The aggregate Euler equation and transaction services of government bonds

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Abstract: We estimate an aggregate consumption Euler equation on a panel of OECD countries allowing for the stocks of government bonds to influence the intertemporal path of private consumption. The results can be interpreted as mildly supportive of recently proposed theories which postulate that government bonds have a liquidity value due to their role in facilitating transactions. We also allow for non-separability between consumption and leisure in utility in a way that distinguishes between the extensive and the intensive margin of employment. The relation between expected changes in consumption and employment at the extensive margin turns out as the most robust feature of the data, while the intensive margin appears to be of limited importance.

JEL: E32, E24, E21, G12, C23

Keywords: Euler equation, liquidity, non-separable utility, dynamic panel, cross-sectional dependence

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1 Introduction

The consumption Euler equation is one of the central building blocks of macroeconomic models. It is the intertemporal first order optimality condition that links the financial return on an asset to the expected rate of change of an investor’s marginal utility of consumption. In a macroeconomic representative agent context, it is thus the condition that establishes the relation between real interest rates and the time path of consumption and thus aggregate spending (e.g. Fuhrer and Rudebusch, 2004, Bilbiie and Straub, 2012).

In the most standard form, consumption Euler equations are derived based on the hypothesis that interest bearing assets are held purely for financial reasons. However, several recent theoretical papers have explored the possibility that some assets, in particular government bonds, are held because they provide additional services to the holder. Particularly, Canzoneri and Diba (2005) have postulated that government bonds offer liquidity services in a way similar to money. The underlying idea is that transactions in the goods market are facilitated by liquid assets in general, and that while money balances are undoubtedly the most liquid of these, other assets like government bonds provide at least some degree of liquidity services, too. Various aspects of models postulating a liquidity role for holding government bonds have been investigated by Canzoneri et al. (2008, 2011), Linnemann and Schabert (2010), and Krishnamurty and Vissing-Jorgensen (2012).

The common feature of these models is that the stocks of liquid assets held, and not only their rates of return, should be related to spending decisions. Thus, for example, an increase in government debt should affect private aggregate consumption not only indirectly through potential effects on the real interest rate, but also directly through the change in the level of liquid assets held. An increase in the stock of liquidity providing assets should, given decreasing marginal returns of transaction services, lower the total return on these assets, which is comprised of the financial rate of return and the liquidity premium. As a consequence, the result of an increase in the stocks of liquid assets on the expected marginal utility of consumption should be similar to a decrease in the real interest rate. In other words, these theories imply that to the extent that government bonds provide transaction services, the level of the stocks of these bonds should enter consumption Euler equations.

The present paper intends to empirically test this hypothesis. Most of the above mentioned papers are theoretical. An exception is Krishnamurty and Vissing-Jorgensen (2012), who show that a reduced form empirical relation exists in US data
between spreads on the interest rate on government bonds over corporate bonds and the outstanding amount of the former. They interpret this as indicative of a liquidity premium that makes holders of government bonds content with a lower financial return because these bonds provide a ‘convenience yield’, i.e. holding them indirectly provides utility due to their transactions role. In this paper, we want to directly test for the appearance of the stock of outstanding government bonds in empirically estimated consumption Euler equations. We start from the observation that the theories that postulate a liquidity providing role of government bonds can all be mapped into a formulation where holdings of government bonds enter the households’ utility function, in direct analogy to the standard money-in-the-utility function approach to the liquidity services provided by money balances. We derive the implications for the consumption Euler equation from this approach and estimate it using a panel of aggregate annual OECD country data.

Apart from the way that the properties of financial assets enter the Euler equation, a more basic decision is how to model the marginal utility of consumption, the expected growth of which constitutes the stochastic discount factor. In the simplest version, with additive separability of consumption from other arguments in the utility function, the marginal utility depends only on consumption itself. However, several authors have argued that the marginal utility of consumption is likely to depend also on employment, since there is evidence that employment and consumption enter utility in a non-additively separable way. Basu and Kimball (2002) show the empirical importance of the growth rate of total hours worked in an estimated Euler equation. More recently, Hall (2009) makes the case for complementarity between consumption and hours, which is also emphasized in Shimer (2010). Kiley (2010) finds this effect to be insignificant in US data, though. However, all of these approaches consider a utility function where households have a choice, apart from consumption, over per capita hours worked which enters in a non-separable way. Empirically, however, it is well known that most short-run fluctuations in labor are along the extensive margin, i.e. are changes in employment, whereas changes in hours per capita play a smaller role. To capture the distinction between an extensive and an intensive margin, we derive our empirical Euler equation starting from a theoretical specification by Eusepi and Preston (2009) who model households as choosing over employment and hours per capita directly, while both potentially affect the marginal utility of consumption due to a non-separable utility function.

Our main results are as follows. In a loglinear approximation to the Euler equation that features effects of government bond stocks and employment change we
find parameter estimates that are qualitatively in line with the predictions of the theories mentioned above. In particular, we find a small but generally statistically significant influence of the stock of public debt on the expected consumption growth rate, while the effect of the real bond interest rate is also significant and compatible with a low, but not implausible elasticity of intertemporal substitution. The latter point is noteworthy in itself, since many previous studies that did not incorporate effects of liquid bonds or consumption-employment complementarity have reported difficulties in finding a sensible relation between the real interest rate and expected consumption or output growth (see Bilbiie and Straub, 2012, and the references therein). Concerning the specifics of the assumed non-separability of consumption and leisure in utility, we find that the effect working through the extensive margin (employment) is the strongest and most robust influence in our data. The intensive margin (hours per worker) is much less important and generally insignificant. We check all of these results with respect to the possibility that the coefficient estimates could be affected by cross-sectional dependence of the error terms. While we do find that there appears to be a limited but discernible amount of spatial correlations, these are not strong enough to affect the results in a noteworthy way.

In sum, we interpret the results as supportive of the view that (i) government bonds are held by the private sector for reasons that go beyond their financial return characteristics, and (ii) there is strong evidence of complementarity between consumption and employment, though only at the extensive margin. However, it should be mentioned that the first of these conclusions is tentative in so far as the direct effect of public bond stocks on the intertemporal consumption profile that is affirmed here could also be brought about by a different mechanism. For example, it might be the case that there is no liquidity role of government bonds, but there are generational effects of finite horizons in the presence of limited altruism, which is known to entail a wealth effect to the stock of government bonds, e.g. Blanchard (1985), Leith and Wren-Lewis (2000), or Leith and von Thadden (2008). The distinction between these two possible interpretations (transaction services of government bonds vs. overlapping generation wealth effects) is not possible within our approach and thus left for future research.

The rest of the paper is organized as follows. Section 2 presents the specification of the estimating equation, section 3 discusses the data and econometric issues. Section 4 presents results, and section 5 concludes.
2 Specification

Assume that a representative household maximizes

\[ E_0 \sum_{t=0}^{\infty} \beta^t f(c_t, e_t, h_t, b_t, m_t), \]

where \( E_0 \) is expectation conditional on period 0 information, \( \beta \in (0, 1) \) is a discount factor, and the arguments of the utility function \( f(.) \) are consumption \( c_t \), employment \( e_t \), hours per employed person \( h_t \) (such that total labor supply is \( e_t h_t \)), and real holdings of government bonds \( b_t \) and real money balances \( m_t \) at the beginning of period \( t \). Thus, this specification generalizes the classic money-in-the-utility function model (Sidrauski, 1967) by assuming that not only real money balances \( m_t \) but also real government bond holdings \( b_t \) are useful to households because both provide liquidity services that facilitate transactions. This assumption is similar to Krishnamurty and Vissing-Jorgensen (2012), who specify that government bonds provide a convenience yield related to their liquidity beyond their purely financial yields. Note that very similar expressions for the Euler equation that we derive below would result if instead we used one of the specifications in Canzoneri et al. (2008, 2011) or Linnemann and Schabert (2010), where either bond holdings enter alongside money balances in a cash-in-advance constraint or where there is a transaction cost function which specifies that both real money and real bond holdings are useful in reducing transaction costs by providing liquidity.

We assume that liquidity enters in a separable way, such that we can write

\[ f(.) = u(c_t, e_t, h_t) + \phi(b_t, m_t), \]

where \( u(c_t, e_t, h_t) \) is a utility function with standard properties, defined over consumption and leisure and hence, given time constraints, over employment, whereas the function \( \phi(b_t, m_t) \) captures the value that consumers attach to real asset holdings over and above their financial returns, since they provide liquidity services. We also assume that \( \phi(b_t, m_t) \) is separable in its two arguments and that (using subindices to denote partial derivatives) \( \phi_b(b_t) \equiv \partial \phi(b_t, m_t)/\partial b_t > 0 \) and \( \phi_{bb}(b_t) \equiv \partial^2 \phi(b_t, m_t)/\partial b_t^2 < 0 \), i.e. there is a positive but decreasing marginal utility of liquidity services (similar assumptions hold for the marginal utility of the liquidity services of money, \( \phi_m(m_t) > 0 \) and \( \phi_{mm}(m_t) < 0 \), though these will not be used henceforth). It seems reasonable to assume that bonds are much less liquid than money, which means that their marginal utility in providing transaction services,
$\phi_b(b_t)$, is relatively small. However, as will become clear below, the extent to which a change in bond holdings affects transaction services, and thus the magnitude of the coefficient with which changes in bond holdings appear in the Euler equation, will depend on $\phi_{bb}(b_t)$, and we do not have any prior concerning the absolute size of this quantity which will be empirically estimated below.

The household’s period budget is

$$w_t e_t h_t + R_{t-1} b_{t-1}/\pi_t = c_t + m_t - m_{t-1}/\pi_t + b_t + \tau_t,$$

(3)

where $w_t$ is the real wage rate, $R_t$ is the gross nominal interest rate on government bonds, $\pi_t = P_t/P_{t-1}$ is the gross inflation rate (where $P_t$ is the price level), and $\tau_t$ is real lump-sum tax payments (capital accumulation could be added without any change in the results, since we focus on the Euler equation for bond demand). Among the first order optimality conditions are

$$\lambda_t = u_e(c_t, e_t, h_t)$$

with $\lambda_t$ being the multiplier on the budget constraint, and the first order condition for bond holdings,

$$\lambda_t = \beta E_t \lambda_{t+1} \frac{R_t}{\pi_{t+1}} + \phi_b(b_t).$$

(4)

This is the basic form of the Euler equation that we wish to study empirically. Note that, as usual, a higher expected real interest rate $\frac{R_t}{\pi_{t+1}}$ raises the marginal utility of consumption $\lambda_t$ relative to its future expected value, and would thus under standard assumptions on the utility function lower current consumption relative to future consumption. In addition, the marginal utility of consumption is affected by the level of real bond holdings through the term $\phi_b(b_t)$. In particular, a higher level of bond holdings (because of $\phi_{bb} < 0$) would lower the marginal utility of consumption (relative to its future expected value), and thus all else equal would increase consumption. This positive effect of bond holdings on private consumption is exploited in the theoretical papers by Linnemann and Schabert (2010) and Canzoneri et al. (2005, 2008, 2011). In the present paper, we want to assess whether there is empirical evidence for such a channel.

In order to make this an estimable equation, we need to further specify the function $u(c_t, e_t, h_t)$. A number of recent studies have argued that consumption and employment are complements (consumption and leisure are substitutes); see e.g. the micro level evidence discussed in Hall (2009). This would entail a non-separable specification, as has been used by Basu and Kimball (2002) or Kiley (2010). These authors use total hours worked $e_t h_t$ as their measure of employment. However, it is often argued that most of the variations in labor input over short time horizons,
i.e. at business cycle frequencies, take place at the extensive margin via changes in employment, rather than at the intensive margin via changes in hours per person. We allow for both the extensive and an intensive margin such that we are able to take an empirical stand at the relative importance of both.

Specifically, we use a formulation due to Eusepi and Preston (2009). It assumes that the household sector can be depicted as a representative family with a fraction of \( e_t \) employed members and a fraction of \( 1 - e_t \) non-employed members. The employed household members work \( h_t \) hours per person, which delivers a disutility of work of \( v(h_t) \), where for the function \( v \) it is assumed that it has positive first and second derivatives, \( v_h > 0, v_{hh} > 0 \). The sub-utility function \( u(c_t, e_t, h_t) \) is thus specified as

\[
u(c_t, e_t, h_t) = e_t v(h_t) \left( \frac{c_t^e}{1 - \sigma} \right) + (1 - e_t) \left( \frac{c_t^n}{1 - \sigma} \right), \quad \sigma > 1, \tag{5}\]

where \( c_t^e \) is consumption of the employed and \( c_t^n \) is the consumption of the non-employed. Consumption employment complementarity implies that \( \sigma > 1 \). Aggregate consumption is defined as

\[ c_t = e_t c_t^e + (1 - e_t) c_t^n. \tag{6}\]

Maximizing (1) subject to (2), (5), (6) and (3) yields the first order conditions

\[
\begin{align*}
 c_t^e & : \lambda_t = v(h_t) \left( c_t^e \right)^{-\sigma}, \\
 c_t^n & : \lambda_t = \left( c_t^n \right)^{-\sigma}, \\
 e_t & : w_t h_t = \frac{\sigma}{\sigma - 1} (c_t^e - c_t^n), \\
 h_t & : w_t = \frac{v_h(h_t) c_t^e}{v(h_t) \sigma - 1},
\end{align*}\tag{7, 8, 9, 10}\]

with \( \lambda_t \) the multiplier on (3), as well as (4), (6), a first order condition for money holdings, the budget constraint (3) and transversality conditions. These completely describe the household sector. Note that, as discussed in Eusepi and Preston (2009), from (9) with \( \sigma > 1 \) equilibrium requires that employed household members consume more than non-employed members, which in turn is compatible with (7) and (8) if there is sufficient disutility to providing market hours in the sense \( v(h_t) > 1 \), which we assume henceforth. Hall (2009) and Eusepi and Preston (2009) point to empirical studies (for the US) that estimate consumption of non-employed to be about 80 to 85 percent on average of the consumption of the employed.
Our aim is to empirically estimate an approximate version of the Euler equation (4). To do so, we take a log-linear approximation around the steady state. Henceforth, symbols without a time subscript denote constant steady state coefficients, and for any variable \( x_t \) we denote its logarithmic deviation from steady state by \( \hat{x}_t = \ln(x_t/x) \). The Euler equation (4) involves the unobservable marginal utility of consumption \( \lambda_t \). To eliminate it, we use the first order conditions (7) and (8), which introduces the equally unobservable consumption levels of the employed \( c_t^e \) and the non-employed \( c_t^n \). These can, however, be related to observable aggregate consumption \( c_t \) by using its definition (6) along with the first order conditions (9) and (10). Using the steps detailed in appendix 7.1, this leads to a log-linear Euler equation in observables,

\[
\begin{align*}
-\sigma \hat{c}_t + \epsilon \left( c^e - c^n \right) \hat{c}_t + \frac{v_h h e c^e c}{v} \hat{h}_t &= \left( 1 - \phi_b \right) E_t \left[ -\sigma \hat{c}_{t+1} \right] + \epsilon \left( c^e - c^n \right) \hat{c}_{t+1} + \frac{v_h h e c^e \hat{h}_{t+1}}{v} \\
&+ \epsilon \left( c^e - c^n \right) \hat{c}_{t+1} + \frac{v_h h e c^e \hat{h}_{t+1}}{v} \\
&+ \left( 1 - \phi_b \right) E_t \left( \hat{R}_t - E_t \hat{R}_{t+1} \right) + \frac{\phi_{bb} b \hat{h}_t}{\lambda}.
\end{align*}
\]  

In the following, we simplify the expression by assuming that the marginal liquidity value of bonds, \( \phi_b \), is a small coefficient, reflecting the assumption that bonds are less liquid than money and thus provide only a small amount of transaction services. This does not eliminate the influence of the level of bond holdings in the Euler equation, however, since the latter depends on the curvature parameter \( \phi_{bb} \). Proceeding thus under the simplification \( \phi_b \approx 0 \) and rearranging, we arrive at the simplified equation

\[
E_t \hat{c}_{t+1} - \hat{c}_t = a_1 \left( E_t \hat{c}_{t+1} - \hat{c}_t \right) + a_2 \left( E_t \hat{h}_{t+1} - \hat{h}_t \right) + a_3 E_t \hat{r}_t + a_4 \hat{b}_t,
\]

where \( a_1 \equiv \epsilon \left( c^e - c^n \right) \), \( a_2 \equiv \frac{v_h h e c^e}{v} \), \( a_3 \equiv 1/\sigma \), and \( a_4 \equiv \frac{\phi_{bb} b}{\sigma \lambda} \), and \( r_t \) denotes the real interest rate on bonds.

The coefficient \( a_1 \) is expected to be positive since the average consumption of employed persons is typically larger than those of non-employed. As mentioned above, this is consistent with consumption-employment complementarity if \( \sigma > 1 \), which in turn implies that the coefficient \( a_3 \) is expected to be positive and smaller than one. The coefficient \( a_2 \) on the hours-per-capita variable depends, among others, on the elasticity of the disutility of providing working hours and is thus expected to be positive as well. The coefficient \( a_4 \) depends on the curvature of the marginal transactions utility of bonds, \( \phi_{bb} \), which should be negative if bond holdings have
decreasing marginal returns in providing transaction services, such that we expect $a_4 < 0$.

3 Data and econometric issues

We estimate the aggregate consumption Euler equation (12) with annual macroeconomic data on a panel of 14 OECD countries comprising the period 1970 to 2011. Data on aggregate private consumption $c$ and the stock of government debt $b$ are mostly from the OECD Economic Outlook database and in some cases from the European Commission’s AMECO database. Data on employment $e$ and per-capita hours $h$ are from the Groningen Growth and Development Centre’s total economy database. The real interest rate $r$ is the OECD’s measure of the yields on long-term government securities, deflated by private consumption price inflation. Appendix 7.2 gives a more detailed description of sources and definitions of data.

We replace expectational terms by appealing to rational expectations, writing $E_t \hat{c}_{t+1} = \hat{c}_{t+1} + v_{t+1}$ where $v_{t+1}$ is the rational expectation error (and likewise for the other variables), and represent the loglinearly approximated variables by log-deviations from time trends (except for the real interest rate, which is not detrended). Denoting $i$ to indicate the country a variable belongs to, denoting first differences by $\Delta$, and introducing a time fixed effect $\gamma_t$ which captures influences which are common to all countries, we get

$$\Delta \hat{c}_{it+1} = \gamma_t + a_1 \Delta \hat{c}_{it+1} + a_2 \Delta \hat{h}_{it+1} + a_3 \hat{r}_it + a_4 \hat{b}_it + \xi_{it+1},$$

where the disturbance term $\xi_{it+1}$ depends on the expectational errors, and is thus correlated with the $(t + 1)$-dated variables, but should be orthogonal to any variable in the date $t$ information set.$^2$ The equation is thus a dynamic panel model with endogenous regressors and requires an instrumental variables approach. We estimate it with a panel GMM approach. Instrument sets used are lagged values of all variables appearing in (13); to check for robustness, we consider different instrument sets. In particular, instrument set I) uses only lag one, II) uses lag one and two, III) uses lag 2 and 3, whereas instrument sets IV) to VI) employ the same lag specifications but using the levels instead of the first differenced variables in the case of consumption, employment, and hours. The GMM estimates are calculated

$^2$In an earlier version of (13), we also included fixed country-specific effects $\alpha_i$, but they turned out to be irrelevant.
as

\[ \arg \min_{a_1, a_2, a_3, a_4, \gamma_t} \sum_t \sum_i^n \sum_j \left[ (\Delta \hat{e}_{it+1} - \gamma_t - a_1 \Delta \hat{e}_{it+1} - a_2 \Delta \hat{h}_{it+1} - a_3 \hat{r}_{it} - a_4 \hat{b}_{it}) z_j \right]^2, \]  

(14)

where the instruments are denoted by \( z_j \). Numerically, we implement a two-stage procedure. In the first step, we start with \( \gamma_t = 0 \ \forall \ t \). The first stage residuals \( \tilde{\xi}_{it} \) provide estimates for the time effects,

\[ \tilde{\gamma}_t := \frac{1}{n} \sum_{i=1}^{n} \tilde{\xi}_{it}, \]

which are plugged into (14) in the second stage to derive updated estimates for \( a_1, a_2, a_3 \) and \( a_4 \).

Further, there may be cross-sectional dependencies in the data if parts of the countries in the sample are subject to correlated disturbances that are not captured in the global time effects. This would call for modifications of both the target function (14) and the asymptotic standard errors. Thus, as a robustness check, we will allow for cross-sectional correlation in the disturbance terms for these using a variant of the method in Arnold and Wied (2012) further below.

### 4 Results

Table 1 shows the point estimates along with asymptotic standard errors.

<table>
<thead>
<tr>
<th>Instrument set</th>
<th>(I)</th>
<th>(II)</th>
<th>(III)</th>
<th>(IV)</th>
<th>(V)</th>
<th>(VI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_1 )</td>
<td>0.880</td>
<td>0.892</td>
<td>0.881</td>
<td>0.848</td>
<td>0.882</td>
<td>0.874</td>
</tr>
<tr>
<td></td>
<td>(0.070)</td>
<td>(0.199)</td>
<td>(0.932)</td>
<td>(0.229)</td>
<td>(0.161)</td>
<td>(0.075)</td>
</tr>
<tr>
<td>( a_2 )</td>
<td>0.199</td>
<td>0.245</td>
<td>0.329</td>
<td>0.179</td>
<td>0.244</td>
<td>0.320</td>
</tr>
<tr>
<td></td>
<td>(0.381)</td>
<td>(0.529)</td>
<td>(1.542)</td>
<td>(0.458)</td>
<td>(0.777)</td>
<td>(1.083)</td>
</tr>
<tr>
<td>( a_3 )</td>
<td>0.128</td>
<td>0.124</td>
<td>0.087</td>
<td>0.127</td>
<td>0.119</td>
<td>0.082</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td>(0.017)</td>
<td>(0.024)</td>
<td>(0.017)</td>
<td>(0.018)</td>
<td>(0.024)</td>
</tr>
<tr>
<td>( a_4 )</td>
<td>-0.007</td>
<td>-0.008</td>
<td>-0.010</td>
<td>-0.007</td>
<td>-0.008</td>
<td>-0.010</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.003)</td>
<td>(0.024)</td>
<td>(0.005)</td>
<td>(0.003)</td>
<td>(0.004)</td>
</tr>
</tbody>
</table>

Table 1: GMM estimates of equation (13); instrument sets as defined in the text.

The estimated coefficients appear to be reasonably robust across instrument
sets, except for the coefficient $a_2$ on the hours per capita variable, which appears insignificant anyway. Note that all coefficients have the signs expected from the theoretical considerations above. The coefficient on employment change, $a_1$, is large and estimated generally precisely (except using instrument set III), as judged by the asymptotic standard errors given in parentheses. This result confirms the specification for the marginal utility of consumption used here as depending on employment. In other words, there is evidence that consumption and employment are non-separable in utility, as first emphasized by Basu and Kimball (2002). Note that this result is in contrast to what Kiley (2010) finds in quarterly US data, while it is the most quantitatively important and robust result in our annual cross-country panel data.

However, whereas Basu and Kimball (2002) use total hours worked as their employment variable, the results in table 1 clearly show that the relevant effect comes from employed persons, whereas the effect of the change in hours per capita which is captured in $a_2$ is positive, but small and generally insignificant. This is consistent with the notion that most short-run variation in total hours comes from changes in the number of employed persons, and not in hours per person. This result confirms the importance of allowing for an extensive margin of employment variations, as in the model by Eusepi and Preston (2009) used to motivate our estimated Euler equation (13).

The coefficient $a_3$ on the real interest rate is also positive, as expected, and estimated relatively precisely. Taken at face value, the estimates point to intertemporal substitution coefficients in consumption at a magnitude between 4 and 8, which appears relatively large compared to standard calibrations used in the business cycle literature. However, it is well known that in many empirical applications it has proven difficult to find any significant effect of real interest rates on aggregate spending decisions at all (e.g. Fuhrer and Rudebusch, 2004; Bilbiie and Straub, 2012; note, however, that these studies use a short-term policy interest rate, while we focus on the long-term government bond rate). The results found here suggest that the theoretically expected effect seems to be present when used in a properly specified version of the aggregate Euler equation, in particular allowing for non-separability between consumption and employment. As a check on this interpretation, we have re-estimated equation (13) leaving out the employment and / or hours variables. The results are shown in table 2 (for instrument set VI; results for other instrument sets are very similar).
Table 2: Effect of leaving out the employment change variables.

As can be seen from the table, the coefficient on the real interest rate becomes smaller and is much less precisely estimated when the employment change variable is left out, in which case $a_3$ is only borderline significant. We thus tentatively conclude that the theoretically important link between real interest rates and aggregate consumption change is readily apparent, though estimated to be of rather small but still plausible magnitude, if the Euler equation is specified in a way that does not suffer from bias due to omitting the important employment change variable.

Finally, and most importantly, the coefficient on the real government debt variable $a_4$ has the predicted negative sign and is quantitatively rather robust across specifications, though it is only borderline significant in some of the instrument sets used. Recall that the appearance of government debt in the Euler equation with a negative sign is interpreted here as evidence for the presence of a non-financial benefit from holding government bonds, such as the liquidity providing role of these bonds that is hypothesized in Canzoneri and Diba (2005) and Canzoneri et al. (2008, 2011). Thus, we see the results shown in table 1 as tentatively confirming the theory. However, the effect is rather small and estimated somewhat imprecisely, and leaving the government debt variable out (not reported) does not change the coefficients of the other variables in a quantitatively important way. Whether small direct effects of government debt on consumption decisions like the ones estimated here would have a sizeable impact on the transmission of shocks in a macroeconomic model calibrated to reflect the estimates in table 1 is an interesting question for future research.

As a robustness check, we now turn to the possibility of spatial dependencies in error terms, which are always a possibility in a cross-country panel approach like ours. Cross-sectional dependencies in the error terms $\xi_{it}$ would call for modifications of both the target function (14) and the asymptotic standard errors. We
therefore fit to the residuals \( \hat{\xi}_{it} \) a spatial panel data model which allows for two kinds of dependencies: dependencies over time are captured by the variance \( \sigma^2_{\mu} \) of country-specific effects whereas the spatial dependence parameter \( \rho \) indicates cross sectional dependencies. Let \( \hat{\xi} \) denote the residual vector. The spatial dependencies are modeled as

\[
\hat{\xi} = \rho W \hat{\xi} + \varepsilon,
\]

where \( W \) is a known spatial weighting matrix and \( \varepsilon \) is the vector of innovations. For \( \rho \neq 0 \) there is spatial dependence in the residuals. We analyze three different specifications of the weighting matrix:

**W_1:** The distance of two countries is given by the inverse of the squared Euclidean distance between the respective time series of debt-to-GDP ratios.

**W_2:** The distance between two countries in year \( t \) is given by the inverse of the squared Euclidean distance between debt-to-GDP ratios of year \( t \).

**W_3:** The countries are split into four groups: anglo saxon (CAN, US, UK, IRL), scandinavian (FIN, SWE, DK), central Europe (D, A, NL) and southern Europe (B, ESP, ITA, FRA). The residual of a given country may then depend on the residuals of other countries within the same group.

Weighting matrix \( W_1 \) produces the same distances for all years whereas \( W_2 \) allows for year-specific distances. Both \( W_1 \) and \( W_2 \) thus attempt to look for spatial patterns that depend on the size of government indebtedness. The grouping of \( W_3 \) is a heuristic split of the countries in the sample according to geographical location. For all of these weighting matrices, there are zeros on the main diagonal, dependencies are only allowed within the same year and the row sums are standardized to one. For estimation and asymptotic standard errors, we follow the approach suggested by Arnold and Wied (2012).

<table>
<thead>
<tr>
<th></th>
<th>( W_1 )</th>
<th>( W_2 )</th>
<th>( W_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\rho} )</td>
<td>0.084</td>
<td>-0.033</td>
<td>0.145</td>
</tr>
<tr>
<td></td>
<td>(0.153)</td>
<td>(0.026)</td>
<td>(0.034)</td>
</tr>
<tr>
<td>( \hat{\sigma}^2_{\mu} )</td>
<td>0.00013</td>
<td>0.00013</td>
<td>0.00012</td>
</tr>
<tr>
<td></td>
<td>(0.0021)</td>
<td>(0.0021)</td>
<td>(0.0019)</td>
</tr>
<tr>
<td>( \hat{\sigma}^2_{\nu} )</td>
<td>0.0002</td>
<td>0.0002</td>
<td>0.0002</td>
</tr>
<tr>
<td></td>
<td>(0.000012)</td>
<td>(0.000012)</td>
<td>(0.000012)</td>
</tr>
</tbody>
</table>

Table 3: Estimates and standard errors for spatial panel model and three different weighting matrices
Table 3 shows the results for instruments set VI; for the other sets of instruments, results are more or less the same. For weighting matrices $W_1$ and $W_2$, the spatial dependence parameter is not significantly different from zero, and for $W_3$, the amount of spatial dependence is small. The estimates of the other two parameters are more or less identical for all three weighting matrices: The estimates for $\alpha_i$ are in line with the assumption that country-specific effects $\alpha_i$ can be assumed to be zero. In contrast, the variance of the idiosyncratic error terms, $\sigma^2_\mu$, is clearly different from zero. Summing up, we conclude that cross-sectional dependencies in the form of the linkages that are allowed by the spatial weighting matrices we consider do not seem to be of noteworthy quantitative relevance in our data. The results presented above in table 1 are thus likely not to be contaminated by unaccounted spatial correlations. As a check, table 4 shows the estimated coefficients and asymptotic standard errors for equation (13) when cross sectional dependencies are accounted for. In the target function (14), for each year the summands are weighted by the inverse of the (estimated) covariance matrix of the disturbance terms corresponding to the spatial model with matrices $W_1$ and $W_3$, respectively. Again, the results in Table 4 are calculated for instruments set (VI), and very similar results would be obtained with the other instrument sets. As can be seen from table 4 in comparison to table 1, the results hardly change when cross sectional dependencies are taken into account. This seems plausible since there appear to be only moderate cross sectional dependencies in the data.

\[
\begin{array}{cccc}
\alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\
0.875 & 0.316 & 0.083 & -0.010 \\
(0.075) & (1.082) & (0.024) & (0.004) \\
0.883 & 0.310 & 0.086 & -0.011 \\
(0.075) & (1.083) & (0.024) & (0.004) \\
\end{array}
\]

Table 4: Estimates and asymptotic standard errors when spatial dependencies of the disturbances are accounted for in (14)

It seems also unlikely that spatial correlations of another form than the ones allowed for so far tinges the results. It is true that tests for zero cross-sectional correlation (calculated from the residuals corresponding to instruments set (VI)) reject the null hypothesis of zero correlation between the disturbances of the 14 countries. This is the case for the $T$-asymptotic test of e.g. Muirhead (1982), p. 151, which builds on the determinant of the sample correlation matrix, as well as for the concentration asymptotic test of Schott (2005) which uses the sum of squared correlation coefficients. The null hypothesis (all $14 \times 13/2 = 91$ population
correlation coefficients are exactly zero) is clearly rejected in both cases: the test statistics are 259.8 for the $T$-asymptotic test (distributed as $\chi^2_{61}$ under $H_0$) and 8.4 for the concentration asymptotic test (null distribution standard normal), respectively. However, the amount of correlation is only moderate: The mean absolute value of the pairwise correlation coefficients is 0.184. Thus, we conclude that the results presented above appear to be unaffected by spatial dependencies.

5 Conclusion

The present paper has estimated an aggregate consumption Euler equation using a panel data set of annual observations on 14 OECD countries. We found that there is a small, but apparently non-negligible effect of the stock of government bonds affecting the expected growth of the marginal utility of consumption. This result can be interpreted as empirical evidence in terms of government bonds providing transaction services and thus having liquidity effects, as emphasized in recent theoretical contributions mentioned in the introduction. However, the effect is not very precisely estimated and appears insignificant in some specifications, depending on the set of instrumental variables used. Further, we have found considerable evidence for complementarity between consumption and employment. Contrary to previous studies, we distinguished between the extensive and intensive margins of employment adjustment. Only the former seems to be important for consumption dynamics, whereas the latter is generally insignificant. All results appear robust to the possible existence of spatial dependencies.

Although the aggregate Euler equation is one of the central relations in dynamic macroeconomics, attempts to estimate it have reported rather limited success so far. Our result that a specification making use of non-separability in utility, combined with an explicit consideration of the extensive employment margin and liquidity effects of assets generally seems to be supported by our data should thus be a stimulus to further research. In particular, it would be interesting to apply our specification to the setting in Fuhrer and Rudebusch (2004) or Canzoneri et al. (2007) which focus on the relation between money market interest rates and expected output growth.
6 References


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7 Appendix

7.1 Derivation of estimating equation

Letting symbols without a time subscript denote steady state constants, and denoting (for any variable $x_t$) its logarithmic deviation from steady state by $\ln(x_t/x)$, we get by loglinearizing the first order conditions (7) to (10):

$$\hat{\lambda}_t = \frac{v_h h_t}{v} - \sigma \tilde{c}_t^e$$  \hspace{1cm} (16)

$$= -\sigma \tilde{c}_t^n,$$  \hspace{1cm} (17)

$$\hat{\omega}_t + \hat{h}_t = \frac{c^e}{c^e - c^n} \tilde{c}_t^e - \frac{c^n}{c^e - c^n} \tilde{c}_t^n,$$  \hspace{1cm} (18)

$$\hat{\omega}_t = \left( \frac{v_h h_t}{v_h} - \frac{v_h h_t}{v} \right) \hat{h}_t + \tilde{c}_t^e,$$  \hspace{1cm} (19)

and by loglinearizing (6) and (4)

$$\hat{c}_t = \frac{e (c^e - c^n)}{c} \hat{c}_t + \frac{ec^e}{c} \tilde{c}_t^e + \frac{(1-e)c^n}{c} \tilde{c}_t^n;$$  \hspace{1cm} (20)

$$\hat{\lambda}_t = (1 - \phi_h) \left( E_t \hat{\lambda}_{t+1} + \hat{R}_t - E_t \tilde{c}_{t+1} \right) + \frac{\phi_h b_t}{\lambda} h_t.$$  \hspace{1cm} (21)

Now eliminate $\hat{\lambda}_t$. From (16) and (17) we have

$$\hat{\lambda}_t = \frac{v_h h_t}{v} \hat{h}_t - \sigma \tilde{c}_t^e = -\sigma \tilde{c}_t^n,$$

$$\Rightarrow \tilde{c}_t^n = -\frac{1}{\sigma} \frac{v_h h_t}{v} \hat{h}_t + \tilde{c}_t^n,$$
and from (20) we get
\[ \hat{c}_t^n = \frac{c}{(1 - e^n)\hat{c}_t} + \frac{e (c^n - c^n)}{(1 - e^n)\hat{c}_t} - \frac{e e^n}{(1 - e^n)\hat{c}_t^e}. \]

Hence, upon combining the last two expressions, we arrive at
\[ \hat{c}_t = \hat{c}_t + \frac{(1 - e^n)}{c} \frac{v h \hat{e} - e (c^n - c^n)}{\sigma} \hat{c}_t. \]

Using this, we get by replacing in (20)
\[ \hat{\lambda}_t = \frac{v h \hat{e} - \sigma \hat{c}_t}{\sigma} \hat{c}_t + \frac{v h e e^n}{c} \hat{h}_t. \]

Substituting this in (21) yields
\[ -\sigma \hat{c}_t + e \frac{(c^n - c^n)}{c} \hat{c}_t + \frac{v h e}{c} \hat{e} \hat{h}_t = (1 - \phi_b) E_t \left[ -\sigma \hat{c}_{t+1} + e \frac{(c^n - c^n)}{c} \hat{c}_{t+1} + \frac{v h e e^n}{c} \hat{h}_{t+1} \right] + (1 - \phi_h) E_t \left( \hat{R}_t - E_t \hat{\pi}_{t+1} \right) + \frac{\phi_h b_{(\hat{e} - \hat{h})}}{\lambda} b_t, \]

as in the main text.

7.2 Data

Data are annual and comprise 1970-2011. The countries in the sample are Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, the Netherlands, Spain, Sweden, the UK and the US. Both the spatial and the time series dimension of the sample are dictated by data availability.

- Data on nominal and real private consumption \( c \) is from the OECD Economic Outlook database.
- The real interest rate \( r \) is constructed from the nominal long-term interest rate on government bonds (OECD Economic Outlook) by subtracting the realized one period ahead inflation rate based on the deflator of private consumption.
- Data on employed persons \( e \) and hours per worker \( h \) is obtained from the Groningen Growth and Development Centre’s total economy database.
- Data on nominal gross government debt is from AMECO. The series for France, Canada, and the Netherlands had missings in AMECO, instead, the OECD
Economic Outlook series on general government gross financial liabilities has been used for these countries. Real government debt is computed by deflating with the implicit price index of private consumption, obtained by dividing nominal through real private consumption.

- Data for Germany are used from 1991 onwards. For prior dates, West German data have been used in the case of the interest rate and government debt, and for the other variables the West German growth rates prior to 1991 have been used to construct a series that matches the 1991 value for total Germany.

- Some other cases: the interest rate for Ireland is missing in the OECD data before 1990, AMECO data have been used. Nominal and real consumption for Ireland are also from AMECO, since they were missing in the OECD database.