A Study of Uncertain Wind Power in Active-Reactive Optimal Power Flow

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Abstract—**Wind power fluctuates with time and it is reasonable to regard it as a random variable. Recently, an active-reactive optimal power flow (A-R-OPF) method in active distribution networks with wind stations has been developed to handle the problem of wind power curtailment (WPC). Since the mentioned method is deterministic, it may fail to handle uncertain wind power (UWP). Therefore, our study in this paper will firstly discuss the issue of UWP and secondly develop a new strategy which can improve the A-R-OPF by considering UWP. The new strategy can be distinguished from the original so that: 1) it considers shorter time intervals, i.e., 15 minutes instead of one hour and 2) it can handle both UWP and WPC simultaneously. The effectiveness of the new strategy is shown by using a real case medium-voltage distribution network.**

Keywords-active-reactive optimal power flow (A-R-OPF); medium-voltage; uncertain wind power (UWP); wind power curtailment (WPC).

I. INTRODUCTION

Uncertain wind power (UWP) is one of many challenges that power system operators face when ensuring optimal and reliable operations. It is a well-known fact that wind power production varies with wind speed and cannot be controlled except by curtailing it [1]. Therefore, wind energy is a partially dispatchable generation source, and consequently curtailment investigation must be considered within a *probabilistic* rather than *deterministic* study context [2].

Recently, a method which is based on so-called chance constrained optimal power flow (OPF) was proposed in [3]. In [3], load power uncertainties were considered as multivariate random variables with a correlated normal distribution, but renewable energy generation (REG) was not considered. In contrast, the forecasted active power outputs of REG units were regarded as normally distributed random variables [4]. Moreover, the uncertainty of load was neglected because the REG, such as wind energy, has more variability than load [5]. It is noted in [4] that no variables were used for curtailing of REG.

More recently, a deterministic active-reactive optimal power flow (A-R-OPF) method in active distribution networks (ADNs) with REG and battery storage systems was developed [6][7][8].

One of the remarkable abilities of the A-R-OPF method is its capability to ensure feasible solutions even with a high penetration of wind power. This is achieved by incorporating a curtailment factor. Note that the problem of A-R-OPF in ADNs [6-8] (*even with a deterministic formulation*) represents a large-scale and complex optimization problem. Therefore, it is expected that considering any uncertain operating conditions [9] will further complicate the problem.

The principle of wind power curtailment (WPC) was also used in [10], where an OPF was formulated as a nonlinear chance constrained optimization problem under non-Gaussian uncertainties. Note that the formulated problem in [10] (*even with a single snapshot*) was solved with a considerable computational time. In addition, the expected wind speed and its variance were assumed to be the same at all wind stations (WSs).

Based on the above literature and distinguished from the previous works in the research area of ADNs [11][12][13], a new strategy to handle UWP and WPC in an ADN is proposed in this paper. The contributions of this work can be summarized as follows:

- Using a shorter time interval (ti) in A-R-OPF, i.e., ti = 15 minutes instead of one hour.
- Introducing a center for environment data which can provide *forecasted* and *actual* wind power production for WSs at different locations in the ADN.
- Developing a new strategy to handle UWP and WPC simultaneously. This ensures the feasibility when considering *actual* wind power production in the deterministic A-R-OPF.

II. DETERMINISTIC A-R-OPF WITH UWP

A. Deterministic A-R-OPF

In [6], an A-R-OPF problem was formulated with forecasted or deterministic input wind power and demand profiles in the time frame of optimization. However, the inaccuracies in these forecasts were not considered as given in [8]. A deterministic A-R-OPF problem can reduce the computational effort on the one hand, but on the other hand it cannot handle the inaccuracies in these profiles. Therefore, our aim here is to overcome such problems by focusing on UWP.
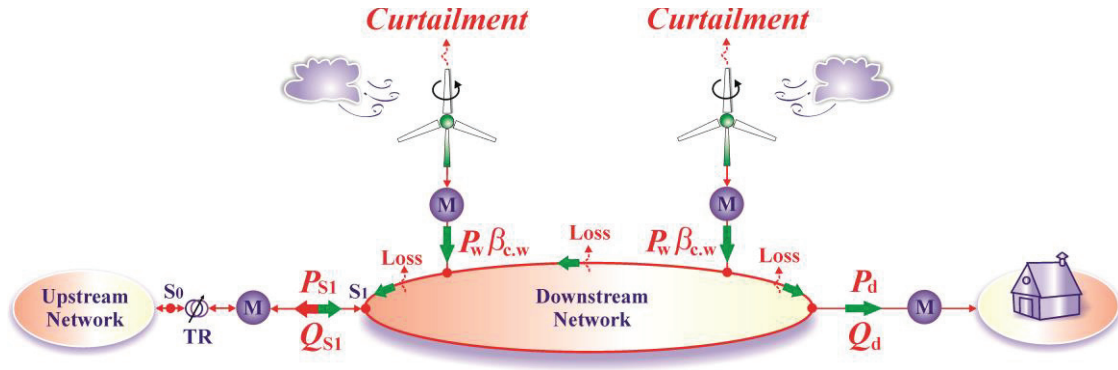
Fig. 1. Illustration of a meter-based method for charging and remunerating different entities connected to a power system [18]. Here, M stands for meter, TR for transformer, S_0 and S_1 for TR primary and secondary side, respectively.

B. Uncertain Wind Power

The idea behind considering UWP in this work can be explained in Fig. 1. Here, it is assumed that there are two WSs with same installed capacities at different locations in an ADN. The produced wind power P_w from each WS can be forecasted from the expected wind speed at each WS. Note that P_w can be different during *ti* even with a relatively small distance between WSs. This is because the forecasted wind speed can be different at each WS.

From another perspective, the forecasted value of P_w during *ti* can also be different from actual wind power production based on, e.g., a probability density function (PDF), as seen in Fig. 2. Here, F stands for forecast (a forecasted value), H for high-side (values higher than forecasted) and L for low-side (values lower than forecasted). Note that P_w in Fig. 2 is assumed normally distributed. The values of F, H and L are also illustrated with time for the two WSs, as shown in Fig. 3.

In Fig. 1, active P_d and reactive Q_d power demand are assumed to follow IEEE-RTS winter season's days [8]. In contrast, active P_{S1} and reactive Q_{S1} power at slack bus S_1 (see Fig. 1) are allowed to be either *positive* or *zero* to avoid any possible generation rejections from reverse power flows (see details in [11]). This means that active and reactive power can be *imported* (in the case of low wind power), but not *exported* (in the case of high wind power) [14][15]. Note that due to system constraints, a wind power curtailment factor ($0 \leq \beta_{c.w} \leq 1$) at each WS is used as a control variable [6], where $\beta_{c.w} = 1$ when no curtailment and $\beta_{c.w} < 1$ otherwise, as seen in Fig. 1.

The objective of A-R-OPF in Fig. 1 is to maximize the total revenue from the wind energy and meanwhile to minimize the total costs of active energy losses in the grid and the total costs of active and reactive energy at bus S_1. Now, the problem of UWP in A-R-OPF is to find a strategy to ensure the feasibility when considering *actual* instead of *forecasted* wind power production.

III. PROPOSED STRATEGY

The proposed strategy in this paper requires a center for environment data as shown in Fig. 4. This center should be able to provide forecasted and actual wind power production of WSs at different locations in an ADN for a given *ti*.
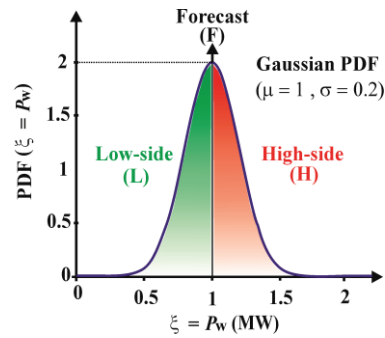

Fig. 2. Illustration of a PDF of wind power. Here, ξ is the random variable, μ is the mean or expectation and σ is the standard deviation.
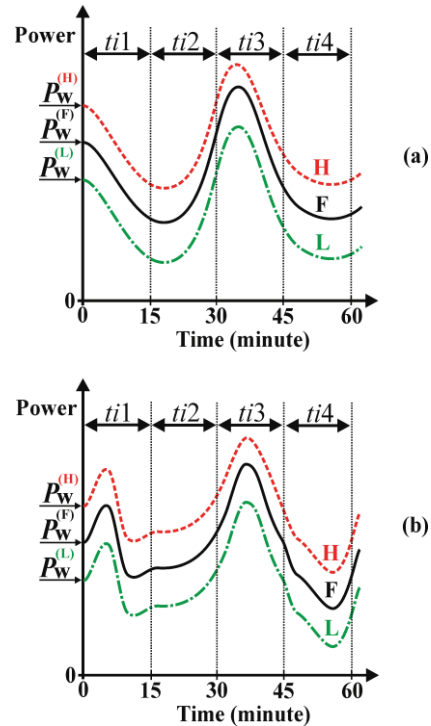

Fig. 3. Illustration of wind power profiles during an hour. Here, (a) and (b) stand for wind power profiles from two different WSs.
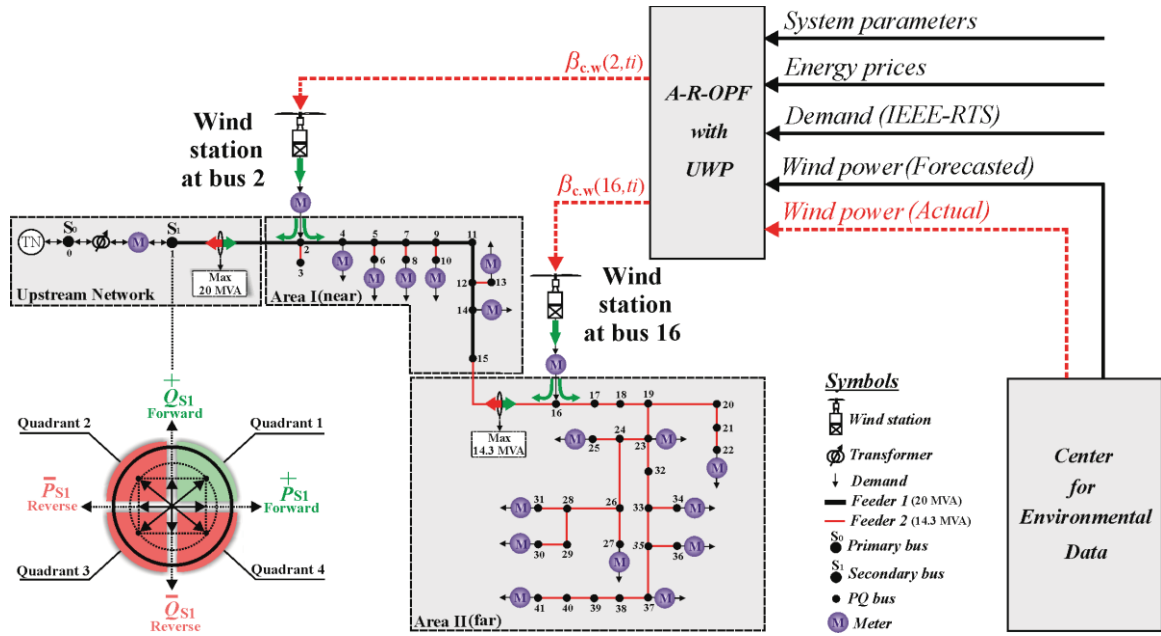
Fig. 4. Distribution network for the case study [17][18].

In this work, *ti* is taken 15 minutes ahead, as seen in Fig. 3. In addition, two WSs are considered to be located at different buses, i.e., bus 2 and bus 16, as seen in Fig. 4. Based on the above considerations, the solution strategy of UWP in A-R-OPF can be summarized by the following steps:

1) Provide system parameters, energy prices and demand as inputs to the A-R-OPF with UWP, as seen in Fig. 4.
2) Provide F(i,ti): forecasted wind power of WS at bus i during *ti*. For example, F(2,1) = $P_w(2,1)$ = 9 MW and F(16,1) = $P_w(16,1)$ = 8 MW, as seen in Fig. 5.
3) Calculate H(i,ti): wind power higher than forecasted of WS at bus i during *ti*. In this paper, we use the following simple formula H(i,ti) = F(i,ti) + $\Delta Pw(i)$. Here, $\Delta Pw(i)$ is defined as a constant power at bus i to get wind power values around the forecasts. Let $\Delta Pw(i) = \Delta Pw(2) = \Delta Pw(16) = 1$ MW. Then, one can simply get H(2,1) = $P_w(2,1)$ = 10 MW and H(16,1) = $P_w(16,1)$ = 9 MW, as seen in Fig. 5. Note that if H(i,ti) > $P_W(i)$, then H(i,ti) = $P_W(i)$. Here, $P_W(i)$ is the rated power of WS at bus i (see Table IV in Appendix).
4) Calculate L(i,ti): wind power lower than forecasted of WS at bus i during *ti*. In this paper, we use the following simple formula L(i,ti) = F(i,ti) − $\Delta Pw(i)$. Let $\Delta Pw(i)$ as defined above, then one can simply get L(2,1) = $P_w(2,1)$ = 8 MW and L(16,1) = $P_w(16,1)$ = 7 MW, as seen in Fig. 5. If L(i,ti) < 0, then L(i,ti) = 0.
5) Solve A-R-OPF for *ti* and all main nine possible scenarios shown in Fig. 5. Then, save obtained results in a *lookup table*, as Table I. Here, $\beta_{c.w}(i,ti)$ is the optimal curtailment factor at bus i during *ti* for all nine scenarios.

It is to note that the required computational time to fill in the lookup table should be small enough in order to ensure the applicability of the proposed strategy.
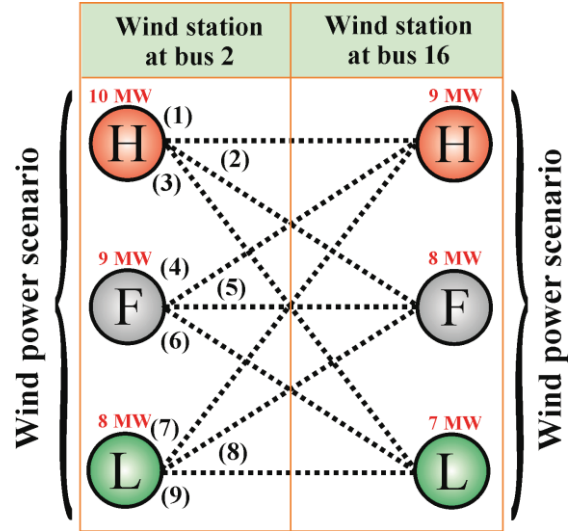

Fig. 5. Illustration of main nine possible scenarios. Here, F stands for forecast (forecasted wind power during *ti*), H for high-side (wind power higher than forecasted during *ti*) and L for low-side (wind power lower than forecasted during *ti*).

Now, provide *actual* wind power (AWP) for WS at bus i during *ti* from the center in Fig. 4. Then, compare it with wind power scenarios (i.e., H-H, H-F, H-L,…) in the lookup table, and choose the optimal control variables $\beta_{c.w}(i,ti)$ based on the following simple rules:

➢ Rule 1: If (H ≥ AWP > F) then consider (AWP = H)
➢ Rule 2: If (F ≥ AWP > L) then consider (AWP = F)
➢ Rule 3: If (L ≥ AWP ≥ 0) then consider (AWP = L)

The application of the above strategy will be further explained in section V.

S04.1

TABLE I. OPTIMIZATION RESULTS FOR THE FIRST TIME INTERVAL AND ALL MAIN NINE POSSIBLE SCENARIOS IN FIG. 5

Scenario		$P_w(2,1)$ (MW)	$P_w(16,1)$ (MW)	$\beta_{c.w}(2,1)$ ---	$\beta_{c.w}(16,1)$ ---	$P_{S1}(1)$ (MW)	$Q_{S1}(1)$ (Mvar)
1	H-H	10	9	0.475	0.278	0	2.53
2	H-F	10	8	0.475	0.313	0	2.53
3	H-L	10	7	0.475	0.357	0	2.53
4	F-H	9	9	0.528	0.278	0	2.53
5	F-F	9	8	0.528	0.313	0	2.53
6	F-L	9	7	0.528	0.357	0	2.53
7	L-H	8	9	0.594	0.278	0	2.53
8	L-F	8	8	0.594	0.313	0	2.53
9	L-L	8	7	0.594	0.357	0	2.53

IV. OPTIMIZATION PROBLEM FORMULATION

Here, we adapt the extended objective function F (1) [15] by changing the discretization from (*1-hour*) to (*15-minutes*) as follows:

$$\max \ F = F_1 - F_2 - F_3 - F_4 \tag{1}$$

where

$$F_1 = C_{pr.p}(ti) \sum_{\substack{i=1 \\ i \in l}}^{N} P_w(i,ti)\beta_{c.w}(i,ti) \tag{2}$$

$$F_2 = C_{pr.p}(ti)\, P_{loss}(ti) \tag{3}$$

$$F_3 = C_{pr.p}(ti)\, P_{S1}(ti) \tag{4}$$

$$F_4 = C_{pr.q}(ti)\, Q_{S1}(ti). \tag{5}$$

It is aimed in (1) to maximize the total revenue from the wind energy F_1, and meanwhile to minimize the total costs of active energy losses in the grid F_2, the cost of active energy at slack bus F_3, and the cost of reactive energy at slack bus F_4, respectively. Here, $C_{pr.p}(ti)$ is the active energy price during ti, $C_{pr.q}(ti)$ is the reactive energy price during ti, $P_{loss}(ti)$ is the active power losses during ti, N is the total number of buses, $P_w(i,ti)$ is the active power of WS at bus i during ti while l stands for the set of WSs, $P_{S1}(ti)$ and $Q_{S1}(ti)$ are the active and reactive power injected at slack bus S_1 during ti, respectively. The control variable of WSs is $\beta_{c.w}(i,ti)$, which represents the curtailment factor of wind power at WS i during ti.

The optimization problem is solved with the general algebraic modeling system (GAMS) [16] where the equality (6) and inequality (7) constraints for the A-R-OPF are as follows (mathematical details can be found in [6]):

$$\left\{\begin{array}{l} \bullet \text{ Active power balance at each bus} \\ \bullet \text{ Reactive power balance at each bus} \end{array}\right. \tag{6}$$

TABLE II. COMPARISON OF COMPUTATION TIME (ON INTEL CORE 2 DUO CPU 3 GHZ AND 4 GB RAM) AND FEASIBILITY STATE IN TWO METHODS

Criterion	A-R-OPF with UWP	A-R-OPF without UWP	Diff.
CPU (sec./ti1) Feasibility	58.348 **feasible**	8.346 **infeasible**	-50.002 (**-599%**)
CPU (sec./ti2) Feasibility	57.6 **feasible**	8.455 **feasible**	-49.145 (**-581%**)
CPU (sec./ti3) Feasibility	57.467 **feasible**	8.452 **feasible**	-49.015 (**-580%**)
CPU (sec./ti4) Feasibility	57.879 **feasible**	8.357 **infeasible**	-49.522 (**-593%**)

TABLE III. OBJECTIVE FUNCTION VALUE IN TWO SCENARIOS

Criterion	Actual wind power	Forecasted wind power	Diff.
$F(\$/ti1)$	82	83	-1(**-1.2%**)
$F(\$/ti2)$	66	83	-17(**-20.48%**)
$F(\$/ti3)$	81	83	-2(**-2.4%**)
$F(\$/ti4)$	80	83	-3(**-3.61%**)

$$\left\{\begin{array}{l} \bullet \text{ Voltage bounds of PQ-buses} \\ \bullet \text{ Active and reactive bounds at slack bus} \\ \bullet \text{ Feeder bounds} \\ \bullet \text{ Bounds of the curtailment factors} \end{array}\right. \tag{7}$$

V. CASE STUDY

The network considered for the case study is taken from [17][18]. It is a typical rural distribution network with 41 buses and 27.6 kV, as shown in Fig. 4, where line thicknesses indicate feeder capacities. The peak power demand of the network is 16.25 MVA [15] and the substation rating is 20 MVA. Two WSs (data in Table IV in Appendix) are located at buses 2 and 16, as seen in Fig. 4. Data of energy prices are given in Table V in Appendix. Slack bus S_1 is considered here with fixed voltage amplitude 1.02 pu [19] and zero voltage angle for all conducted computations.
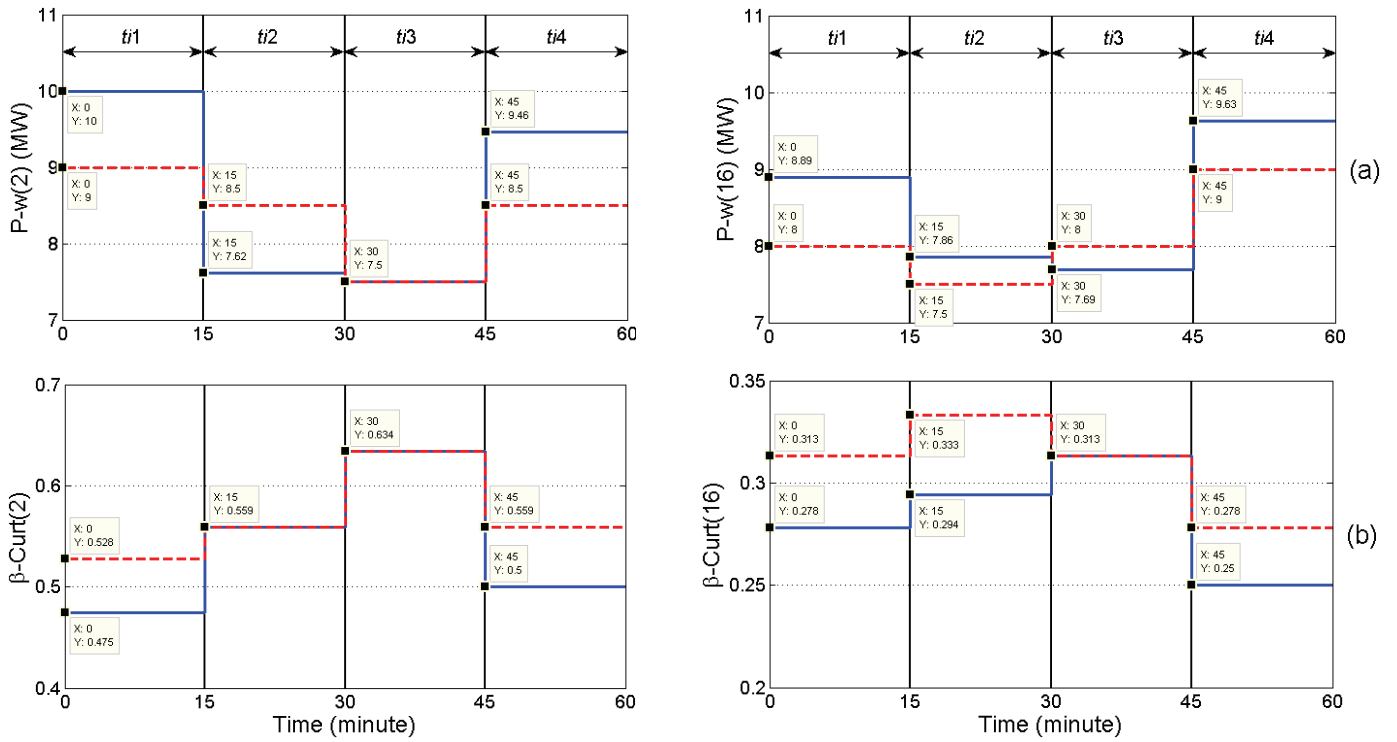
Fig. 6. (a) *Forecasted* wind power profiles (red-dashed) and *actual* wind power profiles (blue-solid) of WS at bus 2 (left column) and WS at bus 16 (right column). (b) Curtailment factors corresponding to *forecasted* wind power profiles (red-dashed) and corresponding to *actual* wind power (blue-solid) of WS at bus 2 (left column) and of WS at bus 16 (right column).
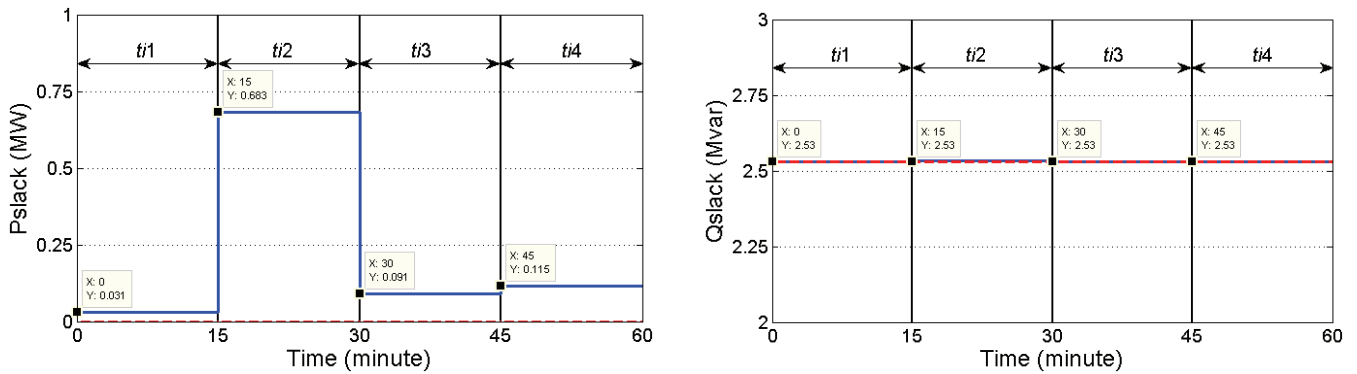

Fig. 7. Slack bus active power (left column) and slack bus reactive power (right column) corresponding to *forecasted* wind power profiles (red-dashed) and corresponding to *actual* wind power (blue-solid).

The results in Table I show that the active power at bus S_1 is always *zero* for all scenarios because of total active power demand (7.234 MW), high wind active power production of WSs (see Table I), and limits on *exported* active power. It can be clearly seen in Table I that high wind power production leads to low values of curtailment factors to ensure a feasible solution. In contrast to active power, the reactive power at bus S_1 is always (2.53 Mvar) for all scenarios (see Table I). This is because of using unity power factors (PFs) of all WSs (see Table IV), total reactive power demand (2.62 Mvar) and the reactive power compensation of feeder capacitive susceptance [18].

The benefits of the proposed strategy can be clearly seen in Table II where the deterministic A-R-OPF fails to achieve

feasible solutions during the first (i.e., *ti*1) and fourth (i.e., *ti*4) time periods. This is because the actual wind power production (see Fig. 6(a)) is higher than forecasted at both WSs. It means the new strategy uses the lowest curtailment factors, i.e., $\beta_{c.w}(2,1) = 0.475$ and $\beta_{c.w}(16,1) = 0.278$ (i.e., H-H) instead of $\beta_{c.w}(2,1) = 0.528$ and $\beta_{c.w}(16,1) = 0.313$ (i.e., F-F) (see Table I and Fig. 6). Of course the new strategy requires more CPU time. Note that the deterministic A-R-OPF can achieve a feasible solution during, e.g., the second time period (i.e., *ti*2) because the actual wind power production from WS at bus 2 is much lower than the production from WS at bus 16.

From economical point of view, the new strategy requires almost a higher active energy import at slack bus S_1 in comparison with the deterministic A-R-OPF as seen in Fig. 7.

For example, the active energy import during the second time period (i.e., $ti2$) is the highest value because the actual wind power production from WS at bus 2 is too lower than expected. The numerical results of the objective function are given in Table III. In Fig. 7, it is clearly seen that the reactive power import is in a weak relation with wind active power curtailment.

VI. Conclusions

This paper discussed the issue of uncertain wind power (UWP) in an active distribution network and proposed a new strategy which can handle both UWP and wind power curtailment (WPC) simultaneously. The new strategy improves the deterministic active-reactive optimal power flow so that on one hand it uses shorter time intervals, i.e., 15 minutes instead of one hour and on the other hand it ensures not only optimal but also feasible solutions.

The benefits of the proposed strategy in handling both WPC and UWP are shown by using a real case medium-voltage network. Beside the advantages of the new strategy, there are some drawbacks in terms of computational effort. Therefore, our future research will focus on tackling such problems by exploring more efficient computational mechanisms.

Appendix

TABLE IV.   Rated Power of WSs (P_W) and Power Factors (PFs)

	Wind stations	
Bus	2	16
P_W (MW)	10	10
PFs	1.0	1.0

TABLE V.   Data of Energy Prices During Time Intervals [8]

ti	$C_{pr.p}$($/MW.$ti$)	$C_{pr.q}$($/Mvar.$ti$)
$ti1$ = 15 min	12.5	3
$ti2$ = 15 min	12.5	3
$ti3$ = 15 min	12.5	3
$ti4$ = 15 min	12.5	3

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