Summary

The thesis „Uniqueness and Regularity for Porous Media Equations with x-dependent Coefficients“ provides regularity and uniqueness results for certain equations occurring in porous medium flow. The main focus lies on x-dependent coefficient functions. Starting from the generalized porous medium equation

$$\partial_t s = \Delta[\Phi(x,s)] \text{ on } \Omega \times (0,T)$$

with $\Phi'(x,0) = 0$ in chapter 4, it is shown that truncations of s are H^1-regular in space and integration by parts formular of the form

$$\int_Q \partial_t s g(\cdot,s)\xi = \int_Q G(\cdot,s)\partial_t \xi \tag{$*$}$$

for certain functions g with primitive G are proved. Both results are essential for the consideration of the main problems of this thesis. In chapter 2, the problem for the discontinuous Richards equation is stated. There one considers, on a domain Ω divided by an interface Γ into two subdomains Ω_l and Ω_r, the Richards equation

$$\partial_t s = \nabla \cdot (\nabla[\Phi_j(s)] + \lambda_j(s)g_j) + f_j \text{ on } \Omega_j \times (0,T)$$

assuming the continuity of flux and pressure across the interface. For this equation, an L^1-contraction and uniqueness result is proven in chapter 5. To this end, $(*)$ is exploited and the method of doubling the variables to obtain a Kato inequality away from $t = 0$ is used. With a Gronwall argument, the Kato inequality is extended up to $t = 0$ and an L^1-contraction could be concluded. In this regime, the new contribution of this thesis is the L^1-contraction result also in the case that $\lim_{s\to 1} \Phi_j'(s) = \infty$. Particularly, Φ_j needs not to be Lipschitz continuous.

In the regime of two-phase flows, we use a standard transformation to obtain a system of the form

$$\begin{aligned}\partial_t s &= \nabla \cdot (\nabla[\Phi(x,s)] - \nabla_x \Phi(x,s) + B(x,s) + D(x,s)u) \\ 0 &= \nabla \cdot u = \nabla \cdot (\lambda(s)\nabla p + E(s))\end{aligned}$$

on $\Omega \times (0,T)$ (see chapter 2). From there on, solution concepts found in the literature are compared and the local Hölder continuity result is stated under the assumption that $\Phi'(x,s) \sim s^{\alpha_0}$ and $\Phi'(x,s) \sim (1-s)^{\alpha_0}$ for s near zero and one, respectively. Particularly, we assume $\alpha_0 > 0$. The local Hölder continuity is proven in chapter 6 using the method of intrinsic scaling. The regularity of truncations of s as well as $(*)$ is essential to justify the calculations presented there. The new contribution of this thesis is the local Hölder continuity for an x-dependent Φ. Particularly, the method of intrinsic scaling needs to be modified since DeGiorgi's lemma can not be applied. Instead a more general Poincaré-type inequality is used.

In the first chapter a brief introduction and derivation of the equations by means of physical principles is provided.