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Article

Against the Impossible Picture: Feynman's Heuristics in his Search for a Divergence-free Quantum Electrodynamics

Adrian Wüthrich
(Institute for Philosophy, University of Bern, Sidlerstrasse 5, CH-3012 Bern, e-mail: awuethr@philo.unibe.ch, and Institute for Philosophy, History of Literature, Science and Technology, Technical University Berlin, Straße des 17. Juni 135, D-10623 Berlin)

Abstract: I review three early steps of the development of Feynman's proposal for a divergence-free quantum electrodynamics and identify the characteristic feature of his heuristics: the search for alternative formulations of the existing theories. Feynman's reformulations always had precise goals, and in each of the three steps one of them was particularly important. Through reformulation, he tried (1) to extend the domain of application of an existing theory, (2) to provide a model to justify the theory's equations, or (3) to reveal assumptions problematic for the existing theory and in this way find amendments to it.

Kexwords: Richard P. Feynman, heuristics, reformulation, means of representation, quantum electrodynamics

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Finding a divergence-free electrodynamics

In the early 1940s, when Richard P. Feynman (1918–88) was a graduate student, one of the most pressing problems facing theoretical physicists was the fact that infinite and, therefore, uninterpretable quantities arose from some of the principles of electrodynamics—in both classical electrodynamics as well as in the early attempts to establish a quantum version of it.[1]

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[1] Some passages in the present text are reprinted from my book (Wüthrich, 2010).

In classical electrodynamics, the difficulties of divergences had been known for some time, and it had been hoped that quantizing the theory would eliminate them. An alternative strategy was to first remove the infinite quantities in the classical theory before attempting to quantize it (see e.g. Dirac 1938; Frenkel 1925).

It is in this context that Feynman began his PhD thesis, with the removal of the divergences in electrodynamics being his superordinate objective (Feynman 2005, p. 2). In the following, I will characterize Feynman's heuristics, i.e. his strategies to achieve this objective. Feynman achieved it through three major steps. In each of them, he formulated problematic hypotheses in alternative ways to achieve goals subordinate to the main objective of removing the divergences.

Step 1: Quantization without Hamiltonian

In his thesis, Feynman adopted the second strategy of first trying to establish a divergence-free classical theory and then proceeding to quantize it. Indeed, together with his supervisor John Archibald Wheeler, Feynman had already developed an alternative theory of electrodynamics with the desired feature, which awaited quantization.[2]

The standard procedure for quantizing a classical theory was to interpret the classical Hamiltonian function as an operator in a Hilbert space of state vectors. This operator would then determine the time evolution of the quantized system. The problem with quantizing the Wheeler–Feynman theory of electrodynamics was that it could not be formulated by specifying a Hamiltonian function (Feynman 2005, p. 5).

In his thesis, Feynman found a way to quantize such theories. However, the domain of application of his procedure remained "non-relativistic throughout" (Feynman 2005, p. 1). Feynman was thus not able to find a satisfactory quantization procedure for the Wheeler-Feynman theory. Only several years after his PhD thesis, and after World War II, did he find ways of dealing with relativistic systems, see Step 2.

Feynman developed his quantization procedure for theories without Hamiltonian starting from a representation of the time evolution of the quantum wave function that he borrowed from Paul Dirac (see e.g. Dirac 1933, p. 68 and Feynman 2005, pp. 26–28). Dirac had found a relation between the classical Lagrangian of a system and the infinitesimal time evolution of the corresponding quantum state vectors. However, since the Hamiltonian function of a system can be constructed from the Lagrangian by Legendre transformations, each system that can be described by a Lagrangian can also be described by a Hamiltonian. By logical contraposition, this means that, when a system is *not* describable by a Hamiltonian, like in the Wheeler–Feynman theory, it is also not describable by a

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[2] Only a summary of a presentation of Wheeler and Feynman's theory had been published by the time Feynman started working on his thesis (see "Minutes of the Cambridge, Massachusetts, Meeting, February 21 and 22, 1941", p. 683). The published accounts of the theory are Wheeler/Feynman (1945); Wheeler/Feynman (1949).

Lagrangian. Dirac's method did, therefore, not provide a solution to Feynman's problem.

However, through an *iterative* application of Dirac's relation, Feynman realized that the wave function and its time evolution could be represented by an integral of an integral of the classical Lagrangian. Moreover, the integral of the Lagrangian was exactly the usual expression for the classical *action*.

Feynman then assumed this representation to be valid also in those cases where only the action but not the Lagrangian existed (like in the Wheeler–Feynman theory). So by representing the quantum dynamics of a system directly using the action function of the corresponding classical system, Feynman was able to generalize the quantization procedure to systems with neither a Lagrangian nor a Hamiltonian.

Here we already see one of the specific purposes of Feynman's alternative formulations. In this instance, Feynman needed the alternative formulation of quantum mechanics to construct a description of important systems that could not be described using standard means:

> "What we have been doing so far is no more than to re-express ordinary quantum mechanics in a somewhat different language. In the next few pages we shall require this altered language in order to describe the generalization we are to make to systems without a simple Lagrangian function of coordinates and velocities." (Feynman 2005, p. 39).

As we will see in two further instances (Steps 2 and 3), the procedure via alternative formulations is characteristic for Feynman's heuristics, i. e. for his way of finding solutions to problems.

Step 2: The model of the quivering electron

After the Second World War, a condensed and revised version of Feynman's thesis was published in the *Reviews of Modern Physics* (Feynman, 1948). While the results of Feynman's thesis were "non-relativistic throughout" (Feynman 2005, p. 1), in the last section of the published article, Feynman provided the correct action functions to include relativistic systems and particles with spin.

However, Feynman was not at all pleased with these treatments of spin phenomena and relativistic particles. Without revealing any details, he let the reader know that he was working on a more satisfactory treatment of these two subjects, which was not yet ready for publication:

> "These results for spin and relativity are purely formal and add nothing to the understanding of these equations. There are other ways of obtaining the Dirac equation which offer some promise of giving a clearer physical interpretation to that important and beautiful equation." (Feynman 1948, p. 387).

Feynman here alluded to his unpublished notes[3] in which he had succeeded, or was about to succeed,[4] in obtaining the quantity that determined the evolution of the wave function in a more satisfactory manner than the "purely formal" way of the published article: In his notes, he showed how to interpret Dirac's differential equation in one dimension as the description of a model of a zigzagging electron, which, by the way, had already been discussed by Gregory Breit (1928) and Erwin Schrödinger (1930).

Feynman reformulated the one-dimensional Dirac equation in "New Variables" (title of one of Feynman's manuscript pages) and considered a discrete version of Dirac's continuous equation on a space-time lattice. Feynman saw that he could obtain basic solutions to the Dirac equation by "path counting": He determined how many paths, on the space-time lattice, were possible for the electron, and how many changes of direction each particular path contained. In order to get the correct solutions, Feynman realized that each change of direction in the path of the particle on the lattice had to be taken into account by a factor $i\epsilon$ in the path counting procedure, where ϵ is the spacing of the space-time lattice. Thus the wave-function, which describes quantum-mechanically an electron and satisfies the Dirac equation, would be a sum of as many terms as there are paths on the lattice leading from a given start to a given end point such that each term contains the factor $i\epsilon$ as many times as there are changes in direction in the given path.

In this way, Feynman obtained the *Green's function* associated with Dirac's equation, and the Green's function served almost exactly the same purpose as did the action in Feynman's alternative formulation of non-relativistic quantum mechanics, which he had developed in his thesis and in the article in the *Reviews of Modern Physics*. Both the exponential of a factor containing the action and the Green's function are the essential quantities which determine the time evolution of the wave function.[5]

Unlike in the final section of the published article, Feynman was thus able to *justify* the action function needed to describe the time evolution of a relativistic electron because he derived the relevant Green's function from a detailed quantitative description of an electron's path on a idealized space-time lattice.

This episode is an instance where Feynman's reformulations follow a second purpose. Feynman found a model system (the zig-zagging electron) and a quantitative description of it (count each corner of the paths by $i\epsilon$) which is equivalent, in one dimension, to Dirac's description of an electron by his equation. Feynman knew the correct action function associated with the Dirac equation but only

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[3] Most of Feynman's manuscripts and letters have been collected by the Archives of the California Institute of Technology. The documents to which I refer here and in the following are reproduced in Wüthrich (2010), Ch. 4. Also Schweber (1994), pp. 406–408 quotes from them.

[4] I could not determine the temporal order of when Feynman was writing his notes and of when he was drafting the publication (Feynman, 1948).

[5] Cf., e. g., equations (3.11) and (4.19) in Wüthrich (2010); see also Wüthrich (2010), pp. 75–77.

through a derivation of the equation from his alternative description did he find a satisfactory justification for it.
Although the specific purpose of Feynman's reformulation here is different from that in Step 1, the episode is again an instance of Feynman's characteristic procedure to find solutions to problems: He wanted to derive the correct action formula from physical considerations instead of only justifying it by alluding to the correctness of the formula's consequences. This case here is a bit special, though, because the reformulation itself of Dirac's electron theory constitutes a solution to this problem and not only a *way to* the solution, as it should be the case for a heuristics in the strict sense of the word.

Against the impossible picture

After having successfully dealt with the one-dimensional Dirac equation, Feynman went on to consider the Dirac equation describing real electrons, that is, electrons moving not just in one spatial dimension but in three spatial dimensions. Like Step 1, this was one of Feynman's attempts to extend the applicability of an existing theoretical treatment, and he planned to resort, again, to his reformulation panacea. In a letter to his student friend Theodore Welton, Feynman wrote:[6]

> "Still my stuff sounds mathematical—& insofar as it is, I still don't understand it—but I will try soon to reformulate [it] in terms of seeing how things look to someone riding with the electron." (see Wüthrich 2010, p. 91).

In fact, the same letter also reminds us of the second purpose (see Step 2) of his reformulations, namely to go beyond mere empirical adequacy. Feynman looked for a physical model to justify the equations, in order to, in Feynman's words, "understand" them better:

> "I am engaged now in a general program of study—I want to understand (not just in a mathematical way) the ideas in all branches of theor. physics. As you know I am now struggling with the Dirac Eqn." (see Wüthrich 2010, pp. 82–83).

Again and again, Feynman's aim was to describe Dirac's well-known equation in alternative ways, for he did not believe that a physical theory was completely specified by its equations. The equations had to be completed by models or "pictures", and several models were possible for the same equations:

> "I find physics is a wonderful subject. We know so very much and then subsume it into so very few equations that we can say we know very

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[6] Feynman's letter was written on a "Monday February 10". Around the time in question, February 10 was a Monday in 1941, 1947 and 1958. The content of the letter makes 1947 the most plausible date.

> little (except these equations—Eg. Dirac, Maxwell, Schrod[inger]). Then we think we have *the* physical picture with which to interpret the equations. But there are so very few equations that I have found that many physical pictures can give the same equations. So I am spending my time in study—in seeing how many new viewpoints I can take of what is known." (see Wüthrich 2010, pp. 90, 92).

Feynman thus sought what luminaries like Dirac would tell him to be impossible. This whole enterprise of devising explanatory models, "pictures" or even mechanisms of quantum phenomena clashes with the usual education in quantum mechanics, which Feynman had received through, among other things, the textbook by Dirac (1935), which says:

> "The methods of progress in theoretical physics have undergone a vast change during the present century. The classical tradition has been to consider the world to be an association of observable objects (particles, fluids, fields, etc.) moving about according to definite laws of force, so that one could form a mental picture in space and time of the whole scheme. This led to a physics whose aim was to make assumptions about the mechanism and forces connecting these observable objects, to account for their behaviour in the simplest possible way. It has become increasingly evident in recent times, however, that nature works on a different plan. Her fundamental laws do not govern the world as it appears in our mental picture in any very direct way, but instead they control a substratum of which we cannot form a mental picture without introducing irrelevancies." (Dirac 1935, p. vi).

Dirac's aversion against "mental pictures", however, did not bother Feynman too much and he declared in his letter to Welton:

> "I dislike all this talk of there not being a picture possible but we only need know how to go about calculating any phenomena." (see Wüthrich 2010, pp. 90, 94).

Feynman knew all too well about the value of having a clear physical interpretation, in terms of an explanatory model, of the mathematical equations of the theory — the "pictures" which Dirac apparently abhorred so much.

"Pictures" or "viewpoints" in this context were not necessarily graphical representations of an abstract theoretical content. For me, the most plausible reading is that of physical interpretations or model systems which would justify or exemplify the relations which are expressed by the equations. However, diagrams were one particularly appropriate way of going beyond what was expressed in the equations and representing the model systems.

Step 3: The right picture

Since the time of his PhD thesis, Feynman knew that the search for different “pictures” was not just an intellectual “pleasure”;[7] it also had precise purposes. In his letter to Welton, Feynman explained his third, and maybe most important motivation:

> “Of course, the hope is that a slight modification of one of the pictures will straighten out some of the present troubles.” (see Wüthrich 2010, pp. 90, 92).

Feynman’s objective was to interpret the known equations in such a way that it becomes clear which assumption in the theory of quantum electrodynamics was causing the troublesome infinities. Once the culprit of the contradiction (between the theory and more general physical principles or uncontested experimental data) had been identified, it should then be possible to resolve the problem by modifying the problematic assumption.

This would achieve Feynman’s superordinate goal of finding a divergence-free quantum electrodynamics; a problem that was with him at least since the days of his PhD thesis, as mentioned in the first section of this article. The way to the solution was again through a reformulation of what had already been known. This is the third and last instance, which I discuss here, of Feynman’s characteristic heuristics.

Feynman further developed the model of a quivering electron into a model where electrons propagate in a more abstract sense of the word. He thereby reduced the whole of quantum electrodynamic phenomena to a single “fundamental interaction” (Feynman 1949, p. 772), which he also represented graphically, i. e. in a more literal sense of “picture”. The *Feynman diagram* (Feynman 1949, Fig. 1) makes particularly apparent the modular structure of Feynman’s theory: Feynman isolated a fundamental process from which every quantum electrodynamic interaction can be built up. Likewise, the mathematical description of the fundamental process isolates the quantity that is the source of all the infinities, at least in Feynman’s approach.

The problematic quantity was a so-called Dirac δ distribution, which, roughly speaking, is zero everywhere except for one point where it is infinite. Feynman replaced this troublesome function by a new function in a way that would give the correct (and finite!) results for important measurable quantities (Feynman 1949, p. 776).

Thus Feynman succeeded in finding an appropriate “picture” that revealed the problematic assumptions of the theory and suggested modifications. Through the suggested modifications he was able to solve, at least partially, the longstanding problems of the infinities in quantum electrodynamics. Thus, the hope expressed in the letter to Welton was finally fulfilled.[8]

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[7] Cf. e.g. Feynman (1948), p. 367.

[8] For more details see Wüthrich (2010) and Wüthrich (forthcoming).

Because Feynman's solution of these problems still relied on a traditional interpretation of the wave-function, his solution was not accepted as the standard way of dealing with these problems. By transferring the graphical means of representation, and the modular structure which Feynman thereby made evident, to a more up-to-date theoretical framework, Freeman Dyson (1949a); Dyson (1949b) provided a more satisfactory solution.

It remains to be investigated in more detail to what extent Feynman's "picture" was essential for Dyson's solution. But I think we tend to underestimate the importance of recognizing the structure of quantum electrodynamic processes which Feynman diagrams represent.[9] Feynman found this structure by devising alternative formulations of unsatisfactory theoretical proposals.

Summary

I divided the early development of Richard Feynman's proposal for a divergence-free quantum electrodynamics into three steps and identified the characteristic feature of Feynman's heuristics: the search for alternative formulations of the existing theories. In each of the three steps, Feynman's reformulations followed precise purposes:

- to extend the domain of application of a theory (Step 1),
- to provide a physical interpretation of the mathematical equations (Step 2),
- to clearly identify and remove the basic problem (Step 3).

The reformulations often involved a "picture" in the sense of a physical interpretation, which Dirac and others believed to be impossible.

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[9] I sketch how Feynman diagrams might represent physical processes elsewhere (Wüthrich, 2012)

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