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Using concrete and visual representations in early algebra teaching

Introduction

Through my teaching of mathematics in a bilingual primary and secondary school I face with the problem that my students, regardless of age, both during the lessons and on the graduation examination, make an amazing number of errors where they should apply the knowledge of elementary algebra. An explanation for these elementary mistakes can be, that these students consider the algebraic symbols and formulas as “letters” instead of expressions of general relationships between numbers or relationships among quantities. I suspect that the roots of this misleading way of thinking could originate from the period of their education when they encountered with algebra for the first time.
Another strong reason that motivated my research is the special position of my students: I teach in a bilingual school in Budapest in Hungary using Hungarian official curriculum. A lot of our students do not have the appropriate command of Serbian language or do not speak it all, so they attend this school in order to learn it. On the other hand, students coming from Serbia do not speak Hungarian. Consequently, we have to use both languages during the lessons.
Due to the reasons mentioned, I decided to extend the period of teaching using concrete manipulative tools and to combine them with more abstract methods. Besides having a beneficial effect on algebra learning this method enriches students’ conception and perspective of mathematics.
The role of the language is complex, explaining activities, thoughts and ideas on the appropriate level of representation and it is also used to bridge over the different levels of representations.

Theoretical background and some algebraic aspects of thinking

According to Bruner (1966) there are three levels of representation: enactive iconic and symbolic level. The algebra teaching usually happens on symbolic level. Our aim was to use the enactive and visual level longer than it is used traditionally.

Gray & Tall (1994) introduced the notion of *procept* as an idea generated by looking at a symbol such as 3+2 both as a process (of addition) and a concept (of sum).

There are different sources from which algebraic expression can take its meaning. One of them is *formal,* an algebraic expression is meaningful if it can be derived from set of axioms. In contrast with formal we use the term *referential* meaning:

An algebraic expression can represent a relationship between numbers in general – we call it *numerical meaning*.

An algebraic expression can recall a relationship among quantities in some situations – we call it situational meaning.

Stripe-arithmetic, the combination of representations

Following the Hungarian curriculum for the 7^{th} grade (13 year old students) I applied my teaching method based on using manipulative tools in March 2004, during 24 lessons of algebra. The class consisted of 7 students. This small number of students enable me to have individual approach to every student. At the beginning students took pre-test and were tested at the end of the school year.

I am going to describe a lesson in order to demonstrate the method by explaining the following identity: a – (b – c) = a – b + c.

The task connected to this identity at the pre-test was:

Find the appropriate expression on the left side without any calculation with the expression on the right side, which have the same value:

	185 – 58 – 9
	58 – 185 + 9
185 – (58 – 9)	185 + 58 – 9
	185 – 58 + 9
	185 + 58 + 9

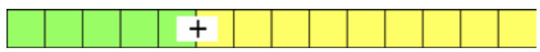
Write a story which will support your choice!

Using mathematical signs write down the rule!

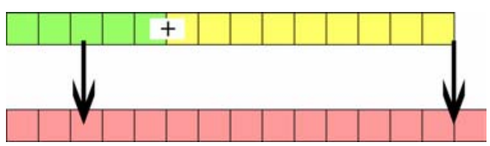
In order to avoid mental calculation I set three digit number in the numerical expressions.

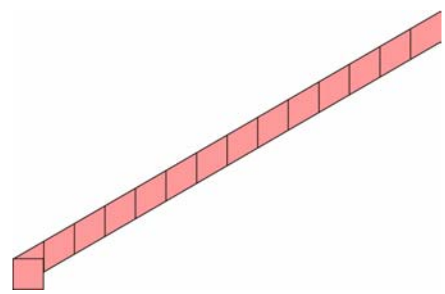
Nobody solved this task rightly, 5 of 7 students chose the first expression on the right side (malrule). Two of them connected the expression on the left side with two different ones on the right side (1st and 3rd), without knowing that the result is uniquely determined.

In the lesson itself the task was to fill in the boxes: 15 – (5 + 9) = 15 – 5 ☐ O. The numbers are represented by colour stripes made of graph paper on which one square represents a unit, which equals one. Different numbers are represented by different colours. To perform addition we connect the stripes using white tape. We represented subtraction by sticking the stripes over the left side of a stripe in a way that the representation of subtrahend is put over the representation of the minuend. After that we fold back the part which is double-layered.
During this performance the notion of procept is adapted through alternative perception of an arithmetical expression as a process on one hand and as a concept on the other.
The students represent numbers 15, 5 and 9 by different stripes. They consider (5+3) as a process. The result of addition is a concept (5+9 i.e. 14).

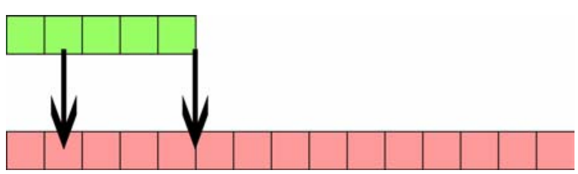


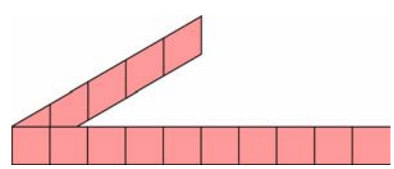
Following this the students considered 15 – (5 + 9) as a process. The result of subtraction is a concept (15 – (5 + 9) i.e. 1).





Having done this they design the other side of the equal sign and through the process of subtraction they get the concept (15 – 5 i.e.10).





Furthermore, by comparison of the concepts 1 and 10, they make the conclusion that they have to "subtract" (as it was described at the beginning of this section) the representation of number 9 from the representation of 15 – 5.
Finally they fill in the boxes and get: 15 – (5 + 9) = 15 – 5 – 9.
Students are obliged to use mental objects when they face with large numbers in tasks as: Fill in the gaps 500 – (100 – 20) = 500 – 100 ☐ O
This is how they step out from the need to use manipulative objects.

One of my students wrote a story (gave referential meaning) explaining this equality:

500 - (100 - 20) = 500 - 100 + 20

Девојчица је имала 500. фт. Одлучило је да купи те књигу која
кошта 100 фт. али су снизили за 20%. Када је платила
књигу остало јој је 500 - (100 - 20) фт.
Сутрадан је опет имала 500 фт. јер је хтела другарици да
купи исту књигу Платила је 100 фт. Удобила је касније 20 фт.
назад јер је књига још на снижењу. Сада има исто пара као
и јуче.

Here is a very liberal try of translation of her story:
A girl had 500 ft. He decided to buy a book which cost 100 ft, but they sell it on sail of 20%. She paid the book and 500 – (100 – 20) ft remained.
Next day she got 500 ft again, and she wanted to buy the same book. She paid 100 ft, and she got 20 ft back because it was still on sale. She has now the same money as yesterday.
Although she made series of serious gramatical and language mistakes (with cases, word order and incomplete sentences) her mathematical understanding of the problem is obvious and can not be denied!

Conclusion

I tried by using concrete objects not only to teach students formal algebraic rules, but also to connect the rules with real situations. Also I tried to promote the ability of constructing referential meaning of algebraic expressions as well as ability of generalization and recognition structures. My intention was also to inspire the creation and adoption of a very important algebraic notion – the notion of procept. Verbalization on its own is demanding task for students and in the case of bilingual school particularly so. The method of stripes was better accepted and more illustrative than any other.

References

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Gray, M. & Tall, D. (1994). Duality, Ambiguity and Flexibility: A Proceptual View of Simple Arithmetic. The Journal for Research in Mathematics Education, 26 (2), 115-141.