnel on the time-trend in the experiment. The experiment was conducted on a single day*

The plots in 3.7 and 3.8 reflect that the carry over and period effects have no significant effects on the observed responses. It should be noted that the two experiments that produce figures 3.7 and 3.8 were conducted independently with two funnels, one ball and one rod. Furthermore, since the results from these two experiments show an indication of trend, we are convinced that we do not need to carry out further experiments in this study. A likelihood explanation for what is happening within the experiment, is that either the funnel is being warmed up by the ball or that the ball gets warmed up by the rod and hence the spinning (running) time of the ball with in the funnel keeps on getting larger as we proceed in the experiment. The later is evident in the results from the second sequence for Funnel 1 (S2F1 in Figure 3.8) which is the last sequence in the conduct of the "abba" experiment conducted over one day (see Figure 3.8). Therefore, we can conclude that there is no other factor except the funnel that is responsible for the observed time-trend in the experiment. It should however be noted that in order to have a comprehensive analysis of the cross-over experiment, more experiments will have to be conducted. This will be an area of interest for future work.

3.3 Experiment with the American type funnel

Examination with the eye during the conduct of the experiments show that the ball behaves in a random manner within the "funnel tip". The running time of the ball bearing in this region keeps changing for different repetition of experiment. Thus, an indication that the trend in the experiment may be due to the behavior of the ball within the funnel tip.

To confirm that the behavior of the ball within the funnel tip is responsible for the trend in the experiment, we constructed another funnel without the tip, this we called the American type funnel (see Figure 3.2 on page 42). This funnel is identical in shape and geometry with the upper part of the funnel with tip (the earlier used funnel) and it is made of aluminium. Using the American type funnel, we repeated the earlier experiments for showing time trend using the same ball and rod as before. The experiments were conducted on the same day keeping all factors settings fixed as before. We started with Funnel 1 by conducting 16 runs as before. After a waiting time of five minutes, we conducted another 16 runs with Funnel 2. In the same manner we conducted another 16 runs with Funnel 3. The obtained running time for funnels 1, 2, and 3 are presented in Figure 3.9.

The computed t_{sp} and p- values (in parenthesis) for the series of the three funnels in Figure 3.9 are $8(p < 0.0001)$, $16.94(p < 0.0001)$, and $19.65(p < 0.0001)$ for funnels 1, 2 and 3, respectively. Therefore, there is trend even in the American type funnel. From the above results, the following are inferred:
(i) The running time keep on increasing as we proceed in the experiment despite changes in funnel (see Figure 3.9). The average running times for the experiment conducted with the first funnel is less than that of the second funnel and that of the second funnel is less than that of the third funnel. This point to our earlier submission that either the ball or the funnel keeps on warming up as we proceed in the experiment. (ii) Two identical funnels behaved differently. (iii) The funnel tip does not have any role to play on the observed trend in the experiment. (iv) Days

do not have any significant influence on the trend in the experiment.

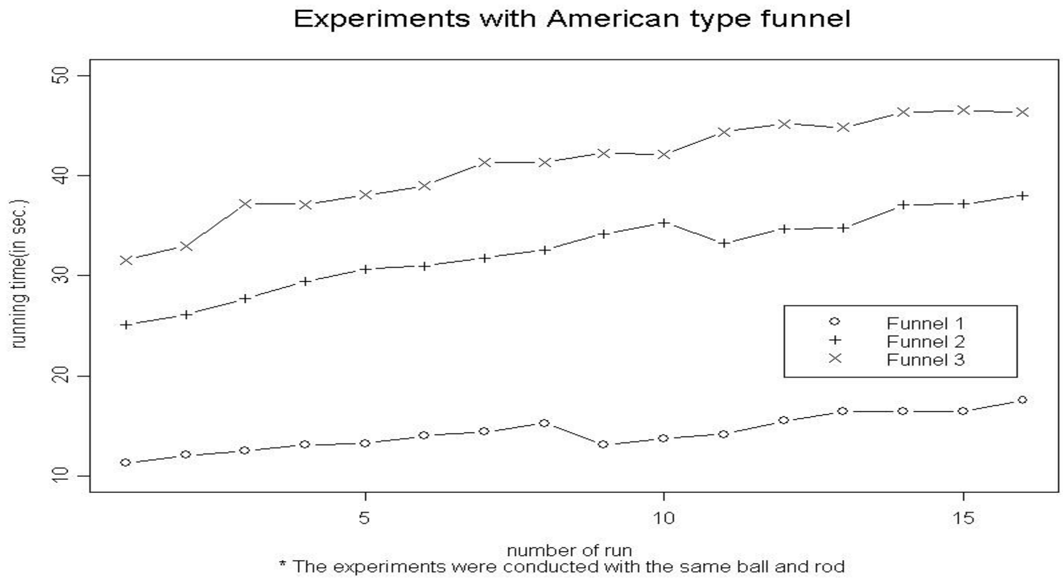


Figure 3.9: *Experimental results with three identical funnels*

The conclusion from the graphical, the factorial analysis, and the cross-over experimental results all lead to one point; the funnel has the highest influence on the trend in the experiments and hence responsible for the observed trend in our exemplified experiment.

To remove the observed time-trend in the funnel experiment, we concentrate on the funnel. This is due to the fact that the analysis on the data from the conducted experiments pointed to the fact that the funnel is responsible for the trend. On cleaning the funnel with a static cloth before conducting the experiment, the observed running time did not show presence of trend. Also, on increasing the time lag between consecutive runs in the experiment from 38 seconds to 120 seconds, the observed running time did not reflects presence of trend. Therefore, using any of the two aforementioned possibilities, the observed time-trend in the experiment was eliminated.

Chapter 4

Comparison of Run Orders of Unreplicated Fractional Factorial Designs

The main focus in this chapter is on the performance of run orders of unreplicated two-levels factorial/fractional factorial designs. However, the approach discussed here can be extended to more than two level factors. Our objective is to compare the performance of these run orders with regards to active contrast in situations where there is a trend influencing the experimental data. A contrast is said to be active if it has a true effect on the behavior of the response. The response used for the comparison is based on the two data sets produced from the funnel experiment where we have observed a strong time-trend as shown in Chapter 3. In order to achieve the set objective, three types of run orders of a fractional factorial design are considered. These are the standard, randomized and systematic run orders for a non-replicated 2^{k-p} experiment, where, $k-p=4$ and therefore the number of runs equals 16.

The chapter is divided into two parts. In the first part, a systematic run order that is linear trend resistant is compared with the standard run order of unreplicated 2^{k-p} design. While in the second part, performance of the standard, randomized, and systematic (time-trends resistant) run orders are measured. The constructed 2^{5-1} designs in Chapter 2 are used to make the comparison. The design in Table 2.1 is taken to be the standard run order, the reduced design in Table 2.3 which is linear time-trends resistant is taken to be the systematic run order, and randomization of the run order of the design in Table 2.1 as the randomized run order. Note that the trend resistant design does not allow estimation of some two factor interactions, the corresponding columns are confounded with the time trend. These two factor interactions are therefore not estimable.

To compare the standard run order and the systematic run order (linear time trend resistant design), **sensitivity** to presence of active contrast of the run orders is used as an evaluation standard. Performance of the standard, randomized, and systematic run orders are measured by **probability of false rejection** of active contrast and **probability of effect detection** of active contrast via simulation studies. Thus, we make use of three evaluation standards for comparison of the run orders.

To determine the three evaluation standards mentioned above to compare the run orders of a fractional factorial design, we use the **censored data approach** in conjunction with the **half normal plot principle**.

The censored data approach is defined as follows. Let y_j, where $j = 1, 2, ...n$ and $n = 2^{k-p}$, denote the response from a 2^{k-p} experiment. Further, let m be a constant value. To censor the data, the constant value m is added to the experimental response y_j for all runs j where the active factor i is at the high level. More precisely,

$$y_j^{(m)} = \begin{cases} y_j + m, & \text{if factor } i \text{ is at the high level } (+) \text{ in run } j \\ y_j, & \text{if factor } i \text{ is at the low level } (-) \text{ in run } j \end{cases} \tag{4.1}$$

where $i \in \{1, 2, ..., n-1\}$ represents the contrast used to censor the data. The new variable from the censored data approach, that is, $y_j^{(m)}$ is referred to as a censor data. For example, using the setting of one contrast, say the first column of a model design matrix, add $m \geq 1$ to the data (y_j) whenever the entries of the first column of the model design matrix of the design under study are at + level, and zero to y_j when the entries of the first column are at - level. For two contrasts, add $m \geq 1$ to y_j whenever the entries of, say, the first or the second columns of the model design matrix are at + levels, $2m$ to y_j whenever the first and the second columns are at + levels, and zero otherwise. For three active contrasts, add $m \geq 1$ to y_j whenever the entries of say, the first or second or third columns of the design matrix are at + levels, $2m$ to y_j when two of the three columns are at + levels, $3m$ to y_j when the first three columns are at + levels, and zero otherwise. In general, if we assume two or more factors to be active, then $y_j^{(m)}$ is derived by adding m for each of the active factors that is at level (+) in the run j. The vector with entries $y_j^{(m)}$, $1 \leq j \leq n$, is called censored observations.

The half normal plot was proposed by Daniel (1959, 1976) and later improved on by Zahn (1975a, 1975b). It is a tool for assessing the significance of contrasts and interpreting unreplicated two-levels fractional factorial designs. The visual inspection of a half normal plot is a popular procedure for interpreting data from unreplicated factorial experiments (Olguin and Fearn, 1997). A random variable, Y, say, is said to have the half normal distribution if its density g can be written as

$$g(y) = \begin{cases} \left[2/\left(\pi\sigma^2\right)\right]^{1/2} exp\left[-y^2/\left(2\sigma^2\right)\right], & y \geq 0 \\ 0, & \text{otherwise} \end{cases} \tag{4.2}$$

If $\sigma^2 = 1.0$, the distribution in (4.2) will give a standard half normal distribution. The procedure for half normal plot for a fractional factorial design is presented as follows. For a 2^{k-p} design, let X represent the model design matrix with $b = 2^{k-p} - 1$ contrasts. Also let $\hat{\beta}_1, \hat{\beta}_2, ..., \hat{\beta}_b$ be the b estimates of contrast effects, where $\hat{\beta}_i$, $1 \leq i \leq b$ is given by $\hat{\beta} = (X'X)^{-1}X'y$, and y is the observed experimental response. Let $|\hat{\beta}_{[1]}| \leq |\hat{\beta}_{[2]}| \leq ... \leq |\hat{\beta}_{[b]}|$ represent the ordered absolute effects for the vector of parameters in the model design matrix X. The half normal plot involves plotting each of the ordered absolute contrast effects $\hat{\beta}_i$ against the corresponding $(i - \frac{1}{2})/b$ percentile of the standard half normal distribution on half normal probability paper. That is, plotting $|\hat{\beta}_{[1]}| \leq ... \leq |\hat{\beta}_{[b]}|$ against $\Phi^{-1}\left(1/2 + (i - 1/2)/(2b)\right), 1 \leq i \leq b,\ b = 2^{k-p} - 1$, where $\Phi^{-1}(.)$ represent the inverse of cumulative distribution function of the normal distribution. Assuming normality of the response, the estimates corresponding to non active contrasts on the half normal plot should form an approximately straight line while the significant contrasts (active contrasts) should appear at a distance as outliers. There are many other versions of half normal plot (see e.g., Zahn 1975b).

The general assumption for the approaches used in this study for comparing the run orders is as follows: (1) When we assume that none of the contrast is active, the original experimental data is used to estimate the contrast effects of the run order. (2) If a contrast is assumed to be active, it is used to censor the data.

4.1 Sensitivity to active contrast

Sensitivity to presence of active contrast is used as a standard to measure the performance of the standard and systematic run orders. To determine the run order that is more sensitive to presence of active contrast, we use some of the columns of the model design matrix X to censor the observed response. This is done by adding a constant value to the response variable when the setting of the column in question is at + levels and zero otherwise as given in Equation (4.1). The new data set $y_j^{(m)}$ is then used to estimate the contrasts (β_i). It is expected that the absolute estimate of the contrast used to censor the response from the experiment (experimental data) should be higher than the absolute estimate of all other contrasts in the model design matrix, and hence should become an outlier to all other contrasts (that is, maximum absolute contrast) on the half normal plot. The algorithm for the procedure is presented in sequence as follows:

(i) Censor the experimental data (response variable) as described in Equation (4.1) using a column of the model design matrix X. This gives a new data set.

(ii) Use the new data set obtained in step (i) to estimate the effect of all the b

contrasts in the model design matrix X.

(iii) Plot the ordered absolute contrast estimates ($|\hat{\beta}_{[1]}| \leq |\hat{\beta}_{[2]}| \leq ... \leq |\hat{\beta}_{[b]}|$) against the corresponding $(i - \frac{1}{2})/b$ percentile of the standard half normal distribution ($\Phi^{-1}(1/2 + (i - 1/2)/(2b))$).

(iv) Record the contrast effect with the maximum absolute estimate on the half normal plot.

(v) If the m value used does not make the contrast used to censor the data the maximum absolute contrast effect on the HNP, increase m by 1 and repeat steps (i-iv).

(vii) Stop the process and record the m as soon as the estimated effect of the contrast used to censor the data is the maximum absolute contrast effect on the half normal plot.

Following the algorithm above, for a chosen run order, we start by adding $m = 1$ to the response variable using the setting of a contrast. If the value $m = 1$ does not make the contrast used to censor the data to be the maximum absolute contrast on the half normal plot for the run order under study, then we use $m = 2$, and so on until we get a m value that makes the estimated effect of the contrast used to censor the data to be declared as the maximum absolute contrast on the half normal plot. A run order that declares the estimated effect of the contrast used to censor the data as the maximum absolute contrast on the half normal plot with the smallest m value is adjudged to be more sensitive to presence of active contrast.

4.2 Simulation Study

The purpose of the simulation study is to compare the behavior of the randomized run order with a systematic order (linear trend resistant design) and the standard run order of an unreplicated fractional factorial designs. For each of the run orders under consideration, we based our simulation study on 10,000 simulations for a 16 runs experiment. In our study, a design identified some contrasts as active whenever the largest absolute value of the test statistic is greater than a given (simulated) critical value at a desired α level of significance. The proportion of simulated designs with false rejection is an estimate of the probability of false rejection (**PFR**) while the proportion of simulated designs with correct detection of an active effect estimates the probability of effect detection (**PED**).

Our analysis is based on an easy formal version of the half normal plot. The half normal plot test statistic for a given set of b contrasts under the null hypothesis of no active contrast, that is, $H_0 : \beta_1 = \beta_2 = ... = \beta_b = 0$, is given by

$$t_{ih} = \frac{\hat{\beta}_i}{\hat{\sigma}_h}, i = 1, 2, ..., b \tag{4.3}$$

Here h represents the statistic used to estimate σ, and $\hat{\sigma}$ is the estimate of the standard deviation of the contrasts. In this study, three statistics are used independently to estimate σ. These are based on the median of absolute contrasts (MAC), the pseudo standard error (PSE) proposed by Lenth(1989), and the asymptotic standard error (ASE) proposed by Dong (1993). Hence, h in Equation (4.3) can take MAC, PSE, and ASE depending on what is used as an estimate of σ. The estimate of the standard deviation of the contrasts based on MAC, PSE and ASE is given by

$$\hat{\sigma}_h = \begin{cases} 1.5 \bullet median|\hat{\beta}_i|, & h = MAC \\ \\ 1.5 \bullet median_{|\hat{\beta}_i| \leq \ 2.5 \bullet \hat{\sigma}_{MAC}} \ |\hat{\beta}_i|, & \text{h=PSE} \\ \\ \sqrt{\frac{1}{z} \sum_{|\hat{\beta}_i| \leq \ 2.5 \bullet \hat{\sigma}_{MAC}} \hat{\beta}_i^2}, & \text{h=ASE} \end{cases} \tag{4.4}$$

respectively. Here z is the number of contrasts with $|\hat{\beta}_i| \leq 2.5 \bullet \hat{\sigma}_{MAC}$.

We say that there is an active factor in the design if the largest of the t_{ih} is larger than a critical value that depends on the number b of contrasts considered.

4.2.1 Probabilities of false rejection and effect detection

The probability of false rejection (PFR) describes the proportion of designs that falsely declare the presence of an active contrast. This is synonymous with the type 1 error (Level of a test). Usually, we allow for a PFR ranging between 1% and 5%. A test is valid, if the true probability of false rejection is not larger than the nominal PFR α. In our simulation study we estimate PFR by the proportion of designs that falsely declare that there is at least one active contrast if in reality there is none.

The probability of effect detection (PED) measures the proportion of designs that rightly declare presence of active contrasts. It is the probability of making a correct decision (power of a test).

The probabilities of false rejection and effect detection are, therefore, taken to be the expected proportion of designs with $\max(t_i) > C(b, \alpha)$, where $C(b, \alpha)$ is an appropriate critical value which depends on the number b of contrasts plotted on the half normal plot and on the desired α level of significance. Hence, the probabilities of false rejection and effect detection of active contrast are given by $PFR = pr(\max|t_h| > C(b,\ \alpha))$, when $m = 0$ is used to censor the data, and $PED = pr(\max|t_h| > C(b,\ \alpha))$, when $m = 1$, or 2, or 3 or more is used to censor the data. Thus,

$$pr(max|t_h| > C(b,\ \alpha)) = \begin{cases} PFR, & \text{if } m = 0 \text{ is used to censor the data} \\ PED, & \text{if } m > 0 \text{ is used to censor the data} \end{cases} \tag{4.5}$$

We used two approaches to estimate the probabilities of false rejection and effect detection of active contrasts for the three run orders under study. The first approach is for the randomized run order, while the second approach is for the standard and systematic run orders. In the first approach, the experimental data is used directly and the test statistics are calculated from several realizations of the randomized ordering. In the second approach, artificial data are generated from a time series model that is derived from the experimental data. The reason for the differences in the data used in the two approaches is explained fully in the description of the approaches. Further, to solve the differences in the data used in the two approaches, an approach we called the harmonized approach is presented later in this section.

4.2.1.1 Probability of False Rejection (PFR)

The two approaches that we used to determine the probability of false rejection are called Approaches **1** and **2**. The first approach (**Approach 1**) is used for the randomized run order while the second approach (**Approach 2**) is used for the standard and systematic run orders.

Approach 1: PFR for randomized run order

In this approach, we created 10,000 artificial designs by permuting the rows of the model design matrix in the standard run order. For each of the permuted designs, we used the same vector of responses. Using this fixed vector, the estimates for the contrasts ($\hat{\beta}_i$) are computed for each randomized run order. The PFR is then estimated by comparing the maximum of the absolute half normal test statistic given in Equation (4.3) with $C(b, \alpha)$ for each design. The algorithm of the procedure is as follows:

(i) Permute the rows of the model design matrix .

(ii) For the permuted model design matrix, obtain the estimate of the contrast effect using the experimental results in the original order.

(iii) Determine the estimate of the error variance $\hat{\sigma}_h$ using any one of the Equations in (4.4).

(iv) Compute the half normal plot test statistic using Equation (4.3).

(v) Determine the maximum of the absolute half normal test statistics ($\max|t_{ih}|$) obtained in (iv).

(vi) Repeat step (i) to (v), 10000 times

(vii) Determine the proportion of designs with the statistic in step (v) greater than $C(b, \alpha)$.

This proportion estimates the PFR for the run order under study.

Permuting the rows of either the standard or the systematic run order will result in a loss of the features of both run orders. For example, permutation of the rows of the systematic run order leads to a run order without a single column being time resistant. Therefore, the above algorithm can only be used for the randomized run ordering. Hence the need for an alternative approach to determine their PFR such that the features of both the standard and systematic run orders will be retained.

The alternative approach, **Approach 2**, uses an artificial set of data generated from a model fitted to the experimental data. Therefore, before **Approach 2** can be employed, we need to first fit an appropriate model to the experimental data set. The ARIMA modelling approach is used to model the experimental data in this study. Thus, we give a brief description of the ARIMA modelling procedure below.

Modelling of Experimental Results

Our intention is to find a good model that will be appropriate to represent the behavior of the experimental results. In this study the univariate Box - Jenkins ARIMA (UBJ - ARIMA) modelling approach (Box and Jenkins; 1976) along with the model building approach of Box, Hunter, and Hunter (1978) are used to find an appropriate model to the experimental data series.

The UBJ - ARIMA method applies only to stationary data series. A stationary time series has a mean, variance, and autocorrelation function that are essentially constant through time. The stationarity assumption simplifies the theory underlying UBJ models and helps to ensure that we can get useful estimates of parameters from a moderate number of observations. If a time series is stationary, then the mean

and variance of any major subset of the series should not differ significantly from the mean and variance of any other major subset of the series. In practice, most time series data are non stationary and hence stationarity requirement may seem quite restrictive. However, quite often non stationary series can be transformed to stationary series through differencing.
A homogenous non-stationary model using the backshift operator B is of the form

$$\phi(B)w_t = \theta(B)a_t, \tag{4.6}$$

where $w_t = (1-B)^d y_t$, y_t is the response variable at time t, $\theta(B)$ represents the MA process operator and is given by $\theta(B) = 1-\theta_1 B-\theta_2 B^2-....-\theta_q B^q$, $\phi(B)$ represents the AR process operator and it is given by $\phi(B) = 1-\phi_1 B-\phi_2 B^2-....-\phi_p B^p$, a_t represents the white noise, and d is the number of times the data series must be differenced to induce a stationary mean (Box, Jenkins, and Reinsel; 1994). It implies therefore, that homogenous non-stationary behaviors can sometimes be represented by a model that calls for the d^{th} difference of the process to be stationary. In practice, d is usually 0, 1, or at most 2 (Pankartz, 1983). The model in Equation (4.6) is an ARIMA process of order p, d, q abbreviated as $ARIMA(p, d, q)$ process. In general the $ARIMA(p, d, q)$ process is described by

$$w_t = \phi_1 w_{t-1} + \phi_2 w_{t-2} + ... + \phi_p w_{t-p} + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - ... - \theta_q a_{t-q}, \tag{4.7}$$

where w_t, ϕ and θ are as defined earlier, p is the AR order, and q is the MA order.

The Box *et.al* (1978) approach for model building include tentative model identification, estimation of model parameter, and diagnosis of fitted model. The two major tools used at the identification stage of model building are the autocorrelation function (ACF) and partial autocorrelation function ($PACF$). The ACF at lag k is the correlation between the observed data, y_t , say, and y_{t+k}. This is given by

$$\rho_k = \frac{E\left[(y_t-\mu)(y_{t+k}-\mu)\right]}{\sqrt{E\left[(y_t-\mu)^2\right]E\left[(y_{t+k}-\mu)^2\right]}} \tag{4.8}$$

where k is the time lag which can take values 0, 1, 2, ... , and $\mu = E[y_t] = E[y_{t+k}]$. One important use of the ACF in modelling is its use in determining whether a series is stationary or not. If the mean of a series is stationary, then the estimated ACF's for the series should drop off rapidly to zero. If the mean of a series is not stationary, the estimated ACF's of the series will drop off slowly towards zero.
The $PACF$ of a process $\{y_t\}$ at lag k, ϕ_{kk}, is defined as the correlation between the adjusted values of y_t and y_{t-k} (Box$et.al$ 1994). This is represented as

$$\phi_{kk} = \frac{E\left[\left(y_t - \hat{y}_t\right)\left(y_{t-k} - \hat{y}_{t-k}\right)\right]}{E\left[\left(y_{t-k} - \hat{y}_{t-k}\right)^2\right]} \tag{4.9}$$

where $\hat{y}_t = \phi_{k-1,1}\ y_{t-1} + \phi_{k-1,2}\ y_{t-2} + ... + \phi_{k-1,k-1}\ y_{t-k+1}$,
$\hat{y}_{t-k} = \phi_{k-1,1}\ y_{t-k+1} + \phi_{k-1,2}\ y_{t-k+2} + ... + \phi_{k-1,k-1}\ y_{t-1}$, and
$\phi_{k-1,j} = \phi_{kj} + \phi_{kk}\phi_{k-1,k-j}, j = 1, 2, ...k - 1$. Thus, ϕ_{kk} measures the correlation between y_t and y_{t-k} after adjusting for the effects of $y_{t-1}, y_{t-2}, ..., y_{t-k+1}$.

At the identification stage, the estimated ACF and $PACF$ are usually compared with the various theoretical characteristics of the common time series model to find a match. The selection of a tentative model depends on the behavior of the computed ACF and $PACF$. The primary distinguishing properties of theoretical $ACF's$ and $PACF's$ for stationary process are stated by Pankratz (1983, pp.123). Using these properties, we can find a match to the behavior of ACF and $PACF$ of any series.

Having tentatively selected a model, the next step is to estimate the parameters of the model. Various approaches have been discussed in the literature for estimating parameters in models for time series data. In this work, the Maximum Likelihood approach which has been proved to reflect all useful information about the parameters contained in the data is used.

The third stage in model building is diagnostic check. The residual ACF is used as a device for testing the independence assumption of the random shocks. The residual ACF is given as

$$\rho_k(\hat{a}) = \frac{E\left[(\hat{a}_t - \overline{a})(\hat{a}_{t+k} - \overline{a})\right]}{E\left[(\hat{a}_t - \overline{a})^2\right]}, \tag{4.10}$$

where $\hat{a}_t = \hat{\phi}(B)w_t\hat{\phi}^{-1}(B)$. A test for adequacy of the fitted model is the chi-squared test for goodness of fit. This is called Ljung-Box test in the literature, see e.g. Ljung and Box (1978). This test is based on all the residual ACF as a set. Given K residual autocorrelations, the hypothesis to be tested is $H_0 : \rho_1(a) = \rho_2(a) = ... = \rho_k(a) = 0$. The test statistic is given by

$$Q = n(n+2)\sum_{j=1}^{K}(n-j)^{-1}r_j^2(\hat{a}), \tag{4.11}$$

where $r_j^2(\hat{a})$ is the estimate for $\rho_k(\hat{a})$ given in Equation (4.10), and n is the number of observations used to estimate the model. The statistic Q follows approximately a chi-squared distribution with $K - v$ degrees of freedom, where v is the number of parameters estimated in the model. If Q is large, it implies that the residual autocorrelation function is significantly different from zero, and the random shocks of the estimated model are probably autocorrelated. We should then consider reformulating the model. If we accept the null hypothesis that the random shocks are independent, it implies that the residual autocorrelation function is not significantly different from zero and therefore the model fitted will be adjudged to be suitable.

Approach 2: PFR for systematic and standard run orders

In this approach, sets of data were generated from the ARIMA model described above. Here, the response is not assumed to be fixed, instead we assumed that the model design matrix is fixed. This is to protect the trend resistance property of the run order for the design under study. Using the data generated from the ARIMA model, we then estimate the effects of the contrasts. The PFR is taken to be the proportion of designs with maximum absolute half normal plot test statistic that is greater than the simulated critical value for a desired α level.

The following steps give the algorithm for Approach **2**:

(i) Generate a set of data from the model fitted to the experimental results.

(ii) Obtain the estimates of the contrasts from the model design matrix using the generated data in (i) and the run order under consideration.

(iii) Determine the estimate of the error variance $\hat{\sigma}_h$ using any one of the Equations in (4.4).

(iv) Compute the half normal plot test statistic using Equation (4.3).

(v) Determine the maximum of the absolute half normal test statistics ($\max |t_{ih}|$) obtained in (iv).

(vi) Repeat step (i) to (v), 10000 times.

(vii) Determine the proportion of designs with the statistic in step (v) greater than $C(b, \alpha)$. This proportion estimates the PFR for the run order under study.

It should be noted that both Approaches **1** and **2** have the same fundamental objective. That is, the proportion of design with maximum absolute half normal plot test statistics greater than the critical value when there is no active contrast. By implication, this is the proportion of designs that falsely declare an active contrast.

4.2.1.2: Probability of Effect Detection (PED)

To determine the PED for the standard, randomized and systematic run orders, we employed the censored data approach by using the setting of the columns in the model design matrix to get a new data set. In this section, again two approaches were used. We called them approaches **A** and **B**. Approach **A** is for the randomized run ordering only while Approach **B** is for the standard and systematic run orders as well. The procedures for the two approaches are similar to those presented in Section 4.2.1.1 with the introduction of an additional step after step (i) of the algorithm for both approaches in Section 4.2.1.1. The additional step is to censor the experimental data using Equation (4.1). In **Approach A**, the experimental results were censored by adding some constant m whenever the corresponding column setting in the model design matrix is at its high level (+) and zero when the setting is at its low level (-), where $m \geq 1$. The new data set (that is, $y_j^{(m)}$) along with the model design matrix of the run order under study is then used to get the proportion of designs that rightly declare active contrasts. The value of m that gives at least 50% of the design that correctly declare presence of an active contrast is also of interest. This is taken to be the m-value for sensitivity to presence of active contrast for the randomized run order. In Approach **B**, the data generated from the fitted model to the original data are used along with the run orders under consideration to obtain the PED. An algorithmic description of the two approaches in steps are presented as follows:

Approach A: PED for randomized run ordering

(i) Permute the rows of the model design matrix.

(ii) For each column (contrast) of the model design matrix X assumed to be active, censor the experimental data using Equation (4.1). Thus, we have a new data set.

(iii) For the permuted model design matrix, determine the estimates of the contrast effects ($\hat{\beta}_i$) using the new data in (ii).

(iv) Determine the estimate of the error variance $\hat{\sigma}_h$ using any one of the Equations in (4.4).

(v) Compute the half normal plot test statistic using Equation (4.3).

(vi) Determine the maximum of the absolute half normal test statistics ($\max |t_{ih}|$) obtained in (v).

(vii) Repeat step (i) to (v), 10000 times.

(viii) Determine the proportion of designs with the statistic in step (vi) greater than $C(b, \alpha)$. This is taken to be the PED for the run order under study.

The above algorithm can be used to achieve two things. One is the sensitivity to presence of active contrast and the second is the PED for the randomized run order. To achieve the first aim, we start by adding $m = 1$ to the response variable using the setting of a contrast as described in Equation (4.1). If the value $m = 1$ does not give a PED of 50% then we proceed to use $m = 2$. If adding $m = 2$ to the experimental data does not give a PED of 50%, then we proceed to use $m = 3$ and so on until we get a m-value that gives at least a PED of 50%. The m-value that gives at least 50% PED is taken to be the m-value for sensitivity of the randomized run order to presence of active contrast.

Approach B: PED for standard and systematic run orderings

(i) Generate a set of data from the model fitted to the series of the experimental results.

(ii) Use a column (contrast) of the model design matrix to censor the generated data using Equation (4.1).

(iii) Obtain the estimates of the contrast effects ($\hat{\beta}_i$) using the censored generated data in (ii).

(iv) Determine the estimate of the error variance $\hat{\sigma}_h$ using any one of the Equations in (4.4).

(v) Compute the half normal plot test statistic (t_{ih}) using Equation (4.3).

(vi) Determine the maximum of the absolute half normal test statistics ($\max|t_{ih}|$).

(vii) Repeat step (i) to (vi), 10000 times.

(viii) Determine the proportion of designs with the statistic in step (vi) greater than $C(b, \alpha)$. This is taken to be the PED for the run order under study.

It should be noted that the number of contrasts to be used for censoring in order to determine the PED using any of the two Approaches **A** and **B**, depends on the number of active contrasts under consideration.

4.2.1.3: Harmonized approach

The harmonized approach is developed to resolve the differences in the data used for the algorithms given earlier for estimating the PFR and PED for the randomized run order. The approach involves the combination of the two approaches for computing the PFR (**Approaches 1** and **2**) and PED (**Approaches A** and **B**). Thus, generated data from an appropriate model fitted to the experimental data along with the permuted rows of the model design matrix of the standard run order are used to estimate the PFR and the PED. The procedures for the approach are presented stepwise below:

(i) Permute the rows of the model design matrix for the standard run order.

(ii) Generate data set from an appropriate model fitted to the experimental data.

(iii) Censor the generated data in step (ii) using Equation (4.1)

(iv) Use the permuted model design matrix in step(i) and the censor data in step (iii) to estimate the contrast effects.

(v) Compute the half normal plot test statistic (t_{ih}) for the estimated effects in (iv) using Equation (4.3).

(vi) Determine the maximum of the absolute half normal test statistics ($\max |t_{ih}|$) obtained in (v).

(vii) Repeat step (i) to (vi), 10000 times.

(viii) Determine the proportion of designs with $\max |t_h| > C(b, \alpha)$. This is taken to be the PFR when $m = 0$ is used to censor the data and as the PED when $m \geq 1$ is used to censor the data.

The principle of the harmonized approach is synonymous with the principle of the approaches for computing the PFR and PED for the standard and systematic run orders.

The graphical display for probabilities of false rejection and effect detection involve plotting of the ordered $max|t_{ih}|$ statistic on the horizontal axis with its corresponding proportion of designs on the vertical axis. Thus, on the graph, the PFR and PED is taken to be 1-η, where η is the point of intersection of the critical value ($C(b, \alpha)$) with the plotted points. It should be noted for clarity that the η value is to be read on the vertical axis.

4.2.2 Simulation of Critical Values ($C(b, \alpha)$)

Two kinds of error rates are mostly of concern when evaluating an approach. These are the individual error rate (IER) and the experimentwise error rate (EER). The IER is the proportion of inactive individual effects declared active, and the EER is the proportion of experiments with at least one inactive effect declared active. In this work the EER is of interest. The simulation of experimentwise error rate (EER) critical values is conducted to obtain the critical values for a given α levels of significance and b number of contrasts. The approach we used for the simulation of the critical value is similar to Ye, Hamada, and Wu (2001) approach, which was earlier proposed by Ye and Hamada (2000) and Loughin (1998) for obtaining EER critical value.

Following Ye, Hamada, and Wu (2001), the calculation of critical values was based on 10,000 samples of b effects generated from the standard normal distribution with mean, μ=0 and standard deviation, σ=1. The procedure used is as follows:

Let $\hat{\beta}_1, \hat{\beta}_2, ..., \hat{\beta}_b$ be the b estimates of factorial effects, under the null hypothesis of no active contrast, that is, $H_0 : \beta_1 = \beta_2 = ... = \beta_b = 0$, the EER critical value at significant level α with b contrasts is the $(1 - \alpha)100$ percentile of the statistic in Equation (4.3). The algorithm used for the simulation of EER critical values is given below:

(i) Generate a set of b estimates of contrasts from a standard normal distribution.

(ii) Compute the estimate of the error variance σ_h using any one of the Equations in (4.4)

(iii) Calculate t_{ih} using equation (4.3).

(iv) Compute $\max|t_{ih}|$.

(v) Repeat steps (i) to (iv), 10000 times to get a set of 10000 $\max|t_{ih}|$ statistics.

(vi) The EER critical value is then the $(\alpha \bullet 10000)^{th}$ largest $\max|t_{ih}|$ statistics.

4.3 Simulation Results

4.3.1 Critical Values

The algorithms described above were implemented using the S-plus programming language. For the critical values ($C(b, \alpha)$), the range of values of α used in this study are 0.01, 0.05, 0.10 and 0.20. The obtained critical values based on the MAC, PSE and ASE as an estimate of the error variance of β_i for the most common sizes of two levels factorial/fractional factorial and Plackett Burman designs used in screening experiment are presented in Table 4.1.

Table 4.1: Simulated EER Critical Values ($C(b, \alpha)$)

	α	b				
		7	11	15	19	31
MAC	0.01	6.0245	5.2713	4.9624	4.7629	4.4726
	0.05	3.9045	3.7236	3.6978	3.6331	3.5966
	0.10	3.1356	3.1434	3.1445	3.1633	3.2036
	0.20	2.4273	2.5712	2.6513	2.7282	2.8305
PSE	0.01	9.7498	7.4204	6.3221	5.8692	5.1624
	0.05	4.8868	4.5607	4.1550	4.1242	3.9012
	0.10	3.6936	3.5721	3.5067	3.4532	3.4512
	0.20	2.4273	2.7381	2.8213	2.8903	2.9694
ASE	0.01	9.2532	6.1387	5.3453	4.9634	4.5379
	0.05	4.8947	4.1488	4.0026	3.8855	3.7684
	0.10	3.6933	3.4987	3.4438	3.4014	3.4063
	0.20	2.1584	2.5108	2.7217	2.8350	3.0126

A visual inspection of the simulated critical values in Table 4.1 show that the PSE based critical values are higher than the other two. This might be due to the fact that $\hat{\sigma}_{PSE}$ has a larger variance (Kunert 1997) and hence, more conservative than the other estimates.

In what follows, we compare our EER critical values based on the PSE in Table 4.1 with the Ye, Hamada, & Wu (2001), Ye & Hamada (2000), and Loughin (1998) critical values. The results are presented in Table 4.2. Henceforth, we will refer to Ye, Hamada, and Wu (2001), Ye and Hamada (2000), and Loughin (1998) critical values as YHW'01, YH'00 and L'98, respectively.

Table 4.2: Comparison of Simulated EER Critical Values Based on PSE

b	α	Our result	Ye,Hamada,and Wu'01	Ye and Hamada'00	Loughin'98
7	0.01	9.7498	9.75	9.715	-
	0.05	4.8868	4.87	4.867	4.878
	0.10	3.6936	3.69	3.689	3.677
	0.20	2.4273	2.42	2.420	2.427
11	0.01	7.4204	7.45	7.412	-
	0.05	4.5607	4.45	4.438	-
	0.10	3.5721	3.56	3.564	-
	0.20	2.7381	2.74	2.738	-
15	0.01	6.3221	6.40	6.446	-
	0.05	4.1550	4.24	4.240	4.242
	0.10	3.5067	3.51	3.507	3.502
	0.20	2.8213	2.84	2.845	2.837
19	0.01	5.8692	5.86	5.884	-
	0.05	4.1242	4.11	4.118	-
	0.10	3.4532	3.48	3.481	-
	0.20	2.8903	2.89	2.896	-
31	0.01	5.1624	5.10	5.095	-
	0.05	3.9012	3.93	3.925	3.910
	0.10	3.4512	3.45	3.450	3.453
	0.20	2.9694	2.98	2.983	2.977

Examination of the results in Table 4.2 show that our results are almost the same with YHW'01, YH'00, and L'98 except for some slight differences for $b = 11$ when $\alpha = 0.05$, $b = 15$ when $\alpha = 0.01, 0.05$, $b = 19$ for $\alpha = 0.10$, and $b = 31$ for $\alpha = 0.01$. Our critical values are slightly higher than others when $b = 11$ for $\alpha = 0.05$ and when $b = 31$ for $\alpha = 0.01$. However, our critical values are slightly less than others when $b = 15$ for $\alpha = 0.01, 0.05$ and $b = 19$ for $\alpha = 0.10$. The YHW'01 has an outstanding higher critical value when $\alpha = 0.01$ for $b = 11$, while YH'00 has an outstanding higher critical value when $\alpha = 0.01$ for $b = 15$. In general, our simulated critical values have the same pattern with those in which comparison were made. That is, the critical values decreases as the number of contrasts (b) increases, also for a particular number of contrasts, the critical values decreases with increase in the level of significance (α).

In a similar way as above, our critical values based on the ASE is also compared with Kunert (1994) critical values which were also based on ASE as an estimate of the error variance. Kunert's (1994) critical values were computed only for the 5% level of significance. His critical values are 3.47, 3.52, 3.56, 3.60, 3.64, and 3.68 for 10, 11, 12, 13 , 14 and 15 contrasts, respectively. Our critical values and Kunert's (1994) critical value for $b = 15$ and 11 are compared. For these two numbers of contrasts, our critical values are higher than those of Kunert (1994). This could be due to the fact that Kunert (1994) used approximations instead of exact calculations in the computation formula for estimating σ_{ASE}.

4.3.2 Performance standard results

We now use the algorithms presented in Sections 4.1 and 4.2 to determine the performance of the three run orders under study with respect to active contrasts. These algorithms are demonstrated on two examples. The two examples used are the experimental results from two repetitions of 16 runs from the funnel experiments presented in Table 3.2 of Chapter 3. We assumed that the data in Table 3.2 were from an unreplicated fractional factorial experiment with 16 observations. The experimental result for machine 1 is used as Example 1 while the result for machine 2 is used as Example 2. Henceforth in this study, the experimental results for machines 1 and 2 will be referred to as cases 1 and 2 data, respectively.

4.3.2.1 Example 1

4.3.2.1a Estimation of Sensitivity For Standard and Systematic Run Orders

To measure the sensitivity to presence of active contrast for the standard run order of an unreplicated 2^{k-p} design, the model design matrix in Table 2.1 and the experimental results for case 1 are used. Using the procedure given in Section 4.1, the case 1 data is censored using the linear time-trend resistant columns (contrasts) settings in Table 2.1. We used the linear trend resistant contrasts of the design in Table 2.1 to censor the data in order to have a fair comparison of the standard ordering with the systematic ordering. However, if any of the contrasts of the standard run order is used to censor the data, the results will not be different from the results obtained here.

In the standard run order, there are eleven linear time-trend resistant columns, therefore, there are eleven different data sets (see Appendix E). For each of the data sets, we obtain the estimates for all the 15 contrasts in the model design matrix. The ordered absolute estimates for the 15 contrasts are then plotted against $\Phi^{-1}\left(1/2 + (i - 1/2)/(2 \bullet 15)\right), 1 \leq i \leq 15$. The half normal plots for the original data and a censored data set based on one of the linear trend resistant column (fifth column) of the design in Table 2.1 are presented in Figure 4.1

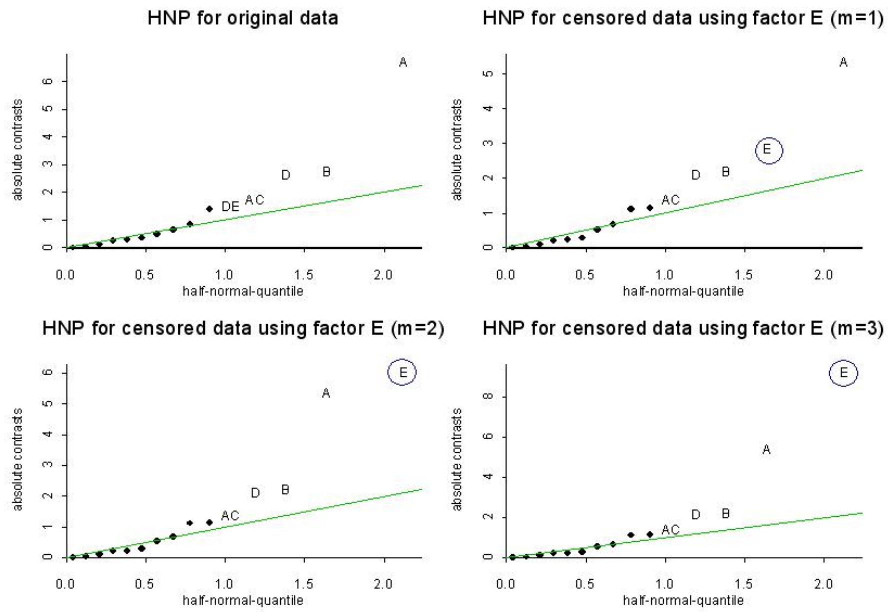


Figure 4.1: *Standard run order half normal plots for the original and censored data. The circled factor in each graph is the factor used as a basis for censoring the data for different m values.*

From Figure 4.1, the observed results are itemized as follows:

a) On the half normal plot for the original data, the estimated effects of contrasts A, B, D, AC, and DE are above the straight line with the estimated effect of factor A clearly above the straight line and hence the maximum absolute contrast.

b) On the half normal plot for the data obtained by adding $m = 1$ to the original experimental data when the setting of factor E (fifth column) in Table 2.1 is at + level and zero otherwise, estimated effects of contrasts A, E, B, D, and AC are above the straight line with the estimated effect of factor A clearly remaining as the maximum absolute contrast.

c) On the half normal plot for the data set obtained with $m = 2$ using the setting of factor E (fifth column) in Table 2.1, estimated effects of contrasts E, A, B, D, and AC are above the straight line with the estimated effect of factor E clearly above the straight line and hence the maximum absolute contrast.

The measure of sensitivity to presence of active contrast was repeated using the setting of the other linear time-trend resistant columns in Table 2.1 to censor the original data. Thus, we have ten new data sets. The half normal plots for the ten new data sets are presented in Figures B11 and B12 in Appendix B. The scenario of the figures in Appendix B reflects that we need as much as $m = 2$ to make the estimated effect contrast used to censor the data to be the maximum absolute contrast on the half normal plot.

For the systematic run order, the design in Table 2.3 along with the experimental results for case 1 are used to determine the m value that makes the estimated effect of the contrast used to censor the data as the maximum absolute contrast on the half normal plot. The design in Table 2.3 has eleven linear time-trend resistant columns. Therefore, there are eleven new data sets (see Appendix C). The half normal plot for this design is called reduced half normal plot in this study. Following the procedure in Section 4.1, the setting of only one column (Factor A) of the design matrix was used to censor the data for illustration. The reduced half normal plot for both the original data and the censored data are presented in Figures 4.2.

From Figure 4.2 the following were observed:

a) For the half normal plot of original data, the estimated effects for contrasts CE, B, C, and BD are seen to be above the straight line with the estimated effect of the two factor interactions contrast CE as the maximum absolute contrast.

b) On the half normal plot for the data obtained by adding $m = 1$ to the original data when the setting of Factor A in Table 2.3 is at $+$ level and zero otherwise, the estimated effects of contrasts A, CE, B, and C are seen to be above the median straight line with the estimated effect of factor A as the maximum absolute contrast.

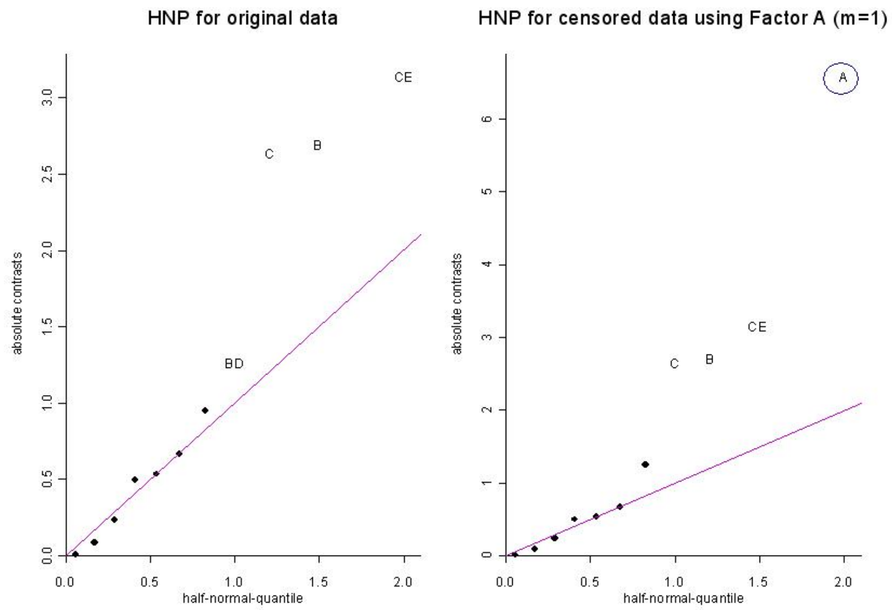


Figure 4.2: *Reduced (systematic run order) half normal plot for the original and censored data. The circled factor in the graph is the factor used as a basis for censoring the data.*

The measure of sensitivity to presence of active contrast was repeated using the setting of the remaining columns (2-11) of the design in Table 2.3 to censor the data. The half normal plots for the obtained data sets (see Appendix C) are presented in Figure B21 in Appendix *B*. The scenario of the graphs in Figure B21 show that, adding $m = 1$ makes the estimated effect of the contrast used for censoring to be the maximum absolute contrast on the half normal plots.

From the obtained results for the measure of sensitivity for both the standard and systematic run orders, the following findings were obtained:

(1) For the standard run order, when $m = 1$ is used to censor the data, the estimated effects of the contrasts that are used for censoring the data are not the maximum absolute contrasts on the half normal plots.

(2) For the standard run order, when $m = 2$ is used to censor the data, about 73% of the HNPs declares the estimated effects of the contrasts that are used

for censoring the data as the maximum absolute contrasts.

(3) On the standard run order HNPs for the data set with $m = 3$, the estimated effects of all the contrasts that are used to censor the data are seen to be the maximum absolute contrasts on the half normal plots.

(4) For the systematic run order, the estimated effects of the contrasts that were used to censor the data are the maximum absolute contrasts on the HNPs for all the data set obtained with $m = 1$ (see circle contrasts in Figure 4.2 and Figure $B21$ in Appendix B).

From the aforementioned points, it is clear that the systematic run order is more sensitive to presence of active contrast than the standard run order.

4.3.2.1.1a Estimation of PFR and PED using the MAC as an estimate of error variance

The proportion of designs with false rejection (PFR) and the proportion of designs with effect detection (PED) are computed using the estimate of error variance based on the median of the absolute contrasts, that is, $h = MAC$ in Equations (4.3) and (4.4).

Estimation of Probability of False Rejection(PFR)

To obtain the proportion of false rejection for the randomized run order, we used the design in Table 2.1 with $b = 15$ estimable contrasts as a starting point of the randomization. For $b = 15$ and $\alpha = 0.05$, a simulated critical value ($C(15, 0.05)$) of 3.6978 (see Table 4.1) was used. Therefore, the proportion of simulated designs with $max|t_{MAC}| > 3.6978$ will estimate the PFR. Permuting the rows of the model design matrix in Table 2.1, and using the experimental result for machine 1 presented in Table 3.2, we estimate the contrast effects and follow the algorithm as stated in **Approach 1**. With 10,000 repetitions, the observed proportion of designs where $max|t_i| > 3.6978$ was 4.7%. We therefore estimate that indeed approximately 5 % of the randomized run orders will falsely give an active contrast. Note that this is already an important result. It supports the view that randomized orderings will keep the nominal level α.

In order to determine the PFR and PED for the standard and systematic run orders, we need to generate data set from a fitted model. Using the procedures in the sub-section of 4.2.1.1, a model was fitted to the experimental data for machine 1 (case 1 data). Since the behavior of y_t and $\widetilde{y}_t = y_t - \overline{y}$ have been found to be the same (Box and Jenkins 1976), we therefore, replace y_t with $\widetilde{y}_t$ in Equations (4.6) through (4.11) in Section 4.2.1.1 for simplicity.

An examination of the plot of the experimental data in Figure 3.2 shows that the series of the data is a non stationary one. On dividing the data of the series into two major subsets (equally half), the obtained means are 23.6225 and 25.2813 for the two subsets. This suggests that the series is non stationary. Also a plot of the estimated ACF in Figure 4.3(a) drops off slowly towards zero, this confirms that the series is a non stationary one.

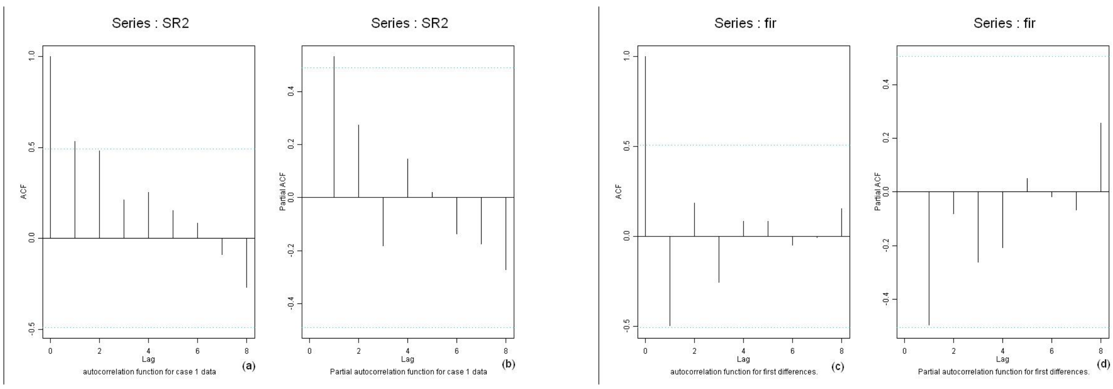


Figure 4.3: *ACF and PACF for the experimental data and the data for the first order difference*

To make the series stationary, we calculate the first differences and find the estimated ACF and $PACF$ for the new series. Figures 4.3(c) and 4.3(d) are plots of the estimated ACF and $PACF$ respectively for the new series. Inspection of

Figure 4.3(c) suggest that the mean is now stationary since the estimated ACF drops off to zero after lag 1. The spike at lag 1 followed by a cut off to zero in the estimated ACF implies that an MA model is appropriate. The plot of the $PACF$ for the new series in Figure 4.3(d) tail off towards zero starting from lag 1. The combination of the features of the ACF and $PACF$ for the first differences suggest an $MA(1)$ model. From the above analysis, we therefore, select an $ARIMA(0,1,1)$ model given by $(1-B)\tilde{y}_t = (1-\theta_1 B)a_t$ as a tentative model for the data series. Using the maximum likelihood approach, the estimate for the $ARIMA(0,1,1)$ model is $\hat{\theta}_1 = 0.3$. This satisfies the invertibility condition, since $|\hat{\theta}_1| < 1$. Therefore, the tentative fitted model is

$$\hat{y}_t = y_{t-1} - 0.3\hat{a}_{t-1} + \hat{a}_t. \tag{4.12}$$

On diagnosing the model in Equation (4.12), the residual ACF cuts off after lag 1. Therefore, the hypothesis that the shocks of the model in Equation (4.12) are independent can be accepted, which implies that the model is appropriate. The computed Q statistic for the residual ACF equals 3.867 with 5 degree of freedom, and p-value of 0.57. The obtained Q value is relatively small when compared to the chi-square quantiles at 5 degree of freedom for 1%, 5%, and 10%. Also the corresponding p-value of the Q value is large enough to accept the hypothesis of no correlation between the residual ACF. Therefore, the model given by Equation (4.12) is statistically adequate representation of the data series for case 1.

The model given by Equation (4.12) will then be used to generate artificial data set to represent the data series in our simulation study for comparing the probabilities of false rejection and effect detection of active contrast for the standard and systematic run orders.

In order to obtain the proportion of false rejections for the standard run order, the design in Table 2.1 was used. For this design the number of contrast (b) is 15, and for $\alpha = 0.05$, $C(15, 0.05)$ from Table 4.1 equals 3.6978 as before. Then 10,000 data sets were generated from the fitted ARIMA (0,1,1) model of Equation (4.12). Following the algorithm of **Approach 2**, the observed proportion of simulated designs with $\max |t_{MAC}| > 3.6978$ equals 0.328. Hence, for 33 % of the simulated observations, the standard run order falsely identified an active contrast. This is catastrophically much, and thus, the standard run order is clearly not usable in the presence of this time trend.

To obtain the proportion of false rejections for the systematic run order, the design in Table 2.3 was used. For this design the number of contrasts, b equals 11. Therefore, we had to use another critical value. For $b = 11$ and $\alpha = 0.05$, the simulated critical value ($C(11, 0.05)$) from Table 4.1 equals 3.7236. Hence, the proportion of simulated designs with $\max |t_{MAC}| > 3.7236$ will estimate the PFR for the systematic run order. Following the algorithm of **Approach 2**, the proportion of simulated designs with $\max |t_{MAC}| > 3.7236$ equals 10.8%. This means that in approximately 11 % of the simulated data-sets, the systematic run order falsely identified an active contrast. Hence, the systematic run order performed better than the standard ordering. However, it seems that the systematic run order does not suffice to provide sufficient protection against the realistic trend considered here.
A plot that displays the empirical distribution function of the ordered $max|t_{MAC}|$ for the systematic, standard, and randomized run orders is presented in Figure 4.4.

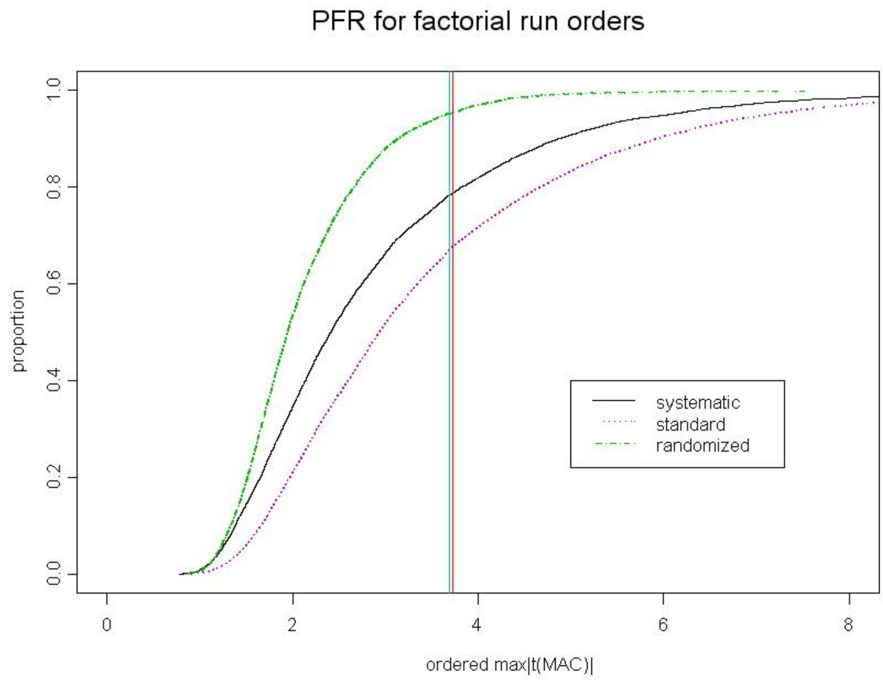


Figure 4.4: *Empirical distribution function for the systematic, standard, and randomized run orders without active contrast. The two vertical lines in the figure represent the critical values. Note that they are very near to each other.*

Estimation of Probability of Effect Detection

The procedure of **Approach A** of Section 4.2.1.2 was used to estimate the PED for the randomized run order. Again, the rows of the design in Table 2.1 were permuted 10,000 times. This implies 10,000 randomized run orderings. For each time that the rows of the design were permuted, the setting of the first column of the run order was used to censor the data. This simulated an effect of factor 1. Then the censored data set was used to obtain estimates for the contrasts of all factors and two factor interactions. When adding $m = 1$ to the data, we observed a PED of 7.3%. When adding $m = 2$, we obtained 36%. Similarly by adding $m = 3$, we obtained 86.2 %. We then continued by assuming two or three active contrasts. For simplicity, we assumed that the active contrasts were all of the same size. The graphical display of the obtained ordered $max|t_{MAC}|$ with the proportion of designs for one, two and three active contrasts are presented in Figure 4.5.

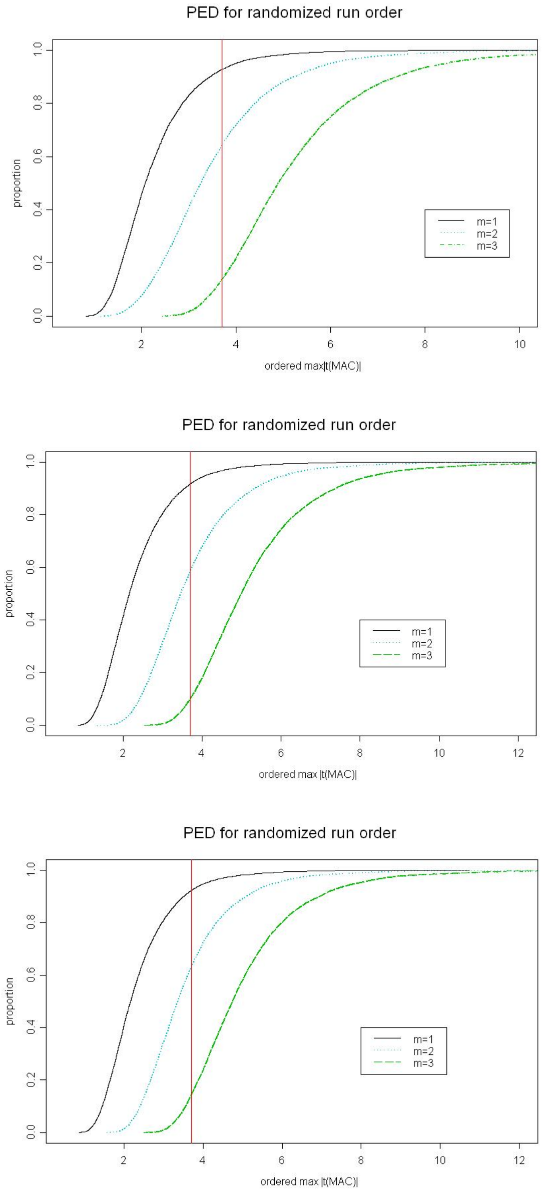
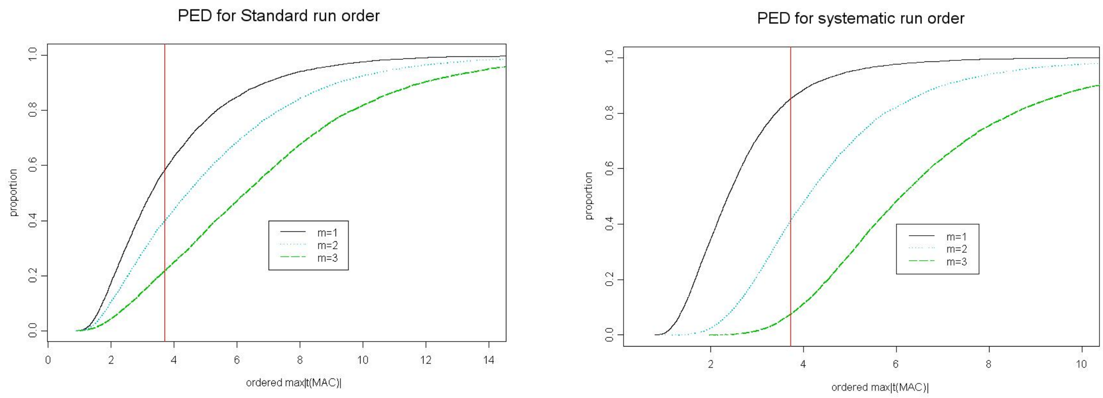


Figure 4.5: *Empirical PED for randomized run order with different sizes of active contrasts. (a) Empirical PED for one active contrast, (b) Empirical PED for two active contrasts, (c) Empirical PED for three active contrasts. The vertical line in each graph represents the critical value.*

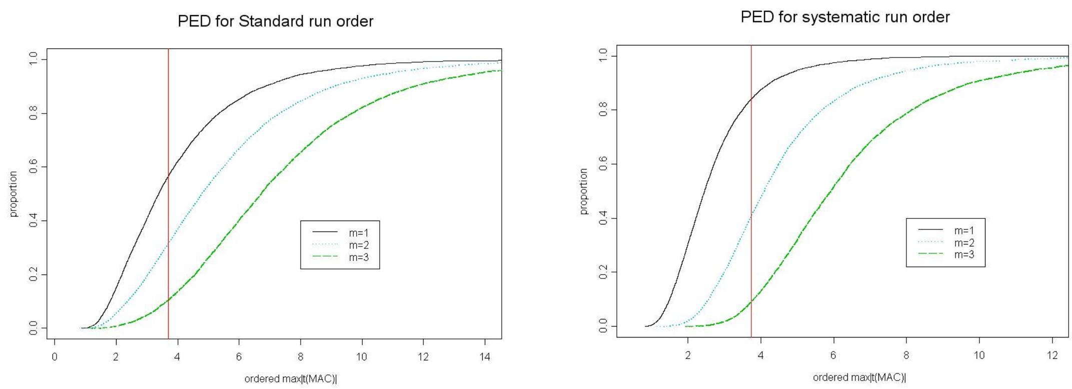
From Figure 4.5, It can be seen that the results for two and three active contrasts did not differ much from the results for one active contrast. Furthermore, using the obtained PED for one active contrast, $m > 2$ gives approximately the desired probability of effect detection for the approximate sensitivity analysis for the randomized run order.

The probability of effect detection (PED) for the standard and systematic run orders was estimated using the design in Tables 2.1 and 2.3, respectively. Following the algorithm of Approach **B**, we generated data sets using the fitted ARIMA (0,1,1) model of Equation (4.12). The generated data sets were then censored and the censored data were used to determine the proportion of designs with one, two, and three active contrasts that correctly identified at least one active contrast.
For one active contrast, we first add $m = 1$ to the generated data when the first column of the model design matrix under study is at + level and zero otherwise. The new data set (censored data) is then used to get maximum of the absolute half normal test statistics which was compared with the simulated critical value 3.6978 for the standard run order and 3.7236 for the systematic run order. The steps are repeated for $m = 2$ and $m = 3$.
For two active contrasts, we add $m = 1$ to the generated data when the first column or the second column of the model design matrix under study are at + level, 2m to the generated data when both the first and second columns are at + level, and zero otherwise. The new data set (censored data) is then used to get the maximum of the absolute half normal test statistics which is thereafter compared with the simulated critical value as before. The steps are repeated for $m = 2$ and $m = 3$.
Similarly for three active contrasts, we add $m = 1$ to the generated data when the first or second or third columns of the model design matrix under study are at + level, 2m when two of the three columns are at + levels, 3m when all the three columns are at + levels, and zero otherwise. The new data set is then used to get the maximum of the absolute half normal test statistics which is thereafter compared with the simulated critical value. The steps are repeated for $m = 2$ and $m = 3$.
Figure 4.6 presents the empirical distribution function for the standard and systematic run orders for one, two, and three active contrasts.

One active contrast



Two active contrasts



Three active contrasts

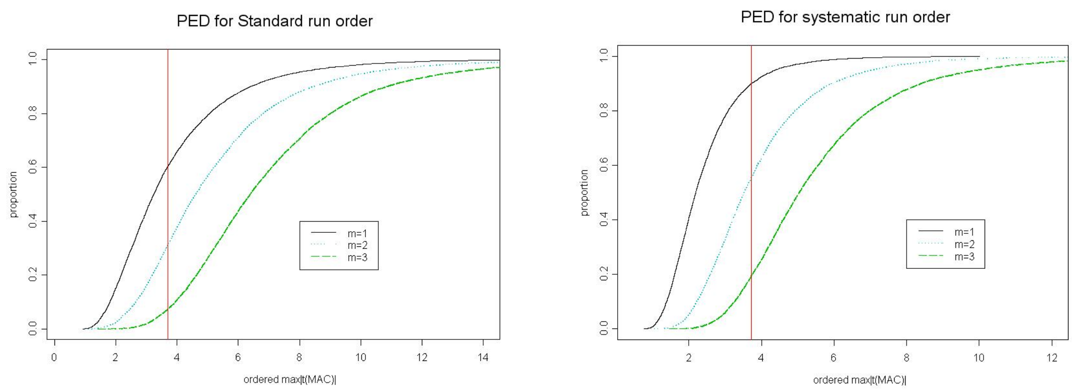


Figure 4.6: *Empirical distribution function of* $max|t_{MAC}|$ *for the standard and systematic run orders with different sizes (m) of active contrast. The vertical line in each graph represents the critical value*

In order to have a fair comparison of the randomized run order with the systematic run order, we did two more series of experiments. First of all, we used the harmonized approach to compute the PFR and PED for the randomized run order. Here, the results were very similar to what we had for the experimental data.
Since the systematic run order did not keep the nominal level $\alpha = 0.05$, it is easy for the systematic run order to get a high estimated power. Thus, the second series of experiment involved the systematic run order where a pseudo critical value was used. The pseudo critical value is then used to compute the PFR and PED for the systematic run order.
The pseudo critical value is taken to be the value of the $max|t_{MAC}|$ that gives the same proportion of false rejections as was derived by the randomized design. Thus, the pseudo critical value is data driven. Unfortunately, a pseudo critical value cannot be determined in practice.
To derive the same PFR of 0.0509 obtained with the harmonized approach for the randomized run, we need a pseudo critical value of 4.6019. This pseudo critical value was then used to compute the PED for the systematic run order. The corresponding proportions could be seen in Figure 4.6 if we moved the vertical line from 3.72 to 4.60.
The PFR and PED for the randomized run order using the harmonized approach and the PFR and PED for the systematic run order using the pseudo critical value yield similar results as those obtained earlier (see Table 4.3 on page 116).

4.3.2.1.1b Estimation of the PFR and PED based on the PSE as an estimate of error variance

The probability of false rejection and the probability of effect detection for the three run orders using the pseudo standard error (PSE) to estimate the standard deviation of the contrasts of the run orders are considered. Here h in Equations (4.3) and (4.4) equals PSE. The approaches for estimating both the PFR and the PED presented in Section 4.2.1 were used. Here the simulated critical values based on the PSE presented in Table 4.1 were used. For $b = 15$ and $\alpha = 0.05$, we used the simulated critical value ($C(15, 0.05)$) of 4.1550 for the standard and randomized run orders. For $b = 11$ and $\alpha = 0.05$, we used 4.5607 ($C(11, 0.05)$) for the systematic run order. Following **Approach 1**, with 10,000 repetition as before, the observed proportion of designs where $max|t_{PSE}| > 4.1550$ equals 33% and 5% for the standard and randomized run orders, respectively. Similarly, the observed proportion of designs where $max|t_{PSE}| > 4.5607$ equals 9% for the systematic run order. The plot of the obtained results for the empirical $PFRs$ for the standard, randomized and systematic run orders are presented in Figure 4.7, while the plot for the obtained results for the empirical PED for the three run orders for one, two and three active contrasts are presented in Figure A11 in Appendix A.1.

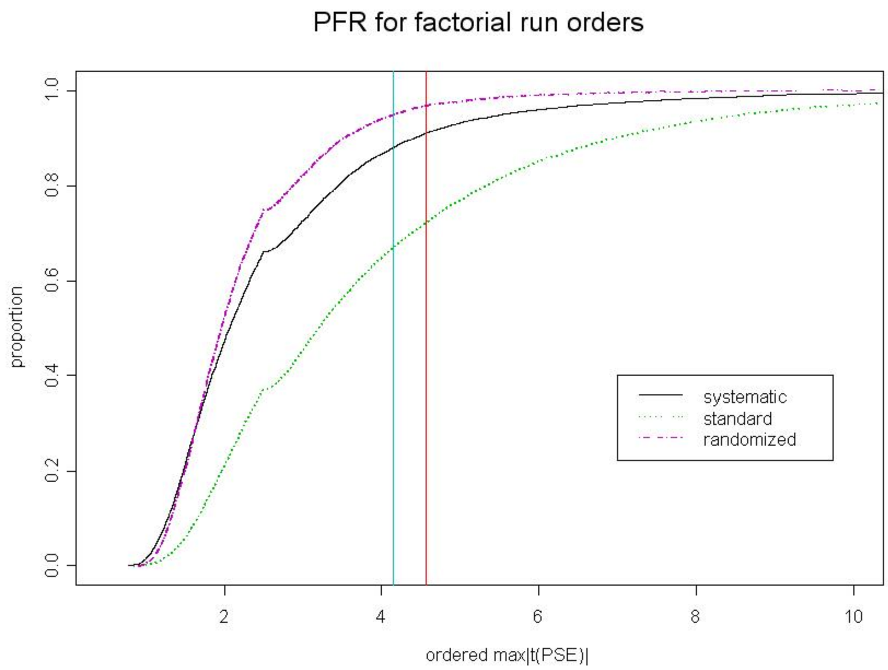


Figure 4.7: *Empirical distribution function of* $max|t_{PSE}|$ *for the standard, randomized, and systematic run orders without active contrast. The blue vertical line represents the critical value for the randomized and standard run orders while the red vertical line represents the critical value for the systematic run oder.*

The PFR and PED values from Figure 4.7 and Figure A11 in Appendix A1 are presented in Table 4.4. Using the harmonized approach, the PFR for the randomized run order equals 0.0585. In a similar way as before, the harmonized approach was used to estimate the PED of the randomized run order for one, two and three active contrasts. To derive the same PFR of 0.0585 obtained with the harmonized approach for the randomized run order, we need a pseudo critical value of 5.2809. This pseudo critical value was then used to compute the corrected PED for the systematic run order. The PFR and PED for the randomized run order using the harmonized approach and the PFR and PED for the systematic run order using the pseudo critical value are documented in the columns marked with an asterisk in Table 4.4 on page 119.

4.3.2.1.1c Estimation of PFR and PED based on the ASE as an estimate of error variance

In this section, we estimated the PFR and PED for the three run orders under study using the adaptive standard error (ASE) proposed by Dong (1993) as an estimate of the standard deviation of the contrasts. That is, $h = ASE$ in Equations (4.3) and (4.4). Following the approaches in section 4.2.1, we estimated the PFR and PED for the run orders under study. Using the simulated critical value based on the ASE presented in Table 4.1, that is, 4.0026 for the standard and randomized run orders $C(15, 0.05)$ and 4.1488 for the systematic run order $C(11, 0.05)$, the obtained $PFRs$ are 37%, 4%, and 12% for the standard, randomized and systematic run orders, respectively. The plot of the obtained results for the probability of false rejection for the three run orders under study are presented in Figure 4.8

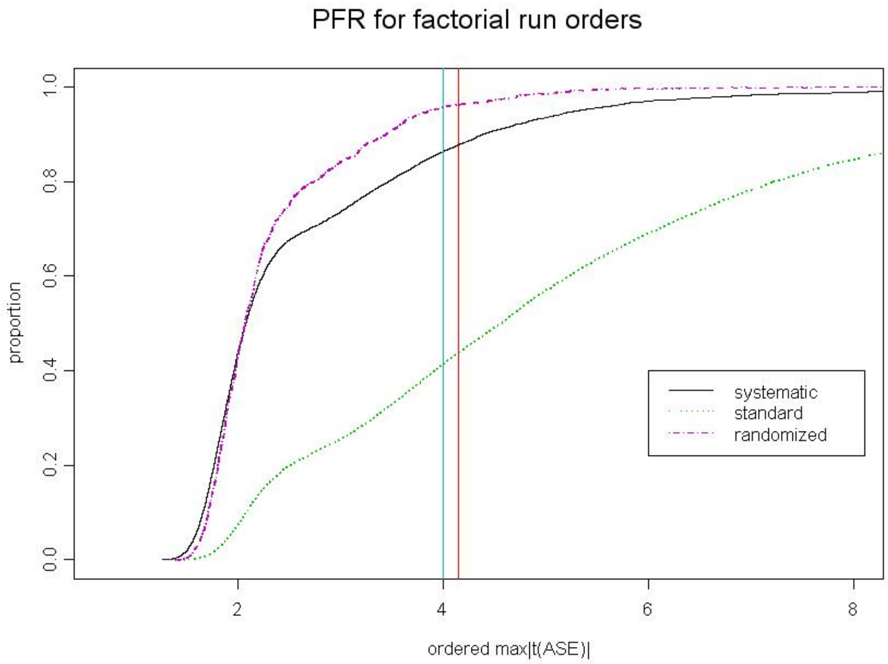


Figure 4.8: *Empirical distribution function of* $max|t_{ASE}|$ *for the standard, randomized, and systematic run orders. The blue and red vertical lines are as earlier defined.*

From the graph in Figure 4.8, it is evident that the randomized run order has the smallest PFR. The PFR obtained with the harmonized approach equals 0.0539. This is almost the same as the PFR obtained with both the MAC and PSE estimated error variance. To derive same PFR of 0.0539 obtained with the harmonized approach for the randomized run order, we need a pseudo critical value of 5.2338 for the systematic run order. This pseudo critical value is then used to compute the corrected PED for the systematic run order. The obtained results are presented in Table 4.5 on page 120. The obtained results for the empirical distribution function of $max|t_{ASE}|$ with one, two, and three active contrasts for the standard, randomized and systematic run orders are plotted in Figure A21 in Appendix A2.

4.3.2.2 Example 2

We repeat all the analyses of Example 1 with another data set. In this example, the experimental result from machine 2 presented in Table 3.2 is used to evaluate the performance of the three run orders under consideration. As in Example 1, the measure of sensitivity to presence of active contrasts, the probabilities of false rejection of active contrasts, and the probabilities of effect detection of active contrasts are estimated.

4.3.2.2a: Estimation of Sensitivity for Standard and Systematic Run Orders

The model design matrix in Table 2.1 was used to measure how sensitive the standard run order is to presence of active contrasts. The experimental case 2 data was censored using the procedure of Section 4.1. The half normal plots for the original data and censored data based on one of the linear trend resistant column (fifth column) of the design in Table 2.1 are presented in Figure 4.9

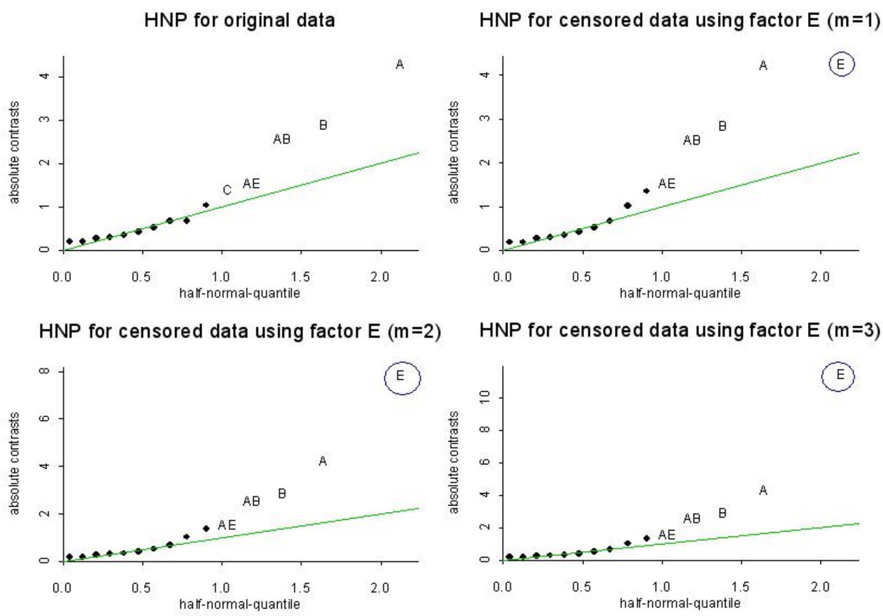


Figure 4.9: *Standard run order half normal plots for the original and censored data. The circled factor in each graph is the factor used as a basis for censoring the data for different m values.*

The observed results from the half normal plots in Figures 4.9 are listed as follows:

a) On the half normal plot for the original data, the estimated effects of contrasts A, B, AB, AE, and C are seen to be above the straight line with the estimated effect of factor A as the maximum absolute contrast.

b) On the half normal plot for the data set obtained with $m = 1$ using the setting of factor E (fifth column) in Table 2.1, the estimated effects of contrasts E, A, B, AB, and AE are above the straight line with the estimated effect of factor E as the maximum absolute contrast.

c) On the half normal plot for the data obtained by adding $m = 2$ to the original data when the setting of factor E (fifth column) in Table 2.1 is at + level and zero otherwise, estimated effects of contrasts E, A, B, AB, and AE are above the straight line with the estimated effect of factor E as the maximum absolute contrast.

The remaining ten trend resistant columns (columns 6-15) of the design in Table 2.1 were also used to censor the data. Thus, there are ten different new data sets (see Tables C4, C5 and C6 in Appendix C). For these ten data sets, the half normal plots for the estimated effects contrasts were plotted, these are presented in Figures B13 and B14 in Appendix B. The resulted HNPs for the eleven data sets show that when $m = 1$ is added to the data, only two out of the HNPs have the contrast effect that were used to censor the data as the maximum absolute contrast. However, all the eleven HNPs have the contrast effect that were used to censor the data as the maximum absolute contrast when $m = 2$ was used.

To measure how sensitive the systematic run order is to presence of active contrast, the design in Table 2.3 was used. Following the algorithm as before, the experimental data was censored using the setting of factor A (first column) in the model design matrix in Table 2.3. The half normal plots for both the original data and the censored data are presented in Figures 4.10

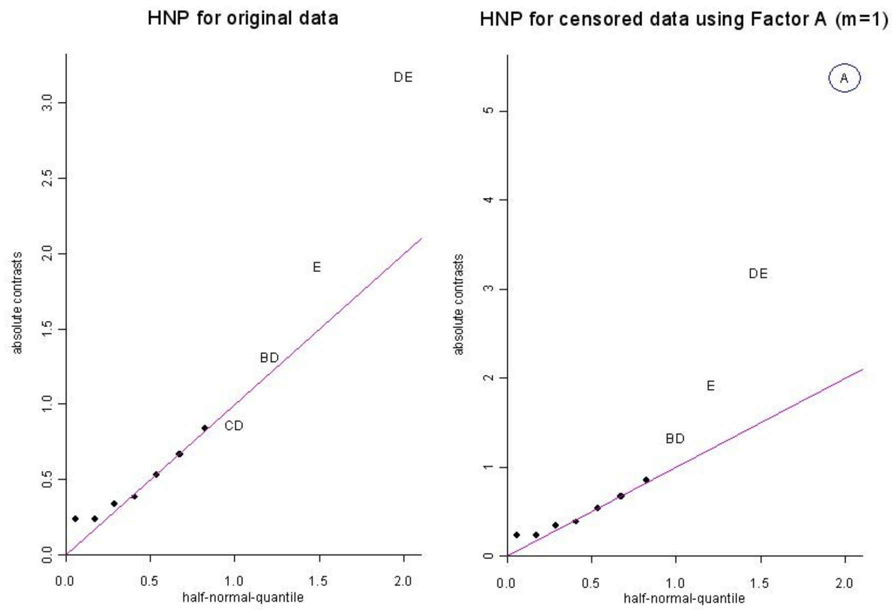


Figure 4.10: *Reduced (Systematic run order) half normal plots for the original and censored data. The circled factor in the figure is the factor used as a basis for censoring the data (case 2)*

From Figure 4.10 the following observations were made:

a) On the half normal plot for the original data, the estimated effects of contrasts DE, E, and BD are seen to be above the straight line with the estimated effect of contrast DE as the maximum absolute contrast.

b) On the half normal plot for the censored data obtained with the setting of Factor A (first column) in Table 2.3 when $m = 1$, the estimated effects of contrasts A, DE, E and BD are seen to be above the straight line with the estimated effect of factor A as the maximum absolute contrast.

As in example 1, the setting of the remaining ten columns of the design in Table 2.3 were used to censor the data. The half normal plots for the new data sets are presented in Figure B21 in Appendix B.

From the plots in Figures 4.9, 4.10 and the Figures in B13, B14 and B22 in Appendix B, the following findings were inferred:

(a) For the standard run order, when $m = 1$ is used to censor the data, the estimated effects of the contrasts that were used for censoring the data are not the maximum absolute contrasts on the half normal plots in 82% of the cases.

(b) For the standard run order, when $m = 2$ is used to censor the data, the estimated effects of the contrasts that were used to censor the data are the maximum absolute contrasts on the HNPs for all the data sets.

(c) For the systematic run order, in 82% of the data sets obtained with $m = 1$, the estimated effects of the contrasts that were used to censor the data are the maximum absolute contrasts.

From the aforementioned points, It is clear that the systematic run order is more sensitive to presence of active contrast than the standard run order. This is similar to the conclusion drawn in example 1.

4.3.2.2.1a: Estimation of PFR and PED using MAC as an estimate of error variance

Estimation of Probability of False Rejection

To obtain the proportion of false rejection for the randomized run order, we used the design in Table 2.1 as a starting point for randomization. Permuting the rows of the model design matrix in Table 2.1, and using the experimental result for machine 2 presented in Table 3.2, we estimated the contrast effects and follow the algorithm as stated in **Approach 1**. Here, h in Equations (4.3) and (4.4) equals MAC. The obtained PFR value is 0.049. similarly, the harmonized approach yields a PFR of 0.0543 for the randomized run order. This verifies once more that approximately 5 % of the randomized run order will falsely give an active contrast.

To evaluate the PFR and PED for the standard and systematic run orders, new data sets were generated from an ARIMA model. The procedures in Section 4.2.1.1 are used to fit model to the case 2 experimental data. On following the procedure, an $ARIMA(0, 1, 2)$ model given by

$$y_t = y_{t-1} - 0.3a_{t-1} + 0.5a_{t-2} + a_t \tag{4.13}$$

was fitted to the data. Following the algorithm of **Approach 2**, data were generated from the $ARIMA(0, 1, 2)$ model of Equation (4.13). The observed proportion of simulated data sets with $max|t_{MAC}| > 3.6978$ for the standard run order equals 0.555. This indicates that with data of this kind, the standard run order will falsely identify an active contrast in about 56 % of all cases. Similarly for the systematic run order, we used the design in Table 2.3. Following the algorithm of **Approach 2**, the observed proportion of simulated data sets with $\max|t_{MAC}| > 3.7236$ is 0.213. This implies that the systematic run order will falsely give an active contrast in about 21% of all cases of data of this kind. Though the obtained PFR for the systematic run order is smaller than the PFR for the standard order, it is still too large. Therefore, the systematic run order does not provide sufficient protection against this kind of trend as modelled here. The plot of the obtained proportion along with the ordered absolute half normal plot test statistic ($max|t_{MAC}|$) of the systematic, standard, and randomized run orders are presented in Figure 4.11

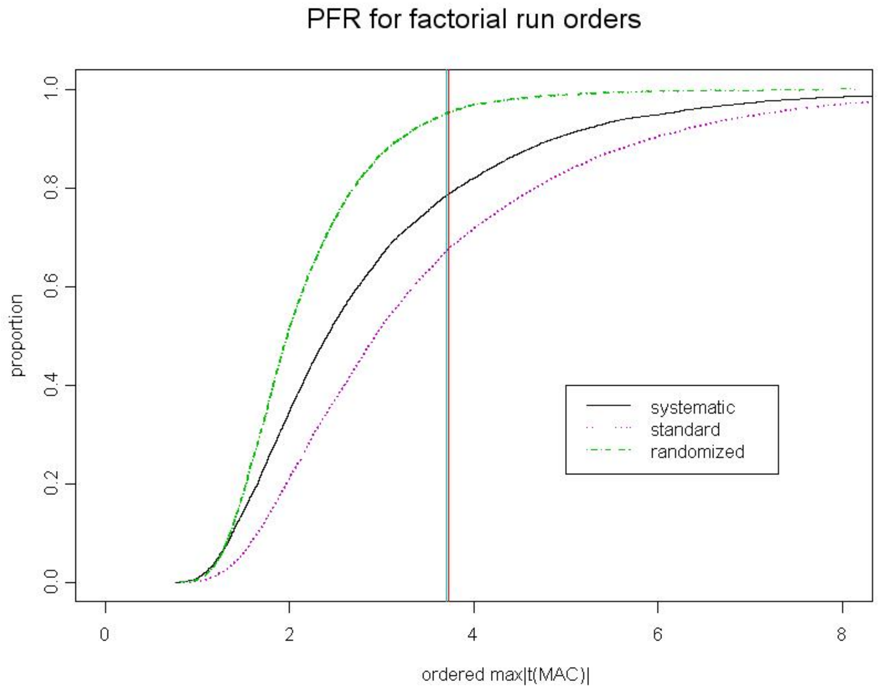
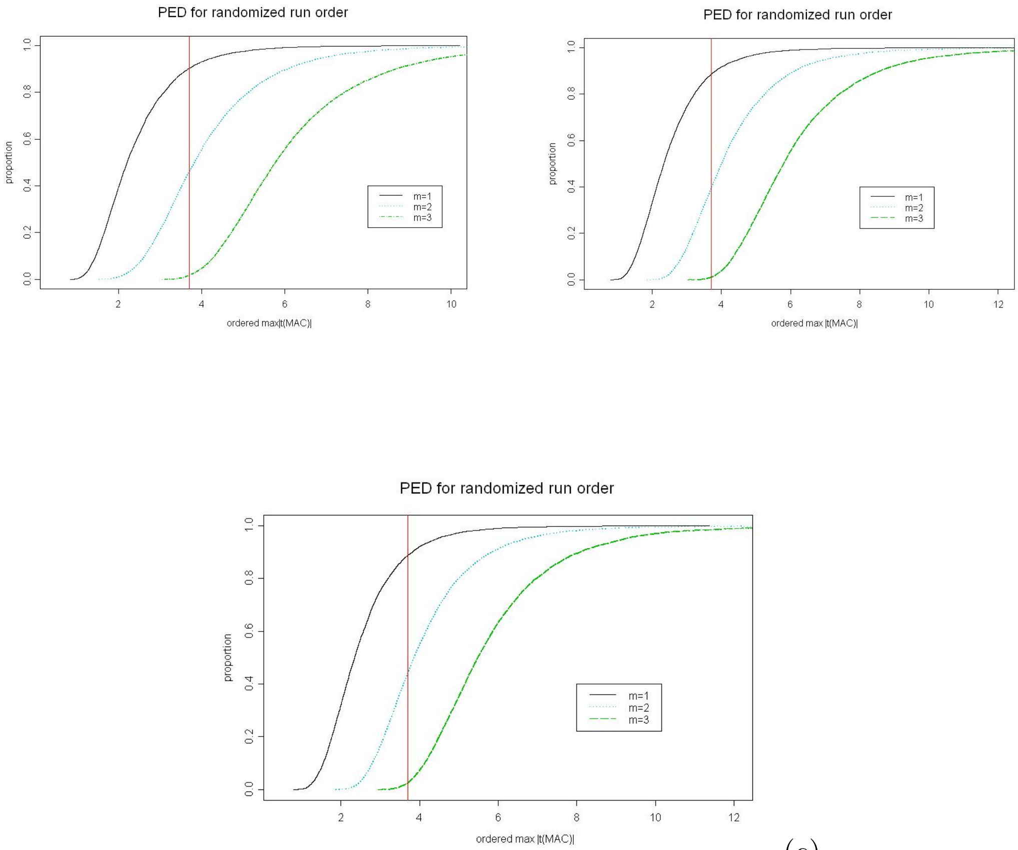


Figure 4.11: *Empirical distribution function for the systematic, standard, and randomized run orders without active contrast. The two vertical lines in the figure are as earlier defined.*

Estimation of Probability of Effect Detection

The procedure of **Approach A** of Section 4.2.1.2 was used to compute the PED for the randomized run order. Using a 0.05 level of significance with $b = 15$, adding $m = 1$ to the data whenever the setting of the first column of the permuted design is $+$, gives a PED of 9.8%, adding $m = 2$ gives 54%, and adding $m = 3$ gives 98.2 %. Thus, adding $m = 2$ gives approximately the desired probability of effect detection for the approximate measure of sensitivity for the randomized run order. Figure 4.12 presents the empirical distribution function of $max|t_{MAC}|$ for the randomized run order for one, two and three active contrasts.

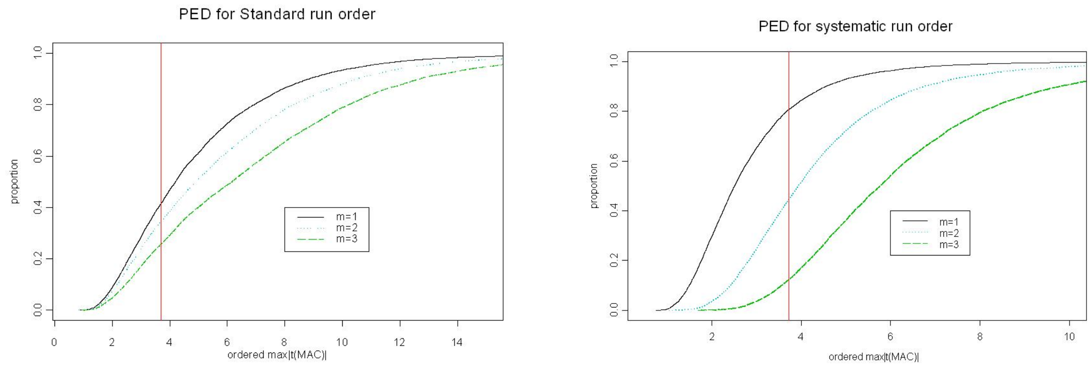


(a) (b) (c)

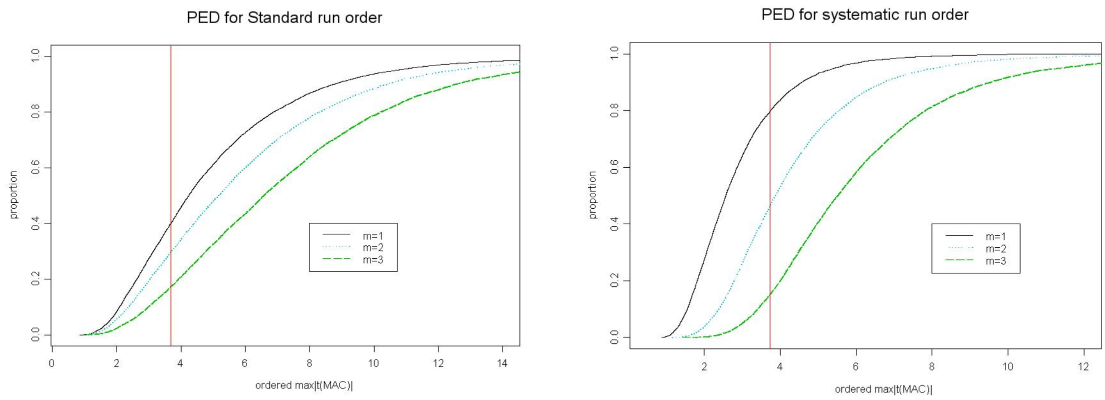
Figure 4.12: *Empirical distribution function of max*$|t_{MAC}|$ *for the randomized run order with different sizes of active contrasts. (a)one active contrast, (b)two active contrasts, (c) three active contrasts. The vertical line in each graph represents the critical value.*

The probability of effect detection (PED) for the standard and systematic run orders was estimated using the data generated from the $ARIMA(0,1,2)$ model of Equation (4.13). Following the algorithm of **Approach B**, the PED for the run orders for one, two, and three active contrasts was determined. The resulted proportion of design along with the ordered max$|t_{MAC}|$ are plotted for different effect sizes for one, two, and three active contrasts for both the standard and systematic run orders. These are presented in Figure 4.13.

One active contrast



Two active contrasts



Three active contrasts

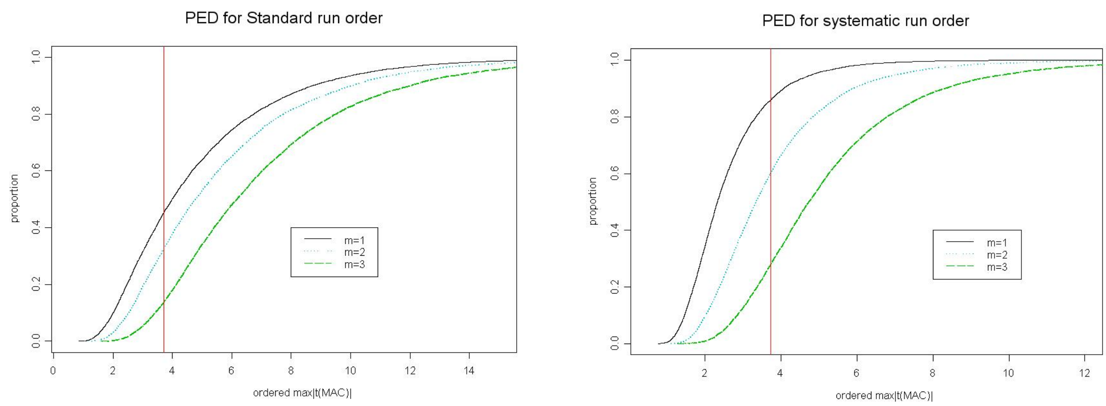


Figure 4.13: *Empirical distribution function of* $max|t_{MAC}|$ *for the standard and systematic run orders with different sizes (m) of active contrasts. The vertical line in each graph represents the critical value*

The harmonized approach yields a PFR of 5% with similar $PEDs$ as those obtained with the other approaches. Using the PFR obtained with the harmonized approach for the randomized run order, the pseudo critical value that we should use for the systematic run order equals 5.8556. Using this pseudo critical value, we computed the PED with different sizes of m for one, two and three active contrasts for the systematic run order. The results are presented in Table 4.3.

4.3.2.2.1b Estimation of PFR and PED using the PSE and ASE as an estimate of error variance

The analyses of the case 2 data and the model fitted to the data. were repeated using the PSE and ASE as an estimate of error variance. The obtained empirical $PFRs$ using the PSE as an estimate of error variance (that is, $h = PSE$ in Equations (4.3) and (4.4)) are 56%, 6%, and 19% for the standard, randomized and systematic run orders, respectively. The ASE estimate (that is, $h = ASE$ in Equations (4.3) and (4.4)) produces a PFR of 59%, 6%, and 23% for the standard, randomized and systematic run orders, respectively. Figure 4.14 presents the graphical display of the results based on the PSE and the ASE.

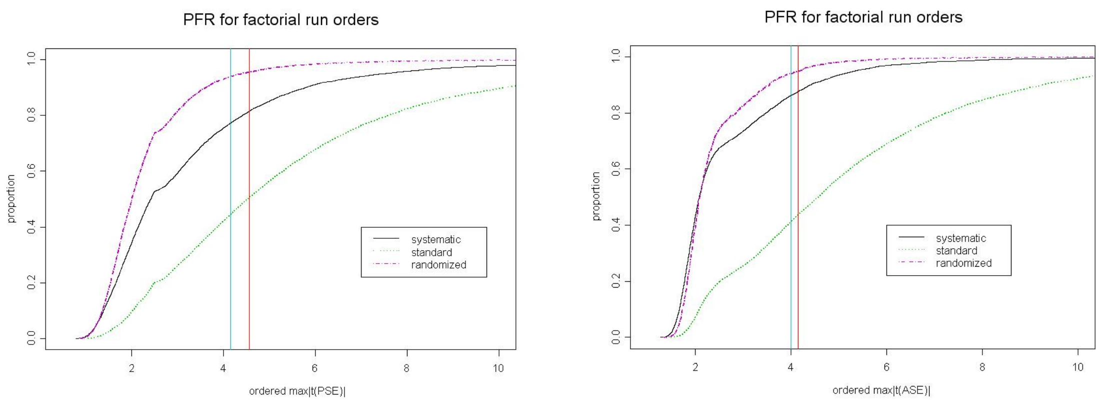


Figure 4.14: *Empirical distribution function for th standard and systematic run orders (LHS= PSE estm., RHS= ASE estm.). The blue and red vertical lines in each graph are as earlier defined.*

In a similar way, we computed the PED with different sizes for one, two and three active contrasts based on the PSE and the ASE. The obtained empirical PED based on the PSE and ASE for the three run orders are presented in Figures A12 in Appendix A1 and in Figures A21 in Appendix A2, respectively.

The harmonized approach was used to compute the PFR for the randomized run order. For the PSE estimate, we have a PFR of 6%, and for the ASE estimate, we have a PFR of 5%. These values are approximately the same as the PFR obtained earlier with approach **1**. The behavior of the obtained PED for the randomized run order using the harmonized approach is similar to those obtained earlier with approach **A** (see Tables 4.3 to 4.5). The pseudo critical value for both the PSE and the ASE equals 7.1249 and 6.6330, respectively. These pseudo critical values were used to compute the PED for the systematic run order with different sizes(m) for one, two and three active contrasts. The obtained results are presented in Table 4.4 for the PSE based estimates and on Table 4.5 for the ASE based estimates.

4.4 Summary and Conclusion for the performance standard simulation results

The obtained probabilities of false rejection and effect detection of the different sizes of active contrasts for the standard, randomized, and systematic run orders based on MAC, PSE and ASE for the two cases used to evaluate the performance algorithms are summarized and presented in Tables 4.3, 4.4 and 4.5, respectively. On the aforementioned tables, the columns marked with an asterisk represent the PFR and PED for the randomized run order obtained with the harmonized approach and the PFR and PED for the systematic run order obtained with the pseudo critical value.

Table 4.3: Summary of The Empirical PFR and PED Based on MAC

[1]

		Case 1					Case 2				
AC	m	SO	RO	RO*	Sys	Sys*	SO	RO	RO*	Sys	Sys*
	0	0.3283	0.0473	0.0509	0.1076	0.0509	0.5553	0.0494	0.0543	0.2130	0.0543
One	1	0.4168	0.0726	0.0944	0.1469	0.0692	0.5850	0.0979	0.0759	0.1919	0.0392
	2	0.6010	0.3587	0.3749	0.5864	0.3858	0.6519	0.5398	0.2293	0.5536	0.1699
	3	0.7829	0.862	0.7056	0.9254	0.7841	0.7420	0.9824	0.4709	0.8764	0.4837
	10	1.00	1.00	0.9995	1.00	1.00	0.997	1.00	0.9887	1.00	1.00
Two	1	0.4338	0.0829	0.1078	0.1601	0.0749	0.5988	0.1150	0.0798	0.2042	0.0355
	2	0.6834	0.4144	0.4159	0.5939	0.3721	0.7006	0.6069	0.2551	0.5356	0.1679
	3	0.8947	0.9013	0.7386	0.9096	0.7587	0.8264	0.9889	0.5059	0.8490	0.4421
	10	1.00	1.00	0.9998	1.00	1.00	0.9999	1.00	0.9905	1.00	0.9995
Three	1	0.3970	0.0778	0.102	0.1004	0.041	0.5486	0.1128	0.0724	0.1409	0.0625
	2	0.6878	0.3703	0.3828	0.4453	0.249	0.6750	0.5585	0.2368	0.4001	0.2313
	3	0.9268	0.8582	0.7051	0.8054	0.5998	0.8641	0.9753	0.4712	0.7230	0.5264
	10	1.00	1.00	0.9995	1.00	1.00	1.00	1.00	0.9880	1.00	1.00

Results on the PFR and PED Based on the MAC

The PFR results on the first row of the Table 4.3 show similarity in the run orders for the two illustrative cases used to evaluate our algorithms. In the two cases (cases 1 and 2), the standard run order performed poorly with as high as 56% probability of false rejection. This is catastrophically high!. The PFR for the randomized run order when both the experimental data and the generated data were used equals

[1] *SO ⇒ Standard run order, RO ⇒ Randomized run order, and Sys ⇒ Systematic run order. RO* columns represents the results obtained from the simulated data for the randomized run order (harmonized approach) and Sys* represents the results obtained by using the pseudo critical value for the systematic run order.*

5%. This is a good result for the performance of the randomized run order since we based our critical value on 5% level (that is, $\alpha = 0.05$). The systematic run order has a PFR value that almost double the size (case 1) and four times higher (case 2) the value obtained for the randomized run order. This is an advantage point in favor of the randomized ordering.

The resulted PFR for the systematic run order with the pseudo critical value has the same PFR with the randomized run order. This is deliberately done so that we can have a fairer comparison of the probability of effect detection for both the randomized and systematic run orders as mentioned earlier. Therefore, the results in columns 3, 5, and 7 for case 1 and columns 8, 10, and 12 for case 2 in Table 4.3 are used to interpret the probability of effect detection for the three run orders under study. These columns give the summarized PED for the standard, randomized and systematic run orders obtained with the generated data from the fitted model in Equations (4.12) and (4.13) for cases 1 and 2, respectively. The results are discussed as follows:

PED with one active contrast:- The standard run order performed better than both randomized and systematic run orders when the effect size equals to one and two (that is, $m = 1$ or 2). This is not surprising since the standard order started with very high level. However with m increased to 3, the three run orders have approximately the same power. Thus, the randomized and systematic run orders have approximately the same power for detecting an active contrast. Also the PED for the three run orders increases with the effect size(m). That is, as the effect size increases, the probability of effect detection also increases.

PED with Two active contrasts:- The pattern of the obtained $PEDs$ are similar as for one active contrast. When $m = 1, 2, 3$, the standard run order has as high power as 89% (case 1, when $m = 3$) and 83% (case 2 when $m = 3$). Also the randomized run order has as high power as 74% (case 1, when $m = 3$) and 51% (case 2, when $m = 3$). Thus, The performance of the randomized and systematic run orders are very close. For the case with $m = 3$, the scenario remains as it is with one active contrast.

PED with Three active contrasts:- The patterns of the PED remain as for one and two active contrasts (see Table 4.3, Figures 4.6 and 4.13).

If we used the results for the systematic run order obtained with the simulated critical value instead of the pseudo critical value to compare the results for the randomized run order, then columns 6 and 11 of Table 4.3 will be used. Using the results on the aforementioned columns, it is clear that the systematic run order has higher power than the randomized run order for one, two, and three active contrasts. However, considering the level for the systematic run order which is as high as 21% (case2) and 11% (case 1), then the resulted power for the systematic run order can not be adjudged to be higher than that of the randomized run order.

Results on the PFR and PED based on the PSE

The results from Figure 4.7 and the Figures in Appendix A1 are summarized into Table 4.4. These results are similar with the results obtained with the MAC (see Table 4.3on page 116). One point that is worth to be mentioned here, is that the obtained PFR with the PSE are slightly higher than the PFR obtained with the MAC. This could be due to the fact that the PSE critical values are higher than the MAC critical values. From the results on Table 4.4, the standard run order still performed poorly with the same percent as obtained with the MAC based estimates. Whereas, the PFR for the randomized run order slightly increases to 6%, but this is not a bad performance for the randomized run order. The relationship of the PFR for the randomized and systematic run orders is as with the results obtained with the MAC based estimates. The PED for the standard, randomized and systematic run orders for one, two, and three active contrasts have the same pattern as those obtained with the MAC based estimates.

Table 4.4: Summary of Empirical PFR and PED Based on PSE

1

		Case 1					Case 2				
AC	m	SO	RO	RO*	Sys	Sys*	SO	RO	RO*	Sys	Sys*
	0	0.3299	0.0506	0.0585	0.0900	0.0585	0.555	0.0614	0.0587	0.1853	0.0587
	1	0.4095	0.0807	0.0944	0.1325	0.0845	0.585	0.114	0.078	0.1707	0.0522
One	2	0.5983	0.3366	0.3558	0.5097	0.3741	0.654	0.5041	0.215	0.4928	0.1741
	3	0.7772	0.8228	0.6817	0.8727	0.7554	0.7404	0.9676	0.4519	0.8296	0.4386
	10	0.9999	1.00	0.9991	1.00	1.00	0.9970	1.00	0.9863	1.00	0.9999
	1	0.4389	0.0887	0.1132	0.1518	0.1027	0.5974	0.1257	0.0826	0.1863	0.0565
Two	2	0.6919	0.4263	0.4244	0.5882	0.4616	0.7061	0.6274	0.2606	0.5466	0.2361
	3	0.8961	0.9155	0.7500	0.9113	0.8283	0.8320	0.9894	0.5177	0.8667	0.5276
	10	1.00	1.00	0.9996	1.00	1.00	0.9999	1.00	0.9912	1.00	0.9999
	1	0.4098	0.0841	0.1108	0.1069	0.0728	0.5613	0.1251	0.0794	0.1360	0.0934
Three	2	0.7282	0.4295	0.4276	0.5223	0.4152	0.7043	0.6458	0.2600	0.4790	0.3908
	3	0.9418	0.9406	0.7599	0.8891	0.8077	0.8899	0.9952	0.5178	0.8242	0.7441
	10	1.00	1.00	0.9999	1.00	1.00	1.00	1.00	0.9923	1.00	1.00

Results on the PFR and PED based on ASE

The results from Figure 4.8 and the Figures in Appendix A2 are summarized in Table 4.5. These results are similar to the results obtained with both the MAC and PSE based estimates. Here the PFR are slightly higher than the PFR obtained with the MAC, but do not have a fixed relationship with the obtained PFR based on the PSE. For example, the PFR for the standard run order based on the ASE are higher than those of the PSE for the two cases, while the PFR for the randomized run order based on the PSE are higher than those of the ASE. A close examination of the obtained PED in Table 4.5 reflect that the randomized run order has higher power than the systematic run order in some cases, while in some cases, both of them have approximately equivalent power. These results are equivalent to the results obtained earlier with the MAC and ASE based estimates.

Table 4.5: Summary of Empirical PFR and PED Based on ASE

1

		Case 1					Case 2				
AC	m	SO	RO	RO*	Sys	Sys*	SO	RO	RO*	Sys	Sys*
	0	0.3708	0.0422	0.0539	0.1233	0.0539	0.5851	0.0588	0.0561	0.2304	0.0561
	1	0.4576	0.0815	0.1026	0.1667	0.0719	0.6200	0.1216	0.0794	0.2045	0.0448
One	2	0.6374	0.4523	0.4439	0.6853	0.4392	0.6777	0.6729	0.2713	0.6211	0.1887
	3	0.8066	0.9998	0.7788	0.9710	0.8645	0.7603	1.00	0.5359	0.9283	0.5360
	10	0.9999	1.00	0.9999	1.00	1.00	0.9976	1.00	0.9941	1.00	1.00
	1	0.4749	0.0812	0.1167	0.1756	0.0882	0.6315	0.1291	0.0835	0.2032	0.0467
Two	2	0.7316	0.4979	0.4894	0.6859	0.5035	0.7293	0.7679	0.2946	0.6005	0.2493
	3	0.9204	0.9980	0.8145	0.9665	0.8958	0.8467	1.00	0.5758	0.9096	0.5938
	10	1.00	1.00	0.9999	1.00	1.00	0.9999	1.00	0.9963	1.00	1.00
	1	0.4243	0.0887	0.1046	0.1038	0.0534	0.5634	0.1181	0.0771	0.1336	0.0670
Three	2	0.7354	0.4198	0.4341	0.5213	0.4137	0.7016	0.6827	0.2625	0.4478	0.3489
	3	0.9551	0.9827	0.7764	0.9065	0.8438	0.8936	1.00	0.5292	0.8017	0.7185
	10	1.00	1.00	0.9999	1.00	1.00	1.00	1.00	0.9945	1.00	1.00

Chapter 5

Conclusion and Discussion of Results

The algorithms presented for the various methods of constructing trend resistant designs are easy and straight forward to implement. Furthermore, the modified version of the reverse foldover algorithm produces a factorial design that is linear trend resistant for all the main effects with a minimum number of factor level changes. This is an improvement in the area of trend resistant designs. Table 5.1 below gives a summary of the results obtained with the algorithms for the various approaches reviewed in this study.

Table 5.1: Summary Table For Methods of Constructing Trend Resistant Designs

Method	Design	No. of linear trend resistant contrasts	No. of factor level changes
DW	2^4	11	37
Foldover	2^4	11	43
Reverse foldover(RF)	2^4	11	53
Modified RF	2^4	11	19
Generalized foldover	2^4	11	43

All the reviewed procedures for the construction of trend resistant factorial/ fractional factorial designs yield the same possible number of linear time-trend resistant contrasts. The summary of the obtained number of possible linear time-trend resistant contrasts for some factorial/ fractional factorial designs using the theorem in Chapter 2 and the obtained number of time-trends contrasts for the constructed linear trend Plackett Burman designs are presented in Table 5.2.

Table 5.2: Maximum Number of Trend Resistant Contrasts

Design	Number of contrasts	No. of linear trend resistant contrasts
$2^3/2^{4-1}$	7	4
$2^4/2^{5-1}$	15	11
$2^5/2^{6-1}$	31	26
PB_{12}	11	4
PB_{12+12}	11	11

The summarized results in Table 5.2 reflect that for 8 runs designs, it is possible to have 57% of the contrasts to be at least linear trend resistant, for 16 runs designs 73%, for 32 runs 83%, for 12 runs PB designs 36%, and for 24 runs PB designs with eleven contrasts 100%. Therefore, for two levels factorial/fractional factorial designs, the best design in terms of linear trend resistance will be 16 runs designs, since with this design it is possible to have as much as 73% of the contrasts to be at least linear trend resistant. For Plackett Burman designs, a price has to be paid for achieving high number of trend resistant columns.

From the results of the experiment which is used as a practical case study to show presence of time trend in a factorial experiment, the following were inferred:

- The running time of the ball bearing gets considerably larger when the ball bearing has run several times. That is, there is a time-trend in the funnel experiment.
- Two or more identical and independent funnels behaved differently.
- Two or more identical and independent rods behaved alike, that is, no differences in the running time within the rod.

– Two identical and independent balls behaved some times differently.

The results from the factorial experiments in section 3.2 show that none of the rod, the ball nor the interaction between ball and rod have a significant effect on the observed time-trend in the experiment. The computed confidence interval for the mean response (sum of differences in the two sequences) for the ball, the rod and their interactions shows that the smallest mean for the sum of differences in the running time for the two sequences in the experiments should be approximately -0.25, and the maximum sum of differences of the running time should be 12.41. The results from the cross over experiment in section 3.3 show that the funnel has the highest influence on the time-trend.
A possible explanation for these results is either that the ball gets warmed-up by the rod and hence the ball takes a longer time to spin within the funnel as we go on in the experiment (increase with time) or that the funnel gets warmed-up by the ball as we proceed in the experiment. From the aforementioned points, we are convinced that the funnel is responsible for the time trend in the exemplified experiment.

One way to eliminate the time trend problem in the funnel experiment is to clean the funnel before conducting the experiments. Another point is to increase the time lag between successive run to say about 120 seconds.

On the results from the funnel experiment, there is the possibility of human error in taking the measurements. Thus, to eliminate the human error, one approach will be to automate the measurement procedure as suggested by an Engineer. This will give more accurate and precise measurements and hence an improvement on the results that are manually measured as done in this study.

From the sensitivity analysis, the obtained results for the standard and systematic run orders to presence of active contrast for the two examples presented in this study are summarized in Table 5.3.

Table 5.3: Summary Table For Sensitivity Analysis

		Standard											systematic										
	m	1	2	3	4	5	6	7	8	9	10	11	1	2	3	4	5	6	7	8	9	10	11
Case1	1	$\star$	$\star$	$\star$	$\star$	$\star$	$\star$	$\star$	$\star$	$\star$	$\star$	$\star$	∇	∇	∇	∇	∇	∇	∇	∇	∇	∇	∇
	2	∇	∇	$\star$	∇	$\star$	∇	∇	$\star$	∇	∇	∇											
case2	1	∇	$\star$	$\star$	$\star$	$\star$	$\star$	∇	$\star$	$\star$	$\star$	$\star$	∇	∇	∇	∇	$\star$	∇	∇	∇	∇	∇	$\star$
	2	∇	∇	∇	∇	∇	∇	∇	∇	∇	∇	∇											

In Table 5.3, ∇ represents the cases where the estimated effect of the contrast used to censor the data is the maximum absolute contrast on the half normal plot and $\star$ represents the cases where the estimated effect of the contrast used to censor the data is not the maximum absolute contrast on the half normal plot. From the results in Table 5.3, we can infer that for the standard order, $m = 2$ makes the estimate effect of the contrasts that were used to censor the data as the maximum absolute contrast on the half normal plot in 73% of the data sets for case 1 and in all the data sets for case 2. Similarly, for the systematic run order, $m = 1$ makes the estimate effect of the contrasts that were used to censor the data as the maximum absolute contrast on the half normal plot in all the eleven data sets for case 1 and in 82 % of the data sets for case 2. Thus, for the two examples used in this study, $m = 1$ makes the contrast used to censor the data to be the maximum absolute contrast on the HNP for the systematic run order, while we obtained $m = 2$ for the standard run order. For the randomized run order, a minimum of $m = 2$ gives the desired PED for sensitivity analysis. Therefore, we can conclude that the systematic run order is more sensitive to presence of active contrast than both the randomized and standard run orders.

The obtained $PFRs$ ($m = 0$) based on the two examples studied show that the randomized run order has the best probability of false rejection out of the three run orders under consideration. Further, the closeness in the empirical PFR for the randomized run order obtained with the experimental data and the generated data confirm the appropriateness of the fitted models to the experimental data.

The obtained PED when m equals 1 or 2 for one active contrast shows that the standard run order has the highest power in both case studies. Also, the results for two and three active contrasts show that the standard run order has the highest PED followed by the systematic run order and then the randomized run order for the situations when m =1, 2, and 3. As mentioned earlier, this is not surprising, since the standard run order started with a very high level.
On the other hand, since our result from the sensitivity analysis shows that before an active contrast will be declared to be active, at least $m = 2$ is needed to censor the data for the randomized run order and $m = 1$ is needed for the systematic run order, then it is sensible to use the PED obtained for these values for comparison. Using these values, that is, $m = 2$ for the randomized order and $m = 1$ for the systematic run order, the obtained probability of effect detection for the randomized run order for one, two and three active contrast(s) are greater than those for systematic run order.
In summary, the randomized run order performed better than the systematic run order. However, the results obtained with the pseudo critical value for the systematic run order reflect that if both the randomized and systematic run order have the same level, then their respective powers are approximately the same irrespective of the method used to estimate the error variance for the contrasts. Thus, the systematic order does not achieve a higher power than the random ordering, when we corrected the critical value to keep the nominal level.

Based on the aforementioned points, the following conclusions are drawn:

- The systematic run order is more sensitive to presence of active contrast than both the standard and randomized run orders.

- When there are no active contrasts, the randomized run order managed to keep the nominal level. The systematic run order was nearer the nominal level than the standard run order, but both did not manage to keep the nominal level.

- For the situation with one active contrast, the systematic run ordering and the randomized order have $PED's$ that are similar to the PED of the standard order, if the size of the active contrast increases.

- when there are two active contrasts, both randomized and systematic run orders produced smaller $PEDs$ than the standard order. The increase of the PED for the randomized order is much less.

- When there are three active contrasts, both randomized and systematic run orders performed alike.

- When we adapted the critical value such that the systematic order kept the nominal level, then the power of the systematic order decreased considerably. In that case the power was no longer higher than the power derived from the randomized order.

It is evident from the two cases used as illustrative example that the randomized run order performed better than the systematic run order with regards to PFR, while the systematic run order performed better than the standard run order.
In summary, the systematic run order is more sensitive to presence of active contrast than the randomized run ordering, while the latter has a more reliable level than the former. In addition, though both the randomized and systematic run orders have similar power for detecting active contrasts, there is no outstanding advantage of the systematic run order over the randomized run order visible in this study.

In general, when factorial/fractional factorial experiments are conducted over sequence of time for quality improvement, randomizing the run order of the design is an appropriate proceeding. However, when randomization procedures are expensive (in time and money) or not feasible, then systematic run orders that are time-trend resistant should be used.

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APPENDICES

Appendix A

EMPIRICAL PED FOR 2^{K-P} RUN ORDERS

A.1 EMPIRICAL PED FOR PSE ESTIMATE.

A.1.1 Empirical PED for standard, randomized and systematic run orders (CASE 1)

Figure A11.1- One active contrast

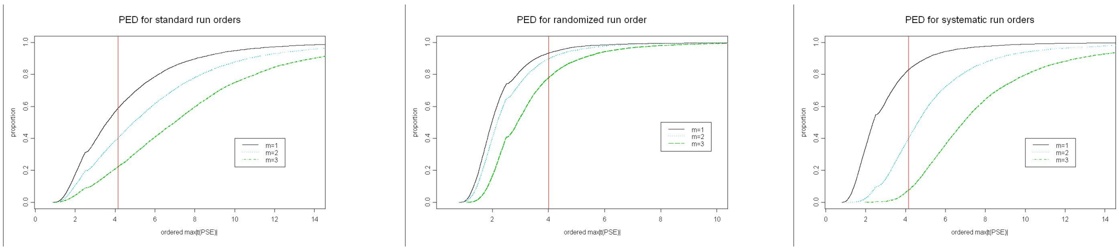


Figure A11.2- Two active contrasts

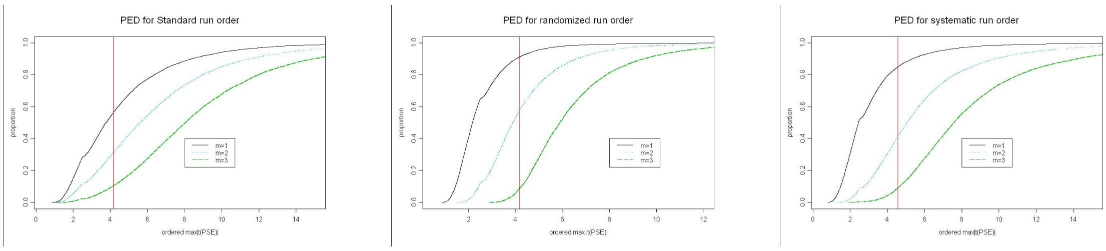
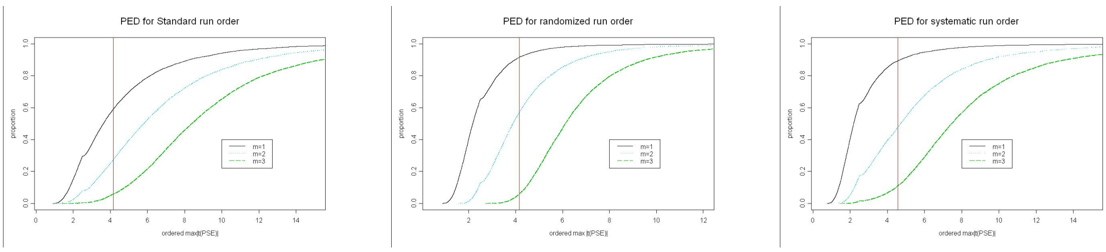


Figure A11.3- Three active contrasts



A.1.2 Empirical PED for standard, randomized and systematic run orders (CASE 2)

Figure A12.1:- One active contrast

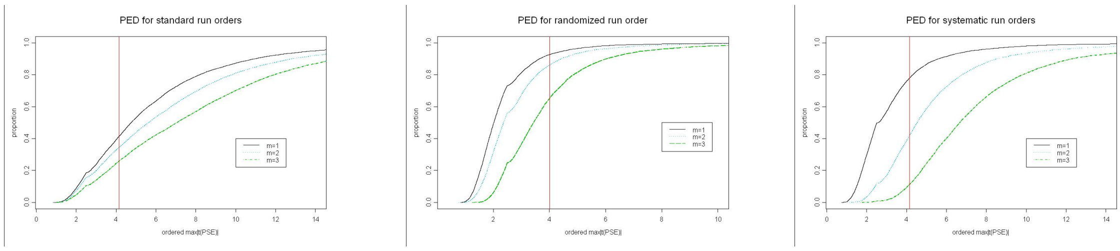


Figure A12.2:- Two active contrasts

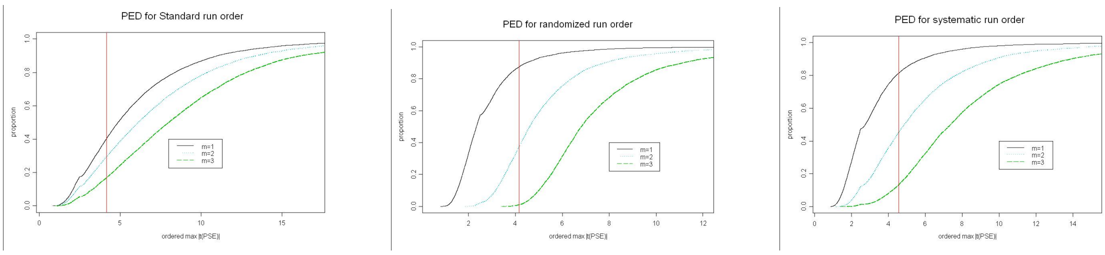
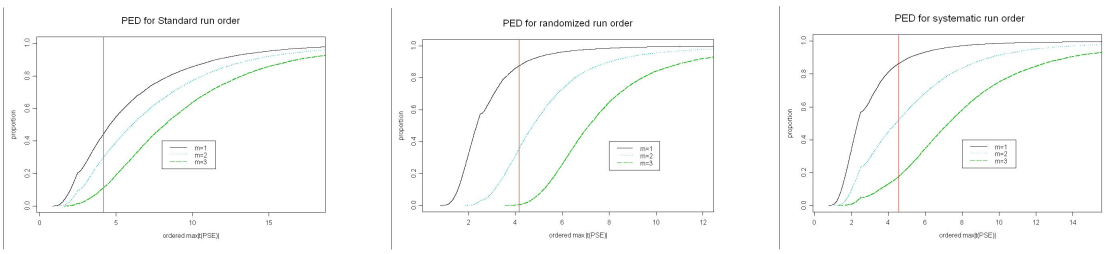


Figure A12.3:- Three active contrasts



A.2 EMPIRICAL PED FOR ASE ESTIMATE.

A.2.1 Empirical PED for standard, randomized and systematic run orders (CASE 1)

Figure A21.1:-One active contrast

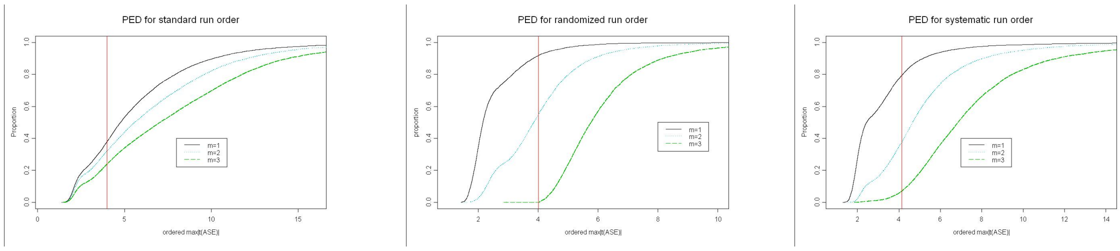


Figure A21.2:-Two active contrasts

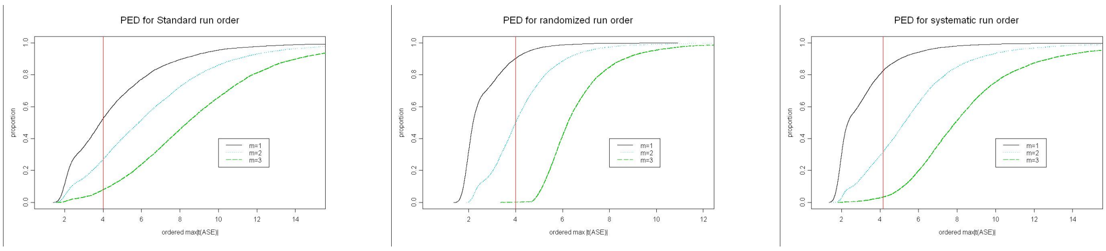
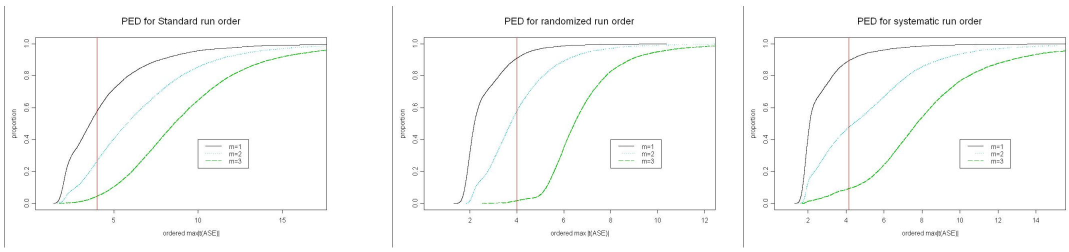


Figure A21.3:-Three active contrasts



A.2.2 Empirical PED for standard, randomized and systematic run orders (CASE 2)

Figure A22.1:-One active contrast

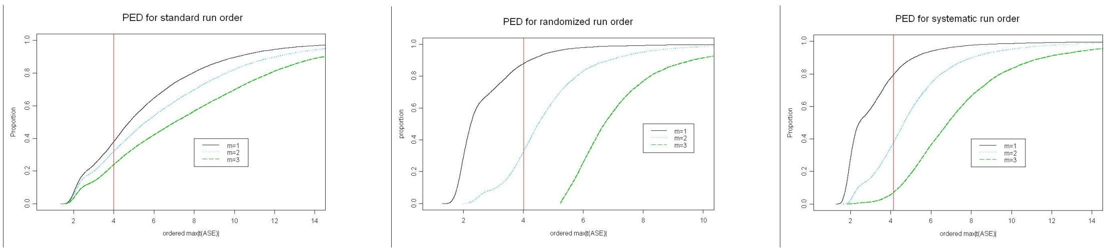


Figure A22.2:-Two active contrasts

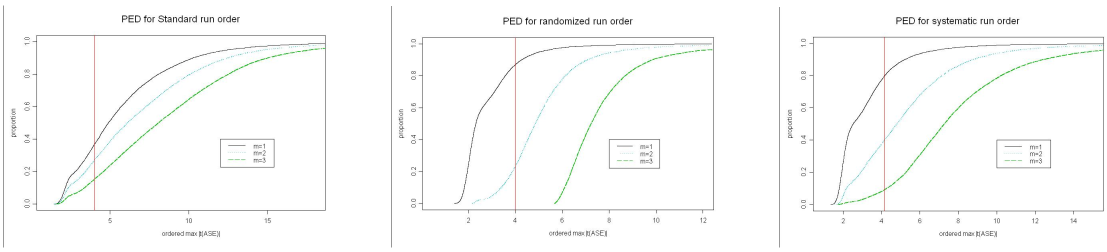
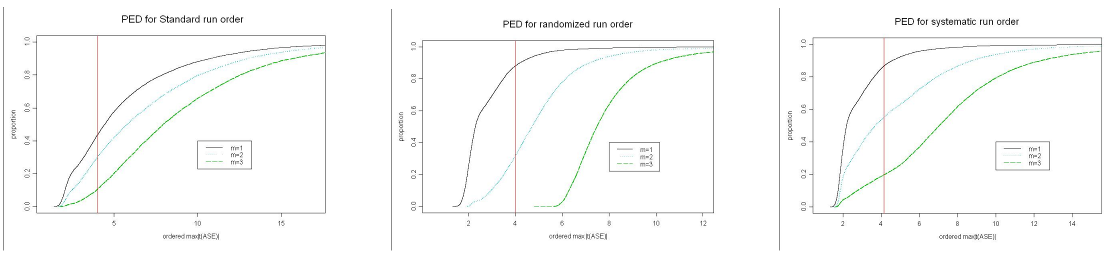


Figure A22.3:- Three active contrasts



Appendix B

HALF NORMAL PLOTS FOR STANDARD AND SYSTEMATIC RUN ORDERS

B.1 Half Normal Plot for Standard run order

Figure B11: HNP for original data and m = 1 data (CASE 1)

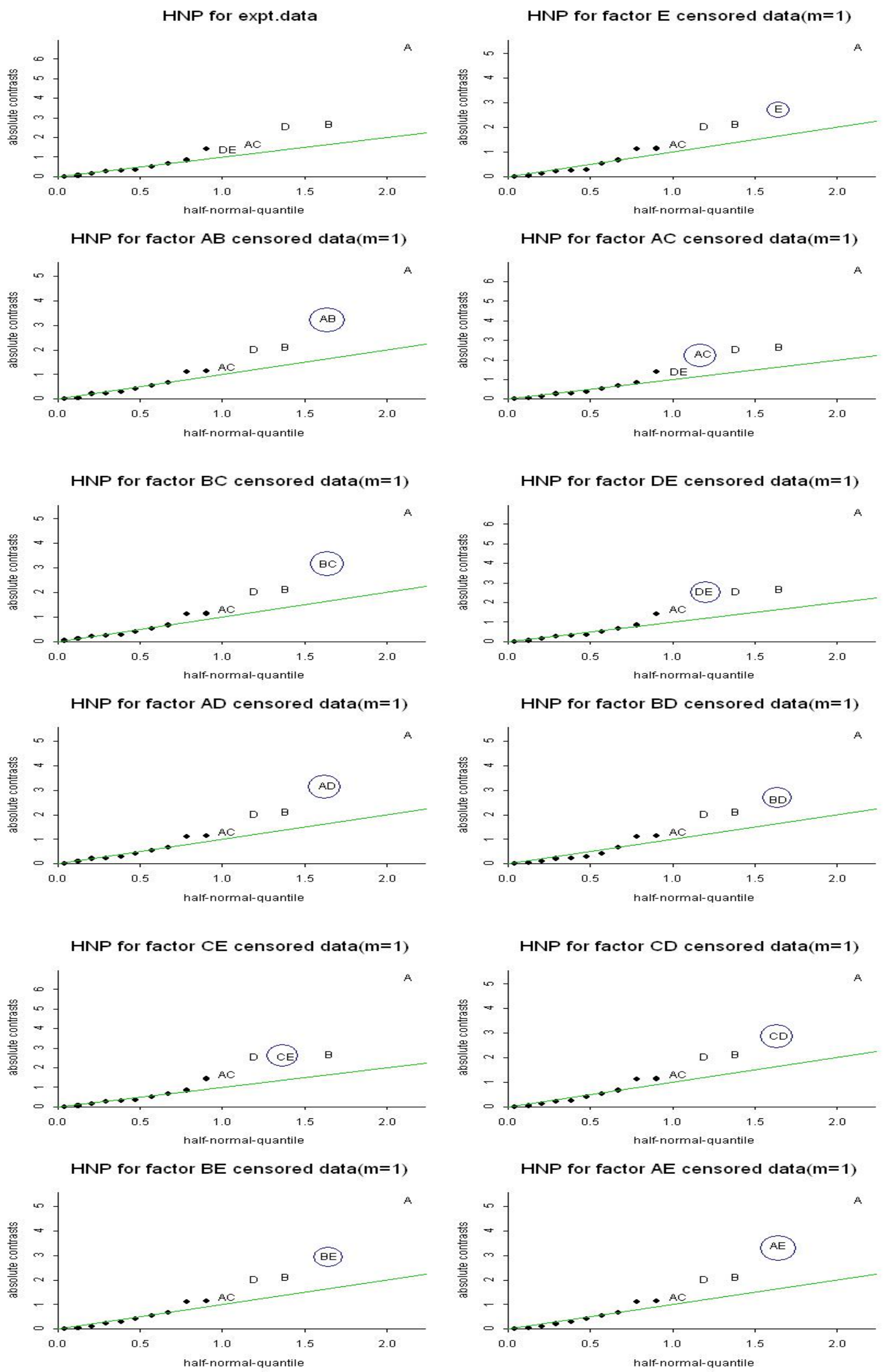


Figure B12: HNP for m = 2 data (CASE 1)

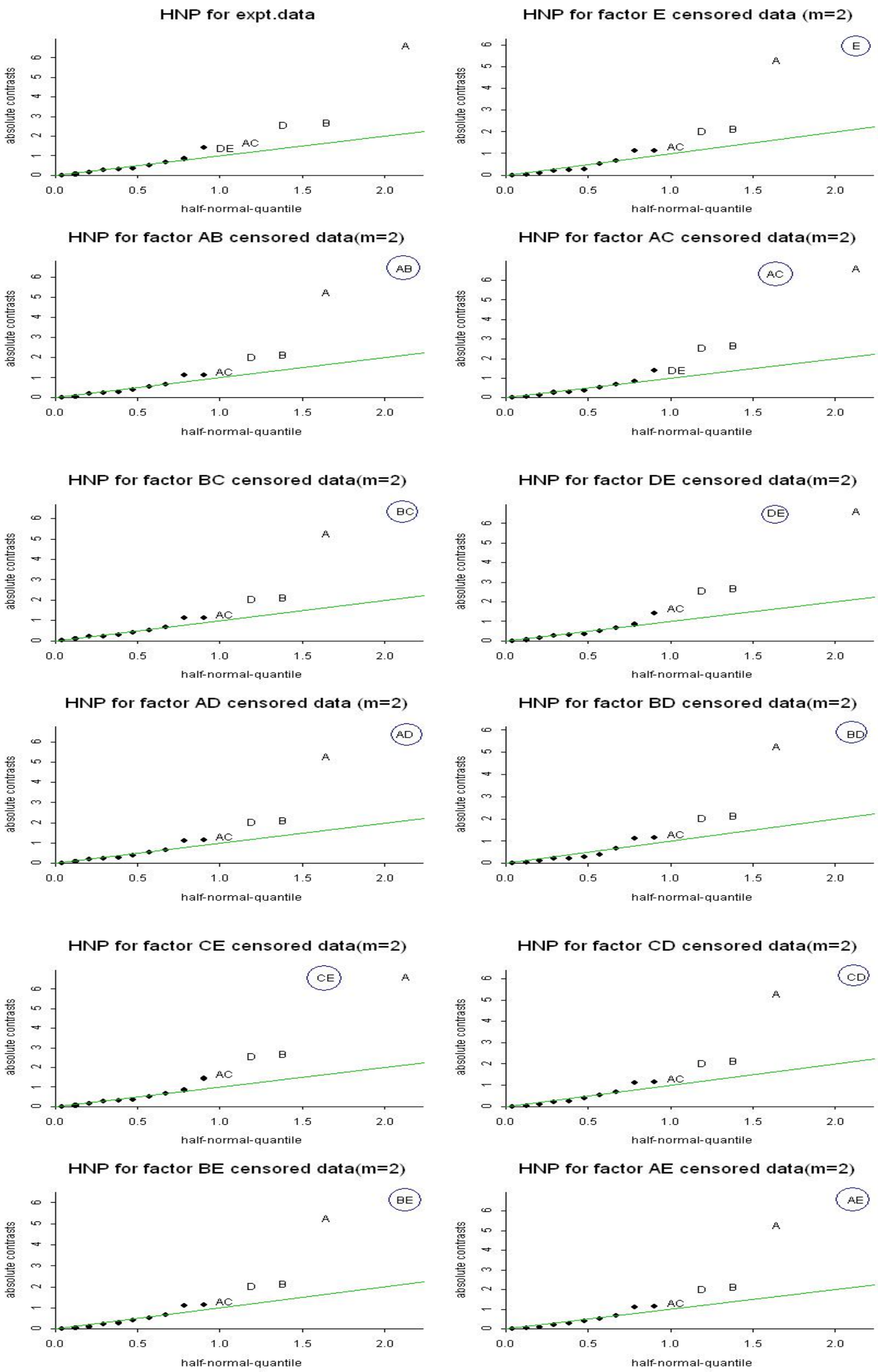


Figure B13: HNP for original data and m = 1 data (CASE 2)

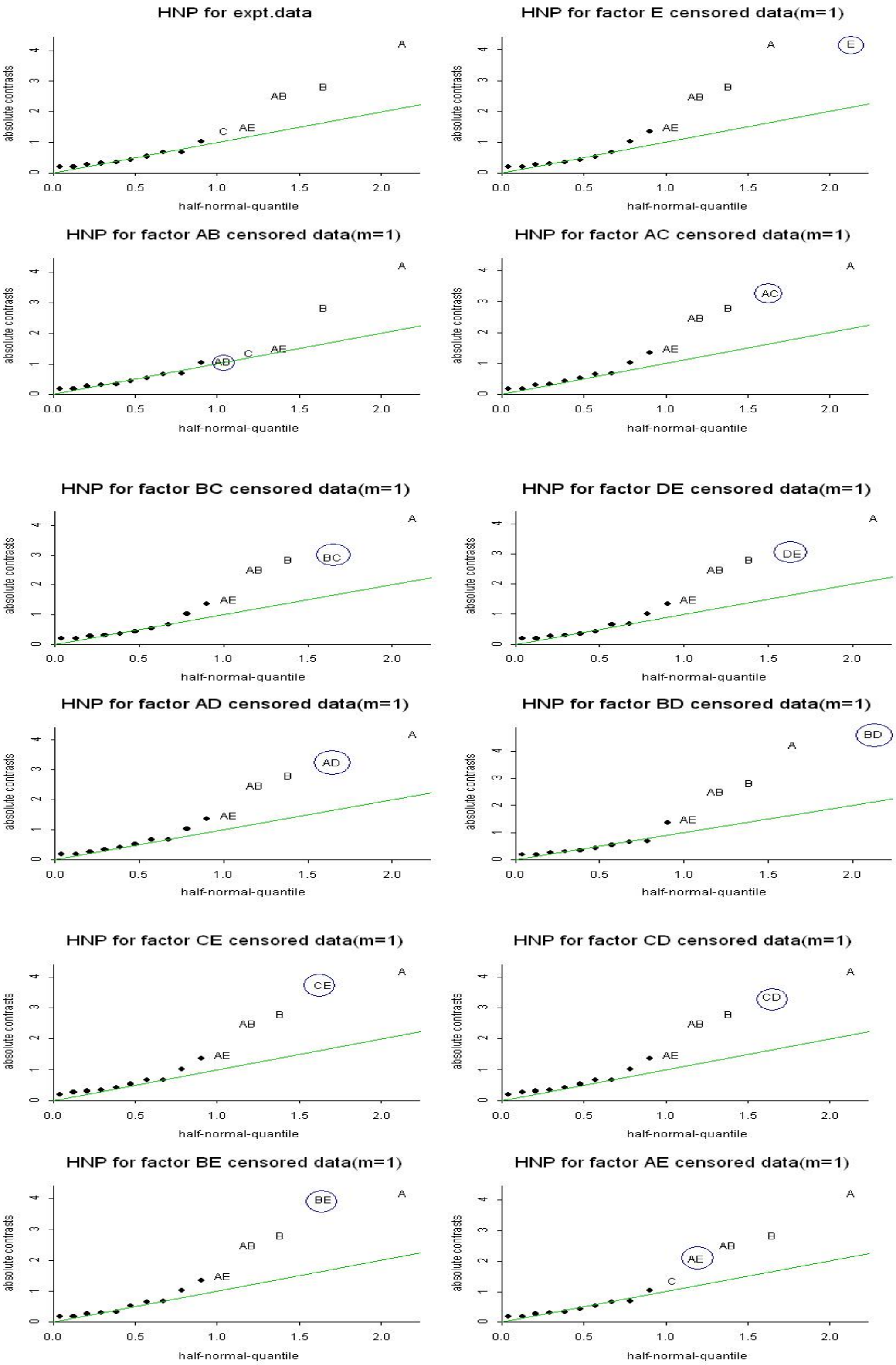
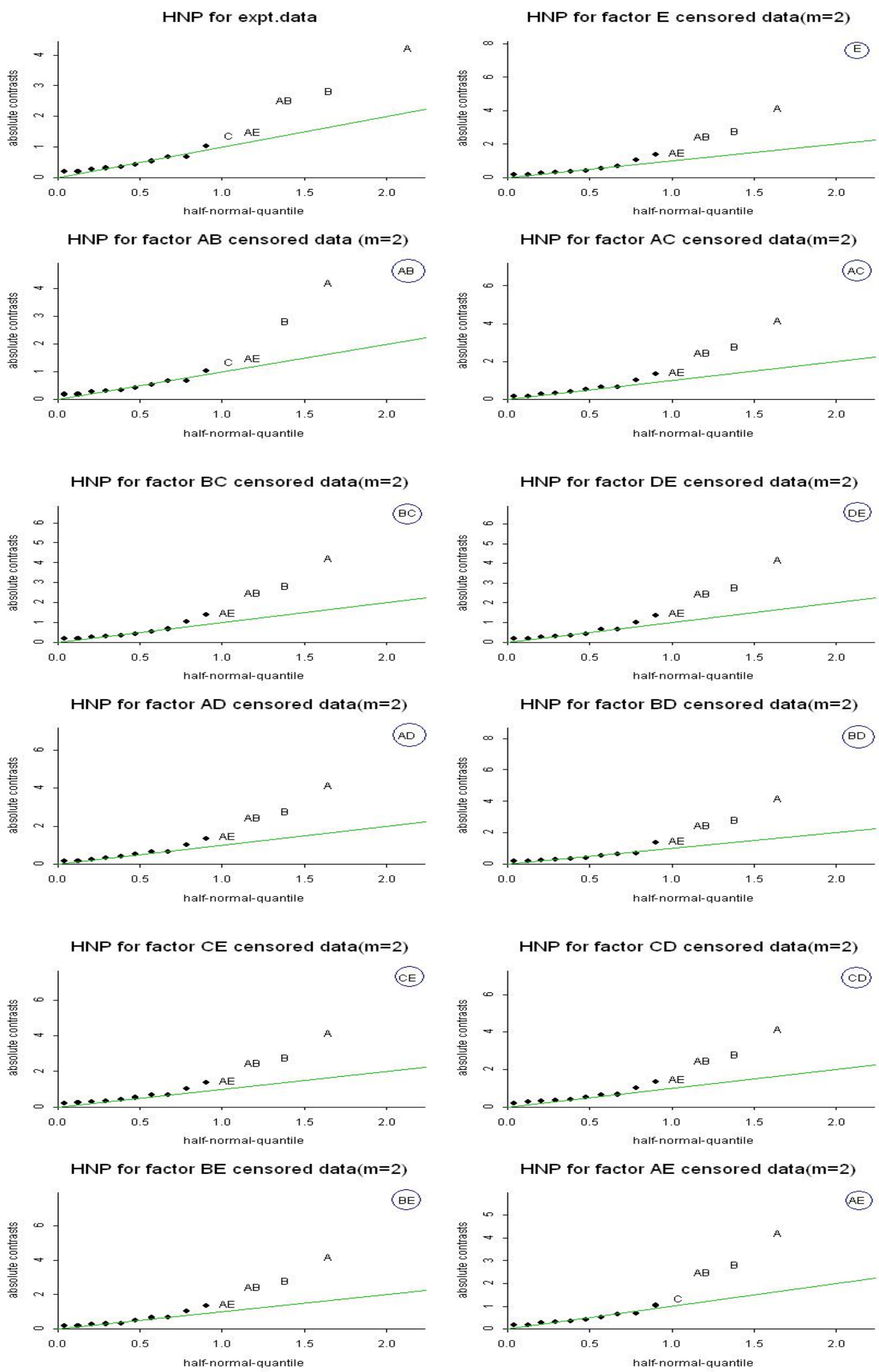


Figure B14: HNP for m = 2 data (CASE 2)



B.2 Half Normal Plot for Systematic Run Order

Figure B21: HNP for original data and $m = 1$ data(CASE 1)

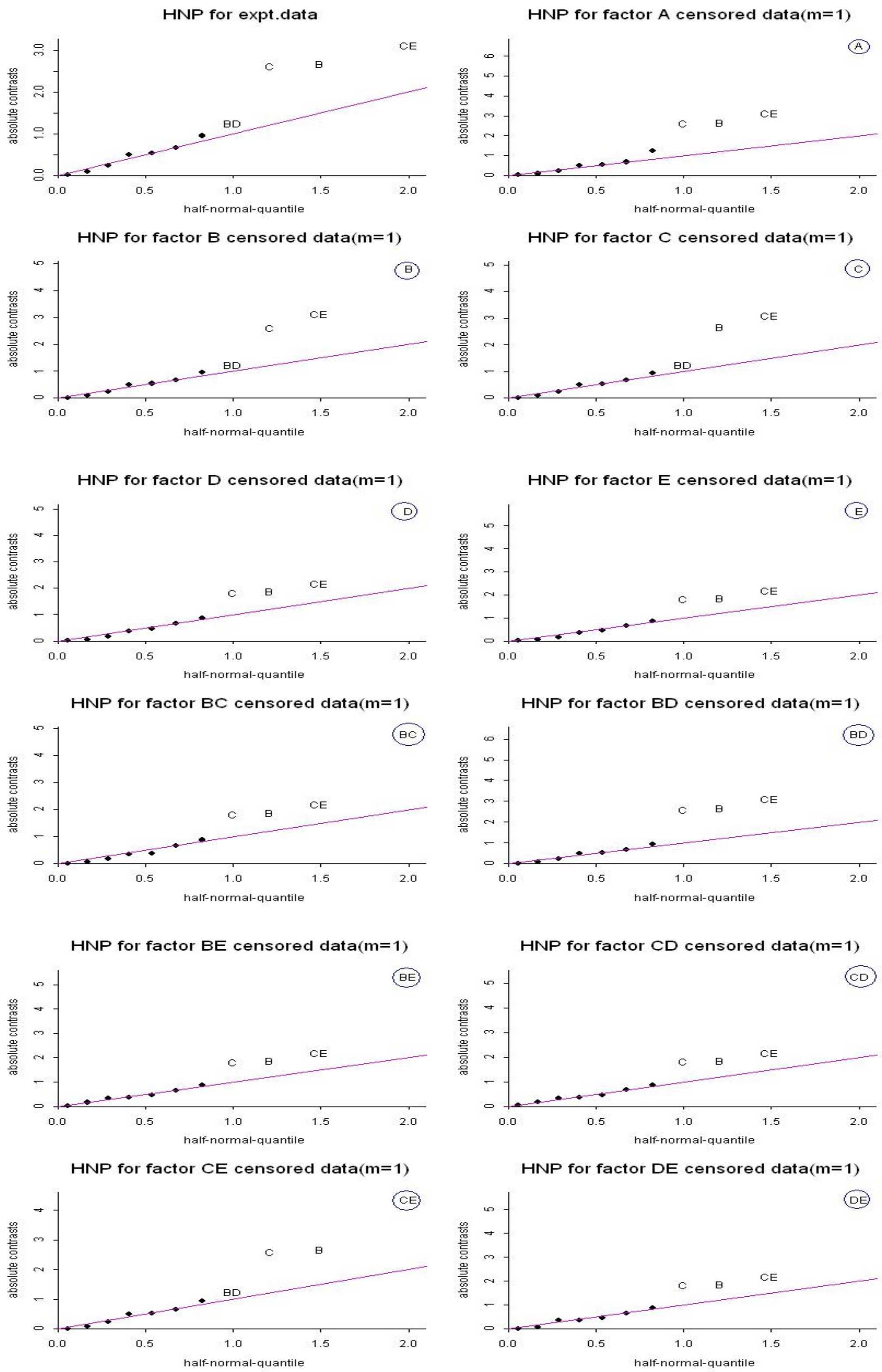
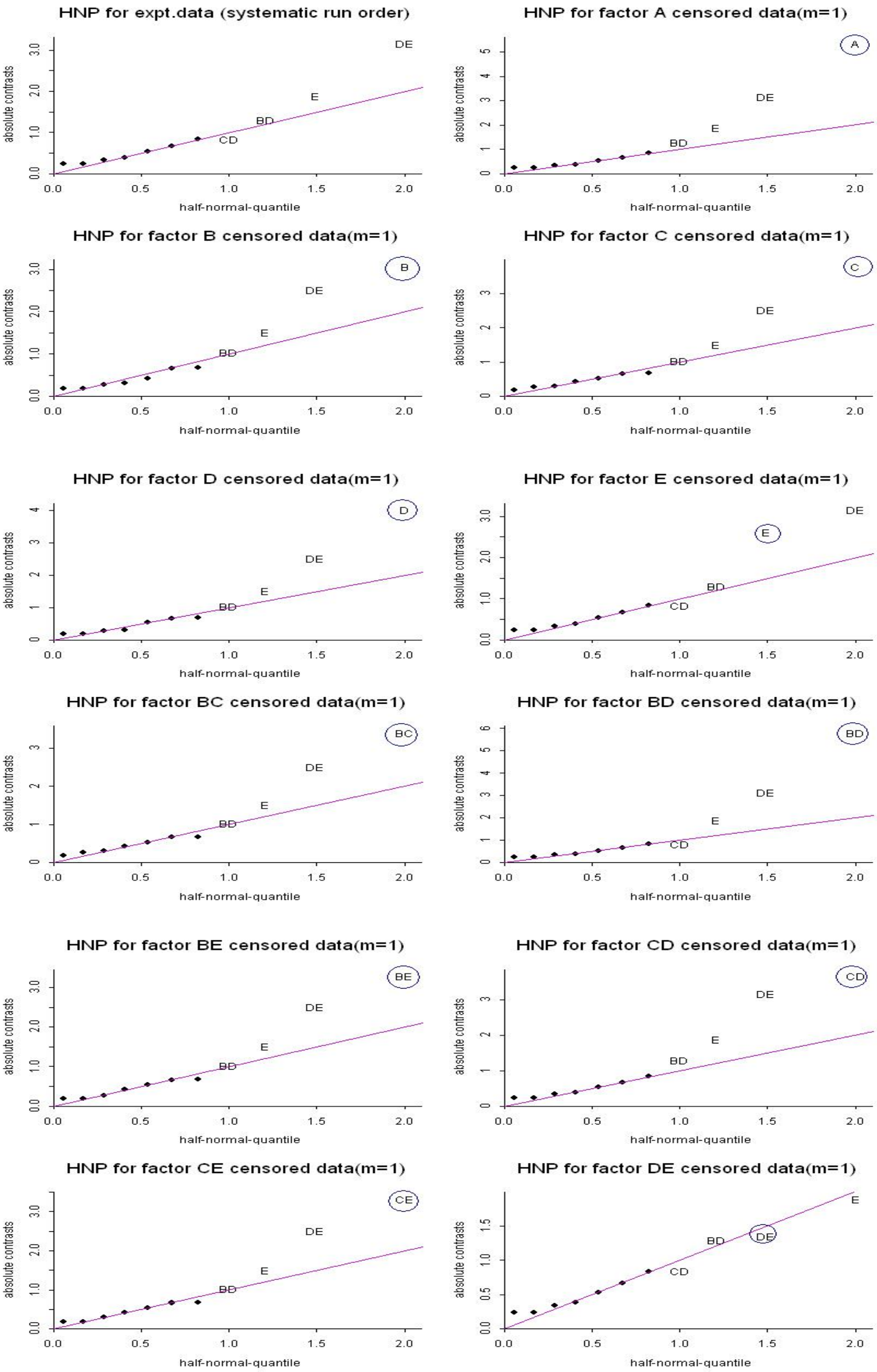


Figure B22: HNP for original data and $m = 1$ data (CASE 2)



Appendix C

CENSORED DATA SETS FOR STANDARD AND SYSTEMATIC RUN ORDERS

C.1 Censored and Uncensored data (CASE 1)

Table C1: Original data and censored data for m =1 for standard run order

	OD	Censored data sets										
Run	case 1	1	2	3	4	5	6	7	8	9	10	11
1	22.13	23.13	23.13	23.13	23.13	23.13	23.13	23.13	23.13	23.13	23.13	23.13
2	23.49	23.49	24.49	24.49	24.49	24.49	23.49	23.49	23.49	23.49	23.49	23.49
3	23.32	23.32	24.32	23.32	23.32	23.32	24.32	24.32	24.32	23.32	23.32	23.32
4	24.26	25.26	25.26	24.26	24.26	24.26	24.26	24.26	24.26	25.26	25.26	25.26
5	23.70	23.70	23.70	24.70	23.70	23.70	24.70	23.70	23.70	24.70	24.70	23.70
6	23.92	24.92	23.92	24.92	23.92	23.92	23.92	24.92	24.92	23.92	23.92	24.92
7	24.07	25.07	24.07	24.07	25.07	25.07	25.07	24.07	24.07	24.07	24.07	25.07
8	24.09	24.09	24.09	24.09	25.09	25.09	24.09	25.09	25.09	25.09	25.09	24.09
9	25.06	25.06	25.06	25.06	26.06	25.06	25.06	26.06	25.06	26.06	25.06	26.06
10	25.36	26.36	25.36	25.36	26.36	25.36	26.36	25.36	26.36	25.36	26.36	25.36
11	24.32	25.32	24.32	25.32	24.32	25.32	24.32	25.32	24.32	24.32	25.32	24.32
12	24.97	24.97	24.97	25.97	24.97	25.97	25.97	24.97	25.97	25.97	24.97	25.97
13	25.03	26.03	26.03	25.03	25.03	26.03	25.03	25.03	26.03	26.03	25.03	25.03
14	26.09	26.09	27.09	26.09	26.09	27.09	27.09	27.09	26.09	26.09	27.09	27.09
15	25.40	25.40	26.40	26.40	26.40	25.40	25.40	25.40	26.40	25.40	26.40	26.40
16	26.02	27.02	27.02	27.02	27.02	26.02	27.02	27.02	26.02	27.02	26.02	26.02

Table C2: Original data and censored data for m =2 for standard run order

Run	OD case 1	Censored data sets 1	2	3	4	5	6	7	8	9	10	11
1	22.13	24.13	24.13	24.13	24.13	24.13	24.13	24.13	24.13	24.13	24.13	24.13
2	23.49	23.49	25.49	25.49	25.49	25.49	23.49	23.49	23.49	23.49	23.49	23.49
3	23.32	23.32	25.32	23.32	23.32	23.32	25.32	25.32	25.32	23.32	23.32	23.32
4	24.26	26.26	26.26	24.26	24.26	24.26	24.26	24.26	24.26	26.26	26.26	26.26
5	23.70	23.70	23.70	25.70	23.70	23.70	25.70	23.70	23.70	25.70	25.70	23.70
6	23.92	25.92	23.92	25.92	23.92	23.92	23.92	25.92	25.92	23.92	23.92	25.92
7	24.07	26.07	24.07	24.07	26.07	26.07	26.07	24.07	24.07	24.07	24.07	26.07
8	24.09	24.09	24.09	24.09	26.09	26.09	24.09	26.09	26.09	26.09	26.09	24.09
9	25.06	25.06	25.06	25.06	27.06	25.06	25.06	27.06	25.06	27.06	25.06	27.06
10	25.36	27.36	25.36	25.36	27.36	25.36	27.36	25.36	27.36	25.36	27.36	25.36
11	24.32	26.32	24.32	26.32	24.32	26.32	24.32	26.32	24.32	24.32	26.32	24.32
12	24.97	24.97	24.97	26.97	24.97	26.97	26.97	24.97	26.97	26.97	24.97	26.97
13	25.03	27.03	27.03	25.03	25.03	27.03	25.03	25.03	27.03	27.03	25.03	25.03
14	26.09	26.09	28.09	26.09	26.09	28.09	28.09	28.09	26.09	26.09	28.09	28.09
15	25.40	25.40	27.40	27.40	27.40	25.40	25.40	25.40	27.40	25.40	27.40	27.40
16	26.02	28.02	28.02	28.02	28.02	26.02	28.02	28.02	26.02	28.02	26.02	26.02

Table C3: Original data and censored data for m =1 for systematic run order

	OD	Censored data sets										
Run	case 1	1	2	3	4	5	6	7	8	9	10	11
1	22.13	23.13	23.13	23.13	23.13	23.13	23.13	23.13	23.13	23.13	23.13	23.13
2	23.49	23.49	24.49	23.49	23.49	23.49	23.49	23.49	23.49	24.49	24.49	24.49
3	23.32	23.32	23.32	24.32	23.32	23.32	23.32	24.32	24.32	23.32	23.32	24.32
4	24.26	25.26	24.26	24.26	25.26	25.26	25.26	24.26	24.26	24.26	24.26	25.26
5	23.70	23.70	23.70	23.70	24.70	23.70	24.70	23.70	24.70	23.70	24.70	23.70
6	23.92	24.92	23.92	24.92	23.92	24.92	23.92	24.92	23.92	23.92	24.92	23.92
7	24.07	25.07	25.07	24.07	24.07	25.07	24.07	24.07	25.07	25.07	24.07	24.07
8	24.09	24.09	25.09	25.09	25.09	24.09	25.09	25.09	24.09	25.09	24.09	24.09
9	25.06	25.06	25.06	25.06	25.06	26.06	26.06	26.06	25.06	26.06	25.06	25.06
10	25.36	26.36	25.36	26.36	26.36	25.36	25.36	25.36	26.36	26.36	25.36	25.36
11	24.32	25.32	25.32	24.32	25.32	24.32	24.32	25.32	24.32	24.32	25.32	24.32
12	24.97	24.97	25.97	25.97	24.97	25.97	25.97	24.97	25.97	24.97	25.97	24.97
13	25.03	26.03	26.03	26.03	25.03	25.03	26.03	25.03	25.03	25.03	25.03	26.03
14	26.09	26.09	27.09	26.09	27.09	27.09	26.09	27.09	27.09	26.09	26.09	27.09
15	25.40	25.40	25.40	26.40	26.40	26.40	25.40	25.40	25.40	26.40	26.40	26.40
16	26.02	27.02	26.02	26.02	26.02	26.02	27.02	27.02	27.02	27.02	27.02	27.02

C.2 Censored and Uncensored data (CASE 2)

Table C4: Original data and censored data for m =1 for standard run order

	OD	Censored data sets										
Run	case 2	1	2	3	4	5	6	7	8	9	10	11
1	21.35	22.35	22.35	22.35	22.35	22.35	22.35	22.35	22.35	22.35	22.35	22.35
2	21.36	21.36	22.36	22.36	22.36	22.36	21.36	21.36	21.36	21.36	21.36	21.36
3	22.31	22.31	23.31	22.31	22.31	22.31	23.31	23.31	23.31	22.31	22.31	22.31
4	21.98	22.98	22.98	21.98	21.98	21.98	21.98	21.98	21.98	22.98	22.98	22.98
5	23.07	23.07	23.07	24.07	23.07	23.07	24.07	23.07	23.07	24.07	24.07	23.07
6	23.29	24.29	23.29	24.29	23.29	23.29	23.29	24.29	24.29	23.29	23.29	24.29
7	22.89	23.89	22.89	22.89	23.89	23.89	23.89	22.89	22.89	22.89	22.89	23.89
8	23.71	23.71	23.71	23.71	24.71	24.71	23.71	24.71	24.71	24.71	24.71	23.71
9	23.18	23.18	23.18	23.18	24.18	23.18	23.18	24.18	23.18	24.18	23.18	24.18
10	23.73	24.73	23.73	23.73	24.73	23.73	24.73	23.73	24.73	23.73	24.73	23.73
11	24.30	25.30	24.30	25.30	24.30	25.30	24.30	25.30	24.30	24.30	25.30	24.30
12	23.30	23.30	23.30	24.30	23.30	24.30	24.30	23.30	24.30	24.30	23.30	24.30
13	23.68	24.68	24.68	23.68	23.68	24.68	23.68	23.68	24.68	24.68	23.68	23.68
14	23.49	23.49	24.49	23.49	23.49	24.49	24.49	24.49	23.49	23.49	24.49	24.49
15	23.51	23.51	24.51	24.51	24.51	23.51	23.51	23.51	24.51	23.51	24.51	24.51
16	24.19	25.19	25.19	25.19	25.19	24.19	25.19	25.19	24.19	25.19	24.19	24.19

Table C5: Original data and censored data for m =2 for standard run order

	OD	Censored data sets										
Run	case 2	1	2	3	4	5	6	7	8	9	10	11
1	21.35	23.35	23.35	23.35	23.35	23.35	23.35	23.35	23.35	23.35	23.35	23.35
2	21.36	21.36	23.36	23.36	23.36	23.36	21.36	21.36	21.36	21.36	21.36	21.36
3	22.31	22.31	24.31	22.31	22.31	22.31	24.31	24.31	24.31	22.31	22.31	22.31
4	21.98	23.98	23.98	21.98	21.98	21.98	21.98	21.98	21.98	23.98	23.98	23.98
5	23.07	23.07	23.07	25.07	23.07	23.07	25.07	23.07	23.07	25.07	25.07	23.07
6	23.29	25.29	23.29	25.29	23.29	23.29	23.29	25.29	25.29	23.29	23.29	25.29
7	22.89	24.89	22.89	22.89	24.89	24.89	24.89	22.89	22.89	22.89	22.89	24.89
8	23.71	23.71	23.71	23.71	25.71	25.71	23.71	25.71	25.71	25.71	25.71	23.71
9	23.18	23.18	23.18	23.18	25.18	23.18	23.18	25.18	23.18	25.18	23.18	25.18
10	23.73	25.73	23.73	23.73	25.73	23.73	25.73	23.73	25.73	23.73	25.73	23.73
11	24.30	26.30	24.30	26.30	24.30	26.30	24.30	26.30	24.30	24.30	26.30	24.30
12	23.30	23.30	23.30	25.30	23.30	25.30	25.30	23.30	25.30	25.30	23.30	25.30
13	23.68	25.68	25.68	23.68	23.68	25.68	23.68	23.68	25.68	25.68	23.68	23.68
14	23.49	23.49	25.49	23.49	23.49	25.49	25.49	25.49	23.49	23.49	25.49	25.49
15	23.51	23.51	25.51	25.51	25.51	23.51	23.51	23.51	25.51	23.51	25.51	25.51
16	24.19	26.19	26.19	26.19	26.19	24.19	26.19	26.19	24.19	26.19	24.19	24.19

Table C6: Original data and censored data for m =1 for systematic run order

	OD	Censored data sets										
Run	case 2	1	2	3	4	5	6	7	8	9	10	11
1	21.35	22.35	22.35	22.35	22.35	22.35	22.35	22.35	22.35	22.35	22.35	22.35
2	21.36	21.36	22.36	21.36	21.36	21.36	21.36	21.36	21.36	22.36	22.36	22.36
3	22.31	22.31	22.31	23.31	22.31	22.31	22.31	23.31	23.31	22.31	22.31	23.31
4	21.98	22.98	21.98	21.98	22.98	22.98	22.98	21.98	21.98	21.98	21.98	22.98
5	23.07	23.07	23.07	23.07	24.07	23.07	24.07	23.07	24.07	23.07	24.07	23.07
6	23.29	24.29	23.29	24.29	23.29	24.29	23.29	24.29	23.29	23.29	24.29	23.29
7	22.89	23.89	23.89	22.89	22.89	23.89	22.89	22.89	23.89	23.89	22.89	22.89
8	23.71	23.71	24.71	24.71	24.71	23.71	24.71	24.71	23.71	24.71	23.71	23.71
9	23.18	23.18	23.18	23.18	23.18	24.18	24.18	24.18	23.18	24.18	23.18	23.18
10	23.73	24.73	23.73	24.73	24.73	23.73	23.73	23.73	24.73	24.73	23.73	23.73
11	24.30	25.30	25.30	24.30	25.30	24.30	24.30	25.30	24.30	24.30	25.30	24.30
12	23.30	23.30	24.30	24.30	23.30	24.30	24.30	23.30	24.30	23.30	24.30	23.30
13	23.68	24.68	24.68	24.68	23.68	23.68	24.68	23.68	23.68	23.68	23.68	24.68
14	23.49	23.49	24.49	23.49	24.49	24.49	23.49	24.49	24.49	23.49	23.49	24.49
15	23.51	23.51	23.51	24.51	24.51	24.51	23.51	23.51	23.51	24.51	24.51	24.51
16	24.19	25.19	24.19	24.19	24.19	24.19	25.19	25.19	25.19	25.19	25.19	25.19