On the Interaction of Non-Convex Investment Technologies and Financial Frictions

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Contents

List of Figures

List of Tables

Preface

This dissertation draws on research I undertook during the one and a half years I was holding a scholarship at the *Graduiertenkolleg "Allokationstheorie, Wirtschaftspolitik und kollektive Entscheidungen"* at the University of Dortmund and later while I was a teaching and research assistant at the chair of public finance. The Graduiertenkolleg is financed by the *Deutsche Forschungsgemeinschaft (DFG).*

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Chapter 1

Introduction

Investment is one of the key figures in determining the business cycle as well as in facilitating long-term economic growth. Despite the relative importance of the topic and the amount of research conducted, economists' knowledge and understanding of micro-level and aggregate investment decisions are far from being conclusive. The neo-classical approach, that assumes the existence of a representative firm which trades off gains in firm value against the (continuous and convex) costs associated with changes in its stock of capital, has proved not very successful empirically. Two major theoretical reasons why the workhorse of the neo-classical investment theory, the quadratic-adjustment cost, q-theoretic model, performs so relatively poorly have been established in the literature.

The first reason put forth, relies on the hypothesis that capital markets are imperfect. This forces firms in their investment activities to consider the investment-funds available to them; and hence, investment decisions not only depend on the fundamental profitability of investment projects. In favor of this view, a number of papers has found empirical evidence for high cash-flows being a driving force in investment decisions. Moreover, this force seems stronger for firms with low investment-funds. However, recent works have questioned these results.[1] The main point of criticism focuses on errors

[1] See Kaplan and Zingales (1997), Cummins et al. (1999), Erickson and Whited (2000)

in measuring fundamental investment incentives, which may mislead econometric findings. Yet, these arguments still typically do not rehabilitate the neo-classical model.

Therefore another reason, why the neo-classical model might fail, has seemed a more promising explanation: If investment is connected to some fixed costs of adjusting the capital stock, then investment activity of firms concentrates at a few points in time. Moreover, firm-expected investment is no longer a linear or concave function in productivity of capital, but is convex. Thus, higher order moments of the marginal productivity distribution (over the firm population of an economy) play an important role in determining aggregate investment.

However, as Gomes (2001) pointed out, without allowing for both financial frictions and non-convex investment-technologies, empirical evidence can hardly discriminate between both theories: Fixed cost of capital adjustment and fixed costs of financial transactions yield similar investment patterns. Moreover, Holt (2003) has argued that financial frictions and fixed costs of adjusting the stock of capital reinforce each other's importance. Hence, in the thesis at hand we merge both the financial frictions and the fixed adjustment costs approach.

Only by allowing for both elements, we are able to differentiate between the financial transaction cost and the fixed adjustment cost model. Moreover, by incorporating both effects we can directly address the seemingly contradictory observation that finance has a quite low influence on the stock of capital whereas it has a strong influence on investment decisions.[2] Additionally, since we employ nonparametric estimation techniques, the empirical results presented in this thesis are free of bias from functional mis-specification. And, as will be shown, this bias can be substantial and may lead to false conclusions if in fact both financial frictions and fixed capital adjustment costs are of importance.

or Gomes (2001).

[2] E.g. Guariglia (1999) finds that liquidity proxies and firm size are not correlated, whereas investment and liquidity are.

However, the relevance of non-convex investment technologies on the macro-level has been taken into question recently by Thomas (2002). She argues that under perfect competition, general equilibrium effects force aggregate investment to behave as if adjustment costs were concave, even though there are fixed costs of adjusting the stock of capital.[3] Yet, the assumption of perfect competition is quite unrealistic for a number of industries. We show in contrast that imperfect competition even amplifies the effects of financial frictions and non-convexities: In duopoly the strategic situation brings up investment behavior that is qualitatively different from both the monopolistic setting with frictions and the duopolistic setting with concave adjustment costs. If e.g. firms are homogeneous enough with respect to their debt-burden or "leverage", a shrinking market may trigger a predatory race for market shares.

One should however mention that most said so far and the major concern of this thesis relates to investment in structures and equipment. Investment is but a much broader phenomenon: A firm invests whenever it trains workers, when it hoards labor during recessions, when it restructures, or when it undertakes R&D. So do workers invest, when they search for a job, train themselves, or spent time and effort in building (social) networks. Needless to say, we can expect investment technologies to differ substantially across these different types of investment. Hence, it is sensible to concentrate on one of these types, and—as stated above—we do so by concentrating on investment in equipment and structures, where fixed costs of investment are a long documented phenomenon.[4]

The following chapter, Chapter 2, extends this introduction by reviewing the investment literature. However, we do not try to give an extensive survey of the literature as there already exists a number of excellent surveys.[5]

[3] Similar results can also be found in Khan and Thomas (2003) and Veracierto (2002).

[4] See e.g. Rothschild (1972).

[5] See e.g. Chrinko (1993), or Hassett and Hubbard (1996) for general surveys, Caballero (1997) for a survey concentrating on fixed adjustment cost works and Hubbard (1998) for a survey of the financial-frictions-in-investment literature.

We rather review the literature to the extent that it builds a foundation for the subsequent analysis. Having introduced to and reviewed models with either non-convex capital-adjustment technologies or financial frictions in the previous chapter, Chapter 3 now provides a first, stylized model of both frictions interacting. Chapter 4 joins the preceding chapters and presents empirical evidence for our hypothesis that the interaction of financial frictions and non-convex technologies plays an important role in determining investment. Chapter 5 now puts this evidence into a strategic perspective, and analyzes the interaction of financial frictions and non-convex technologies in a duopolistic setting. In a sense this chapter changes the focus from a macroscopic to a more microeconomic point of view. To allow Chapter 5 to be read independently, it is organized mostly self-contained. So someone who is only interested in a theory of predatory behavior based upon financial frictions and capital irreversibility may concentrate on this chapter. Chapter 6 gives some concluding remarks. Three appendices follow and present the technical proofs, which were omitted in the preceding chapters.

Chapter 2

Reviewing the literature: Financial frictions or non-convex adjustment cost

2.1 Tobin's-q approach and before

One of the earliest contributions to the (empirical) investment literature is the accelerator model of Clark (1917). Or, to account for the serial correlation of investment, the flexible accelerator model (Clark 1944, Koyck 1954):

$$I_t = \alpha \sum_{\tau=0}^{n} \beta_\tau \Delta Y_{t-\tau} \tag{2.1}$$

This model was motivated by the assumption that the long-run optimal stock of capital K^* was a fraction α of the output Y produced. So investment I was assumed to be a distributed lag process of changes in optimal capital.

Although the approach was empirically rather successful, the absence of prices was a major criticism. Hence, starting from the optimization problem of a perfectly competitive firm that faces no adjustment costs and produces with a constant returns to scale technology, Jorgenson (1963) proposed the neoclassical theory of investment. Again, there also was a (ad hoc) flexible version (Hall and Jorgenson, 1967). In these models, K^* is replaced by the first-order condition

$$K^* = \alpha \frac{Y}{P_k} \tag{2.2}$$

where P_k is the price–respectively user-cost–of capital so that

$$I_t = \alpha \sum_{\tau=0}^{n} \beta_\tau \Delta \left(\frac{Y_{t-\tau}}{P_{k,t-\tau}} \right). \tag{2.3}$$

However, modifying this model to

$$I_t = \alpha \sum_{\tau=0}^{n} \left(\beta_{1\tau} \Delta Y_{t-\tau} P_{k,t-\tau}^{-1} + \beta_{2\tau} Y_{t-\tau} \Delta P_{k,t-\tau}^{-1} \right) \tag{2.4}$$

Eisner (1969) showed that the cost of capital coefficient, $\beta_{2\tau}$, was quite low and statistically insignificant once it was not imposed to equal the one of production, $\beta_{1\tau}$. Additionally, the flexible neo-classical model suffered from the same ad-hockery as the flexible accelerator model, since there was no clear theoretical underpinning for the distributed lags.[1] Moreover, one should note that production is not determined on the firm level in such a setting with perfect competition.[2] Consequently, the seminal contribution of Tobin (1969) and Brainard and Tobin (1968) was the first one that constituted a full model of investment. The basic idea was that in a general equilibrium the ratio of firm value over the replacement costs of the firm's assets should be equal to 1 in the absence of adjustment costs. This ratio then became well known as "average-q" or "Tobin's q". In its ad-hoc flexible version the now widely used investment equation was

$$I_t = \sum_{\tau=0}^{n} \beta_\tau q_{t-\tau}. \tag{2.5}$$

Later, the theoretical contributions of Abel (1979) and Hayashi (1982) showed

[1] Hall and Jorgenson (1967) propose delivery lags to be the driving force behind the dynamics, but their explicit model is a static one.

[2] See Hayashi (1982, p. 213)

that the neoclassical investment model with convex adjustment cost could be represented by a q-model in which Tobin's q had to be replaced by the fraction of the marginal internal value of new capital over its replacement costs. Additionally, Hayashi showed that, when firms are price-takers and the technology is linear homogeneous, Tobin's-q remains a sufficient statistic for investment. As marginal-q is—unlike Tobin's-q—unobservable, this is an important result for empirical work.

Hayashi's model—here presented in discrete time as in Barnett and Sakellaris (1999)—reads as follows: Let $F(K_t, \mu_t)$ denote a production function that is linear homogeneous in capital, K, and depends on a stochastic productivity parameter μ. Moreover, let $A(I_t, K_t, u_t) \cdot P_{k,t}$ be an adjustment cost function which also is homogeneous of degree one in capital and investment and also depends on a stochastic term u. Consequently, the optimization problem of the firm—assuming no depreciation for simplicity—can be represented by the following Bellman equation:

$$V(K_t, \mu_t, u_t, P_{k,t}) = \max_{I_t} \left\{ \begin{array}{c} F(K_t, \mu_t) - A(I_t, K_t, u_t) \cdot P_{k,t} \\ +\beta \mathbb{E}_t \left[V\left(K_t + I_t, \mu_{t+1}, u_{t+1}, P_{k,t+1}\right)\right] \end{array} \right\}. \tag{2.6}$$

$\mathbb{E}_t$ denotes the expectations operator conditional on information available at time t. β is the discount-factor.

Define $i_t := \frac{I_t}{K_t}$ and denote items relative to capital by small letters. Due to our homogeneity assumptions, V is linear in capital K_t. A division of (2.6) by K_t now yields:

$$v(\mu_t, u_t, P_{k,t}) = \max_{i_t} \left\{ \begin{array}{c} f(\mu_t) - a(i_t, u_t) \cdot P_{k,t} \\ +\beta \mathbb{E}_t \left[(1 + i_t)\, v\left(\mu_{t+1}, u_{t+1}, P_{k,t+1}\right)\right] \end{array} \right\}. \tag{2.7}$$

The first order condition for optimal investment then is

$$\frac{\partial a(i_t, u_t)}{\partial i_t} \cdot P_{k,t} = \beta \mathbb{E}_t \left[v\left(\mu_{t+1}, u_{t+1}, P_{k,t+1}\right)\right]. \tag{2.8}$$

If individual shocks to productivity, μ_t, and to adjustment costs, u_t, are uncorrelated with shocks to the price of capital, we obtain by dividing by $P_{k,t}$:

$$\frac{\partial a(i_t, u_t)}{\partial i_t} = \beta \mathbb{E}_t \left[\frac{v(\mu_{t+1}, u_{t+1}, P_{k,t+1})}{P_{k,t+1}} \right] \mathbb{E}_t \left[\frac{1}{1+\dot{P}_{t+1}} \right], \tag{2.9}$$

where $\dot{P}_t$ denotes the inflation rate, $\frac{P_{k,t+1}}{P_{k,t}} - 1$. Let a be quadratic, i.e. $a(i_t, u_t) = \frac{1}{2\alpha_1}(\alpha_0 + \alpha_1 i_t + u_t)^2$ and define average q as $q_{t+1} := \frac{v(\mu_{t+1}, u_{t+1}, P_{k,t+1})}{P_{k,t+1}}$. Then we can rewrite (2.9) as

$$\alpha_0 + \alpha_1 i_t + u_t = \beta \mathbb{E}_t \left[\frac{q_{t+1}}{1+\dot{P}_{t+1}} \right] =: \beta \hat{q}_{t+1}. \tag{2.10}$$

Therefore, investment is theoretically directly related to q. However, it is related to the *expected* value of *future* q for which two proxies are at hand as Barnett and Sakellaris (1999) argue: q_t and q_{t+1}. As the original model of Hayashi (1982) was presented in continuous time, most authors have used q_t in empirical applications. However, the correlation between q_t and $\hat{q}_{t+1}$ is not clear-cut. For that reason Barnett and Sakellaris propose to use q_{t+1} instead, which is an unbiased estimator, as

$$q_{t+1} = \hat{q}_{t+1} + \xi_{t+1}; \ \mathbb{E}_t(\xi_{t+1}) = \mathbb{E}_t(\xi_{t+1}\hat{q}_{t+1}) = 0. \tag{2.11}$$

The structural parameters $\alpha_{0,1}$ can then be estimated by regressing q_{t+1} on i_t. Previous studies typically solved (2.10) for i_t and replaced $\hat{q}_{t+1}$ by q_t. So the estimated equation was:

$$i_{jt} = \gamma_0 + \gamma_1 q_{jt} + \nu_{jt}. \tag{2.12}$$

However, such an estimation has modest success: The estimated parameters yield implausible high adjustment costs, and changes in q explain only about 5% of the variation in investment rates in firm-panels.[3] Moreover, additional scale variables such as cash-flow or sales have a significant influence on in-

[3] See for example Blundell et al. (1992).

vestment, though there is no additional role for them in the above model.[4]

Indeed, Barnett and Sakellaris (1999) show that the estimated parameters become much more plausible, once (2.10) is directly estimated replacing $\hat{q}_{t+1}$ by its realization. Nevertheless, marginal R^2 remains as low as 3% in their regressions. Respectively R^2 raises to 8% when lagged cash-flow enters the regression, which is still considerably small. Of course, when explaining such relatively large fraction of variation, the cash-flow parameter is highly significant.

2.2 Incorporating financial frictions

The lack of empirical success of q-theoretic models and the superiority of accelerationists' models let researchers look for a theoretical reasoning to include cash-flow or other sales based variables besides q in the empirical models. Already in the 1950s Meyer and Kuh (1957) had argued that financial considerations of firms could result in an accelerationist model. Yet, after the famous contribution of Modigliani and Miller (1958) most economists believed in the dichotomy of financial and real decisions of firms. The progress in capital-market theory, achieved during the late 1970s and 1980s,[5] gave a renewed theoretical ground for the inclusion of financial variables in investment regressions.

The seminal contribution of Fazzari et al. (1988) triggered a new research

[4]See Summers (1981), who argues, why these variables should not be included in a structural estimation. Similar Fazzari, Hubbard and Petersen (1988) argue that under a perfect capital market assumption, no other scale variables should appear significant in the regression.

[5]Since the famous "lemon market" paper of Akerlof (1970) it has been well noticed that because of informational asymmetries external finance can be more expensive than internal one. Jaffee and Russell (1976) provide a model of asymmetric information that leads to credit rationing. Stiglitz and Weiss (1981) argue, that due to moral hazard and adverse selection credit rationing may occur. Gale and Hellwig (1985) relate capital market imperfections to costly-state-verification in case of bankruptcy.

The Myers and Majluf (1984) model, directly based on the "lemons" consideration, shows that new shares can only be issued at a discount so that external equity finance is more expensive just as external debt financing had been shown to be.

agenda which was joined in by others in a number of subsequent papers.[6] Denoting cash-flow by CF, the canonical estimating equation became:

$$i_{jt} = \gamma_0 + \gamma_1 q_{jt} + \gamma_2 \frac{CF_{jt}}{K_{jt}} + \nu_{jt}. \tag{2.13}$$

Fazzari et al. now split their sample on apriori grounds in three subgroups. These subgroups had the purpose to reflect the likelihood of a firm being financially constrained in its investment decisions. For each subgroup (2.13) is then estimated independently and the estimated parameters are compared across the groups. If considered as a formal test, the neoclassical model is rejected if $\gamma_2 \neq 0$. Yet, a rejection of the neo-classical model does not necessarily imply the existence of financial frictions. Other deviations from the neo-classical model are e.g. adjustment costs other than to capital, market power, or fixed costs of adjustment. However, Fazzari et al. argue that, if the rejection is due to financial frictions, γ_2 can be expected to be larger for a financially constrained sample of firms than for an unconstrained one. Hence, a test for financial frictions uses the differences between firm groups.

Following this procedure a number of studies emerged. These studies each used different econometric procedures or different groupings: Fazzari et al. (1988) grouped firms by their dividend payout behavior. They showed that those firms which payed least dividends reacted strongest to changes in cash flow. Gilchrist (1991) has confirmed Fazzari et al.'s results using similar sample splittings. Calmoris and Hubbard (1995) use a tax-reform as a natural experiment to identify the shadow value of internal funds. They group the firms by this estimated shadow price and find a significantly higher cash-flow estimate for the firms that valued internal funds higher. Hoshi et al. (1991) obtain similar results analyzing a sample of Japanese firms, splitting the sample on the basis of membership in a "keiretsu", a large industrial group. The procedure of Lammont (1997) is quite analogous. He uses a drop in oil-prices to estimate the different response of investment of non-

[6]See Hubbard (1998) for an extensive survey.

oil branches in oil and non-oil companies. He found a significant decrease in investment of the non-oil-branches of oil companies which is not present for the non-oil companies. Moreover, controlling for other factors, liquidity constraints can be identified as the driving force behind this phenomenon. In the same spirit, Ber et al. (2002) uses the differential impact of monetary policy on exporting and non-exporting firms to identify liquidity constraints. As exporting firms have better access to international capital markets, they can be expected to be much less influenced by monetary policy through the credit channel than other firms. Indeed, this is what Ber et al. find.[7]

Gilchrist and Himmelberg (1995) proceed similar to Fazzari et al. (1988), but split their sample of manufacturing firms on the basis of a multitude of indicators, e.g. if the firm has a bond rating or a commercial paper program or on firm size. Moreover, they explicitly take the dynamic and interdependent structure of the data into account.

In all these papers, typically for the unconstrained group γ_2 is indeed smaller than for the constrained group, and sometimes even $\gamma_2 = 0$ cannot be rejected for firms considered as unconstrained. Hence, this evidence supports the view that capital market imperfections are an important factor in determining investment.

However, there is a number of reasons why q might measure fundamental investment incentives with error and if this measurement-error differs between "constrained" and "unconstrained" firms, the obtained evidence may be flawed.

In the denominator the value of the stock of capital reported may not equal the replacement costs.One reason for this could e.g. be a difference between economic and reported depreciation. In the nominator of q there is e.g. an error when the market value of a firm may be subject to stock

[7] Note, however, that Ber et al.'s paper is not based as straightforward on a q-theoretic model as the other papers mentioned. The estimation equation they used is augmented by a number of other regressors than Tobin's q. Their main point of focus is the interaction term of the short-term (aggregate) interest-rate and the amount of export-orientation of a firm.

market bubbles or stem from other capitalized quasi-rents besides the one from capital.[8] Additionally, average q and marginal q differ, when firms have market power. Hence Whited (1992), Bond and Meghir (1994) and Hubbard et al. (1995), test the neoclassical model on the basis of the first-order-condition Euler-equations. They also can confirm the results of Fazzari et al. (1988). The basic idea of these Euler equation approaches is as follows: If the Lagrange multiplier for the liquidity variable appears significant in the estimated first-order condition, the neoclassical model can be rejected. Yet, the Euler-equation approach has been criticized for the lack of statistical power.[9]

On the other hand, the original approach by Fazzari et al. (1988) has been criticized for only testing the neoclassical model as a whole. Kaplan and Zingales (1997) argue that the sensitivity of investment to liquidity does not necessarily increase with net-worth when capital markets are imperfect. Moreover, they argue that typically the empirical studies carried out mostly rejected the neoclassical model not only for the constrained firms but for the whole sample. Kaplan and Zingales use the as-constrained-classified subsample of Fazzari et al. and split it up into in 5 groups based on the firms' financial reports. Although for all groups the neoclassical model is rejected, there is no clear-cut structure in the sensitivity. Rather, the firms which appear to be least likely constrained exhibit the largest investment-sensitivity to cash flow. However, Fazzari et al. (2000) note that Kaplan and Zingales' sample is rather small so that splitting the sample in five subgroups may result in a substantial small sample bias. Moreover, Kaplan and Zingales' sorting itself is rather subjective and firms identified as financially constrained are likely to be in financial distress. This also might bias their results.

Nevertheless, Kaplan and Zingales' paper was very influential in the sense that it emphasized the problem of how to interpret positive cash-flow coefficients. One possible alternative interpretation—which still is somewhat

[8] See e.g. Merz and Yashiv (2002) for a discussion of this topic, when labor markets are incomplete.

[9] See e.g. Oliner et al. (1995).

related to Fazzari et al.'s (1988) one—are non-value-maximizing managers (Jensen, 1986). Blanchard et al. (1994) use cash-windfalls as a natural experiment to test the asymmetric information model against the agency model of Jensen and find better support for the agency model. Typically, even if average q is low managers keep cash-windfalls within the firm and invest them in fixed assets rather than use them to reduce debt or repurchase shares. So this interpretation still links the estimates to some informational imperfections, but here within the firm. More recently, product market imperfections, respectively decreasing returns-to-scale in production, have been put forward as a potential cause of the positive empirical cash-flow influence.

Recall that only under a constant returns-to-scale production technology and perfect competition marginal-q can be replaced by average q. Hence $\gamma_2 = 0$ tests these assumptions jointly with the assumption of a perfect capital market.

Cooper and Ejarque (2001) provide numerical examples by generating artificial data for firms with a decreasing returns-to-scale technology. When they estimate (2.13) from their generated data cash-flow appears as a significant regressor although the capital market has been assumed to be perfect. A very similar analysis has been carried out by Alti (2003). He shows that even the differences between the sample-splittings can be reproduced by the decreasing-returns assumption. He does so by splitting his artificially generated data by a similar rule as Fazzari et al. (1988) apply to their data. Abel and Eberly (2002) provide a fairly general theoretical underpinning for these results: They show analytically that investment of a monopolist with perfect access to finance reacts positively to average q even in the absence of adjustment costs. Furthermore, the monopolist's investment also reacts positively to cash-flow. Moreover, Abel and Eberly show that in this setting cash-flow sensitivity of fast-growing firms are larger. Empirically, these firms are the no-dividend paying small firms and are the ones typically considered as financially constrained in the previous literature.

Closely related to the results concerning market power are the findings of

Gomes (2001) who simulates a general equilibrium model of investment and financing decisions. In summary, he finds positive estimates for cash flow to be neither sufficient nor necessary for capital market imperfections. Noteworthy, in one alternative specification Gomes (2001, pp. 1278) also picks up the idea of fixed financial transaction costs and concludes that empirically the resulting investment behavior can hardly be distinguished from the behavior of a firm facing fixed capital adjustment costs. This is unusual insofar as most theoretical models which employ the imperfect capital market framework at the aggregate level emphasize net-worth effects on the marginal cost of capital and hence rely on the above mentioned (more sophisticated) asymmetric information microeconomic models of capital market imperfections.[10]

What drives Gomes main results? On the one hand, a positive cash-flow effect (in the absence of financial frictions) is obtained since average-q measures marginal-q imperfectly. Therefore, cash-flow provides additional information (on this measurement error). On the other hand firm value and hence q not only captures the fundamental profitability of incremental investment, but also the value liquidity has when capital markets are imperfect. Therefore, a high q can simply reflect the financial flexibility a firm has, so that additional liquidity variables have a lesser informational content.[11]

This problem has been observed before by Gilchrist and Himmelberg (1998), who construct a structural VAR-model, that controls for the dynamics and interdependency of fundamental and financial variables that determine investment. Gilchrist and Himmelberg disentangle the effect of both factors on firm-value and find a significant fraction of cash-flow influence in investment-decisions simply reflects fundamental productivity. Neverthe-

[10] See e.g. Bernanke et al. (1996).

[11] Note that the assumptions of Gomes' (2001) model are somewhat problematic. His basic idea is that a firm which cannot finance its investment from current cash flow has to rely on external finance and pay a premium for these funds. However, in such setting, firms would have an incentive to precautionarily save. Hence and as Blinder (1988) argues, liquidity is rather to be measured as a stock which—for example—proxies the line of credit. This view of "liquidity" also is more suitable to capture the ideas of the capital markets literature that highlights the role of net-value, e.g. Bernanke et al. (1998).

less, they also find an independent influence of the financial factor in the investment decisions. Insofar, they still are able to reject a broad class of neo-classical models. Moreover, the influence of the financial factor is found to be stronger for the group of firms apriori considered as financially constrained.

In summary, evidence of financial frictions still is mixed, but one has to be careful in selecting measures of liquidity and fundamental investment incentives.

2.3 Measurement errors or financial frictions?

The argument put forth in the last section, based on economic theory and highlighting the imperfectness of average-q as a proxy for marginal-q, is just one possible reason for a classical measurement error that might actually drive the results of the empirical financial constraints literature. Other possible reasons that have been discussed in the literature are stock market bubbles or misreported respectively falsely calculated replacement costs of capital.[12]

As a remedy for measurement errors induced by stock-market bubbles Cummins et al. (1999) suggest to construct a measure of fundamental firm value based on analysts' net-income forecasts ECF_{it} for each firm.

As this highlights the problems involved in dealing with the measurement error problem in q-theoretic models, we will discuss their procedure in more detail: Cummins et al. define a variable "real Q", $\widehat{Q}_{it}$, which is computed on the basis of analysts' expectations $\widehat{ECF}_{it}$ of firm net-income and its expected net-income growth-rates. These items are available through I/B/E/S. Scaling

[12]See for example Erickson and Whited (2000, 2001 and 2002) and Cummins et al. (1999) for a discussion.

all items by the replacement value of capital, they compute $\widehat{Q}_{it}$ as:

$$\widehat{Q}_{it} = \sum_{j=1}^{n} \alpha^j \widehat{ECF}_{it+j-1}. \tag{2.14}$$

This formula is of course motivated by the idea that fundamental firm value is the sum of discounted net-income. "Real Q" is then used in (2.13) instead of Tobin's q. Using GMM-techniques they obtain more reasonable estimates for the parameter of "real Q", when this parameter is interpreted in terms of a quadratic adjustment cost function. Moreover, they find no significant influence of cash-flow. These results have been replicated by Bond and Cummins (2001), who also provide a more detailed theoretical underpinning. Furthermore, Bond and Cummins find that lagged sales and Tobin's q also appear as insignificant additional regressors in this setting. So, they conclude, the neo-classical model captures investment behavior quite well, once rational bubbles are controlled for. However, in a sense their empirical model just replicates the findings of Jorgenson (1963), so that Eisner's (1969) criticism might well apply:

First of all, $\widehat{Q}_{jt}$ and cash-flows are notably correlated,[13] so that one might also specify $\widehat{Q}_{jt} = \beta \frac{CF_{jt}}{K_{jt-1}} + u_{jt}$. As a result the estimation equation becomes

$$i_{jt} = \gamma_0 + \gamma_1 \left(\beta \frac{CF_{jt}}{K_{jt-1}} + u_{jt} \right) + \gamma_2 \frac{CF_{jt}}{K_{jt-1}} + \nu_{jt}. \tag{2.15}$$

If the variance of u is small, there is a strong collinearity problem. Indeed the very large confidence bounds reported may indicate such a situation. Moreover, using instruments as a cure for the potential *measurement error problem* makes the potential *collinearity problem* even worse, if the instruments are not correlated with u_{jt}. And even if one imposes $\gamma_2 = 0$, as Bond and Cummins (2001) do in one of their regressions, the interpretation of their results is not clear-cut. If u_{jt} and user-cost of capital are highly correlated,

[13] See figure 1 in Cummins et al. (1999).

we obtain Jorgenson's neoclassical model; if they are not, we are back to the accelerationist-model.[14]

Moreover, more important and besides the issues raised by Gomes (2001), the structural interpretation of Cummins et al. (1999) is correct if and only if β is chosen correctly, i.e. there is no systematic correlation between cash-flow and the difference of marginal-q and $\widehat{Q}_{jt}$. If there is such a difference, γ_2 will be estimated incorrectly, perhaps as insignificant although it might be significant in the "true" marginal-q model.

A sufficient condition for this to happen would be systematic errors in the constructed analysts' forecasts of $\widehat{Q}_{jt}$ which are correlated with current cash flow. To be more precise, suppose $\widehat{Q}_{jt}$ is formed (by the analysts) as a linear function of marginal q, a purely random measurement error, and maybe an additional effect of cash-flow:

$$\widehat{Q}_{jt} = \theta_1 \frac{CF_{jt}}{K_{jt-1}} + \theta_2 q_{jt} + u_{jt} \quad . \tag{2.16}$$

Using this equation in (2.13) now yields:

$$I_{jt} = \gamma_{0i} + \frac{\gamma_1}{\theta_2}\widehat{Q}_{jt} + \left(\gamma_2 - \frac{\theta_1}{\theta_2}\beta_1\right)\frac{CF_{jt}}{K_{jt-1}} + \left(\nu_{jt} - \frac{\gamma_1}{\theta_2}u_{jt}\right) \tag{2.17}$$

Obviously, the structural parameters are not identified unless one is willing to assume fixed values for $\theta_{1,2}$, e.g. $\theta_1 = 0$ and $\theta_2 = 1$, which means there is no systematic error in forming a measure of marginal-q from the analyst's cash-flow forecast.

Yet, there are a number of reasons why $\theta_1 > 0$: If, for example, a firm also has other (quasi-)rents but the one from capital to exploit, cash-flow will be serially correlated to a greater extent than marginal-q is.[15] Another reason could be costly information. If information is costly to acquire, analysts

[14] If e.g. analysts have some knowlege about (non-profitable) investment projects, that will increase net-income, u_{jt} clearly will measure investment, but it will not measure fundamental incentives.

[15] Merz and Yashiv (2002) analyse the case, where the labour market is imperfect. They show, that the labor force in place significantly effects firm-value and investment decisions.

will concentrate on information easy to obtain and omit the more expensive information. As a result—of the omitted variable problem—analysts will rationally overemphasize current and lagged cash-flow. Alternatively, if analysts have incentives to trade off accuracy of their forecasts in favor of optimism,[16] their cash-flow forecasts will be biased. If additionally the tolerated inaccuracy depends on the realized cash-flow, we obtain the stated effect.

The problem of mismeasured marginal-q has been addressed by Erickson and Whited (2000, 2001, 2002), too. Instead of replacing observed average-q by some related measure, they use higher order moment conditions to tackle the measurement error problem more directly. Depending on the way Tobin's-q is computed, Erickson and Whited (2002) find that the empirical measure Tobin's-q explains between 44% and 65% of the variation in true but unobservable average-q.[17] Moreover, once measurement errors are controlled for, Erickson and Whited (2000) find no additional influence of cash-flow on investment. In line with Gomes (2001) argument they explicitly state that this result does not necessarily imply the absence of financial frictions. Insofar Erickson and Whited's results rather suggest that it is necessary to obtain more direct measures of expected marginal productivity of capital and of the financial status of a firm, if one wishes to carry out estimations that can be interpreted structurally.

2.4 Investment and fixed adjustment cost

Another possible argument for the empirical failure of the neo-classical investment model stems from the fact that adjustment costs of capital may

[16] Hong and Kubik (2003) provide evidence that analyst who provide upwardly biased earnings forecasts are more likely to be promoted. Hence, analysts have an incentive to do so.

[17] In their setting average-q is defined as the unobservable variable driving investment. This may of course be different to the explicit definition as "true" value over replacement costs.

not be a continuous convex function.[18]

There are a number of reasons why adjustment cost could be non-convex. The typically considered cases are irreversibilities and fixed costs: Investment may be (partially) irreversible due to asymmetric information in the market for used capital goods. Fixed costs of investment could for example arise because the installation of a new assembly line in an existing factory requires to halt current production. Another form fixed costs of investment can for example take are overhead costs involved in planning and carrying out an investment project.

In all these cases, as Caballero and Leahy (1996) show, there needs not be a monotonic relation between marginal-q and investment; and paradoxically average-q better predicts investment than marginal-q does. However, average-q no longer is a sufficient statistic for investment.

The following model briefly sketches this idea: Suppose a firm completely finances the capital stock by debt (capital is leased). Moreover, assume that *profits* $\Pi(x, K)$ are linear homogeneous in capital given the measure of capital imbalance x, which is affected by investment. Then,

$$\Pi(x_t, K) \quad : \quad = \pi(x_t) K_t, \tag{2.18}$$

$$x_{t+1} \quad : \quad = i_t + x_t + \mu_t \tag{2.19}$$

with i.i.d. productivity-shocks μ_t. Now, assume that the costs of adjusting the stock of capital are a fixed-fraction of the capital previously installed and are borne by the equityholders, i.e.

$$a(i) := \begin{cases} w & \text{if } i \neq 0 \\ 0 & \text{if } i = 0. \end{cases} \tag{2.20}$$

Then (2.7) simplifies to

$$v(x_t) = \max_{i_t} \left\{ \pi(x_t) - \mathbb{I}_{\{i_t \neq 0\}} w + \beta \mathbb{E}_t \left[v(i_t + x_t + \mu_t) \right] \right\}. \tag{2.21}$$

[18] Caballero (1997) gives a good overview over the literature considering this situation.

Now, for notational convenience define

$$\hat{v}(x_t) = \beta \mathbb{E}_t [v(x_t + \mu_t)]. \tag{2.22}$$

The optimal "capital imbalance" x^* is implicitly defined by the first-order condition

$$\left[\frac{\partial \hat{v}(x)}{\partial x}\right]_{(x=x^*)} = 0. \tag{2.23}$$

Therefore, optimal investment is given by $i_t = (x^* - x_t)$. However, the firm will only invest if

$$w \leq \hat{v}(x^*) - \hat{v}(x_t) \tag{2.24}$$

holds.[19] One can show, that v is quasi-concave, when π is quasi-concave. Thus (2.24) implicitly defines two thresholds (l, u) for investment such that the firm invests if and only if $x_t \geq u$ and disinvests if $x_t \leq l$. Hence, in the presence of fixed adjustment costs a firm lumps adjustment to avoid paying the fixed adjustment costs too often. This is reflected by the inaction range (l, u) in fundamental investment incentives. Yet, if $x_t \notin (l, u)$ the firm invests and q_{it} is constant as $q := \hat{v}(x) = const$ holds. Therefore, there is no direct and obvious link between the volume of investment and Tobin's-q. If, however, the setting is generalized, this conclusion does not necessarily hold anymore, as Abel and Eberly (1994) show in their model that remerges q-theory and the theoretical literature on lumpy investment.

The "lumpy investment" model itself has proved quite successful empirically: Doms and Dunne (1998) analyze the investment-series of those firms contained in the Longitudinal Research Database (LRD), which documents investment pattern of 12 000 US manufacturing plants from 1972 to 1989.

To do so, they calculate the fraction of the total equipment investment of each establishment that is carried out in a single year. They find that on average the year with the largest plant-year investment accounts for more than

[19] Caballero and Engel (1999) show, that a very similar model is well defined. Here, we take the assumptions made, for heuristic reasons. See chapter 3 for a more elaborated version of this model.

a quarter of the total cumulative investment of a plant over the whole period. Moreover, more than half of the establishments exhibits capital growth close to 50 % in a single year. Employing the same database, Cooper et al. (1999) estimate the hazard-rate of a plant exhibiting an investment spike. They estimate this hazard-rate as a function of the time elapsed since the last spike occurred and find evidence for a time-increasing hazard-rate. This points towards non-convex costs of adjustment.

A more direct approach has been taken by Caballero et al. (1995): Also using the LRD, they estimate a measure of "mandated investment", which is the difference between optimal and realized capital and takes the role of q. They construct "mandated investment" by imposing a long-run cointegrating relation between earnings, capital employed and the user cost of capital. In a second step Caballero et al. estimate the adjustment hazard as a function in "mandated investment" by spline smoothing. They find a convex relationship between "mandated investment" and the actual realized investment rate, i.e. an upward sloping hazard. In contrast, the convex-cost framework would predict investment to be concave or linear in "mandated investment".

Additional evidence from other data (COMPUSTAT) has been provided by Abel and Eberly (2002), who test their (1994) q-theoretic augmented adjustment cost model. Indeed they find evidence for an inaction range and hence especially for fixed costs.[20] Goolsbee and Gross (2000) estimate a model very similar to the one of Caballero et al. (1995), but they employ detailed data from the airline industry. Goolsbee and Gross find significant non-linearities in the adjustment costs, when they allow for capital-heterogeneity. If they, however, aggregate the data to the firm-level, the non-linearities partly wash out. Moreover, the estimated adjustment costs become upwards biased as by aggregation parts of the fixed costs are perceived as marginal costs by the econometric procedure. Because of that result, data on the lowest level of aggregation available should be used to estimated adjustment

[20] Barnett and Sakellaris (1998) who tried to test that model earlier, could not find evidence for non-convex costs. However, it has been noted that this might well be due to unobserved heterogeneity, that they do not control for.

costs.

Yet, in most cases the level of micro-detail Goolsbee and Gross have in their data is not available to the researcher, so that one can only test for convex adjustment cost using firm-level-aggregate data. Data, for which Goolsbee's and Gross' result strikingly shows the lack of power a test for non-convex adjustment cost might have. Accordingly one will often accept the hypothesis of convex adjustment-cost, although adjustment-costs are in fact non-convex.

To avoid this data problem and to obtain estimates from macro-data, Caballero and Engel (1999) have developed an empirical model of stochastic fixed adjustment costs that can work out the form of adjustment costs from the time-series behavior of aggregate investment.

In this model, the range of inaction varies from period to period and the probability of a firm to invest is given by a hazard function that is upward sloping in "mandated investment". They employ this model to two-digit aggregate data and find both an upward sloping hazard-rate and a superior within and out-of sample behavior of this model compared to the linear (partial adjustment) model. Interestingly, when estimating their model they do not have to rely on any explicit measure of mandated investment. Instead, they calculate the stationary mandated investment distribution of their model in the absence of aggregate shocks and obtain the parameters distribution of shocks to productivity and adjustment-costs by maximizing the likelihood of the sectoral investment series. Basically, their model extends the above one to cases, where the fixed cost of investment, w_t, is i.i.d. distributed. Hence the probability of an investment(-spike) for a firm is

$$\Lambda(x) = prob\left[w_t \leq (\hat{v}(x^*) - \hat{v}(x))\right]. \tag{2.25}$$

Given the stochastic model for w and μ, both Λ and v can be calculated.

An alternative econometric approach to obtain structural estimates of the underlying adjustment costs has been followed by Cooper and Haltiwanger (2002), using an indirect inference method. In a first step, they use employment and sales data to generate a measure of capital-productivity. In a second step, they regress investment on a polynomial of this measure and its lagged value. Finally, they obtain the structural estimates by carrying out the estimation of their second step for simulated investment models (with different types of adjustment costs). The estimates of the structural parameters are then identified as those, which minimize the distance between the estimates from a simulated model and the previously estimated empirical one. They find that a model which mixes both convex and non-convex adjustment costs fits the data best.

2.5 Towards merging both strands

Up until very recently the typical strategy was to analyze each single reason why the neo-classical model might fail separately at a time. So far, the fixed or more generally speaking the non-convex adjustment costs model has seen least criticism. However, do non-convex adjustment costs solely explain the strong empirical rejection of the neo-classical model, really? Thomas' (2002) results casts some doubt on this: In aggregate investment general equilibrium effects mostly wash-out the influence of higher moments of the fundamental investment incentives-distribution, that is caused on the micro-level by fixed costs.

Moreover, if there are transaction costs of finance, debt-ceilings, or net-worth effects in financing costs, the estimates of fixed costs may well be biased upwards compared to their structural values: Holt (2003) presents a real-options model of investment in which equity-funds cannot be raised after the firm is founded and in which equity serves as a buffer-stock for bad outcomes, i.e. equity has to be always positive. In his model financial variables amplify the influence of fundamental ones. Put differently, fundamentals and finance work as complementaries in determining investment.

Empirically this has recently been confirmed by Whited (2002), who shows that the hazard-rates of firms to exhibit an investment spike are substantially "flatter" for firms which are financially constrained. Hence, the average time between two spikes is much larger for financially constrained firms.[21] Figure 2.1 illustrates this.

Moreover, only when both financial frictions and fixed costs of capital adjustment are allowed for in the (econometric) model, one can discriminate between the various models of financial frictions. Furthermore, as Gomes (2001) claims, the fixed transaction and the fixed adjustment costs model may yield similar *investment patterns.* Hence, only by allowing for financial

[21] Independently developed, a comparable idea is the basis of the analysis in chapeters 3 and 4, which are based on working papers (Bayer, 2002 and 2003) previously available. The results are similar to the ones of Whited (2002), but obtained using a different procedure.

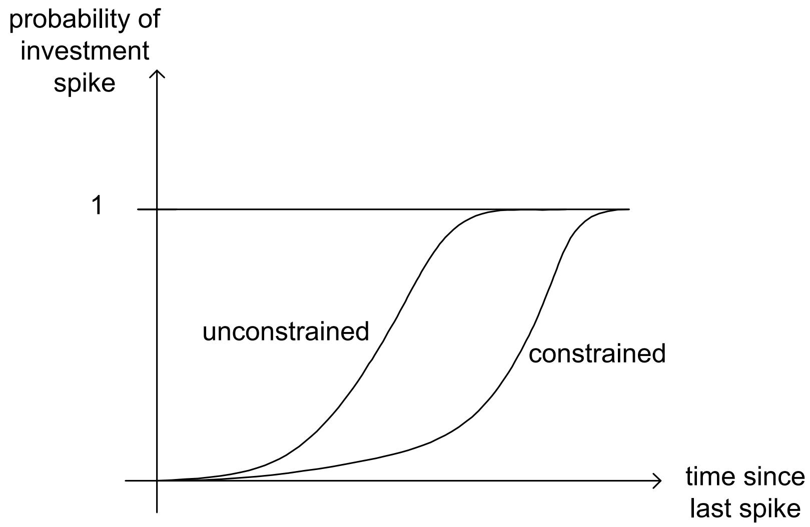

Figure 2.1: Hazard-rates for constrained and unconstrained firms

frictions and fixed adjustment costs, we can separately test for both *structural* (technological) assumptions.

Therefore, we will analyze investment behavior under both financial frictions and non-convex adjustment costs simultaneously in the following chapters. How to measure financial frictions is anything but straightforward. Mostly cash-flow has been employed as a proxy. Yet, already at the very beginning of the financial frictions literature, and commenting the seminal paper of Fazzari et al. (1988), Blinder (1988, p. 199) stated that it is probably a stock of liquidity like the line of credit which influences investment. This claim holds true in both the transaction-cost and the net-worth models. So, Blinder concludes it would be preferable to use a stock-based measure and that a flow measure of liquidity could be a misleading proxy. We share this view and use the equity-ratio as such proxy throughout this thesis.[22]

[22]Strictly speaking, the equity-ratio is a stock-based relative measure. By using a relative specification, we assume that influence of size is approximately constant (or zero) over time. But even if varying size effects are present, using the relative measures avoids picking up size with the liquidity proxy.

This also has a number of additional advantages. Firstly, only with a stock-measure of liquidity one can distinguish between different types of financial frictions using their differential impact on short- and long-run behavior of investment: Suppose fixed transaction costs of finance are the driving force behind the dependence of investment on liquidity. Then we can expect firms to have an optimal stock of capital independent of their financial status, whereas investment heavily depends on liquidity. If, however, there is a net-worth effect on the marginal cost of capital, one can expect the optimal-stock of capital to depend on the financial status, while investment is only indirectly influenced. In fact, apriori evidence seems to support the transaction costs hypothesis rather than the net-worth one.[23] Secondly, the problem of fundamental investment incentives being correlated with the measure of liquidity can be expected to be less pronounced using a stock-measure than with a flow-measure like the cash-flow.

2.6 Synopsis and conclusion

Before proceeding we should sum up the main points again:

Although the q-model of investment is theoretically well-founded, its empirical success is modest at best. Q-models typically yield estimates of adjustment costs that are way to large and hence imply very implausible half-lifes of capital imbalances. Moreover, the fit of these regressions is typically very low. Conversely, this leaves much room for other factors to explain investment.

One of these other factors is cash-flows. By many authors this has been interpreted as evidence for financial frictions playing an important role in firm-level investment decisions. Especially backed up by inter-firm differences in the influence of cash-flows, this view has some appeal.

[23]E.g. Guariglia (1999) reports firm size and stock-measures of liquidity to be uncorrelated. Mairesse et al. (1999) report in their survey the influence of financial variables in investment to die out rapidly over time.

However, a measurement-error in q (whose existence can be soundly justified from theory) could as well generate the significant positive cash-flow coefficient. Moreover, this virtual influence even varies across firms in accordance to "a priori" measures of financial frictions. Gomes (2001) even shows that positive cash-flow coefficients in a q-model are neither sufficient nor necessary for the existence of financial frictions. By and large, he thereby renders the whole approach unsuitable. Additionally he shows that the fixed transaction-cost of finance and the fixed adjustment-cost of real investment models are observationally virtually equivalent, if the other friction is not controlled for.

As a consequence, an analysis has to be based on a more direct measure fundamental investment incentives. Moreover, one has to focus much more on the interaction of frictions and test the various models along those lines. Instead, most studies so far have tried to establish a direct influence of a financial variable.

Fixed-adjustment cost models always reflect the adjustment costs in the frequency of adjustment. If we view liquidity as a measure of options to adjust the stock of capital, crossbreeding financial frictions and fixed-adjustment cost models yield an influence of finance on this frequency, as we will see in the following chapter. This influence can be used to discriminate between and to test against alternative models.

Therefore, combining the fixed adjustment-costs and the financial frictions frameworks allows to discriminate both, within various financial-frictions models, and between the financial frictions and the fixed adjustment-cost models. Moreover it obviously avoids a number of problems of omitted variables and misspecification.

Chapter 3

A first model of interacting frictions

3.1 Firm-level investment

We will start off with presenting and discussing the representative *problem* of a firm which is at the same time subject to financial constraints and fixed adjustment costs.[1] For simplicity and as in Caballero and Engel (1999), the industry modelled in this chapter shall consist of a large number of monopolistically competitive single-plant and single-product firms, which all are subject to a limited liability constraint. The investment decision is modelled in discrete time. Each firm faces an infinitely elastic supply of all factors. At the beginning of each period all uncertainty about that period is resolved and is common knowledge from then on. Thereafter, each firm decides upon investment.

[1]Note that typically the investment-functions in investment problems with fixed cost are not aggregable. Hence we can only discuss the typical investment problem–of course given a parameterization of the production technology, market structure etc. If these parameters vary across firms, this makes the aggregation problem more severe. This is quite analoguous to the aggregation problem in consumer-theory.

Moreover, when relating investment to consumer theory it should be noted that under finacial frictions, the "law of supply" obviously does no longer necessarily hold, as firms then face a budget constraint.

3.1.1 Adjustment technology and financial constraint

If a firm wants to change its capital stock it has to pay some fixed costs; all other factors may be adjusted without costs. At the end of every period each firm has to pay back its last period's debt plus interest, has to pay for any new purchased capital goods and for all other factors. Moreover, it can issue new debt and pay out dividends.

Apart from the non-convexity in the investment technology, firms face a capital market imperfection: As in Gilchrist and Himmelberg (1998) and as an extremely simplified version of the pecking order of finance theory, there shall be a no-new-equity constraint, i.e. firms are unable to issue new shares or to have negative dividends once founded (which is in the absence of taxation equivalent on the margin):

Assumption 3.1: Once founded, firms are unable to issue new equity. Especially, dividend payments D_t must be non-negative at any point in time: $\forall t : D_t \geq 0$.

This simplified version of the pecking order theory is necessary to keep the model tractable. Nevertheless, the general results should not change if this assumption were replaced by a more complex version as in Gomes (2001).

Moreover and more important, this assumption is not strongly contradicted by empirical findings as e.g. Friedman (1982) shows empirically that firms hardly use any external equity finance at all. Moreover this assumption also has theoretical support: Fries et al. (1997) show how full collataralization- and "no new equity"-constraints may theoretically arise as an industry equilibrium.

Secondly, we assume that a firm must declare bankruptcy immediately if it has a negative book value of equity (at any point of time). Under assumption 3.1 this is equivalent to a full collataralization constraint, i.e., the amount of debt a firm may issue is limited by the actual stock of capital depreciated and discounted for one period.[2] To see the equivalence, suppose

[2] This ensures that the firm always has a positive present value in the following period

a firm would borrow above the constraint. Then the firm would be bankrupt next period. Hence, either all assets were transferred to the debtholders who continue operations or debt burden had to be renegotiated. Any combination of the two would contradict assumption 3.1. However, both procedures would imply a sure loss either for the stock- or the debtholders. Thus, at least one party will never allow for any increase in debt beyond that ceiling.

Assumption 3.2: The maximum debt B_{t+1} (repayable in period t+1) a firm may issue in period t is restricted by the book value of the stock of capital K_t, depreciated (with rate δ) and discounted (at the interest rate on debt r) for one period: $\frac{B_{t+1}}{K_t} \leq \frac{(1-\delta)}{1+r\left(\frac{B_{t+1}}{K_t}\right)}$.

The third, last, and weakest assumption regarding capital market imperfections is that the interest rate on debt r only depends on the leverage. As Gilchrist and Himmelberg (1998), we assume r to be homogenous of degree zero in B and K, and to be weakly increasing in B. This does not rule out r to be independent of B and K.

Assumption 3.3: $r = r\left(\frac{B_{t+1}}{K_t}\right)$ and $r'\left(\frac{B_{t+1}}{K_t}\right) \geq 0$.

3.1.2 Periodic sales

Now let K^* denote the frictionless stock of capital of a firm, i.e., the stock of capital that would be chosen in the absence of fixed costs for investment and capital market imperfections. Let K be the actual capital employed.

The semi-reduced function of earnings per-period (EBIT), Π, shall be linear homogenous in the frictionless stock of capital K^* and can be written as:[3]

$$\Pi(z, K^*) = \pi(z)K^*, \tag{3.1}$$

and will stop production before going bankrupt. Alternatively, this assumption can be viewed as a simplification of Hart and Moore's (1994) debt capacity model.

[3]This assumption and the assumptions below with respect to π are e.g. fulfilled if demand is iso-elastic and the production function is Cobb-Douglas. See Caballero and Engel (1999) for details.

with z denoting the capital-imbalance at the beginning of each period before investment takes place, i.e. the ratio of actual capital employed to frictionless capital, $\frac{K}{K^*}$. The profit, π, per unit of frictionless capital shall be strictly concave and fulfill the (Inada) conditions $\pi(0) = 0$ and $\lim_{z \to 0} \pi'(z) = +\infty$. Moreover, assume $\lim_{z \to +\infty} \pi'(z) < \psi\delta$ to make profits bounded.

When a firm invests, it faces a stop of production, the duration of which is determined by the random variable $w \in]0,1[$ representing the fraction of the period used for the installation of the new capital—similar to the adjustment costs assumption in Caballero and Engel (1999). So the costs, A, of adjusting the capital stock are given by:

$$A(z^o, K^*, w) := w\pi(z^o)K^*, \tag{3.2}$$

where z^o denotes the capital-imbalance after adjustment, $z^o := \frac{K_t + Investment_t}{K_t^*}$. Note that in the presence of depreciation the firm will typically invest up to a larger stock of capital than the frictionless optimal one, K^*.

3.1.3 Dynamics of the stochastic variables

So far there are no restrictions on the stochastic dynamics of the random variables K_t^* and w_t. Both variables together completely determine firm heterogeneity and investment dynamics, so any assumptions on these variables are crucial. A minimal assumption for keeping the model tractable is that both variables exhibit the Markov property.[4] Furthermore, we shall assume that w_t is i.i.d. and that K_t^* follows a geometric random walk (with drift d). The innovations ξ_t to K_t^* are normally distributed and serially uncorrelated (although possibly correlated across firms):

$$\frac{K_t^*}{K_{t-1}^*} = \exp\left(d + \xi_t\right). \tag{3.3}$$

[4] If shocks to productivity would be serially correlated, the analysis would just be complicated. Managers would have a true optimal stock of capital that is different from the one that maximizes current profits. This would make the financing problem more pronounced.

3.1.4 Capital market and the firm's objective

Firms are assumed to be risk-neutral. Therefore, they seek to maximize the expected, discounted dividend stream. They do so by choosing some capital-imbalance z^o (respectively the amount of capital employed, $z^o K_t^*$) and the amount of debt used to finance production, B_{t+1}.

In order to finance investment, a firm can either cut back dividend payments D_t or raise debt B_{t+1}. As assumed, a firm is unable to sell any new shares or raise equity by negative dividends (assumption 3.1). Moreover, the amount of debt a firm can issue is limited by the actual stock of capital employed (assumption 3.2). Additionally, the interest rate is a function of $b_t := \frac{B_t}{K_{t-1}}$ and is weakly increasing in b_t (assumption 3.3).[5] Therefore, dividend payments, D_t, are given by

$$\begin{aligned} D_t &= D(z^o, B_{t+1}, K_t^*, w_t, z_t, B_t) \\ &:= \Pi(z^o, K_t^*) - A(z^o, K_t^*, w_t)\mathbb{I}_{\{z^o \neq z_t\}} - K_t^*(z^o - z_t) + B_{t+1} - (1 + r_t)B_t \end{aligned} \tag{3.4}$$

in which $\mathbb{I}$ is an indicator function. Moreover, let the time constant discount factor be denoted by ψ and the value of the firm be denoted by V. Then the following Bellman equation determines both V and the optimal investment policy:

$$\begin{aligned} &V(K_t^*, w_t, z_t, B_t) = \\ &\quad \max_{(z^o, B_{t+1}) \in X} \left\{ D(z^o, B_{t+1}, K_t^*, w_t, z_t, B_t) + \psi \mathbb{E}_t[V(K_{t+1}^*, w_{t+1}, z_{t+1}, B_{t+1})] \right\}. \end{aligned} \tag{3.5}$$

[5]That the interst-rate for bonds increases does not follow from our model but is an assumption. And as we have explicitely ruled out bancruptcies, debt is even risk-free. Hence, the assumption in its strong form itself is somewhat inconsistent with the model. Yet, to rule out risky debt is only to simplify and concentrate the analysis. Introducing another risk-term that enters after the investment decisions are made and adding bancruptcy costs for the debt-holders would generate an upward sloping interest-function, but would only complicate the analysis a lot.

In this expression $X := X(K_t^*, w_t, z_t, B_t)$ is the correspondence of financially feasible capital-imbalance and debt pairs. $\mathbb{E}_t$ denotes the expectations operator, conditional on information available at time t.

Let the ratio of equity to capital (slightly abusing notation here) denoted by e_t, which is a function of b_t, i.e. $e_t := e(b_t) = 1 - \frac{(1+r(b_t))}{1-\delta} b_t$.[6] To reduce the number of state variables and to obtain a more convenient formulation of the problem at hand, we subtract the book value of equity from V and divide by K_t^*. This defines a new value function $v := \frac{V}{K_t^*} - e_t z_t$, which is the difference between "market-value" and book-value of equity relative to the frictionless stock of capital. As both e_t and K_t^* are determined before the optimal policy decision is taken, maximizing v and maximizing V yield the same optimal policy.

Now, define firm value $\tilde{v}$ as v if the capital imbalance is not altered by investment in the current period. Defining $c(z^o, b^o) := \frac{\pi(z^o)}{z^o} + (b^o - 1)$ and rearranging terms,[7] we obtain

$$\begin{aligned} \tilde{v}(z, b^o) &:= c(z, b^o) + (1-\delta)\psi z e(b^o) \\ &\quad + \psi \mathbb{E}_t \left\{ v \left[w_{t+1}, z \frac{1-\delta}{\exp(d+\xi_{t+1})}, e(b^o) \right] \exp(d + \xi_{t+1}) \right\}. \end{aligned} \tag{3.6}$$

For capital-imbalance and debt pairs ("plans") with strictly positive capital we define Y to be the correspondence of financially feasible plans in terms of z^o and b^o :

$$\begin{aligned} &Y(w_t, z_t, e_t) := \\ &\quad \left\{ (z^o, b^o) \in \mathbb{R}_{++} \times \mathbb{R}_+ | z^o - e_t z_t - \pi(z^o)[1 - w_t \mathbb{I}_{\{z^o \neq z_t\}}] \leq z^o b^o \leq z^o \widehat{b} \right\}. \end{aligned} \tag{3.7}$$

[6] See Appendix A.1 for details.
[7] Again, see Appendix A.1.

$\widehat{b}$ denotes the maximum debt-to-capital ratio

$$\widehat{b} := \sup \left\{ b \in \mathbb{R}_{++} |\ b \leq \frac{1-\delta}{1+r(b)} \right\}.$$

The following Lemma proves to be useful in order to write the Bellman equation (3.5) in terms of z^o and b^o in a short and accessible form:

Lemma 3.1 *(a) Y is non-empty and*

(b) employing zero capital is suboptimal, i.e.

$$\max_{(z^o,b^o)\in Y} \tilde{v}(z^o,b^o) - \pi(z^o)w_t\mathbb{I}_{\{z^o\neq z_t\}} > \psi\mathbb{E}_t[v(w_{t+1},0,0) > 0$$

Proof. *(a) As, $e_t, z_t \geq 0$ also $e_t z_t \geq 0$ holds. Thus to proove (a), it is sufficient to show, that*

$$\exists \underline{z}(w_t) : \underline{z} - \pi(\underline{z})[1 - w_t\mathbb{I}_{\{\underline{z}\neq z_t\}}] \leq 0.$$

Because $\lim\limits_{x\to 0}\pi'(x) = +\infty$ and $\frac{\pi(x)}{x} \geq \pi'(x)$ (since π is concave), this "self-financing" $\underline{z}$ always exists.

(b) Using $\underline{z}$ from part (a) a firm can always pay out a larger dividend than e_t and can also set $b^o = b_{t+1} = 0$ as well. By paying a larger dividend in the current period and having the same debt as if it was to stop production but with a larger stock of capital, the expected value for $t+1$ must be larger than $E_t[v(w_{t+1},0,0)]$, so that

$$\tilde{v}(\underline{z},0,w_t,z_t,b_t) > \psi\mathbb{E}_t[v(w_{t+1},0,0)]$$

follows. Because the plan $\forall t : z_t^o = \underline{z}(w_t)$ is always feasible and leads to positive dividends, $v(\cdot)$ must be bounded from below by a positive real number, so that $\psi\mathbb{E}_t[v(w_{t+1},0,0)] > 0$. ■

Because of the above Lemma, firms never stop production completely respectively declare bankruptcy.[8] Therefore, the optimal policy is always an element of Y and thus the Bellman equation defining v is given by:

$$v(w_t, z_t, e_t) = \max_{(z^o, b^o) \in Y(w_t, z_t, e_t)} \tilde{v}(z^o, b^o) - \pi(z^o) w_t \mathbb{I}_{\{z^o \neq z_t\}} \ . \tag{3.8}$$

3.1.5 Adjustment process

In general, Y is only upper-hemicontinuous, thus v might not be continuous everywhere. The lack of continuity arises because of the fixed adjustment costs: If a firm employs a large stock of capital and is heavily indebted, it may find itself unable to repay the debt obligations if the capital-imbalance or the debt level rise marginally. Therefore it will be necessary to distinguish two cases when describing firm level investment:

(a) The firm is in danger of becoming insolvent: this happens, if

$$1 - e_t - \frac{\pi(z_t)}{z_t} > \widehat{b} \ . \tag{3.9}$$

In this case the firm has a negative cash flow and cannot sustain the actual level of capital employed by issuing new debt. Therefore, it has to (heavily) cut back production to increase its average productivity. In consequence, a firm always disinvests if in financial distress.

(b) The firm is not in danger of becoming insolvent:

Then, denoting the (optimal) capital-imbalance after adjustment with z^* and the ratio of debt to capital after adjustment with b^*, a firm adjusts its stock of capital (i.e. it invests) in period t if the expected increase in

[8]This result seems a contradiction to empirical facts at a first glance, i.e. of course in reality firms do declare bankcruptcy and are shut down. However, Lemma 1 should not be taken literally as "firms never disappear". Basically the Lemma states, that the monopoly power of a firm is always of some value, which would be lost upon bancruptcy. Hence, the Lemma may better be interpreted as "brands never disappear", which surely comes closer to reality than the former interpretation.

discounted value outweighs the adjustment costs. That is if:

$$
\begin{aligned}
0 \quad &\geq \max_{b^o \in Z(z_t,e_t)} \tilde{v}(z_t, b^o) - \tilde{v}(z^*, b^*) + \pi(z^*) w_t \qquad (3.10) \\
Z(z_t, e_t) \quad &: \quad = \left\{ b^o \in \mathbb{R}_{++} | 1 - e_t - \frac{\pi(z_t)}{z_t} \leq b^o \leq \widehat{b} \right\}
\end{aligned}
$$

or equivalently

$$
w_t \leq \min_{b^o \in Z} \left\{ \frac{\tilde{v}(z^*, b^*) - \tilde{v}(z_t, b^o)}{\pi(z^*)} \right\} . \qquad (3.11)
$$

As shown in the appendix, the value of a firm that adjusts,

$$
\tilde{v}(z^*(w_t, e_t), b^*(w_t, e_t)) - w_t \pi(z^*(w_t, e_t)) , \qquad (3.12)
$$

is monotonically decreasing in w_t, so that for every (e_t, z_t) there exists an unique $\bar{w}$ such that

$$
\tilde{v}(z^*(w_t, e_t), b^*(w_t, e_t)) - w\pi(z^*(w, e_t)) \gtreqless \max_{b^o \in Z} \tilde{v}(z_t, b^o) \text{ for } w \lesseqgtr \bar{w} . \qquad (3.13)
$$

3.2 Cross-sectional investment

3.2.1 Aggregation

Having reduced the firm's investment decision to a comparison of two values, it is now possible to define for every (e_t, z_t) a critical value Ω which is the largest value of the stoppage duration w_t for which the firm chooses to invest.[9]

$$
\Omega(e_t, z_t) := \begin{cases} \overline{w}(e_t, z_t) & \text{if } Z(z_t, e_t) \neq \emptyset \\ 1 & \text{if } Z(z_t, e_t) = \emptyset \end{cases} \qquad (3.14)
$$

As only contemporary state variables matter for the aggregation, the time indices of state variables are suppressed henceforth. Let $K_t^A, I_t^A, K_t(e, z), I_t(e, z)$

[9] Since w is always smaller than one, $\Omega(z, e) = const \geq 1$ implies that a firm adjusts independent of its realized w. Furthermore, as a firm always disinvests if it is in danger of becoming insolvent ($:\Leftrightarrow Z = \emptyset$), $\Omega(z, e) = 1$ is a sensible value if $Z = \emptyset$.

denote the aggregate stock of capital, aggregate gross investment, and the stock of capital and gross investment of firms with capital-imbalance z and equity-to-capital-ratio e (sectoral aggregates) in period t, respectively. Moreover, let $G(w)$ be the distribution of w. Then the investment hazard can be defined as $\Lambda(e, z) := G(\Omega(e, z))$ and we can define

$$\bar{z}^*(e, z) := \Lambda(e, z)^{-1} \int^{\Omega(e,z)} z^*(w, e, z) dG(w), \tag{3.15}$$

which is the average optimal capital-imbalance of firms that invest conditional on having an equity-ratio e and capital-imbalance z before investment.

At time t investment of firm j with capital-imbalance z_{jt} and equity e_{jt}, which adjusts indeed, is given by

$$I_{jt} = \left[z^*(w_{jt}, e_{jt}, z_{jt}) - z_{jt}\right] K_{it}^* = \left[\frac{z^*(w_{jt}, e_{jt}, z_{jt})}{z_{jt}} - 1\right] K_{jt} \; . \tag{3.16}$$

Therefore, with these quantities at hand, the expected (cross-sectional) investment conditional on (e, z) can be expressed as

$$\mathbb{E}[I_t(e, z)] = K_t(e, z) \left[\frac{\bar{z}^*(e, z)}{z} - 1\right] \Lambda(e, z). \tag{3.17}$$

Since adjustment-cost shocks w are i.i.d., the cross-sectional average investment rate $i(e, z)$ follows directly from (3.17):

$$i(e, z) := \frac{I_t(e, z)}{K_t(e, z)} = \left[\frac{\bar{z}^*(e, z)}{z} - 1\right] \Lambda(e, z) \tag{3.18}$$

Differentiating average investment i with respect to the equity-ratio e yields an interesting decomposition of the effect of a change in the leverage (if $\frac{\partial \bar{z}^*}{\partial e}$ and $\frac{\partial \Omega}{\partial e}$ exist):[10]

$$\frac{\partial i(e,z)}{\partial e} = \underbrace{\frac{\Lambda(e,z)}{z}\left(\frac{\partial \bar{z}^*(e,z)}{\partial e}\right)}_{\text{level-effect}} + \underbrace{\frac{(\bar{z}^*(e,z)-z)}{z}\frac{\partial \Lambda(e,z)}{\partial e}}_{\text{frequency-effect}} \tag{3.19}$$

While the first term represents a long-run or level effect of the equity-ratio, the second term represents an only short run or frequency effect. The latter effect is only short-run since an increase in Λ decreases the variance of the cross sectional capital-imbalance and therefore later on decreases the probability of investment. Due to this frequency effect investment can be more sensitive to the financial situation than the optimal stock of capital is:

Theorem 3.1 *(a) If $[i(e,z)+\Lambda(e,z)]$ is large enough—but possibly smaller than one—then the investment rate is more sensitive to the equity-ratio than the optimal stock of capital, i.e. the elasticity of $\bar{z}^*$ w.r.t. the equity-ratio is smaller then the semi-elasticity of investment w.r.t. the equity-ratio:*

$$\frac{\partial i(e,z)}{\partial \ln(e)} \geq \frac{\partial \bar{z}^*(e,z)}{\partial e}\frac{e}{\bar{z}^*}. \tag{3.20}$$

(b) In an environment around $(e,\bar{z}^*(e,z))$ we have

$$\frac{\partial^2 i(e,\bar{z}^*(e,z))}{\partial e \partial z} \Bigg/ \frac{\partial i(e,z)}{\partial e} \leq 0. \tag{3.21}$$

Proof. *(a) From equation* (3.18) *we obtain*

$$\frac{i}{\Lambda(e,z)} = \frac{(\bar{z}^*(e,z)-z)}{z}.$$

[10] Proposition A.4 in the appendix shows that $\frac{\partial \Omega}{\partial \bar{e}}$ exists almost everywhere (a.e.).

Using this to rewrite (3.19) *in terms of elasticities* η, *we obtain*

$$\begin{aligned}
\eta_e^i &= \frac{e}{i(e,z)}\frac{\Lambda(e,z)}{z}\left(\frac{\partial\bar{z}^*(e,z)}{\partial e}\right)+\frac{e}{i(e,z)}\frac{(\bar{z}^*(e,z)-z)}{z}\frac{\partial\Lambda(e,z)}{\partial e} \\
&= \frac{\Lambda(e,z)}{i(e,z)}\frac{e}{z}\left(\frac{\partial\bar{z}^*(e,z)}{\partial e}\right)+\eta_e^\Lambda=\frac{\left[\frac{\bar{z}^*(e,z)}{z}-1+1\right]\Lambda(e,z)}{i(e,z)}\eta_e^{\bar{z}^*}+\eta_e^\Lambda \\
&= \frac{i(e,z)+\Lambda(e,z)}{i(e,z)}\eta_e^{\bar{z}^*}+\eta_e^\Lambda. \qquad (3.22)
\end{aligned}$$

If $[i(e,z)+\Lambda(e,z)]+i\,(e,z)\,\frac{\eta_e^\Lambda}{\eta_e^{\bar{z}^*}}\geq 1$, *this yields*

$$\frac{\partial i(e,z)}{\partial\ln(e)}=\eta_e^i\cdot i(e,z)=[i(e,z)+\Lambda(e,z)]\,\eta_e^{z^*}+\eta_e^\Lambda\cdot i(e,z)\geq\eta_e^{z^*} \qquad (3.23)$$

(b) Suppose $\frac{\partial i(e,z)}{\partial e}>0\ (<0)$, *now let* e *increase marginally then we have for some firms an increase (decrease) in realized* z. *Now assume contradictory* $\frac{\partial^2 i(e,\bar{z}^*(e,z)))}{\partial e\partial z}>0\ (<0)$, *then investment rates would rise (fall) further, contradicting* $\bar{z}^*$ *to be optimal. Thus the stated inequality must hold.* ■

Remark 3.1 *Differentiating* (3.23) *with respect to* $\ln(z)$ *yields the following:*

$$\begin{aligned}
\frac{\partial^2 i(e,z)}{\partial\ln(e)\partial\ln(z)} &= \frac{\partial}{\partial\ln(z)}\left[[i(e,z)+\Lambda(e,z)]\,\eta_e^{z^*}+\eta_e^\Lambda\cdot i\right] \qquad (3.24) \\
&= \frac{\partial\,[i(e,z)+\Lambda(e,z)]}{\partial\ln(z)}\eta_e^{z^*}+\frac{\partial^2\bar{z}^*}{\partial z\partial e}\frac{e}{\bar{z}^*}z+i\frac{\partial\eta_e^\Lambda}{\partial\ln(z)}+\eta_e^\Lambda\frac{\partial i}{\partial\ln(z)}
\end{aligned}$$

As z *and* e *enter* Y *only multiplicatively and as* Y *determines* $z^*(w,e,z)$, $\frac{\partial^2\bar{z}^*}{\partial z\partial e}$ *can be approximated by* $\frac{\partial^2\bar{z}^*}{\partial e^2}\frac{e}{z}$. *Therefore, we can state:*

$$\begin{aligned}
\frac{\partial^2 i(e,z)}{\partial\ln(e)\partial\ln(z)} &\simeq \frac{\partial[i(e,z)+\Lambda(e,z)]}{\partial\ln(z)}\eta_e^{z^*}+\frac{\partial^2\bar{z}^*}{\partial e^2}\frac{e^2}{\bar{z}^*}+i\frac{\partial\eta_e^\Lambda}{\partial\ln(z)}+\eta_e^\Lambda\frac{\partial i}{\partial\ln(z)} \\
&= \frac{\partial\eta_e^{z^*}}{\partial\ln(e)}+\frac{\partial i(e,z)}{\partial\ln(z)}\left[\eta_e^{z^*}+\eta_e^\Lambda\right]+\left[\frac{i}{\Lambda(e,z)}+\eta_e^{z^*}\right]\frac{\partial\Lambda(e,z)}{\partial\ln(z)}. \qquad (3.25)
\end{aligned}$$

Although the sign of these terms is not clear from analytic grounds, we would expect for negative $\ln(z)$ *the effect of equity on the optimal capital stock to* *be decreasing in e, the adjustment hazard to be decreasing in z and the expected investment to be a decreasing function of z, too. Hence, intuitively $\frac{\partial^2 i(e,z)}{\partial \ln(e)\partial \ln(z)} < 0$ if i is positive.*

3.2.2 Discriminating between our model and alternatives

The above theorem and remark are central in discriminating between the model of this chapter and both, the Myers and Majluf (1984) pecking-order of finance and the liquidity-dependent cost of capital (but convex adjustment cost) models: In the liquidity-dependent cost of capital models with convex adjustment costs the long-run effect is clearly dominant.[11] Any effect of liquidity on the speed of adjustment in these models is only a second-order effect. Slower adjustment marginally saves internal funds, so that the marginal gains of faster adjustment and the marginal-costs of internal funds have to be equalized. Therefore, liquidity can have an influence on the adjustment speed only via the second-order derivative of the costs of capital with respect to liquidity. In the fixed adjustment cost model however, investment is an extramarginal decision. For a given fixed cost of investment, a change in liquidity hence may render some adjustments unprofitable, so that there is a first-order effect of liquidity on the adjustment speed.

In the pecking-order model there is an important short run effect of liquidity independent of the form of adjustment costs. In these models when adjustment costs are concave typically three regimes of firm-finance emerge.[12] These are stylized in figure 3.1.

The firms with a high z (low productivity of investment) are financially unconstrained, rely on internal finance, and their investment decision is in-

[11] However, note that in a model with convex costs z^o has to be defined somewhat differently. In this case it is the capital imbalance at which a firm would not actively change the capital imbalance by investing. Yet and although firms adjust their capital imbalnce in the short-run towards this level, outside its long run equilibrium level (where e is endogenuous) z^o is never actually reached by active investment.

[12] See Gomes (2001), Bond and Meghir (1994), or Whited (1992) for details.

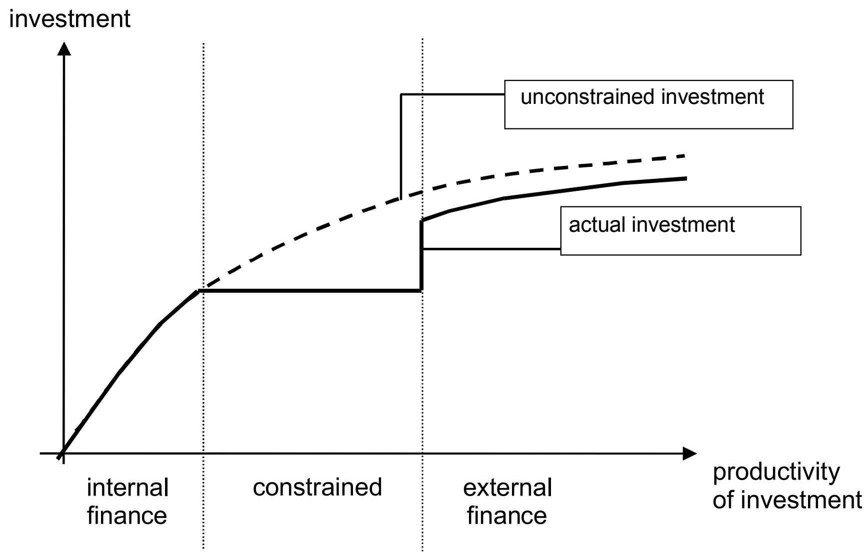

Figure 3.1: A stylized version of the pecking-order model

dependent of their liquidity constraint. Firms with low z rely on external finance and depending on the form of transaction costs, investment of these firms can be sensitive to liquidity. Firms with intermediate z are strictly constrained by liquidity and a change in liquidity changes investment. Firms with high productivity rely on external finance. For these firms increasing equity either has no effect or actually reduces the sensitivity of investment with respect to fundamentals, because these firms become liquidity constrained when equity rises. This can happen since the gains of obtaining external finance get smaller, the larger the internally financed amount of investment is. Therefore, the cross-derivative $\frac{\partial^2 i(e,z)}{\partial \ln(e) \partial \ln(z)}$ would be positive. Consequently, Theorem 3.1(b) and Remark 3.1 can be used to test the model of this paper against (simpler) pecking-order alternatives.[13] Another possibility to dis-

[13]This argument, however, only holds if transaction costs in a pecking-order model are deterministic. If transaction costs are stochastic, cross-sectional aggregation of the short run investment function may be governed by the *distribution* of transaction costs, which - if chosen from a set of *arbitrary* distributions - can have a dynamics similar to the one of our model as a result.

criminate between fixed cost of investment and fixed cost of external finance models is the investment behavior for $z > 1$: Fixed disinvestment costs imply a range of inactivity, while transaction costs of finance yield immediate disinvestment, if firms can hold financial assets.

Whether within our model the debt-ceiling or the liquidity dependence of the cost of capital is more important can be evaluated by comparing $\frac{\partial i}{\partial z} \times \frac{\partial \bar{z}^*}{\partial e}$ and $\frac{\partial i}{\partial e}$. If only the liquidity dependence is of importance, $\frac{\partial i}{\partial z} \times \frac{\partial \bar{z}^*}{\partial e}$ and $\frac{\partial i}{\partial e}$ should be close to equal. In this case a change in the equity ratio alters the discount factor. This is similar to an increase in z, i.e. the typical investment project is smaller. Yet, there still is an effect on investment frequency. Only this effect is similar to the effect a marginal change in z would have, smaller investment projects only pay at lower investment costs.

If the debt-ceiling is the important financial friction, $\frac{\partial i}{\partial e}$ can be expected to exceed $\frac{\partial i}{\partial z} \times \frac{\partial \bar{z}^*}{\partial e}$ substantially. Here, liquididity corresponds to a number of investment options a firm can expect to have at most over some given period of time. The smaller the number of options is, the larger the value of each option will be. This option-value adds another factor to the fixed cost of adjusting the stock of capital.

In that sense, even if we later estimate the investment function non-parametrically and even if the estimated derivatives have no structural interpretation in the form of coefficients of an adjustment-cost function, we can identify the various investment models using first- and higher-order derivatives. Moreover, in its general formulation, the model nests the alternative models as it only partially differs with respect to the "test objects" mentioned.

3.3 Aggregate investment and its dynamics

One obtains the time-series of aggregate investment by aggregating over the distribution of (e, z) pairs. Aggregate investment can therefore be expressed as:

$$I_t^A = \int \int i(e,z) K_t(e,z) f(e,z,t) dedz \tag{3.26}$$

where f denotes the joint density of (e,z).

To obtain a simplified expression for the aggregate investment equation, we make the following assumption:[14]

Assumption 3.4: Let $\frac{K_t(e,z)}{K_t^A}$ and $i(e,z)$ be independent in (e,z).

Then we get for the aggregate investment rate $\frac{I_t^A}{K_t^A}$:

$$\frac{I_t^A}{K_t^A} \simeq \int \int \left[\frac{\bar{z}^*(e,z)}{z} - 1\right] \Lambda(e,z) f(e,z,t) dedz. \tag{3.27}$$

Given equation (3.27), aggregate investment is fully determined by the distribution of equity and the conditional distribution of capital-imbalances. The dynamics of the aggregate investment series is then determined by the transition from $f(\cdot,t)$ to $f(\cdot,t+1)$. Now let—as in Caballero et al. (1995, p. 35) and without loss of generality—the sequence of shocks/adjustment be:

1. adjustment as described
2. idiosyncratic shocks ξ
3. aggregate shock v_t and depreciation δ.

Given this sequence, we can now describe the transition from $f(\cdot,\cdot,t-1)$ to $f(\cdot,\cdot,t)$. We do so by proceeding backwardly beginning with step 3.

Given adjustment and idiosyncratic shocks, aggregate shocks and depreciation still alter the capital imbalance and the equity ratio. If e_1 is the equity ratio before depreciation, then the equity ratio after depreciation is

$$e_2 = \frac{e_1 - \delta}{(1-\delta)}$$

[14] See Caballero and Engel (1999, p. 816) for a detailed numerical justification of a closely related assumption.

so that

$$e_1 = e_2\,(1-\delta) + \delta. \tag{3.28}$$

Analoguously, we obtain

$$z_2 = \frac{K_1\,(1-\delta)}{K_1^* \exp(v_{t-1})} = z_1 \frac{1-\delta}{\exp(v_{t-1})} \tag{3.29}$$

and hence

$$z_1 = z_2 \frac{\exp(v_{t-1})}{1-\delta}. \tag{3.30}$$

Therefore, expression (3.28) and (3.30) give the transformations the capital imbalance and the equity-ratio of a given firm undergo by step 3. Let the joint distribution of (e, z) after adjustment and idiosyncratic shocks be denoted by $\widetilde{f}_1$. For f we then obtain.

$$f(e,z,t) = \widetilde{f}_1\left(\delta + e(1-\delta), z\frac{\exp(v_{t-1})}{1-\delta}, t-1\right). \tag{3.31}$$

For step 2, let $g\,(\xi)$ denote the density of the productivity shocks ξ. Then—using the same reasoning as for the aggregate shocks and denoting the distribution after adjustment by $\widetilde{f}_2$—we obtain

$$\widetilde{f}_1(e,z,t\text{-}1) = \int \widetilde{f}_2(e, z\exp(d+\xi), t\text{-}1) g(\xi) d\xi. \tag{3.32}$$

As adjustment itself is governed by the stochastic stoppage duration, adjustment is a stochastic variable itself. So let $\dot{K}_{jt} := 1 + i_{jt}$ denote the growth rate of capital and let $\dot{E}_{jt}$ be the growth-rate of the equity-ratio. Moreover, let $H\left(\dot{E}, \dot{K} \middle| e, z\right)$ denote their conditional distribution. This distribution governs step 1, the adjustment-process. The distribution $\widetilde{f}_2$ of (e, z) after adjustment is generated by a convolution of f in period $t-1$ and H:

$$\widetilde{f}_2(e,z,t\text{-}1) = \int dH(o,p|\tfrac{e}{o}, \tfrac{z}{p}, t\text{-}1) f(\tfrac{e}{o}, \tfrac{z}{p}, t\text{-}1) do dp. \tag{3.33}$$

Since w_t are i.i.d., H is stationary. Combining (3.31) - (3.33) yields

$$f(e,z,t) = \int dH(o,p|\tfrac{e}{o},\tfrac{z}{p}) f\left(\delta + \left(\tfrac{e}{o}+\tfrac{o}{p}\right)(1-\delta), \tfrac{z}{p}\tfrac{\exp(d+\xi+v_{t-1})}{(1-\delta)}, t\text{-}1\right) g(\xi) d\xi dodp \ . \tag{3.34}$$

Therefore, the aggregate investment series can be characterized as a generalized Markov-chain. Note that because of the presence of aggregate shocks f is non-stationary.[15]

Our model maps the parameters at the firm level now to some density dH. Therefore and as in Caballero and Engle (1999), even with only aggregate data—and given an initial distribution $f(e,z,0)$—one could still obtain parameter estimates by choosing the firm level parameters as to maximise the likelihood of the aggregate investment and equity-ratio data. However, in this thesis we will pursue an empirical approach using firm-level data.

3.4 Brief Summary

As we will begin the following chapter with a relatively extended discussion of the empirical implications of our model, we shall only very briefly summarize the main results of this chapter: The key result of our model was to introduce a frequency-of-adjustment interpretation of the effect liquidity has on investment. It explains seemingly contradictory evidence concerning the influence of liquidity on investment decisions in the short and on stocks of capital in the long-run. Moreover, we provided implications of the model for

[15] If f were stationary in the absence of aggregate shocks, the density of disequilibria and equity to capital ratios through which a firm goes over its lifetime would be ergodic (Caballero and Engel (1992)). Without aggregate shocks f would be stationary, if the operator T defined below converges (to a non-degenerated density).

$$\begin{aligned}(Tf)(e,z) \ :&= \int dH(o,p|e\cdot p+\frac{o}{p},\frac{z}{p}|e\cdot o, z\cdot p) \\ &\times f\left(\left(e\cdot p+\frac{o}{p}+\delta\right)(1-\delta),\frac{z}{p}\frac{\exp(d+\xi)}{(1-\delta)}\right) g(\xi) d\xi dodp\end{aligned}$$

the derivatives of the investment function which can be used to test our model against a broad range of alternatives. Furthermore, we have shown how the aggregate investment series driven by the distributions of equity-ratios and capital imbalances.

We leave an extended summary of the main implications—focused on its empirical content—of our model to the beginning of next chapter, where our model will be applied to German and UK firm panel-data.

Chapter 4

Empirical evidence

With the initial considerations of the last chapter at hand, we can now turn towards empirically assessing the importance of our claim: financial factors and non-convexities amplify each other's importance. We will do so by analyzing both a UK and a German dataset, which both contain accounting data at the company-level. Yet, or of course, we will also check the existence of an influence and its importance for both factors themselves. Before proceeding, however, we should reconsider our main theoretical results.

4.1 Summing up and extending the theoretical considerations

Like before, let k^* denote the log of the stock of capital a company would hold in the absence of any adjustment costs. As argued, this stock of capital may depend on the (log) equity-ratio e in the presence of some influence of equity on the cost of finance, i.e. the cost of capital. Thus, $k^* = k^*(e, \xi)$, in which ξ denotes the productivity of capital. Let k be the log of the capital actually employed. Now define the capital imbalance

$$x := z^* - z = \varkappa + k^* - k, \tag{4.1}$$

where $\varkappa + k^*$ is the dynamically optimal (long-run) capital-stock in the presence of adjustment costs. Parameter $\varkappa$ captures the optimal capital imbalance, i.e. the capital-imbalance that is optimally installed to account for depreciation and trends in productivity. For example, if there is depreciation, a firm will adjust to a larger stock-of capital than it would hold in the absence of adjustment-costs. The firm does so to minimize the average distance between the capital-stock and its optimal level.

Now suppose the firm faces only fixed costs of investment; then it will invest up to $(\varkappa + k^*)$ if it decides to invest. Therefore, investment would equal mandated investment as in (3.16) and hence is given by

$$I = \exp\left(\varkappa + k^*\right) - \exp\left(k\right). \tag{4.2}$$

Again, let $\Lambda(x\left(e\right), e)$ be the hazard rate of investment for a firm with capital imbalance x and equity e. The expected investment rate i is then given by

$$i\left(x, e\right) := \mathbb{E}\left(\frac{I}{\exp\left(k\right)}\right) = \Lambda(x\left(e\right), e)\left[\exp\left(x\left(e\right)\right) - 1\right] \tag{4.3}$$

If investment is not lumpy but continuous, then Λ can be interpreted as the fraction of mandated investment that is actually realized. As discussed in the previous chapter, Λ is a function of both x and e: Internal funds may not only change costs of capital but also affect the speed of adjustment itself and thus interact with productivity in a more complex way.

In order to assess to what extent financial considerations enter the firm's decision process mainly by a flexibility or a cost-of-capital argument we can apply our decomposition of the effect of a marginal change in equity on investment, obtained in the previous chapter in (3.19):

$$\frac{\partial i\left(x, e\right)}{\partial e} = \underbrace{\frac{\partial i\left(x, e\right)}{\partial x}\frac{\partial x}{\partial e}}_{\text{level-effect}} + \underbrace{\frac{\partial \Lambda}{\partial e}\left[\exp\left[x\left(e\right)\right] - 1\right]}_{\text{frequency-effect}}. \tag{4.4}$$

Thus, the effect of a marginal change in equity on investment can be decomposed into a "level"-effect and a "frequency"-effect. The "frequency"-effect reflects changes in the speed of adjustment to the target stock of capital. The "level"-effect results as the change in the optimal stock of capital induced, multiplied by the marginal propensity to invest upon changes in productivity. The optimal stock of capital may for example be changed by e through altering the cost of capital or the (implied) managerial discount-factor.

Given this decomposition, we can test two hypothesis how the availability of (accumulated) internal funds influences investment activity. The first hypothesis reflects the long run neutrality of finance:

H_0^0 : Internal funds have no effect on the optimal stock of capital, a company holds. This is equivalent to $\frac{\partial x}{\partial e} = 0$.

The second hypothesis accounts for the influence of equity on the investment process, this is:

H_0^1 : Investment reacts to changes in internal funds only because the optimal stock of capital is changed, i.e. $\frac{\partial i(x,e)}{\partial e} = \frac{\partial i(x,e)}{\partial x}\frac{\partial x}{\partial e}$.

If H_0^0 cannot be rejected, the Modigliani-Miller theorem holds in the long run. However, this still allows for the possibility that finance influences the transition path of the stock of capital, i.e. hypothesis H_0^1 is rejected. This could for example be due to fixed costs of external equity finance or short-run ceilings on debt-ratios.[1] Note that (only) if H_0^0 holds true, H_0^1 simplifies to $\frac{\partial i(x,e)}{\partial e} = 0$.

Moreover, we can test the way fundamental and financial variables interact in the investment decision. This—as laid out in Chapter 3—enables us to discriminate between the "convex-adjustment but fixed financial transaction costs", the "net-worth effect in finance with fixed adjustment costs" and the "both fixed adjustment and transaction costs" model.[2]

[1] See Gomes (2001) for a discussion.

[2] See sections 4.2.5.3, 4.3.4 and 4.3.5.

Since productivity of capital is unobservable, a proxy for the marginal productivity of capital or directly for fundamental investment incentives is needed. To obtain such a proxy, one could of course rely on Tobin's-q. However, the drawbacks of this measure have been discussed in detail in Chapter 2. Therefore, for the German data, we follow a technique closest to the one used by Cooper and Haltiwanger (2002): As we have data on firm-specific wages available, we will use first-order employment conditions, to determine the productivity of capital and hence the optimal stock of capital. In contrast to Cooper and Haltiwanger, however, we allow the technology to vary across firms.

For the UK-data we do not have any wage data available, but detailed data on firm-specific subsidies. This makes the procedure of Caballero et al. (1995) an attractive option, as we have some variation in observed marginal cost of capital, even if we control for firm and time fixed-effects.

However, our analyses of the UK and the German sample not only differ in the way the optimal stock-of-capital measure is generated. For the UK data we present full estimates of the investment functions as well as average derivative estimates. To avoid any longueurs, for the German dataset only the derivative estimates, the "parameters" of interest, are reported. This gives room to address possible endogeneity problems associated with the liquidity variable. We will begin with the analysis of the UK data.

4.2 Evidence from a sample of UK firms

4.2.1 Measuring the capital imbalance

Alternatively to the two approaches mentioned above, we could of course proceed with the full-information maximum-likelihood approach of Caballero and Engel (1999).[3] However, when micro-panel data is available, as in our

[3] To do so we would need to estimate the distributions involved in (3.34). The distribution of equity can be estimated without use of any theoretical model with a suitable parametric or nonparametric estimator. However, one has to generate B and Ω for any

case) the two-step approach of Caballero et al. (1995) is more flexible. We follow this approach and estimate z as a proxy of fundamental investment incentives in a first step. Thereafter, we regress investment on this proxy and the equity-ratio, to obtain the (short-run) expected investment function. It seems useful to take this approach, since it is applicable to a much broader class of models that incorporate nonlinear adjustment-/investment-functions and/or capital-market-imperfections, i.e. this estimation approach *nests* the alternative models we want to test against.

The first intermediate goal of this section is hence to construct an estimator for the capital-imbalance z. In contrast to Caballero et al. (1995), it cannot be assumed that the desired capital is proportional to the stock of capital K^* that a plant would hold in the absence of *adjustment costs*. From lemma 2, we know that z and e enter only multiplicatively in z^*. Taking logs of all variables except for $i\,(e,z)$ (without changing notation), we then can write the optimal capital imbalance, as defined in the previous chapter, as a function in two arguments. Abusing notation slightly, we replace $z^*(w,e,z)$ by $z^*(w,e+z)$. Neglecting the differential effect of e and z on the composition of $\bar{z}^*$, we then can also write $\bar{z}^*$ as a function of $(e+z)$. Assuming $\bar{z}^*$ is differentiable, a Taylor-approximation of $\bar{z}^*(e+z)$ around $\bar{z}^*(0)$—neglecting higher-order-derivatives—yields for desired stock of capital $\tilde{k}$ in logs (by the definition of z^*):[4]

$$\bar{z}_i^*(e_{it}+z_{it}) = \tilde{k}_{it} - k_{it}^* = \alpha_{i0} + \beta_i(e_{it}+z_{it}). \tag{4.5}$$

For an isoelastic production/sales function (in logs: $y_{it} = \psi_{it}^0 + \psi_i^k k_{it}$) we

G from our Chapter 3 model and choose a starting distribution $f(\cdot,0)$. Then one is able to maximize the likelihood of the system by choosing an optimal G (Caballero and Engel, 1999). Though this approach does not rely on a specific measure of fundamental investment incentives, specifying an initial distribution $f\,(\cdot,0)$ and the distribution of shocks to productivity and adjustment costs can be problematic. Moreover, as a very structural approach it does not nest all the alternative models we want to test against.

[4]This means, we approximate $\bar{z}^*$ around the average optimal stock of capital a firm would like to hold that has a book value of equity equal to the frictionless optimal stock of capital.

obtain from the first order condition for k_{it}^*:[5]

$$-z_{it} = k_{it}^* - k_{it} = \frac{1}{1-\psi_i^k}\left[\ln(\psi_i^k) + y_{it} - k_{it} - \theta_i \widetilde{co}_{it}\right], \tag{4.6}$$

where k_{it} denotes log-capital employed, y_{it} denotes log-sales, $\widetilde{co}_{it}$ denotes log cost-of-capital and b_i denotes the elasticity of sales to capital. Now combining (4.5) and (4.6) yields:

$$\begin{aligned}\frac{\bar{z}_{it}^* - z_{it}}{1-\beta_i} &= \alpha_{i1} + \eta_i\left[y_{it} - k_{it} - \theta_i \widetilde{co}_{it}\right] + \tfrac{\beta_i}{1-\beta_i} e_{it} \qquad (4.7)\\ \text{with } \alpha_{i1} \quad : &= \tfrac{\alpha_{i0}}{1-\beta_i} + \frac{\ln(\psi_i^k)}{1-\psi_i^k} \text{ and } \eta_i := \frac{1}{1-\psi_i^k}\end{aligned}$$

In this equation $\bar{z}_{it}^* - z_{it}$ gives the log of the ratio of dynamically optimal capital to capital currently employed. If firms adjust their stock of capital over time, we can expect $\bar{z}_{it}^* - z_{it}$ to be mean reverting. y, k and $\widetilde{co}$ are most likely to be non-stationary. The equity-ratio e_{it} in the opening balance is predetermined. If $\bar{z}_{it}^* - z_{it}$ is not only mean-reverting but also stationary, there must be a cointegration relation and θ and β can be estimated from a panel-cointegration regression (Caballero et al., 1995, p. 15).[6,7]

Regarding (β_i, θ_i) we assume the cointegrating vector to be homogeneous (at least amongst industries Ind_j). Thus $(\beta_i, \theta_i) = (\beta, \theta)\,\forall i \in Ind_j$. Furthermore, since the number of parameters is larger than the number of variables, one needs to approximate ψ_i^k by the cost share of equipment capital (Caballero et al., 1995, p. 15).

Given consistent estimators $\widehat{\theta}, \widehat{\beta}$ (and $\widehat{\alpha}_{i0}$) it is then possible to compute z and estimate $i\,(e, z)$, $f(e, z, t)$ non-parametrically.

[5] See appendix B.1 for details, especially for the definition of b_i.

[6] Obviously it would be preferable to test for a cointegration relation being present in the data. However, as the panel data we have is large only along the cross-section- and not along the time series dimension, such a test would be meaningless. Nevertheless, estimating the cointegrating vector should still be possible, as this is even possible in pure cross-sections (Madsen, 2001).

[7] Strictly speaking, of course we only estimate a long-run covariance. This covariance reflects the cointegration relation if there is such a relation, which is what we assume.

4.2.2 Estimation procedure

For estimating the cointegration relation (4.8), Phillips' and Moon's (1999) "full-modified panel cointegration estimator" (henceforth PFM-OLS) is used. This estimator is $\sqrt{n}T$-consistent, asymptotic normal and corrects for possible endogeneity of the regressors. However, the consistency result is obtained by letting T (the time series dimension) and n (the number of observations per individual) tend to infinity sequentially, i.e. it is a consistent estimator for samples which are larger along the time-series than along the cross-sectional dimension. Nevertheless, the estimator should still be superior to OLS. Moreover and mentioned before, Madsen (2001) shows, how inference on the cointegrating vector can even be obtained from a cross-section in a similar framework.

Another drawback of the PFM-OLS estimator is that it is formulated for balanced panels with integrated regressors only. The data we have is an unbalanced panel and at least for e we would rather assume it to be an I(0) process. However, the PFM-OLS estimator is a generalization of the full-modified OLS estimator of Phillips and Hansen (1990) and Phillips (1995) and hence we expect the results of Phillips (1995) to carry over to the panel-case as well, i.e. the estimator is $\sqrt{nT}$-consistent and asymptotic normal for parameters of stationary regressors. The standard errors will thus be calculated in analogy to the time-series case.

To account for the unbalancedness, the unbalanced-panel equivalents of all items that appear in the formula of the estimator are calculated.[8] For inference the average number of observations per firm is used. The equation we use for estimation is:

$$\eta_i\left(k_{it}-y_{it}\right)=\alpha_{i1}+\gamma_t-\theta\eta_i\widetilde{co}_{it}+\kappa e_{it}+u_{it},\ \kappa:=\frac{\beta}{1-\beta}. \tag{4.8}$$

[8] For example when estimating the short-run covariance matrix, we calcualte the covariance for every firm in the sample using the firm-specific number of observations and then average over firms.

For inference in the I(1)-regressor case we calculate confidence bounds on the basis of T being the average number of observations per firm.

The time-dummies γ_t have been added for two reasons: The first reason is that the data used covers a period of large shocks to inflation, thus measuring the real interest rate correctly is something to be concerned about. The second reason is that we do not have data on taxational shocks—common to all firms. The usual within transformation will be used to remove time and individual effects.

With the estimator of θ at hand, we can use (4.6) to calculate end-of-period t capital imbalances $\hat{z}_{t+1}$. Of course, we would need to know the beginning-of-period capital imbalance. Nevertheless, even with the consistent estimates generating beginning of period capital imbalances is problematic. Of course one could simply subtract investment from $\hat{z}_{t+1}$, which would yield beginning-of-period z consistently with our model. However, subtracting the regressand in generating the regressor is obviously problematic.[9] Instead, we will simply take $\widehat{z}_t$, as a proxy for z_t. The innovation to productivity and costs will then however cause a measurement error, which may bias our estimates for the investment-function towards zero. Yet, there are no obvious instruments available to correct for this problem.

After removing individual averages and time-specific effects, we obtain for our model:

$$\begin{aligned}\frac{\widehat{z}_{it+1}}{\eta_i} = \left(k_{it} - \overline{k}_{i.} - \overline{k}_{.t} + \overline{k}_{..}\right) - \left(y_{it} - \overline{y}_{i.} - \overline{y}_{.t} + \overline{y}_{..}\right) \\ + \widehat{\theta}_{(T,n)}\left(\widetilde{co}_{it} - \overline{\widetilde{co}}_{i.} - \overline{\widetilde{co}}_{.t} + \overline{\widetilde{co}}_{..}\right)\end{aligned} \quad (4.9)$$

As already mentioned before, in a second step, the investment rate i_{jt} is regressed non-parametrically on $(e_{jt}, \widehat{z}_{jt})$. To compare the short- and long-run behavior, and to compare the results with other empirical studies, average derivatives of $i(e, z)$ are estimated, as well. These are the counterparts to the coefficients estimated in linear models. However, as we only observe average z on the company level, but not for every single plant, and also because of

[9] In an earlier version, this approach has been taken and the results were qualitatively the same.

the measurement error described above, the second-order derivative of the investment function in z will be underestimated if the function is convex, i.e. the function will appear to be less curved than it is.

The nonparametric estimation procedure, that will be employed will basically be a generalized-nearest neighborhood estimator. However, accounting for firm-specific effects is not as straightforward in the nonparametric case, as it is in the parametric one (Ullah and Roy, 1998). According to our specification in (4.9) the investment function shall meet the following assumption:

Assumption 4.1: For each firm j at time t investment i is given by

$$i_{jt}(z_{jt}, e_{jt}) = a_{0j} + b_{0t} + i(z_{jt} - (a_{zj} + b_{zt}), e_{jt} - (a_{ej} + b_{et})) + v_{jt} \ . \quad (4.10)$$

Moreover, $\mathbb{E}(i(\cdot, \cdot)) = 0$ (to identify i). a_{xj} and b_{xt} are individual- and time-fixed effects.

Under this assumption the function i can be directly estimated using within-transformed quantities only. Alternatively, the nonparametric first-derivative estimator of Ullah and Roy (1998) could be applied. To obtain the investment function one has to integrate over the derivatives. For estimating average derivatives however, we will also apply the estimator of Ullah and Roy.

4.2.3 Data

The UK-data we employ is the BSO-dataset of the Cambridge/DTI Database.[10] This database contains annual accounting data from UK companies from 1976 to 1990. 50494 company-year observations are included in the data. About half of them come from manufacturing firms. For the subsequent analysis the dataset has been restricted to companies of the manufacturing sector with positive fixed capital and positive equity. Moreover, only

[10] See Goudi et al., 1985.

firms with 5 or more consecutive observations available remain in the sample. After removing outliers (see below), the sample contains 7147 observations from 915 different firms.[11]

The BSO dataset contains capital and investment data for land and buildings as well as for tools and machinery. Since reported depreciation rates for machinery are about 40%, we restrict the analysis to land and buildings.[12] All data have been deflated to 1975 prices using the retail-price-index (RPI).

The user-cost of capital are computed as the average reported depreciation rate (on land and buildings) for each firm $\overline{\delta}_i$ plus the real interest-rate (reported in table 1). But then a fraction s_{it} of the investment spending is payed for by subsidies. In consequence, this fraction has to be subtracted from the cost:

$$\widetilde{co}_{it} = \ln((\overline{\delta}_i + r_t)(1 - s_{it})). \tag{4.11}$$

The real interest series is obtained by subtracting annual inflation rate (on the basis of RPI) from 3-month Euro-sterling deposit rates, see table 4.1. Due to the logarithmic transformation the real-interest rate shocks are not completely removed by the time-dummies.

The data on subsidies is problematic as we obtain $s_{it} \geq 1$ for a few firm-years. Therefore, all these observation are removed by discarding all observations with cost-of capital below the 4% and above the 99.5% quantile.[13] Moreover, apriori it is doubtful that the *yearly average* subsidy fraction s_{it} is equal to the *expected marginal* fraction of subsidies. In this sense, there could

[11] There are missing observations for few firms due to the way outliers are removed, i.e. we have an observation in $t-1$ and in $t+1$ but the observation in t is removed. The within-transformation we use takes care of this, but we simply neglect this fact for the following regressions. More strictly, we either would have to take the whole time series for the firm out of the sample or at least treat observations after the outlier and before differently. However, in any case we would lose quite many observations.

[12] Although our model might still hold true for fast depreciating capital goods, we would need data at a higher frequency to sensibly analyse the data. At a depreciation rate of 50% capital goods are replaced on average every second year on a regular basis, if the stock of capital stays constant. Hence, we can expect to hardly find any influence of fixed costs in yearly data.

[13] Additionally, we remove observations with the 0.05% highest or lowest costs after the within transformation.

Table 4.1: Nominal interest-rate i, real interest-rate r, change in the RPI, deflator P

year	1975	76	77	78	79	80	81	82	
i [%]	11.4	13.5	8.9	10.4	13.9	16.7	13.9	12.3	
$\frac{RPI_{t+1}}{RPI_t}$ [%]	17	16	9	13	17	12	9	5	
r [%]	-4.75	-2.18	-0.09	-2.28	-2.65	4.22	4.53	6.93	
P	1	1.17	1.357	1.479	1.672	1.956	2.191	2.388	
year	83	84	85	86	87	88	89	90	91
i [%]	10.1	10.0	12.2	11.0	9.7	10.3	13.9	14.8	11.5
$\frac{RPI_{t+1}}{RPI_t}$ [%]	5	5	4	4	5	6	8	7	5
r [%]	4.89	4.72	7.91	6.70	4.48	4.07	5.46	7.26	6.22
P	2.507	2.632	2.764	2.875	2.990	3.139	3.327	3.594	3.845

theoretically be a measurement error problem present in this specification. However, there are no obvious instruments available and the actually estimated coefficient looks in no way biased downwards, as it is insignificantly different from its theoretical value of 1.

Table 4.2 reports descriptive statistics for the variables of the sample that we use. The within transformation is calculated before taking first differences, and thus before loosing the first observation for each firm. Hence, the regressors do not have mean zero exactly.

In chapter 2 we summarized the recent literature on q-theoretic empirical investment models which has highlighted the role of measurement errors. Although the analysis does not rely on q, in two other systematic ways a measurement error might be present in the cost series. The first is a systematic risk effect, the second would be a difference between our constructed real interest series and the real interest rate on debt. If firms have different idiosyncratic risks, their costs of capital are different. If risk enters capital costs multiplicatively, however, the fixed effects perfectly control for this (if risk is constant over time), otherwise they still should do most to remove the measurement-error bias.[14] The same argument also holds true for differences

[14] Note that obviously, if financial frictions determine the cost of capital, our measure of costs measures with an error. However, the residual is just explained by the parameter of

Table 4.2: Descriptive statistics of the quantities used (within-transformed)

full sample				removing first obs. (as in PFM-OLS)				
	min	max	std		mean	min	max	std
ηy	-5.655	3.438	0.326	$\eta\,(y-k)$	0.001	-5.648	4.956	0.383
ηk	-3.389	6.302	0.486	$\eta\widetilde{co}$	-0.004	$-.4233$	0.627	0.062
$\eta\widetilde{co}$	$-.4233$	0.627	0.075	e	0.0005	-2.414	1.585	0.209
e	-2.414	1.585	0.210					
η^a	0.617	1.000	0.026	before individual demeaning				
z^b	-5.680	4.775	0.358	z	-4.08	-10.66	1.716	0.796
i	-10.37	0.778	0.514	i^c	0.339	-0.936	8.115	0.571

[a] Not demeaned

[b] After removing individual effects, the correlation of capital-imbalance and equity-ratio is about 0.032

[c] Gross-investment, calculated as annual differences of reported capital stocks.

between the real-interest series used and the relevant real-interest series for firm debt.

As a last preliminary check, we test for independence of investment rates and equity. For the full dataset, i.e. without restricting to more than 5 consecutive observations, Ahmad and Li's (1997) nonparametric kernel-based test yields a test statistics of 10.0775. Therefore, the hypothesis of independent equity and investment rate distributions can be rejected far below the 0.1% level.

4.2.4 Long-run optimal stock of capital

Table 4.3 reports estimates of (4.8) for the whole dataset as well as for the industries for which many observations are available. These are Food (21), Chemicals (26), Non-electrical engineering (33), Electrical Engineering (36), Paper, Printing & Publishing (48) and Transport & Communication (50). The standard errors are given in parentheses.

The estimated long-run elasticity of capital with respect to the user-costs is with 0.9752 statistically insignificantly different from 1. The elasticity with

equity in our regression.

Table 4.3: Estimates from the cointegration regression (PFM-OLS, Within)

full sample	κ	θ	$\sum T_i n_i$	By industries[c] Ind.		κ	θ	$\sum T_i n_i$
PFM-OLS[a]	0.079	0.98	5944	21	PFM	0.191	1.73**	471
prelim. OLS	0.070	1.08			OLS	0.136	1.340	554
std. err. I(1)	0.026	0.07		26	PFM	0.108	1.119	760
std. err. I(0)[b]	0.035	0.14			OLS	0.097	0.738	897
OLS	0.082	0.71	7147	33	PFM	0.020	0.32**	836
					OLS	-0.019	0.075	1065
				36	PFM	0.164	0.957	534
					OLS	0.209	0.443	626
				48	PFM	-0.037	0.991	410
					OLS	-0.047	0.570	493
				50	PFM	0.296	0.627	378
					OLS	0.156	0.637	482

[a]Only observational units have been used for which (outliers removed) 5 or more observations are available.
[b]The standard errors are obtain as panel analogues to Phillips (1995, p. 1033ff).
[c]Full industry-sample OLS estimates are reported in brackets ( ).
** Significantly different from the remaining-sample-estimate at the 5%-level - based on PFM-OLS estimate under the assumption of an I(1)-regressor.

respect to the equity-ratio is with $\frac{\kappa}{\kappa+1} = 0.077$ statistically significant under both the I(0) and I(1) assumption for e.[15] If the equity-ratio influences the long-run stock of capital through influencing the marginal cost of finance (interest rate on debt), these estimates imply an elasticity of the cost of finance with respect to the equity-ratio that is equal to $\frac{\kappa}{\theta} \simeq 0.081$, which is only moderate.

The OLS-estimates for the elasticity with respect to equity is slightly lower than the PFM-OLS-estimate. This is quite in line with what one would expect: High realizations of z induce losses and will therefore lower the equity-ratio. Other explanations, such as older firms being less productive and more equity financed are mostly controlled for by the fixed effect.

[15]If e exhibits lag-dependency, however, the parameter-estimate of β may be biased. In section 4.3 this issue is discussed in more detail using the German dataset. For the German data, we show the bias from lag-dependency to be negative, if present.

Although there are differences among subsample estimates these are insignificant, except for two estimates. Therefore, we conclude the model to be reasonably specified as a homogeneous panel.

Compared to the estimates of Caballero et al. (1995) the PFM-OLS estimates of the cost-elasticity are much closer to its neoclassical-theory value of 1. Moreover, the variation between the industry specific estimates is slightly smaller.

With the regression estimates it is now possible to construct a time series $\widehat{z}_{it}$ for each firm. The series is constructed using all 7147 observations.[16]

4.2.5 Investment behavior

4.2.5.1 Density and conditional expectations estimates

Since the structure of our investment model is highly nonlinear, and the functional forms of the profit function and distributions involved are not quite clear, a nonparametric estimation is most fruitful.[17]

Figure 4.1 shows the distribution of within transformed investment rates conditional on the capital-imbalance $\widehat{z}$. The distribution has been estimated with a normal-kernel estimator and a fixed window width of $0.12 \times n^{-\frac{1}{6}}$, both z and i have been standardized. The relationship is as expected: Firms with lower z invest more. More surprisingly and in line with the model the distribution of investment rates has two peaks for high mandated investments.

For firms with an unproductive capital stock, we find three peaks in the distribution, one where no disinvestment occurs, one with partial (unconstrained optimal) disinvestment and one peak for nearly full disinvestment,

[16] However, 3% of the observations were discarded as outliers with respect to the within transformed investment-rate. We do so by removing the top 0.5 % observations and the bottom 2.5 %.

[17] The non-parametric estimates presented in this section were generated in autumn 2000. At that time, computational effort of the variable window width estimator still played some role . This forced us to rely on less computational intense estimation techniques, such as k-nn estimators. At the time, the estimates presented in section 4.3 were generated (Spring 2003), this issue was of much lesser importance, as the computer system available then was about 10 times faster.

being again perfectly in line with the model. However, these findings are on the boundary of the support, where the estimator becomes less reliable.

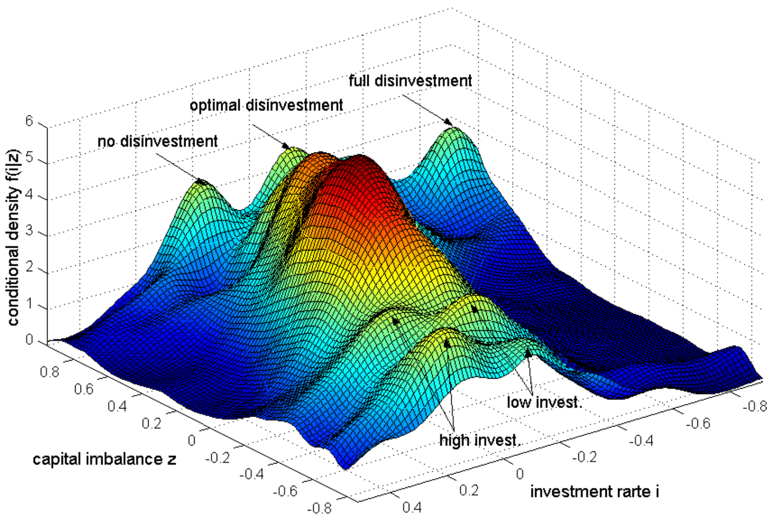

Figure 4.1: Density of the investment-rate distribution conditional on z

However the distributional findings are not completely supportive: Most firms do by far not remove their capital-imbalance completely in case they invest. One explanation for this could be time-to build constraints which cause investment to be spread over two accounting periods. Another explanation is that many of the firms in our sample are large firms. These are themselves aggregates of many establishments. Therefore, given z, the model presented above would yield an approximate investment rate of $\frac{n}{m}i(e,z)$ for a company with m independent establishments of which n establishments adjust. The distribution of n would then be given by a binomial distribution B with:[18]

$$P(n|z,e) = B(n|m, G(\Omega(e,z))$$

When the number m of establishments per firm is large enough we would expect to hardly ever see zero investment rates for larger z in our model.

Moreover, if heterogeneous capital goods indeed matter, aggregation over these goods will result in a downward bias of second order derivatives. (Goolsbee and Gross, 2000).

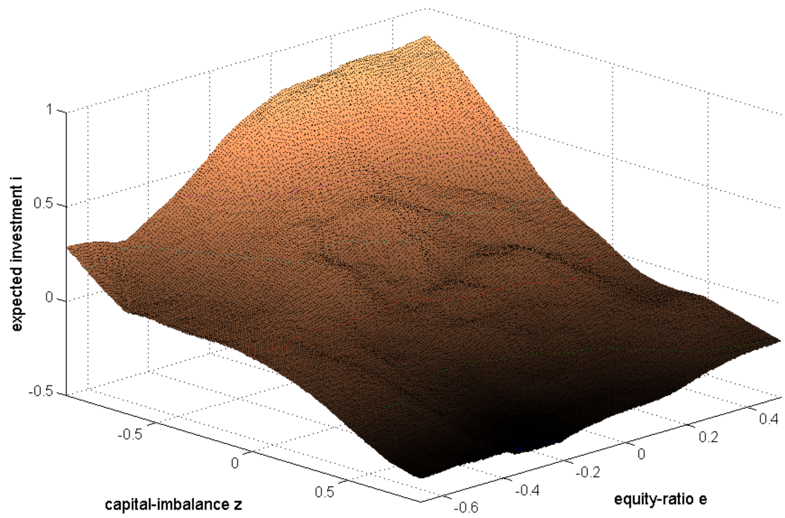

Figure 4.2: Expected investment-rate i conditional on e and z

Figure 4.2 presents the expected investment rates conditional on the log equity-to-capital ratio and mandated investment. For the estimation a 360-nearest neighborhood (k-NN), local linear, Epanechnikov-kernel estimator[19]

[18]Note that, investment decisions in heterogeneous capital goods as in Goolsbee and Gross (2000) or for different establishments or plants of one company would - strictly speaking - not be independent in our model, as investment decisions are linked through their effect on the balance sheet.

[19]This means, when estimating the expected investment rate $\mathbb{E}(i)$ at (e,z) we drop all,

has been used. The choice of $k = 360$ is approximately equivalent to twice the fixed window-width used for density estimation.[20] The window-width has been chosen by "eye-balling", but tends to slightly undersmooth the conditional expectation estimator on the center of the support. Especially for second order-derivative estimation, this problem becomes much more apparent. Therefore, when estimating second-order derivatives (see below), a combination of 720-nearest-neighborhood and fixed window-width (of $h = 4n^{-1/10}$) kernel is used. This effectively approximates variable window-width estimators, but is computationally much faster.

Figure 4.2 shows that the investment function is nonlinear in a manyfold sense: For a large equity-ratio, investment is a convex function of the capital-imbalance, highly indebted firms investment mostly only disinvest and for these firms investment is concave in fundamentals. In our model this is a result of the concavity of the earnings-function and the fixed costs. A higher equity-ratio raises investment, but only when fundamentals allow so.[21] Therefore, this results supports our pecking-order, fixed-adjustment-cost model. Moreover, for very low productivity equity-ratio first has to reach some threshold to influence investment, i.e. to stop sharp disinvestment. Again this is very much in line with our pecking-order, fixed-cost model. And as a last (but not really surprising) deviation from the linear model, the second order derivative of equity seems to be negative, at least for low z.

To better visualize the effect of ignoring the important interaction of equity and capital-imbalance and thus the non-linear structure for investment,

but the 360 data points that are nearest to (e, z) in Euklidean distance.

Then we estimate $\mathbb{E}(i)$ at (e, z) running a weighted least-squares regression, for which the weights are calculated using an Epachenikov-kernel function. See Pagan and Ullah (1999) for details.

[20] The optimal k for k-NN estimators and the optimal window-width h are linked by $k = nh^{4/6}$ in the two dimensional case (Pagan and Ullah, 1999, p. 91).

[21] Note that we have not included any long-run effect of equity in generating the capital imbalance estimates.

the generalized additive model

$$\mathbb{E}[i|(e, z)] = m_1(z) + m_2(e) \tag{4.12}$$

has also been estimated.

Figures 4.3 and 4.4 now present the estimates for the generalized additive model. As the identifying restriction in a generalized additive model is arbitrary, we employ different restrictions for figures 4.3 and 4.4: While in figure 4.3 $\mathbb{E}[m_2(e)] = 0$ was used, in figure 4.4 $\mathbb{E}[m_1(z)] = 0$ is chosen. This makes it easier to visualize the relative changes in the investment rate caused by changes in z and e.

Still there is some sign of non-linearity of investment in z. However, this is very minor, as in general the function is close to linear. Clearly this shows, how misleading it can be if one neglects (non-linear and) cross-effects. This bias becomes even worse for the estimate of $m_2(e)$ in figure 4.4:

For low equity-ratios there is *no* effect, whereas for high equity-ratios the effect is clearly positive. However, we have seen a clearly positive effect for low equity as soon as productivity is high enough. Moreover from figure 4.2, this effect is *decreasing* if equity rises, while in figure 4.4 it is *increasing* in e. This no-effect result, when equity is low, has appeared in earlier studies in the form of *apriori* as financially constrained considered firms reacting less on increases in cash flow (Kaplan and Zingales, 1997). Therefore, figure 4.4 sheds some new light on these results, too: It may well be that a misspecification bias drives this evidence that has been brought forward as an argument against financial constraints playing a prominent role in investment decisions.

The effect of the within transformation can be seen, when comparing figure 4.4 to figure 4.5 and 4.6, where we report the estimates without transformation. This allows to use the full dataset and not only firms with at least five observations, therefore, the full dataset has been used for the estimates in figure 4.5. For figure 4.6 the restricted dataset was used.

Interestingly, the effect seems to be U-shaped. This could be due to some underlying economic structure or to the unreliability of nonparametric esti-

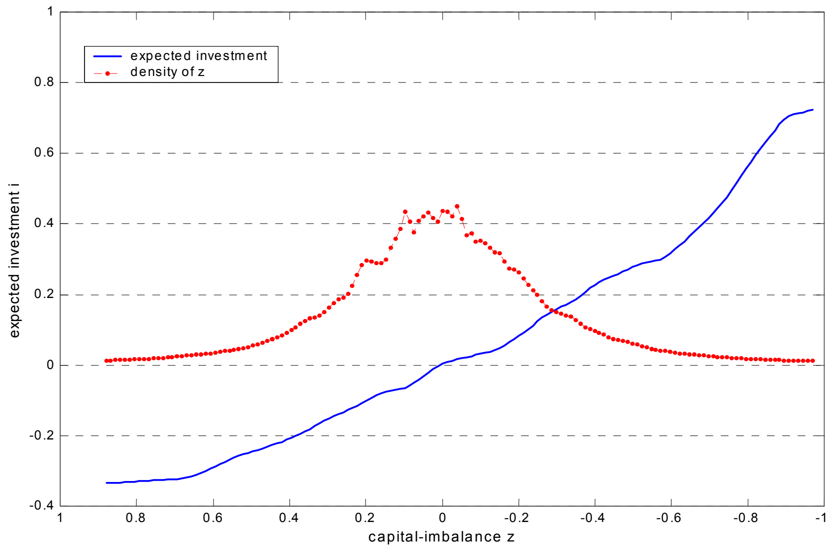

Figure 4.3: Generalized additive model: $m_1(z)$

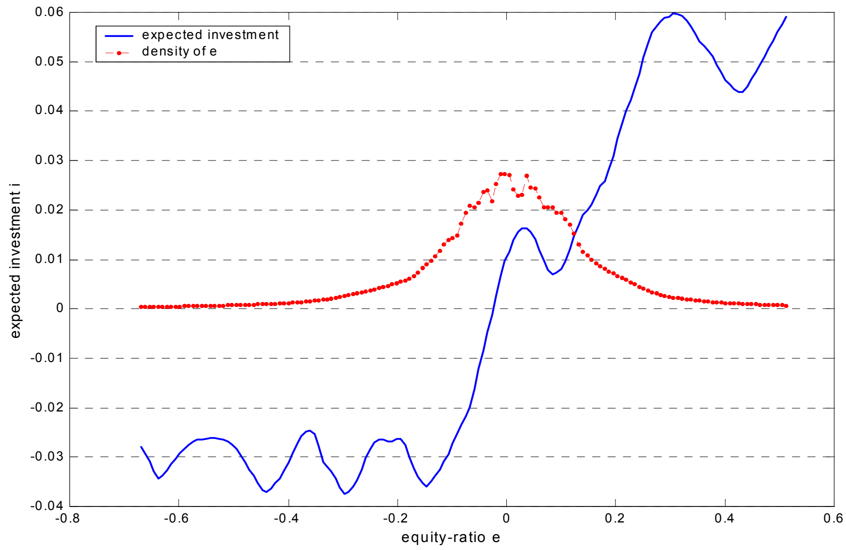

Figure 4.4: Generalized additive model: $m_2(e)$

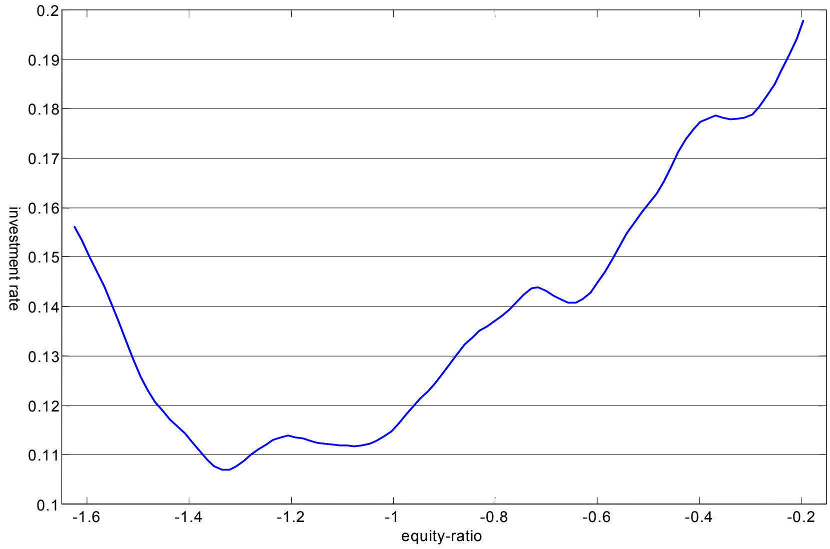

Figure 4.5: Genralized additive model, without transformation, full sample: $m_2(e)$

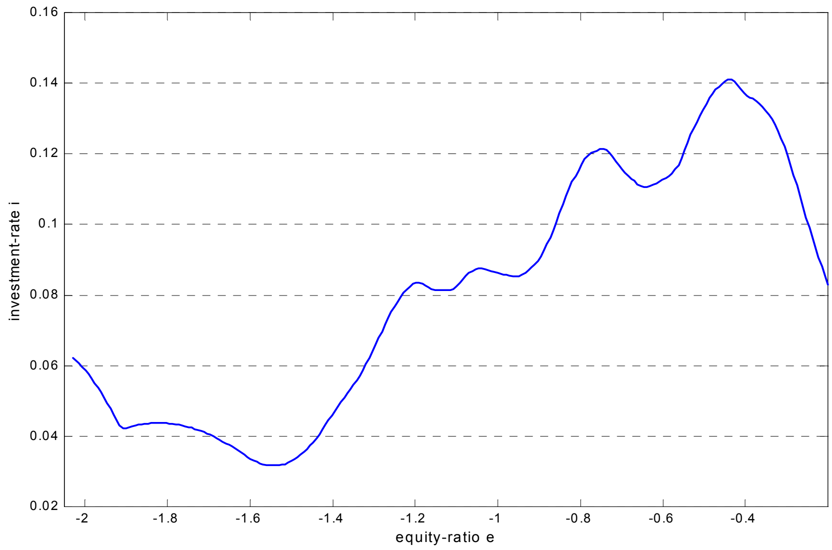

Figure 4.6: Generalized additive model, without transformation, restricted sample: $m_2(e)$

mates near the boundary of the support. If there is an economic explanation for this, it has to rely on firm- or industry-characteristics of the very indebted firms. A theoretical explanation for this will be brought forward in chapter 5.

4.2.5.2 Average derivative estimators

A major drawback of the nonparametric estimates are their wide confidence bounds. Thus, to draw more reliable conclusions, it is necessary to estimate average derivatives of $i(z,e) = \mathbb{E}(i|z,e)$ directly. This yields much closer confidence bounds because nonparametric average-derivative estimators converge with parametric rates of convergence (Rilstone, 1991). The estimates are reported in tables 4.4 and 4.5.

Several nonparametric estimators for the average derivative are available in our panel data setting (Ullah and Roy, 1998). E.g. for the local linear estimator, there are immediately two procedures to estimate the derivative at hand. The local linear estimator is basically a locally weighted version of an OLS estimator. Typically it is derived from a first order Taylor expansion around the point (z,e), at which the function is estimated, i.e.

$$i_{jt} = \mathbb{E}(i|z,e) + (z_{jt} - z)\, b_z\,(z,e) + (e_{jt} - e)\, b_e\,(z,e) + u_{jt} \quad . \tag{4.13}$$

Therefore, we can numerically generate $b^* := \partial_{(z,e)}\widehat{\mathbb{E}}(i|z,e)$ or alternatively take $b^{**} := \left(\widehat{b}_z, \widehat{b}_e\right)$ as their direct estimates. The weights are generated according to the kernel-function chosen, with $K = diag(K(z,e))$. When we denote a vector of ones by ι, the matrix of regressors by

$$X := [(z_{jt} - z) \;\; (e_{jt} - e)]_{t=1..T_j}^{j=1..N} \tag{4.14}$$

and the vector of observed investment-rates by $\overrightarrow{i}$, we obtain:

$$[\widehat{\mathbb{E}}(i|z,e) \;\; b^{**}(z,e)] = \beta^{OLS}(K^{1/2}\overrightarrow{i}, K^{1/2}\,[\iota \;\; X]). \tag{4.15}$$

Both estimators are asymptotically normally distributed, but have a different variance. Moreover, in small samples the bias is different and they perform differently. In most cases the numerical estimator b^* should be preferable (Ullah and Roy, 1998). However, a drawback of this estimator is that the variance of its averaged form is not yet known (Pagan and Ullah, 1999).

In our panel data setting, additionally the fixed effects estimator of Ullah and Roy (1998) is available.[22] This estimator b^{FE} is locally weighted OLS of:

$$\begin{aligned} i_{jt} - \bar{i}_{j.} - \bar{i}_{.t} + \bar{i}_{..} &= \left(z_{jt} - \overline{z}_{j.} - \overline{z}_{.t} + \overline{z}_{..}\right) b_z\left(z, e\right) \\ &\quad + \left(e_{jt} - \overline{e}_{j.} - \overline{e}_{.t} + \overline{e}_{..}\right) b_e\left(z, e\right) + u_{jt} - \overline{u}_{j.} - \overline{u}_{.t} + \overline{u}_{..} \end{aligned} \tag{4.16}$$

The weights are different to those used for the pooled estimator in our setting. For the pooled estimator we used the within-transformed data to measure the distance of the evaluation point and the observations, while the fixed effects estimator uses the original data. The advantage of this estimator is that assumption 4.1 is no longer needed to let the estimates be interpretable in a sensible manner. All we need to assume is the investment function to exhibit fixed idiosyncratic and time effects and to be otherwise homogenous among the firms in the original (not within transformed) quantities. As we will see below, results do not qualitatively depend on the estimator chosen.

The average derivative estimators are generated as the mean of the pointwise estimates. Average-derivatives over a subset of observations are calculated by using the conditional mean (conditional on the observation falling into the subsample) and not by re-estimating for the subsample.

The cross- and higher-order-derivatives are computed as numerical estimates of these quantities using (4.13). As for the first-order derivative numerical estimator the asymptotic variance is not yet known.

For the second-order-derivatives the undersmoothing of the nearest neighborhood estimator in the center of the distribution becomes a more apparent

[22] Note that we also use within-transformed data in estimating (4.13).

problem. Therefore, we generate the weights K in (4.15) as average of a 720 nearest-neighborhood Epanechnikov kernel and a fixed window-width Epanechnikov kernel. The fixed window width is calculated as $h = n^{-1/10}$. The implied window width d_k of the nearest neighborhood estimator varies, depending on the evaluation point x_i at which we estimate. We set $d_k(x_i)$ equal to the maximum distance (to x_i) of the 720 points of data being nearest to x_i. The weight (kernel) of a point of data x is hence calculated as

$$K(x|x_i) = \frac{1}{2}\left[K_1\left(\frac{x_i - x}{d_k(x_i)}\right) + K_2\left(\frac{x_i - x}{h}\right)\right].$$

This is computationally very efficient and gives a variable window-width, which is neither too small on the border nor on the center of the support, so that the estimator should not be too heavily under- or oversmoothed. Furthermore and within a certain range, window-width selection has been reported to be of a lesser issue for average derivative estimators, as they are generally not too sensitive to window-width selection,[23] as long as the averages are not dominated by a few outliers.

4.2.5.3 Average derivative estimates

The average derivative estimates for all four estimators are reported in table 4.4. Although the various estimators yield quantitatively slightly different estimates, they qualitatively do not differ: Both the equity-ratio and the capital-imbalance have a significant effect. Moreover and more surprisingly, the derivative with respect to the equity-ratio is about twice as large as the elasticity of the optimal stock of capital with respect to the equity-ratio. The difference is both economically and statistically significant. This also holds true if one requests higher levels of significance to account for the dependence of the short-run and long-run estimates.[24] Therefore, for the DTI-Database,

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[23] See Pagan and Ullah (1999).

[24] Taking the standard deviations as reported above and the asymptotic distributions as approximation, the 5% one-sided upper confidence bound for β is 0.1368, while the lower 5% one-sided confidence bound for $\overline{b}_e^{**}$ is 0.1425. If both estimators were independent, this

Table 4.4: Average first-order derivatives of the investment rate $i(e,z)$

Number of observations: $N = 6950$					std. deviation
derivative	$\overline{b}^{*}$	$\overline{b}^{**}$	$\overline{b}^{FE}$	$\overline{b}^{*}_{720,fix}$ [a]	for $\overline{b}^{**}$
$-\frac{\partial \widehat{i}}{\partial z}$	0.5057	0.5146	0.5341	0.5137	0.0068
$\frac{\partial \widehat{i}}{\partial e}$	0.1555	0.1588	0.1782	0.1450	0.0097

[a] estimated with a mixture of fixed window-width and 720-NN-Kernel.

Table 4.5: Average second-order derivatives of the investment rate $i(e,z)$

derivative	full Sample[a]	low z[b]	high e[c]
$\frac{\partial^2 i}{\partial z^2}$	0.2931	0.3838	0.3565
$\frac{\partial^2 i}{\partial e^2}$	0.1079	-0.106	-0.150
$\frac{\partial^2 i}{\partial e \partial z}$	0.1078	-0.123	0.2257
N	6922	2284	2353

[a] e and z within the 0.5% and 99.5% quantiles .

[b] e within the 0.5% and 99.5%, z within the 0.5% and 33% quantiles.

[c] z within the 0.5% and 99.5%, e within the 66% and 99.5% quantiles.

we can reject both the hypothesis H_0^0 and H_0^1, i.e. finance matters in both the short and the long-run but influences the frequency of adjustment much stronger, than it influences the optimal stock of capital.

Table 4.5 reports the higher-order derivative estimates and reveals some interesting results as well: We find a clear evidence for convexity in z for both the total sample average and the local averages over low z. Moreover and as predicted by the theoretical model, for low z the cross-derivative is clearly negative. Therefore, all empirical results are in line with the investment model that includes fixed adjustment cost and capital market imperfections.

Due to the slow rate of convergence of nonparametric multivariate density estimators, a meaningful test of forecasting performance of the generalized Markov-chain-model proposed in chapter 3.3. does not seem to be achievable with an unbalanced panel of only about 7000 observations over 13 years.

would of course equal an one-sided 0.25% test.

Therefore, a forecasting performance test remains open to further research. Maybe a series estimator for the investment function could help to overcome the need for very large data samples.

4.3 Evidence from a sample of German firms

So far from our analysis of the UK data the main claim of this thesis seems well supported. To provide additional empirical support of our hypothesis we will now analyze a German company accounting dataset, the "Bonner Stichprobe". Moreover, we will also address some econometric issues left out in the last section, i.e. especially we want to account for the lagged dependency of the equity-ratio. Additionally, we will complement the nonparametric analysis with a non-structural parametric one and also will present results for the long-run estimates from a number of different estimators. Furthermore, the analysis of the German data complements the analysis of the UK one, as we use a differently constructed measure of fundamental incentives.

4.3.1 Brief description of the data

The "Bonner Stichprobe" is a sample of annual company accounts of German companies. Most of these companies are large listed stock companies. The data covers the time-period 1960 to 1997. The panel is unbalanced and contains 694 companies (observational units) and 18943 observations in total. Thus, the average time in the sample is 28.7 years.

The data bank includes complete profit- and loss-statements as well as annual accounting data. Moreover, for the very most company-years data on average wages and salaries as well as the number of employees is reported.

Unfortunately, after firms which are holdings, multi-corporate companies, or business trusts are removed from the sample, sample-size falls substantially. Additionally, we have to drop a few firm-years for which data seemed inconsistent with usual accounting standards (e.g. negative depreciation, very high appreciation). This leaves us with a sample of about 10000 observations. Although—as Goolsbee and Gross (2000) report—assuming a homogeneous capital good biases the estimated investment function towards a linear specification, it is necessary to do so in this section, since many firms do not report stock and depreciation of land and buildings and machinery separately.

If removing a single observation (due to data inconsistency) splits a firm-series in two parts which are long enough to be sensibly analyzed, the second part of the series is identified as a different firm. If the missing observation separated the series in a very short and a longer one, the short one was completely removed, i.e. only firms with five or more consecutive observations remain in the sample. Additionally, single observations were removed, if the investment rate differed from the mean by 5 times the standard deviation (removing 11 observations), differed from the firm specific log-equity ratio by 4 standard deviations (39 observations), or if the turnover-change differed from the mean by 6-times the standard deviation (14 observations). Moreover, firms were excluded, if their average wage-share or proxied average cost-of-capital share[25] exceeded 70% (removing 111 observations). This leaves us with 449 firms and a total of 9731 observations, making an average of 21.67 accounting years per firm.

The stock of capital series has been generated using the perpetual inventory method, investment, wages and profit were deflated using the producer-price index for investment goods.

Investment-ratios exhibit moderate excess skewness and kurtosis, reflecting the fact, that most firms in the sample are aggregates of many plants. However, investment is typically a highly concentrated activity only at the plant level. Using the widely employed cut-off value of 30% for the definition of an investment spike, we find that 17.4% of all firm years exhibit an investment-spike, accounting for 36.1 % of all investment.

As the time-dimension compared to the number of observational units is only moderate and the sample is unbalanced, the Breitung-Meyer (1994)

[25] See section 4.3.3.

Table 4.6: Descriptive Statistics "Bonner-Stichprobe"

Variable	Obs.	Mean	Std. Dev.	Min	Max
investment-rate	9770	0.210	0.120	-0.140	0.8254
capital	9770	164.97	491.50	.03631	6893.2
equity-ratio	9770	.40287	.1395	.01561	0.9371
real wage	9770	14.803	4.969	1.7556	36.908
total value added (turnover)	9770	464.70	1415.4	0	20584
No. Employees	9770	6009.0	17813	4	215800

Table 4.7: Breitung-Meyer Unit-Root Tests

Variable	estim. root	sign. of estim. root ≥ 1
log No. Employees	1.034	1
log turnover	1.010	1
log capital	1.008	1
log equity-ratio	0.965	0
$\hat{\Pi}$	1.010	1
x	0.965	0

test is employed to test for unit-roots. The hypothesis of a unit root cannot be rejected for capital, revenues, the number of employees, and for the measure of capital-productivity $\hat{\Pi}$, which is derived below. We can however reject the unit-root hypothesis for the equity-ratio. Also for the cointegration regression—specified below—we can reject the null of a unit-root in the cointegration error (and thus the null of no cointegration).

4.3.2 Empirical model

Again, for the empirical model assume that revenues Y are generated by capital K and labor L according to a Cobb-Douglas function, which is subject to decreasing returns to scale,[26] i.e. $\varepsilon_i^L + \varepsilon_i^K < 1$ and

$$Y = \Xi L^{\varepsilon_i^L} K^{\varepsilon_i^K}. \tag{4.17}$$

[26]The assumption of decreasing returns to scale is well supported by the data, for every firm in the sample $\varepsilon_i^L + \varepsilon_i^K < 1$ holds.

Furthermore, note that the parameters ε_i^L and ε_i^K will be firm-specific.

Ξ is the parameter characterizing productivity. If labor is perfectly flexible adjustable and w is the wage, we obtain

$$wL = \varepsilon_i^L Y \tag{4.18}$$

and thus

$$Y = \left[\Xi \left(\frac{\varepsilon_i^L}{w} \right)^{\varepsilon_i^L} \right]^{\frac{1}{1-\varepsilon_i^L}} K^{\frac{\varepsilon_i^K}{1-\varepsilon_i^L}}. \tag{4.19}$$

Take the equity-ratio as not influenced by the level of the capital stock, then the long-run optimal stock of capital k^* is determined by the first order condition that equates marginal revenues and marginal costs mc.[27] The optimal (log) stock of capital is thus driven by marginal costs and total capital-productivity Π,

$$\Pi_{it} := \frac{\ln(\Xi_{it}) + \varepsilon_i^L \left(\ln\left(\varepsilon_i^L\right) - \ln(w_{it}) \right)}{1 - \varepsilon_i^L}. \tag{4.20}$$

This gives for the optimal capital

$$k_{it}^* = \frac{\left(1 - \varepsilon_i^L\right)}{1 - \left(\varepsilon_i^L + \varepsilon_i^K\right)} \left[\Pi_{it} - \ln(mc_{it}) \right]. \tag{4.21}$$

Yet, this equation has still to be operationalised. Assume, productivity Ξ is random, e.g. it could follow a geometrical Brownian motion, then Π is in principle computable from estimates of (4.17). And if marginal costs were known, k_{it}^* could be calculated. However, true marginal cost mc_{it} of capital for each firm i at time t are not directly observable. Nevertheless, under our assumptions from the previous section, the gap between optimal and realized stock of capital can still be identified. These assumption are especially:

[27] If larger firms would have different access to capital (e.g. through an equity channel) then of course mc would need to be corrected for this.
Suppose $mc = r_t + \mu_i + \theta k$, then the correctly measured optimal stock of capital k^{**} would be $k_{it}^{**} = \left(1 + \theta \frac{\left(1-\varepsilon_i^L\right)}{1-\left(\varepsilon_i^L+\varepsilon_i^K\right)}\right)^{-1} k_{it}^*$.

1. k_{it} and k_{it}^* are cointegrated.[28]

2. Log-marginal costs split up in an aggregate component r_t (e.g. the risk-free interest rate), a firm specific component μ_i (e.g. due to firm-specific risk) and the influence of the equity-ratio $\gamma_1 e_{it}$ - which is common across firms.

Thus, we can estimate r_t, μ_i, γ_1 and γ_2 from a panel cointegration regression between k and k^*, using preliminary obtained estimated $\hat{\varepsilon}_i^L, \hat{\varepsilon}_i^K$ and $\hat{\Pi}_{it}$ and

$$k_{it}^* = \frac{1 - \hat{\varepsilon}_i^L}{1 - \left(\hat{\varepsilon}_i^L + \hat{\varepsilon}_i^K\right)} \left[\gamma_1 \hat{\Pi}_{it} - (r_t + \mu_i) + \gamma_2 e_{it}\right]. \tag{4.22}$$

Note, that this specification explicitly allows for a certain degree of technological heterogeneity among firms.

Once estimates of the optimal stock of capital are obtained, these can be used in estimating the investment function. As argued before, the functional form of investment is not clear a priori and highly depends on the distribution of adjustment cost shocks,[29] the technology, the structure of financial markets,[30] etc.. Therefore, again a flexible estimation technique such as a nonparametric local-linear regression is most appropriate. Yet, we will only present our estimates of average derivatives, as these are the "parameters" of main interest. Additionally we will also estimate the polynomial expression given below for the investment-rate i. However, to control for firm specific adjustment costs and firm specific access to capital markets (determining the long-run equity-ratio), only deviations from firm-specific means $\bar{x}_{j.}$ are considered:

$$i_{jt} - \bar{\imath}_{j.} = \sum_{p=0}^{P} \sum_{q=0}^{Q} a_{p,q} \left(x_{jt} - \bar{x}_{j.}\right)^p \left(e_{jt} - \bar{e}_{j.}\right)^q + v_{jt}. \tag{4.23}$$

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[28] Of course this assumption can be tested, and we actually can reject the unit-root hypothesis for the cointegration error x. (See table 2)

[29] See Caballero and Engel (1999) for a detailed discussion.

[30] See e.g. Gomes (2001).

To compute capital imbalances x_{jt}, we take the optimal capital stock of period t relative to the reported stock of capital at the end of period $t-1$.

Given estimates for ε_i^L and ε_i^K, there are two estimators at hand for Π. One is the direct estimate, taking Ξ_{it} as the residuals of (4.17) and using these together with the estimate for ε_i^L in (4.20). However, Π can also be estimated indirectly using the optimality condition for labor, as Cooper and Haltiwanger (2002) do. This estimate is

$$\widetilde{\Pi}_{it}^{ind} := \ln\left(w_{it}L_{it}\right) - \frac{\hat{\varepsilon}_i^K}{1-\hat{\varepsilon}_i^L} k_{it}. \tag{4.24}$$

Just as Cooper and Haltiwanger state for their data, the indirect estimator is less volatile. However, this estimator relies on the hypothesis that labor could be perfectly flexibly adjusted, which is likely to be wrong for the German labor-market.

Nevertheless, this estimator will—even due to the inflexibility—perform better than the residual based one, if there is a transitory as well as a persistent component in the shocks to productivity. E.g. $\ln\left(\Xi_{it}\right)$ could be an ARIMA process affected by a firm-specific fixed effect, a random walk component (driven by aggregate and individual shocks) and a transitory component υ_{it}. This transitory component should of course not matter for any employment decisions, if capital and labor are not immediately productive and adjustment is not costless.[31]

The direct estimator would include υ_{it} as an error term, while the indirect one would correctly measure the productivity expected by the firm (if managers can observe υ_{it}). For transitory shocks to the real wages ($w_{it} = \bar{w}_{i.} + \bar{w}_{.t} + \omega_{it}$) the same line of argument applies. As wages in the long-run cannot differ across firms unless they do so due to firm specific reasons, we can easily identify the transitory shocks as ω_{it} and remove them from the data. However, this only holds true if working conditions remain relatively constant across firms over time.

[31] See Barnett and Sakellaris (1999) for an analoguous argument on for Tobin's q.

Nevertheless, it could and should in general be assessed by a technique similar to the one developed by Erickson and Whited (2001) which estimator has the lower measurement error. Hence, which estimator is superior is in the end an empirical question. So this issue is analogous to the problem of the quality of different measures of Tobin's-q. However, in the non-linear setting applied here, the application of Erickson and Whited's method is not straightforward. Moreover, the amount of data required for this procedure is quite large. Thus, a more heuristic approach is taken: If the estimator $\hat{\Pi}$ has a low measurement error, first of all γ_1 should be close to 1, and second the estimate for x should be informative for the investment regression. In the empirical specification employed, it turns out, that the direct estimate $\widetilde{\Pi}_{it}^{dir}$ is more informative for the investment regression, whereas the indirect one gives more realistic estimates for γ_1. However, simply taking $\hat{\Pi}_{it} := \frac{1}{2}\left(\widetilde{\Pi}_{it}^{dir} + \widetilde{\Pi}_{it}^{ind}\right)$ performs very close to the respective superior estimate for both the cointegration- and the investment-regression.

4.3.3 Long-run optimal stock of capital

Inspecting the firm-specific average wage-shares, $\frac{1}{T}\sum_t \frac{w_{it}L_{it}}{Y_{it}}$, or the firm specific mean fraction of revenues which compensates for depreciation, $\frac{1}{T}\sum_t \frac{\delta_{it}K_{it}}{Y_{it}}$, substantial differences can be found across firms. Figure 4.7 presents a kernel density estimate of the time-average wage-shares of firms.

Therefore, we can expect firms to be heterogeneous with respect to their technology. Hence, using any pooled or panel estimator on

$$\ln(Y_{it}) = \varepsilon_i^L \ln(L_{it}) + \varepsilon_i^K \ln(K_{it}) + \ln(\Xi_{it}) \tag{4.25}$$

would yield biased estimates of the productivity shock. Estimating the productivity for each firm is again not really feasible due to data-limitations, as in any case a dynamic estimator[32] and therefore more observations would

[32] Especially note that estimates just using the levels will be inconsistent. Even taking first differences will still not remove all correlation between $(\Delta k_t, \Delta l_t)$ and the productivity

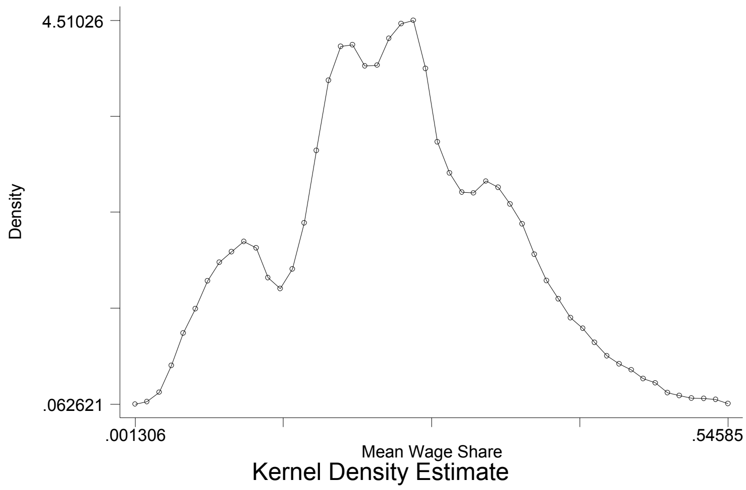

Figure 4.7: Firm–specific average wage shares

be needed. However the expenditure shares are still valid estimates for ε_i^L and ε_i^K.[33] To compute the expenditure-shares for capital, the firm average-depreciation-rates plus a time constant real interest-rate of 3% was used.

Factor productivity $\hat{\Pi}$ is then estimated as described above. The coin-

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innovation $\Delta \ln (\Xi)_t$. This however means that one would need at least lagged differences as instruments, meaning a loss of 2 observations per firm, and thus limiting the analysis to firms with at least 5 observations (leaving no degree of freedom in the equations for some firms then).

[33] To see this, note that L ,Y and K are endogenuous unit-root processes, all driven in the long-run by Ξ. Thus (8) gives—loosely speaking—a cointegration relation. However, then OLS for each firm on this equation is super-consistent, but collapses to $\varepsilon_i^L = \frac{1}{T_i} \sum \frac{w_{it} L_{it}}{Y_{it}}$ if we can write the estimation equation with a "heteroscedastic" error term

$$w_{it} L_{it} = \varepsilon_i^L Y_{it} + Y_{it} v_{it}.$$

If the error were to come in multiplicatively, the geometric mean were apprpriate. Note that using the geometric mean instead, does not alter the results significantly.

Still, Caggese (2003, p. 10) has argued that this procedure will lead to biased results, if labor and capital employment decisions are constrained by some third variable, e.g. by financial constraints. In our case r might depend on the finacial conditions. Nevertheless, as we find a only very minor influence of finance on the long-run stock of capital, the bias can be expected to matter only marginally. More formally, our estimates are consistent under H_0^0, the hypothesis we test.

tegration relation (4.22) can then be estimated using the Panel-Dynamic-OLS-Estimator (PDOLS) following Kao and Chiang (2000), the Panel-Full-Modified-OLS-Estimator (PFM-OLS) of Phillips and Moon (1999) or OLS controlling for fixed effects. However, allowing for heterogeneous short-run dynamics in the PDOLS regression would mean the inclusion of at least 2300 parameters (for 2 lags and 2 leads of first difference). Therefore, in estimating the PDOLS estimator, a homogeneous short-run dynamics is assumed in all but two models (PDOLS-Ind), in which industry specific[34] short-run-dynamics is allowed for. All regressions control for fixed time and firm-effects. Table 4.8 presents the main results.

Table 4.8: Single-Stage Cointegration regressions

Estimator and Model	PDOLS-1	PDOLS-2	PDOLS-Ind-2	PDOLS-Ind-1
$\gamma_1 \left(\hat{\Pi}\right)$	0.7626***	0.7404***	0.7442***	0.7717***
γ_2 (log equity-ratio)	-0.0386	-0.0275	-0.0144	-0.0175
No. of Parameters	39	43	378	535
No. of Observations	6612	7447	7447	6612^a
No. of Firms	383	416	383	412

Estimator and Model	PFM-OLS	PFM-OLS	OLS	FM-std. err. I(1)
$\gamma_1 \left(\hat{\Pi}\right)$	0.6938***	0.6009***	0.6442	0.02768
γ_2 (log equity-ratio)	-0.0345	-0.0484*(e_{t-1})	0.03813	0.02544
No. of Parameters	39	39	39	std. err. I(0)
No. of Observations	9289	8823	9731	γ_1 .02802
No. of Firms	442	442	442	γ_2 .02763

***/**/* indicate significance at the 1/5/10% level

Model PDOLS-(Ind-)1 includes 3 lags and leads of $\Delta\Pi$, whereas model PDOLS-(Ind-)2 only includes 2 lags and leads, but of both Δe and $\Delta\Pi$. Standard errors are calculated on the basis of the FM-Estimator, using the average number of observations per firm. Standard errors are generated for

[34]The industry variable provided in the Bonn Database has been used for classification. This variable splits up the database in 52 different industries. Note however, that this variable does not coincide with SIC.

both, the case where the regressors are I(1) and the case where the regressors are I(0). Significance is indicated on the I(1) basis.

A few remarks on these results are worthwhile. First of all and although γ_1 is significant in all estimations and the estimates are reasonably large, it is clearly smaller than 1. This means, capital productivity seems to be either measured with a (stochastic) error or to be simply deterministically overestimated. Especially in the presence of fixed adjustment costs, we would rather expect the dynamically optimal stock of capital to exceed the static one, i.e. $\gamma_1 \geq 1$. Nevertheless, $\gamma_1 < 1$ does not necessarily mean that there is a severe bias in our later fundamental investment incentives estimate. It could well be a fixed percentage of revenues, that has to be attributed to something not modelled, which deterministically drives up the productivity measure. Another explanation could be that wages endogenously react to increases in productivity in the long-run—a fact, that is also not modelled. Again if a fixed fraction of the productivity increase runs into wage-increases, the estimates should not be severely biased. At any rate, the estimates for γ_1 are well in line with the estimates Caballero et al. (1995) obtained for their cost-of capital proxy.[35]

For the estimate of γ_2 evidence is mixed assuming the log equity-ratio is I(1) . However, the unit-root test clearly rejects this hypothesis. Using a formula analogous to the general one provided in Phillips (1995)[36] standard errors get slightly larger (the I(0) estimates are of course not super-consistent). Then, hypothesis H_0^0, which means the equity-ratio has no influence on the optimal-stock of capital, cannot be rejected. In any case, all except the OLS-estimate have a negative sign, saying higher equity ratios lead to lower optimal stocks of capital, which is contradictory to most of the earlier empirical financing-constraints literature. Also this seems inconsistent with a "wealth effect on the cost-of-capital" explanation as e.g. in the theoretical macro-model of Bernanke et al. (1998). However, still the regression results

[35] However, if adjustment costs strongly dampen the variation of capital, it is well known, that our estimator will underestimate γ_1 by construction (Caballero, 1997, p. 8).

[36] See equation (14), Phillips (1995, p. 1038).

have to be interpreted with care, as the equity-ratio may be endogenous. Fixed effects of different "baseline" access to capital markets are accounted for by controlling for firm-specific effects. The PDOLS and the PFMOLS account for the endogeneity of the equity-ratio due to shocks to the stock of capital. If e.g. larger firms have better access to equity markets and have reasons of holding larger amounts of equity, which are not related to costs, our estimators control for this. Firms could e.g. have an incentive to increase their equity holdings, if a larger stock of equity would yield more flexibility with their investment decisions. This could economicly explain the difference between FM and OLS estimate.

However, if there is lag-dependency in the equity-ratio (for which there seems to be evidence), or if the contemporaneous shocks to equity and capital are correlated, the PFM-OLS estimator $\hat{\gamma}_2^+$ is likely to be biased. Yet, $\hat{\gamma}_1^+$ remains asymptotically unbiased in all these cases. To remove the contemporaneous correlation, one can replace e_t by e_{t-1} and argue that beginning of period liquidity determines managerial discount factors. The resulting estimates are reported in column PFM-OLS (e_{t-1}). The estimate for γ_2 becomes smaller and is now weakly significant. However, if there is lag dependency, still the estimator remains biased.

Since γ_1 can be estimated consistently in any case, there is another way to obtain estimates of γ_2. If k and k^* are indeed cointegrated, $k_{it} - \gamma_1 \Pi_{it} - r_t - \mu_i$ can be expressed as

$$\hat{x}_{it} := k_{it} - \hat{\gamma}_1^+ \hat{\Pi}_{it} + \hat{r}_t + \hat{\mu}_i = \gamma_2' e_{it} + \gamma_2'' e_{it-1} + C^* (L) \xi_{it}, \tag{4.26}$$

with a moving-average error-term $C^* (L) \xi_{it}$ on the right hand side and a the stationary cointegration error on the left hand. Depending on, whether the beginning-of-period equity-ratio or the end-of-period (respectively during period) equity determines the managerial discount rate, γ_2' or γ_2'' is zero. However, the equity-ratio depends of course on its previous realization and past and current capital imbalances, so that we have as a second equation

$$e_{it} = \rho e_{it-1} + \eta_1 \hat{x}_{it} + \eta_2 \hat{x}_{it-1} + \nu_{it}. \tag{4.27}$$

If now $\eta_1 = \gamma_2'' = 0$ and

$$\forall j \geq 0 : cov\left(\xi_{it-j}, \nu_{it}\right) = 0, \tag{COV}$$

or $\gamma_2' = 0$ and

$$\forall j \geq 0 : cov\left(\xi_{it-j}, \nu_{it-1}\right) = 0, \tag{COV*}$$

the parameters in (4.27) can be consistently estimated recursively as all regressors are predetermined then.[37] In a second stage (4.26) can be estimated, using the fixed-effects OLS-residual $\hat{\nu}_{it}$ as an instrument for e_{it}.

Since we would rather assume current capital imbalances to influence the current equity ratio, but not the other way round, models with $\gamma_2' = 0$ are the preferred ones, yet this comes at the price that the assumption (COV*) is more restrictive: If x measures fundamental investment incentives imperfectly and current residual changes in equity reflect productivity, assumption (COV*) will be wrong and our estimates will be biased upwards.

Hence, table 4.9 presents the two-stage estimates for both models. Again, we cannot reject hypothesis H_0^0. However, the estimated coefficient for equity becomes larger. For estimating the investment-function, we use the PDOLS-Ind-1 model with $\gamma_2' = 0$. Since the resulting estimates for the influence of equity are among the larger ones, this makes our test of the frequency effect,

[37]Strictly speaking, this is only true if $T \to \infty$, as we use fixed effects OLS. Therefore in small samples, our estimates are biased. However, we are mainly interested in generating an instrument that is orthogonal to the within transformed variables, but contains information on e_t. Hence, one should not interprete the estimates of (4.27) structurally.

To avoid this probelm at least for the estimation of (4.27) we additionally estimate a IV-regression of this equation in first differences, in which Δe_{it-1} is instrumented by e_{it-2} and Δe_{it-2}. Yet, we only obtain $\Delta \nu_{it}$ as error-term, which may be correlated with ξ_{it} under assumption (COV) as ξ_{it} and ν_{it-1} may be correlated. Hence, we use $\Delta \nu_{t+1}$ as instrument for e_t. The results are reported under IV-PDOLS, but are not significantly different from the ones obtained by fixed effects OLS. Nevertheless, notice that for (4.26) the small sample bias only vanishes if one assumes $\forall s, t : cov\left(\xi_{is}, \nu_{it}\right) = 0$.

i.e. the test of H_1^0, more conservative.

Now, how do these estimates correspond to an investment-function? In any steady state, $\frac{I}{K} = const$ needs to hold. Suppose for simplicity investment to be a linear function in the capital imbalance and the log-equity-ratio. Of course, what we measure is not k^* itself, but the steady-state stock of capital k^S. Define $i := i(\hat{x}, e)$. As by construction $\frac{\partial \hat{x}}{\partial e} = -\frac{\partial k^S}{\partial e}$, the implicit function theorem yields

$$\frac{\partial i\left(\hat{x}^S, e\right)}{\partial \hat{x}} \cdot d\hat{x} + \frac{\partial i\left(\hat{x}^S, e\right)}{\partial e} \cdot de = 0 \Rightarrow \frac{\partial k^S}{\partial e} = \frac{\frac{\partial i\left(\hat{x}^S, e\right)}{\partial e}}{\frac{\partial i\left(\hat{x}^S, e\right)}{\partial \hat{x}}}. \tag{4.28}$$

Of course, if investment would be linear in $\hat{x}$ and equity e, $\frac{\partial i}{\partial e}$ and $\frac{\partial i}{\partial \hat{x}}$ were constants and the average derivatives later estimated would equal these constants. As we will see, the estimated adjustment process is quite slow, with $\frac{\partial i}{\partial x} \approx 0.14$.[38] Therefore, if the linear specification was correct, from our long-run analysis, $\frac{\partial i}{\partial e}$ can be expected to be very small.

4.3.4 Investment behavior

4.3.4.1 Parametric Analysis

To analyze the investment decisions, with our German data "mandated investment" is generated as the difference between log optimal capital at time t and end-of-period stock of capital in period $t-1$. To construct

$$x_{it} := k_{it}^* - k_{it-1} = \hat{\gamma}_1^+ \hat{\Pi}_{it} + \hat{\gamma}_2^{IV} e_{it-1} - (\hat{r}_t + \hat{\mu}_i) - k_{it-1}, \tag{4.29}$$

the estimates of $\hat{\gamma}_1^+$, $\hat{\gamma}_2^{IV}$ from PDOLS-Ind-1 are used. This specification has one of the largest influences of equity for the long-run regression. This makes the test of H_0^1 more conservative. Fixed effects are then removed as in (4.23). This controls for inter-firm differences in the optimal capital imbalance, i.e. different target values for the capital employed. Yet, it is

[38] This estimate implies a half-life of 4.27 years of technological shocks.

Table 4.9: Two-Stage Cointegration regressions

Model	$\gamma_2'' = \eta_1 = 0$	$\gamma_2' = 0$
Estimator.	PFM-OLS	PFM-OLS
γ_1 [a]	0.6938***	0.6009***
γ_2	-0.0118	0.0187
η_1	0 (assumed)	-0.0177
η_2	0.0119	0.0285**
ρ	0.8057***	0.8058***
No. of Observ.	9364	8897
Estimator.	PDOLS-Ind-1	PDOLS-Ind-1
γ_1 [a]	0.7717***	0.7714***
γ_2	-0.0074	0.0310
η_1	0 (assumed)	-0.0067
η_2	0.0107	0.0164
ρ	0.8056***	0.8056***
N	8897	6720
Estimator	PDOLS-2	PDOLS-2
γ_1 [a]	0.7404***	0.7404***
γ_2	-0.0091	0.0287
η_1	0	-0.0086
η_2	0.1121	0.0184
ρ	0.8056***	0.8056***
N	6720	6720
Estimator	IV-PDOLS-Ind-2	IV-PDOLS-Ind-2
γ_1 [a]	0.7442***	0.7442***
γ_2	0.0475	-0.0311
η_1	0 (assumed)	-0.0176
η_2	0.0058	0.0068
ρ	0.5436***	0.5458***

***/**/* indicate significance at the 1/5/10% level,

[a] Std. Err. from first stage PFM-OLS is 0.02833

not obvious how to estimate $\hat{r}_t$. The coefficients of time-dummies used in the cointegration regression would also pick up the state of aggregate mandated investment (which is to the most extent driven by productivity). However, aggregate mandated investment should not be subtracted from the individual mandated investment as both together determine the actual investment of a firm. Hence, we project the series of the time-specific effects obtained from the cointegration-regression on a series of real-interest rates and take these projections as estimate of $\hat{r}_t$.[39] The equity-ratio used in the regressions is the equity-ratio taken from the opening balances. Table 4.10 presents the regression-results.[40]

Essentially, the approach taken in equation (4.3) generalizes the error-correction model

$$\Delta k_{it} = \alpha \left(x_i^* - x_{it} \right) + \sum_{j=1}^{L} \gamma_j \left(\Delta \chi_{it-j} \right) + \phi_{it}$$

in which $\chi := \begin{pmatrix} k & \Pi & e \end{pmatrix}^T$. The generalization is derived as α is taken to be a function (here approximated by a polynomial) of (e, x). Neglecting the short run dynamics, we obtain

$$i_{jt} = \left[\alpha_0 + \sum_{j=1}^{p} \sum_{k=1}^{q} \alpha_{jk} \left(x_{it} - \bar{x}_{i.} \right)^j \left(e_{it} - \bar{e}_{i.} \right)^k \right] \left(\left(x_i^* - \bar{x}_{i.} \right) - \left(x_{it} - \bar{x}_{i.} \right) \right) + \phi_{it}.$$

The capital imbalance x^* that is chosen upon investment is typically different from the average $\bar{x}_{i.}$ since depreciation deterministically increases the amount of mandated investment between adjustments.

The parametric estimates show only a moderate degree convexity of the investment function with respect to the capital imbalance. And so the aver-

[39] The correlation between the real-interst-rates and the time-specific effects is quite low. this reflects the fact, that the aggregate (average) capital-imbalance is mainly driven by productivity respectively demand-shocks, which vary more than the real interest-rate.

[40] To preclude that our results are driven by extreme observations of mandated investment, we remove all observations from the sample which deviate by more than 4 standard deviations from the firm-specific average in the capital-imbalance measure.

Table 4.10: Short-Run Parametric Estimates

	Model 1		Model 2	
Variable	Coefficient	(Std. Err.)	Coefficient	(Std. Err.)
x	0.1495***	0.0058	0.1504***	0.0042
x^2	0.0650***	0.0112	0.0662***	0.0111
x^3	-0.0416*	0.0231	-0.0429***	0.0073
x^4	-0.0415***	0.0113	-0.0414**	0.0110
x^5	-0.0204	0.0162	–	–
e	0.0246*	0.0091	0.0162***	0.0045
e^2	0.0666**	0.0258	0.0329**	0.0132
e^3	-0.1430	0.0796	–	–
e^4	-0.0792*	0.0592	–	–
e^5	0.2718	0.1274	–	–
xe	0.0358***	0.0130	0.0347***	0.0122
$(xe)^2$	-0.0630	0.0439	-0.0840**	0.0398
xe^2	0.0152	0.0288	–	–
x^2e	0.0240	0.0216	–	–
$const$	-0.0060***	0.0014	-0.0052***	0.0013
Adj. R^2	0.1977	–	0.1973	–
No. Obs.	8973	–	8973	–

***/**/* significant at the 1/5/10% level

age second order derivative $\frac{\partial^2 i}{\partial x^2}$ equals only 0.126.[41] Moreover, the investment function becomes concave when x is about as large as one standard deviation. However, this should not be taken at face value as evidence against the fixed adjustment cost model. It rather reflects the fact, that most companies in the sample are multi-establishment / multi-plant firms, so their individual investment function rather equals an average over many investment functions of different plants with mean capital imbalance x. Due to this fact—and as e.g. Whited (2002) or Goolsbee and Gross (2000) argue—the observed investment function becomes less curved.

In fact it is the rather stark effect of equity, that would be at odds with a convex cost model, where there would be no frequency effect of equity

[41] Average parametric derivatives are calculated by differencing the estimated function (Model 2) and then averaging over the observation-wise calculated derivatives.

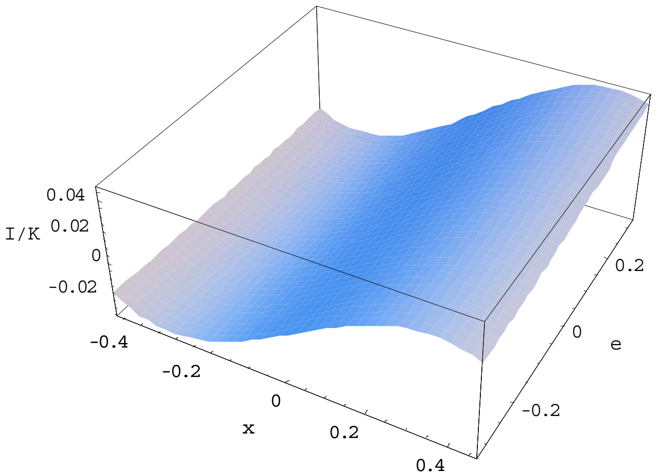

Figure 4.8: Investment function, e and x between 1.5 std. errors

on investment. This is also reflected by an important interaction between fundamental capital imbalance and the financial variable. The fundamental influence is the strongest one and explains most of the (explained) variation. Moreover, the adjusted R^2 is notably large for an investment regression, pointing towards a reasonable quality of $\hat{x}$ in measuring investment incentives.[42] Moreover, as the employed estimation procedure is close to the one of Cooper and Haltiwanger (2002), we might interpret the larger R^2 as additional evidence for substantial technological heterogeneity (which is one of the major differences between this paper and their one). Figure 4.8 plots the shape of the estimated investment function for 1.5 standard deviations around the means of x and e.

In Table 4.11 average derivatives for the parametric model are reported. The results are in line with the frequency-effect interpretation of short term influences of equity on investment introduced in Chapter 3: Equity has a

[42]Typical R^2 statistics in most (homogeneous) investment regressions (using q or some other estimator for productivity) range in between 5 and 10%. See for instance Cooper and Haltiwanger (2002) or Barnett and Sakellaris (1999).

much larger effect if there are strong fundamental investment incentives anyway. Interestingly, the Kaplan and Zingales (1997) result, that apriorily financially constrained firms are less sensitive to changes in liquidity is replicated. Firms with below "normal" equity exhibit a far lower average derivative with respect to equity.

Table 4.11: Parametric estimates of average derivative $\frac{\partial i}{\partial e}$

	$x \leq 0$	$x > 0$
$e \leq 0$	-0.0001	0.0110
$e > 0$	0.0194	0.0328

4.3.4.2 Non-parametric Analysis

As the parametric analysis naturally depends on the choice of the functional form, also a nonparametric analysis has been employed. Additionally, this allows to obtain direct inference on the derivatives of *expected* investment with respect to finance and fundamentals. To analyze the data non-parametrically, the data are pooled after individual fixed effects have been removed. As the (average) derivatives of the investment-function are the main points of interest, we concentrate on estimating these derivatives only. The derivatives are calculated by employing a local linear kernel-estimator to the data. For the derivative estimators, we have discussed the various alternatives in the section about the UK-data. Here, we will use the following two: One is Li et al.'s (1998) (analytic) estimator from the local linear regression. As before, the average derivative is computed by taking the sample average over the pointwise estimates $\begin{pmatrix} \hat{\beta}_x(x) & \hat{\beta}_e(e) \end{pmatrix}$, which are generated by weighted least squares on

$$i(x_j, e_j) = m(x, e) + \beta_x(x)(x_j - x) + \beta_e(e_j - e) + u_j. \tag{4.30}$$

The weights themselves are computed using a kernel-function.[43] Alternatively, numerical derivative (of i w.r.t. argument s) can be used, which is obtained (at point t) as

$$\widetilde{\beta}_s(t) = \frac{\hat{m}\left(x + \frac{1}{2}h_{t,s}\mathsf{e}_s\right) - \hat{m}\left(x - \frac{1}{2}h_{t,s}\mathsf{e}_s\right)}{h_{t,s}}, \tag{4.31}$$

where $h_{t,s}$ is the (variable) window-width used to generate kernels at evaluation point t, e_k is the s-th unit-vector, and $\hat{m}$ again is the weighted least squares estimate.[44] Again, average derivatives are computed as sample-means of $\widetilde{\beta}(t)$. Both the analytic and the numerical average derivative estimator converge with parametric rates.

As kernel, a Gaussian-product kernel has been employed. To generate window width $h_{t,s}$, the adaptive two-stage estimator for the window width was used, starting with a fixed window width of $\sigma_s n^{-1/4}$, in the first stage, in which σ_s stands for the standard deviation of argument s.[45]

Table 4.12 reports average derivative estimates for both the direct estimators and the numerical ones. Additionally, the mean of all estimates that fall inside the interval of ± 5 standard deviations around the previously calculated mean are reported as $\overset{\simeq cens}{\beta}$. These are less affected by outliers generated by undersmoothing and low density in the tails of the distribution.

The overall speed of adjustment, measured by the derivative of the investment rate with respect to the capital imbalance x is with 0.137 again rather low. This speed of adjustment is equivalent to an overall half-life of an capital imbalance of 4.7 years. In comparison to that, the mean effect of equity is with 0.018 quite substantial. An effect of that size in the linear model (4.28) would result in a long-run elasticity of approximately 0.131. This is far larger than the previously obtained long-run estimate. Moreover, the equity effect

[43] Note that this estimator is asymptotically equivalent to the Rilstone (1991) estimator.

[44] In most cases the numerical estimator has better small sample (and asymptotic bias) properties (Ullah and Roy, 1998). However, its asymptotic variance is not yet known (Pagan and Ullah, 1999).

[45] Note that in comparison to pointwise derivative estimations, this choice of window width leads to substatial undersmoothing.

Table 4.12: Average nonparametric derivative estimates

(a): Full sample

	$\bar{\beta}_x$	$\bar{\beta}_e$	$\tilde{\bar{\beta}}_x$	$\tilde{\bar{\beta}}_e$	$\tilde{\bar{\beta}}_x^{cens}$	$\tilde{\bar{\beta}}_e^{cens}$
overall	0.137	0.018	0.134	0.020	0.138	0.018
std. error[46]	0.0009	0.0013	–	–	–	–

(b): Stratified

Case		$\bar{\beta}_x$	$\bar{\beta}_e$	$\tilde{\bar{\beta}}_x$	$\tilde{\bar{\beta}}_e$	$\tilde{\bar{\beta}}_x^{cens}$	$\tilde{\bar{\beta}}_e^{cens}$
$x > 0$	$e > 0$	0.163	0.043	0.152	0.050	0.159	0.044
	$e \leq 0$	0.137	-0.021	0.126	-0.018	0.129	-0.016
$x \leq 0$	$e > 0$	0.124	0.043	0.121	0.043	0.127	0.039
	$e \leq 0$	0.126	-0.003	0.141	-0.002	0.141	-0.002

measured is an effect additionally to the one equity has on x. Hence, testing H_0^1 means testing the significance of $\beta_e = 0$. And therefore, we are clearly not able to reject H_0^1 and can expect second order and cross effects to be important and of a significant amount. This is validated by looking at the sample stratified by values of e and x, especially the numerical ones:

If equity is high, firms react 21% faster to positive mandated investment, than they do if equity is low.[47] For low mandated investment there is no such strong effect. Interestingly, the effect, equity has, again seems to be rather U-shaped, if there is high mandated investment. An explanation for this could be a Brander and Lewis (1986 and 1988) type strategic effect, which we will discuss in chapter 5. Using the robust estimator, that calculates the averages without using "outliers", the overall picture remains the same.

4.3.5 Discussion

Now as we have also used the "Bonner Stichprobe" to test hypothesis H_0^0 and H_0^1 - the "long-run" and "short-run versions" of the Modigliani and Miller theorem, if one likes - we can state the following, with respect to these hypothesis:

[47] The halve-life is 3.9 years for high equity, whereas it is 4.7 years for low equity.

1. H_0^0 can only be rejected on fairly weak grounds, so in the long-run finance does not seem to matter (a lot).

2. Since the estimated short-run influence of equity, measured by the average derivative, is both substantial and significant, H_0^1 has to be rejected. This holds true, as we only tested for *additional* short-run influences of liquidity on investment.

Hence, the question arises, why there is the additional short-run effect that has been found. Inspecting the short-run parametric and nonparametric estimates, we find a substantial interaction of finance and fundamentals in determining investment.

To further condense the results and to give them a more intuitive appeal, Table 4.13 presents (geometric) means of pointwisely calculated half-lifes of capital imbalances. These are calculated as $\frac{\ln 0.5}{\ln\left(1-\hat{\beta}(x)\right)}$. As the pointwise derivatives exhibit large variation, and sometimes obtain negative values, the derivatives are reestimated with a three times larger window width. Again the larger x, the faster is investment, and if firms wish to invest, more equity speeds up investment. Therefore, a pure "net-wealth effect on cost-of-capital" model can be rejected. Reading Table 4.13 linewise, we find some important additional evidence for the non-convex adjustment cost and financial frictions, but against a model that only incorporates fixed transaction cost in finance. Without analyzing second-order-effects, these models are typically hard to discriminate empirically (Gomes, 2001). The evidence we find is, that investment is "more convex", i.e. has a stronger curvature, in the fundamentals, if equity is high. However, under the pure transaction cost model, the reverse would hold true. Thus, our results could be summed up as: adjustment costs are non-convex and finance influences the speed of adjustment substantially.

So far, we omitted any short run dynamics to keep the empirical model and the theoretical one as closely together as possible. This gave our empirical model a structural interpretation. One may, however, be tempted to

Table 4.13: Average half-lifes of capital-imbalances (in years)

	$x \leq 0$	$x > 0$
$e \leq 0$	5.06	4.41
$e > 0$	5.19	4.07

argue that the non-linearities found are a mere result of the omitted dynamic links between changes in productivity, capital, and the equity-ratio. While a structural interpretation for including the lagged change in the stock of capital could be a delivery lag, an interpretation for other short-run dynamics is far from obvious. Moreover, even if we find a significant short-run dynamics, this could well be due to an imperfect approximation of the true functional form which is picked up by the first-differences of the equity-ratio and productivity. Table 4.14 presents the regression results from a model as in Table 4.10 augmented by some short-run dynamics.

Though the point estimates change, the overall structure of the estimated error-correction, i.e. investment function, remains the same. Hence, our results seem—at least to a certain extent—robust to the inclusion of short-run dynamics. However, the levels of significance of the terms involving equity drop.

4.4 Summary of the empirical results

In chapter 3 we presented a stylized model of financial frictions and fixed adjustment-cost. In chapter 4 we now have tested this model against alternatives. To do so, we analyzed two samples of company accounts, one from the UK and one from Germany. Both samples comprised a large number of company accounting data of (large) companies. And for both, we generated a measure of fundamental investment incentives and took the equity-ratio as an indicator of the financial status of a firm. Although, the way we generated the fundamental investment-incentive measure was different for both data, qualitatively we obtained quite similar results.

Table 4.14: short-run parametric estimates, dynamics-augmented

Variable	Model 1 Coefficient	(Std. Err.)	Model 2 Coefficient	(Std. Err.)
Δk_{it-1}	0.1627***	0.0064	0.1630***	0.0064
Δe_{it}	-0.0204**	0.0064	-0.0210***	0.0064
Δe_{it-1}	0.0276***	0.0073	0.0281***	0.0073
$\Delta \Pi_{it}$	0.0340***	0.0073	0.0340***	0.0073
$\Delta \Pi_{it-1}$	0.0458***	0.0074	0.0459***	0.0074
$(k_{it}^* - k_{it-1})$	0.1586***	0.0060	0.1580***	0.0044
$(k_{it}^* - k_{it-1})^2$	0.0394***	0.0116	0.0405***	0.0115
$(k_{it}^* - k_{it-1})^3$	-0.0579**	0.0251	-0.0458***	0.0081
$(k_{it}^* - k_{it-1})^4$	-0.0255*	0.0132	-0.0237*	0.0128
$(k_{it}^* - k_{it-1})^5$	0.0067	0.0191	–	–
e_{it-1}	0.0194**	0.0091	0.0105**	0.0049
e_{it-1}^2	0.0604**	0.0260	0.0306**	0.0134
e_{it-1}^3	-0.1302	0.0805	–	–
e_{it-1}^4	-0.0809	0.0595	–	–
e_{it-1}^5	0.2252	0.1310	–	–
$(k_{it}^* - k_{it-1})\, e_{it-1}$	0.0239*	0.0135	0.0205	0.0128
$[(k_{it}^* - k_{it-1})\, e_{it-1}]^2$	-0.0550	0.0501	-0.0972**	0.0432
$(k_{it}^* - k_{it-1})\, e_{it-1}^2$	0.0373	0.0315	–	–
$(k_{it}^* - k_{it-1})^2\, e_{it-1}$	0.0084	0.0240	–	–
$const$	-0.0094***	0.0013	-0.0088***	0.0013
Adj. R^2	0.2637		0.2635	
No. Obs.	8153		8153	

***/**/* significant at the 1/5/10% level

Note that $k_{it}^* := \hat{\gamma}_1^{+}\hat{\Pi}_{it} + \hat{r}_t + \hat{\mu}_i + \gamma_2'' e_{it-1}$, $x := (k_{it}^* - k_{it-1})$

One major outcome of our theoretical model was to identify the difference between a short-run effect of liquidity on the frequency of investment and a long-run effect on the optimal stock of capital. While the standard agency or oligopoly models of investment and finance[48] predict only a strong

[48] See e.g. Myers (1977) or Brander and Lewis (1986). Moreover, very most of modern macroeconomic literarure, which emphasizes the role of a financial accelerator mechanism, builds upon a long-run influence of finance via the cost-of-capital. Examples are: Bernanke et al. (1995, 1998), Céspedes et al. (2000) or Devereux and Lane (2001).

In contrast, our findings are more in line with the earlier literaure such as Meyer and Kuh (1957, pp. 198). However, other modern macroeconomic models of financial frictions influence of liquidity on the chosen stock of capital, the model of chapter 3 rather suggested a strong short-run effect. This stronger short-run than long-run influence of the equity-to-capital ratio on investment, is exactly what is empirically found, so that in this point the empirical evidence supports our theoretical model. In detail, we have found the long-run influence of finance on the stock of capital to be only small but significant for the UK-sample and for the German sample the influence was even mostly insignificant.[49] However, the short-run influence on investment, we find, is well significant and comparably large in both samples; but again the measured role of finance is larger for the UK-sample.

Moreover, empirically we find the investment rate to be a highly nonlinear function of the capital-imbalance (investment opportunities) and equity (liquidity). With the UK-data we have shown, that because of this non-linearity even only imposing additive separability leads to a severe error. This error could well be the cause of the puzzling finding reported in the literature that "*apriori* unconstrained" firms react stronger to changes in their financial variables than constrained ones. For further empirical research, this finding therefore suggests a need to estimate investment equations in a generalized error correction framework[50] as we did.

The results from the UK-data are complemented with our evidence from the "Bonner Stichprobe". Here, we find, that the half-life of a shock to the capital imbalance is not only substantially smaller in high-equity states, but also, that the investment-function is stronger curved if equity is large.

These higher-order and cross-effects help to discriminate between the (pure) financial (fixed) transaction cost, the net-worth, the (pure) fixed capital-adjustment cost and our mixed model. The comparably low influ-

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rather rely on versions of the pecking-order theory of finance, so e.g. Aghion et al. (2001).

[49]These results are somewhat similar to the results Bond et al. (1999) have reported earlier. They analyzed both a sample of German and UK firms using identical estimation techniques on both samples, so that their results are better comparable between contries. In result, they found a much larger and more significant influence of cash-flow for their UK firm-sample than they could find for their (comparable) German one.

[50]See e.g. de Jong (2001).

ence finance has on the equilibrium stock of capital does not well support the net-worth model. The transaction and convex adjustment-cost model is however at odds with an investment-function that is "more convex" when equity is high. In this setting, which yields a pecking order of finance, theory would rather suggest the investment-function to be more curved when equity is low. And as for the pure fixed adjustment cost model, the significant influence of equity on investment can be put as evidence against it.

To obtain these results, we have followed an econometric approach that estimates the long-run relationships of capital, productivity and costs in a first step. The short-run dynamics (or error correction function) have then been estimated nonparametrically. From a complementary parametric analysis, we have seen that our approach is well capable to explain a large fraction in company-level investment. Hence, we can conclude that our proxy of fundamental investment-incentives, which we have obtained from the long-run relation specified, is of reasonable quality. Moreover, and since the proxy for the fundamental incentives we used for the German data, explicitly allows for heterogeneity, the good relative quality of the proxy suggests that there is indeed a substantial degree of technological heterogeneity across firms—which may be considered as a side-result of our analysis.

For questions of economic policy, the findings of our analysis suggest, e.g. that the central bank not only should try to observe the distribution of capital-imbalances, but also consider the financial situation of firms in order to predict policy implications. More specifically both effects cannot be considered separately as the magnitude of each effect depends on the state of the other variable. Especially, we find firms with high equity to adjust to shocks much faster. If this itself leads to a rather stabilizing or destabilizing effect remains open for further research. However, if we only concentrate on adverse shocks, an economy which is simultaneously hit by a crisis of company-finance and an adverse aggregate shock will recover much slower than the financially unconstrained economy. In any case, an economy with many firms being in a high equity state should have the stock of capital more efficiently distributed among firms.

Therefore, another example where our results have policy implications are (corporate) tax reforms. There, the impact of the reform on the costs of retaining earnings have to be taken into account as well. A rise in the average equity-to-capital ratio, induced by such a reform, would not only raise investment in the short run but also—at least in the partial model presented in chapter 3—increase efficiency as average (absolute) capital-imbalances fall.

Interlude
Monopolistic competition and oligopoly

In the proceeding chapters, we assumed the market structures firms operate in are monopolistically competitive. In fact, we only needed that firms face decreasing returns to scale. Monopolistic competition is just one way to rationalize this, without introducing strategic action to the model. Decreasing returns to scale are also well compatible with perfect competition.

Anyway, this allowed us to have firms with positive profits—respectively positive variable gross margins—analyzed in an environment of many firms. So, we could use laws of large numbers and the expectations operator consistently with our model, thereby deriving expressions for average industry-investment.

Whether monopolistic competition, perfect competition or pure oligopoly is the more realistic way to think about market structure has a strong history of lively debate since the Chamberlin (1933) first introduced the idea of monopolistic competition.[51] We used monopolistic competition merely to abstract from the influence of strategic considerations to keep our model tractable. In a way adding imperfect competition to the framework developed

[51] See Samuelson (1967) and Bain (1967) for reviews. Central contributions to the debate how to differentiate empirically between those concepts were Bishop (1952, 1953 and 1955) Heiser (1955), Fellner (1953), Chamberlin (1953).

in Chapter 3 serves as some sort of comparative statics.[52]

For homogeneous oligopolies the products of each competitor are each perfect substitutes for the consumers. The inverse demand function is defined as

$$p_i = p\left(\sum_{j=1}^{N} x_j\right). \tag{4.32}$$

If the products are differentiated, which is the basis for monopolistic competition, the inverse demand function takes the more general form[53]

$$p_i = p_i\left(x_i \left| \mathbf{x}_{-i}\right.\right). \tag{4.33}$$

Monopolistic competition now is a situation, where firm i can rightly neglect the influence of any other single firm on $\mathbf{x}_{-i}$, the output of other firms. Hence it can also neglect its own influence on others. In contrast to perfect competition, the influence of x_i on p_i is but not negligible.[54]

Which model of market-structure is appropriate for any given industry is in the end an empirical question.[55] Most probably some markets are

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[52] However, note that we do not simply adopt the previously derived framework to duopoly. Again this would be intractable and hence it would be excessively hard to reach decisve conlusions.

[53] See e.g. Shaked and Sutton (1982) for a model of product differentiation.

[54] Hart (1985) presents a model that gives monopolistic competition as an arbitrary good approximation of the equilibrium of an economy with many firms and heterogeneous consumers.

Dixit and Stiglitz (1977) provide a model based on the assumption of a representative consumer.

[55] The classical debate initiated by Bishop (1952), how to measure market structure has focused on the the the size of cross elasticities of demand in comparison to the elasticity of demand itself. However, there is not neccessarily a such strong relation between cross-elasticities, numbers of firms in the market and the elasticity of demand as suggested by Bishop (1952, 1953, and 1955).

Suppose e.g. there is a representative consumer with a Cobb-Douglas utility function over a (very large) number of goods. However, her effective preferences shall be only formulated over the set of goods that are indeed produced I.

$$U(x) = \prod_{i \in I} x_i^{\alpha_i}. \tag{4.34}$$

Then given any number of firms in the economy producing an arbitrary set of goods I, oligopolistic (differentiated or not), some are near monopolistically competitive, some are regulated or natural monopolies, and some are near perfectly competitive.

An analysis of the impact financial frictions and non-convex investment-technologies have on duopoly—as a complement to our analysis under monopolistic competition in Chapter 3—therefore seems to be a valid research-goal. Also, by moving perspective from monopolistic competition, respectively perfect-competition with decreasing returns to scale to duopoly, we zoom to the very microeconomic analysis of investment decisions.

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given total consumption C, this representative consumer demands

$$x_i^* \left(p_i, \mathbf{p}_{-i}\right) = \frac{\alpha_i}{\sum_{j \in I} \alpha_j} \frac{C}{p_i}. \tag{4.35}$$

Therefore, the elasticity of x_i w.r.t. its own price is $E_{ii} = -1$ and the elasticity of x_i w.r.t. price j is $E_{ij} = 0$! Moreover, as there are monopoly rents to exploit, every new entrant (or operating firm) will rather invent a new product, than imitating an existing one. It now depends on the number of goods the consumer differentiates at most, the relation of costs of entering a new market and a market for an established good, etc. whether monopolistic competition, differentiated oligopoly, a number of different homogeneous oligopolies or perfect competition emerges.

Chapter 5

Investment timing and predatory behavior in duopoly with debt

5.1 Introducing strategic considerations

5.1.1 Non-convexifying imperfect competition vs. convexifying perfect competition

Our analysis of company investment decisions in the previous chapters has highlighted the importance of the interaction between non-convexities and financial frictions. However, Thomas (2002), Khan and Thomas (2003) and Veracierto (2002) have recently questioned the relevance of the non-convex adjustment-costs framework for macroeconomic data. Their basic idea is that on the aggregate two effects smooth out the non-linearities. The first effect is the direct one from aggregation itself: in the aggregate time-series domain typically the higher-order moments of the "mandated investment" distribution evolve slowly. The other, less obvious reason for the minor relevance of the non-convex framework in the aggregate is the convexifying effect of (perfect-) competition and general equilibrium.

This chapter contrasts these perfect competition results, with evidence from imperfect competition. Our model in Chapter 3 focused on the interaction of financial fractions and non-convex investment technologies purely within a firm and therefore, we assumed monopolistic competition to abstract from strategic aspects of this interaction between firms. Similarly, the assumption of perfect competition abstracts from strategical aspects. Yet, for many industries if not monopolistic competition then oligopoly seems the more realistic way to model competition.

Therefore, we present a model of duopoly in which capital-market imperfection, irreversibility of investment, and imperfect goods-market competition interact. This interaction brings about investment patterns which are neither present in the monopolistic competition, irreversible investment framework nor in the imperfect competition, reversible investment one.

Our model presented in this chapter also addresses the broader question which impact non-convex adjustment cost and capital market imperfections have on the strategic situation of a firm. And this strategic perspective proves fruitful. Our model contributes to a number of strands in the literature, such as the investment, the debt-in-industry-equilibrium, the strategic real options, or the predatory behavior literature. To keep this chapter mostly self-contained we will discuss the relation of our model to all these strands of the literature in detail in this introductory section.

Most models of interdependent financial and real investment decisions can neither replicate the observed debt-ratios, which are rather modest compared to the substantial tax benefits of debt,[1] nor a non-monotone influence of leverage ratios on investment.[2] The model of this chapter can explain both

[1] E.g. Schowalter (1999, p. 327) finds in a sample of 1641 manufacturing firms that the average debt to asset ratio is about 0.25, whereas Fischer et al. (1989) obtain debt ratios somewhat between 0.3 and 0.7 as optimal debt ratios in their capital choice model.

[2] An example for this non-monotone relationship are our results in the previous chapter, presented in figure 4.5 and 4.6 and table 4.12. An alternative example is the paper of Busse (2001), who reports that price wars between airlines are predicted best as non-linear (threshold functions) in flow measures of liquidity.

Brander and Lewis (1988) provide an explanation for this finding, in cases when there are bankruptcy costs which are proportional to the "magnitude" of bankruptcy. However, these empirical findings by the strategic situation of indebted firms in a duopolistic real options setting of investment.

Moreover, our model offers an explanation for predatory behavior without defining predatory behavior as deviation from tacit collusion, without relying on learning or network effects, and without relying on Folk theorem like reputational arguments. It rather builds upon a real options approach. This approach models predatory investment as the attempt of one firm driving a competitor out of the market if the loss from an early exercise of the investment option is more than compensated by a gain from the expected earlier exit of a competitor.

5.1.2 Imperfect competition and debt

Since the seminal paper of Brander and Lewis (1986), economists have drawn attention to the strategic effects of debt on competition.[3] As described in chapter 2, most of the empirical literature on the link between financial situation and investment decisions of firms suggests that increasing debt reduces investment. In Brander and Lewis contribution however, larger debt leads to fiercer competition, so that firms invest and produce more. In a monopolistic setting, in which equity but not debt is the marginal source of finance, we can easily build a model that replicates the general empirical findings (Jou, 2001). However, in a duopoly with equity being the marginal source of finance, endogenous bankruptcy decisions render predatory investment-strategies possible.

Although Brander and Lewis (1988) already describe how predatory behavior may emerge in a two period setting with bankruptcy, a dynamic model of finance, investment timing, and predatory behavior has not been formulated yet. In this sense the present chapter extends the Brander and Lewis contribution to cases in which investment is subject to non-convex costs of

their model as a static one cannot explain periods of predatory behavior triggered by changes in the environment.

[3] See e.g. Maksimovic (1990), Maksimovic and Zechner (1991) and Fries et al. (1997).

adjustment. In contrast to the Brander and Lewis (1988) model however, declaring bankruptcy is a truly endogenous decision, as the owners of the firm are allowed to pay for the firm's obligations with private funds, if this is advantageous for them. Therefore, bankruptcy is declared only when optimal and not necessarily, once a itself firm has insufficient funds.

In this (general) setting, changes in market conditions may trigger predatory behavior in highly leveraged industries, but debt does not necessarily make output market competition fiercer in general. This also is a central difference to Brander and Lewis' contributions, where predatory behavior simply is fiercer competition in all states of nature and not a different policy-regime triggered by exogenous changes in the environment.

Moreover, the model of this chapter can also be read as a contribution to the debt-in-industry-equilibrium literature. This literature typically has either ruled out the possibility of predatory investment by the assumption of free market entry—which makes predatory investment unprofitable—or assumed myopic behavior from the outset.[4]

5.1.3 Imperfect competition, real options and predatory behavior

Our model also extends the literature on real options in oligopoly to cases with predatory investment. So far, most of this literature has ignored the strategic effects of debt.[5] Nevertheless, debt can have important strategic effects and, especially in the irreversible investment framework, influences investment decisions substantially: If a firm is subject to limited liability, its owners have to decide to finance negative cash flows from private funds or to default on the firms obligations in adverse states, i.e. to declare bankruptcy. As declaring bankruptcy often leads to an irreversible and complete exit of that firm from the market, a bankruptcy decision influences the payoff of the

[4]See for instance Fries et al. (1997) or Maksimovic and Zechner (1991).
[5]See for example Huisman and Kort (1999), Sparla (2001), or Weeds (2001).

competitor and thus the value of the competitor´s investment. Therefore, the option to declare bankruptcy and the possibility of driving a competitor to bankruptcy (and thereby out of the market) may alter the prices of investment options significantly. The decision to declare bankruptcy obviously depends on the relative debt-burden a firm faces. Thus, investment not only can be used to drive the competitor to bankruptcy by lowering prices, but can also serve as a commitment device not to declare bankruptcy by altering the relative debt-obligation.

Therefore, allowing for predatory investment yields new insights and clearly distinguishes our model from existing work on exit decisions in duopoly in a real options framework: Sparla (2001) discusses partial but irreversible capacity reductions, only. Depending on the parameters of aggregate demand firms may end up in a war-of-attrition or a preemption game. However, as firms are assumed to be unable to increase capacity, predatory behavior does not emerge.

Joaquin and Khanna (2001) allow for both irreversible investment and exit decisions in their model of potential competition. Yet, predatory investment cannot occur in their model, due to their assumptions on revenues and costs: (rational) exit of the competitor imposes a *loss* on the remaining firm.

This chapter combines the real options approach with the strategic effects of debt literature. The basic setting is similar to the one of Jou (2001), who models a potential monopolist (or myopic investor) that enters the market and at the same time issues debt to finance investment.

In contrast to Jou, we model a duopoly. Both firms are assumed to operate with a given capacity and are indebted from the very beginning. Both firms can irreversibly increase capacity, but do not have any possibility of raising or lowering debt-levels. Therefore, debt is taken to be exogenous. The corporate tax-system is assumed to be classical, i.e. there is no shareholder´s relief for corporate taxes, whereas interest payments are completely tax-deducible. This gives a tax-advantage of debt.

Although throughout this chapter "debt" will be interpreted as a special financial obligation, by the way "debt" is to be modelled, all results equally apply to any fixed running cost (e.g. overhead costs), on which a firm may default. As a result, the model of this chapter gives an explanation for predatory behavior itself in a wide class of situations, and is thus a contribution to the literature on predatory behavior, too. In contrast to existing models, that relate financial situation of firms to predatory behavior (e.g. Bolton and Scharfstein, 1990, or Glazer, 1994), predatory outcomes in our model can be described as regimes that depend on the fluctuating state of demand. Insofar, predatory behavior emerges from time to time, triggered by changes in market-conditions.

In contrast to many contributions in this field that study price wars,[6] predatory behavior is not defined as a deviation from tacit collusion, but as investment at market conditions which yield negative net-present value for investment unless the endogenous nature of the exit decisions of the competitors is taken into account.[7] This leads to additional investment activity when the aggregate state of demand worsens. Hence, the present paper is related to and extends the results of Grenadier's (1996) real options-model on investment-cascades to a non-collusive setting.

Moreover, our model also does not rely on the assumption that information is asymmetric among competitors, but only on the imperfectness of information.

In line with Fershtman and Pakes' (2000) model heterogeneity of firms with respect to their fixed costs is one of the driving factors in the presented model. Other models of predatory behavior that do not rely on asymmetric information typically build upon network[8] or learning-curve effects (Cabral and Riordan, 1994 and 1997). Besides the work of Fershtman and Pakes, the empirical paper of Busse (2002) is most closely related. Busse finds that

[6]See Ordover and Saloner (1989) for a summary or for more recent contributions Fershtman and Pakes (2000) or Busse (2002).

[7]In this respect, our model is closest to Milgrom and Roberts (1982) paper.

[8]See Athey and Schmutzler (2001) for a fairly general model of investment and increasing dominance, that includes network-industries as a special case.

leverage is one of the main determinants for starting a price war in the airline industry, and that the probability of starting a price war reacts in a quite non-linear fashion to changes in the financial situation of a firm.

5.1.4 Some main results anticipated

Our model will partly consist of equations that cannot be solved analytically, and so numerical simulations are presented where analytical solutions are not available. Nevertheless, it will be shown analytically as well as numerically that parameter constellations exist which lead to predatory behavior, i.e. firms invest not because investment itself has a positive present value, but to drive the competitor out of the market. However, the occurrence of predatory behavior in equilibrium depends on the "competitiveness" of the market: When adjustment costs are high or demand is sufficiently elastic, predatory investment never occurs.

The numerical analysis yields furthermore that already a credible threat of predatory behavior lowers price triggers for investment substantially. However, predatory behavior only emerges in highly indebted industries. If predatory behavior does not occur in equilibrium, increasing debt increases the price trigger for investment of the firm increasing its debt. The price trigger for the other firm decreases. Therefore, only if the firm that changes its debt were the follower in equilibrium, increasing debt would delay investment. Furthermore, competition and the possibility of predatory investment generally lowers the value of debt and therefore might explain the lower leverage ratios observed in practice, when compared with those predicted by the contingent claims literature.[9]

We will proceed as follows: Section 5.2 outlines the model and presents the basic assumptions. Section 5.3 presents the expressions that determine the value of equity and price triggers for bankruptcy when both firms have already exercised their investment option. Section 5.4 derives the value of

[9] See e.g. Fischer et al. (1989).

equity and the price-triggers for investment and bankruptcy when firms may invest. Section 5.5 shortly states the conditions which determine debt-value. Section 5.6 presents our numerical results and section 5.7 summarizes. Detailed proofs are available in appendix C.

5.2 Model setup

We model a duopoly with quantity-competition and stochastic demand fluctuations in continuous time t, $t \in [0, \infty[$. Total production given, the price process $(P_t)_{t \geq 0}$ is assumed to be a geometrical Brownian motion and shall be given by

$$\begin{aligned} P_t &= D(Q_t) Y_t \quad , & (5.1) \\ dY_t &= Y_t \left(\mu dt + \sigma dB_t \right) \quad . & (5.2) \end{aligned}$$

B_t denotes a standard Brownian motion and Q_t denotes aggregate industry production, such that

$$Q_t = q_{1,t} + q_{2,t} \quad ,$$

with $q_{i,t}$ denoting production of firm i at time t. For the sake of simplicity, we assume that output is solely produced by a capital good which does not depreciate. Moreover, we assume that both firms already operate in the market with some initial production $\underline{q}_i$. However, both firms may irreversibly invest and increase production to $\overline{q}_i$ at cost C_i. This increase in production is assumed to be instantaneous. At time $t = 0$ each firm issues debt of unstated maturity with associated coupon payments b_i. Thereafter, a firm may not change its debt. Since we want to model heterogenous firms, we assume the "flow leverage" of the two firms to differ. With "flow leverage" the ratio $l_i(q_i) := \frac{b_i}{q_i D(q_i)}$ of debt payments to monopoly earnings at the state of demand normalized to $Y_t = 1$ is meant. Without loss of generality assume firm 2 is the firm with the higher leverage once both firms have invested.

Assumption 5.1: $l_1(\overline{q}_1) < l_2(\overline{q}_2)$.

For notational convenience, we define a function Δ for the relative price-change induced by a change in aggregate supply.

$$\Delta_{Q_1,Q_2} := \frac{P_t(Q_2)}{P_t(Q_1)} = \frac{D(Q_2)}{D(Q_1)} \tag{5.3}$$

The following assumption ensures, that price levels always exist, so that investment is profitable:

Assumption 5.2: Investment increases revenues of the investing firm, regardless if the other firm has invested or not, i.e.

$$D(\underline{q}_i + q_{-i})\underline{q}_i < D(\overline{q}_i + q_{-i})\overline{q}_i \tag{5.4}$$

As the tax-system is assumed to be classical, losses are fully offset. Therefore, there is a tax advantage of debt, and at tax-rate τ instantaneous net earnings of firm i are given by

$$(1-\tau)(q_{i,t}P_t(Q_t, Y_t) - b_i) \quad . \tag{5.5}$$

However, firms may default and declare bankruptcy at any point in time, as they are assumed have limited liability. If bankruptcy is declared, the coupon payments are stopped, the firm leaves the market, and its assets are transferred to the creditors and sold at price λq_i. Although in a more general context λ may well be determined endogenously, here it will be treated as exogenous. Moreover, λ will only matter for determining the market value of debt and thus will not influence any decisions of the equityholders after debt has been issued.

Furthermore, we assume that the firm is unable to temporarily suspend production. Finally, equityholders are assumed to have unlimited external resources. The risk-adjusted discount rate is ρ, which shall be larger than μ, i.e. $\rho > \mu$.

As Sparla (2001) argues, if the drift μ is strong compared to the variance σ^2, the probability that firms will not exit in finite time is strictly positive.

However, this causes notational inconvenience as one root of the "fundamental quadratic equation" (see below) has to be "adjusted" to derive the correct value functions, see Sparla (2001) for details. To avoid this difficulty the drift is assumed to be not excessively large, i.e. $|\mu| < \frac{\sigma^2}{2}$.

Under these assumptions, the roots of the so called "fundamental quadratic equation" (see e.g. Dixit and Pindyck, 1994) are given by

$$\beta_{1,2} = \frac{1}{2} - \frac{\mu}{\sigma^2} \pm \sqrt{\left[\frac{1}{2} - \frac{\mu}{\sigma^2}\right]^2 + \frac{2\rho}{\sigma^2}} \quad , \tag{5.6}$$

which implies $\beta_1 > 1$ and $\beta_2 < 0$ and $\beta_1 + \beta_2 = 1 - 2\frac{\mu}{\sigma^2} > 0$.

Therefore, just as in Jou (2001, p. 72), the general solutions for the value of equity $E_i(P, b, q_i, q_{-i})$ and the value of debt $B_i(P, b, q_i, q_{-i})$ for firm i are given by the following equations:[10]

$$\begin{aligned} E_i(P, b_i, q_i, q_{-i}) &= (1-\tau)\left[q_i \tfrac{P}{\rho-\mu} - \tfrac{b_i}{\rho}\right] + a_{i1}(q_i, q_{-i})P^{\beta_1} \qquad (5.7)\\ &\quad + a_{i2}(q_i, q_{-i})P^{\beta_2}, \\ B_i(P, b_i, q_i, q_{-i}) &= \frac{b_i}{\rho} + c_{i1}(q_i, q_{-i})P^{\beta_1} + c_{i2}(q_i, q_{-i})P^{\beta_2}. \qquad (5.8) \end{aligned}$$

For notational convenience, b is dropped from the list of arguments of the value function, as it will remain constant throughout the subsequent analysis.

5.3 Value and bankruptcy without investment option

The derived formula for the value of equity still contains two unknowns a_{i1} and a_{i2}. These have to be solved for by deriving further conditions that reflect the optimality of investment-plans. Therefore, strategic considerations have an important influence on these parameters. This means the parameters a_{i1} and a_{i2} will in general vary with the state of production (q_i, q_{-i}).

[10] See appendix C.1 for details. As usual, we denote by firm $-i$ the competitor of firm i.

We should hence discuss in a bit more detail the timing structure of the game. At each point in time an active firm may chose to

(1) invest and increase capacity to $\bar{q}_i$ if it has not invested yet,

(2) declare bankruptcy and become inactive from then on,

(3) or do nothing and wait.

As a result, the number of possible states of production (q_1, q_2) complicate the analysis a lot. The following two sections will derive and specify equity-value at the various states (q_i, q_{-i}), which also index subgames. We do not assume which firm invests first, but let this be determined in equilibrium. Therefore, at the time the first firm invests the other firm must be at least indifferent between investing somewhat earlier or becoming the second firm that invests. We discuss the behavior of both firms as second-mover first. This allows us to obtain the valuation of the second-mover position for both competitors. This valuation is crucial in the competition for the position of the first-mover.

To some extent, this discussion cannot avoid some technical complexity. The most technical proofs are presented in appendix C.

5.3.1 The monopolist's case

Once one of the firms declares bankruptcy, the other firm becomes a monopolist. Therefore, we also have to consider the monopoly situation, although we want to model a duopoly. We start with determining the value functions of a monopolist who has already exercised its investment option. As our model is—for the case of a monopolist that has invested—similar to Jou´s (2001) investment and financing model, we can primarily rely on his result to obtain the value functions.

Proposition 5.1 *Having invested, the monopolist´s value of equity and debt* *is given by*

$$E(P,\overline{q},0) = (1-\tau)\left(\overline{q}\frac{P}{\rho-\mu}-\frac{b}{\rho}\right)+\frac{b\,(1-\tau)}{\rho(1-\beta_2)}\left(\frac{P}{P^{exit}_{\overline{q},0}}\right)^{\beta_2}, \quad (5.9)$$

$$B(P,\overline{q},0) = \frac{b}{\rho}+\left(\lambda\overline{q}-\frac{b}{\rho}\right)\left(\frac{P}{P^{exit}_{\overline{q},0}}\right)^{\beta_2}. \quad (5.10)$$

Here, we denote by $P^{exit}_{\overline{q},0}=\frac{\beta_2(\rho-\mu)}{(\beta_2-1)\rho}\frac{b}{\overline{q}}$ *the trigger price to declare bankruptcy.*

Proof. *Denote the revenues process by* $\widetilde{P}:=\overline{q}P$. *This process has exactly the same properties as the price process in Jou (2001). The proposition then follows straightforward from Jou´s Proposition 1.* ■

5.3.2 The duopolists' case

When both firms have invested, so that we are in state $(\bar{q}_1,\bar{q}_2)$, both firms still have to decide whether and when to declare bankruptcy. However, a priori it is not obvious which firm will declare bankruptcy first. But since we have assumed the two firms to be differently leveraged in the above introduced sense, the only Markov-perfect equilibrium of the resulting exit game is the one where the higher leveraged firm exits at its monopoly exit price. This is shown by the proposition below:

Proposition 5.2 *In all Markov-perfect equilibria in pure strategies of the* $(\bar{q}_1,\bar{q}_2)$*-subgame (exit after investment), the firm with the higher leverage (firm 2) chooses its monopoly exit price as the price trigger for bankruptcy* $P^{exit,2}_{\overline{q}_2,\overline{q}_1}=P^{exit,2}_{\overline{q}_2,0}$, *whereas firm 1 chooses as exit-price trigger some* $P^{exit,1}_{\overline{q}_1,\overline{q}_2}\in\left]\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1}{}^{-1}P^{exit,1}_{\overline{q}_1,0},\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_2}{}^{-1}P^{exit,2}_{\overline{q}_2,0}\right[$.

Proof. *See appendix.* ■

With the equilibrium exit strategies given by Proposition 5.2, which determine who stays in the market, we can now compute the equilibrium value functions for equity and debt in a duopoly when both firms have exercised their investment option. As firm 2´s behaves myopic, its value functions are the same as the monopolist's ones. However, the possible exit of firm 2 changes firm 1´s value. Therefore, the value functions of firm 1 need to be determined anew and the "option values" a_{11}, a_{12}, c_{11} and c_{12} of (5.7) and (5.8) have to be calculated.

Firstly, when prices tend to infinity, the bankruptcy option becomes worthless. This leads to $a_{11} = c_{11} = 0$.[11] Secondly, when the price tends towards the exit price of firm 2, the following value-matching conditions must hold:

$$E_1(P^{exit,2}_{\overline{q}_2,0}, \overline{q}_1, \overline{q}_2) = E(\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1} P^{exit,2}_{\overline{q}_2,0}, b_1, \overline{q}_1, 0) \tag{5.11}$$
$$B_1(P^{exit,2}_{\overline{q}_2,0}, \overline{q}_1, \overline{q}_2) = B(\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1} P^{exit,2}_{\overline{q}_2,0}, b_1, \overline{q}_1, 0) \tag{5.12}$$

This now yields for c_{12} and a_{12} after some algebraic calculations:

$$c_{12}(\overline{q}_1, \overline{q}_2) = \left(\lambda \overline{q}_1 - \frac{b_1}{\rho}\right) \left(\frac{\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1}}{P^{exit,1}_{\overline{q}_1,0}}\right)^{\beta_2}, \tag{5.13}$$

$$a_{12}(\overline{q}_1, \overline{q}_2) = g \cdot \frac{(1-\tau)}{1-\beta_2} \frac{b_1}{\rho} \left(\frac{1}{P^{exit,1}_{\overline{q}_1,0}}\right)^{\beta_2}, \tag{5.14}$$

with g being defined as

$$g := \underbrace{\left(\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1}\right)^{\beta_2}}_{<1} \underbrace{-\beta_2 \left(\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1} - 1\right)}_{>0} \left(\frac{\overline{q}_1 b_2}{\overline{q}_2 b_1}\right)^{1-\beta_2} > 1.$$

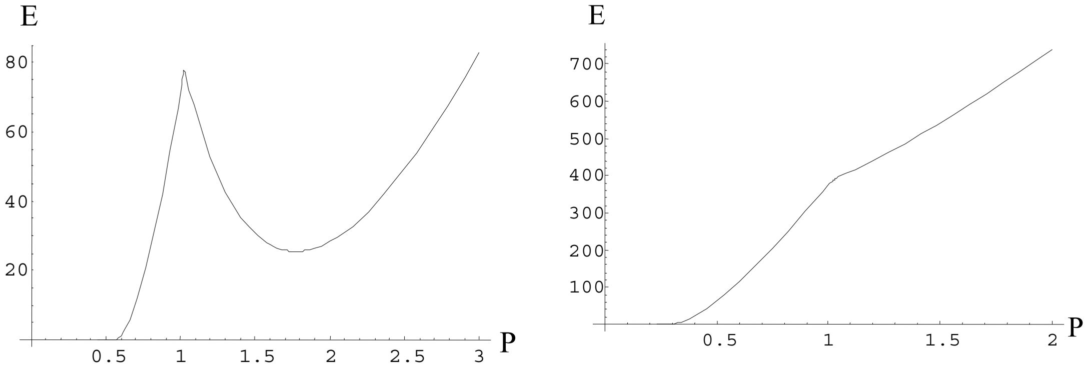
The stated inequalities are shown to hold in the appendix. With these terms at hand, we obtain for the value of equity and debt of firm 1 the following expressions

$$\frac{E_1(P, \overline{q}_1, \overline{q}_2)}{(1-\tau)} = \left(\overline{q}_1 \frac{P}{\rho-\mu} - \frac{b_1}{\rho}\right) + g \cdot \frac{b_1}{(1-\beta_2)\rho} \left(\frac{P}{P^{exit}_{\overline{q}_1,0}}\right)^{\beta_2}, \tag{5.15}$$

$$B_1(P, \overline{q}_1, \overline{q}_2) = \frac{b_1}{\rho} + \left(\lambda \overline{q}_1 - \frac{b_1}{\rho}\right) \left(\frac{P}{P^{exit,1}_{\overline{q}_1,0}}\right)^{\beta_2} \left(\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1}\right)^{\beta_2}. \tag{5.16}$$

[11] See Jou (2001) for details.

Figure 5.1: Equity-value of firm 1



(a) both $\frac{b^1}{\overline{q}^1}$ and $\Delta_{\overline{q}^1+\overline{q}^2,\overline{q}^1}$ large

(b) both $\frac{b^1}{\overline{q}^1}$ and $\Delta_{\overline{q}^1+\overline{q}^2,\overline{q}^1}$ small

As one can now easily see, the presence of a competitor who leaves the market at a higher trigger price adds a factor g in (5.9) to the price of the exit-option. This factor is composed of the costs of postponed exit $\left(\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1}\right)^{\beta_2}$ and the gain, when firm 2 exits, which is a "hedge" against bad states. This hedge outweighs the cost of waiting and in total increases equity-value. Moreover, value of firm 1´s equity now exhibits a kink where firm 2 exits and it no longer necessarily monotonously increases in P (see figures 5.1(a),(b) and appendix for details).

5.4 Value, optimal investment and bankruptcy

To describe investment behavior in the model presented here, some assumptions are necessary to keep the model tractable. Some of them were mentioned above. However, further discussion of these assumptions is still open.

First of all, as debt payments are fixed, investment—financed by external equity—is the only way to decrease leverage. Although this seems too great a constraint at first glance, it is very much in line with the findings of Fries et al. (1997), who find that a small equityholder finds only extramarginal changes in leverage advantageous. Therefore, if only marginal changes are feasible by issuing or buying back marginal debt, investment—as an extramarginal change—is the only possibility to lower leverage.

Moreover, as leverage is changed by investment, the order of leverage ratios and therefore the order of exit prices may be reversed by investment. Furthermore, as we have made no assumptions about the sizes of the investment projects, the order of leverage ratios may differ before and after investment anyway. Therefore, we have to consider a number of sub-cases which depend on the parameters of the model.

A further issue concerns predatory behavior, i.e. firms might invest just to drive the price down to increase the probability of the other firm defaulting. Obviously, the possibility of profitable predatory investment again depends on the parameter-values and the order of leverage ratios.

As no assumptions concerning which firm invests first shall be made, the two possible asymmetric orders of movement have to be considered. The order of movement will later be determined endogenously in equilibrium. However, as with ruling out equal leverage ratios, we assume simultaneous investment of both firms (or collusive strategies) to be ruled out by prohibitive costs of simultaneity.[12]

In order to obtain the sequence of price triggers for bankruptcy by the same reasoning used in Proposition 5.2 and to obtain a benchmark case, we start with the investment and bankruptcy decisions of a monopolist. We will leave the calculation of debt-values open for a later section.

5.4.1 Investment- and bankruptcy-decisions of a monopolist

For a monopolist holding an investment and a bankruptcy option, we will label its bankruptcy trigger price as $P^{exit}_{\underline{q},0}$ and its investment price-trigger as $P^{inv}_{\underline{q},0}$. The two following value-matching conditions must hold for the equity-

[12] Instead we could as well assume firm sales to be decreasing if both firms invest simultaneously, i.e. $\Delta_{\underline{q}_1+\underline{q}_2,\overline{q}_1+\overline{q}_2}\overline{q}_i < \underline{q}_i$. However, such assumption would be stronger.

value function:

$$E(P^{exit}_{\underline{q},0}, \underline{q}, 0) = 0, \tag{5.17}$$

$$E(P^{inv}_{\underline{q},0}, \underline{q}, 0) = E(\Delta_{\underline{q},\overline{q}} P^{inv}_{\underline{q},0}, b, \overline{q}, 0) - C. \tag{5.18}$$

Together with the following smooth-pasting conditions these equations fully determine equity-value and price triggers. The two smooth-pasting conditions are:

$$\frac{\partial E(P^{exit}_{\underline{q},0}, \underline{q}, 0)}{\partial P} = 0, \tag{5.19}$$

$$\frac{\partial E(P^{inv}_{\underline{q},0}, \underline{q}, 0)}{\partial P} = \frac{\partial E(\Delta_{\underline{q},\overline{q}} P^{inv}_{\underline{q},0}, \overline{q}, 0)}{\partial P}. \tag{5.20}$$

Now, combining (5.7),(5.17) and (5.19) , an algebraic expression for the value of equity can be derived after some calculations:

$$\begin{aligned} E(P, \underline{q}, 0) &= (1-\tau)\left[\underline{q}\frac{P}{\rho-\mu} - \frac{b}{\rho}\right] \\ &+ \frac{(1-\tau)}{\beta_2-\beta_1}\left(\frac{\underline{q}P^{exit}_{\underline{q},0}}{\rho-\mu} - \frac{b}{\rho}\right)\left[\beta_1\left(\frac{P}{P^{exit}_{\underline{q},0}}\right)^{\beta_2} - \beta_2\left(\frac{P}{P^{exit}_{\underline{q},0}}\right)^{\beta_1}\right] \\ &+ \frac{(1-\tau)}{\beta_2-\beta_1}\frac{\underline{q}P^{exit}_{\underline{q},0}}{\rho-\mu}\left[\left(\frac{P}{P^{exit}_{\underline{q},0}}\right)^{\beta_2} - \left(\frac{P}{P^{exit}_{\underline{q},0}}\right)^{\beta_1}\right] . \end{aligned} \tag{5.21}$$

The trigger prices $P^{inv}_{\underline{q},0}$ and $P^{exit}_{\underline{q},0}$, however, have to be calculated numerically from equations (5.18) and (5.20).

5.4.2 Investment- and bankruptcy-decisions in duopoly when one firm has invested

As mentioned above, both the order of movement where firm 1 invests first and firm 2 invests second and the reversed one have to be considered before one is able to determine which firm invests first in equilibrium. The firm investing first will be called "leader" while the other firm will be called "follower". We begin with discussing the behavior of the follower. As the case where firm 2 is the follower is the simpler one, we start with discussing this case.

Firm 2 as follower

Proposition 5.3 *As a follower, firm 2 behaves myopically, i.e. just as a monopolist would behave that faces the same demand function as firm 2.*

Proof. *Investment causes no continuous costs. Therefore, equity value at* $\underline{q}$ *(before investment) must be smaller than equity value at* $\overline{q}$ *(after investment)– statet differently: the investment option must be worth less than the increase in revenues after investment,* $(\Delta_{\bar{q}_1+\underline{q}_2,\bar{q}_1+\bar{q}_2} \cdot \overline{q}_2 - \underline{q}_2)P$*. Thus, we obtain* $P^{exit,2}_{\underline{q}_2,\overline{q}_2} > P^{exit,2}_{\overline{q}_2,0}$*. Moreover, proposition 5.2 yields that firm 2 leaves first after investment and behaves just as a monopolist would do. Thus, firm 2 can also not credibly threaten to exit second before investing, i.e. firm 2 leaves first and therefore cannot influence the (exit) behavior of firm 1.* ■

So, as follower—just like in other games with preemption (e.g. Weeds, 2001), firm 2, the higher leveraged firm, does not need to take into account the strategic situation. Therefore, we obtain equations similar to the situation of a monopolist, determining the investment and bankruptcy price-triggers.

Firm 1 as follower

If firm 1 is the follower the strategic situation changes: In contrast to the simpler situation of firm 2, firm 1's actions affect the probability of firm 2 declaring bankruptcy. Hence, the strategic situation of firm 1 is much richer and firm 1 may invest not because it is "fundamentally" profitable but because this makes the exit of firm 2 more likely. As we have seen, firm 2 does not have this opportunity.

Taking into account the possible exit of firm 2, a couple of different cases have to be considered. For the moment, take $P^{exit,2}_{\overline{q}_2,\underline{q}_1}$ to be given. $P^{exit,2}_{\overline{q}_2,\underline{q}_1}$ will be later endogenously determined in equilibrium of course. The two cases differ with respect to the number of price-triggers for investment. In the first case, there is a unique (high) price-trigger so that firm 1 invests if and only if the price gets larger than this trigger. In the other case there are two trigger prices a high and a low one. Investment at the low price-trigger is predatory.

However, to calculate the price-trigger(s) it is necessary to determine in a first step if predatory investment is profitable. To do so, we define two auxiliary "equity-value" functions. The first function to be defined is the equity value firm 1 would have if it could not investment. This is similar to the case where both firms have already invested but $\overline{q}_1$ is replaced with $\underline{q}_1$ and the exit-price trigger of firm 2 is replaced with $P^{exit,2}_{\overline{q}_2,\underline{q}_1}$. $\widetilde{E}_1(P,\underline{q}_1,\overline{q}_2)$ will denote this function. Given the exit price-trigger of firm 2 $P^{exit,2}_{\overline{q}_2,\underline{q}_1}$, one can easily decide if $\widetilde{E}_1(P,\underline{q}_1,\overline{q}_2)$ is similar to firm 1's or firm 2's value when both firms have invested. This function obviously defines a lower bound for $E_1(P,\underline{q}_1,\overline{q}_2)$.

It hence is *necessary* for predatory investment to be profitable that

$$\widetilde{E}_1(P,\underline{q}_1,\overline{q}_2) + C_1 = E_1(\Delta_{\underline{q}^1+\overline{q}^2,\overline{q}^1+\overline{q}^2}P,\overline{q}_1,\overline{q}_2)$$

has more than one solution in P. The largest solution to this equation also defines a lower bound for the non-predatory investment price-trigger.[13]

Lemma 5.1 *(i) Other things equal*

$$\widetilde{E}_1(P,\underline{q}_1,\overline{q}_2) + C_1 = E_1(\Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}P,\overline{q}_1,\overline{q}_2) \tag{5.22}$$

has at most three solutions in $P > \max\{P^{exit,2}_{\overline{q}_2,\underline{q}_1}, P^{exit,1}_{\underline{q}_1,\overline{q}_2}\}$. *We denote the solutions with* $P^*(< P^{**})(< P^{***})$ *respectively and the set of solutions by* S.

[13] A remark to notation is necessary at this point: The solution to an equation of the form $lhs(P(Y)) = rhs(P(Y))$, like equation (5.22), is noted as a "price level" for which the equation is solved. However, this is slightly incorrect, as we rather obtain solutions in Y. E.g. the solution might be at a price-level, lower than the exit-price of one of the firms. Therefore, the exit-decision has to be taken into account, since it causes prices to rise. So we normalize for induced changes in price. Nevertheless we stick to the notion as "price level", as it is easier to interpret.

Moreover, if P is large the following inequality holds:

$$\widetilde{E}_1(P, \underline{q}_1, \overline{q}_2) + C_1 < E_1(\Delta_{\underline{q}_1+\overline{q}_2, \overline{q}_1+\overline{q}_2} P, \overline{q}_1, \overline{q}_2). \tag{5.23}$$

Proof. *See appendix.* ■

Although equilibrium exit-price-triggers will at first be derived in the next subsection, individual optimality already puts a restriction to the exit-price-triggers, as the following Lemma shows. This Lemma proves useful in discussing the existence of predatory investment in our model.

Lemma 5.2 *(i) If firm 2 leaves the market first $P^{exit,2}_{\overline{q}_2,0} < P^{exit,2}_{\overline{q}_2,\underline{q}_1}$ holds.*
(ii) Moreover, in all cases $P^{exit,2}_{\overline{q}_2,\underline{q}_1} < \Delta^{-1}_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2} P^{exit,2}_{\overline{q}_2,0}$.

Proof. *See appendix.* ■

The second auxiliary equity-value function to be defined, is the value equity would have, if investment was not allowed for prices below $\max(S)$. This prohibits predatory investment, if $P^* > \Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}{}^{-1} P^{exit,2}_{\overline{q}_2,0}$. We denote this second auxiliary function by $\widehat{E}_1(P, \underline{q}_1, \overline{q}_2)$. Note that—given the exit strategy of firm 2—this function is well defined by the usual smooth-pasting and value-matching conditions.[14]

With these auxiliary functions at hand, three possible investment schemes can now be distinguished:

1. A situation may occur, where firm 2 exits at quite high prices in situation $(\overline{q}_1, \overline{q}_2)$, the demand is very inelastic,[15] and the costs of investing are low, so that the value-gain of equity of firm 1 in situation $(\overline{q}_1, \overline{q}_2)$ is always larger than the costs of investing. This however means that firm 1 would *invest at any price*. This is the case if for all $P > P^{exit,2}_{\overline{q},\underline{q}}$

$$E_1(\Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2} P, \overline{q}_1, \overline{q}_2) > C_1 + \widetilde{E}_1(P, \underline{q}_1, \overline{q}_2).$$

[14] See appendix for details.

[15] This means prices react strongly to changes in total output.

Therefore, in this case

$$E_1(P, \underline{q}_1, \overline{q}_2) \equiv E_1(\Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2} P, \overline{q}_1, \overline{q}_2) - C_1 . \tag{5.24}$$

2. If $\#S = 1$ or $\#S > 1$ but

$$\widehat{E}_1(P, \underline{q}_1, \overline{q}_2) = E_1(\Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2} P, \overline{q}_1, \overline{q}_2) - C_1 \tag{5.25}$$

has one and only one solution in $\left[P^{exit,2}_{\overline{q}_2,\underline{q}_1}, \infty\right[$, we have

$$E_1(P, \underline{q}_1, \overline{q}_2) = \widehat{E}_1(P, \underline{q}_1, \overline{q}_2) \quad .$$

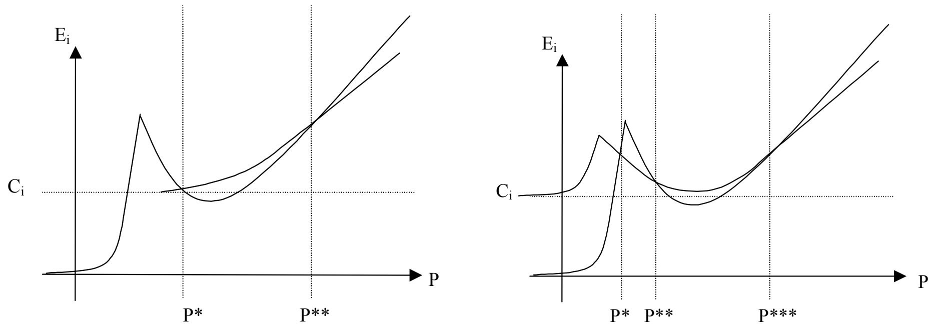
This will usually be the non-predatory behavior case. However, if $P^{exit,2}_{\overline{q}_2,\underline{q}_1} < \Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}{}^{-1} P^{exit,2}_{\overline{q}_2,0}$, we may obtain $P^* < \Delta^{-1}_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2} P^{exit,2}_{\overline{q}_2,0}$, which is predatory, though we will not refer to this case as predatory investment.

3. If $\#S > 1$ and (5.25) has more than one solution in $\left[P^{exit,2}_{\overline{q}_2,\underline{q}_1}, \infty\right[$, firm 1 will predatorily invest, i.e. firm 1 invests at low prices to crowd firm 2 out of the market. However, the exact structure of the value-function of firm 1 depends on the number of solutions to (5.25):

   (a) If (5.25) has two solutions, we will get an Investment/ No-Investment/ Investment scheme, i.e. a low price-trigger for which investment occurs and a high price-trigger for investment and a region of inactivity in between. See figure 5.2(a). We can then obtain both price-triggers and the equity value by applying standard boundary and smooth-pasting conditions for investment. However, for the predatory-investment price-trigger $P^{pred,1}_{\underline{q}_1,\overline{q}_2}$ (low price-trigger) the smooth-pasting condition needs not to hold and the border solution $P^{pred,1}_{\underline{q}_1,\overline{q}_2} = \Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}{}^{-1} P^{exit,2}_{\overline{q}_2,0}$ may well be obtained.

   (b) If (5.25) has three solutions, the situation gets even more complex: If occasionally P is very low, firm 1 will not invest, but will invest

Figure 5.2: Solutions to (5.22)



(a) two solutions (b) three solutions

as prices rise. However, smooth-pasting conditions need not to hold in this situation. See figure 5.2(b). Starting between P^{**} and P^{***}, we obtain the same Investment/ No-Investment/ Investment scheme obtained under (a). Again note the possibility of a border-solution for the predatory-investment price-triggers.

To simplify the following analysis we shall rule out the latter case by assumption:

Assumption 5.3: The costs of investment shall be such that firm 2 will never invest at a price level that would lead to a type 3(b) predatory investment case with very low prices, where firm 1 would predatorily invest as soon as prices rise.

The following proposition discusses the general possibility of predatory investment. The exact equations describing equity value and determining price-triggers will be derived afterwards.

Proposition 5.4 *(i) If demand is sufficiently elastic, i.e.* $\forall Q_1, Q_2 : \Delta_{Q_1,Q_2} \approx 1$, *or if demand is not too inelastic and the costs of investment* C_1 *are sufficiently high, then predatory investment never occurs.*
(ii) If $g > \left(\frac{\overline{q}_1 b_1}{\overline{q}_2 b_2}\right)^{1-\beta_2}$ *and*

(a) if firm 1 exits first when it had no investment option and if

$$\widetilde{P}^{exit,1}_{\underline{q}_1,\overline{q}_2} < \Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}{}^{-1} P^{exit,2}_{\overline{q}_2,0}, \tag{5.26}$$

then there exists an investment-cost C_1, so that (5.22) *has multiple solutions.*

(b) if firm 2 exits first and if for the right-hand partial derivative $\frac{\partial E_1}{\partial P^+}$

$$\frac{\partial \widetilde{E}_1}{\partial P}\left(\Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}{}^{-1} P^{exit,2}_{\overline{q}_2,0}, \underline{q}_1, \overline{q}_2\right) > \frac{\partial E_1}{\partial P^+}\left(P^{exit,2}_{\overline{q}_2,0}, \overline{q}_1, \overline{q}_2\right) \Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}$$

holds, then there exists the cost of investing C_1, so that predatory investment occurs.

Proof. *See appendix.* ■

In case, predatory investment indeed occurs, the exit value-matching condition for firm 1 has to be modified to

$$E_1^{pred}(P^{pred,1}_{\underline{q}^1,\overline{q}^2}, \underline{q}_1, \overline{q}_2) = E_1(\Delta_{\underline{q}^1+\overline{q}^2,\overline{q}^1} P^{pred,1}_{\underline{q}^1,\overline{q}^2}, \overline{q}_1, \overline{q}_2) - C_1 \quad . \tag{5.27}$$

Let $\frac{\partial}{\partial P^+}$ denote the right-hand partial-derivative. Due to Lemma 2 (and since we have to allow for a corner solution) we obtain for the necessary (smooth pasting) condition the following generalized expression:

$$\forall P \geq \Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}{}^{-1} P^{exit,2}_{\overline{q}_2,0} :$$
$$\left(\frac{\partial E_1^{pred}}{\partial P}(P^{pred,1}_{\underline{q}^1,\overline{q}^2}, \underline{q}_1, \overline{q}_2) - \frac{\partial E_1}{\partial P^+}(P^{pred,1}_{\underline{q}^1,\overline{q}^2}, \overline{q}_1, \overline{q}_2)\Delta_{\underline{q}^1+\overline{q}^2,\overline{q}^1}\right)\left(P^{pred,1}_{\underline{q}^1,\overline{q}^2} - P\right) \leq 0. \tag{5.28}$$

Corollary 5.1 *The predatory investment-price trigger $P^{pred,1}_{\underline{q}^1,\overline{q}^2}$ is always strictly larger than $P^{exit,2}_{\overline{q}_2,\underline{q}_1}$, $P^{pred,1}_{\underline{q}^1,\overline{q}^2} > P^{exit,2}_{\overline{q}_2,\underline{q}_1}$.*

Proof. *Follows straightforward from Lemma 5.2 and the definition of the predatory investment-price trigger.* ■

Equilibrium exit-strategies

The exit strategy of firm 2 was taken to be given so far. However, exit strategies have to be determined as an equilibrium of both firms competing to monopolize the market.

Independently of the (q_i, q_{-i})-state equilibrium exit strategies can be characterized as follows:

Definition 5.1 *A vector of (q_i, q_{-i})-state-contingent price-triggers $\bar{P}_i^{\#}$ is a* ***Markov-perfect best-response in pure strategies*** *of firm i, given the vector of price-triggers $P_{-i}^{\#}$ of firm $-i$, if for all (q_i, q_{-i}) not to exit before the declared price-trigger is credible*

$$\forall P > \bar{P}_{q_i,q_{-i}}^{exit,i} : E_i\left(P, q_i, q_{-i}|\bar{P}_i^{\#}, P_{-i}^{\#}\right) > 0. \qquad \text{(limited liability)}$$

Secondly, it also must be credible not to preempt on the own proposed price triggers for investment, i.e.

$$\begin{aligned} \forall P \leq \bar{P}_{\underline{q}_i,q_{-i}}^{inv,i} &: E_i\left(P, \underline{q}_i, q_{-i}|\bar{P}_i^{\#}, P_{-i}^{\#}\right) \geq E_i\left(P, \overline{q}_i, q_{-i}|\bar{P}_i^{\#}, P_{-i}^{\#}\right) - C_i \\ \forall P \geq \bar{P}_{\underline{q}_i,q_{-i}}^{pred,i} &: E_i\left(P, \underline{q}_i, q_{-i}|\bar{P}_i^{\#}, P_{-i}^{\#}\right) \geq E_i\left(P, \overline{q}_i, q_{-i}|\bar{P}_i^{\#}, P_{-i}^{\#}\right) - C_i. \end{aligned} \qquad \text{(no preemption)}$$

For all price-triggers $P_i^{\#}$ (which may include predatory investment-triggers) that fulfill the above credibility constraints, the proposed price-triggers need to be optimal, i.e.

$$\begin{aligned} E_i\left(\bar{P}_{q_i,q_{-i}}^{exit,i}, q_i, q_{-i}|\bar{P}_i^{\#}, P_{-i}^{\#}\right) &\geq E_i\left(\bar{P}_{q_i,q_{-i}}^{exit,i}, q_i, q_{-i}|P_i^{\#}, P_{-i}^{\#}\right) \\ E_i\left(\bar{P}_{q_i,q_{-i}}^{inv,i}, q_i, q_{-i}|\bar{P}_i^{\#}, P_{-i}^{\#}\right) &\geq E_i\left(\bar{P}_{q_i,q_{-i}}^{inv,i}, q_i, q_{-i}|P_i^{\#}, P_{-i}^{\#}\right). \end{aligned} \qquad \text{(optimality)}$$

Definition 5.2 *A* ***Markov-perfect equilibrium in pure strategies*** *is a pair of vectors $\left(P_i^{\#}, P_{-i}^{\#}\right)$, so that each is a best response of the other.*

Thus, a firm that defaults first, uses the value-matching and smooth pasting condition

$$E_i(P^{exit,i}_{q_i,q_{-i}}, q_i, q_{-i}) = \frac{\partial E_i}{\partial P}(P^{exit,i}_{q_i,q_{-i}}, q_i, q_{-i}) \stackrel{!}{=} 0 \tag{5.29}$$

to determine its own exit price-trigger. If a firm expects to leave second, the value function of this firm is flat in its own exit price-trigger. Thus, this firm is indifferent about the level of its own exit price-trigger on the margin. Therefore, there always is a multiplicity of equilibria which only differ at the (virtual) exit price-trigger of the firm which exits second.

Now denote by $P^{ind,i}_{q_i,q_{-i}}$ the largest exit-price-trigger of firm i that lets the limited liability constraint of firm $-i$ hold with equality for some P'. If firm i chooses an exit price trigger below $P^{ind,i}_{q_i,q_{-i}}$, firm $-i$ cannot credibly threaten to exit later as the limited liability constraint would bind. Thus, firm $-i$ exiting at the myopic price-trigger and the other firm choosing an exit price-trigger smaller than $P^{ind,i}_{q_i,q_{-i}}$ will be an equilibrium, if $P^{ind,i}_{q_i,q_{-i}}$ fulfills the other equilibrium conditions of firm i. Note that $P^{ind,i}_{q_i,q_{-i}} \geq \Delta^{-1}_{q_i+q_{-i},q_{-i}} P^{exit,-i}_{q_{-i},0}$ always holds since firm $-i$ would immediately exit in state $(q_{-i}, 0)$ at a price below $\Delta^{-1}_{q_i+q_{-i},q_{-i}} P^{exit,-i}_{q_{-i},0}$.

The following proposition describes all equilibria of the exit-game depending on the parameters of the actual environment.

Proposition 5.5 *(i) If for firm i the myopic exit price-trigger $P^{m\_exit,i}_{q_i,q_{-i}}$, obtained from* (5.29)*, is smaller than $P^{ind,i}_{q_i,q_{-i}}$, then firm $-i$ choosing $P^{m\_exit,-i}_{q_{-i},q_i}$ and firm i choosing a lower price-trigger is an equilibrium of the (q_i, q_{-i}) stage.*
(ii) If $\left[\Delta_{q_i+q_{-i},q_i}{}^{-1} P^{exit,i}_{q_{-i},0}, P^{ind,i}_{q_i,q_{-i}}\right] = \emptyset$, then firm i chooses $P^{m\_exit,i}_{q_i,q_{-i}}$ as the exit-price-trigger in all equilibria of the (q_i, q_{-i}) stage.
(iii) If $P^{m\_exit,-i}_{q_{-i},q_i} > P^{ind,i}_{q_i,q_{-i}}$ and $\left[\Delta_{q_i+q_{-i},q_i}{}^{-1} P^{exit,i}_{q_{-i},0}, P^{ind,i}_{q_i,q_{-i}}\right] \neq \emptyset$, then firm i choosing some $P^{exit,i}_{q_i,q_{-i}} \in \left[\Delta_{q_i+q_{-i},q_i}{}^{-1} P^{exit,i}_{q_{-i},0}, P^{ind,i}_{q_i,q_{-i}}\right]$ and firm $-i$ choosing $P^{m\_exit,-i}_{q_{-i},q_i}$ is an equilibrium of the (q_i, q_{-i}) stage, if this yields no incentive to predatorily invest for firm $-i$.
(iv) If both firms have an incentive (and option) to predatorily invest, instead

of choosing their myopic exit price-trigger, then both firm preempt on predatory investment.[16]
(v) If firm i predatorily invests, firm $-i$ cannot credibly threaten to deviate from choosing the myopic exit price-trigger to hinder i in investing predatorily.

Proof. *See appendix.* ■

Remark 5.1 *Part (iii) of the last proposition gives rise to the problem of multiplicity of equilibria. For the numerical simulations the firm with the larger $P^{ind,i}_{q_i,q_{-i}}$ is selected as the one, who leaves second.*
This selection can be motivated by the following idea: Suppose firm $-i$ chooses an exit price-trigger $\bar{P}$ marginally larger than $P^{ind,i}_{q_i,q_{-i}}$. Is the choice of $P^{ind,i}_{q_i,q_{-i}}$ still credible then? Of course not for the firm with the lower $P^{ind,i}_{q_i,q_{-i}}$, as $P^{ind,i}_{q_i,q_{-i}} < \bar{P} < P^{ind,-i}_{q_{-i},q_i}$, so that firm i would find it optimal to leave before $\bar{P}$ is actually reached, even when (falsely) expecting to leave second.
Conversely, this rule can be interpreted as a notion of conservativeness in the following sense: Suppose the shareholders of firm i imagine the worst case, i.e. firm $-i$ defaults just one logical second before $i's$ (proposed) trigger price is reached. Then only if the proposed exit price trigger is larger than the own indifference price trigger, i has no incentive to exit before.
Another motivation for this rule would be that firms stepwise and sequentially undercut each other's exit price-triggers before the actual game commences.

5.4.3 Investment- and bankruptcy-decisions in duopoly when no firm has invested yet

We shall now turn to the investment decision of the leader. Here however, the problem becomes more complex, as there may be a preemption game for both, predatory investment and "fundamental" investment decisions. However, if second mover advantages are not too strong, one can obtain a relatively simple rationale for the investment trigger prices.

[16] *See section 5.4.3*

To avoid further complication, we will make the following assumption according to the price-level (and investment costs) in $t = 0$:

Assumption 5.4: At the initial price-level P_0 at least one firm finds it unprofitable to invest and both firms find it unprofitable to declare bankruptcy. Moreover, P_0 lies between the preemption thresholds $\left(P_{pre}^{pred,i}, P_{pre}^{inv,i}\right)$, which will be define below, i.e.

$$\min_{i=1,2}\{P_{pre}^{pred,i}\} < P_0 < \max_{i=1,2}\{P_{pre}^{inv,i}\} \quad .$$

Non-predatory investment

To disentangle the interrelated decisions at the early stage of the game (when both firms still do not have invested) non-predatory investment is analyzed separately in a first-step. The exit and investment strategy of the respective other firm are assumed to be given for the moment. Just as we did when firm 1 was the follower, we construct a hypothetical value-function for the leading firm i. For this hypothetical value-function we again assume firm i would not have any investment option. Hence, this function exhibits a kink at the price-trigger where the follower, firm $-i$, invests.

Non-predatory investment of a leader (with the leader's role preassigned) is again defined as investment, that occurs at prices larger than the largest price-level P^* for which this hypothetical value function intersects with the value at output $(\overline{q}_i, \underline{q}_{-i})$ less investment costs.[17] However, if this intersection is at a higher price than the investment price of firm $-i$, then firm i cannot profitably invest in a non-predatorily before firm $-i$ invests. Therefore, under the assumption that firm $-i$ does not predatorily invest, we obtain the following value-matching conditions for our second hypothetical value-function:

$$\widehat{E}_i(P_{\underline{q}_{-i},\underline{q}_i}^{exit,-i}, \underline{q}_i, \underline{q}_{-i}) = E_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\underline{q}_i} P_{\underline{q}_{-i},\underline{q}_i}^{exit,-i}, \underline{q}_i, 0) \qquad (5.30)$$

[17] Again a case where investment is profitable at any price may occur. However, we will not further comment on this possible case.

if firm i chooses an exit price-trigger larger than the one of firm $-i$ and

$$\widehat{E}_i(P^{exit,i}_{\underline{q}_i,\underline{q}_{-i}}, \underline{q}_i, \underline{q}_{-i}) = 0 \tag{5.31}$$

otherwise. If firm i non-predatorily invests, we obtain

$$\widehat{E}_i(P^{inv,i}_{\underline{q}_i,\underline{q}_{-i}}, \underline{q}_i, \underline{q}_{-i}) = E_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\overline{q}_i+\underline{q}_{-i}} P^{inv,i}_{\underline{q}^i,\underline{q}^{-i}}, \overline{q}_i, \underline{q}_{-i}) - C^i \tag{5.32}$$

and if firm $-i$ invests first, we obtain

$$\widehat{E}_i(P^{inv,-i}_{\underline{q}_{-i},\underline{q}_i}, \underline{q}_i, \underline{q}_{-i}) = E_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\underline{q}_i+\overline{q}_{-i}} P^{inv,-i}_{\underline{q}_{-i},\underline{q}_i}, \underline{q}_i, \overline{q}_{-i}). \tag{5.33}$$

The investment price-trigger $P^{inv,i}_{\underline{q}_i,\underline{q}_{-i}}$ is then determined as the solution $P^{inv,i}_{\underline{q}_i,\underline{q}_{-i}} \in \Theta := \left[P^{pred,-i}_{\underline{q}_{-i},\underline{q}_i}, P^{inv,-i}_{\underline{q}_{-i},\underline{q}_i}\right]$ to the generalized smooth-pasting condition

$$\forall P \in \Theta : \left(\frac{\partial \widehat{E}_i}{\partial P}(P^{inv,i}_{\underline{q}_i,\underline{q}_{-i}}, \underline{q}_i, \underline{q}_{-i}) - \frac{\partial E_i}{\partial P}(P^{inv,i}_{\underline{q}_i,\underline{q}_{-i}}, \overline{q}_i, \underline{q}_{-i}) \cdot \Delta_{\underline{q}_i+\underline{q}_{-i},\overline{q}_i+\underline{q}_{-i}}\right) \times \left(P^{inv,i}_{\underline{q}_i,\underline{q}_{-i}} - P\right) \leq 0. \tag{5.34}$$

Proposition 5.5 can be used to find an exit-equilibrium, given the investment strategies of firm $-i$. If predatory investment at the early stage of the game is possible, can be checked by expressions analogous to those obtained in the case when firm 1 is the follower.

Equilibrium investment

The investment strategies of firm $-i$ obviously cannot be taken as given, i.e. we have to determine the exit- and investment-equilibrium at the early stage of the game simultaneously. Remember that we were merely interested in competitive outcomes. Thus, we assumed that collusive investment strategies are not feasible.

We define as Huisman and Kort (1999) a function $\Phi_i(P)$ which represents the advantage of taking the role of the leader and investing at P instead of becoming the follower, when the other firm invests at price P.

$$\Phi_i(P) := \underbrace{E_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\overline{q}_i+\underline{q}_{-i}}P, \overline{q}_i, \underline{q}_{-i}) - C}_{\text{Value as leader}} - \underbrace{E_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\underline{q}_i+\overline{q}_{-i}}P, \underline{q}_i, \overline{q}_{-i})}_{\text{Value as follower}} \tag{5.35}$$

Note that this expression is independent of exit decisions in the early stage of the game and is thus well defined, although we have not identified the exit equilibrium, yet.

To describe the root-behavior of $\Phi_i(P)$ a bit more notation has to be introduced. We denote the state-$(\underline{q}, \underline{q})$ price at which firm i invests as follower by $\overline{P}_i$, i.e.

$$\overline{P}_i = \Delta^{-1}_{\underline{q}_i+\underline{q}_{-i},\underline{q}_i+\overline{q}_{-i}} P^{inv,i}_{\underline{q}_i,\overline{q}_{-i}}$$

analogously we define $\underline{P}_i$ as the price at which firm i quits as follower, invests predatorily, or firm $-i$ quits, whichever happens first.

Proposition 5.6 *(i) On* $M := [\max_{j=1,2}\{\underline{P}_j\}, \min_{j=1,2}\{\overline{P}_j\}]$

$$\Phi_i(P) = 0 \tag{5.36}$$

has at most three solutions.
(ii) If $\overline{P}_i \leq \overline{P}_{-i}$*, then* $\Phi_i(P) = 0$ *has at most two solution on* M *and only one additional one for* $\overline{P}_i \leq P \leq \overline{P}_{-i}$*, namely* $\overline{P}_{-i}$ *with* $\Phi_i'(\overline{P}_{-i}) < 0$.
(iii) If $\overline{P}_i > \overline{P}_{-i}$ *, then* $\Phi_i(\overline{P}_i) = 0$ *and* $\Phi_i(P) < 0$ *for all* $\overline{P}_i > P > \overline{P}_{-i}$.
(iv) If $\Phi_i(P) = 0$ *has two solutions on* M *in case (ii) or three solutions in case (iii), then* $\Phi_i(\max\{\underline{P}_i\}) > 0$*. Moreover, there can only exist an additional solution on* $\min_{j=1,2}\{\underline{P}_j\} < P < \max_{j=1,2}\{\underline{P}_j\}$ *if* $\underline{P}_i < \underline{P}_{-i}$.

Proof. *See appendix.* ■

If there are two solutions to $\Phi_i(P) = 0$ in case (ii) (respectively three solutions in case (iii)) we will call the smaller one the preemption-threshold for predatory investment and the larger one preemption-threshold for non-predatory investment. They define a threshold at which firm i is just indifferent between being the leader and being the follower. We will denote these thresholds by $P_{pre}^{pred,i}$ and $P_{pre}^{inv,i}$ respectively. If there are less solutions, we only define a non-predatory preemption threshold.

If in case (ii) there is no solution to (5.36) on M then there is a solution for smaller P since the leader´s payoff is negative at its effective exit price, while it is zero for this firm being follower. We then denote this solution by $P_{pre}^{inv,i}$. Moreover, we set $P_{pre}^{pred,i} = \overline{P}_i$. This is the only case in which $P_{pre}^{inv,i} > P_{pre}^{pred,i}$.

If in case (iii) $\Phi_i(P) < 0$ for all $P < \overline{P}_i$, we set $P_{pre}^{inv,i} = \overline{P}_{-i}$.

A typical problem that arises in timing games of (dis-)investment is the multiplicity of equilibria associated with Fudenberg and Tirole´s (1985) notion of perfect timing game equilibria. The non-uniqueness typically arises if none of the firms has an unilateral incentive to invest, while both firms have an incentive to preempt each other.[18] However, this non-uniqueness disappears when we look at the renegotiation-proof equilibria only and make the following additional assumption.

Assumption 5.5: Let i be the firm with the larger $\overline{P}_i$, then the largest solution to (5.36) for firm $-i$ shall be larger, than the optimal unconstrained[19] investment price trigger of firm i. Moreover at least for one firm there exists an (interior) solution for $P_{pre}^{inv,i}$ on M.

This assumption assures that second-mover advantages not being "too strong"

Proposition 5.7 *Under Assumption 5.5 the only Markov-perfect equilibrium for the preemption game for non-predatory investment is that the firm*

[18] See e.g. Weeds (2001) or Sparla (2001).

Indeed, in some numerical simulations (not reported) we found non-renegotiation-proof predatory equilibria. Renegotiation-prooveness is but a strong assumption on rationality.

Therefore, we might in reality observe a circular situation of the following form: One agent takes pre-emptive, predatory action in threat of a predatory action of the other agent. This other agent however has no incentive to undertake that action as long as the first agent does not take action. The first agent however takes action as she is threatened. Triggered off by a sunspot, the situation escalates.

[19] This means $P_{\underline{q}_{-i},\underline{q}_i}^{inv,-i}$ is set to infinity in calculating the price-triggers.

with the smaller $P_{pre}^{inv,i}$ *takes the lead, and chooses an investment-price trigger, which is a solution to* (5.34)*, where* $P_{\underline{q}_{-i},\underline{q}_i}^{inv,-i} = P_{pre}^{inv,-i}, P_{\underline{q}_{-i},\underline{q}_i}^{pred,-i} = P_{pre}^{inv,i}$*. We denote this investment price trigger by* $P_{\underline{q}_L,\underline{q}_{-L}}^{inv,L}$*.*

Proof. *First note that at* $P_{\underline{q}_L,\underline{q}_{-L}}^{inv,L}$ *firm L indeed prefers to be the leader, moreover because of assumption 5.5 we have a preemption game for non-predatory investment: Suppose* τ *denotes the stopping-time associated with the optimal investment-price trigger of firm* i*. Then firm* $-i$ *has an incentive to invest at time* $\tau - \epsilon$*, as long as the price is above its preemption threshold. Therefore, when* $P_t \in [P_{pre}^{pred,-L}, P_{pre}^{inv,-L}]$ *firm* $-L$ *prefers to be the follower, while at* $P_{\underline{q}_L,\underline{q}_{-L}}^{inv,L}$ *firm L profitable invests.* ∎

However, an existing preemption threshold for predatory investment does not necessary imply predatory investment to occur in a renegotiation-proof Markov-perfect equilibrium:

Proposition 5.8 *There can only occur predatory investment in a renegotiation-proof Markov-perfect equilibrium, if at least one firm can profitably invest predatorily, without assuming that the other firm invests predatorily, i.e. a* $P' < P_{\underline{q}_L,\underline{q}_{-L}}^{inv,L}$ *exists, such that*

$$E_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\overline{q}_i+\underline{q}_{-i}}P', \overline{q}_i, \underline{q}_{-i}) - C \geq \widehat{E}_i(P', \underline{q}_i, \underline{q}_{-i}), \tag{5.37}$$

where $\widehat{E}_i$ *denotes the equity value without the possibility of predatory investment. If* (5.37) *has indeed such a solution, we will call predatory investment for firm* i *to be "fundamentally profitable".*

Proof. (5.37) *defines a kind of net-present value rule to predatory investment: The option to predatorily invest exists if and only if predatory investment can give a net-present-value gain. If predatory investment is not profitable for both firms unless the other firm predatorily invests, then both firms can renegotiate not to invest predatorily.* ∎

Proposition 5.9 *A solution to* (5.37) *implies that a predatory investment preemption threshold for firm* i *exists, i.e. there cannot be second-mover advantages for profitable predatory investment independent of how low* P *gets.*

Proof. *See appendix.* ■

Therefore, if both firms have a predatory-investment preemption-threshold price, or if the non-predatory investment threshold of one firm is smaller than the predatory one of the other firm, we have a preemption game for predatory investment. However, both firms only enter this game, if at least one firm finds predatory investment fundamentally profitable. If only for one firm a predatory-investment preemption threshold is defined, and if this firm finds predatory investment fundamentally profitable, then it will predatorily invest, indeed. In this case it sets the predatory investment-price trigger according to a generalized smooth pasting condition analogous the one derived for firm 1 as follower.

Proposition 5.10 *(i) If there is a preemption game for predatory investment and* $P_{pre}^{pred,i} < P_{pre}^{inv,-i}$ *for both firms, the only Markov-perfect equilibrium (outcome) is that the firm with the higher* $P_{pre}^{pred,i}$*, takes the lead for predatory investment and invests at a price-trigger which is a solution to a version of* (5.34) *that is modified by defining* $P_{\underline{q}_{-i},\underline{q}_i}^{inv,-i} := P_{pre}^{pred,i}$, $P_{\underline{q}_{-i},\underline{q}_i}^{pred,-i} := P_{pre}^{pred,-i}$ *and by using the appropriate value matching conditions.*
(ii) If there is a preemption game for predatory investment, and $P_{pre}^{pred,i} > P_{pre}^{inv,-i}$ *for firm i, and one of the firms has an unilateral incentive to invest in* $\left[P_{pre}^{inv,-i}, P_{pre}^{pred,i}\right]$,[20] *then in all Markov-perfect equilibria firm* $-i$ *invests predatorily at* $P_{pre}^{pred,i}$.
(iii) If there is a preemption game for predatory investment and $P_{pre}^{pred,i} > P_{pre}^{inv,-i}$ *for one of the firms, and none of the firms have an unilateral incentive to invest on* $\left[P_{pre}^{inv,-i}, P_{pre}^{pred,i}\right]$ *, then in all renegotiation-proof Markov-perfect equilibria firm i predatorily invests at its unconstrained optimal predatory investment price-trigger or at* $P_{pre}^{pred,-i}$*, whichever is the higher price.*

Proof. *See appendix.* ■

Corollary 5.2 *If both firms wish to predatorily invest, then the equilibrium*

[20] *This means the unconstrained optimal investment price-trigger falls in this interval.*

predatory investment price-trigger is strictly smaller than the equilibrium normal investment price-trigger.

The equilibrium exit strategies are determined analogously to the follower case. Therefore, we now have all equations that determine the equilibrium price-triggers of our model and so we can obtain numerical results and numerically check the importance of the strategic situation modelled in this chapter.

5.5 Value of debt

For completeness, we briefly state the two value-matching conditions which generally characterized the value of debt:

$$B_i(P^{exit,i}_{q_i,q_{-i}}, b_i, q_i, q_{-i}) = \lambda q_i \quad , \tag{5.38}$$

$$B_i(P^{Trig}, b_i, q_{i1}, q_{-i1}) = B_i(\Delta_{q_{i1}+q_{-i1},q_{i2}+q_{-i2}} P^{Trig}, b_i, q_{i2}, q_{-i2}). \tag{5.39}$$

P^{Trig} denotes an arbitrary decision price trigger that does not lead to an immediate exit of firm i and at which capacities are changed from (q_{i1}, q_{-i1}) to (q_{i2}, q_{-i2}). However, as we do not aim at deriving optimal leverage strategies, debt values are not reported in the numerical results.

5.6 Numerical results

Numerical solutions have been calculated for different parameter values. Table 5.1 contains the parameter-values for the non firm-specific parameters. In all calculations an isoelastic specification for the inverse demand-function has been used. Two base cases are considered. One to discuss the impact of changes in the leverage, and one to study the influences of the elasticity of demand and the tax rate.

Tables 5.2, 5.3 and 5.4 report the results for those two base cases. First of all, investment price-triggers for the duopoly and the monopoly differ very

Table 5.1: General Parameter Values, Base Cases

Tax Rate τ	0.3
Discount Rate ρ	0.05
Drift μ	0.03
Variance σ^2	0.1
Inverse-Demand	$Q^{-\xi}$

Table 5.2: Base Case I (a), (b): $\xi = 0.9$

	Firm 1 (a)	Firm 1 (b)	Firm 2
Production before Investment	18	17.9	18
Production after Investment	20	20	20
Investment-Costs	120	120	120
Coupon-Payment [Debt]	49.9	49.9	50
MONOPOLY			
Exit-Price Trigger after investment	0.4496	0.4496	0.4505
Exit-Price Trigger before investment	0.4980	0.4980	0.4990
Investment-Price Trigger	99.844	95.337	99.844
DUOPOLY			
Both Invested, Exit-Price Trigger	Firm 2 exits	Firm 2 exits	0.45049
Firm 1 Follower: Exit-Price Trigger	0.4867	0.4888	Firm 1 exits
Firm 1 Follower: Investment-Price Trigger	15.872	15.061	Firm 1 inv.
Firm 2 Follower: Exit-Price Trigger	Firm 2 exits	Firm 2 exits	0.4881
Firm 2 Follower: Investment-Price Trigger	Firm 2 inv.	Firm 2 inv.	17.48
Preemption Threshold Non-Predatory Investment	4.57881	4.36741	4.895 (4.873)
Unilateral Incentive to predatorily Invest as Leader	No	No	Yes
Firm 1: Predatory Investment-Price Trigger	0.611624	0.6206	Firm 1 inv.

significantly[21].

For the follower, this is just the standard result of Cournot competition. For the preemption threshold of the leader, this is the usual effect of the strong first-mover advantages of the Stackelberg-leader. These first-mover advantages are additionally amplified in the presence of predatory behavior or if the order of exit is reversed by investment. These first-mover advantages drive the results of our base cases.[22] The (tax adjusted) net-present-value

[21] This is also true for the trigger values of Y, which can be obtained by rescaling the price triggers by 1.86607 for the leader and 1.95912 for the follower, setting $D(\overline{q}) = 1$.

[22] Note, that although interior solutions for the non-predatory investment price-trigger

Table 5.3: Base Case I (c): $\xi = 0.8$

	Firm 1	Firm 2
Production before Investment	17	18
Production after Investment	21	20
Investment-Costs	160	120
Coupon-Payment [Debt]	70	70
MONOPOLY		
Exit-Price Trigger after investment	0.601	0.631
Exit-Price Trigger before investment	0.7296	0.695
Investment-Price Trigger	34.75	49.77
DUOPOLY		
Both Invested, Exit-Price Trigger	Firm 2 exits	0.631
Firm 1 Follower: Exit-Price Trigger	0.693	Firm 1 exits
Firm 1 Follower: Investment-Price Trigger	9.293	Firm 1 inv.
Firm 2 Follower: Exit-Price Trigger	Firm 2 exits	0.680
Firm 2 Follower: Investment-Price Trigger	Firm 2 inv.	15.924
Preemption Threshold Non-Predatory Investment	3.579	3.109
Unilateral Incentive to predatorily Invest as Leader	No	Yes
Firm 1: Predatory Investment-Price Trigger	Firm 2 inv.	0.786

price triggers, P^{NPV}, for investment in duopoly in the two base cases were (according to rule 1 below) 5.7 (8.2) and 7.6 (10.9) respectively.[23] If one assumes the other firm would invest, unless firm i invests (rule 2), then the net-present-value rule yields a tax-adjusted price trigger of $P^{NPV} = 4.5$ in base case I (a), which is only slightly below the equilibrium investment-price trigger of firm 1.

Moreover, in calculating the adjusted net-present value price-trigger of

were allowed for, in the cases reported, the leader always invested at the preemption threshold of the follower.

[23]

$$P^{NPV}_{Rule\_1} = \frac{\rho C_i}{(1-\tau)\left(\overline{q}_i \Delta_{\underline{q}_i+\underline{q}_{-i},\overline{q}_i+\underline{q}_{-i}} - \underline{q}_i\right)} \tag{5.40}$$

$$P^{NPV}_{Rule\_2} = \frac{\rho C_i}{(1-\tau)\left(\overline{q}_i \Delta_{\underline{q}_i+\underline{q}_{-i},\overline{q}_i+\underline{q}_{-i}} - \underline{q}_i \Delta_{\underline{q}_i+\underline{q}_{-i},\underline{q}_i+\overline{q}_{-i}}\right)} \tag{5.41}$$

Table 5.4: Base Case II: $\xi = 0.7$

	Firm 1	Firm 2
Production before Investment	18	18
Production after Investment	20	20
Investment-Costs	160	160
Coupon-Payment [Debt]	40	50
MONOPOLY		
Exit-Price Trigger after investment	0.3604	0.4505
Exit-Price Trigger before investment	0.3963	0.4951
Investment-Price Trigger	43.9427	43.9679
DUOPOLY		
Both Invested, Exit-Price Trigger	Firm 2 exits	0.4505
Firm 1 Follower: Exit-Price Trigger	Firm 2 exits	0.4584
Firm 1 Follower: Investment-Price Trigger	19.59	Firm 1 inv.
Firm 2 Follower: Exit-Price Trigger	Firm 2 exits	0.4863
Firm 2 Follower: Investment-Price Trigger	Firm 2 inv.	19.73
Preemption Threshold Non-Predatory Investment	9.055	9.4218
Unilateral Incentive to predatorily Invest as Leader	No	No
Firm 2: Exit-Price Trigger before Investment	Firm 2 exits	0.4974

$P^{NPV} = 4.5$, the investment of firm 2 when the price-process reaches the follower's price-trigger is ignored. Therefore the "true" net-present-value rule investment-price trigger is larger than 4.5. In that sense, we can conclude that in some cases the threat of being forced to exit first outweighs the gains from waiting.

Comparing the results for the three sub-cases of Base Case I—(a) Firm 1 has lower leverage before and after investment, (b) Firm 1 has a higher leverage before, but lower leverage after investment and (c) the size of the investment projects differs between Firm 1 and 2—shows that the firm with the higher initial, the firm with the higher post-investment leverage, and the firm with the lower leverage in both states can prey in equilibrium. Therefore, our model includes not only the cases studied by Busse (2001) but also the cases in which the financially healthier firm preys.

Table 5.5 now reports the effects of a change in leverage of firm 1 (relative to Base Case I(a)). In states, in which investment does neither change the

Table 5.5: Effects of firm 1's debt on investment-price triggers

debt	$P^{inv,1}_{\underline{q}_1,0}$	$P^{inv,1}_{\underline{q}_1,\bar{q}_2}$	$P^{inv,1}_{pre}$	$P^{inv,2}_{pre}$
49.9*	99.8436	15.8715	4.57881	4.89525
49	99.8422	17.2948	6.0871	6.52665
48	99.8407	17.2942	6.08645	6.52679
47	99.8392	17.2935	6.08581	6.52694
40	99.8294	17.2894	6.08161	6.5279
35	99.8232	17.2868	6.07892	6.52851

* Firm 1 exits first as follower and also predatorily invests as leader.

Table 5.6: Effects of firm 2's debt on investment-price triggers

debt	$P^{inv,2}_{\underline{q}_2,0}$	$P^{inv,2}_{\underline{q}_2,\bar{q}_1}$	$P^{inv,2}_{pre}$	$P^{inv,1}_{pre}$
50	133.102	23.2316	8.57345	8.20269
55	133.108	23.2526	8.61058	8.1743
60	133.115	23.2749	8.64987	8.14432
70	133.13	23.3233	8.73448	8.07987
80	133.147	23.3762	8.8266	8.00988

ordering of exit price triggers, nor predatory investment would occur on the second stage, the effect of debt on investment-price triggers are rather minor.

Moreover, as firm 1 becomes leader in equilibrium, the probability of investment in duopoly shrinks with larger debt levels, as long as debt levels stay intermediate.

However, if both firms become more similarly leveraged, debt starkly influences investment decisions. If firm 2 can expect that firm 1 leaves the market first (in case firm 2 becomes the leader) then–as we have seen–first-mover advantages become very strong. The first-mover advantages can be even strong enough to let firm 1 in equilibrium invests below the simple net-present-value price trigger.

Therefore, the model presented in this chapter might indeed explain the U-shaped investment-debt relation found in our empirical analysis of the previous chapter. Figure 5.3 shows the Φ_i functions for both firms corresponding to the first base case.

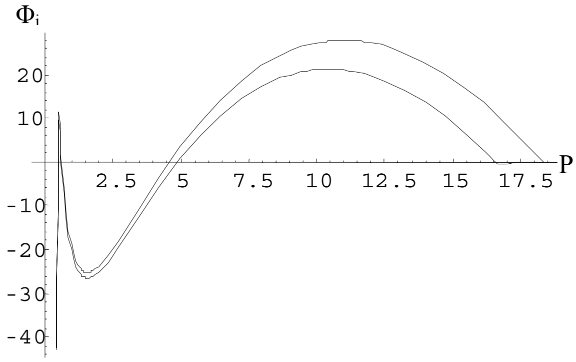


Figure 5.3: Gain, Φ_i, from becoming the leader, Base Case I (a)

Table 5.6 reports the investment price triggers for different debt-levels of firm 2 (with the base case II specifications). For firm 2 investment becomes more likely for lower debt levels, which is in line with Jou´s (2001) findings for monopolists and empirical evidence. Interestingly, different debt levels influence firm 2´s investment decision much stronger in a duopoly than in monopoly.

Unlike in a static model of Cournot-competition, here increasing the elasticity of demand make investment of the leader more likely, whereas the investment of the follower becomes less likely, as shown in table 5.7. Here again the strong first-mover advantages drive the result. This may serve as an independent test of the model.

Table 5.8 presents the price triggers for different corporate tax rates. We use Base Case II as the reference-case, because in Case I, firm 1 would invest immediately as follower for low tax rates. However, note that the effect of tax-rates of course only comes in, when tax rates do not lower investment costs by the same percentage, i.e. given debt, taxes are neutral if $C_i(\tau) = (1-\tau) C_i$.

At last one may wonder if the results hinge on the near symmetry of both firms in the cases studied. Yet, although only results for firms that are relatively symmetric were reported, solutions have been calculated for cases more asymmetric in production or investment costs, too. In more asymmetric

Table 5.7: Effects of the elasticity of demand on investment-price triggers

elasticity	$P^{inv,1}_{\underline{q}_1,0}$	$P^{inv,2}_{\underline{q}_2,0}$	$P^{inv,1}_{\underline{q}_1,\bar{q}_2}$	$P^{inv,2}_{\underline{q}_2,\bar{q}_1}$	$P^{inv,1}_{pre}$	$P^{inv,2}_{pre}$
−0.5	26.1318	26.1671	17.0162	17.1305	9.98429	10.3272
−0.6	32.8083	32.8389	18.2142	18.34	9.50913	9.86668
−0.7	43.9427	43.9679	19.5916	19.729	9.05488	9.42182
−0.8	66.2228	66.2419	21.1916	21.3402	8.61981	8.99124
−0.9	133.09	133.102	23.0725	23.2316	8.20269	8.57345

Table 5.8: Effects of the tax rate on investment-price triggers

tax rate	$P^{inv,1}_{\underline{q}_1,0}$	$P^{inv,2}_{\underline{q}_2,0}$	$P^{inv,1}_{\underline{q}_1,\bar{q}_2}$	$P^{inv,2}_{\underline{q}_2,\bar{q}_1}$	$P^{inv,1}_{pre}$	$P^{inv,2}_{pre}$
0.1	34.2003	34.2304	15.2379	15.399	6.98867	7.41972
0.2	38.462	38.4898	17.1425	17.2921	7.8937	8.29367
0.3	43.9427	43.9679	19.5916	19.729	9.05488	9.42182
0.4	51.2517	51.2744	22.8573	22.9816	10.6	10.9317
0.5	61.4864	61.5063	27.4294	27.5396	12.7591	13.0528

cases the results do not change much, except for predatory outcomes to be more likely, i.e. we obtain predatory equilibria also for cases with a greater difference in leverage ratios or more elastic demands.

Generally predatory outcomes are likely if firm 1 is small compared to firm 2 and both are heavily leveraged.

5.7 Summary of the results of chapter 5

At the end of this chapter, we may briefly summarize. We analyzed a real options model of a duopoly. This model simultaneously allowed for both irreversible investment and exit decisions. The duopoly was modelled in continuous time, firms were assumed to hold debt because of tax advantages, and were allowed to default on their obligations at no cost to the equityholders.

We showed how endogenous bankruptcy decisions alter the strategic situation significantly: Firms may invest not because investment is fundamentally profitable, but because this makes the exit of the competitor more likely. This, however, affects the value of the investment options themselves.

Therefore, in the presented model debt not only induces an agency problem, but also has a negative strategic effect, which reduces the value of debt. This might explain the low debt-ratios found in practice compared to the substantial tax benefits of debt.

Moreover, a discontinuous effect of debt on investment incentives was found. For moderate levels of debt, investment tends to decrease with increasing debt. However, if debt levels get large enough, investment-incentives become very strong, as both firms seek to become leader and monopolize the market when revenues decrease in the presence of adverse shocks. This strategic effect of debt might explain the non-monotone influence of debt on investment we found in chapter 4. Therefore, our model also explains predatory behavior in a dynamic setting with neither relying on an asymmetry of information among competitors, nor on learning-curve or network effects. Our numerical examples show that in equilibrium both the firm with the higher and the firm with the lower debt may prey.

Chapter 6

Concluding Remarks

Although we have discussed the main results of each chapter at the end of it, some concluding remarks may still be justified.

The goal of this work was modest. We wanted to show that financial frictions and non-convex adjustment costs interact in a non-trivial and important way, when firms decide upon investment behavior. To do so, we began with building a model of a monopolistically competitive firm which faces both financial constraints and fixed costs of investment. We found the financial status of the firm to be influential on both the adjustment speed and the optimal stock of capital in this model. Maybe, one should summarize the idea behind this effect in the following very stylized way: Suppose a firm completely leases its capital. Given an amount of liquidity and given a fixed cost of adjusting the stock of capital, this firm is employed with a certain number of options to change the stock of capital. The larger this number, the smaller the value of each of the options. However, the option-value adds to the fixed cost of adjustment. Hence, adjustment is the more frequent, the larger liquidity is.

To test our hypothesis, we analyzed company-account data from Germany and the UK. Indeed, we could find only a weak influence of finance on the long-run optimal stock of capital. Yet, we found the influence of finance on investment to be quite substantial. Moreover, greater liquidity also amplified the reaction of investment to changes in fundamentals. Therefore, our theoretical point appears to be well supported by the data.

On the basis of these results, we have built a model of duopoly in which two firms are both equipped with an irreversible investment and a bankruptcy option. As we have seen, these two options and competition interact in a complex manner. This interaction can trigger predatory investment in adverse states of demand. Moreover, we have seen this interaction to speed up investment more than the effect of competition alone would do.

So we might say we have reached our goal to a considerable extent. However, it is still not clear, how the proposed investment model would actually behave in a macroeconomic setting. Would the financial frictions exacerbate shocks to the economy or would they rather smooth the reaction of firms to those shocks over time? Suppose an economy is hit by an adverse shock, that drives down firm liquidity as well as productivity. On the one hand, firms would adjust their stock of capital more slowly than without the financial shock. Hence, there would be a lesser financial-multiplier effect, caused by a drop in the price of capital goods. On the other hand, the economic downturn would appear to be more persistent. Compared to the convex-cost framework the reaction of aggregate investment, may yet still be stronger, if the shock is large and many firms would be forced to fully adjust their stock of capital.

Another field of further research, related to our findings, could look for the interaction of non-convex adjustment cost and other rigidities than only an imperfect financial market. For example studying labor demand of firms with non-convex adjustment costs in both capital and labor would surely be a topic of interest.[1] Again these interdependent demands could be influenced by financial market imperfections.

Our duopoly model might be extended in several directions: The choice of debt may be endogenised and collusive behavior and simultaneous action may be allowed for. This might especially be of interest, as Glazer (1994)

[1] See Sakellaris (2001) for an exploratory study of this topic.

finds debt to make collusive behavior more likely in a two-period setting of the Brander and Lewis (1986) model. Moreover, market entry as a special case may be worthwhile to study. Another interesting extension of the duopoly model would be to allow for a multitude of investment options or to analyze welfare issues.

However, these questions go beyond both, the aim and scope of the present thesis.

Appendix A

Appendix to Chapter 3

In this appendix, we derive the Bellman equation which is central to our model in chapter 3. Thereafter, we show the existence and uniqueness of a solution to this equation. At the end of this appendix, some properties of the induced optimal-policy function are discussed.

A.1 Deriving the Bellman Equation

All variables, functions etc. are defined as in the main text, unless stated differently. The correspondence, X, of financial feasible capital-imbalance and debt pairs is given by:

$$\begin{aligned} X\left(K_t^*, w_t, z_t, B_t\right) &= \left\{ \begin{array}{c} z^o, B_{t+1} \in \mathbb{R}_+^2 | D(z^o, B_{t+1}, K_t^*, w_t, z_t, B_t) \geq 0 \\ \wedge B_{t+1} \leq \hat{b} z^o K_t^* \end{array} \right\} \\ &= \left\{ z^o, B_{t+1} \in \mathbb{R}_+^2 \;\middle|\; \begin{array}{c} (1 + i(\frac{B_t}{K_{t-1}}))B_t - \Pi(z^o, K_t^*)[1 - w_t \mathbb{I}_{\{z^o \neq z_t\}}] \\ +K_t^*(z^o - z_t) \leq B_{t+1} \leq \hat{b} z^o K_t^* \end{array} \right\} \end{aligned} \tag{A.1}$$

Dividing the expression by K_t^* and using $b_t := \frac{B_t}{K_{t-1}}$ and for the stock of capital before investment $K_{t-1} = \frac{K_t}{1-\delta}$ yields:

$$\hat{X}\left(K_t^*, w_t, z_t, b_t\right) := \\ = \left\{ z^o, b^o \in \mathbb{R}_{++} \times \mathbb{R}_+ \left| \begin{array}{c} \frac{(1+r(b_t))}{1-\delta} b_t z_t - \pi(z^o)[1 - w_t \mathbb{I}_{\{z^o \neq z_t\}}] \\ +(z^o - z_t) \leq b^o z^o \leq \hat{b} z^o \end{array} \right. \right\} \cup \{(0,0)\} \tag{A.2}$$

To obtain a more accessible form define e_t to be the equity-ratio in the opening balance and thus

$$e_t := e(b_t) = 1 - \frac{(1 + r(b_t))}{1 - \delta} b_t.$$

Define furthermore $c(z, b)$ to be the cash flow per unit of capital (including cash flow from newly issued debt, and "costs" for "buying back" the capital stock), that is

$$c_t := c(z^o, b^o) = \frac{\pi\left(z^o\right)}{z^o} + \left(b^o - 1\right).$$

We then get for $\hat{X}$:

$$\hat{X}\left(K_t^*, w_t, z_t, b_t\right) = \\ \left\{ z^o, b^o \in \mathbb{R}_{++} \times \mathbb{R}_+ \left| \begin{array}{c} e(b_t)\frac{z_t}{z^o} + c(z^o, b^o) - w_t \mathbb{I}_{\{z^o \neq z_t\}} \frac{\pi(z^o)}{z^o} \geq 0 \\ \wedge\ 0 \geq \left(b^o - \hat{b}\right) \end{array} \right. \right\} \cup \{(0,0)\}. \tag{A.3}$$

Next define

$$Y\left(w_t, z_t, e_t\right) := \left\{ z^o, b^o \in \mathbb{R}_{++} \times \mathbb{R}_+ \left| \begin{array}{c} e_t \frac{z_t}{z^o} + c(z^o, b^o) - w_t \mathbb{I}_{\{z^o \neq z_t\}} \frac{\pi(z^o)}{z^o} \geq 0 \\ \wedge\ 0 \geq \left(b^o - \hat{b}\right) \end{array} \right. \right\}. \tag{A.4}$$

For $e_t = e(b_t)$ we have $\hat{X} = Y \cup \{(0,0)\}$.

Now denote the value-function by V. For notational convenience define $Y := Y\left(w_t, z_t, e(b_t)\right)$. Then V is determined by the following Bellman equation:

$$
\begin{aligned}
&V(K_t^*, w_t, z_t, b_t) \\
&:= \max_{(z^o,b^o)\in X(K_t^*,w_t,z_t,b_t)} \left\{ D(z^o, b^o, K_t^*, w_t, z_t, b_t) + \psi \mathbb{E}_t \left[V(K_{t+1}^*, w_{t+1}, z_{t+1}, b^o) \right] \right\} \\
&= K_t^* \max_{\substack{(z^o,b^o)\\ \in Y}} \left\{ e(b_t)z_t + c(z^o,b^o)z^o - w_t \mathbb{I}_{\{z^o \neq z_t\}} \pi(z^o) + \psi \mathbb{E}_t \left[\frac{V(K_{t+1}^*, w_{t+1}, z_{t+1}, b^o)}{K_t^*} \right] \right\} \\
&= K_t^* \left[e(b_t)z_t + \max_{\substack{(z^o,b^o)\\ \in Y}} \left\{ \begin{array}{c} c(z^o,b^o)z^o - w_t \mathbb{I}_{\{z^o \neq z_t\}} \pi(z^o) + \\ +\psi \mathbb{E}_t \left[\left(\frac{V(K_{t+1}^*, w_{t+1}, z_{t+1}, b^o)}{K_{t+1}^*} - e(b^o)z_{t+1} + e(b^o)z_{t+1} \right) \frac{K_{t+1}^*}{K_t^*} \right] \end{array} \right\} \right] \\
&= K_t^* \left[e(b_t)z_t + \max_{\substack{(z^o,b^o)\\ \in Y}} \left\{ \begin{array}{c} c(z^o,b^o)z^o - w_t \mathbb{I}_{\{z^o \neq z_t\}} \pi(z^o) + \psi e(b^o) z^o (1-\delta) \\ +\psi \mathbb{E}_t \left[\left(\frac{V(K_{t+1}^*, w_{t+1}, z_{t+1}, b^o)}{K_{t+1}^*} - e(b^o)z_{t+1} \right) \frac{z^o(1-\delta)}{z_{t+1}}) \right] \end{array} \right\} \right]
\end{aligned} \tag{A.5}
$$

The first equality follows from the linear homogeneity in K^* of the function D and the linearity of the $\mathbb{E}_t-$operator, the second equality stems from the fact, that $e(b_t)z_t$ is no function of (z^o, b^o) and is thus not affected by the maximization. The third follows from the linearity of $\mathbb{E}_t$ and the definition of z_{t+1}. (3.3) yields $\frac{K_{t+1}^*}{K_t^*} = \exp(d + \xi_{t+1})$ and $z^o(1-\delta) = z_{t+1} \cdot \exp(d + \xi_{t+1})$. Now define $v_t := \frac{V(K_t^*, w_t, z_t, b_t)}{K_t^*} - e(b_t)z_t$. Due to the homogeneity of V, v does not depend on K_t^*. Thus we obtain:

$$
v(w_t, z_t, e(b_t)) := \max_{\substack{(z^o,b^o)\in \\ X(1,w_t,z_t,b_t)}} \left\{ \begin{array}{c} c(z^o,b^o)z^o - w_t \mathbb{I}_{\{z^o \neq z_t\}} \pi(z^o) + \psi e(b^o) z^o (1-\delta) \\ +\psi \mathbb{E}_t [v(w_{t+1}, z^o \frac{(1-\delta)}{\exp(d+\xi_{t+1})}, b^o) \exp(d + \xi_{t+1})] \end{array} \right\} \tag{A.6}
$$

That maximizing v leads to an equivalent policy to maximizing V has been discussed in the main text.

A.2 Existence and uniqueness

From now on time-indices will be suppressed. Due to Lemma 3.1 we can drop "no production" from the set of alternatives X and express the value function v as

$$v(w,z,e) = \max_{(z^o,b^o)\in Y(w,z,e)} \left[\begin{array}{c} c(z^o,b^o) - w\pi(z^o) I_{\{z^o\neq z_t\}} + (1-\delta)\psi e(b^o) z^o \\ +\psi \iint \left[v\left(\epsilon, z^o \frac{1-\delta}{\exp(d+\xi)}, e(b^o)\right) \exp(d+\xi) \right] dF(\xi) dG(\epsilon) \end{array} \right] \tag{A.7}$$

or respectively as:

$$v(w,z,e) = \begin{cases} \max\{v_{no\ \mathrm{adj}}(z,e), v_{\mathrm{adj}}(w,z,e)\} & \text{for } Z \neq \emptyset \\ v_{\mathrm{adj}}(w,z,e) & \text{for } Z = \emptyset \end{cases} \tag{A.8}$$

with

$$\begin{aligned} v_{no\ \mathrm{adj}}(z,e) &:= \max_{b^o\in Z} \left\{ \begin{array}{c} c(z,b^o) + (1-\delta)\psi e(b^o) z \\ +\psi \iint \left[v\left(\epsilon, z\frac{1-\delta}{\exp(d+\xi)}, e(b^o)\right) \exp(d+\xi) \right] dF(\xi) dG(\epsilon) \end{array} \right\} \\ v_{\mathrm{adj}}(w,z,e) &:= \max_{\substack{(z^o,b^o) \\ \in Y(w,z,e)}} \left\{ \begin{array}{c} c(z^o,b^o) - w\pi(z^o) + (1-\delta)\psi e(b^o) z^o \\ +\psi \iint \left[v\left(\epsilon, z^o\frac{1-\delta}{\exp(d+\xi)}, e(b^o)\right) \exp(d+\xi) \right] dF(\xi) dG(\epsilon) \end{array} \right\}. \end{aligned}$$

Assumption A.6: $\mu_\xi := \psi E[\exp(\xi + d)] < 1.$[1]

Lemma A.1 *Consider the operator T defined by posing $(Tv)(w,z,e)$ equal to the right hand side of (A.7). This operator is defined on the set B of all real-valued, a.e. continuous and bounded functions with domain $D =$* $[0,1] \times \mathbb{R}_+ \times \mathbb{R}_+$
Then the mapping T (a) preserves boundedness, (b) preserves continuity a.e.,

[1] This assumption is equivalent to assumption A.6 in Caballero and Engel (199, p. 811). Assume ξ is normally distributed with variance σ^2. Then this assumption is equivalent to $\exp\left(d + \frac{\sigma^2}{2}\right) < 1 + r$. Approximately, this is $r - d > \frac{\sigma^2}{2}$. Economicly this means that productivity and hence value of a given stock of capital grows at a smaller rate than the market rate of return.

Suppose, this assumption would not hold and neglect adjustment costs for the moment. It is easy to see that a firm could obtain infinite expected value by choosing a stock of capital that is small enough to reproduce its depreciation plus the interest rate in the first period. In the next period it can be expected, that this stock of capital (depreciated capital replaced) generates a positive profit, which grows at a larger rate than the interest rate.

In this sense, assumption 1 is an equilibrium condition for the capital-market.

and (c) satisfies Blackwell´s conditions.

Proof. ***(a)*** *To show that T preserves boundedness, one has to show that for any bounded function u $(Tu)(\cdot)$ is bounded.*
Consider $u \in B$, that is bounded from above by $\overline{u}$ and bounded from below by $\underline{u}$, then$(Tu)(\cdot)$ is bounded from above because

$$\begin{aligned}
(Tu)(w,z,e) &\leq \mu_{\xi}\overline{u} + \sup_{(z^o,b^o)\in Y(w,z,e)} \left\{c(z^o,b^o) - w\pi(z^o)\mathbb{I}_{\{z^o\neq z_t\}} + (1-\delta)\psi e\left(b^o\right)z^o\right\} \\
&\leq \mu_{\xi}\overline{u} + \sup_{0\leq z^o, 0\leq b^o\leq\widehat{b}} \left\{c(z^o,b^o) + (1-\delta)\psi e\left(b^o\right)z^o\right\} \\
&= \mu_{\xi}\overline{u} + \sup_{0\leq z^o, 0\leq b^o\leq\widehat{b}} \left\{\left[1-\psi\left(1+r(b^o)\right)\right]bz^o + \pi(z^o) - \left(1-\psi(1-\delta)\right)z^o\right\} \\
&\leq \mu_{\xi}\overline{u} + \sup_{0\leq z^o, 0\leq b^o\leq\widehat{b}} \left\{\left(1-\psi\right)z^o + \pi(z^o) - \left(1-\psi(1-\delta)\right)z^o\right\} \\
&= \mu_{\xi}\overline{u} + \sup_{0\leq z^o} \left\{\pi(z^o) - \psi\delta z^o\right\}.
\end{aligned}$$

The first inequality reflects the boundedness of u. The second inequality results from dropping adjustment costs.
Using the definitions for c and e we obtain the first equality. The third inequality now follows from $r(b^o) \geq 0, 0 \leq \widehat{b} < 1$.
The last supremum is bounded, because $\pi(z^o) - \psi\delta z^o$ obtains its maximum. This follows from our concavity assumption on π and our assumption on the first derivative of π that leads to

$$\lim_{z^o\to 0}\pi'(z^o) - \psi\delta > 0 > \lim_{z^o\to\infty}\pi'(z^o) - \psi\delta.$$

That $(Tu)(\cdot)$ is bounded from below follows from

$$\begin{aligned}
(Tu)(w,z,e) &\geq \mu_{\xi}\underline{u} + \sup_{(z^o,b^o)\in Y(w,e,z)} \left\{c(z^o,b^o) - w\pi(z^o)\mathbb{I}_{\{z^o\neq z_t\}} + (1-\delta)\psi e\left(b^o\right)z^o\right\} \\
&> \mu_{\xi}\underline{u}.
\end{aligned}$$

The last inequality follows directly from Lemma 3.1–the optimality of no-bankruptcy.

(b) *Firstly, for every* u *that is continuous a.e. the parameter integrals in (A.7) are continuous. So the function that is maximized is continuous. Secondly, both* Y *and* Z *are continuous correspondences except for the equity and capital imbalance pairs* $(e, z) \in A := \{(m, n) | m = (1 - \widehat{b}) - \frac{\pi(n)}{n}$. *These are the points at which* Z *switches to being empty, the* (e, z) *pairs at which a marginal decrease in equity will force the firm to adjust capital to avoid bankruptcy. Therefore, the maximization fullfils the assumptions of Berge's thorem at all points outside* A. *Hence,* $(Tu)(w, z, e)$ *is continuous for all points* $(e, z) \notin A$. *Now, as* A *is a curve in* R^2 *and so has measure* 0, (Tu) *is continuous a.e.*

(c) *To show that* T *satisfies Blackwell´s conditions, one first notes that if* $f_1, f_2 \in B$ *and if* $\forall(w, z, e) \in D : f_1(w, z, e) \leq f_2(w, z, e)$, *then (because* $\exp(d + \xi) > 0$) *the expected value in (A.8) preserves the inequality, and so does the max-function. Thus*

$$(Tf_1)(w, z, e) \leq (Tf_2)(w, z, e)$$

Straightforward algebra yields

$$(Tf + a)(w, z, e) = (Tf)(w, z, e) + \mu_\xi a$$

Assumption A.6 now yields the second Blackwell condition. ■

Proposition A.1 *Equation (A.7) has exactly one solution (which belongs to B).*

Proof. *Lemma A.1 yields that* T *defines a contraction mapping on the metric space* B *with a modulus strictly smaller than one. The existence and uniqueness now follows from the contraction mapping theorem (See Theorem 3.2 in Stockey, Lucas and Prescott, 1989)* ■

A.3 Optimal policy

Now define the following functions related to the solution of the Bellman equation of $v\,(w, z, e)$ in Chapter 3.

$$J(z, b, w) := c(z, b) - w\pi(z) + (1-\delta)\psi e\,(b)\,z + I(z, b) \qquad \text{(A.9)}$$

$$I(z, b) := \psi \iint \left[v\left(\epsilon, z\frac{1-\delta}{\exp(d+\xi)}, e\,(b)\right) \exp(d+\xi) \right] dF(\xi)dG(\epsilon) \qquad \text{(A.10)}$$

Lemma A.2 *The function $J(z, b, w)$ is bounded from above, so that $\sup\limits_{(z^o,b^o)\in Y(w,e,z)} J(z, b^o, w)$ is finite.*

Proof. *As v satisfies the Bellman-equation, it must be bounded. However, since*

$$v\,(w, z, e) = \max_{(z^o,b^o)\in Y(w,e,z)} J(z^o, b^o, w) + w\pi(z^o)\mathbb{I}_{\{z^o=z\}} \geq \begin{cases} \max\limits_{(z^o,b^o)\in Y(w,z,e)} J(z^o, b^o, w) \\ \max\limits_{b^o\in Z(z,e)} J(z, b^o, w) + w\pi(z) \end{cases}$$

always hold, J must be bounded, too. ■

Corollary A.1 *$\widehat{J}_{\max}(z, e) := \sup\limits_{b^o\in Z(z,e)} J(z, b^o, w) + w\pi(z)$ and $J_{\max}(w, z, e) := \sup\limits_{(z^o,b^o)\in Y(w,z,e)} J(z^o, b^o, w)$ are finite, too.*

Lemma A.3 *(a) J and $J_{\max}$ are strictly monotonously decreasing in w.*
(b) $J(z, e, w) + w\pi(z)$ is independent of w,
(c) $J_{\max}$ and $\widehat{J}_{\max}(z, e)$ (and therefore v, too) are monotonously increasing in e.

Proof. *(a) For any $w_1, w_2 \in [0; 1] : w_1 < w_2$ we have:*

$$\begin{aligned} J(z, b, w_2) &= c(z, b) - w_2\pi(z) + (1-\delta)\psi e\,(b)\,z + I(z, b) \\ &< c(z, b) - w_1\pi(z) + (1-\delta)\psi e\,(b)\,z + I(z, b) \\ &= J\,(z, b, w_1) \end{aligned}$$

And since $Y_2 := Y(w_2, z, e) \subset Y(w_1, z, e) =: Y_1$, *we get:*

$$\begin{aligned} J_{\max}(w_2, z, e) &= \max_{(z^o,b^o)\in Y_2} J(z^o, b^o, w_2) \leq \max_{(z^o,b^o)\in Y_1} J(z^o, b^o, w_2) \\ &< \max_{(z^o,b^o)\in Y_1} J(z^o, b^o, w_1) = J_{\max}(w_1, z, e) \end{aligned}$$

(b) This follows directly from the definition of J*,*

(c) Since both Z *and* Y *strictly grow with* e *this follows straightforwardly.* ■

Proposition A.2 *Define for* $Z \neq \emptyset$ *as an implicit function* $\overline{w}(z, e)$ *by*

$$J_{\max}(\overline{w}, z, e) - \widehat{J}_{\max}(z, e) = 0 \tag{A.11}$$

Then firms adjust if their current adjustment cost factor w *is smaller than* $\overline{w}(z, e)$ *or if* $Z = \emptyset$.

Proof. *That a unique* $\overline{w}(z, e)$ *equating* $J_{\max}$ *and* $\widehat{J}_{\max}$ *exists, follows from* $J_{\max}(0, z, e) \geq \widehat{J}_{\max}(z, e)\ \forall(z, e)$ *together with the monotonicity of* $J_{\max}$ *in* w. *As argued in the main text firms adjust if* $Z = \emptyset$ *or* $J_{\max}(w, z, e) > \widehat{J}_{\max}(z, e)$. *Since* $J_{\max}$ *is monotonously decreasing in* w *this inequality holds if and only if* $w < \overline{w}$. ■

Proposition A.3 $J(z, b, w)$ *is analytic and thus the set* Q *of* $(z^o, b^o) \in \widehat{Y}(w, e) \cap \{z^o \geq \underline{z}\}$ *(Lemma 3.1) such that* $J(z, b, w) = J_{\max}(w, z, e)$ *is a non-empty set with a finite number of points.*

Proof. *To show that* J *is analytic it suffices to show that* I *is analytic. Therefore note that* I *can be written as a convolution of an a.e. continuous function and a normal density:*

$$\begin{aligned} I(z, b) &:= \psi \int K(z, b, \xi) dF(\xi) \\ K(z, b, \xi) &:= \int \left[v\left(\epsilon, z\frac{1-\delta}{\exp(d+\xi)}, e(b)\right) \exp(d+\xi) \right] dG(\epsilon) \end{aligned}$$

However, as K *is integrable, the convolution of* K *and a normal density is analytic (see e.g. Theorem 9 on p. 59 in Lehmann (1986)).*

As J is analytic is must be continuous, too. Since $Y(w,z,e)$ is compact this ensures that J attains its maximum within a non-empty compact set Q. Since J is analytic the maxima are isolated, so that Q contains a finite number of elements. ■

Proposition A.4 *Suppose π is analytic, then the function $\overline{w}(z,e)$ is analytic on the open and convex set $C :=$ $\left\{(e,z) \in R_+^2 | e > (1-\widehat{b}) - \frac{\pi(z)}{z}\right\}$, of which A is the border. Therefore, $\Omega(z,e)$ is analytic a.e. and so has derivatives of all order on $R_+^2 \backslash A$*

Proof. *From proposition A.3 we know that J is analytic on C. As all functions involved in the construction of Z and Y are analytic (when π is analytic), so must be $J_{\max}$ and $\hat{J}_{\max}$.(The hessian of the associated Lagrange-Kuhn-Tucker problem has maximal rank). As (A.11) implicitly defines $\bar{w}$, its differentiabilty of all order follows from $J_{\max}$ and $\hat{J}_{\max}$ being analytic together with the implicit function theorem.* ■

Appendix B

Appendix to Chapter 4

B.1 Approximation of the optimal stock of capital for the UK data

Suppose a firm produces with a production function that is of the Cobb-Douglas-type and with two Input factors K and L.[1] Then taking logarithms:

$$y_{it} = \xi_{it} + \varepsilon_i^K k_{it} + \varepsilon_i^L l \quad ; \quad \varepsilon_i^K + \varepsilon_i^L < 1.$$

Denoting wages by w and interest-rates by r, the first-order condition for labor yields

$$l_i^* (k_{it}) = \ln \left(\varepsilon_i^L\right) + y_{it} (k_{it}, l^* (k_{it})) - \ln (w_{it})$$

Hence, we can rewrite the production function in a semi-reduced form as an iso-elastic function only of capital.

$$\begin{aligned} y_{it} &= \psi_i^0 + \psi_i^k k_{it}, \\ \psi_i^k &:= \frac{\varepsilon_i^K}{1-\varepsilon_i^L}, \psi_{it}^0 = \frac{\xi_{it} + \varepsilon_i^L \left[\ln \left(\varepsilon_i^L\right) - \ln (w_{it})\right]}{1-\varepsilon_i^L} \end{aligned}$$

[1] The derivation of z follows Caballero et al. (1995, p. 37).

Now the first-order condition for k determines k^*

$$\ln\left(\psi_i^k\right) + y_{it}\left(k_{it}^*\right) - \ln\left(r\right) = k_{it}^*$$

Replacing $y\left(k^*\right)$ by an exact Taylor-expansion y around k_{it} now yields

$$\ln\left(\psi_i^k\right) + y_{it}\left(k_{it}\right) + \psi_i^k\left(k_{it}^* - k_{it}\right) - \ln\left(r\right) = k_{it}^*$$

Subtracting $\left(1-\psi_i^k\right)k_{it}$ from both sides and rearranging terms, we obtain

$$\ln\left(\psi_i^k\right) + y_{it}\left(k_{it}\right) - k_{it} - \ln\left(r\right) = \left(1-\psi_i^k\right)\left(k_{it}^* - k_{it}\right)$$

$$-z_{it} := \left(k_{it}^* - k_{it}\right) = \frac{1}{\left(1-\psi_i^k\right)}\left[\ln\left(b_i\right) + y_{it}\left(k_{it}\right) - k_{it} - \ln\left(r\right)\right]$$

B.2 Data Appendix to the "Bonner Stichprobe" (Section 4.2)

The dataset used is the "Bonner Stichprobe", a sample of annual company accounts of German companies. To the very most, these companies are large listed stock companies. The sample covers the time-period 1960 to 1997. The panel is unbalanced and contains 694 companies (observational units) and 18943 observations in total. Thus, the average time in the sample is 28.7 years.

The sample includes complete profit- and loss-statements as well as further annual accounting data. Moreover, for the very most company-years data on average wages and salaries as well as the number of employees is reported.

Unfortunately, after firms which are holdings, multi-corporate companies, or business trusts are removed from the sample, sample-size falls substantially. Additionally, we have to drop firm-years for which data seemed inconsistent with usual accounting standards (e.g. negative depreciation, very high appreciation). This leaves us with a sample of about 10000 observations.

If removing a single observation (due to data inconsistency) splits a firm-series in two parts which are long enough to be sensibly analyzed, the second part of the series is identified as a different firm. If the missing observation separated the series in a very short and a longer one, the short one was completely removed, i.e. only firms with five or more consecutive observations remain in the sample. Additionally, single observations were removed, if the investment rate differed from the mean by 5 times the standard deviation (removing 11 observations), differed from the firm specific log-equity ratio by 4 standard deviations (39 observations), or if the turnover-change differed from the mean by 6-times the standard deviation (14 observations). Moreover, firms were excluded, if their average wage-share or proxied average cost-of-capital share exceeded 70% (removing 111 observations). This leaves us with 449 firms and a total of 9731 observations, making an average of 21.67 accounting years per firm. In many cases series for "land and buildings" and "machinery" were not reported separately over the full sample period. Therefore "capital" is identified as "total tangible fixed assets" ("Sachanlagevermögen").

Depreciation rates were generated as reported depreciation relative to the reported stock of capital before depreciation. For a number of firm-years the data contains capital sales as well as gross investment. For some firm years only investment net of capital sales are reported. The stock of capital used for the analysis was generated by the perpetual inventory method. Investment was deflated by the producers-price index for investment goods. To account for sales of capital, we assume that in case capital is sold, the capital stock of each vintage is reduced by the same fraction. Thus, we obtain for the capital series (in real terms):

$$K_{it} = K_{it-1}\left(1-\delta_{it}\right)\left(1-\frac{CS_{it}}{\widehat{K}_{it}+CS_{it}}\right)+\frac{I_{it}}{P_t}, t > T_i$$

$$K_{iT_i} = \frac{\widehat{K}_{iT_i}}{P_{T_i}}.$$

Here $\widehat{K}_{it}$ is the reported stock of capital of firm i at time t. CS_{it} are reported capital-sales and I_{it} is reported investment, P_t is the price-index, T_i is the year when firm i joins the sample. Wages, profits etc. were also deflated using the producer-price index for investment goods as well.

By using the perpetual inventory method, problems induced by a change in accounting standards in 1987 are partly eluded. However, the perpetual inventory method leads to different (mostly larger) stocks of capital than reported. Thus the book-value of equity was adjusted as well.

Appendix C

Appendix to Chapter 5

In the following appendix, we first derive the functional form of the value function involved in the model of chapter 5. Thereafter the proofs which were omitted in the main text of chapter 5 are presented.

C.1 Deriving the value functions

Treating $E_i(P, b_i, \overline{q}_i, \overline{q}_{-i})$ as an asset value and using (5.2) yields according to Itô´s Lemma for the firms expected gain in value (capital gain):

$$\mathbb{E}\left[\frac{dE_i(P, b_i, q_i, q_{-i})}{dt}\right] = \frac{1}{2}\sigma^2 P^2 \frac{\partial^2 E_i(P, b_i, q_i, q_{-i})}{\partial P^2} + \mu P \frac{\partial E_i(P, b_i, q_i, q_{-i})}{\partial P} \tag{C.1}$$

This expected capital gain plus the dividend $(1-\tau)[q_i P - b_i]$ should be equal to the normal return $\rho E_i(P, b_i, q_i, q_{-i})$ to prevent any arbitrage profits from arising. This yields the differential equation

$$\begin{aligned} &\rho E_i(P, b_i, q_i, q_{-i}) = \\ &\quad \frac{\sigma^2}{2} P^2 \frac{\partial^2 E_i(P, b_i, q_i, q_{-i})}{\partial P^2} + \mu P \frac{\partial E_i(P, b_i, q_i, q_{-i})}{\partial P} + (1-\tau)[q_i P - b_i] \end{aligned} \tag{C.2}$$

A particular solution to this equation is

$$E_i(P, b_i, q_i, q_{-i}) = (1-\tau)\left[q_i \frac{P}{\rho - \mu} - \frac{b_i}{\rho}\right] \tag{C.3}$$

The complementary solution involves terms in the form P^{β}, for each solution β to the fundamental quadratic equation

$$\beta^2 \frac{\sigma^2}{2} + \beta\left(\mu - \frac{\sigma^2}{2}\right) - \rho = 0 \tag{C.4}$$

as given in (5.6). (See Dixit and Pindyck (1994) for details.)

C.2 Proofs of the propositions of the main text

C.2.1 Proof of Proposition 5.2

Lemma C.1 *Under the assumptions of our model*

$$g \geq h := \left[\left(\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1}\right)^{\beta_2} - \beta_2\left(\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1} - 1\right)\left(\frac{\overline{q}_1 b_2}{\overline{q}_2 b_1}\right)^{1-\beta_2} \Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_2}^{-(1+b)}\right] \geq 1 \tag{C.5}$$

holds for all $\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1} \geq 1 \geq \Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_2}^{-\beta_1}$.

Proof. *For notational convenience we denote* $a := \Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1} - 1$; $b := -\beta_2 > 0$ *and* $r := \frac{\overline{q}^1 b^2}{\overline{q}^2 b^1}$. *By definition of flow leverage, we obtain* $r = \frac{l_2}{l_1}\frac{\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_2}}{\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1}}$. *Therefore, we have* $\frac{r}{\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_2}}(a+1) = \frac{l_2}{l_1} > 1$ *by assumption, so* $\frac{r}{\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_2}} > \frac{1}{a+1}$ *follows. Rewriting* h *yields:*

$$\begin{aligned} h(a) &= \left(\frac{1}{a+1}\right)^b + bar^{1+b}\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_2}^{-(1+b)} && \text{(C.6)} \\ &\leq \left(\frac{1}{a+1}\right)^b + bar^{1+b} = g(a) && \text{(C.7)} \end{aligned}$$

and

$$\frac{\partial h(a)}{\partial a} = b\left[\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_2}^{-(1+b)} r^{1+b} - \left(\frac{1}{a+1}\right)^{1+b}\right] = b\left(\frac{1}{a+1}\right)^{1+b}\left[\left(\frac{l_2}{l_1}\right)^{1+b} - 1\right] > 0 \tag{C.8}$$

Now, note that $h(0) = 1$ which completes the proof. ■

Lemma C.2 *For any $P^{exit,2}_{\overline{q}_2,\overline{q}_1}$ and $P \geq P^{exit,2}_{\overline{q}_2,\overline{q}_1} \geq \Delta^{-1}_{\overline{q}_1+\overline{q}_2,\overline{q}_2} P^{exit,2}_{\overline{q}_2,0} > 0$ equity value of firm 1 is positive.*

Proof. *To obtain negative equity values dependent on choosing $P^{exit,2}_{\overline{q}_2,\overline{q}_1}$, a necessary condition would be that $\exists \hat{P}^{exit,2}_{\overline{q}_2,\overline{q}_1} > 0 : \min_{P>0} \{E_1(P, b_1, \overline{q}_1, \overline{q}_2)\} \leq 0$. Using* (5.11) *we obtain as a general solution for the equity-value of firm 1:*

$$\frac{1}{1-\tau} E_1(P, b, \overline{q}, \overline{q}_2) = \frac{\overline{q}P}{\rho-\mu} - \frac{b}{\rho} + OpV P^{\beta_2} \tag{C.9}$$

$$OpV := \left(\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1} - 1\right)\frac{\overline{q}}{\rho-\mu}\left(\hat{P}^{exit,2}_{\overline{q}_2,\overline{q}_1}\right)^{1-\beta_2} + \frac{1}{1-\beta_2}\frac{b}{\rho}\left(\Delta^{-1}_{\overline{q}_1+\overline{q}_2,\overline{q}_1} P^{exit,1}_{\overline{q}_1,0}\right)^{-\beta_2}. \tag{C.10}$$

Therefore, we obtain for the price $P_{\min}$ which minimizes value:

$$P_{\min} = OpV^{\frac{1}{1-\beta_2}}\left[-\frac{1}{\beta_2}\frac{\overline{q}}{\rho-\mu}\right]^{-\frac{1}{1-\beta_2}}. \tag{C.11}$$

Note that this price may be smaller than $\hat{P}^{exit,2}_{\overline{q}_2,\overline{q}_1}$. Using $P_{\min}$ to calculate the minimal equity-value $E_{\min}$ of firm 1 yields.

$$\begin{aligned}
E_{\min} \quad &: = \frac{1}{1-\tau} E_1(P_{\min}, b_1, \overline{q}_1, \overline{q}_2) \\
&= \tfrac{\overline{q}}{\rho-\mu} OpV^{\frac{1}{1-\beta_2}}\left[-\tfrac{1}{\beta_2}\tfrac{\overline{q}}{\rho-\mu}\right]^{-\frac{1}{1-\beta_2}} - \frac{b}{\rho} + OpV^{1+\frac{\beta_2}{1-\beta_2}}\left[-\tfrac{1}{\beta_2}\tfrac{\overline{q}}{\rho-\mu}\right]^{-\frac{\beta_2}{1-\beta_2}} \\
&= \left[(-\beta_2)^{\frac{1}{1-\beta_2}} + (-\beta_2)^{\frac{\beta_2}{1-\beta_2}}\right]\left[\frac{\overline{q}}{\rho-\mu}\right]^{-\frac{\beta_2}{1-\beta_2}} OpV^{\frac{1}{1-\beta_2}} - \frac{b}{\rho}.
\end{aligned} \tag{C.12}$$

Since OpV is increasing in $\hat{P}^{exit,2}_{\overline{q}_2,\overline{q}_1}$ so is $E_{\min}$. Therefore, the smallest minimal equity value is obtained at $\hat{P}^{exit,2}_{\overline{q}_2,\overline{q}_1} = \Delta^{-1}_{\overline{q}_1+\overline{q}_2,\overline{q}_2} P^{exit,2}_{\overline{q}_2,0} = \Delta^{-1}_{\overline{q}_1+\overline{q}_2,\overline{q}_2} r P^{exit,1}_{\overline{q}_1,0}$ (with r defined as above in the previous Lemma). Substituting this back in (C.10) *we obtain (making further use of the definition of $P^{exit,1}_{\overline{q}_1,0}$)*

$$\begin{aligned}
OpV_{\min}\left(P^{exit,1}_{\overline{q}_1,0}\right)^{\beta_2} &= \left(\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1}-1\right)\frac{\overline{q}^1 P^{exit,1}_{\overline{q}^1,0}}{\rho-\mu}\left(\Delta^{-1}_{\overline{q}_1+\overline{q}_2,\overline{q}_2}r\right)^{1-\beta_2}+\tfrac{1}{1-\beta_2}\tfrac{b}{\rho}\left(\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1}\right)^{\beta_2}\\
&= \left(\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1}-1\right)\frac{\overline{q}\frac{-\beta_2(\rho-\mu)}{(1-\beta_2)\rho}\frac{b}{\overline{q}}}{\rho-\mu}\left(\Delta^{-1}_{\overline{q}_1+\overline{q}_2,\overline{q}_2}r\right)^{1-\beta_2}+\tfrac{1}{1-\beta_2}\tfrac{b}{\rho}\left(\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1}\right)^{\beta_2}\\
&= \left[-\beta_2\left(\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1}-1\right)\left(\Delta^{-1}_{\overline{q}_1+\overline{q}_2,\overline{q}_2}r\right)^{1-\beta_2}+\left(\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1}\right)^{\beta_2}\right]\tfrac{1}{1-\beta_2}\tfrac{b}{\rho}\\
&= h\frac{1}{1-\beta_2}\frac{b}{\rho}. \qquad \text{(C.13)}
\end{aligned}$$

This now yields for $P^{\min}_{\min}$:

$$\begin{aligned}
P^{\min}_{\min} &= h^{\frac{1}{1-\beta_2}}\left[\frac{\frac{1}{1-\beta_2}\frac{b}{\rho}}{-\frac{1}{\beta_2}\frac{\overline{q}}{\rho-\mu}}\right]^{\frac{1}{1-\beta_2}}\left(P^{exit,1}_{\overline{q}_1,0}\right)^{-\frac{\beta_2}{1-\beta_2}} \qquad \text{(C.14)}\\
&= h^{\frac{1}{1-\beta_2}}\left[P^{exit,1}_{\overline{q}_1,0}\right]^{\frac{1}{1-\beta_2}}\left(P^{exit,1}_{\overline{q}_1,0}\right)^{-\frac{\beta_2}{1-\beta_2}} = h^{\frac{1}{1-\beta_2}}P^{exit,1}_{\overline{q}_1,0}.
\end{aligned}$$

Thus, the global minimum of the equity value is

$$\begin{aligned}
\frac{1}{1-\tau}E_1(P^{\min}_{\min},b_1,\overline{q}_1,\overline{q}_2) &= h^{\frac{1}{1-\beta_2}}\frac{\overline{q}_1 P^{exit,1}_{\overline{q}_1,0}}{\rho-\mu}-\tfrac{b_1}{\rho}+h\tfrac{1}{1-\beta_2}\tfrac{b_1}{\rho}\left[P^{exit,1}_{\overline{q}_1,0}\right]^{-\beta_2}\left[h^{\frac{1}{1-\beta_2}}P^{exit,1}_{\overline{q}_1,0}\right]^{\beta_2}\\
&= h^{\frac{1}{1-\beta_2}}\left[\frac{\overline{q}_1 P^{exit,1}_{\overline{q}_1,0}}{\rho-\mu}-\tfrac{b_1}{\rho}+\tfrac{1}{1-\beta_2}\tfrac{b_1}{\rho}\right]+\left(h^{\frac{1}{1-\beta_2}}-1\right)\frac{b_1}{\rho}\\
&= \left(h^{\frac{1}{1-\beta_2}}-1\right)\frac{b_1}{\rho}>0. \qquad \text{(C.15)}
\end{aligned}$$

The last equality follows from the definition of $P^{exit,1}_{\overline{q}_1,0}$, whereas the inequality is result of Lemma C.1. This completes the proof. ■

Proposition C.1 (Proposition 5.2 main text) *In all Markov-perfect equilibria in pure strategies of the $(\bar{q}_1,\bar{q}_2)$-subgame (exit after investment), the firm with the higher leverage (firm 2) chooses its monopoly exit price as the price trigger for bankruptcy $P^{exit,2}_{\overline{q}_2,\overline{q}_1}=P^{exit,2}_{\overline{q}_2,0}$, whereas firm 1 chooses any price*

$P^{exit,1}_{\overline{q}_1,\overline{q}_2}$, *such that* $P^{exit,1}_{\overline{q}_1,\overline{q}_2} \in \widetilde{P} :=]\Delta(\overline{q}_1+\overline{q}_2,\overline{q}_1)^{-1}P^{exit,1}_{\overline{q}_1,0}, \Delta(\overline{q}_1+\overline{q}_2,\overline{q}_2)^{-1}P^{exit,2}_{\overline{q}_2,0}[$.

Proof. *First note that under the proposed equilibrium strategy firm 2 never becomes a monopolist. Therefore, only the actual price and not the quantity of the competitor matters for firm 2, so that firm 2 behaves myopic. Thus, the value of firm 2 under the proposed strategy is zero at* $P^{exit,2}_{\overline{q}_1,\overline{q}_2}$, *which then is indeed the optimal trigger price.*

Secondly, we have to show that firm 2 cannot profitably choose an exit price trigger smaller than $P^{exit,1}_{\overline{q}_1,\overline{q}_2}$. *To see this, suppose firm 2 chooses a lower price trigger. Hence, it becomes a monopolist after firm 1 exits. However, as the price after firm 1 has left the market is still below firm 2´s monopoly exit trigger, the value associated with this strategy must be negative.*

To see that $P^{exit,1}_{\overline{q}_1,\overline{q}_2}$ *is indeed an equilibrium trigger price, first note that, if firm 2 chooses* $P^{exit,2}_{\overline{q}_2,\overline{q}_1}$, *all trigger prices below* $P^{exit,2}_{\overline{q}_2,\overline{q}_1}$ *yield the same payoff given* P. *Because of the above Lemmata this payoff must be positive. Moreover, it cannot be rational to exit earlier than firm 2 as firm 1 would then forego monopoly profits, i.e. the positive payoffs.*

Choosing any price-trigger outside $\widetilde{P}$ *for firm 1, however, cannot be part of an equilibrium strategy as firm 2 could profitably deviate and set a price slightly smaller and obtain monopoly profits.*

Last, we have to show, that $\widetilde{P}$ *is non-empty. This now follows straightforward from our assumption regarding the leverage ratios.* ■

C.2.2 Proof of Lemma 5.1

Lemma C.3 (Lemma 5.1 main text) *(i) Other things equal*

$$\widetilde{E}_1(P, b_1, \underline{q}_1, \overline{q}_2) + C_1 = E_1(\Delta_{\underline{q}^1+\overline{q}^2,\overline{q}^1+\overline{q}^2}P, b_1, \overline{q}_1, \overline{q}_2) \tag{C.16}$$

has at most three solutions in $P > \max\{P^{exit,2}_{\overline{q}_2,\underline{q}_1}, P^{exit,1}_{\underline{q}_1,\overline{q}_2}\}$. *We denote the solutions with* $P^{*}(< P^{**})(< P^{***})$ *respectively and the set of solutions by* S.

(ii) The sign of $\frac{\partial\left[\widetilde{E}_1(P,b_1,\underline{q}_1,\overline{q}_2)-E_1(\Delta_{\underline{q}^1+\overline{q}^2,\overline{q}^1+\overline{q}^2}P,b_1,\overline{q}_1,\overline{q}_2)\right]}{\partial P}$ *changes from one solution to the next.*

(iii) If the number of solutions is odd, we have

$$\left.\frac{\partial\left[\widetilde{E}_1(P,b_1,\underline{q}_1,\overline{q}_2)-E_1(\Delta_{\underline{q}^1+\overline{q}^2,\overline{q}^1+\overline{q}^2}P,b_1,\overline{q}_1,\overline{q}_2)\right]}{\partial P}\right|_{P=P^*}<0;$$

(iv) if there are two solutions, we have

$$\left.\frac{\partial\left[\widetilde{E}_1(P,b_1,\underline{q}_1,\overline{q}_2)-E_1(\Delta_{\underline{q}^1+\overline{q}^2,\overline{q}^1+\overline{q}^2}P,b_1,\overline{q}_1,\overline{q}_2)\right]}{\partial P}\right|_{P=P^*}>0.$$

Proof. *(i) Due to Lemma C.1, if firm 2 exits first,* $\widetilde{E}_1(P,b_1,\underline{q}_1,\overline{q}_2)$ *exhibits a kink, and this kink must be at a smaller* P *than the kink in* $E_1(\Delta_{\underline{q}^1+\overline{q}^2,\overline{q}^1+\overline{q}^2}P,b_1,\overline{q}_1,\overline{q}_2)$*. Define the continuous function*

$$f(P):=\widetilde{E}_1(P,b_1,\underline{q}_1,\overline{q}_2)+C_1-E_1(\Delta_{\underline{q}^1+\overline{q}^2,\overline{q}^1+\overline{q}^2}P,b_1,\overline{q}_1,\overline{q}_2). \tag{C.17}$$

In case $\Delta_{\underline{q}^1+\overline{q}^2,\overline{q}^1+\overline{q}^2}P^{exit,1}_{\underline{q}_1,\overline{q}_2}<P^{exit,2}_{\overline{q}_2,0}$ *it has the following functional form (for* $P>\max\{P^{exit,2}_{\overline{q}_2,\underline{q}_1},P^{exit,1}_{\underline{q}_1,\overline{q}_2}\}$*):*

$$f(P)=\begin{cases} x_{11}P+x_{12}P^{\beta_2}+C & \text{if } \Delta_{\underline{q}^1+\overline{q}^2,\overline{q}^1+\overline{q}^2}P<P^{exit,2}_{\overline{q}_2,0}\\ x_{21}P+x_{22}P^{\beta_2}+C & \text{if } \Delta_{\underline{q}^1+\overline{q}^2,\overline{q}^1+\overline{q}^2}P\geq P^{exit,2}_{\overline{q}_2,0}\end{cases}. \tag{C.18}$$

Otherwise

$$f(P)=x_{31}P+x_{32}P^{\beta_2}. \tag{C.19}$$

Hence, f *must be either concave or convex on each subset. Consequently* $f(P)=0$ *can have at most four solutions.*
Now note that as sales are increasing with investment $\lim_{P\to+\infty}f(P)=-\infty$*. Moreover, consider that* f *keeps its functional form until firm 1 exits in monopoly. As* $\widetilde{E}_1(P,b_1,\underline{q}_1,\overline{q}_2)\geq 0$*, we can conclude* $f(P)>0$ *at the normalized price where firm 1 exits in monopoly. Therefore, the number of solutions to* $f(P)=0$ *must be odd on the set of price-levels larger than the*

monopoly exit-price. However, $\left\{P \,\middle|\, P > \max\{P^{exit,2}_{\overline{q}_2,\underline{q}_1}, P^{exit,1}_{\underline{q}_1,\overline{q}_2}\}\right\}$ *is subset of this set. This completes the proof.*
(ii)-(iv) follow trivially. ■

C.2.3 Proof of Lemma 5.2

Lemma C.4 (Lemma 5.2 main text) *(i) If firm 2 leaves the market first* $P^{exit,2}_{\overline{q}_2,0} < P^{exit,2}_{\overline{q}_2,\underline{q}_1}$ *holds.*
(ii) Moreover, in all cases $P^{exit,2}_{\overline{q}_2,\underline{q}_1} < \Delta^{-1}_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2} P^{exit,2}_{\overline{q}_2,0}$.

Proof. *(i) The first inequality follows from the fact that the possible investment of firm 1 lowers equity-value of firm 2. Thus,* $P^{exit,2}_{\overline{q}_2,0} < P^{exit,2}_{\overline{q}_2,\underline{q}_1}$.
(ii) We firstly show that

$$\forall P < P^{inv,1}_{\underline{q}_1,\overline{q}_2} : E_2(\Delta^{-1}_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2} P, \overline{q}_2, \underline{q}_1) > E_2(P, \overline{q}_2, \overline{q}_1).$$

$\Delta^{-1}_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2} P$ *maps situation* $(\overline{q}_2, \overline{q}_1)$ *to* $(\overline{q}_2, \underline{q}_1)$ *prices and situation* $(\overline{q}_2, \underline{q}_1)$ *differs from* $(\overline{q}_2, \overline{q}_1)$ *for firm 2 only in the different prices which correspond to the same* Y_t.
Therefore, the decrease in equity value of firm 2 that is caused by the existence of an investment option of firm 1 is always smaller than the decrease caused by investment itself. Thus, the stated inequality follows. This inequality itself implies that the equity-value of firm 2 at price $\Delta^{-1}_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2} P^{exit,2}_{\overline{q}_2,0}$ *must be positive, because*

$$E_2(\Delta^{-1}_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2} P^{exit,2}_{\overline{q}_2,0}, \overline{q}_2, \underline{q}_1) > E_2(P^{exit,2}_{\overline{q}_2,0}, \overline{q}_2, \overline{q}_1) = 0.$$

Thus, $P^{exit,2}_{\overline{q}_2,\underline{q}_1} < \Delta^{-1}_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2} P^{exit,2}_{\overline{q}_2,0}$*, which concludes the proof.* ■

C.2.4 Proof of Proposition 5.4

Proposition C.2 (Proposition 5.4 main text) *(i) If demand is sufficiently elastic, i.e.* $\forall Q_1, Q_2 : \Delta_{Q_1,Q_2} \approx 1$*, respectively if demand is not too inelastic and the costs of investment* C_1 *are sufficiently high, then predatory invest-*

ment never occurs.
(ii) If $g > \left(\frac{\overline{q}_1 b_1}{\overline{q}_2 b_2}\right)^{1-\beta_2}$ *and*

(a) if firm 1 exits first when it had no investment option and if

$$\widetilde{P}^{exit,1}_{\underline{q}_1,\overline{q}_2} < \Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}{}^{-1} P^{exit,2}_{\overline{q}_2,0},$$

then there exists an investment-cost C_1*, so that* (5.22) *has multiple solutions.*

(b) if firm 2 exits first and if for the right-hand partial derivative $\frac{\partial E_1}{\partial P^+}$

$$\frac{\partial \widetilde{E}_1}{\partial P}\left(\Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}{}^{-1} P^{exit,2}_{\overline{q}_2,0}, \underline{q}_1, \overline{q}_2\right) > \frac{\partial E_1}{\partial P^+}\left(P^{exit,2}_{\overline{q}_2,0}, \overline{q}_1, \overline{q}_2\right) \Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}$$

holds,[1] *then there exists the cost of investing* C_1*, so that predatory investment occurs.*

Proof. *(i) As* $\Delta_{\overline{q}_1+\overline{q}_2,\overline{q}_1} \to 1$ *the value of a monopolist and of the value a duopolist (after investment) converge and firm 1 cannot gain anything by firm 2´s exit. Therefore, predatory investment must become unprofitable.*
(ii) First note from the proof of Lemma C.2 that if $g > \left(\frac{\overline{q}_1 b_1}{\overline{q}_2 b_2}\right)^{1-\beta_2}$ *and if* $P' \in \left(P^{exit,2}_{\overline{q}_2,0}, P_{\min}\right)$*, then* $\frac{\partial E_1}{\partial P}(P', \overline{q}_1, \overline{q}_2) < 0$*. Moreover,* $P^{exit,2}_{\overline{q}_2,0}$ *corresponds to the same* Y *as* $\Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}{}^{-1} P^{exit,2}_{\overline{q}_2,0}$ *does in situation-*$\left(\underline{q}_1, \overline{q}_2\right)$.
(a) If firm 1 exits first if not predatorily investing in situation $\left(\underline{q}_1, \overline{q}_2\right)$*, equity value is upward sloping and* $\widetilde{P}^{exit,1}_{\underline{q}_1,\overline{q}_2} < \Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}{}^{-1} P^{exit,2}_{\overline{q}_2,0}$ *by assumption. Thus, shifting* $\widetilde{E}_1 + C_1$ *by altering* C_1 *yields a solution of* (5.22) *on*

$$\left]\Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}{}^{-1} P^{exit,2}_{\overline{q}_2,0}, \Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}{}^{-1} P_{\min}\right[$$

[1] *This condition means that an increase in the state of demand,* Y*, affects the value of a firm with low capacity stronger than the firm with high capacity. This is possible if the hedging effect from the other firm potentially leaving is relatively strong, i.e. value decreases when demand increases. Then, for a large firm the potential gain from a price increase is of course larger.*

and another one on $\left]\Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}{}^{-1}P_{\min},+\infty\right[$.
(b) If firm 2 would exit first, due to Lemma 2, the peak in $E_1\left(P,\overline{q}_1,\overline{q}_2\right)$ *lies at* $\Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}{}^{-1}P^{exit,2}_{\overline{q}_2,0} > P^{exit,2}_{\overline{q}_2,\underline{q}_1}$. *Consider the costs* C' *that yield*

$$\widetilde{E}_1\left(P,\underline{q}_1,\overline{q}_2\right)+C' < E_1\left(\Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}P,\overline{q}_1,\overline{q}_2\right) \tag{C.20}$$

for all $P \geq \Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}{}^{-1}P^{exit,2}_{\overline{q}_2,0}$ *except for one point* P' *(see figure C.1). At this point either both functions are tangentially or* $P' = \Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}{}^{-1}P^{exit,2}_{\overline{q}_2,0}$. *The assumption on the derivative rules out the latter case.*
Therefore, at costs C' *firm 1 invests for all prices* $P \geq \Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}{}^{-1}P^{exit,2}_{\overline{q}_2,0}$ *and*

$$\hat{E}_1\left(P,\underline{q}_1,\overline{q}_2\right) \equiv E_1\left(\Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}P,\overline{q}_1,\overline{q}_2\right)-C'.$$

Now take costs to be equal to $C'+\varepsilon$, $\varepsilon>0$. *Assuming that there is only one price trigger for investment* P^{inv}, *will lead to a contradiction: For this trigger* $P' \leq P^{inv}$ *holds. However, defining the stopping-time* $\tau\left(P_1\right) := \inf\{t \in \mathbb{R} | P_t = P_1\}$ *the difference in value for* $\Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}{}^{-1}P^{exit,2}_{\overline{q}_2,0} < P < P'$ *evaluates as*

$$\begin{aligned}
& \hat{E}_1\left(P,\underline{q}_1,\overline{q}_2|C'\right)-\hat{E}_1\left(P,\underline{q}_1,\overline{q}_2|C'+\varepsilon\right) \\
= \ & \mathbb{E}\left[\int_0^{\tau\left(P^{inv}\right)}\left(E_1\left(\Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}P_t,\overline{q}_1,\overline{q}_2\right)-C'-\widetilde{E_1}\left(P_t,\overline{q}_1,\overline{q}_2\right)\right)e^{-\rho t}dt\right| P_0=P\Bigg] \\
\geq \ & \mathbb{E}\left[\int_0^{\tau(P')}\left(E_1\left(\Delta_{\underline{q}_1+\overline{q}_2,\overline{q}_1+\overline{q}_2}P_t,\overline{q}_1,\overline{q}_2\right)-C'-\widetilde{E}_1\left(P_t,\overline{q}_1,\overline{q}_2\right)\right)e^{-\rho t}dt\right| P_0=P\Bigg]>0.
\end{aligned}$$

Both inequalities follow from (C.20) *(and* $\tau(P^{inv}) \leq \tau\left(P'\right)$ *). Thus, a marginal change in costs would lead to a non-marginal drop in value (the last integral does not depend on* ε*), whereas this would not be true for a system of two price-triggers of investment depending on* ε. *Thus for* ε *small enough two price-triggers are optimal.* ■

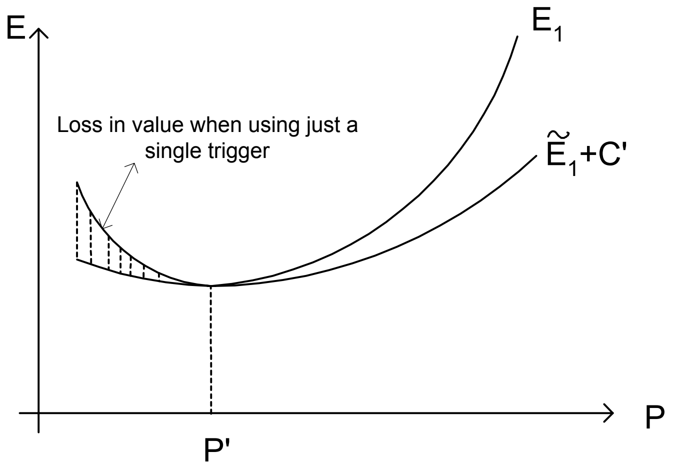


Figure C.1: Loss in value from a single trigger

C.2.5 Proof of Proposition 5.5

Proposition C.3 (Proposition 5.5 main text) *(i) If for firm i the myopic exit price-trigger* $P^{m\_exit,i}_{q_i,q_{-i}}$ *obtained from* (5.29)*, is smaller than* $P^{ind,i}_{q_i,q_{-i}}$*, then firm* $-i$ *choosing* $P^{m\_exit,-i}_{q_{-i},q_i}$ *and firm i choosing a lower price-trigger is an equilibrium of the* (q_i, q_{-i}) *stage.*
(ii) If $\left[\Delta_{q_i+q_{-i},q_i}{}^{-1}P^{exit,i}_{q_{-i},0}, P^{ind,i}_{q_i,q_{-i}}\right] = \emptyset$*, then firm i chooses* $P^{m\_exit,i}_{q_i,q_{-i}}$ *as the exit-price-trigger in all equilibria of the* (q_i, q_{-i}) *stage.*
(iii) If $P^{m\_exit,-i}_{q_{-i},q_i} > P^{ind,i}_{q_i,q_{-i}}$ *and* $\left[\Delta_{q_i+q_{-i},q_i}{}^{-1}P^{exit,i}_{q_{-i},0}, P^{ind,i}_{q_i,q_{-i}}\right] \neq \emptyset$*, then firm i choosing some* $P^{exit,i}_{q_i,q_{-i}} \in \left[\Delta_{q_i+q_{-i},q_i}{}^{-1}P^{exit,i}_{q_{-i},0}, P^{ind,i}_{q_i,q_{-i}}\right]$ *and firm* $-i$ *choosing* $P^{m\_exit,-i}_{q_{-i},q_i}$ *is an equilibrium of the* (q_i, q_{-i}) *stage, if this yields no incentive to predatorily invest for firm* $-i$*.*
(iv) If both firms have an incentive to predatorily invest, instead of choosing their myopic exit price-trigger, then both firm preempt on predatory investment.
(v) If firm i predatorily invests, firm $-i$ *cannot credibly threaten to deviate from choosing the myopic exit price-trigger to hinder i in investing predatorily.*

Proof. *(i) Firm value is increasing in the exit price of the competitor. If undercutting the price-trigger of firm i is not credible even when i chooses the myopic price-trigger, then $-i$ cannot threaten to exit second.*
(ii) If leaving second always yields positive equity value for all credible exit price triggers of the competitor i, then firm $-i$ will leave second, as this increases value at the myopic exit price-trigger. Note that since $P^{ind,i}_{q_i,q_{-i}} \geq \Delta^{-1}_{q_i+q_{-i},q_{-i}} P^{exit,-i}_{q_{-i},0}$, the interval can never be empty for both firms.
(iii) This was largely discussed in the main text, it remains to mention that if firm $-i$ invests predatorily, this decreases equity value below the value obtained by behaving myopically, since the competitor will only invest predatorily (if not preempting) if she expects to leave second after investment.
(iv) See main text.
(v) If firm -i would exit earlier it would forgo profits, therefore, this is not credible. Exiting later is also not credible. If firm -i exits later, payoff becomes negative. Thus, firm -i cannot credible threaten to set an exit price different to the one determined by the smooth-pasting conditions if firm i predatorily invests. ■

C.2.6 Proof of Proposition 5.6

Lemma C.5 *Let $f(P) = x_0 + x_1 P + x_2 P^{\beta_1} + x_3 P^{\beta_2}$; $P > 0$ and $x_1 > 0$, $x_2 < 0$, then f has at most three roots. Moreover if at P' $f(P') > 0$, there can only be two roots of f for $P < P'$.*

Proof. *We have to consider two cases:*
Case 1: $x_3 \leq 0$, then f is concave and therefore has at most two roots.
Case 2: $x_3 > 0$. We firstly show that the second derivative changes its sign at most once:
Suppose $f''(P^*) = P^{*-2}\left[\beta_1(\beta_1-1)x_2 P^{*\beta_1} + \beta_2(\beta_2-1)x_3 P^{*\beta_2}\right] = 0$. Then

$$\begin{aligned} f'''(P^*) = &-2P^{*-3}\left[\beta_1(\beta_1-1)x_2 P^{*\beta_1} + \beta_2(\beta_2-1)x_3 P^{*\beta_2}\right] \\ &+ P^{*-2}\left[\beta_1(\beta_1-1)\beta_1 x_2 P^{*\beta_1} + \beta_2(\beta_2-1)\beta_2 x_3 P^{*\beta_2}\right] \quad . \end{aligned} \tag{C.21}$$

However, the first term is zero and therefore:

$$f'''(P^*) = P^{*-2}[\underbrace{\beta_1(\beta_1-1)\beta_1}_{>0}\underbrace{x_2P^{*\beta_1}}_{<0} + \underbrace{\beta_2(\beta_2-1)\beta_2}_{<0}\underbrace{x_3P^{*\beta_2}}_{>0}] < 0\ . \quad \text{(C.22)}$$

This implies that the second derivative changes its sign at most once, dividing the function in a convex and a concave part. Now suppose $f(P^*) < 0$*, then there may be two roots larger than* P^**. However, as* $f(P^*) < 0$*, at the next smallest root* $f'(P)$ *must be negative, but as* f *is convex , there are no more roots. The case* $f(P^*) \geq 0$ *follows analogously.* ■

Lemma C.6 $\Phi_i(P)$ *can be represented by* $\Phi_i(P) = x_{0i} + x_{1i}P + x_{2i}P^{\beta_1} + x_{3i}P^{\beta_2}$ *for* $P \in M := [\max_{j=1,2}\{\underline{P}_j\}, \min_{j=1,2}\{\overline{P}_j\}]$*; with* $x_{1i} > 0$*,* $x_{2i} < 0$*. Moreover,* $\Phi_i(P)$ *is also continuous on* $[\min_{j=1,2}\{\underline{P}_j\}, \max_{j=1,2}\{\overline{P}_j\}]$

Proof. *First note that* $\Phi_i(P)$ *has the stated functional form since it is a difference of functions of the type given in* (5.6) *(which are analytic on* M*). It is clear, that the followers equity value must be a convex function. Moreover, the leader´s value decreases by the potential entry of the follower, therefore* $x_2 < 0$*. That sales are increased by investment implies* $x_1 > 0$*. Continuity follows from the value-matching conditions* ■

Proposition C.4 (Proposition 5.6 main text) *(i) On*
$M := [\max_{j=1,2}\{\underline{P}_j\}, \min_{j=1,2}\{\overline{P}_j\}]$

$$\Phi_i(P) = 0 \quad \text{(C.23)}$$

has at most three solutions.
(ii) If $\overline{P}_i \leq \overline{P}_{-i}$*, then* $\Phi_i(P) = 0$ *has at most two solution on* M *and only one additional one for* $\overline{P}_i \leq P \leq \overline{P}_{-i}$ *namely* $\overline{P}_{-i}$ *with* $\Phi_i'(\overline{P}_{-i}) < 0$*.*
(iii) If $\overline{P}_i > \overline{P}_{-i}$*,* $\Phi_i(\overline{P}_i) = 0$ *and* $\Phi_i(P) < 0$ *for all* $\overline{P}_i > P > \overline{P}_{-i}$*.*
(iv) If $\Phi_i(P) = 0$ *has two solutions in case (ii) or three solutions in case (iii) on* M *then* $\Phi_i(\max\{\underline{P}_i\}) > 0$*. There may only exist an additional solution for* $\min_{j=1,2}\{\underline{P}_j\} < P < \max_{j=1,2}\{\underline{P}_j\}$ *if* $\underline{P}_i < \underline{P}_{-i}$*.*

Proof. *(i) follows straightforward from the last two Lemmata.*

(ii) At $\overline{P}_i$ firm i invests as follower, therefore

$$E_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\underline{q}_i+\overline{q}_{-i}} P, b_i, \underline{q}_i, \overline{q}_{-i}) = E_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\overline{q}_i+\overline{q}_{-i}} P, b_i, \overline{q}_i, \overline{q}_{-i}) - C \quad \forall P \geq \overline{P}_i. \tag{C.24}$$

This implies $\Phi_i(P) > 0$ if $\overline{P}_i \leq P < \overline{P}_{-i}$ and $\Phi_i(\overline{P}_{-i}) = 0$, since sales of the leader are larger before the follower has invested and

$$E_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\overline{q}_i+\underline{q}_{-i}} \overline{P}_{-i}, b_i, \overline{q}_i, \underline{q}_{-i}) - C = E_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\overline{q}_i+\overline{q}_{-i}} \overline{P}_{-i}, b_i, \overline{q}_i, \overline{q}_{-i}) - C. \tag{C.25}$$

(iii) At $\overline{P}_{-i}$ firm $-i$ invests as follower, therefore

$$\begin{aligned} \forall P \geq \overline{P}_{-i} : E_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\overline{q}_i+\underline{q}_{-i}} P, b_i, \overline{q}_i, \underline{q}_{-i}) - C& \\ = E_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\overline{q}_i+\overline{q}_{-i}} P, b_i, \overline{q}_i, \overline{q}_{-i}) - C.& \end{aligned} \tag{C.26}$$

Therefore, as long as $P < \overline{P}_i$ firm i finds it unprofitable to invest as follower and obtain $E_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\overline{q}_i+\overline{q}_{-i}} P, b_i, \overline{q}_i, \overline{q}_{-i}) - C$. Hence, the right-hand term must be smaller than the equity value of i being the follower when $P < \overline{P}_i$.

(iv) $\Phi_i(\max_{j=1,2}\{\underline{P}_j\}) > 0$ follows from the continuity of Φ_i and in case (ii) from $\Phi_i\left(\bar{P}_i\right) > 0$, respectively $\Phi_i\left(\bar{P}_{-i}\right) < 0$ in case (iii). Define $f(P)$ as stated in Lemma C.5. Now suppose that firm i exits at $\underline{P}_i \geq \underline{P}_{-i}$, then $\Phi_i(P) > f(P)$ for $P < \underline{P}_i$, since the value of firm i as follower is a convex function with derivative zero at $\underline{P}_i$. Therefore, for $\Phi_i(P)$ to have an additional root, $f(P)$ must have an additional root, too. However, due to Lemma C.5, $f(P)$ cannot have that additional root.

If firm i predatorily invests at $\underline{P}_i$. Then for all $\min_{j=1,2}\{\underline{P}_j\} < P < \max_{j=1,2}\{\underline{P}_j\}$

$$\Phi_i(P) = E_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\overline{q}_i+\underline{q}_{-i}} P, b_i, \overline{q}_i, \underline{q}_{-i}) - E_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\overline{q}_i+\overline{q}_{-i}} P, b_i, \overline{q}_i, \overline{q}_{-i}) > 0 \tag{C.27}$$

because the sales of the follower are lower, and firm $-i$ exits later. ■

C.2.7 Proof of Proposition 5.9

Lemma C.7 *If firm i invests predatorily as follower, then $\Phi\left(\underline{P}_i\right) > 0$.*

Proof. *At the price where firm $-i$ exits after firm i has predatorily invested as follower $\Phi = 0$ must hold. However, in state $(\overline{q}_i, \overline{q}_{-i})$ firm $-i$ will exit later than in state $\left(\overline{q}_i, \underline{q}_{-i}\right)$. Moreover, prices are lower before $-i$ exits if i is the follower. Therefore, firm i's value as leader must be larger than firm i's value as follower at the price at which firm i predatorily invests as follower, i.e.*

$$E_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\overline{q}_i+\underline{q}_{-i}}\underline{P}_i, b_i, \overline{q}_i, \underline{q}_{-i}) - C \geq E_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\overline{q}_i+\overline{q}_{-i}}\underline{P}_i, b_i, \overline{q}_i, \overline{q}_{-i}) - C \quad . \tag{C.28}$$

■

Lemma C.8 *Only a firm i that effectively stays in the market longer than its competitor in state $\left(\underline{q}_i, \overline{q}_{-i}\right)$ could have second mover advantages of profitable predatory investment.*

Proof. *Suppose that firm i leaves the market first in state $\left(\underline{q}_i, \overline{q}_{-i}\right)$. Furthermore, suppose firm $-i$ invests at P' and at P' predatory investment would be profitable for firm i, too, i.e.*

$$E_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\overline{q}_i+\underline{q}_{-i}}P', b_i, \overline{q}_i, \underline{q}_{-i}) - C \geq \widehat{E}_i(P', b_i, \underline{q}_i, \underline{q}_{-i}) \quad . \tag{C.29}$$

Investment of firm $-i$ will lower prices and therefore decrease the revenues of firm i compared to the situation $\left(\underline{q}_i, \underline{q}_{-i}\right)$. Then, as firm i leaves first, it cannot gain of any investment-induced change in the probability of firm $-i$ exiting. Therefore, its value as follower must be less than $\widehat{E}_i(P', b_i, \underline{q}_i, \underline{q}_{-i})$, so that there cannot be any second mover advantages, i.e. $\widehat{E}_i(P', b_i, \underline{q}_i, \underline{q}_{-i}) > E_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\underline{q}_i+\overline{q}_{-i}+}P', b_i, \underline{q}_i, \overline{q}_{-i})$. ■

Lemma C.9 *If firm i does not predatorily invest as follower, then*

$$E_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\underline{q}_i+\overline{q}_{-i}}P, b_i, \underline{q}_i, \overline{q}_{-i}) \leq \widehat{E}_i(P, b_i, \underline{q}_i, \underline{q}_{-i}) \quad .$$

Proof. *The option of firm $-i$ investing decreases firm i´s value to a lesser extent than the drop in revenues caused by investment itself does. Because of that $\widehat{E}_i(P, b_i, \underline{q}_i, \underline{q}_{-i}) < E_i(\Delta_{\underline{q}_i+\underline{q}_{-i},\underline{q}_i+\overline{q}_{-i}} P, b_i, \underline{q}_i, \overline{q}_{-i})$ can only hold, if firm i would have to expect some costs not included in $\widehat{E}_i$ for low P or if there would be any expected gain in revenues associated with investment of firm $-i$. However, as firm i as follower was assumed to not predatorily invest, there cannot be any expected gain, since firm $-i$'s investment decreases the probability of firm $-i$ leaving the market. Moreover, there are no costs ignored in deriving $\widehat{E}_i$ (except for the cost of possible predatory investment, which - if profitable - increases value). Therefore, the stated inequality follows.* ■

Proposition C.5 (Proposition 5.9 main text) *A solution to* (5.37) *implies that a predatory investment preemption threshold for firm i exists, i.e. there cannot be second-mover advantages for profitable predatory investment independent of how low P gets.*

Proof. *In case firm i exits first or exists last, but does not predatorily invest, the proposition follows straightforward from the last two lemmata. Hence, we only need to discuss the case where firm i would predatorily invest as follower. However, due to Lemma C.8 $\Phi\left(\underline{P}_i\right) > 0$. Therefore, and because of Proposition C.4(iv) we either have first-mover advantages of investment for all P, or we have an preemption threshold price for predatory investment which is larger than $\underline{P}_i$.* ■

C.2.8 Proof of Proposition 5.10

Proposition C.6 (Proposition 5.10 main text) *(i) If there is a preemption game for predatory investment and $P_{pre}^{pred,i} < P_{pre}^{inv,-i}$ for both firms, the only Markov-perfect equilibrium (outcome) is that the firm with the higher $P_{pre}^{pred,i}$ takes the lead for predatory investment and invests at a price-trigger which is a solution to a version of* (5.34) *that is modified by defining $P_{\underline{q}_{-i},\underline{q}_i}^{inv,-i} := P_{pre}^{pred,i}$, $P_{\underline{q}_{-i},\underline{q}_i}^{pred,-i} := P_{pre}^{pred,-i}$ and by using the appropriate value matching conditions.*

(ii) If there is a preemption game for predatory investment, and $P_{pre}^{pred,i} > P_{pre}^{inv,-i}$ *for firm* i*, and one of the firms has an unilateral incentive to invest on* $\left[P_{pre}^{inv,-i}, P_{pre}^{pred,i}\right]$ *, then in all Markov-perfect equilibria firm* $-i$ *invests predatorily at* $P_{pre}^{pred,i}$*.*
(iii) If there is a preemption game for predatory investment and $P_{pre}^{pred,i} > P_{pre}^{inv,-i}$ *for one of the firms, and none of the firms have an unilateral incentive to invest on* $\left[P_{pre}^{inv,-i}, P_{pre}^{pred,i}\right]$ *, then in all renegotiation-proof Markov-perfect equilibria firm* i *predatorily invests at its unconstrained optimal predatory investment price-trigger or at* $P_{pre}^{pred,-i}$*, whichever is the higher price.*

Proof. *(i) Suppose firm* i *wishes to invest at a price* $P' < P_{pre}^{pred,-i}$*. Define* τ *to be the corresponding stopping time. Then firm* $-i$ *would have an incentive to preempt and invest at a smaller price at time* $\tau - \varepsilon$*. Therefore, predatorily investing below* $P_{pre}^{pred,-i}$ *cannot be part of an equilibrium. However, at prices between* $P_{pre}^{pred,-i}$ *and* $P_{pre}^{inv,-i}$ *firm* $-i$ *wishes to become follower and so will not preempt. Moreover at prices below* $P_{pre}^{pred,i}$ *firm* i *wishes to become leader, therefore the described solution* P^* *to the generalized smooth pasting condition* (5.34) *(allowing for border and non-border solutions) is indeed optimal for firm* i*, given firm* $-i$ *would invest as soon as prices hit the preemption thresholds. Hence, this is indeed an optimal strategy for firm given that firm* i *would invest at all prices lower than* P^**.*
(ii) If one of the unconstrained investment price triggers—say for firm j*— lies between* $P_{pre}^{pred,i}$ *and* $P_{pre}^{inv,j}$*, then not only threatening to invest is for firm* j *credible, but also threatening to not invest is not credible. Thus, the firms wish to preempt until* $P_{pre}^{pred,i}$ *is reached, where firm* i *is indifferent between becoming leader or follower. As investment-price trigger, firm* $-i$ *will clearly choose (if this is possible) its unconstrained-optimal predatory-investment price trigger, which determined by a smooth-pasting condition or otherwise* $P_{pre}^{pred,i}$ *as investment price-trigger.*
(iii) We only need to argue that investing on $[P_{pre}^{pred,i}, P_{pre}^{inv,-i}]$ *cannot be renegotiation-proof. Suppose one firm would invest at* $P^* \in [P_{pre}^{pred,i}, P_{pre}^{inv,-i}]$*. Then, since neither firm has an unilateral incentive to invest at some price* $P \in [P_{pre}^{pred,i}, P_{pre}^{inv,-i}]$*,*

both firms would find it profitable to renegotiate and sign an incentive compatible contract that investment should be carried out at the proposed price-triggers for predatory and non-predatory investment. ■

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Ich versichere, daß ich diese Dissertation selbständig verfaßt habe. Bei der Erstellung der Arbeit habe ich mich ausschließlich der angegebenen Hilfsmittel bedient. Die Dissertation ist nicht bereits Gegenstand eines erfolgreich abgeschlossenen Promotions- oder sonstigen Prüfungsverfahrens gewesen.

Dortmund,