Experimental verification and comparison of analytical and FE models for calculation of a Bitter solenoid

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07.05.2013

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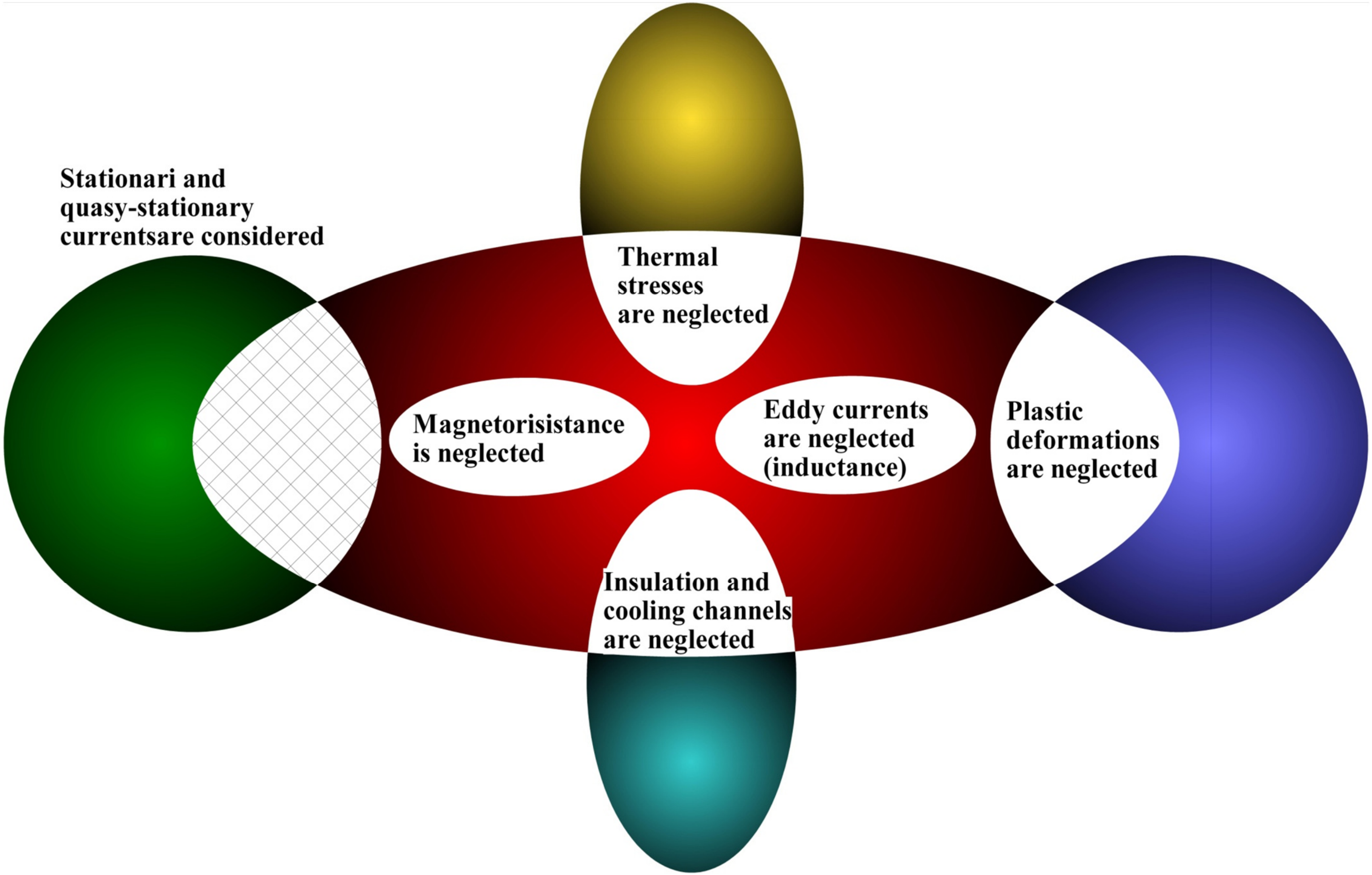
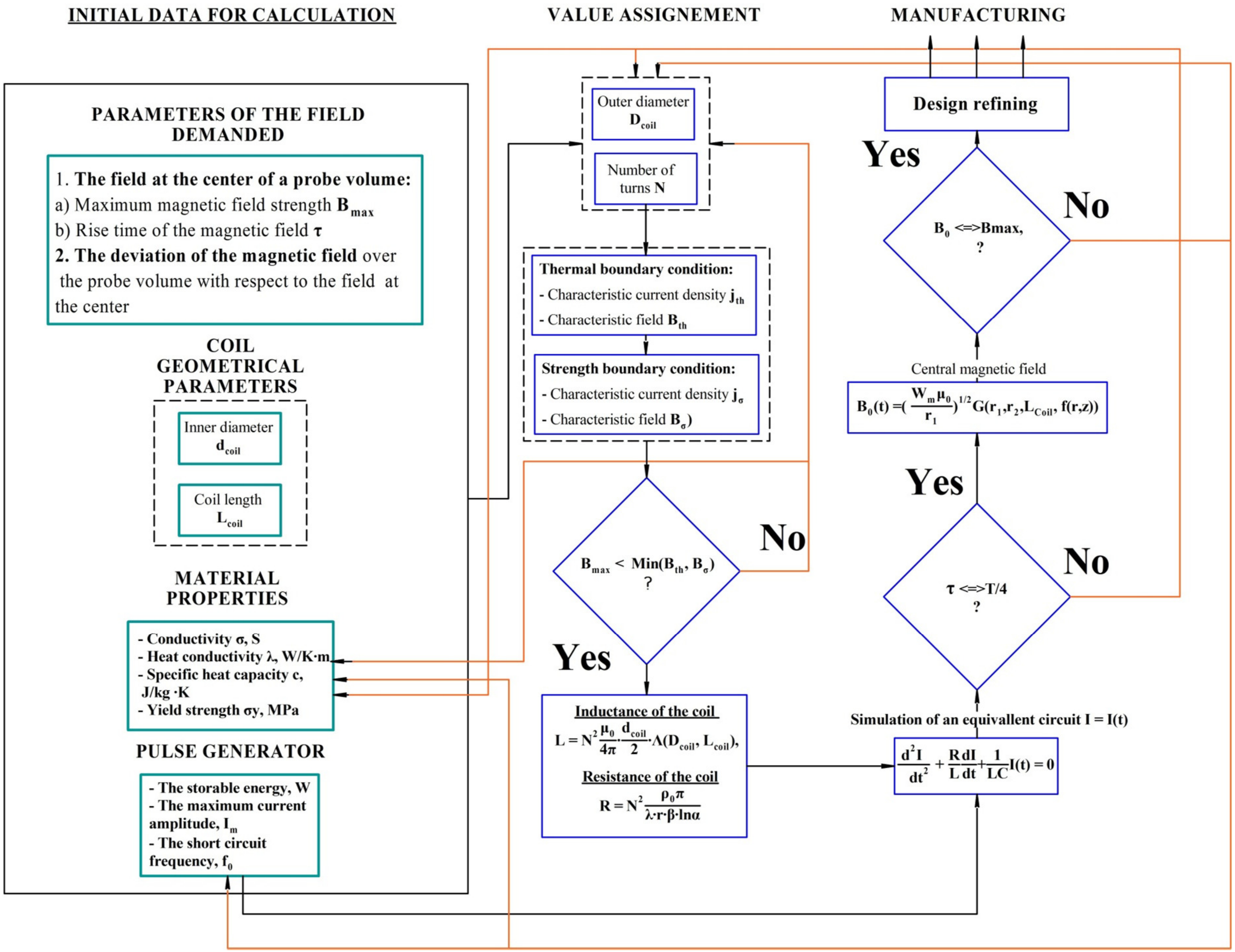
Analytical calculation of a Bitter solenoid

1. Limitations of the approach



2. Calculation methodology



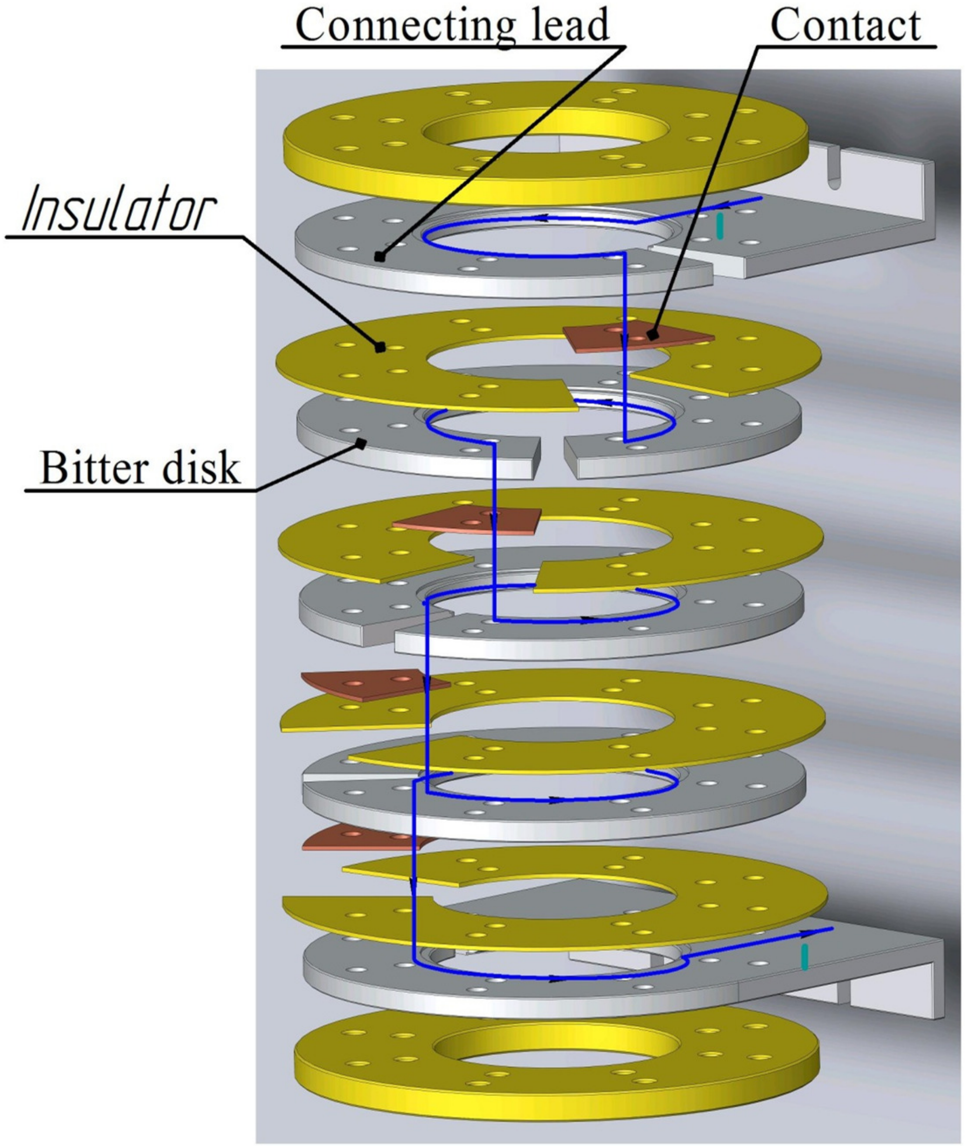
3. Determination of input data

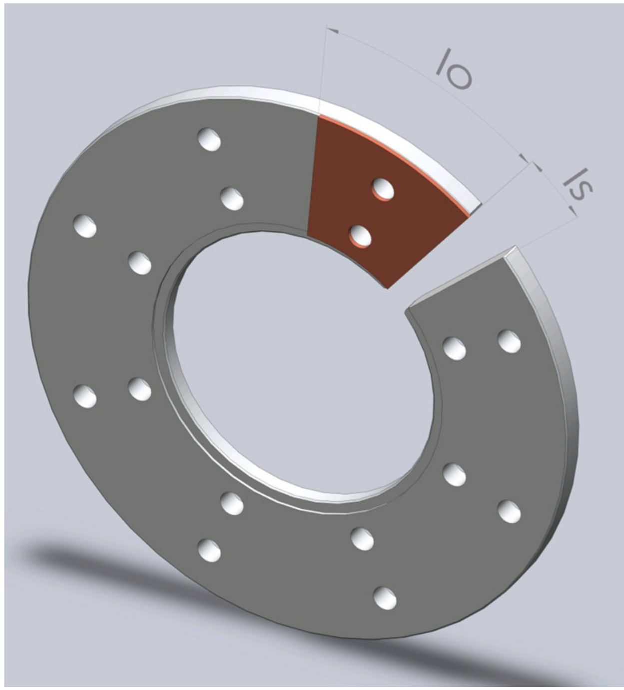
1. Material properties:
a) Conductivity σ [MS]
b) Specific heat capacity c [J/kg·K]
c) Working temperatures: initial T_i and final T_f [K]
d) Mass density ρ [kg/m^3]
e) Yield strength σ_y [MPa]

2. Value assignment for an outer radius r_2 and a number of turns N

3. To take into account a symmetry breakdown in a real coil

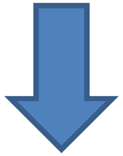
3. Determination of input data





Ideal symmetrical disk = 360°

Real disk = 360° - l_0 - l_s



Nominal number of turns must be reduced to an effective ideal number of turns having a cylindrical symmetry.

4. Boundary conditions: characteristic current densities and fields

1. Thermal BC takes into account only heating of the solenoid

Material integral must be as high as possible:

$$Fmat(Ti, Tf) := \int_{Ti}^{Tf} \rho m \cdot \frac{c(T)}{\rho(T)} \, dT$$

Material	$F_{Mat}(T_i,T_f)$ [10^{16} A^2s/m^4]	
	77 K – 400 K	77 K – 700 K
Cu	9,42	
Al	4,58	
C17510	4,45	
C17200	1,30	
AerMet 100	0,27	0,46
AISI 316	0,15	0,26

$$jth := \sqrt{\frac{Fmat(Ti, Tf)}{tpulse \cdot \zeta}}$$

$$Bth := \mu 0 \cdot \xi \cdot r1 \cdot \sqrt{\frac{F(Ti, Tf)}{tpulse \cdot \zeta}}$$

$$B0 := \min(Bth, B\sigma) \cdot \ln(\alpha)$$

2. Strength BC takes into account only mechanical strength of a material

$\sigma max = \sigma y$

ξ – filling factor of the coil

$\alpha = r2/r1$ – form – factor of the coil

$$j\sigma := \sqrt{\frac{\sigma max}{\mu 0}} \cdot \frac{1}{\xi \cdot r1 \cdot \sqrt{\ln(\alpha)}}$$

$$B\sigma := \sqrt{\mu 0 \cdot \sigma max} \cdot \frac{1}{\sqrt{\ln(\alpha)}}$$

5. Calculation of the inductance and active resistance of the coil

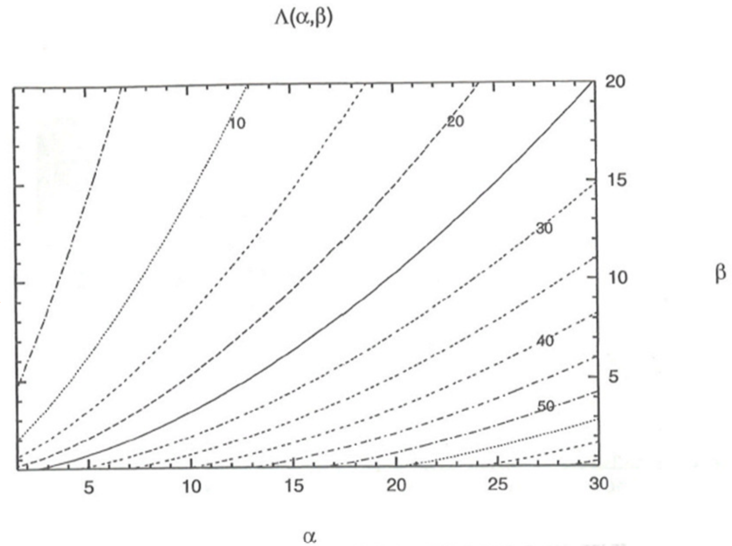
A) Inductance

$$Lcoil := \eta^2 \frac{\mu0 \cdot r1 \cdot \Lambda(\alpha,\beta)}{4 \cdot \pi}$$

$\beta = l_{coil}/2r_1$ – form – factor of the coil

η – effective number of turns

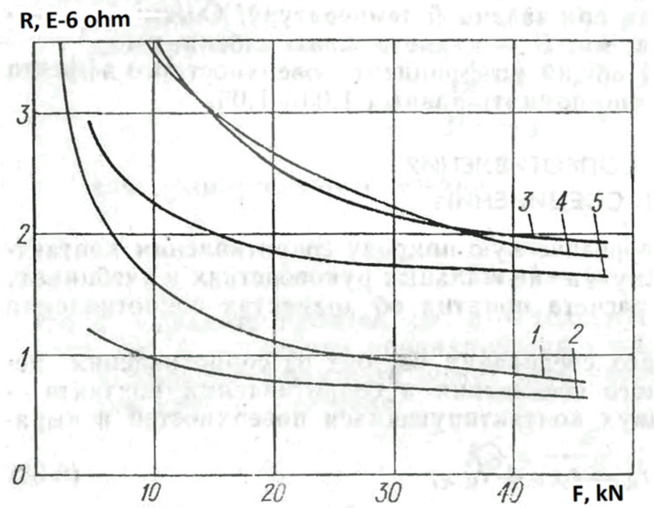
Λ(α;β) – self – inductance factor



B) Resistance

$$Rdc := \eta^2 \cdot \frac{\rho \cdot \pi}{\xi \cdot r1 \cdot \beta \cdot \ln(\alpha)}$$

Rcontact



1 – Cu-Cu contact, oxidized surface 2 – Cu-Cu contact, clean surface
3 – Contact between Cu-Cr-Zn plates, oiled surfaces,
4 – Cu-Cu, clean surface,
5 – Contact between Cu-Cr-Zn plates, clean surface,

$$Rac := \frac{a \cdot (Rdc + Rcontact) \cdot \sqrt{\omega d \cdot \frac{\mu0}{2 \cdot \rho}}}{u}$$

a – cross-section of a turn [m^2]

u – circumference of a turn [m]

ωd – current angular frequency [Hz]

6. Simulation of an equivalent RLC circuit

Differential equation of electromagnetic damped oscillation can be easily solved at initial conditions defined by a pulsed generator:

C – capacitance [F]

U – discharge voltage[V]

Given

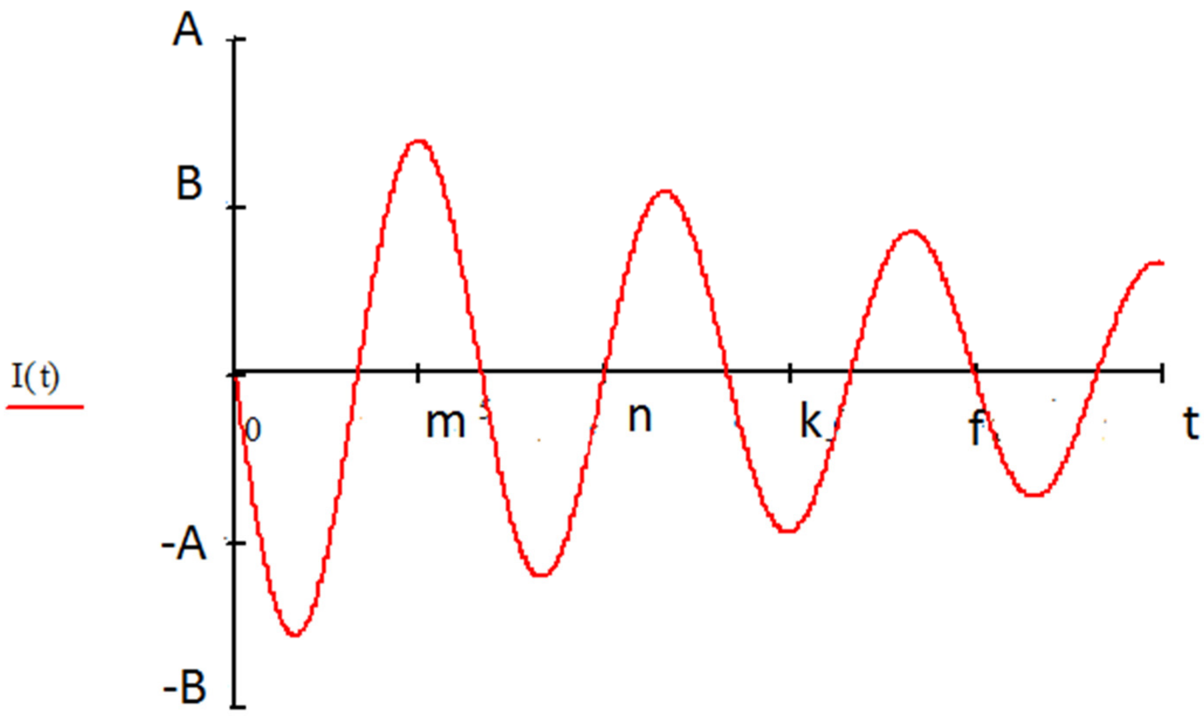
$$\frac{d^2}{dt^2}q(t) + 2\cdot\beta1\cdot\left(\frac{d}{dt}q(t)\right) + \omega^2\cdot q(t) = 0$$

$$q(0) = C\cdot U$$

$$q'(0) = 0$$

$$q := \mathrm{Odesolve}(t, t1)$$

$$I(t) := \frac{d}{dt}q(t)$$



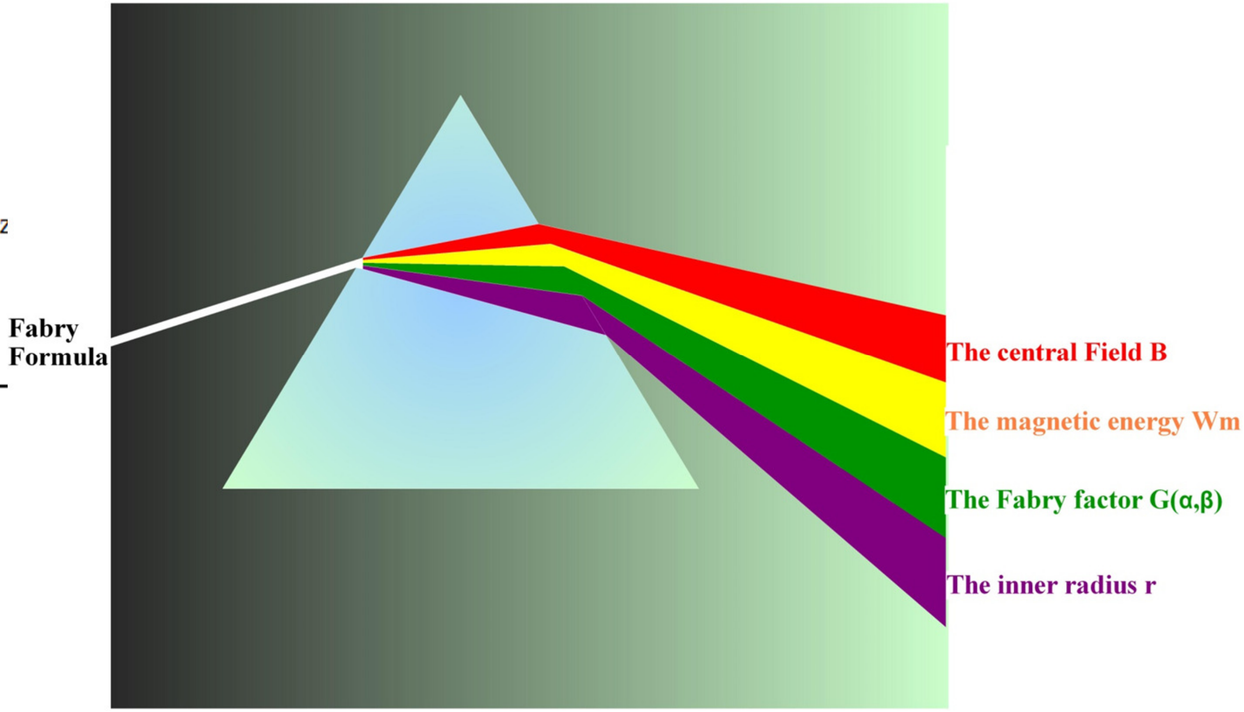
On this step a verification of the frequencies demanded and calculated takes place

7. Calculation of the central magnetic field

$$f(r,z) := \frac{r1}{r}$$ Current density distribution

A – cross-section

$$B0(t) := \sqrt{Wm(t)\cdot\frac{\mu0}{r1}}\cdot\sqrt{2\frac{\pi}{\Lambda(\alpha,\beta)}}\cdot\frac{\displaystyle\int\!\!\int_A r^2\cdot\frac{f(r,z)}{\left(r^2+z^2\right)^{\frac{3}{2}}}\,dr\,dz}{\displaystyle\int\!\!\int_A f(r,z)\,dr\,dz}$$

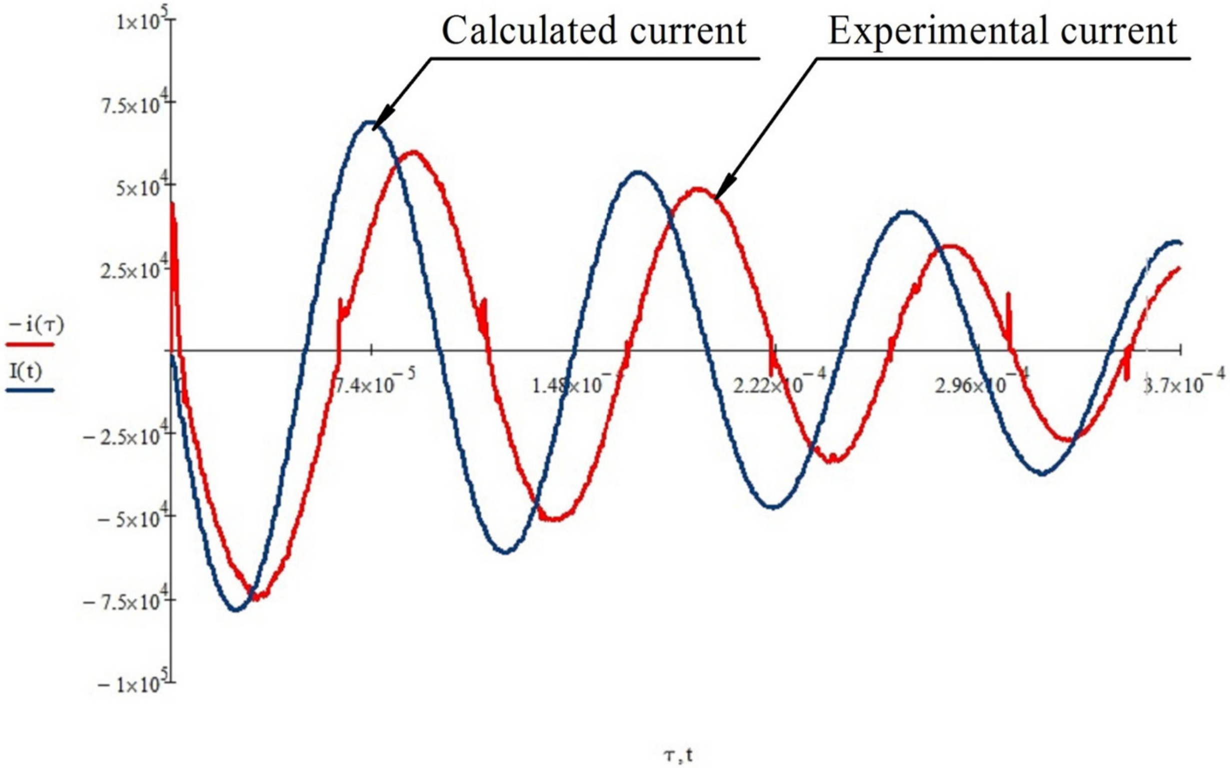


After the integration a closed form expression was obtained so a convenient calculation can be made for any sizes and current courses:

$$B0(t) := \sqrt{Wm(t)\cdot\frac{\mu0}{r1}}\cdot\frac{\ln\left(\frac{lcoil}{r1}+\sqrt{\frac{lcoil^2}{r1^2}+1}\right)}{lcoil\cdot\ln\left(\frac{r2}{r1}\right)}$$

8. Verification of the methodology on an example of BWI Bitter solenoid

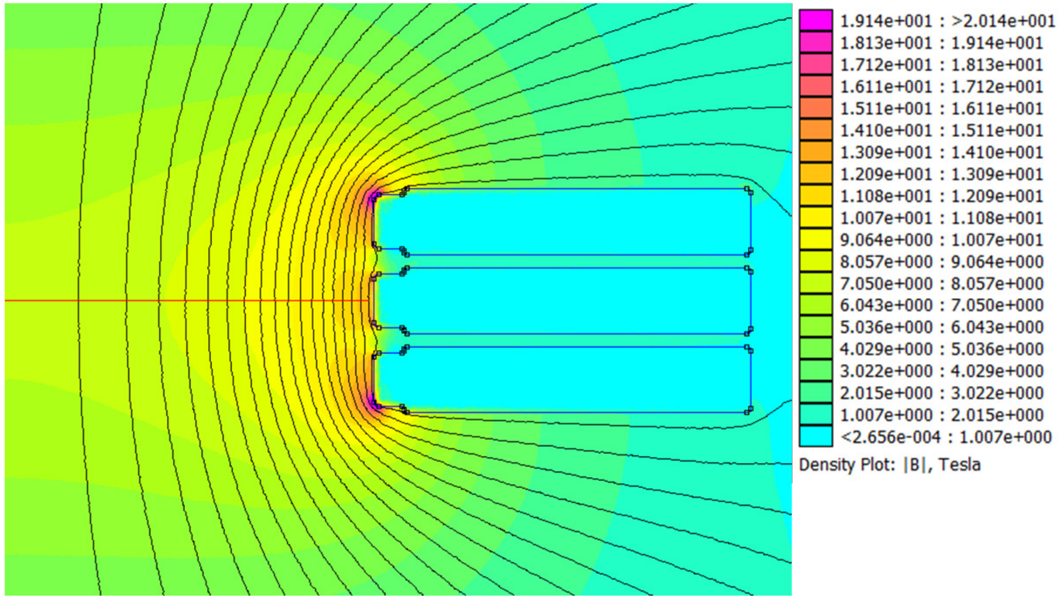
The initial conditions C = 130e-6 F, U = 10 kV and I(0)=0 correspond to the experiment:

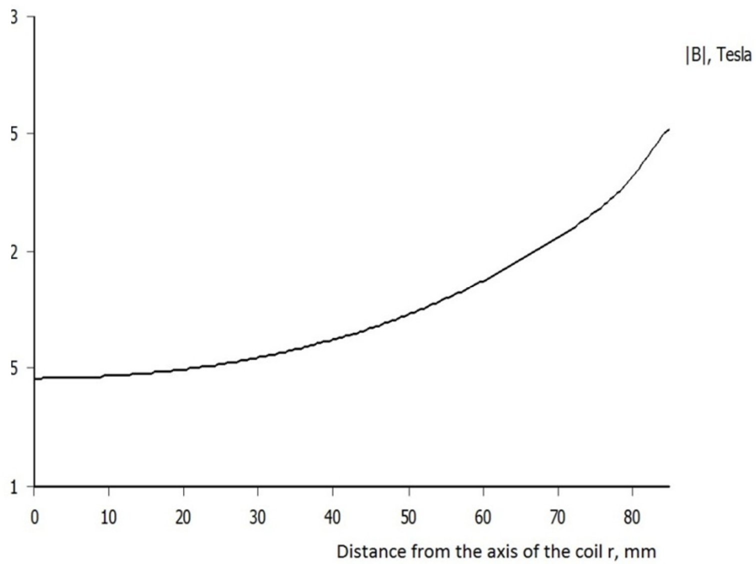


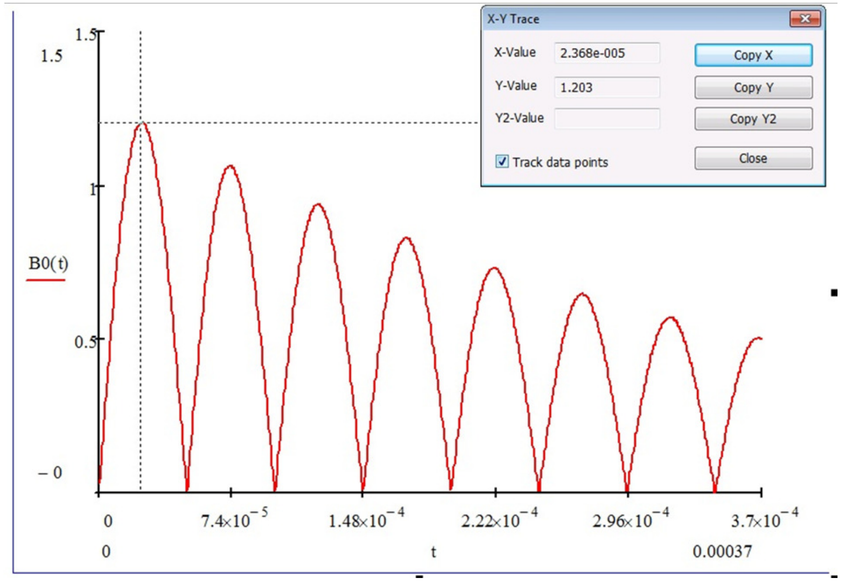
Parameter	R·10e-3 [Ohm]	L·10e-9 [H]
Experimental	7.441	1504
Calculated	6.588	1838
Relative error	11.5%	22%

A comparison of the graphs allows to draw a conclusion of satisfying errors between the experimental and calculated curves.

9. Modeling of the magnetic field in the probe volume of the BWI coil using FEMM



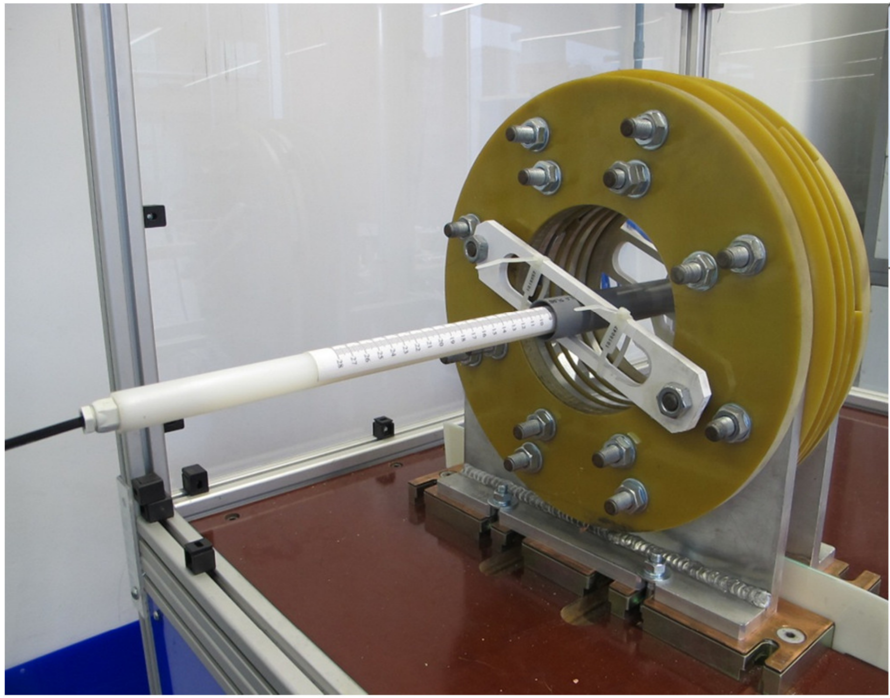


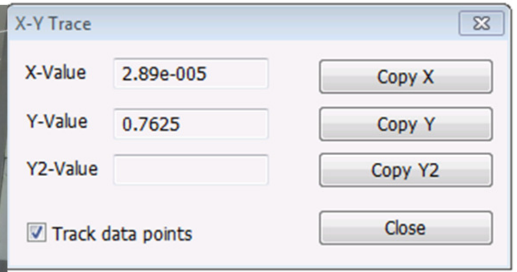


B-field [T]	
FEMM	1.4
Analytical model	1.2
Relative error, %	14.3

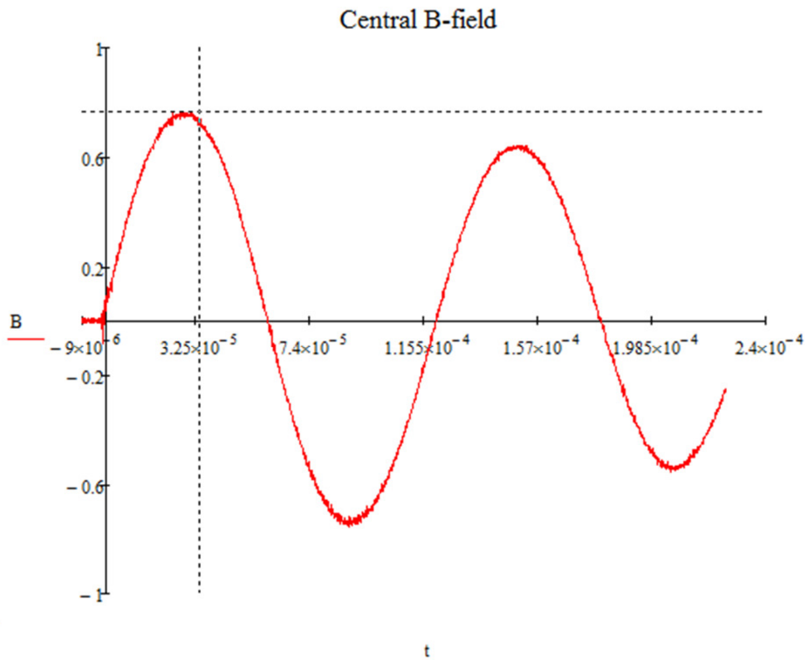
The results are close!

10. Field verification using a measuring coil





$$B := \frac{U}{6.4}$$



The B-field measured differs from both analytical and numerical calculation:

B-field [T]	
FEMM	1.4
Analytical model	1.2
Relative error, %	14.3
B measured	0.76…1.9