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Neutrino masses and flavor symmetries

Three different aspects of the challenge of model building

Neutrino masses and flavor symmetries - Three different aspects of the challenge of model building

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- Ivo de Medeiros Varzielas and Daniel Pidt. "UV completions of flavour models and large θ_{13}". In: *JHEP* 1303 (2013), p. 065. DOI: 10.1007/JHEP03(2013)065. arXiv: 1211.5370 [hep-ph]

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Summary

Up to this point, no striking evidence for the theoretical nature of physics beyond the Standard Model has been found. However, there are many theoretical arguments and experimental observations why an extension of the Standard Model is necessary. In this work we focus on one of them, i.e. the existence of neutrino masses and mixing and their origin. The structures and hierarchies observed in the lepton sector show a large discrepancy to the well established quark sector. To motivate these differences we will rely on models based on flavor symmetries. After introducing a few general concepts, like Supersymmetry and the seesaw mechanism, we discuss three different classes of these. In each chapter we highlight one additional topic involved in the challenge of model building. We begin with a framework employing the symmetry A_4. Based on two existing supersymmetric models, guided by the now excluded paradigm of exact tribimaximal-mixing, we propose two viable modifications, illustrating that it can still be a useful starting point. While minimal ultraviolet completions of the old models exist, we discuss the interplay and restrictions of next-to-minimal completions and modifications of the general field content, employed to arrive at these viable versions. Afterwards we proceed to the symmetry $\Delta(27)$ in the context of geometrical CP violation, an interesting implementation of spontaneous CP violation. We start by presenting a concept to remedy the known shortcomings of the existing first viable model of the quark sector. Then we apply the lessons learned to construct a first existence proof for an implementation covering all known fermions featuring geometrical CP violation. Finally we conclude with a generic class of models which is only constrained by a general flavor hypothesis. In this part R-parity violation leads to viable neutrino structures. The number and size of the associated couplings is then reduced to a predictive set via the flavor hypothesis. Here we emphasize the differences between tribimaximal and realistic models and the question of neutrino mass hierarchy - normal or inverted?

Zusammenfassung

Bis zu diesem Punkt, hat sich kein eindeutiger Hinweis auf die Natur der Physik jenseits des Standardmodells gefunden. Es existieren jedoch viele theoretische Argumente und experimentelle Beobachtungen, dass eine Erweiterung des Standardmodells zwingend notwendig ist. In dieser Arbeit konzentrieren wir uns auf einen Aspekt, die Existenz von Neutrinomassen und -mischung und deren Ursprung. Die beobachteten Strukturen im Leptonen-Sektor unterscheiden sich hierbei komplett von den Strukturen im etablierten Quark-Sektor. Um diese Differenzen zu motivieren greifen wir auf Flavor-Symmetrie Modelle zurück. Nachdem wir einige grundlegende Konzepte, wie Supersymmetrie und den Seesaw-Mechanismus eingeführt haben, diskutieren wir drei Klassen von diesen. In jedem Kapitel beleuchten wir zusätzlich ein weiteres Konzept, das in der Modellkonstruktion eine Rolle spielt. Wir beginnen mit Modellen, die die Symmetrie A_4 verwenden. Basierend auf zwei existierenden supersymmetrischen

Modellen, welche vom jetzt ausgeschlossenen Paradigma der exakten tribimaximalen Mischung inspiriert waren, schlagen wir zwei aktuelle Modifikationen vor und zeigen damit, dass es immer noch ein nützlicher Ausgangspunkt sein kann. Auf Grundlage der bereits vorliegenden minimalen, ultravioletten Vervollständigungen der alten Modelle, diskutieren wir den Zusammenhang und die Einschränkungen fast minimaler Vervollständigungen und der Modifizierung der generellen Feldstruktur, die Verwendung finden um zu diesen realistischen Versionen zu gelangen. Wir fahren fort mit der Symmetrie $\Delta(27)$ im Zusammenhang mit geometrischer CP-Verletzung, einer interessanten Implementierung der spontanen CP-Verletzung. Zu Beginn präsentieren wir ein Konzept, das die bekannten Schwachstellen des ersten vorgelegten realistischen Models für die Quarks ausbessert. Danach bauen wir auf dem dort gelernten auf um den Existenzbeweis für eine erste Klasse von Modellen zu präsentieren, welche den kompletten Fermionensektor abdeckt und geometrische CP-Verletzung beinhaltet. Wir schließen mit einer generischen Art von Modellen, welche nur von einer simplen Flavor-Hypothese eingeschränkt ist. In diesem Teil führt die Verletzung der R-Parität zu den gewünschten Neutrinostrukturen. Die Zahl und Größe der beteiligten Kopplungen wird hierbei von der Flavor-Hypothese auf ein prädiktives Maß reduziert. In diesem Zusammenhang betonen wir den Unterschied zwischen tribimaximalen und realistischen Modellen und die Frage der Neutrionmassen-Hierarchie - normal oder invertiert?

Contents

1. Introduction

Particle physics rarely delivers a breaking news noticed not only by the scientific community, but also by the general public. July 4, 2012 marked such a day when the ATLAS and CMS collaborations announced the discovery of a massive spin-0 particle in a press conference at CERN. The last missing piece of a highly successful theory describing the interaction of fundamental particles, called Standard Model was finally found after decades of searching. But while the Standard Model-like properties of this so called Higgs boson were confirmed later, open issues still remain.

The Standard Model has several shortcomings, and it is clear that it has to be superseded by a different theory at some energy scale. The limit in this regard is 10^{19} GeV, which marks the point at which the quantum effects of gravity have to be taken into account. The LHC has been constructed as a general purpose experiment to discover any new physics beyond the Standard Model around the TeV scale. Unfortunately no striking indication of this has been found yet.

Arguably one of the most obvious features the Standard Model is missing, related to an even smaller scale, is the need to accommodate massive neutrinos which are the focus of this work. These are in turn closely related to the flavor puzzle. The standard model contains three generations of fermions with a large mass hierarchy and completely different mixing in the quark and lepton sector.

In the following section 1.1 we will give a short recap of the theoretical background of the Standard Model. First we will discuss the relevant gauge symmetries before introducing particle masses and mixing via the process of electroweak symmetry breaking in which the Higgs boson plays a crucial role. Section 1.2 will then discuss the most general additional concepts and mechanisms relevant to this work, namely Supersymmetry and neutrino mass generation. This will be accompanied by a short overview of the current experimental status quo in these fields. The next three chapters will then deal with concrete models based on additional flavor symmetries to generate the aforementioned neutrino masses and mixing.

In chapter 2 we present next to minimal ultraviolet completions of two models based on the symmetry A_4. The original minimal ultraviolet completions have been proposed with a very special mixing pattern, called tribimaximal-mixing, in mind. Exact tribimaximal-mixing is now excluded by experimental data, but it might still provide a good starting point to build realistic models like the ones we propose. We illustrate the process how these ultraviolet completions are build. Discussing the restrictions and difficulties encountered along the way also highlights one of the advantages of ultraviolet complete models; the additional constraints lead to a higher predictivity.

The models in chapter 2 only deal with the leptonic part of the fermions. In Chapter 3 we then turn to models featuring geometrical CP violation employing a $\Delta(27)$ triplet. We first discuss a modification of a viable existing model for the quarks and then proceed to combine this with working structures for the lepton sector. Overall this combines to a proof of existence for models of geometrical CP violation accounting for masses and mixing of the complete fermion sector.

Finally, chapter 4 introduces a different class of models to generate neutrino masses in a Supersymmetry framework without the explicit need for additional neutrino fields - R-parity violation. The challenge here is to explain the smallness of the introduced couplings and to reduce the relevant number to a set leading to predictive models. We show how to achieve this by introducing a very generic idea for the involved (unspecified) flavor symmetry. We discuss models leading to both inverse hierarchy and normal hierarchy for tribimaximal-mixing and large θ_{13}. Contrary to the models of chapter 2, in these models the realistic scenario does not necessarily arise as a perturbation of tribimaximal-mixing.

1.1. The Standard Model

1.1.1. Gauge group

The Standard Model (SM) of particle physics [5, 6, 7, 8] is the established status quo in the theoretical description of the interactions of fundamental particles, which are governed by the strong, the electromagnetic (EM) and the weak force. During its development, many bold predictions have been made. One of them for example being the postulation of the existence of the charm quark [9, 10]. It has been verified to an astonishing degree after years of experimental searches and tests including the arguably most precise measurement ever made, the anomalous magnetic moment of the muon $g-2$.[1] With the measurement of a massive spin-0 particle announce by the ATLAS and CMS collaborations [12, 13], the last piece of the puzzle finally fell into place.

Mathematically the SM is implemented as a local gauge quantum field theory represented by the unitary product group

$$\mathsf{G}_{\mathsf{SM}} = \mathsf{SU}(3)_C \times \mathsf{SU}(2)_L \times \mathsf{U}(1)_Y. \tag{1.1}$$

The fields of the theory can be categorized in three groups. The first is given by the fermion fields which are used to represent matter particles. Equation (1.1) is the product of *local* gauge symmetries, with generators depending on the Minkowski space time coordinate x. Therefore the introduction of these fields in combination with the requirement to keep the Lagrangian invariant at all points give rise to the second group, the gauge boson fields. Physically they can be interpreted as particles mediating the forces of the associated symmetry group. The

[1] For a review of up to date values for almost all involved quantities, see e.g. [11].

		$SU(3)_C$	$SU(2)_L$	Y
Gluons	G_a	**8**	**1**	0
$SU(2)_L$ gauge fields	A_i	**1**	**3**	0
$U(1)_Y$ gauge field	B	**1**	**1**	0
Quarks	$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix}$	**3**	**2**	$+1/3$
	u_R, c_R, t_R	**3**	**1**	$+4/3$
	d_R, s_R, b_R	**3**	**1**	$-2/3$
Leptons	$L_L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	**1**	**2**	-1
	e_R, μ_R, τ_R	**1**	**1**	-2
Higgs field	ϕ	**1**	**2**	1

Table 1.1.: The field content of the SM.

final field is a scalar field which is necessary for the construction of mass terms for the fermion fields and will also play a role in the generation of several gauge boson masses.

The group $SU(3)_C$ describes the dynamics governed by the strong force. It acts on fields with a so called *color* charge which is illustrated by the use of the subscript C. As a result the theory is christened as Quantum Chromodynamics (QCD). The matter fields charged under this symmetry are called quarks. There are three knows generations of them each containing quarks of two distinct electrical charges. These are usually referred to as **u**p, **d**own, **c**harm, **s**trange, **t**op and **b**ottom. The three different color charges of the quarks are labeled as red, green and blue. The eight generators of $SU(3)_C$ are constructed from the well known Gell-Mann matrices. The associated eight mediators carry (anti-)color charge and are called **g**luons. The fields will be referred to as G_a ($a = 1, 2, ..., 8$).

The name of these mediators is related to an interesting experimental observation. There are no free quarks. This is a result of the combination of two features of the theory. The symmetry is non-Abelian and the gluon fields themselves carry a charge under the symmetry and are massless. The larger the distance between two quarks (the lower the energy scale), the more energy is required to further increase it. The effect is called confinement. Therefore, we only observe particles which are uncharged (singlets) under $SU(3)_C$. These are called hadrons, which are separated into Mesons - a combination of quark and antiquark - and Baryons - a combination of three quarks. On the other end of the spectrum at very high energy scales (short distances) QCD can be considered as almost free. The result is called asymptotic freedom. This allows for the application of perturbation theory at high energy scales, while one has to rely on different, usually less explored/developed techniques for lower energy scales. On of them e.g. being QCD on the lattice [14].

The product of the remaining two symmetry groups is the basis of the famous Glashow-Weinberg-Salam (GWS) theory. The major part of this thesis will deal with physics related to

Fermions				Bosons	
Leptons		Quarks		Vector	Scalar
ν_e	e	u	d	γ	h
ν_μ	μ	c	s	Z^0, $W^\pm$	
ν_τ	τ	t	b	g	

Table 1.2.: Bosons and fermions constituting the particle content of the SM.

this part of the SM gauge group. The GWS theory unifies the EM and the weak force to one called electroweak (EW). This unification was a major theoretical breakthrough and many efforts have been made to further unify it with QCD to a grand unified theory. Unfortunately none of these could be established via experimental observation so far.

The weak interaction, tied to $\mathsf{SU}(2)_L$ only acts on left-chiral fermion fields with a left chiral spinor Ψ_L having the property $\gamma_5\Psi_L = \Psi_L$. These fields are assigned to $\mathsf{SU}(2)_L$ doublets and labeled with the index L while the right chiral fields are assigned to singlets with the index R. For the quarks the doublets are labeled as Q_L, while for the leptons we use the notation L_L. The fermion singlets will be denoted by the respective small case letter.[2] Analogously to the concept of spin in classical quantum mechanics, the weak isospin is introduced. Its third component I_3 is related to the electric charge Q and the $\mathsf{U}(1)_Y$ hypercharge Y via the relation

$$Y = 2(Q - I_3). \tag{1.2}$$

In a single $\mathsf{SU}(2)_L$ doublet for the known particles, the electric charge between the upper and the lower component varies by one unit of the electron charge. The group $\mathsf{SU}(2)_L$ has three generators τ_i (with $i = 1, 2, 3$). The associated three gauge fields constituting a triplet are referred to as A_i, while the single gauge field of $\mathsf{U}(1)_Y$ is denoted as B.

The field content of the standard model is completed with the scalar Higgs field ϕ which is assigned to be a $\mathsf{SU}(2)_L$ doublet with a positive hypercharge of one and no color. This field acquires a nontrivial vacuum expectation value (VEV) which leads to the breakdown of $\mathsf{SU}(2)_L \times \mathsf{U}(1)_Y$ which is the main topic of the following subsection 1.1.2.

An overview of all SM fields and their assignments under the SM gauge group is presented in table 1.1.

1.1.2. Massive particles

Introducing masses to the theory described in subsection 1.1.1 is a nontrivial task. In fact the Nobel price in physics of 2013 was awarded to Peter W. Higgs and François Englert, for their major contributions [15, 16, 17] to this topic. The challenge arises from the requirement of $\mathsf{SU}(2)_L$ invariance of the Lagrangian. Due to the multiplet assignments, including a mass term

[2] Note that the SM does not contain any right-handed neutrinos.

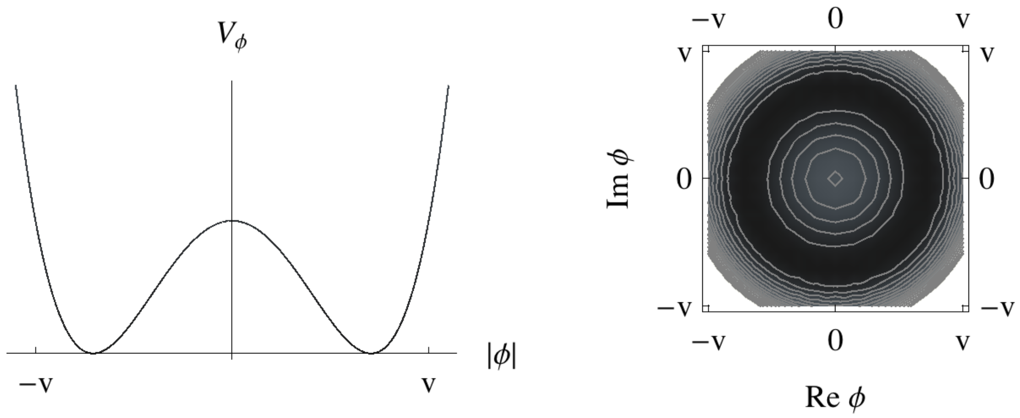

Figure 1.1.: Illustrations of the Higgs potential symmetries. On the right side the color indicates a rising V_ϕ from black to white. Grey lines correspond to contours of the same values with equal spacing.

for an arbitrary fermion field f of table 1.1

$$\mathscr{L}_{SM} \supset -m_f \overline{f}_L f_R \tag{1.3}$$

with a constant m would break this invariance and is thus not allowed. The solution to this problem is to generate such a term dynamically. In the SM this is achieved via the now famous Higgs mechanism of electroweak symmetry breaking (EWSB). The key ingredient is the introduction of a scalar field with a non-trivial charge under $\mathrm{SU}(2)_L$, the aforementioned ϕ.

Associated with scalar particles comes a scalar potential. For the single field ϕ this potential V_f is given by

$$V_\phi = -\frac{1}{2}\lambda\left(|\phi|^2 - \frac{1}{2}v^2\right)^2, \tag{1.4}$$

with the signs of the constants λ and v fixed by the requirement of the potential to be bounded from below. Studying this so-called Mexican hat potential is very helpful in illustrating the process of ESWB. For a complex field the potential of equation (1.4) has two classes of extrema, one single unstable point at the center and a stable ring of minima around it with radius v and it is symmetrical with respect to rotations around and reflections on the z-axis, as can be seen in figure 1.1. Now, as a complex doublet, ϕ has four degrees of freedom. Breaking the symmetry by letting ϕ acquire an arbitrary VEV $|v|$ breaks the latter symmetry. Due to the remaining rotation invariance, the doublet can always be rotated to a basis where

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ v+h \end{pmatrix}, \tag{1.5}$$

where h is the physical Higgs boson. In the language of gauge groups, this is formulated as

$$\mathsf{SU}(2)_L \times \mathsf{U}(1)_Y \to \mathsf{U}(1)_{\mathrm{EM}} \tag{1.6}$$

where the remaining rotation symmetry after the breakdown of EW symmetry is called EM symmetry. Inserting this VEV in a term

$$\mathscr{L}_{SM} \supset -y_f \phi \overline{f}_L f_R, \tag{1.7}$$

where y_f is a Yukawa-coupling, now leads to a mass term with the required form of equation (1.3). The actual mass is determined by the strength of the coupling constant and the Higgs-VEV.

The process of EWSB further leads to gauge boson masses. Technically however, these arise in a different way compared to the fermions. The SM Lagrangian contains the term with the covariant derivative D_μ of the Higgs field

$$\mathscr{L} \subset \left|D_\mu \phi\right|^2. \tag{1.8}$$

Introducing the EW coupling constants g and g', this derivative is defined as

$$D_\mu = \left[\delta_\mu - \frac{ig}{2}\tau^a A_\mu^a - \frac{ig'}{2} Y B_\mu\right]\phi. \tag{1.9}$$

The expansion of this Lagrangian term and the insertion of the Higgs-VEV then lead to the three massive physical Bosons $W^\pm$ and Z^0 and the massless photon γ. In the context of the Goldstone theorem the massive Bosons are interpreted to 'eat' three Goldstone bosons and the massless photon is protected by the remaining $\mathsf{U}(1)_{\mathrm{EM}}$ of equation (1.6). Note that the field content of the SM and the Higgs mechanism do not lead to massive Neutrinos due to the lack of the necessary right handed fields.[3] All the physical particles of the SM are summarized in table 1.2.

1.1.3. Fermion mixing

While the interaction vertices of the SM mediate only between fermions of one generation, in nature we observe phenomena switching between these generations. An obvious example are several Meson decays in the hadron sector. The origin of this can be explored by examining the weak charged current we observe. For instance for quarks u and d this is given as

$$W_\mu^+ \overline{u}_L \gamma^\mu d_L. \tag{1.10}$$

In experiments we only measure mass eigenstates. The EW current on the other hand mediates between weak eigenstates and obviously the two bases defined by these eigenstates do not

[3] This is true at the renormalizable level. Non-renormalizable operators will be discussed later.

match. To account for this we introduce the unitary complex 3×3 matrices V_u and V_d which transform u and d from the mass basis to their weak counterparts u', respective d'

$$u' = V_u u \tag{1.11}$$
$$d' = V_d d. \tag{1.12}$$

These can be used to rewrite equation (1.10) in terms of the weak eigenstates

$$W^+_\mu \overline{u}'_L \left(V_u V_d^\dagger\right) \gamma^\mu d'_L \equiv W^+_\mu \overline{u}'_L V_{\text{CKM}} \gamma^\mu d'_L, \tag{1.13}$$

which defines the CKM-matrix V_{CKM}. It is named after its three inventors. Cabibbo first postulated mixing between the first two quark generations. The associated mixing angle θ_C is therefore referred to as the Cabibbo angle [18]. This was later generalized by Kobayashi and Maskawa for all three generations [19] and awarded with the noble price of physics in 2008.

The CKM matrix can be parametrized in terms of three real angles and one complex phase. The standard parametrization for such a matrix is denoted as

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}. \tag{1.14}$$

Here s_{ij} and c_{ij} refer to the sine and cosine of the mixing angle of the generation indices and δ_{13} is the complex phase. It is this complex phase of V_{CKM}, which is the only origin of CP violation (CPV) in the standard model, a concept which will receive further attention in chapter 3. Experimentally, V_{CKM} is nearly diagonal. One possible approximation in frequent use was introduced by Wolfenstein [20] and reads

$$V_{\text{CKM}} \approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}, \tag{1.15}$$

where λ is identified as θ_C.

To conclude this section we turn our focus to trying the same exercise in the lepton sector. In terms of weak eigenstates, analogously to the quark case, the charged current here reads

$$W^+_\mu \overline{\nu}'_L \left(U_\nu V_L^\dagger\right) \gamma^\mu e'_L \equiv W^+_\mu \overline{\nu}'_L U_{\text{PMNS}} \gamma^\mu e'_L, \tag{1.16}$$

which defines the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix U_{PMNS}. However, since neutrinos are not massive in the standard model, one has always the freedom to choose $U_\nu = V_L^\dagger$ leading to no physical mixing. This is not compatible with the phenomena, that we observe in nature. These in fact point to a mixing matrix which is not even close to diagonal, contrary to the quark sector. Current best fit values for a global fit of the mixing angles taken from [21] indicate

$$\sin^2\theta_{12} = 0.30, \tag{1.17}$$

$$\sin^2\theta_{23} = 0.41, \tag{1.18}$$

$$\sin^2\theta_{13} = 0.023 \tag{1.19}$$

and require at least two massive neutrinos. The famous tribimaximal-mixing (TBM) matrix [22]

$$V_{\mathrm{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{1.20}$$

remains a good first order approximation, although exact TBM mixing is now excluded, since the value of θ_{13} has to deviate from its TBM value which demands $\theta_{13} = 0$. First concrete evidence for this was presented in 2011 by the experiments T2K in Japan [23] and MINOS in the United States [24]. Both are long-baseline accelerator experiments which searched for $\nu_\mu \to \nu_e$ events. This initial findings were later confirmed by other experiments like Daya Bay, Double Chooz and RENO.[4]

Oscillations only tell us about two mass squared differences. Best fit values, also included in [21] are

$$\Delta m^2_{21} = 7.50 \times 10^{-5}\,\mathrm{eV}^2, \tag{1.21}$$

$$\Delta m^2_{31} = +2.473 \times 10^{-3}\,\mathrm{eV}^2, \tag{1.22}$$

or

$$\Delta m^2_{32} = -2.427 \times 10^{-3}\,\mathrm{eV}^2, \tag{1.23}$$

where $\Delta m^2_{ij} = m_i^2 - m_j^2$ is defined by the three neutrino mass eigenvalues m_i. The ambiguity of the last two of these reflects the fact that in the neutrino sector, the hierarchy of these masses is not yet completely determined. The two possible scenarios $m_1^2 < m_2^2 < m_3^2$ and $m_3^2 < m_1^2 < m_2^2$ are referred to as normal hierarchy (NH) and inverse hierarchy (IH).[5] For these masses so far only upper bounds are known. The most stringent one is derived from cosmology and after the latest Planck results [25] given by

$$\sum m_i < 0.23\,\mathrm{eV}. \tag{1.24}$$

While this bound involves many uncertainties, similar bounds, i.e. from neutrinoless double beta decay, indicate the same order of magnitude.

[4] Published Results for all relevant experiments can again be found in [11].

[5] The loosely defined case where all of them are in the same order of magnitude is usually called degenerate.

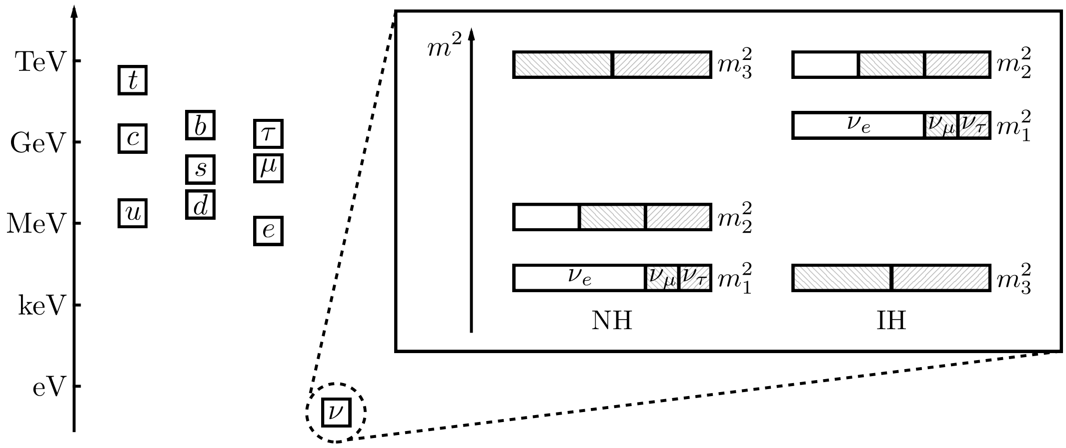

Figure 1.2.: A sketch of the known fermion masses. The logarithmic mass scale on the left indicates the huge range and hierarchies covered by the three generations of up-type quarks, down-type quarks, charged leptons and neutrinos (from left to right). Since the status of neutrinos is not visible at this scale, a zoom is provided which illustrates the relations of the squared mass eigenvalues, and highlights the ambiguity of their normal or inverse order and the large mixing compared to the quark sector. TBM is displayed to highlight the correspondence between the squared mass eigenvalues and the squared columns of equation (1.20).

The sketch of figure 1.2 summarizes the situation of fermion masses and mixing and hints at the huge challenges involved in the construction of a realistic theory. One big question is why are all known fermions replicated in three generations and how are they related? The gauge groups of the standard model provide no answer, since they only discriminate by quantum numbers which are the same for all particles belonging to the same family. Even putting aside neutrino masses for a moment, why is the hierarchy between the masses spanning almost five orders of magnitude? Another issue is the observed fermion mixing. Why is it so different in the quark and lepton sector, with one mixing matrix almost diagonal and the other one almost of tribimaximal form? Finally, why are the mass hierarchies and scales in the quark sector different compared to the lepton sector? With this open questions we conclude, that an appealing theory needs to expand the symmetry content of the SM in a way that distinguishes between quarks and leptons, and their respecting generations. Since these additional symmetries mediate between different flavors they are often called flavor symmetries, or horizontal[6] symmetries.

Such symmetries will be the topic of the following chapters, but since neutrino masses are

[6]This naming may seem a little bit odd due to the layout we choose to present the particle content so far. It's origin lies in displaying the generations of particles from left to right, therefore symmetries relating these have to mediate in the horizontal direction.

such a special challenge we would like to quickly summarize some general mechanisms how to extend the SM to include these.

1.2. Beyond the Standard Model

1.2.1. Supersymmetry

As we pointed out, to contribute to a viable theoretical description of nature one necessarily needs to go beyond the Standard Model (BSM). Many different concepts to do so and a countless number of models have been proposed in the past. But since all the models discussed in this work are implemented in a so called supersymmetric framework, we take the liberty of introducing just one broad concept - Supersymmetry (SUSY), a symmetry relating bosons and fermions.

There are several arguments, why this might be a promising extension of the SM but here we choose to only sketch a motivation based on symmetry considerations. These have proven to be invaluable in the past, and will be (albeit in a different manifestation) heavily used in the original part of this work. The largest continuous spacetime symmetry we have observed so far is the one of the Poincaré group. It's algebra is defined in terms of the hermitian generators of the unitary operators of four translations and homogeneous Lorentz transformations. These are denoted as P_μ, respective $M_{\mu\nu}$. One can now ask the question how to expand this symmetry further. As it turns out this is hardly possible on the basis of physically reasonable assumptions. The Coleman-Mandula theorem [26] tells us that it is not possible to combine the Poincaré algebra with any other continuous symmetry in a nontrivial way to a Lie algebra. I.e. for any generator of an additional Lie algebra T the relation

$$[T, P_\mu] = [T, M_{\mu\nu}] = 0 \tag{1.25}$$

holds. The Haag-Łopuszánski-Sohnius theorem [27] however directs us to a unique way out - Z_2-graded Lie algebras. It proves, that such an algebra is the most general continuous symmetry of the S-Matrix when two requirements are fulfilled. First, the odd generators of this symmetry belong to the representations[7] $(1/2, 0)$ and $(0, 1/2)$ of the homogeneous Lorentz group and second, the even generators commute with the other generators of the symmetry group. Now identifying even generators as bosonic and odd generators as fermionic, this means that we are able to add relevant fermionic generators of the above mentioned representations. An object with exactly these properties is an operator Q which generates transformations between fermions and bosons

$$Q\,|\text{boson,fermion}\rangle = |\text{fermion,boson}\rangle\,. \tag{1.26}$$

[7] Note that $\mathsf{SU}(2)_+ \otimes \mathsf{SU}(2)_-$ is homomorph to $\mathsf{SL}(1,3)$ and thus any relevant representation can be described by an 'angular-momentum' index pair (j_1, j_2).

Further investigation of this operator leads to the algebraic relations

$$\{Q, Q\} \propto P_\mu, \tag{1.27}$$

$$[Q, P_\mu] = 0, \tag{1.28}$$

$$[Q, M_{\mu\nu}] \propto Q, \tag{1.29}$$

and thus expands the S-Matrix symmetry including the Poincaré symmetry.[8] This concept is called SUSY. Note that in principle it is possible to add more than one generator satisfying the Haag-Łopuszánski-Sohnius theorem. In this work however, we only discuss SUSY with one generator which is usually referred to as $N = 1$ SUSY.

Single particle states are irreducible representations of the SUSY algebra and called supermultiplets. Two properties of theses are, that all particles in such a multiplet are required to have the same gauge quantum numbers and that the number of bosonic degrees of freedom equals the number of fermionic degrees of freedom. The fermionic and bosonic states of a supermultiplet are referred to as superpartners. The simplest possible way to construct a supermultiplet is combing a Weyl fermion with a complex scalar field, this is called chiral supermultiplet. The next to simplest possibility is constituted by a massless spin-1/2 Weyl fermion and a massless spin-1 boson. This is labeled as a vector multiplet. Every particle of a SUSY extension of SM particle content falls into one of these categories. The superpartners of the SM fermions are usually indicated by prefacing an s to the original name and applying a tilde to the mathematical symbol, so e.g. an neutrino (ν) becomes a sneutrino ($\tilde{\nu}$). The superpartners of bosons are named by appending an -ino. Note that for each chiral fermion of the SM, the associated superfield must provide the same number of fermion and boson degrees of freedom. For the $\mathsf{SU}(2)_L$ doublet this requires two complex scalar fields, consequently called left sfermions and labeled with the index L, while for the singlets one right sfermion field with the index R is sufficient.

A geometric interpretation of SUSY, which is suitable to a quantum field theory, is possible in superspace. This relies on the additional use of Grassmann variables and the details of this will not be discussed here. The takeaway message is, that in this context it makes sense to talk about superfields. This is the expression we will use from now on.

1.2.2. The minimal supersymmetric Standard Model

While there are many different implementations of supersymmetric extensions of the SM, we will only introduce one of them, which is called Minimal supersymmetric Standard Model (MSSM). This is, roughly speaking, constructed by introducing a superpartner for every SM field of table 1.1, group them into superfields, parametrize the symmetry breaking and stop at this point, except for one other necessary addition to the superfield content.

[8] In this short discussion we have omitted the spinorial indices of Q since the technical details of SUSY are not discussed in this work. For a more thorough introduction, see e.g. [28], or one of the many available textbooks like [29].

		$SU(3)_C$	$SU(2)_L$	Y
Gauge fields	G_a	$\mathbf{8}$	$\mathbf{1}$	0
	A_i	$\mathbf{1}$	$\mathbf{3}$	0
	B	$\mathbf{1}$	$\mathbf{1}$	0
Matter fields	Q_i	$\mathbf{3}$	$\mathbf{2}$	$+1/3$
	$\overline{U}_i$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-4/3$
	$\overline{D}_i$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$+2/3$
	L_i	$\mathbf{1}$	$\mathbf{2}$	-1
	$\overline{E}_i$	$\mathbf{1}$	$\mathbf{1}$	2
Higgs fields	H_1	$\mathbf{1}$	$\mathbf{2}$	-1
	H_2	$\mathbf{1}$	$\mathbf{2}$	1

Table 1.3.: Superfield content of the MSSM.

sleptons		squarks		Gaug- and Higgsinos
$\tilde{\nu}_e$	$\tilde{e}_L$, $\tilde{e}_R$	$\tilde{u}_L$, $\tilde{u}_R$	$\tilde{d}_L$, $\tilde{d}_R$	$\tilde{g}_a$
$\tilde{\nu}_\mu$	$\tilde{\mu}_L$, $\tilde{\mu}_R$	$\tilde{c}_L$, $\tilde{c}_R$	$\tilde{s}_L$, $\tilde{s}_R$	$\tilde{\chi}_1^\pm$, $\tilde{\chi}_2^\pm$
$\tilde{\nu}_\tau$	$\tilde{\tau}_L$, $\tilde{\tau}_R$	$\tilde{t}_L$, $\tilde{t}_R$	$\tilde{b}_L$, $\tilde{b}_R$	$\tilde{\chi}_1^0$, $\tilde{\chi}_2^0$, $\tilde{\chi}_3^0$, $\tilde{\chi}_4^0$

Table 1.4.: List of sparticles of the MSSM omitting antisfermions.

To establish the necessary notations and concepts, we will delve into some of the intricacies of this a little bit more deeply in the following. As previously stated we start by introducing scalar superpartners for all SM fields. For quarks this would be e.g.

$$\tilde{q}_{iL} = \begin{pmatrix} \tilde{u} \\ \tilde{d} \end{pmatrix}_L, \tilde{u}_{iR} \text{ and } \tilde{d}_{iR}, \tag{1.30}$$

were i is a generation index for the corresponding quark family. With the help of these and the SM fields we define the left chiral superfields Q_i, $\overline{U}_i$ and $\overline{D}_i$ following the requirements stated in subsection 1.2.1. Analogously we proceed in the leptonic sector by promoting the SM fields into the superfields L_i and $\overline{E}_i$.

The gauge sector is handled in a similar way. For each gauge field of the SM we introduce a vector superfield, additionally containing the spin-1/2 Majorana superpartners. We simply name these G_a, A_i and B replacing the notation of subsection 1.1.1.

After skimming over this part, the supersymmetric Higgs sector requires a little more attention. The reason is the way how Yukawa interactions arise in supersymmetric theories. These are derived from the superpotential W which is constrained to only contain left or right chiral superfields to be holomorphic. Thus it is not possible to use one Higgs doublet to derive mass terms for both up- and down-quarks, like it was the case in the SM. So to construct a viable theory we need to include a second Higgs field with opposite hypercharge Y. We call these two Higgs fields h_1 and h_2 and group them with the necessary superpartners into the superfields H_1 and H_2. The overall superfield content of the MSSM is summarized in table 1.3.

It is important to point out at least one additional property of the MSSM. In the SM Lagrangian lepton and baryon numbers L and B are conserved accidentally. This is not the case when writing down the most general superpotential of the MSSM superfields obeying the SM gauge group. Therefore an additional conserved quantum number called R-parity (R_P) is introduced in the MSSM, to prevent e.g. rapid proton decay. Additionally using the spin S this is defined for any particle as

$$R_P \equiv (-1)^{3(B-L)+2S}. \tag{1.31}$$

Note that via this definition all particles are R-parity even and all sparticles are R-parity odd. This prevents the lightest MSSM sparticle to decay, which is therefore stable. The superpotential of the MSSM W_{MSSM} is then given by

$$W_{MSSM} = \mu H_1 H_2 - f^e_{ij} H_1 L_i \overline{E}_j - f^d_{ij} H_1 Q_i \overline{D}_j - f^u_{ij} H_2 Q_i \overline{U}_j, \tag{1.32}$$

where the f_{ij} are the corresponding 3×3 Yukawa matrices and the parameter μ has mass dimension one. The scale of μ is required to be of the order of the weak scale to include EWSB. Interesting consequences arise in this place when R_P is dropped. This will be discussed in subsection 1.2.3.

As mentioned SUSY must be broken. It turns out that this breaking can not be spontaneous in the MSSM. It has to be introduced explicitly by adding soft[9] SUSY breaking terms $\mathscr{L}_{\text{soft}}$ to the supersymmetric terms $\mathscr{L}_{\text{SUSY}}$ so that overall

$$\mathscr{L} = \mathscr{L}_{\text{SUSY}} + \mathscr{L}_{\text{soft}}. \tag{1.33}$$

The details of this process and the necessary EWSB are beyond the scope of this short introduction.

In the end this leads to two charged mass eigenstates, called charginos $\tilde{\chi}^{\pm}_{1,2}$, four neutralinos $\tilde{\chi}^0_{1,2,3,4}$ and five Higgs bosons h, H, A, $H^{\pm}$, where the indices correspond to the ordering of the masses from lightest to heaviest. Minimization of the Higgs potential and EWSB involve the two VEV's

$$\langle h_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \quad \langle h_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2. \end{pmatrix} \tag{1.34}$$

These are related to the W and Z masses of the SM and their ratio defines

$$\tan\beta \equiv v_2 / v_1. \tag{1.35}$$

Overall the particle content of the MSSM is given by the particle content of the SM with an enlarged Higgs sector and the superparticles. These are completed with the addition of the gluon superpartners called gluinos g_a. The sparticle content is summarized in table 1.4.

[9] This means that every field operator has a mass dimension less than four.

1.2.3. Generating neutrino masses

The previous discussion has still not addressed our main problem - how to include massive neutrinos? However, it has given us a hint of one possible starting point in supersymmetric models. But before discussing this, it is in order to derail a little bit and to point out the effects of one unique feature of neutrinos. They are the only fermions we observer at a low energy scale that are electrically neutral. This allows us to illustrate the status of lepton masses after EWSB via the effective Lagrangian

$$\mathscr{L}_{\text{leptons}} = -\overline{l_L}\mathbf{m_l} l_R - \frac{1}{2}\nu_L^\top C\mathbf{m}_\nu \nu_L + \text{H. c.}, \tag{1.36}$$

where we denote the corresponding charged lepton fields as $l_{L/R}$. The charge conjugation matrix C is defined by $C = i\gamma^2\gamma^0$ and m_L and m_ν are arbitrary (potentially complex) 3×3 mass matrices. An additional requirement for the latter one is to be symmetric. They are called Majorana masses and violate Lepton number by two units.[10] Note that Majorana masses allow for two additional phases in U_{PMNS} compared to the parametrization of equation (1.14). Clearly, the $\mathsf{SU}(2)_L$ structure of these terms is different compared to the Dirac ones. This indicates that they result from a separate mechanism. Thus it can be considered natural to relate them to a different scale, which is a key ingredient to arrive at a satisfying description of neutrinos. There are numerous ways to realize this effective low energy picture of neutrino masses. We will discuss the most prominent ones, which are relevant to this work, in the following.

In supersymmetric models one has the option to drop the concept of R-parity conservation which was introduced in the MSSM by hand without any deeper theoretical background anyway. The most rigorous way to do so is to add all terms that are now allowed to the superpotential of equation (1.32).[11] Since lepton number conservation is no longer an issue anyway, one can start by replacing each occurrence of H_1 by L_i since both superfields share the same quantum numbers and add these terms to the superpotential. As it turns out there is one more operator which is allowed that violates baryon number. Overall the new R-parity violating (RPV) superpotential terms W_{RPV} are

$$W_{\text{RPV}} = -\mu_i L_i H_2 + \frac{1}{2}\lambda_{ijk} L_i L_j \overline{E}_k + \lambda'_{ijk} L_i Q_j \overline{D}_k + \frac{1}{2}\lambda''_{ijk}\overline{U}_i\overline{D}_j\overline{D}_k, \tag{1.37}$$

introducing three new couplings μ_i with mass dimension one, nine Yukawa couplings λ_{ijk}, 27 Yukawa couplings λ'_{ijk} and nine Yukawa couplings λ''_{ijk}.[12] The introduction of these terms

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[10] An experimental confirmation of this would be the observation of neutrinoless double β decay, which is a heavily explored topic but not confirmed so far. See e.g [30, 31, 32, 33, 34] for published results and [35, 36, 37] for proposed sensitivities. A more complete overview can be found in one of the many recent review talks. See e.g. [38].

[11] R-parity violation (RPV) can also be introduced spontaneously, this was first proposed by Aulakh and Mohapatra [39] and will not be further mentioned here. For a thorough introduction to the concept of RPV see e.g [40].

[12] The number of λ (λ'') couplings is reduced due to the requirement of $\mathsf{SU}(2)_L$ ($\mathsf{SU}(3)_C$) invariance.

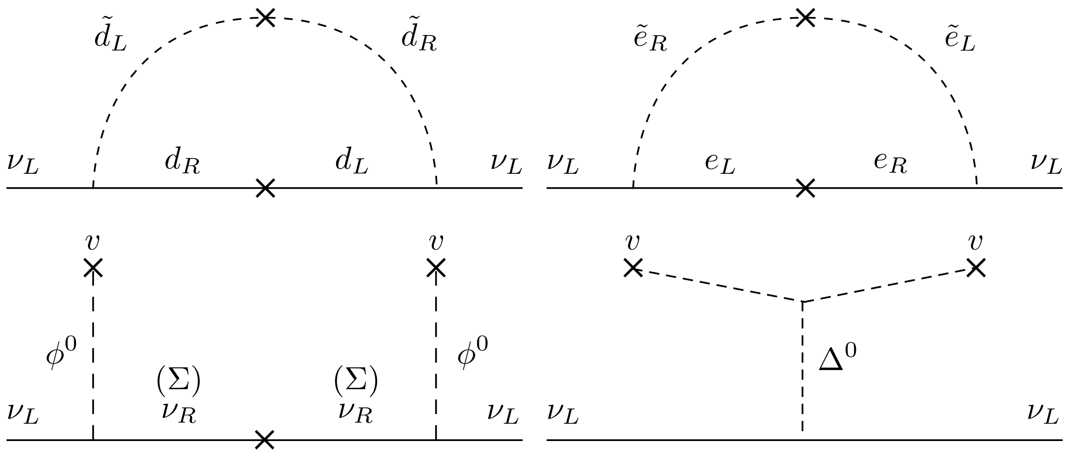


Figure 1.3.: Exemplary Feynman diagrams for neutrino Majorana mass terms. The upper row illustrates (s)quark and (s)lepton loops for R-Parity violating SUSY. The lower row corresponds to seesaw Type I (Type III) on the left and seesaw Type II on the right. The crosses indicate chirality flips and VEV insertions, while suitable couplings and generation indices are implicitly understood.

then leads to associated terms in the soft part of the Lagrangian, and is the most general way to parametrize RPV.

When both lepton and baryon number violating couplings are present at the same time, this may lead to rapid proton decay which is not observed in nature. However, there are many possible concepts to avoid this, e. g. based on symmetries. This is a topic discussed in [41].

Exploring the consequences of equation (1.37) one can immediately see that neutrino (Majorana) masses proportional to $\mu_i \mu_j$ arise at tree level. This already points out one of the challenges to generate viable neutrino masses in this context – to motivate why these couplings are small enough to satisfy the very tight bounds for neutrino masses.

At tree level, these couplings generate only one non vanishing neutrino mass eigenvalue. Thus, to construct a viable model in this context one has to necessarily go to the loop level. Numerous kinds of diagrams arise from combinations of the L violating couplings. These are discussed in more detail in chapter 4. To illustrate the concept figure 1.3 already depicts some of them. This hints to the second challenge, to reduce the number of involved couplings to small subset for a predictive theory – an issue addressed by flavor symmetries.

A second route to arrive at the low energy picture of equation (1.36) related to a different scale M is the non-renormalizable Weinberg operator

$$\mathcal{O}_{\mathrm{W}} = \frac{(L_L \phi)(L_L \phi)}{M}, \tag{1.38}$$

with a resulting low energy neutrino mass scale

$$m_\nu \propto \frac{v^2}{M}. \tag{1.39}$$

Given $M \gg v$, now this allows to elegantly explain why neutrino masses are so small compared to the scale m_l of the charged lepton masses which is proportional to v.

The simplest way to achieve this is to introduce additional fields coupling to the lepton and the Higgs doublet. The Weinberg operator then arises by integrating out the degrees of freedom of these heavy fields. This realization of small neutrino masses is called seesaw mechanism. Most seesaw models fall into one of three classes, named Type I [42, 43, 44, 45], Type II [46, 47, 48, 49, 50] and Type III [51]. They are defined by the properties of the mediating particles. In Type I models this is a heavy fermion which is a singlet under G_{SM}, e.g. a heavy right handed neutrino ν_R. The other two types employ $\mathsf{SU}(2)_\mathsf{L}$ triplets. In the case of Type II this is a scalar Δ, and in the case of Type III a fermion Σ.[13] Feynman diagrams exemplary for this model can also be found in figure 1.3. Note that the seesaw mechanism is not necessarily tied to SUSY. However, it is also used frequently in supersymmetric models.

[13] There are other, more exotic seesaw schemes, see e.g. [52].

2. Two A_4 lepton models

In this chapter we will discuss two BSM models addressing the challenge to give a realistic description of the lepton sector. As previously stated, we will add additional symmetries to the gauge group G_{SM} and extend the field content. The models employ the aforementioned seesaw mechanism to generate the neutrino mass hierarchy and are supersymmetric.

The general idea is to assign all fields to suitable irreducible representations of the overall symmetry and to construct invariants under it. The new structures, mainly arising from the new symmetry, are then used to explain the observed lepton mixing. The multiplication rules of the used irreducible representations will be essential in this context.

In this chapter the supersymmetric nature of the models is not the essential feature, we will loosely refer to the superfields as fields. Further we will denote the representations as multiplets of the according dimension and use the term singlet for the invariant and non-trivial singlets for the other one-dimensional irreducible representations.

This chapter is based on the previously published article [2]:

Ivo de Medeiros Varzielas and Daniel Pidt
UV completions of flavour models and large θ_{13}
JHEP 1303 (2013), p.065
arXiv:1211.5370

2.1. The group A_4 and ultraviolet completions

The models presented here employ the discrete non-abelian symmetry group A_4. They are ultraviolet (UV) completions of two previous effective models [53, 54] and [55] which predicted exact TBM at leading order (LO). This is achieved by a specific spontaneous breaking of A_4. In correspondence to the names of the authors, we will denote the original models as the Altarelli-Feruglio (AF) and Altarelli-Meloni (AM) model. Before discussing the concrete implementation, a few comments on the symmetry group and the benefits of UV completions are in order.[1]

The symmetry A_4 has three one-dimensional and one three-dimensional irreducible representations which we will label as $\mathbf{1}$, $\mathbf{1'}$, $\mathbf{1''}$ and $\mathbf{3}$. To understand the way the model is built it is important to establish the used multiplication rules in the particular basis chosen. For an

[1] For a more thorough introduction of A_4 and an overview of other symmetries and their application, see e.g. [56, 57, 58].

arbitrary representation $\mathbf{r}$ the relation

$$\mathbf{1}\times\mathbf{r}=\mathbf{r}\times\mathbf{1}=\mathbf{r} \tag{2.1}$$

holds. Further, for the other products of one-dimensional representations one finds

$$\left.\begin{aligned}\mathbf{1}'\times\mathbf{1}'=\mathbf{1}''\\ \mathbf{1}''\times\mathbf{1}''=\mathbf{1}'\\ \mathbf{1}'\times\mathbf{1}''=\mathbf{1}''\times\mathbf{1}'=\mathbf{1}\end{aligned}\right\}\propto\alpha_1\beta_1. \tag{2.2}$$

Here and in the following α_i and β_i denote the i-th component of the first and second multiplet of the product respectively. The singlet products with the triplet are

$$\mathbf{1}'\times\mathbf{3}=\mathbf{3}\propto\begin{pmatrix}\alpha\beta_3\\ \alpha\beta_1\\ \alpha\beta_2\end{pmatrix}, \tag{2.3}$$

$$\mathbf{1}''\times\mathbf{3}=\mathbf{3}\propto\begin{pmatrix}\alpha\beta_2\\ \alpha\beta_3\\ \alpha\beta_1\end{pmatrix} \tag{2.4}$$

and the product of two triplets is

$$\mathbf{3}\times\mathbf{3}=\mathbf{3}_S+\mathbf{3}_A+\mathbf{1}+\mathbf{1}'+\mathbf{1}''\begin{cases}\mathbf{1}\propto\alpha_1\beta_1+\alpha_2\beta_3+\alpha_3\beta_2\\ \mathbf{1}'\propto\alpha_3\beta_3+\alpha_1\beta_2+\alpha_2\beta_1\\ \mathbf{1}''\propto\alpha_2\beta_2+\alpha_1\beta_3+\alpha_3\beta_1\\ \mathbf{3}_S\propto\dfrac{1}{3}\begin{pmatrix}2\alpha_1\beta_1-\alpha_2\beta_3-\alpha_3\beta_2\\ 2\alpha_2\beta_2-\alpha_1\beta_2-\alpha_2\beta_1\\ 2\alpha_3\beta_3-\alpha_3\beta_1-\alpha_1\beta_3\end{pmatrix}.\\ \mathbf{3}_A\propto\dfrac{1}{2}\begin{pmatrix}\alpha_2\beta_3-\alpha_3\beta_2\\ \alpha_1\beta_2-\alpha_2\beta_1\\ \alpha_3\beta_1-\alpha_1\beta_3\end{pmatrix}\end{cases} \tag{2.5}$$

The two subscripts A and S refer to the symmetric and antisymmetric ways to construct a triplet again from two triplets, which will be essential for the following models. We will use curly brackets to indicate contractions to different one dimensional A_4 representations, i.e. for two generic triplets A and B, $\{AB\}\propto\mathbf{1}$, $\{AB\}'\propto\mathbf{1}'$ and $\{AB\}''\propto\mathbf{1}''$.

Even though exact TBM is excluded it is still a LO good starting point to build a realistic model. In effective models the deviations from TBM can be accounted for by the inclusion of next to leading order (NLO) terms. In this context UV completions can play an important role, they introduce additional messenger fields to construct a fundamental theory. Integrated out, these messenger fields provide the effective terms of the theory. The motivation behind this is not merely the wish for a more aesthetically pleasing theory but the fact, that these completions can lead to an improved predictivity compared to the effective models. This has been shown e.g. in [59] and is achieved by limiting the set of effective terms which arise at the lower scale.

For the AF and AM models this has been shown with the minimal UV completions presented in [60]. However, they were guided by the paradigm of TBM. The completion of the AF model predicts exact TBM and is therefore excluded. The minimal AM completion on the other hand, allows for deviations from TBM, but we find its parameters quite constrained by the currently measured mixing angles. So clearly there is a motivation to investigate next-to-minimal completions of these models, accounting the experimental status quo.

There is a multitude of sources from which the needed deviations may arise. Namely, these are the charged lepton sector (e.g. [61, 62]), Dirac (e.g. [63]) or Majorana masses (in type I seesaw models, as in [64]), or VEV alignment (e.g. [65]). A very recent proposal combining type I seesaw with either type II or III can be found in [66], for a general review, see e.g. [67]. The details of this are explored in the following two sections.

Generally, both presented models will employ the MSSM with additional (chiral) superfields. There are two classes of these fields which are discriminated by their charge under a global symmetry $\mathsf{U}(1)_R$, which is part of the SUSY embedding and contains R-parity as a discrete subgroup.[2] The additional fields with $\mathsf{U}(1)_R$ charge 0 will be called flavons, while the fields with $\mathsf{U}(1)_R$ charge 2 will be denoted as driving fields. Since the ordinary matter fields have charge 1, only the flavons can couple to these. Further note that the scalar components of the driving fields do not receive a VEV, since they only appear linearly in the superpotential terms.[3] Further details of their assignments and the additional messenger fields will be discussed in sections 2.2 and 2.3.

In section 2.2 we explore modifications of the AF model. We compare possible next to minimal completions with modifications of the flavon sector. A similar comparison will then be presented for possible modifications of the AM model in section 2.3, which allows to relax the tight constraints for the minimal completion.

2.2. Completions of $\mathsf{A}_4 \times \mathsf{Z}_3 \times \mathsf{U}(1)_{\mathsf{FN}}$ models

The foundation of the models discussed in this section is the supersymmetric implementation of the AF model [54]. Its flavor symmetry is given by the product $\mathsf{A}_4 \times \mathsf{Z}_3 \times \mathsf{U}(1)_{\mathrm{FN}}$. All three symmetries play a different role in the construction of the models. The spontaneous breaking of A_4 leads to exact TBM at LO, while the Z_3 separates the neutrino sector and the charged lepton sector and prevents unintended couplings. The final $\mathsf{U}(1)_{\mathrm{FN}}$ implements the Froggatt-Nielsen (FN) mechanism [68]. The basic idea is to introduce a global $\mathsf{U}(1)$ as a flavor symmetry.

[2] The reason behind this SUSY embedding is that it simplifies the construction of the scalar potential, which allows to explain the vacuum.

[3] Strictly speaking this is only true while SUSY is still intact.

The VEV of an associated field is then used to explain the observed hierarchy of the charged fermion Yukawa couplings, in our case the charged lepton ones.

The minimal completion of the AF model presented in [60] is elegant and has the rather unique feature of predicting exact TBM leading to its exclusion by present experimental results. This combination motivates us to see how next to minimal completions perform. Since the original model is of seesaw Type I, all of the aforementioned tools are at our disposal.[4] The field content relevant to the discussion is presented in tables 2.1, 2.2 and 2.3 for the matter fields, the flavons and the FN messengers respectively. Before delving into the details, we would like to make a few comments on these. As discussed later in this section the field $\tilde{\xi}$ in table 2.2 is only present in the original AF model and its minimal completion. It is replaced by ξ' in the new model that we present here. Further we relabeled the fields ϕ_T and ϕ_S of the original papers as ϕ_l and ϕ_ν for the sake of a consistent notation. The messengers of the completion are generically denoted as χ. They are FN messengers with R charge 1 like the leptons and thus do not couple to the driving fields. The right-handed neutrino field is labeled as ν^c and should not be confused with the left-handed neutrinos contained in l.

2.2.1. Modifications of the completion

The superpotential w for the original AF model can be split up into three parts, the neutrino superpotential w_ν, the charged lepton superpotential w_l and the driving superpotential w_d,

$$w = w_l + w_\nu + w_d. \tag{2.6}$$

For the following discussion keep in mind, that we use curly brackets to represent the A_4 contractions $\{a\} \sim \mathbf{1}$, $\{a\}' \sim \mathbf{1}'$ and $\{a\}'' \sim \mathbf{1}''$ and that the field $\tilde{\xi}$ will be replaced in the improved models later on. The first two parts are given by

$$w_\nu = y \left\{l \nu^c\right\} h_u + \left(x_A \xi + \tilde{x}_A \tilde{\xi}\right) \left\{\nu^c \nu^c\right\} + x_B \left\{\phi_\nu \nu^c \nu^c\right\} \tag{2.7}$$

and

$$w_l = \frac{y_e}{\Lambda^3} \theta^2 e^c \left\{\phi_l l\right\} h_d + \frac{y_\mu}{\Lambda^2} \theta \mu^c \left\{\phi_l l\right\}' h_d + \frac{y_\tau}{\Lambda} \tau^c \left\{\phi_l l\right\}'' h_d. \tag{2.8}$$

The origin of the latter one is the following renormalizable superpotential of the minimal UV completion w_l^{UV}. Using the subscript A to refer to the different messenger pairs (c.f. table 2.3) it can be written as

$$\begin{aligned} w_l^{\mathrm{UV}} = {} & M_{\chi_A} \left\{\chi_A \chi_A^c\right\} + h_d \left\{l \chi_\tau^c\right\} + \tau^c \left\{\phi_l \chi_\tau\right\}'' + \theta \mu^c \chi_1 + \theta e^c \chi_3 \\ & + \left\{\phi_l \chi_\tau\right\}' \chi_1^c + \left\{\phi_l \chi_\tau\right\} \chi_2^c + \theta \chi_2 \chi_3^c. \end{aligned} \tag{2.9}$$

[4] The model recently proposed in [66] is very similar to what we will present. There, the authors use the seesaw type I to implement TBM and then combine it with one of the other two seesaw types to deviate from it.

	ν^c	l	e^c	μ^c	τ^c	h_d	h_u
A_4	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}''$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}$
Z_3	ω^2	ω	ω^2	ω^2	ω^2	1	1
$U(1)_{FN}$	0	0	2	1	0	0	0
$U(1)_R$	1	1	1	1	1	0	0
$U(1)_Y$	0	$-1/2$	$+1$	$+1$	$+1$	$-1/2$	$+1/2$

Table 2.1.: Assignments of matter fields relevant to the discussion of the AF model.

	θ	ϕ_l	ϕ_ν	ξ	$\tilde{\xi}$	ξ'	ϕ_l^0	ϕ_ν^0	ξ^0
A_4	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$
Z_3	1	1	ω^2	ω^2	ω^2	ω^2	1	ω^2	ω^2
$U(1)_{FN}$	-1	0	0	0	0	0	0	0	0
$U(1)_R$	0	0	0	0	0	0	2	2	2
$U(1)_Y$	0	0	0	0	0	0	0	0	0

Table 2.2.: Assignments of flavons and driving fields relevant to the discussion of the AF model. The original AF model and the minimal completion only employ $\tilde{\xi}$ and do not contain ξ'. The opposite is the case for the new model we propose.

	χ_τ	χ_1	χ_2	χ_3	χ_τ^c	χ_1^c	χ_2^c	χ_3^c
A_4	$\mathbf{3}$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}''$	$\mathbf{1}$	$\mathbf{1}$
Z_3	ω	ω	ω	ω	ω^2	ω^2	ω^2	ω^2
$U(1)_{FN}$	0	0	0	-1	0	0	0	$+1$
$U(1)_R$	1	1	1	1	1	1	1	1
$U(1)_Y$	-1	-1	-1	-1	$+1$	$+1$	$+1$	$+1$

Table 2.3.: The FN messengers for the AF model. They are the same as presented in [60].

The masses of the messengers are expected to be similar and their scale is generically denoted as M_χ. Superdiagrams for the charged lepton mass terms can be found in section A.1 of the appendix.

The final part of the superpotential is

$$
\begin{aligned}
w_d = M\left\{\phi_l^0\phi_l\right\} + g\left\{\phi_l^0\phi_l\phi_l\right\} + g_1\left\{\phi_\nu^0\phi_\nu\phi_\nu\right\} + g_2\tilde{\xi}\left\{\phi_\nu^0\phi_\nu\right\} + g_3\xi^0\left\{\phi_\nu\phi_\nu\right\} \\
+ g_4\xi^0\xi\xi + g_5\xi^0\xi\tilde{\xi} + g_6\xi^0\tilde{\xi}\tilde{\xi}.
\end{aligned} \tag{2.10}
$$

This part gives rise to the VEV structure, since the associated alignment conditions can be derived from it. These are

$$\frac{\partial w}{\partial\phi_{\nu1}^0} = g_2\tilde{\xi}\phi_{\nu1} + \frac{2}{3}g_1\left(\phi_{\nu1}^2 - \phi_{\nu2}\phi_{\nu3}\right) = 0, \tag{2.11}$$

$$\frac{\partial w}{\partial\phi_{\nu2}^0} = g_2\tilde{\xi}\phi_{\nu3} + \frac{2}{3}g_1\left(\phi_{\nu2}^2 - \phi_{\nu1}\phi_{\nu3}\right) = 0, \tag{2.12}$$

$$\frac{\partial w}{\partial\phi_{\nu3}^0} = g_2\tilde{\xi}\phi_{\nu2} + \frac{2}{3}g_1\left(\phi_{\nu3}^2 - \phi_{\nu1}\phi_{\nu2}\right) = 0, \tag{2.13}$$

$$\frac{\partial w}{\partial\xi^0} = g_3\left(\phi_{\nu1}^2 + 2\phi_{\nu2}\phi_{\nu3}\right) + g_4\xi^2 + g_5\xi\tilde{\xi} + g_6\tilde{\xi}^2 = 0, \tag{2.14}$$

and

$$\frac{\partial w}{\partial\phi_{l1}^0} = M\phi_{l1} + \frac{2}{3}g_1\left(\phi_{l1}^2 - \phi_{l2}\phi_{l3}\right) = 0, \tag{2.15}$$

$$\frac{\partial w}{\partial\phi_{l2}^0} = M\phi_{l3} + \frac{2}{3}g_1\left(\phi_{l2}^2 - \phi_{l1}\phi_{l3}\right) = 0, \tag{2.16}$$

$$\frac{\partial w}{\partial\phi_{l3}^0} = M\phi_{l2} + \frac{2}{3}g_1\left(\phi_{l3}^2 - \phi_{l1}\phi_{l2}\right) = 0. \tag{2.17}$$

Additionally to the trivial solution, this allows the VEV structure

$$\langle\phi_\nu\rangle \propto (1,1,1), \quad \langle\phi_l\rangle \propto (1,0,0), \quad \langle\xi\rangle \neq 0, \quad \langle\tilde{\xi}\rangle = 0. \tag{2.18}$$

This VEV structure and the alignment equation play a crucial role in modifications of the model. In fact it is a delicate task to change or introduce terms that contribute to these, as the consequences can be vanishing VEVs or a completely different, non-viable VEV alignment. This can, for example, be seen by enabling ϕ_l to appear in the ϕ_ν^0 terms. An example of this at the non-renomalizable level is $\left\{\phi_\nu^0\phi_l\right\}\xi\xi$. The construction of this term in an UV complete theory requires only one new field η. It has to transform as ω under the Z_3 (c.f. table 2.2) and allows for the new vertex $\left\{\phi_\nu^0\phi_l\right\}\eta$ and the mass term $\xi^0\eta$. If η acquires a non-vanishing VEV this vertex alone mixes ϕ_l into the alignment equations of ϕ_ν^0. But additionally, even for $\langle\eta\rangle = 0$ new combinations with $\xi^0\xi\xi$ appear. At the non-renormalizable level this manifests itself in

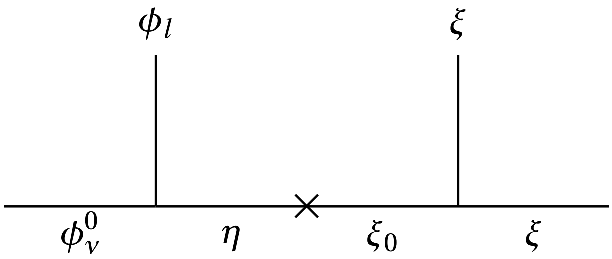


Figure 2.1.: Topology for $\phi_\nu^0 \phi_l \xi \xi$.

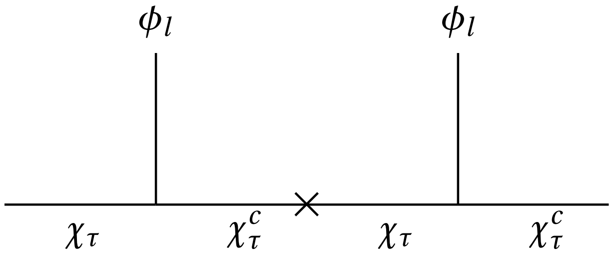


Figure 2.2.: Topology for $\chi_\tau^c \chi_\tau \phi_l \phi_l$.

the term $\{\phi_\nu^0 \phi_l\} \xi \xi$ via the topology displayed in figure 2.1. The altered alignment equations (c.f. equations (2.11)-(2.14)) are then given by

$$\frac{\partial w}{\partial \phi_{\nu 1}^0} = g_2 \tilde{\xi} \phi_{\nu 1} + \frac{2}{3} g_1 \left(\phi_{\nu 1}^2 - \phi_{\nu 2}\phi_{\nu 3}\right) + g_8 \phi_{l1} \eta = 0, \tag{2.19}$$

$$\frac{\partial w}{\partial \phi_{\nu 2}^0} = g_2 \tilde{\xi} \phi_{\nu 3} + \frac{2}{3} g_1 \left(\phi_{\nu 2}^2 - \phi_{\nu 1}\phi_{\nu 3}\right) + g_8 \phi_{l3} \eta = 0, \tag{2.20}$$

$$\frac{\partial w}{\partial \phi_{\nu 3}^0} = g_2 \tilde{\xi} \phi_{\nu 2} + \frac{2}{3} g_1 \left(\phi_{\nu 3}^2 - \phi_{\nu 1}\phi_{\nu 2}\right) + g_8 \phi_{l2} \eta = 0, \tag{2.21}$$

$$\frac{\partial w}{\partial \xi^0} = g_3 \left(\phi_{\nu 1}^2 + 2\phi_{\nu 2}\phi_{\nu 3}\right) + g_4 \xi^2 + g_5 \xi \tilde{\xi} + g_6 \tilde{\xi}^2 + g_7 \eta = 0. \tag{2.22}$$

A quick check with the original configuration $\langle \phi_l \rangle \propto (1,0,0)$ and $\langle \phi_\nu \rangle \propto (1,1,1)$ immediately uncovers that the old VEVs do not fulfill these new constraints. A more torough investigation reveals, that preserving $\langle \phi_\nu \rangle \propto (1,1,1)$ requires $\langle \phi_l \rangle \propto (1,1,1)$. This is not compatible with equations (2.15)-(2.17), an issue that can not be resolved by small perturbations of the structure of $\langle \varphi_\nu \rangle$. A more detailed discussion of modifications to the alignment sector will follow within the $A_4 \times Z_4$ framework in section 2.3.

Now to the charged leptons. In this sector the effective terms are already renormalizable. The effect of the vertex $\{\chi_\tau^c \chi_\tau \phi_l\}$ is already discussed in [60]. As shown in figure 2.2 the non-renormalizable term $\chi_\tau^c \chi_\tau \phi_l \phi_l$ is enabled by merely employing this renormalizable vertex

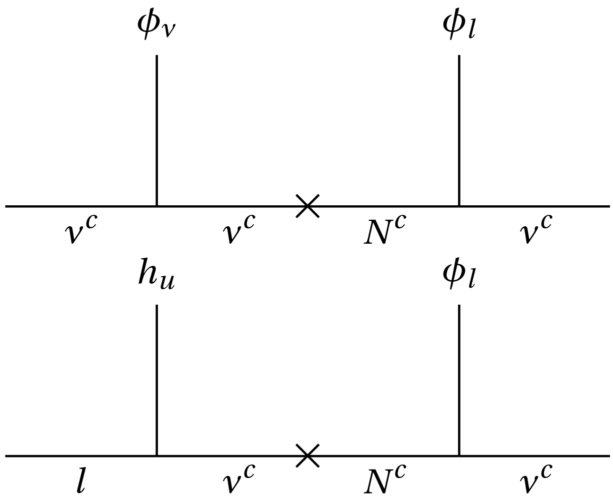


Figure 2.3.: Topologies contributing to neutrino masses, enabled by N^c.

twice. It does not affect the leptonic mixing. Thus, charged lepton terms are not a promising source to explain a large mixing angle θ_{13}.

In the neutrino sector, expanding the available set of Majorana terms requires the introduction of one additional R-charge 1 field N^c with a Z_3 charge ω and otherwise mimicking the present ν^c (i.e. $U(1)_Y = 0$). This allows for the vertex $\{\nu^c\phi_l N^c\}$ and the mass term $\nu^c N^c$. One possible non-renormalizable term which can now be constructed is e.g. $\nu^c\phi_\nu\phi_l\nu^c$[5] (see the upper topology in figure 2.3). Additionally the effective Dirac mass term $l h_u \phi_l \nu^c$ (allowed by $A_4 \times Z_3 \times U(1)_{\mathrm{FN}}$, but not present in the minimal completion) now arises from the lower topology in figure 2.3, so overall this amounts to an extended seesaw realization. Here, the new contribution δm added to the original m_D is the relevant part. In the flavor basis, it is of the form

$$\delta m = \begin{pmatrix} 2a & 0 & 0 \\ 0 & b-a & 0 \\ 0 & 0 & -b-a \end{pmatrix}. \tag{2.23}$$

Note that in this structure the new parameter a preserves the μ-τ symmetry included in TBM. Overall, its effects lead to undesired, significant changes of θ_{12} when a and b are not fine-tuned while the influence on the other two angles is negligible. Therefore the addition of N^c does not lead to an experimentally viable modification of the model.

At this stage we conclude that next-to-minimal completions of the AF model are not very successful in generating large θ_{13}. Further modifications of the completion, introducing a

[5]This term can also be constructed by introducing an SU(2) doublet messenger. This messenger would further contribute to the charged lepton mass terms. However, this contributions would merely redefine the already existing terms and thus yield no new opportunities to modify the existing terms.

multitude of additional fields by employing more than one extra mediator, leading to viable mixing are certainly possible. However, they further and further sacrifice the main benefit of having an explicit completion, which is the increase in predictivity. Therefore we will not consider these.

2.2.2. Modifications of the flavons

A more promising origin of a viable modification of the AF model is its flavon field content. A natural starting point here is the field $\tilde{\xi}$. In the original framework it played the role of a duplicate flavon of ξ, that was necessary to obtain the desired VEV alignment. However, the algebra provided by A_4 allows for structures not leading to TBM. Thus replacing $\tilde{\xi}$ by a nontrivial singlet ξ' provides access to new relevant terms, while preserving the same number of fields of the original minimal UV complete model. In the following we will show that this leads to a viable phenomenology and can be understood as the construction of a minimal completion for the different effective model presented in [69]. Since, this replacement is the only change in this respect of the field and symmetry content of the model, it was already summarized in table 2.2 and the messengers can again be found in table 2.3.

This replacement does not influence the charged lepton and UV completion parts of the superpotential (e.g. equations (2.8) and (2.9) remain valid). It only modifies the driving and the neutrino parts to

$$
\begin{aligned}
w_d = {} & M\left\{\phi_l^0\phi_l\right\} + g\left\{\phi_l^0\phi_l\phi_l\right\} + g_1\left\{\phi_\nu^0\phi_\nu\phi_\nu\right\} + g_2\xi\left\{\phi_\nu^0\phi_\nu\right\} \\
& + g_3\xi'\left\{\phi_\nu^0\phi_\nu\right\}'' + g_4\xi^0\left\{\phi_\nu\phi_\nu\right\} + g_5\xi^0\xi\xi
\end{aligned}
\tag{2.24}
$$

and

$$
w_\nu = y\left\{\nu^c l\right\} h_u + \left(x_A\xi + x'_A\xi'\right)\left\{\nu^c\nu^c\right\} + x_B\left\{\phi_\nu\nu^c\nu^c\right\},
\tag{2.25}
$$

which replace equations (2.10) and (2.7). What are the effects enabled by these changes? Recall, that $\tilde{\xi}$ was necessary in the original model to generate a nontrivial VEV structure from the minimization of the potential. As we demonstrated previously, any modification related to the alignment conditions is a delicate task and we have to make sure that the desired VEV structure is preserved. Comparing equations (2.24) and (2.10) reveals that both the old $\tilde{\xi}$ and the new ξ only couple to ϕ_ν and not to ϕ_l. Thus the related alignment equations (2.15)-(2.17) still apply. In combination with no changes in the charged lepton superpotential, this yields a diagonal charged lepton sector after symmetry breaking, which was also the case in the original model. Differences arise in the alignment conditions of ϕ_ν due to the changed terms and A_4 contractions. The old conditions (c.f. equations (2.11)-(2.14)) are replaced by

$$\frac{\partial w}{\partial \phi^0_{\nu 1}} = g_2 \xi \phi_{\nu 1} + g_3 \xi' \phi_{\nu 3} + \frac{2}{3} g_1 \left(\phi^2_{\nu 1} - \phi_{\nu 2}\phi_{\nu 3}\right) = 0, \tag{2.26}$$

$$\frac{\partial w}{\partial \phi^0_{\nu 2}} = g_2 \xi \phi_{\nu 3} + g_3 \xi' \phi_{\nu 2} + \frac{2}{3} g_1 \left(\phi^2_{\nu 2} - \phi_{\nu 1}\phi_{\nu 3}\right) = 0, \tag{2.27}$$

$$\frac{\partial w}{\partial \phi^0_{\nu 3}} = g_2 \xi \phi_{\nu 2} + g_3 \xi' \phi_{\nu 1} + \frac{2}{3} g_1 \left(\phi^2_{\nu 3} - \phi_{\nu 1}\phi_{\nu 2}\right) = 0, \tag{2.28}$$

$$\frac{\partial w}{\partial \xi^0} = g_4 \left(\phi^2_{\nu 1} + 2\phi_{\nu 2}\phi_{\nu 3}\right) + g_5 \xi^2. \tag{2.29}$$

This leads to the VEV structure

$$\frac{\langle \phi_l \rangle}{M_\chi} = (u, 0, 0), \quad \frac{\langle \phi_\nu \rangle}{M_\chi} = c_b (u, u, u), \quad \frac{\langle \xi \rangle}{M_\chi} = c_a u, \quad \frac{\langle \xi' \rangle}{M_\chi} = c'_a u, \tag{2.30}$$

$$u = -\frac{3}{2}\frac{M}{g}, \quad c_b^2 = -\frac{g_5 g_3^2}{3 g_4 g_2^2} c'^2_a, \quad c_a = -\frac{g_3}{g_2} c'_a. \tag{2.31}$$

The parameter c'_a remains undetermined and again M_χ generically denotes the messenger masses. Small perturbations of this VEV structure are incompatible with the alignment equations. This configuration is very similar to the original one so, contrary to our previous example, in this case the original, desired structure is preserved.

After ensuring that these fundamental building blocks are still in place, we now turn to modifications enabled by ξ' in the neutrino sector. The Dirac mass structure does not change compared to [54]

$$M_D = y v_u \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \tag{2.32}$$

where v_u denotes the VEV $\langle h_u \rangle$. On the other hand, the new terms in equation (2.25) modify the structure of the Majorana mass, which now reads

$$M_M = x_A c_a u M_\chi \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + x'_A c'_a u M_\chi \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} + \frac{1}{3} x_B c_b u M_\chi \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}. \tag{2.33}$$

The second term in this equation is introduced by the new flavon ξ' and as previously mentioned this structure is one of the possible A_4 contractions which is not used for TBM models.

Consequently, it provides the gateway to a large mixing angle θ_{13}. Indeed one can check that it leads to the effective neutrino mass matrix presented in [69]

$$M_\nu = a\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} + b\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + c\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix} + d\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}. \tag{2.34}$$

The translation of the generic parameters a, b, c and d to the parameters of our complete model is

$$a = \frac{y^2 v_u^2}{M_\chi u}\frac{x_B c_b}{-x_A^2 c_a^2 + x_A c_a x'_A c'_a - x'^2_A c'^2_a + x_B^2 c_b^2}, \tag{2.35}$$

$$b = \frac{y^2 v_u^2}{3M_\chi u}\frac{3x'^2_A c'^2_a + \left(x_A c_a + x'_A c'_a\right) x_B c_b + x_B^2 c_B^2}{x_A^3 c_a^3 + x'^3_A c'^3_a - x_B^2 c_b^2 \left(x_A c_a + x'_A c'_a\right)}, \tag{2.36}$$

$$c = \frac{y^2 v_u^2}{M_\chi u}\frac{x_A c_a - x'_A c'_a}{x_A^2 c_a^2 - x_A c_a x'_A c'_a + x'^2_A c'^2_a - x_B^2 c_b^2}, \tag{2.37}$$

$$d = -\frac{y^2 v_u^2}{M_\chi u}\frac{x'_A c'_a}{x_A^2 c_a^2 - x_A c_a x'_A c'_a + x'^2_A c'^2_a - x_B^2 c_b^2}. \tag{2.38}$$

For this structure the matrix diagonalizing M_ν can be expressed by an additional rotation applied to the TBM matrix V_{TBM}

$$U_{\text{PMNS}} = V_{\text{TBM}}\begin{pmatrix}\cos\theta & 0 & \sin\theta\\ 0 & 1 & 0\\ -\sin\theta & 0 & \cos\theta\end{pmatrix}, \tag{2.39}$$

with

$$V_{\text{TBM}} = \begin{pmatrix}\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0\\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}}\\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}\end{pmatrix} \tag{2.40}$$

and

$$\tan 2\theta = \frac{\sqrt{3}d}{-2c+d}. \tag{2.41}$$

Clearly, enforcing $d = 0$ leads to TBM. A well known fact in this case is that M_ν is invariant under $\mathsf{Z}_2 \times \mathsf{Z}_2$ symmetries with well defined matrices in the flavor basis. Introducing structures like the last term of equation (2.34) breaks this invariance but only one part of it. Since one of the two Z_2 symmetries is related to the $\mu-\tau$ symmetry of TBM which is clearly not respected by the new term, only the second one remains. This type of situation with one residual Z_2

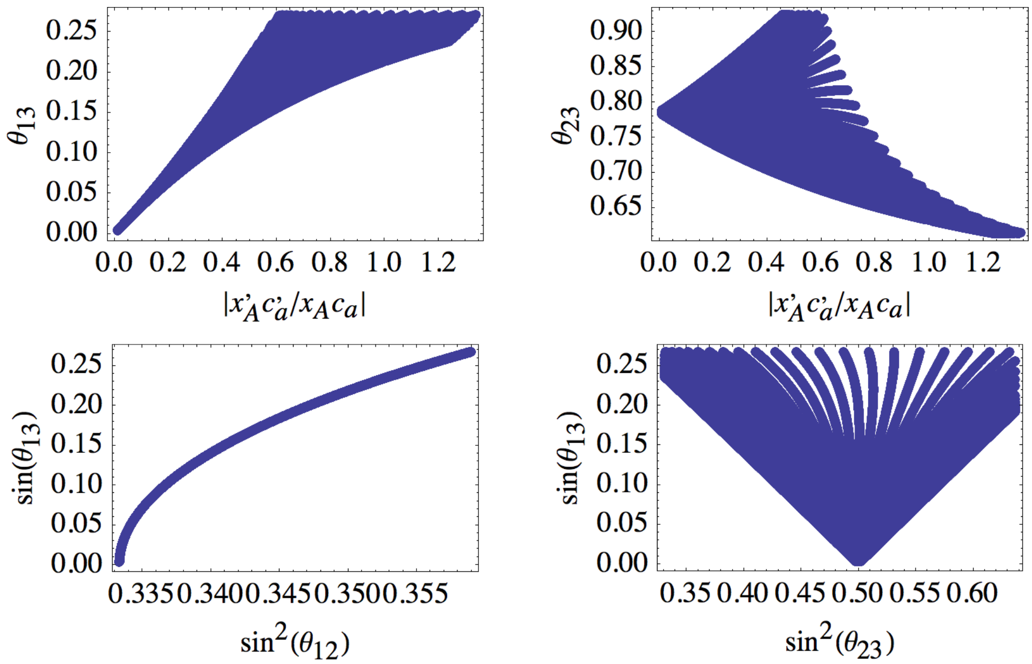


Figure 2.4.: Deviations of θ_{13} and θ_{23} from TBM values for complex parameters (top row). Correlation of θ_{13} to θ_{12} and θ_{23} (bottom row).

symmetry was discussed recently in [70]. In the case we present here the model generates trimaximal mixing [71, 72, 73, 74]

$$|U_{e2}| = \frac{1}{\sqrt{3}}, \quad |U_{e3}| = \frac{2}{\sqrt{6}}|\sin\theta|, \quad |U_{\mu 3}| = \left| -\frac{1}{\sqrt{6}}\sin\theta - \frac{1}{\sqrt{2}}\cos\theta \right|. \tag{2.42}$$

Our findings for the neutrino mixing angles are summarized in figure 2.4. There we display the angles as functions of the parameters of equation (2.33) and the correlations between the different angles. Our results confirm the LO results presented in [69] for their effective generic model. The crucial improvement is, that in our case the results are not only valid at leading order. We provided a renormalizable model with an explicit UV completion including type I seesaw and FN messengers. Thus the predictions of figure 2.4 are protected against corrections from higher order terms allowed in the effective model. We achieve this while preserving the simplicity of the original AF completion (presented in [60]). No expansion of the symmetry content or an enlarged set of fields is needed. For this new, complete framework we have demonstrated, that it leads to a viable VEV alignment and to desirable structures for the charged leptons and neutrinos.

Remembering that A_4 not only provides two different triplet contractions to singlets but three (c.f. equation (2.5)) reveals one final option to modify the framework. The models we discussed covered the 1 and the $1'$ but it is interesting to briefly consider the effects of an additional $1''$. At the effective level [69] already demonstrated that this is superfluous. The structure associated with the $1''$ flavon is

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{2.43}$$

It would expand equation (2.34), but can simply be absorbed in a redefinition of a, b, c, d as the respective structures are not linearly independent. In fact

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}. \tag{2.44}$$

In our model the change enabled by such a new flavon occurs in the Majorana mass term and still has to go trough the whole seesaw mechanism before it manifests itself in the light neutrino masses, which complicates the discussion. Equation (2.33) is modified by adding a new coefficient x''_A and the structure of equation (2.43). This can be reabsorbed without loss of generality by redefinition of $x_A c_a \to x_A c_a - x''_A c''_a$, $x'_A c'_a \to x'_A c'_a - x''_A c''_a$, $x_B c_b \to x_B c_b - 3x''_A c''_a$ and adding a contribution proportional to the identity matrix (which shifts the overall mass scale by $3x''_A c''_a$ but does not affect the mixing). Recall that due to seesaw mechanism these coefficients are related to a, b, c, d in a non-linear way (c.f. equations (2.35)-(2.38)). For this reason it is convenient to consider the ratio $d/c = x'_A c'_a/(x_A c_a - x'_A c'_a)$. As both of our original Majorana coefficients, $x_A c_a$ and $x'_A c'_a$, get redefined by $x''_A c''_a$, the effect of the redefinition translates into a linear change of d/c through the shift in the redefined $x'_A c'_a$ and modifies e.g. equation (2.41), which defines the deviation from TBM.

2.3. Completions of $\mathsf{A}_4 \times \mathsf{Z}_4$ models

The framework discussed in this section builds upon the AM model [55]. Again the foundation of the model is A_4. The other part of the overall symmetry group is simplified compared to the previous section. In this case the separation of the charged lepton and neutrino sector, allowing for the breaking in two different directions and the charged lepton mass hierarchies are implemented via a single Z_4. This has to be compared with the product $\mathsf{Z}_3 \times \mathsf{U}(1)_{\mathrm{FN}}$ of the AF model. Tables 2.4 and 2.5 list the field and symmetry content of the original AM effective model, and table 2.6 contains the messenger content of its minimal completion as proposed in [60]. Later on we will modify the flavon content again by introducing an additional non-trivial singlet.

	ν^c	l	e^c	μ^c	τ^c	h_d	h_u
A_4	3	3	1	1	1	1	1
Z_4	-1	i	1	i	-1	1	i
$U(1)_R$	1	1	1	1	1	0	0
$U(1)_Y$	0	$-1/2$	$+1$	$+1$	$+1$	$-1/2$	$+1/2$

Table 2.4.: Assignments of matter fields relevant to the discussion of the AM model.

	ϕ_l	ξ'	ϕ_ν	ξ	ϕ_l^0	ϕ_ν^0	ξ^0
A_4	3	$1'$	3	1	3	3	1
Z_4	i	i	1	1	-1	1	1
$U(1)_R$	0	0	0	0	2	2	2
$U(1)_Y$	0	0	0	0	2	2	2

Table 2.5.: Assignments of flavons and driving fields relevant to the discussion of the AM model.

	χ_τ	χ_1	χ_2	χ_3	χ_τ^c	χ_1^c	χ_2^c	χ_3^c
A_4	$\mathbf{3}$	$\mathbf{1}''$	$\mathbf{1}'$	$\mathbf{1}''$	$\mathbf{3}$	$\mathbf{1}'$	$\mathbf{1}''$	$\mathbf{1}'$
Z_4	i	-1	-1	$-i$	$-i$	-1	-1	i
$U(1)_R$	1	1	1	1	1	1	1	1
$U(1)_Y$	-1	-1	-1	-1	$+1$	$+1$	$+1$	$+1$

Table 2.6.: The FN messengers for the AM model. They remain the same as presented in [60].

2.3.1. Modifications of the completion

At this stage, without the additional field, the neutrino superpotential is

$$w_\nu = y_\nu \left\{ l\nu^c \right\} h_u + (M + a\xi) \left\{ \nu^c \nu^c \right\} + b \left\{ \phi_\nu \nu^c \nu^c \right\}, \tag{2.45}$$

which is composed by a Dirac mass term followed by Majorana mass terms. Note that the structure is very similar to the one of the AF model, with the simple replacements $y \to y_\nu$, $\left(x_A\xi + \tilde{x}_A\tilde{\xi}\right) \to (M + a\xi)$ and $x_B \to b$. The driving superpotential associated to the alignment conditions now reads

$$\begin{aligned} w_d = {} & M \left\{ \phi_0^\nu \phi_\nu \right\} + g_1 \left\{ \phi_0^\nu \phi_\nu \phi_\nu \right\} + g_2 \xi \left\{ \phi_0^\nu \phi_\nu \right\} + g_3 \xi^0 \left\{ \phi_\nu \phi_\nu \right\} + g_4 \xi^0 \xi^2 + M_\xi \xi^0 \xi \\ & + M_0^2\, \xi 0 + h_1 \xi' \left\{ \phi_l^0 \phi_l \right\}'' + h_2 \left\{ \phi_l^0 \phi_l \phi_l \right\}. \end{aligned} \tag{2.46}$$

For the terms related to ϕ_ν a quick comparison of this equation with equation (2.10) reveals the A_4 structure of the contractions remains unchanged. Simply replace $g_2\tilde{\xi}$ with $(M + g_2\xi)$. Thus the alignment of $\langle \phi_\nu \rangle \propto (1,1,1)$ is expected to stay the same. The interesting effects arise for ϕ_l and ξ'. There we have

$$\frac{\partial w}{\partial \phi_{l1}^0} = 2h_2(\phi_{l1}^2 - \phi_{l2}\,\phi_{l3}) + h_1\,\xi'\,\phi_{l3} = 0, \tag{2.47}$$

$$\frac{\partial w}{\partial \phi_{l2}^0} = 2h_2(\phi_{l2}^2 - \phi_{l1}\,\phi_{l3}) + h_1\,\xi'\,\phi_{l2} = 0, \tag{2.48}$$

$$\frac{\partial w}{\partial \phi_{l3}^0} = 2h_2(\phi_{l3}^2 - \phi_{l1}\,\phi_{l2}) + h_1\,\xi'\,\phi_{l1} = 0. \tag{2.49}$$

The most relevant difference to the previously discussed framework is due to ξ'. It aligns $\langle \phi_l \rangle$ in the $(0,1,0)$ direction replacing the previous $(1,0,0)$ direction. Overall the new VEVs are

$$\frac{\langle \phi_l \rangle}{M_\chi} = (0,u,0)\,, \qquad \frac{\langle \phi_\nu \rangle}{M_\chi} = \varepsilon'\,(1,1,1)\,. \tag{2.50}$$

For the AF model the minimal completion presented in [60] leads to a diagonal charged lepton mass matrix. However, this is not the case in the present framework. The messenger content of table 2.6 already enables off-diagonal terms since it allows for the term $\left\{\phi_\nu \chi_\tau \chi_\tau^c\right\}$. This leads a non-diagonal charged lepton mass matrix

$$m_l = \begin{pmatrix} m_e & (-c_s + c_a)\varepsilon' m_\mu & (-c_s - c_a)\varepsilon' m_\tau \\ (-c_s + c_a)\varepsilon' m_e & m_\mu & 2c_s\varepsilon' m_\tau \\ \varepsilon'(-c_s - c_a) m_e & 2c_s\varepsilon' m_\mu & m_\tau \end{pmatrix}, \tag{2.51}$$

where ε' is the VEV of ϕ_ν as in equation (2.50) and c_a, c_s are the superpotential parameters governing the respective $\left\{\phi_\nu \chi_\tau \chi_\tau^c\right\}$ antisymmetric and symmetric A_4 invariants (c.f. equation (2.5)). To better illustrate the structure and order of magnitude of the (non-symmetric)

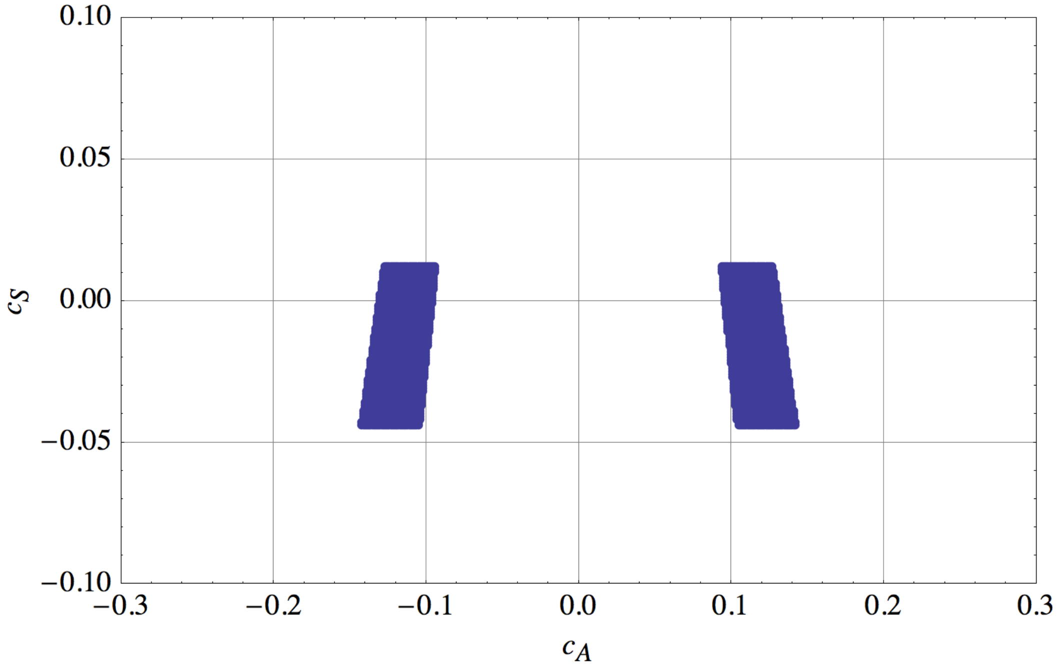


Figure 2.5.: Range of c_A and c_S allowed by present mixing angle data for the minimal AM completion [60] ($\varepsilon' = 1$). Note the different scales.

matrix we have written the entries in terms of the the charged lepton masses. More details on this can be found in section A.1 of the appendix and in [60][6]

In combination with the TBM structure in the neutrino sector this already allows to introduce the necessary modification. The deviations can be found by diagonalizing $m_l m_l^\dagger$. The most influential entry in the generation of a large mixing angle θ_{13} is the 13 entry of equation (2.51). While manipulating this entry one has to be simultaneously very careful about changes related to the 23 entry. They rapidly lead to modifications of the two other mixing angels and drive them out of the allowed 3σ ranges. The dependencies of equation (2.51) lead to the expectation, that for a given ε', c_s is constrained to be very close to 0, while c_a is bounded by the 3σ range allowed for θ_{13}. This is confirmed by a more careful numerical analysis for $\varepsilon' = 1$ and real couplings. The results for the allowed paramter space are displayed in figure 2.5 in the (c_a, c_s) plane. There we imposed the requirement for all mixing parameters to stay in their allowed 3σ ranges.[7] Indeed, the parameter c_s has to be very close to zero, while the allowed range for c_a is symmetric. Note the order of magnitude difference between the two parameters. Figure 2.5 is a very good illustration of the virtues of UV completions. By

[6] Note that they use a different convention for the mass matrices compared to this work.
[7] For complex parameters the analysis becomes more complicated but the same reasoning remains valid.

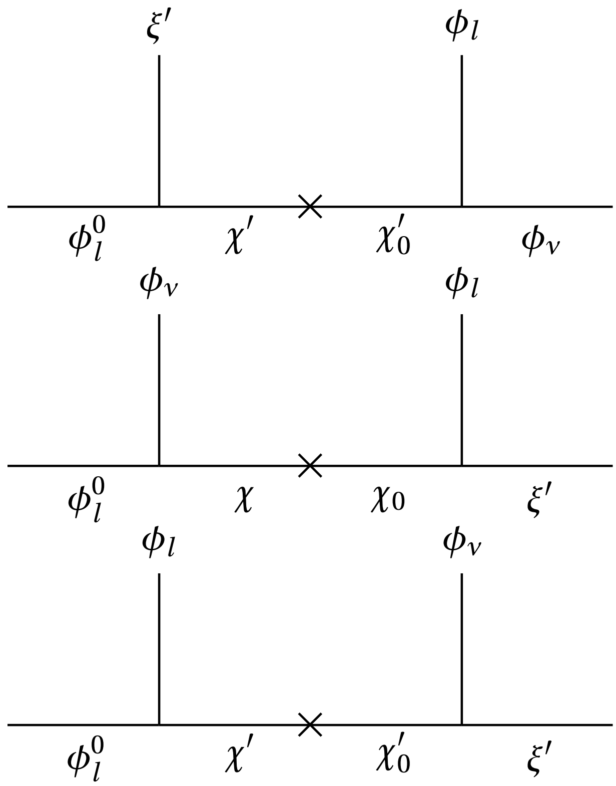


Figure 2.6.: Topologies for $\xi' \{\phi_l^0 \phi_l \phi_\nu\}''$.

measuring the mixing angles one can directly probe parameters of the superpotential and the messenger sector, related to a much higher scale.

So the original minimal completion is still viable. However, as we have demonstrated the bounds imposed by recent experimental data severely constrain the allowed parameter space and lead to an unexplained, to some extent unnatural, hierarchy. This serves as a motivation to check how next-to-minimal completions fare. The following discussion has the same basic structure as the previous section 2.2. As promised, this time we will provide a more in-depth look at the issues related to perturbing the the VEV alignment. The same reasoning applies to the framework of section 2.2 too. The main problem is related to the R-parity containing $\mathsf{U}(1)_R$ symmetry. In each messenger pair needed for effective alignment terms, one of the two fields is required to have R-charge 2. Often this results in the addition of a new alignment field accompanied by new minimization conditions, which lead to a complete different VEV alignment or only allow for the trivial solution.

Explicitly, we will discuss the non-renormalizable terms $\xi' \{\phi_l^0 \phi_l \phi_\nu\}''$ and $\{\phi_l^0 \phi_l \phi_l \phi_\nu\}$. The latter one allows for different A_4 contractions and both are invariant under A_4. To construct UV complete versions of these terms, there are different choices of suitable messengers depending

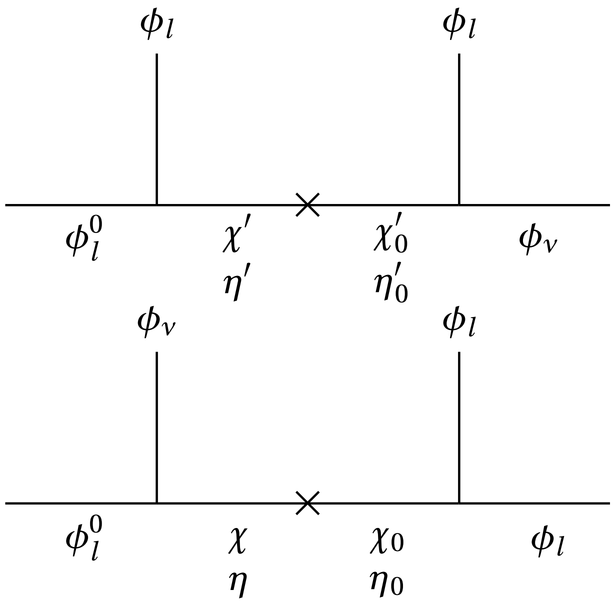


Figure 2.7.: Topologies for $\{\phi_l^0\phi_l\phi_l\phi_\nu\}$.

on the topology. There are three topologies for $\xi'\left\{\phi_0^l\phi_l\phi_\nu\right\}''$ and two for $\{\phi_l^0\phi_l\phi_l\phi_\nu\}$, displayed in figure 2.6 and figure 2.7.

To enable at least one of these, the introduction of new messengers is necessary. In case of a pair, the two messengers are required to be in A_4 representations of the same dimension and the two possible choices are triplets and singlets. For the triplet messenger pairs we have on the one hand χ, χ_0 with respective Z_4 charges -1 and -1 and on the other hand χ', χ_0' with the charges $i, -i$. The singlet options are η, η_0 with respective Z_4 charges -1 and -1 or alternatively η', η_0' with $i, -i$. We use the subscript zero to indicate the messenger fields with

pair	A_4	Z_4	$\mathsf{U}(1)_R$
χ, χ_0	triplet	$-1, -1$	$0, 2$
χ', χ_0'	triplet	$i, -i$	$0, 2$
η, η_0	singlet	$-1, -1$	$0, 2$
η', η_0'	singlet	$i, -i$	$0, 2$

Table 2.7.: Properties of new messenger pairs necessary to enable the topologies $\xi'\left\{\phi_l^0\phi_l\phi_\nu\right\}''$ and $\{\phi_l^0\phi_l\phi_l\phi_\nu\}$. Some of these messengers can be identified with fields of the original content, i.e. $\chi_0 \equiv \phi_l^0$ and for a suitable A_4 singlet choice additionally $\eta' \equiv \xi'$.

R-charge 2 for each pair.[8] Some of these messengers can be identified with existing fields. For instance, assuming a suitable A_4 singlet choice for η'_0 allows the field ξ' to serve as η'. Additionally, the triplet χ_0 can be identified as ϕ_l^0. See table 2.7 for a summary of the new messengers.

Adding an an R-charge 2 triplet like χ'_0 leads to three new minimization constraints which have to be simultaneously fulfilled with equations (2.47)-(2.49). This is an example for a too tightly constrained scenario, since it can be verified, that this multitude of requirements only allows for the trivial, vanishing solution for the VEVs. The same applies to the pair χ', χ'_0. Contrary to this, the singlet choice of an R-charge 2 field such as η_0 in combination with η, leads to non-trivial VEVs. The new constraint added is

$$\frac{\partial w}{\partial \eta_0} = \eta + \{\phi_l \phi_l\} = 0 \tag{2.52}$$

where $\{\phi_l \phi_l\} = \phi_{l1}^2 + 2\phi_{l2}\phi_{l3}$. This constraint is satisfied for $\langle \eta \rangle = 0$ and $\langle \phi_l \rangle \propto (0,1,0)$. So the desired configuration is preserved. But the singlet messengers only allow for the topology where the A_4 contractions are $\{\phi_l^0 \phi_\nu\}\{\phi_l \phi_l\}$ and since $\{\phi_l \phi_l\} = 0$, this results in an unmodified model at the effective level only adding additional parameters.

The remaining choice is the one of singlets with Z_4 charges $i, -i$. In this case the most economic option identifies $\eta' \equiv \xi'$. This only requires the addition of one new field η'_0 which transforms as a $1''$ under A_4. This enables the new terms $\eta'_0 \xi' + \eta'_0 \{\phi_l \phi_\nu\}'$ and relates the VEV of η' and the VEVs of the triplet flavons. This is harmless since previously $\langle \xi' \rangle$ was a free parameter. At the effective level this gives rise to the term $\{\phi_l^0 \phi_l \phi_l \phi_\nu\}$. But the A_4 representations chosen only allow for the contraction $\{\phi_l^0 \phi_l\}'' \{\phi_l \phi_\nu\}'$, where $\langle \{\phi_l \phi_\nu\}' \rangle \propto \langle \xi' \rangle$ due to $\partial w / \partial \eta'_0 = 0$. Confronting this with the second terms of equations (2.47)-(2.49) leads to the conclusion that this choice merely amounts in a redefinition of h_1, yielding no new effects for the mixing. Placing the same pair in a different irreducible singlet representation of A_4 is also not viable. This would modify equations (2.47)-(2.49) to a structure similar to equations (2.26)-(2.29). Thus, both triplet VEVs would be aligned in the $(1,1,1)$ direction.

The last two modifications we will discuss only introduce one new messenger instead of a pair. Adding an R-charge 2 A_4 1 or $1'$ like η'_0 without a partner does not alter equations (2.47)-(2.49). However, fulfilling the new alignment condition $\partial w / \partial \eta'_0 = 0$ requires either $\{\phi_l \phi_\nu\} = 0$ or $\{\phi_l \phi_\nu\}' = 0$ respectively for the 1 and $1''$ choice for η'^0. In combination with the other alignment conditions, this again only allows the trivial solution. The final option, dispensing the R-charge 2 field of the pair, requires a new triplet messenger χ which transforms as -1 under Z_4. This adds the terms $\{\phi_l^0 \chi\} + \{\phi_l^0 \chi\}\xi + \{\phi_l^0 \chi \phi_\nu\}$ to the superpotential and enables both topologies $\xi' \{\phi_0^l \phi_l \phi_\nu\}''$ and $\{\phi_l^0 \phi_l \phi_l \phi_\nu\}$ (c.f. figures 2.6 and 2.7). They add terms to the existing alignment conditions without adding new equations. If $\langle \chi \rangle = 0$ the contributions arise from the enabled contractions of the non-renormalizable terms $\{\phi_l^0 \phi_\nu \phi_l\}'' \xi' + \{\phi_l^0 \phi_\nu \phi_l \phi_l\}$.

[8] The R-charge 2 fields of the original field content use 0 as a superscript.

The extra terms modifying alignment equations (2.47)-(2.49) force $\langle\phi_l\rangle$ to be aligned in the same direction as $\langle\phi_\nu\rangle$, yielding no viable phenomenology.

This leads us to the conclusion, that although [55] presents the NLO VEVs as $\langle\phi_\nu\rangle = \varepsilon(1+dw, 1+dw, 1+dw)$, $\langle\phi_l\rangle = (dx, u+dy, dz)$, at least next-to-minimal completions do not allow these small, general perturbations of the VEV alignment.

2.3.2. Modifications of the flavons

After excluding modifications of the original completion as an attractive source of deviations, we are again left with two remaining options. The first of these is the straightforward extension of the Dirac or Majorana sector of the model, by introducing the effective terms $l h_u \phi_l \nu^c$, $\nu^c \phi_\nu \phi_l \nu^c$ respectively. However, it can be easily verified via tables 2.4 and 2.5 that these terms are not Z_4 invariant. Implementing them would require at least four extra insertions of non-trivial Z_4 charged flavons (ϕ_l or ξ'). This would go hand in hand with the obligation to add multiple new messengers pushing the model well beyond the limit of next-to-minimal completions we consider in this work. More insertions of ϕ_ν also do not lead to a satisfying modification. As ν^c would be the only messenger involved, this would just boil down to the seesaw mechanism of the minimal completion.

Finally, we consider modifications of the flavon content. This goes analogously to section 2.2. We start by adding another non-trivial A_4 singlet. This time we rely on a field transforming as $1''$ which we consequently call ξ''. It is easy to see that this field has to transform trivially under Z_4. Exploring its effects demonstrates the constraining power of UV completions. In case of an effective implementation this would enable one additional NLO term for every term involving two or more A_4 triplets. In the UV complete model however, the only added superpotential terms are

$$\{\nu^c \nu^c\}' \xi'' + \{\phi_\nu^0 \phi_\nu\}' \xi'' + \{\chi_\tau^c \chi_\tau\}' \xi'' . \tag{2.53}$$

The effects of the first term are very similar to what was discussed in the end of section 2.2. The second term has the potential to modify the alignment, but a quick check reveals a comparable structure to equations (2.26)-(2.29). Thus the term $\{\phi_\nu^0 \phi_\nu\}' \xi''$ preserves the alignment $(1,1,1)$ for ϕ_ν.

What is new compared to section 2.2 is the third term which yields additional contributions to the mass of the χ_τ^c, χ_τ messenger pair. This modifies equation (2.51), the already non-diagonal charged lepton mass matrix. Labeling the coupling corresponding to the last term of equation (2.53) as c_χ, the mass now reads

$$m_l = \begin{pmatrix} m_e & (-c_s + c_a)\varepsilon' m_\mu & (-c_s - c_a + c_\chi)\varepsilon' m_\tau \\ (-c_s + c_a + c_\chi)\varepsilon' m_e & m_\mu & 2c_s \varepsilon' m_\tau \\ \varepsilon'(-c_s - c_a) m_e & (2c_s + c_\chi)\varepsilon' m_\mu & m_\tau \end{pmatrix}. \tag{2.54}$$

Consequently, the phenomenological differences of this models arise from the combination of the non-trivial charged lepton mixing matrix with the modified Majorana mass term, where

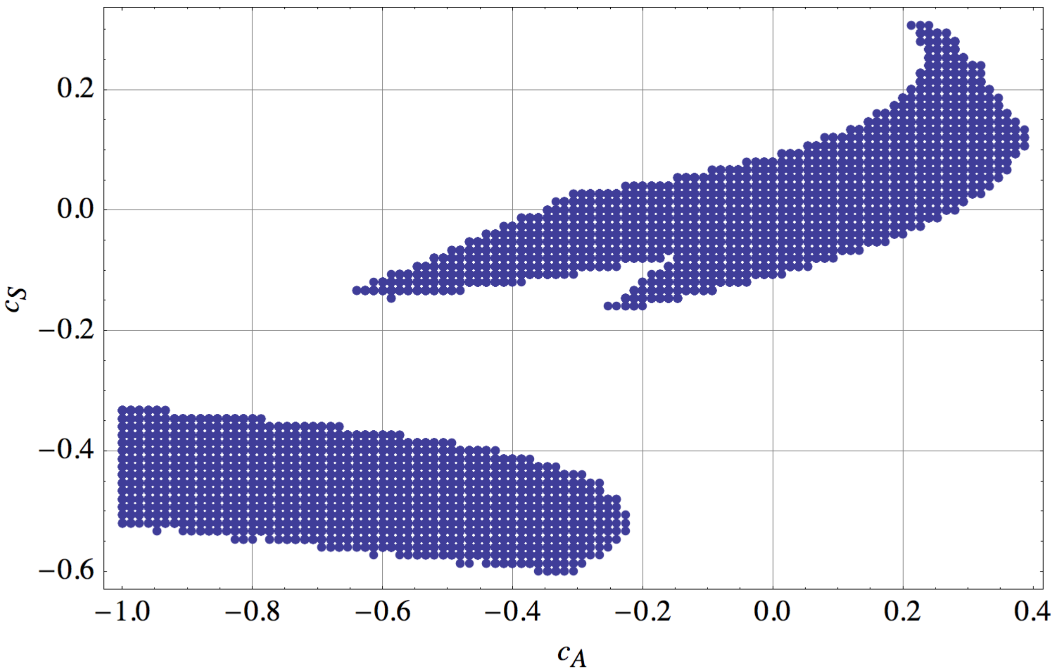


Figure 2.8.: Range of c_A and c_S allowed by present mixing angle data for the modified AM model ($\varepsilon' = \varepsilon'' = 1$).

the new flavon ξ'' adds the structure of equation (2.43). The overall effects of the modification are illustrated in figure 2.8 where we present the now relaxed ranges for the allowed parameter space in the c_a-c_s plane. This has to be compared with figure 2.5. As expected this allows to avoid the somewhat unnatural hierarchy of these two parameters, leading to a more appealing framework in which both can be of the same order.

2.4. Conclusions

In this chapter we discussed UV completions of two different A_4 models, which are guided by the concept of TBM. We investigated the possibility to obtain a large mixing angle θ_{13} as a deviation from this strict mixing scheme.

In case of the $\mathrm{A}_4 \times \mathrm{Z}_3 \times \mathrm{U}(1)_{\mathrm{FN}}$ model, the originally proposed minimal completion predicts exact TBM and thus is already excluded. We illustrated that next-to-minimal completions appear to be a poor choice to solve this issue. Instead we demonstrated, that the minimal completion in combination with a modified flavon content, which replaces one singlet A_4 flavon with a non-trivial one, preserves the elegance of the original completion while it allows to accommodate for experimental data.

On the other hand, the existing completion of the $\mathsf{A}_4 \times \mathsf{Z}_4$ model is still viable. However, the recent more precise measurements impose very strict constraints and a hierarchy of the involved parameters without any theoretical motivation. Analogously to the first discussed model, we started to study in detail the prospects of completion modifications to arrive at a more satisfying model. Again we concluded, that these are not very attractive and instead a more appealing alternative is to use the minimal completion for a model including a non-trivial A_4 singlet flavon. This relaxes the tight constraints and allows for related parameters of the same order.

The discussions of both symmetry groups share the conclusion, that minimal and next-to-minimal completions can lead to an increased predictivity compared to the effective, non-renormalizable ones. This translates from the paradigm of TBM, as presented in [60], to more realistic scenarios. We were indeed able to highlight how these allow to rule out several possibilities.

3. A model of all fermions featuring geometrical CP violation

After discussing the leptonic sector in chapter 2 we are now going to expand the scope to include the quarks too. We will shift our focus from UV completions to a different topic relevant to the SM and its extensions, the origin of CPV. As discussed in subsection 1.1.3 there is only one source of CPV in the SM. At the Lagrangian-level it is explicitly introduced via complex Yukawa couplings. In this chapter we discuss minimal models, instead implementing it via the spontaneous breaking of a symmetry. A concept, well motivated and heavily employed in this work.

In section 3.1 we will start by introducing spontaneous CP violation and discuss geometrical CP violation as an elegant way to implement it via a discrete symmetry. In this context we highlight why models with three Higgs doublets based on the flavor symmetry $\Delta(27)$ can be considered as a minimal framework. We then proceed to discuss the relevant properties of the group $\Delta(27)$ and establish the necessary notation for the following discussion of flavor models with geometrical CP violation. We start by proposing an improvement to the model of [75] in section 3.2, which first demonstrated how viable quark mixing can be implemented in this context. We then proceed to present a first existence proof of a model which employs the same structures for the quark sector and implements viable structures in the lepton sector, finally covering all fermions in section 3.3.

This chapter is based on the previously published articles [3] and [4]:

Ivo de Medeiros Varzielas and Daniel Pidt
Towards realistic models of quark masses with geometrical CP violation
J.Phys. G41 (2014), p.025004
arXiv:1307.0711

Ivo de Medeiros Varzielas and Daniel Pidt
Geometrical CP violation with a complete fermion sector
JHEP 1311 (2013), p.206
arXiv:1307.6545

3.1. Geometrical CP violation and the group $\Delta(27)$

Recall that the implementation of CPV in the SM occurs via the explicit introduction of complex Yukawa couplings in the Lagrangian. While this is a very general approach, it is rather ad hoc and doubles the number of parameters compared to real couplings. Albeit at the physical level, many of the complex phases can be removed by redefinitions of the fields[1], a more compelling solution, in our opinion, is the idea to tie CPV to the spontaneous breakdown of a symmetry. Indeed this has been proposed for non-Abelian gauge groups [76, 77] and can thus be related to EWSB. This framework features a CP conserving Lagrangian while CPV enters the picture only through complex VEVs of scalar fields. This concept is called spontaneous CP violation (SCPV).

Aside from fitting nicely to the breaking of the SM gauge group this has other attractive effects for BSM scenarios. For instance, it solves the strong CP problem [78, 79, 80, 81, 82, 83, 84, 85] and can lead to an alleviated SUSY CP problem [86, 87]. Further SCPV is a requirement for the embedding in perturbative string theory [88, 89, 90].

This helps to motivate the inclusion of CP violating quantities but is not sufficient to necessarily lead to CPV at the end of the day. One has to further ensure, that no unitary transformation

$$\phi_i \to \phi'_i = U_{ij}\phi_i \tag{3.1}$$

for the Higgs fields can be found, that fulfills the relation

$$U_{ij}\,\langle\phi_j\rangle^* = \langle\phi_i\rangle \tag{3.2}$$

and leaves the Lagrangian invariant. In this case CP would still be conserved. Another problem is that generally the complex phases of these fields depend on arbitrary parameters of the scalar potential, which can be numerous themselves depending on the BSM model.

These problems have already been tackled and can be solved by finding an additional symmetry of the potential which leads to phases that are precisely determined by the properties of this symmetry [91]. This calculable phases are called *geometric* and thus lead to the name geometrical CP violation (GCPV). It has further been shown, that these complex phases are stable against radiative corrections [92, 93].

The additional symmetry required for GCPV can neatly be identified with a horizontal discrete flavor symmetry. As discussed in chapter 2, this can also solve the problem of neutrino masses. The authors of [91] already pointed out that at least three Higgs doublets are required and found an interesting solution for the symmetry group $\Delta(27)$.

In [94] it is illustrated that $\Delta(27)$ can indeed be considered as a minimal framework for GCPV. The argument goes as follows. One starts with the most general $\mathrm{SU}(2)\times\mathrm{U}(1)$ potential for three Higgs fields. The only solution for the minimization of this potential with a calculable phase is the trivial one. Thus it is necessary to forbid and/or relate different couplings for all these terms. The most elegant justification for this is a discrete non-Abelian symmetry. The authors then start by choosing the smallest one, which is S_3. However S_3 alone is not sufficient to yield interesting solutions. A further reduction of parameters is needed. This can be achieved by

[1] For a 3×3 Yukawa matrix of Dirac fermions, like in the SM, only one complex phase remains.

the combination of S_3 with cyclic symmetries Z_i to forbid terms. The combination of terms that leads to a viable solution can be preserved for for a suitable charge assignment under $Z_3 \times Z_3$. For this assignment the group $S_3 \ltimes (Z_3 \times Z_3) \equiv \Delta(54)$ leads to the same scalar potential as $Z_3 \ltimes (Z_3 \times Z_3) \equiv \Delta(27)$ which is a finite subgroup of $\Delta(54)$.[2] Thus, it is the minimal basis required for a GCPV model.

This leads to the two VEV configurations already found in [91], which are

$$\langle\phi\rangle = \frac{v}{\sqrt{3}}\left(1,\omega,\omega^2\right) \tag{3.3}$$

$$\langle\phi\rangle = \frac{v}{\sqrt{3}}\left(\omega^2,1,1\right), \tag{3.4}$$

with $\omega \equiv \exp(2\pi i/3)$ for the triplet ϕ. The latter VEV is the more relevant one for model building in the context of SCPV. The reason is, that for this configuration the complex phase can not be removed by an additional symmetry of the potential, and therefore guarantees the manifestation of CPV. This VEV structure in combination with the group $\Delta(27)$ will be the basis for the models proposed in this chapter.

Analogously to chapter 2 we are now going to introduce the irreducible representations and multiplication rules relevant to the discussed models. A thorough analysis of the general flavor group $\Delta(3n^2)$ including all the details and derivations can be found e.g. in [95].[3] The group $\Delta(27)$ has nine irreducible representations of dimension one, which we will label as $1_{i,j}$ with $i,j = 0,1,2$. Of these nine only three are relevant to the construction of models featuring GCPV since the construction of the remaining six singlets involves the complex phase ω which is at odds with the goal to construct a CP invariant Lagrangian. These are the singlets $1_{0,j}$. The multiplication rule for their products is

$$1_{0,i} \times 1_{0,j} = 1_{0,i+j\mathrm{mod}3}, \tag{3.5}$$

where we denote the invariant as $1_{0,0}$. Further we have two three-dimensional representations $3_{0,1}$ and $3_{0,2}$ which serve as triplet and anti-triplet. The product of a pair of identical triplets yields three of the other kind of triplets, i.e.

$$3_{0,j} \times 3_{0,j} = 3_{0,k}^{(i)} + 3_{0,k}^{(ii)} + 3_{0,k}^{(iii)} \begin{cases} 3_{0,k}^{(i)} \propto (\alpha_1\beta_1, \alpha_2\beta_2, \alpha_3\beta_3) \\ 3_{0,k}^{(ii)} \propto (\alpha_2\beta_3, \alpha_3\beta_1, \alpha_1\beta_2)\,, \\ 3_{0,k}^{(iii)} \propto (\alpha_3\beta_2, \alpha_1\beta_3, \alpha_2\beta_1) \end{cases} \tag{3.6}$$

with $j,k = 1,2$ and $j \neq k$ and again α and β denoting the components of the first respective second multiplet. The last missing ingredient is the construction of singlets out of the two

[2] The relevant properties are preserved in all discrete subgroups of $SU(3)$ which are of the type $Z_3 \ltimes (Z_n \times Z_n) \equiv \Delta(3n^2)$ or $S_3 \ltimes (Z_n \times Z_n) \equiv \Delta(6n^2)$ with $n \in 3\mathbb{Z}$.

[3] A similar treatment of the group $\Delta(6n^2)$ can be found in [96].

different triplets. It is given by

$$3_{0,1} \times 3_{0,2} = \sum_{i,j} 1_{i,j} \begin{cases} 1_{0,0} \propto (\alpha_1\beta_1 + \alpha_2\beta_2 + \alpha_3\beta_3) \\ 1_{0,1} \propto (\alpha_1\beta_2 + \alpha_2\beta_3 + \alpha_3\beta_1) \\ 1_{0,2} \propto (\alpha_1\beta_3 + \alpha_2\beta_1 + \alpha_3\beta_2) \\ \quad \dots \end{cases}, \tag{3.7}$$

where we constrained ourselves to the presentation of the relevant singlets. This completes the establishment of the notation for the remaining chapter. The colon between the indices is only used in this part to clarify the structure. It will be omitted in the following sections.

3.2. Towards a realistic quark model

Now, after presenting the general idea and the necessary building blocks, we are going to discuss actual models arising from this foundation. The appealing implementation of SCPV via GCPV has received further attention in recent years [97, 98, 75, 99, 100, 101] and it is in order to point out a few achievements in this field.

We already discussed parts of [94] in the previous section but the authors also identified promising leading order structures for the quark sector. However, it remained a very challenging task to produce viable masses and mixing patterns for the fermions. On the one hand, the only parameter coming from the symmetrical VEV structure of equation (3.4) is its magnitude v, which is shared by all components. On the other hand, the calculable phase ω is very sensitive to further extensions of the scalar content. However, the authors of [75] finally managed to present a working explicit model of GCPV for the quark sector. For the lepton sector, one possible approach is discussed in [101]. Reference [97] contributed considerations on the stability of the calculable phase under non-renormalizable terms in the scalar potential and [102] also discussed the $\Delta(27)$ case. Generalizations of the framework to other symmetry groups and different calculable phases are presented in [99, 98].

Since in this chapter we will rely on the viability of the structures discussed in [75], we would like to address the two shortcomings of this model. The first problem is that the overall symmetry offers no explanation for the large hierarchy of the quark Yukawa couplings. In this work however, we always use the symmetry group to at least explain parts of the fermion hierarchies, so this is not satisfying. The other issue of the model is that it has to rely on the combination of a $\Delta(27)$ triplet scalar and at least one scalar transforming as a non-trivial singlet under $\Delta(27)$. This was necessary to accommodate realistic CKM mixing. The problem is that this endangers the crucial feature of GCPV. To preserve it the couplings mixing these two scalars have to be assumed as being negligible. This premise might be considered natural in some sense but not enforcing it by a symmetry jeopardizes the central point of models building on this.[4]

[4]The same issue is present in the model [101] discussing the lepton sector. There it is even amplified by the presence of two non-trivial $\Delta(27)$ singlet scalars.

	Q_1	Q_2	Q_3	u^c	d^c	H	φ	θ
$\Delta(27)$	1_{00}	1_{00}	1_{02}	3_{01}	3_{02}	3_{01}	1_{00}	1_{02}
$\mathsf{U}(1)_F$ or Z_N	3	2	0	0	0	0	-1	-2

Table 3.1.: The symmetry and field content of the model we adopted to improve [75].

Generally, there is a vivid activity in the field of discrete flavor groups related to CP symmetries and violation [103, 104, 105, 106, 107, 108, 102, 109, 110, 111, 112, 113, 114, 115, 116]. It is not constrained to the particular case of GCPV and thus serves as a motivation that this aspect should be covered by related models explaining the fermion masses and mixing. Thus we will proceed by proposing the appropriate improvements to the framework of [75].

3.2.1. The model

There is a single solution for the two previously discussed issues of the model presented in [75]. Namely this is to increase the overall symmetry of the model. We propose to add either a single continuous $\mathsf{U}(1)_F$ or a single discrete Z_N subgroup. By assigning suitable charges to the different quark fields and non-trivial $\Delta(27)$ singlet scalars, it is possible to implement a FN mechanism and to forbid undesired couplings at the same time. While the former explains the observed hierarchy the latter ensures that GCPV is not spoiled.

We start the presentation of the field content of our model with the Higgs field. It is a $\Delta(27)$ 3_{01} triplet with the VEV structure of equation (3.4)[5], i.e.

$$\langle H_i \rangle = v(\omega, 1, 1)\,. \tag{3.8}$$

This is accompanied by the second class of triplets, the $\mathsf{SU}(2)_L$ singlets u^c and d^c which transform as 3_{01} and 3_{02} respectively under $\Delta(27)$. This allows them to be contracted with the Higgs triplets to suitable singlets 1_{0j}. These structures in combination with the three generations of $\mathsf{SU}(2)_L$ doublet quarks Q_i are the foundation of the quark mass terms. These quark field are $\Delta(27)$ singlets with different $\mathsf{U}(1)_F$ (Z_N) charges to implement the FN mechanism, (i.e. the charges are 3, 2 and 1 for Q_1, Q_2 and Q_3). This is balanced out by the two new SM gauge singlets φ and θ we add. Their respective charges under $\Delta(27)$ and $\mathsf{U}(1)_F$ are 1_{00}, -1 and 1_{02}, -2. This concludes the field content of our model, which is summarized in table 3.1 and mostly preserves the $\Delta(27)$ assignments of [75].

A short comment before we explain the resulting structures in more detail. We will generally use M and M_α to denote mass matrices, where α is a letter indicating the associated fermions. The model we propose does not lead to the same $MM^\dagger$ structures for M_u and M_d that are presented in [75]. However, the employed structures also populate all off-diagonal entries and thus our solution is equally viable.

[5]Note that it is not important whether the VEV of H contains the ω and the one of $H^\dagger$ the ω^2 or vice versa.

Now to the Lagrangian construction. The generic forms involved in building invariants for the up and down quark sector are $Q_i(H^\dagger u^c)_{0j}$ and $Q_i(Hd^c)_{0j}$. The brackets indicate the $\Delta(27)$ contraction of the two triplets to a singlet $\mathbf{1}_{0j}$ which is then combined with one of the $\mathsf{SU}(2)_L$ quark doublets Q_i again to a $\Delta(27)$ singlet $\mathbf{1}_{0k}$. These expressions can generally have a non-trivial charge under $\mathsf{U}(1)_F$ (Z_N). This provides the opportunity to alleviate the hierarchy in the quark Yukawa couplings. As previously explained, we consider this as essential for a framework adding a flavor symmetry to the SM. With the assignments of table 3.1, the charge is different for each quark generation and thus has to be (eventually) canceled by additional insertions of φ and θ to build an overall invariant term. This implements a generalized FN mechanism. The structure of the Lagrangian is further constrained by the $\Delta(27)$ assignments of these fields. For the down quarks it reads

$$\begin{aligned}\mathscr{L}_d = {} & y_3 Q_3 (Hd^c)_{01} + y_2 Q_2 (Hd^c)_{00}\varphi^2 + y_1 Q_1 (Hd^c)_{00}\varphi^3 \\ & + p_2 Q_2 (Hd^c)_{01}\theta + p_1 Q_1 (Hd^c)_{01}\varphi\theta \\ & + h_3 (HH^\dagger) Q_3 (Hd^c)_{01} + h_2 (HH^\dagger) Q_2 (Hd^c)_{00}\varphi^2 \\ & + h_1 (HH^\dagger) Q_1 (Hd^c)_{00}\varphi^3 .\end{aligned} \tag{3.9}$$

Here the mass scale suppressions of all non-renormalizable terms are omitted and implicitly understood to be absorbed by the associated y_i, p_i and h_i coefficients. This is done for the sake of a simple notation, since contrary to chapter 2 we are not going to discuss an UV complete version of the framework here. In such a model the mass scale suppressions would be provided by the respective FN messenger masses. While the explicit construction is beyond our intended scope, figure 3.1 illustrates two examples of how the effective term $Q_2(Hd^c)_{01}\theta$ may arise in a complete model.

In principle other non-renormalizable terms are allowed. In case of using the $\mathsf{U}(1)_F$ this requires adding invariant combinations under this symmetry like $\theta^\dagger\theta$ or $\theta^\dagger\varphi^2$, leading to terms such as $Q_3(Hd^c)_{00}\theta^\dagger\varphi^2$. However, the first of these invariants ($\theta^\dagger\theta$) merely redefines existing terms since it is a $\Delta(27)$ invariant itself. Terms involving the second invariant ($\theta^\dagger\varphi^2$) are always suppressed by three or more additional field insertions compared to the associated terms of equation (3.9) and thus negligible. In case of using the discrete symmetry instead, one can check that the lowest viable N for the cyclic group is 4 (otherwise Q_1 becomes neutral). With the non-negative assignments 3 for φ and 2 for θ this allows for an additional invariant insertion of two fields, i.e. θ^2 enabling e.g. $Q_3(Hd^c)_{00}\theta\theta$. Anyhow, this third contribution, which is not present in the $\mathsf{U}(1)_F$ implementation, can be easily pushed to higher orders by choosing a larger N for the discrete symmetry.

The Lagrangian of equation (3.9) leads to to the down quark mass matrix

$$M_d = M + M_p + M_h , \tag{3.10}$$

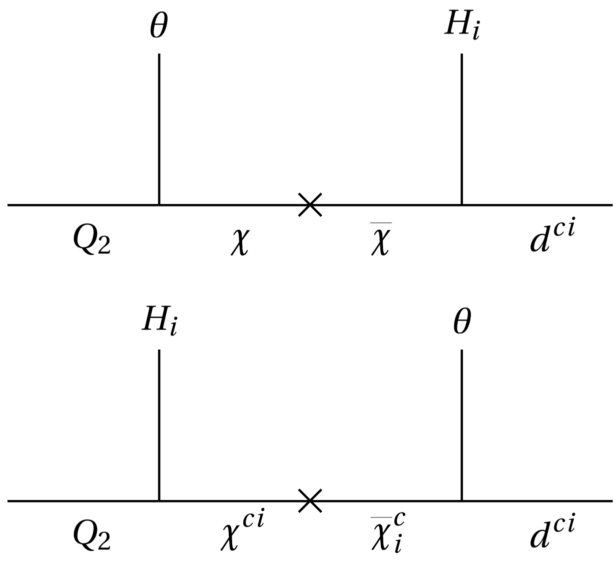


Figure 3.1.: Diagrams of possible $Q_2(Hd^c)_{01}\theta$ topologies. The χ,$\bar{\chi}$ messengers are $\mathsf{SU}(2)_L$ doublets, whereas the χ^{ci}, $\bar{\chi}_i^c$ are $\Delta(27)$ triplets. The index i is used to indicate the triplet components.

with

$$M = v\begin{pmatrix} y_1\omega & y_1 & y_1 \\ y_2\omega & y_2 & y_2 \\ y_3 & y_3 & y_3\omega \end{pmatrix}, \tag{3.11}$$

$$M_p = v\begin{pmatrix} p_1 & p_1 & p_1\omega \\ p_2 & p_2 & p_2\omega \\ 0 & 0 & 0 \end{pmatrix} \tag{3.12}$$

and

$$M_h = v^3\begin{pmatrix} h_1 & h_1\omega^2 & h_1\omega^2 \\ h_2 & h_2\omega^2 & h_2\omega^2 \\ h_3\omega^2 & h_3\omega^2 & h_3 \end{pmatrix}. \tag{3.13}$$

This structure is sufficient to yield viable CKM mixing. To see how this arises, it is instructive to discuss the role of each involved matrix. M combines the first three straightforward Yukawa terms of the Lagrangian. The problem with this structure alone is revealed by examining $MM^\dagger$. Using the identity $1+\omega+\omega^2 = 0$ it can be written as

$$MM^\dagger = 3v^2\begin{pmatrix} y_1^2 & y_1 y_2 & 0 \\ y_1 y_2 & y_2^2 & 0 \\ 0 & 0 & y_3^2 \end{pmatrix}, \tag{3.14}$$

the same structure that was present in [75]. It can easily be verified that this matrix only has rank two. Obviously, more terms are needed to enable the addition of off-diagonal entries to generate realistic CKM mixing.

These structures are introduced in our model via the additional field θ, which transforms non-trivially under $\Delta(27)$ (1_{02}). It enables the terms in the second line of equation (3.9) (and thus M_p). There θ combines with both Q_1 and Q_2 (both 1_{00}) again to 1_{02} and has to be finally linked to the contraction $(Hd^c)_{01}$ to build an invariant. Since $y_3 Q_3 (Hd^c)_{01}$ was already present, we now have terms which combine with the $(Hd^c)_{01}$ contraction of the Higgs field for each generation of quarks. Via the $\langle H_1 \rangle$ VEV this now gives a contribution with the complex phase ω in the same (i.e. third) column of the mass matrix. Analogously to the previous discussion it can be shown that this is sufficient to fill in the missing (13) and (23) entries of equation (3.14) and leads to a fully populated CKM matrix. This happens in a very similar way in [75]. In their model the (13) and (23) entries arose from $Q_3(Hd^c)_{00}$ interfering with $Q_2(Hd^c)_{00}$, $Q_2(Hd^c)_{00}(HH^\dagger)$ and $Q_1(Hd^c)_{00}$, $Q_1(Hd^c)_{00}(HH^\dagger)$, whereas in our case the responsible interference stems from $Q_{2,1}(Hd^c)_{01}$ with $Q_3(Hd^c)_{01}$ (and $Q_3(Hd^c)_{01}(HH^\dagger)$ once M_h is included).

Albeit, there is still one missing piece with only M and M_p considered, the complex phase (although present in both) would not survive the construction of $M_d M_d^\dagger$. This problem is solved by the remaining term of equation (3.9) leading to M_h. It is directly responsible to generate a complex CKM matrix. This was also the case in [75]. Note that for M_h we have constrained ourselves to only include contributions from the possible $\Delta(27)$ invariants leading to a new structure. All other contributions can be absorbed in the already present structures by employing the relation $1+\omega+\omega^2 = 0$ again.

The Lagrangian for the up quark sector is

$$\begin{aligned}\mathscr{L}_u = x_3 Q_3 (H^\dagger u^c)_{01} + x_2 Q_2 (H^\dagger u^c)_{00}\varphi^2 + x_1 Q_1 (H^\dagger u^c)_{00}\varphi^3 \\ + q_2 Q_2 (H^\dagger u^c)_{01}\theta + q_1 Q_1 (H^\dagger u^c)_{01}\varphi\theta.\end{aligned} \tag{3.15}$$

Now because $H^\dagger$ is the 1_{02} we have $(u^c H^\dagger)_{01} = u_1^c H_3^\dagger + u_2^c H_1^\dagger + u_3^c H_2^\dagger$, compared to the down sector where we had $(Hd^c)_{01} = H_1 d_3^c + H_2 d_1^c + H_3 d_2^c$. This leads to the mass matrix

$$M_u = v \begin{pmatrix} x_1\omega^2 & x_1 & x_1 \\ x_2\omega^2 & x_2 & x_2 \\ x_3 & x_3\omega^2 & x_3 \end{pmatrix} + v \begin{pmatrix} q_1 & q_1\omega^2 & q_1 \\ q_2 & q_2\omega^2 & q_2 \\ 0 & 0 & 0 \end{pmatrix}. \tag{3.16}$$

Due to the stronger hierarchy in the up sector, the contributions to the CKM matrix are irrelevant and the Yukawa couplings can easily fit the required three up quark masses.[6]

[6]Note that analogously to the down quark sector, only implementing the first term of equation (3.16) leads to a rank two matrix and consequently considering the additional term is not optional but mandatory.

3.2.2. Scalar potential

With the charge assignments of table 3.1 the scalars H, φ and θ lead to the potential

$$
\begin{aligned}
V(H,\varphi,\theta) =& m_\varphi^2 \varphi\varphi^\dagger + m_\theta^2 \theta\theta^\dagger + m_H^2 \left[H_i i^\dagger\right] \\
&+\lambda_\varphi (\varphi\varphi^\dagger)^2 + \lambda_\theta (\theta\theta^\dagger)^2 + \lambda_{\varphi\theta} (\varphi\varphi^\dagger)(\theta\theta^\dagger) + \lambda_1 \left[(H_i H_i^\dagger)^2\right] \\
&+\lambda_2 \left(H_1 H_1^\dagger H_2 H_2^\dagger + H_2 H_2^\dagger H_3 H_3^\dagger + H_3 H_3^\dagger H_1 H_1^\dagger\right) \\
&+\lambda_3 \left(H_1 H_2^\dagger H_1 H_3^\dagger + H_2 H_3^\dagger H_2 H_1^\dagger + H_3 H_1^\dagger H_3 H_2^\dagger + \text{h.c.}\right) \\
&+ \left(\lambda_{\varphi H} \varphi\varphi^\dagger + \lambda_{\theta H} \theta\theta^\dagger\right) \left[H_i H_i^\dagger\right] ,
\end{aligned}
\tag{3.17}
$$

where $\lambda_3 > 0$ is responsible for the GCPV solution of equation (3.8). Details on this can be found in the original paper [91] or in [98, 100]. Discussing why this solution is preserved shows that it is necessary to choose different $\mathsf{U}(1)_F$ (Z_N) charges for θ and φ. Otherwise the $\Delta(27)$ contractions of these two singlet with $HH^\dagger$ would enable the renormalizable, phase-dependent term $(HH^\dagger)(\varphi\theta^\dagger)$ and therefore spoil GCPV. On the other hand all terms involving $\theta\theta^\dagger$ or $\varphi\varphi^\dagger$ are harmless since these combinations transform as the trivial singlet under $\Delta(27)$ ensuring that no additional phase dependency is introduced. Instead, all terms with this effect are pushed to the non-renormalizable level. Consider e.g. terms with $HH^\dagger$ and a single θ, that can enable additional non-trivial Higgs contractions. They would require at least one additional insertion of $(\varphi^\dagger)^2$ in order to be invariant under the extra $\mathsf{U}(1)_F$ symmetry.

We use the scalars φ or θ in the FN mechanism to generate the hierarchies in the quark Yukawa couplings once suitable messengers are introduced.[7] This can be problematic since the scales involved in the determination of their VEVs have to be larger than the scale of EWSB. Fortunately, equation (3.17) separates any masses and quartic couplings of terms involving either φ or θ (or both) and terms involving H. Thus a different choice of the scales can be considered natural in our framework.

With the parameters of the presented scalar potential it is easy to accommodate a decoupling scenario in which the additional $\Delta(27)$ singlet scalars have masses of order TeV (as already demonstrated in [75]). In this limit it is easy to realize that only one of the CP even states associated with H is light. This requires no fine-tuning and allows this Higgs to serve as the recently discovered light Higgs boson. An even more interesting phenomenology can arise when the parameters are chosen in a way that two CP even states are light. In that scenario h_a avoids detection due to not having $h_a VV$ couplings with the SM vector bosons V and h_b has a mass of 125-126 GeV and SM-like couplings. Only h_c is heavier.

[7] As we mentioned earlier this is beyond the scope of this work. An illustration of the possible, involved topologies can be found in figure 3.1.

	Q_1	Q_2	Q_3	u^c	d^c
$\Delta(27)$	1_{00}	1_{00}	1_{02}	3_{02}	3_{02}
$\mathsf{U}(1)_F$	3	2	0	$-p$	$-p$

Table 3.2.: The quark field and symmetry content of the models discussed in section 3.3.

	L	e^c	μ^c	τ^c
$\Delta(27)$	3_{01}	1_{00}	1_{00}	1_{02}
$\mathsf{U}(1)_F$	$f_L(p)$	$f_e(p)$	$f_\mu(p)$	$f_\tau(p)$

Table 3.3.: The lepton field and symmetry content of the models discussed in section 3.3.

3.2.3. Conclusions

In summary, we succeeded to address both issues of the framework presented in [75]. While it first proved that viable CKM mixing can be realized in combination with a minimal implementation of GCPV, it was not able to explain the hierarchies of the Yukawa couplings and relied on a non-trivial $\Delta(27)$ singlet jeopardizing the calculable phase. We point the way toward a more realistic model by enlarging the symmetry content to include either an additional $\mathsf{U}(1)_F$ or a discrete Z_N and expand the field content with two scalars charged under it. One of these is a non-trivial singlet under $\Delta(27)$ and its charge under the additional symmetry enforces that GCPV is not spoiled by its presence (addressing one of the original issues). At the same time both scalars can acquire VEVs larger than the EW scale. This is possible due to a separation between their mass and quartic couplings and the Higgs fields in the scalar potential. In combination with suitably charged quark doublets, this is used to enact a FN-like mechanism and justifies the hierarchy present in the Yukawa couplings. At the same time our improved model preserves the interesting properties of the scalar sector described in [75].

3.3. A complete picture of all fermions

After the establishment of a satisfying quark sector, we now turn our focus to the lepton part and will investigate the viability of several possible extensions. While [101] pointed out interesting structures in this sector it does not include the other fermions. We would like to extend the isolated focus in this field to finally cover the whole picture.

One of our goals is to develop a lepton framework that is not only viable by itself, but fully compatible with the quark structures proposed in [75] and section 3.2. To achieve this, we intend to keep the $\mathsf{U}(1)_F$ FN symmetry introduced in the previous section. By finding viable lepton structures that are consistent with feasible quark structures, we provide an existence proof of a complete model of fermion masses and mixing featuring GCPV.

We start with an explanation why the most straightforward extensions of the $\Delta(27)$ GCPV

	H	ϕ	φ	θ
$\Delta(27)$	$\mathbf{1}_{00}$	$\mathbf{3}_{01}$	$\mathbf{1}_{00}$	$\mathbf{1}_{02}$
$\mathsf{U}(1)_F$	0	p	-1	-2

Table 3.4.: The scalar field and symmetry content of the models.

framework do not readily work. In the lepton sector we want to construct the invariants $HL\tau^c$ for the charged lepton masses and the effective operator $H^\dagger H^\dagger LL$ to generate neutrino masses, where H and L are the respective SM scalar and lepton doublet.[8] Previously, H transformed as a $\mathbf{3}_{01}$ under $\Delta(27)$.[9] $\Delta(27)$ invariance of the effective neutrino mass term $H^\dagger H^\dagger LL$ now requires the lepton doublet to transform as the same triplet representation as H (c.f. equations (3.5)-(3.7)). Consequently, invariance of the charged lepton mass term $HL\tau^c$, also forces the $\mathsf{SU}(2)_L$ lepton singlets to transform as $\mathbf{3}_{01}$. It has already been demonstrated that the resulting mass structures are not viable [91, 94]. One way to avoid this pitfall is to enlarge the field content of the model. Multiple ways are viable, e.g. [101] chose to introduce additional $\mathsf{SU}(2)_L$ doublets ζ transforming as $\Delta(27)$ singlets. This enabled the neutrino mass operator $\zeta H^\dagger LL$, with L transforming as the conjugate triplet ($\mathbf{3}_{02}$). The charged lepton $\mathsf{SU}(2)_L$ singlets e^c, μ^c, τ^c transformed as singlets under $\Delta(27)$. However, this model only covered the lepton sector and has its issues too, as we already pointed out in section 3.2.

Instead we explore a different path. We abandon the idea to assign H as a triplet and chose it to be a trivial $\Delta(27)$ singlet. This is accompanied by the introduction of a new $\mathbf{3}_{01}$ triplet ϕ (a SM singlet) and amounts to the field and symmetry content displayed in tables 3.2-3.4. As a result now the scales of breaking the SM gauge group (related to H) and of breaking $\Delta(27)$ and CP (related to ϕ) are separated. On the one hand, this has the disadvantage that the rich phenomenology of the $\mathsf{SU}(2)_L$ doublet scalars, sketched in the end of section 3.2 and in [75], no longer is present. On the other hand, this provides the opportunity for a higher scale of CP breaking. A requirement which may be imposed by future implementations of the framework also addressing the subject of the baryon asymmetry of the universe (e.g. via leptogenesis).

3.3.1. Modified quark structures

With this separation of H and ϕ in place, we are now free to assign L as a $\mathbf{3}_{01}$. This allows us to build neutrino mass terms like $H^\dagger H^\dagger (LL\phi)$ or $H^\dagger H^\dagger (LL\phi^\dagger\phi^\dagger)$ with potentially viable structures. However, before we discuss this in detail, we have to ensure that two building blocks of the framework are still in place with our modification.

First of all we have to arrange that the VEV structure of equation (3.8) is not spoiled. As

[8]Again, for the discussion in this section the necessary mass scale suppressions for effective terms are omitted and implicitly understood. As discussed previously in chapter 2 or e.g. in [60, 114, 117, 118], these are related to the masses of the messenger fields associated to the UV completion of the model, or the seesaw mechanism.

[9]Choosing $\mathbf{3}_{02}$ instead does not influence the argumentation.

previously mentioned this is the case, as long as no cubic terms of ϕ appear in the scalar potential. Because ϕ is a SM singlet we now have to enforce this by a different symmetry. In the presented models this happens via the FN symmetry that we already have. To preserve the VEV alignment we assign a non-trivial charge to ϕ which is suitable to forbid any cubic terms. There are multiple viable choices and we label this charge as p. As indicated by tables 3.2-3.4 this has consequences for the charges of most fermion fields. We will discuss this when we arrive at the specific models.

The second preliminary remark is that we have to apply further minor modifications to the quark structures of section 3.2 and [75]. There the invariants were $H^\dagger Q u^c$ and HQd^c, with H, u^c and d^c respectively 3_{01}, 3_{01} and 3_{02} under $\Delta(27)$. In comparison now we have $H^\dagger Q u^c \phi$ and $HQd^c\phi$, with H, u^c, d^c and ϕ respectively 1_{00}, 3_{02}, 3_{02} and 3_{01} under $\Delta(27)$. This results in a slight change of the up quark matrix. Because u^c is the 3_{02} in the invariant contraction (with ϕ, not $\phi^\dagger$) the first structure of equation (3.16) is altered to

$$M_u = v \begin{pmatrix} x_1\omega & x_1 & x_1 \\ x_2\omega & x_2 & x_2 \\ x_3 & x_3 & x_3\omega \end{pmatrix} . \tag{3.18}$$

The structure of section 3.2 can be recovered by replacing ω with ω^2 and swapping the entries (32) and (33). Analogously $HQd^c\phi(\phi\phi^\dagger)$ replaces the previous invariant $HQd^c(HH^\dagger)$.[10] However, overall no differences arise in the ability to accommodate viable masses for all quarks and realistic CKM mixing, thus leading to an equally appealing framework. For the details regarding the involved structures we refer to the according discussion in section 3.2 and [75].

We would like to conclude the introduction of our new, complete framework with a short recapitulation of its essentials. A FN mechanisms is implemented either via a continuous $\mathsf{U}(1)_F$ or a discrete Z_N. The scalar sector is extended with a $\Delta(27)$ triplet scalar ϕ, which has to be charged non-trivially under this symmetry to enforce GCPV since it is a SM singlet.[11] This charge p has consequences for other fields of the model and will be specified in the actual implementation of models. The SM singlets φ and θ serve as FN fields and have the charges 1_{00}, -1 and 1_{02}, -2 under $\Delta(27)$ and the other respective symmetry. Finally, the $\mathsf{SU}(2)_L$ breaking Higgs field is now a trivial $\Delta(27)$ singlet. The details of the changed scalar potential will be addressed in subsection 3.3.4.

3.3.2. Leptonic structures

After establishing this solid foundation, we now focus to build models featuring a viable lepton sector upon this base, that is compatible with the proposed quark structures. One general intricacy of the framework we discuss is the charged lepton mass matrix. Since we choose L to transform as a $\Delta(27)$ triplet the hermitian combination $M_l M_l^\dagger$ is not diagonal at leading order.

[10] This invariant was required to introduce a complex phase to the CKM matrix.

[11] The requirement is to forbid the terms ϕ^3, $\phi^3\theta$, $\phi^3\theta^\dagger$, $\phi^3\varphi$ and $\phi^3\varphi^\dagger$.

For the generic, most important invariants, like $H\left[(L\phi^\dagger)l_i^c\right]$ and the assignments 1_{00}, 1_{00} and 1_{02} for the $\mathsf{SU}(2)_L$ singlet leptons e^c, μ^c and τ^c, the contractions generally look like

$$H\left[y_3(L\phi^\dagger)_{01}\tau^c + y_2(L\phi^\dagger)_{02}\mu^c(\theta^2) + y_1(L\phi^\dagger)_{00}e^c(\theta^3)\right], \tag{3.19}$$

giving rise to the mass matrix

$$M_l = \begin{pmatrix} y_1\omega^2 & y_2 & y_3 \\ y_1 & y_2 & y_3\omega^2 \\ y_1 & y_2\omega^2 & y_3 \end{pmatrix}, \tag{3.20}$$

where we have reabsorbed the VEVs into the y_i.[12] The hermitian combination now reads

$$M_l M_l^\dagger = \begin{pmatrix} y_1^2+y_2^2+y_3^2 & y_1^2\omega^2+y_2^2+y_3^2\omega & y_1^2\omega^2+y_2^2\omega+y_3^2 \\ y_1^2\omega^2+y_2^2+y_3^2\omega & y_1^2+y_2^2+y_3^2 & y_1^2+y_2^2\omega+y_3^2\omega^2 \\ y_1^2\omega^2+y_2^2\omega+y_3^2 & y_1^2+y_2^2\omega+y_3^2\omega^2 & y_1^2+y_2^2+y_3^2 \end{pmatrix}. \tag{3.21}$$

There is a simple analytic expression for the matrix V_l diagonalizing this, i.e.

$$V_l = \frac{1}{\sqrt{3}}\begin{pmatrix} \omega^2 & \omega & 1 \\ 1 & \omega & \omega^2 \\ 1 & 1 & 1 \end{pmatrix}. \tag{3.22}$$

Note that V_l has no free parameters. The three Yukawa couplings y_i are fixed by the charged lepton masses.

Before we proceed with a detailed discussion of the neutrino related invariants, we would now like to shed a little more light on the general obstacles encountered in the construction of a viable lepton model within these limits. The $\Delta(27)$ contractions for the Majorana neutrino mass invariant $H^\dagger H^\dagger(LL\phi)$ include structures that can easily generate TBM for the neutrinos. While this was generally a good starting point up until now, in this case the mixing would be completely screwed up by the non-diagonal form of V_l, due to $U_{\text{PMNS}} = V_l^\dagger U_\nu$. For a compatible solution with the TBM neutrino structures, a contraction of L with a different triplet field aligned as $(1,0,0)$ would be required. Vice versa, we could chose to assign the $\mathsf{SU}(2)_L$ doublets L_1, L_2, L_3 as the $\Delta(27)$ singlets 1_{00}, 1_{01}, 1_{02} and instead build the triplet from e^c, μ^c, τ^c. This would yield a leading order neutrino mass matrix with (11), (23) and (32) as the only non-vanishing entries. In combination with V_l from equation (3.22) this $\mu-\tau$ interchange would provide a very promising leading order structure for the neutrino mixing matrix. Unfortunately, these assignments do not preserve the structure of equation (3.22), lead to a diagonal charged lepton sector (c.f. the discussion of the quark sector) and thus yield no viable mixing. Again the solution would be the contraction with a different alignment

[12] Note that although [101] employed $\Delta(27)$ singlets e^c, μ^c, τ^c too, the specific choice of the suitable singlet for each field differ.

field, i.e. this time the contraction of LL with a field $\propto (1,0,0)$. Both cases would amount to a well-proven strategy to implement TBM-like models with a discrete symmetry - the breaking of neutrino and charged lepton sector in two different directions. The models discussed in chapter 2 rely on this too. However, both also involve the introduction of additional triplet scalars which is at odds with the main feature of this framework, the enforcement of GCPV. Consequently this is not an option and a more creative approach is needed.

Our proposal relies on the Majorana invariants of the type $H^\dagger H^\dagger (LL\phi)$. The algebra of equation (3.6) allows the construction of two distinct triplets 3_{02} from LL (two combinations are symmetric). Each of these can be further contracted with ϕ to build all three available singlets (c.f. equation (3.7)) amounting to six different singlets overall. The term under consideration clearly requires the singlet contractions $H^\dagger H^\dagger (LL\phi)_{00}$ and the two different parameters available for this are not sufficient to build viable neutrino structures. This requires the introduction of auxiliary (spurion) fields. Analogously to the singlets of the previous chapter 2 we label them as ξ, ξ' and ξ'', indicating their representations 1_{00}, 1_{01} and 1_{02}. They all share the same FN charge which has to be the opposite of $H^\dagger H^\dagger (LL\phi)$ to build an overall invariant. These auxiliary fields can be constructed from the physical fields φ and θ. We address this in more detail in subsection 3.3.3. As discussed, each of these spurions provides two distinct invariants, i.e.

$$H^\dagger H^\dagger \xi \left[z_1 (L_i L_i \phi_i)_{00} + z_4 (L_i L_j \phi_k)_{00}\right], \tag{3.23}$$
$$H^\dagger H^\dagger \xi' \left[z_2 (L_i L_i \phi_i)_{02} + z_5 (L_i L_j \phi_k)_{02}\right], \tag{3.24}$$
$$H^\dagger H^\dagger \xi'' \left[z_3 (L_i L_i \phi_i)_{01} + z_6 (L_i L_j \phi_k)_{01}\right], \tag{3.25}$$

where the ijk invariant denotes the symmetric contraction, i.e.

$$(L_i L_j \phi_k)_{00} = L_2 L_3 \phi_1 + L_3 L_1 \phi_2 + L_1 L_2 \phi_3, \tag{3.26}$$
$$(L_i L_j \phi_k)_{01} = L_2 L_3 \phi_3 + L_3 L_1 \phi_1 + L_1 L_2 \phi_2, \tag{3.27}$$
$$(L_i L_j \phi_k)_{02} = L_2 L_3 \phi_2 + L_3 L_1 \phi_3 + L_1 L_2 \phi_1. \tag{3.28}$$

The corresponding mass structures read

$$M_\xi = \begin{pmatrix} z_1\omega^2 & z_4 & z_4 \\ z_4 & z_1 & z_4\omega^2 \\ z_4 & z_4\omega^2 & z_1 \end{pmatrix}, \tag{3.29}$$

$$M_{\xi'} = \begin{pmatrix} z_2 & z_5\omega^2 & z_5 \\ z_5\omega^2 & z_2 & z_5 \\ z_5 & z_5 & z_2\omega^2 \end{pmatrix}, \tag{3.30}$$

$$M_{\xi''} = \begin{pmatrix} z_3 & z_6 & z_6\omega^2 \\ z_6 & z_3\omega^2 & z_6 \\ z_6\omega^2 & z_6 & z_3 \end{pmatrix}. \tag{3.31}$$

This concludes the general setup of our first phenomenological scan for viable parameter space. With V_L fixed by the charged lepton masses, equations (3.23)-(3.25) leave us with six parameters z_i to fit the neutrino related parameters. Even though the structures provided by equations (3.29)-(3.31) are fairly constraining, we were able to identify feasible regions of parameter space resulting in mixing angles and mass squared differences satisfying the 3-σ bounds of [21]. The obtained solutions include examples of both neutrino mass orderings, i.e. IH and NH (c.f. table A.1 in the appendix for an illustration of the resulting couplings).

For the setup of our second scan, we also include the operator $H^\dagger H^\dagger (L\phi^\dagger)(L\phi^\dagger)$. The relevant contractions are

$$H^\dagger H^\dagger \left[A(L\phi^\dagger)_{00}(L\phi^\dagger)_{00} + B(L\phi^\dagger)_{01}(L\phi^\dagger)_{02} \right], \tag{3.32}$$

introducing the parameters A and B. This allows us to constrain the set of spurion fields required to obtain viable results from three to two. We performed a scan for each of the three possible combinations. Again, for all three classes of models we were able to find large regions of viable parameter space leading to both hierarchies. Exemplary coupling patterns can be found in tables A.2, A.3 and A.4 in the appendix, which also contains considerations concerning the measurement of the involved fine-tuning.

3.3.3. Specific models

After demonstrating the potential of the general framework, we are now going to delve into the details of building concrete models. By assigning suitable $\mathsf{U}(1)_F$ charges to the fields the FN mechanism is implemented and the auxiliary fields of the scans can be constructed from the fields θ and φ.

First to the simple part, the quark sector. There, the assignment of table 3.1 uses the charges of u^c and d^c to balance out the charge of ϕ, which we labeled p. The different choice for the three quark generations Q_i in combination with the FN fields then enables the viable quark structures discussed in the according section, including the required FN suppression.

The more intricate part is the lepton sector. First of all note, that not all solutions for the effective parameters z_i obtained in the scans are equally appealing. Of these six parameters, two are always related to the same spurion, i.e. (z_1, z_4) to ξ, (z_2, z_5) to ξ' and (z_3, z_6) to ξ''. Since each spurion corresponds to a physical field combination of θ and φ, a large hierarchy between couplings from the same pair is unnatural. The same argument holds for A and B. First we constrained ourselves to solutions with all couplings within one order of magnitude. Sample hits for the different classes of models can be found in tables A.1-A.4 in section A.2 of the appendix. To build two actual models, we modify this constraint a little bit by considering two solutions which allow the combination (z_1, z_4) to be roughly one order of magnitude larger than (z_2, z_5), (z_3, z_6) and (A, B). For exact numerical values, see table A.5.

invariants	L	ϕ	e^c	μ^c	τ^c
$1_{00}+1_{01}$	-1	-4	3	1	-3
$1_{00}+1_{02}$	1	4	9	7	3

Table 3.5.: Specific $\mathsf{U}(1)_F$ charges for two sample models that are considered natural in terms of hierarchies. The column invariants refers to the enabled $\Delta(27)$ contractions of FN field combinations.

For the two models we find the invariants

$$\begin{aligned}
&H^\dagger H^\dagger (\theta^3)^\dagger \left[A(L\phi^\dagger)_{00}(L\phi^\dagger)_{00} + B(L\phi^\dagger)_{01}(L\phi^\dagger)_{02}\right] \\
+&H^\dagger H^\dagger \theta^3 \left[z_1(L_iL_i\phi_i)_{00} + z_4(L_iL_j\phi_k)_{00}\right] \\
+&H^\dagger H^\dagger \theta^2\varphi^2 \left[z_2(L_iL_i\phi_i)_{02} + z_5(L_iL_j\phi_k)_{02}\right],
\end{aligned} \tag{3.33}$$

and

$$\begin{aligned}
&H^\dagger H^\dagger \theta^3 \left[A(L\phi^\dagger)_{00}(L\phi^\dagger)_{00} + B(L\phi^\dagger)_{01}(L\phi^\dagger)_{02}\right] \\
+&H^\dagger H^\dagger (\theta^3)^\dagger \left[z_1(L_iL_i\phi_i)_{00} + z_4(L_iL_j\phi_k)_{00}\right] \\
+&H^\dagger H^\dagger (\theta^2\varphi^2)^\dagger \left[z_3(L_iL_i\phi_i)_{01} + z_6(L_iL_j\phi_k)_{01}\right]
\end{aligned} \tag{3.34}$$

respectively. Compared to equations (3.23)-(3.25) and equation (3.32), ξ, ξ', ξ'' are now replaced by θ and φ. Thus, these spurions can indeed be identified as combinations of the physical FN fields. In both models the combination (z_1, z_4) is the only one appearing in invariants with eight field insertions. All remaining combinations require nine fields and thus, the resulting hierarchy between them is justified. The $\mathsf{U}(1)_F$ assignments are listed in table 3.5. The general idea behind the charge assignments for L and ϕ is to make the fields $H^\dagger H^\dagger(L\phi^\dagger)(L\phi^\dagger)$ invariant in combination with either θ^3 or $(\theta^3)^\dagger$ resulting in overall nine fields for the (A, B) terms.[13]

3.3.4. Scalar potential

Finally, after constructing viable structures for fermion masses and mixing, it is time to discuss the full scalar potential to ensure that GCPV manifests itself and to see how all this comes together in the end. Due to our careful choice of the FN charge p for ϕ, no cubic terms of this field can appear in the potential. At the same time the charges of φ and θ guarantee, that no other phase dependent invariants, like $(\phi^\dagger\phi)_{01}\theta$, $(\phi^\dagger\phi)_{02}\theta^\dagger$ are allowed, which would spoil

[13]There are two other possible choices where $H^\dagger H^\dagger(L\phi^\dagger)(L\phi^\dagger)$ has the same overall FN charge as $H^\dagger H^\dagger(LL\phi)$. However, these correspond to $p = 0$ allowing for cubic ϕ terms in the scalar potential. This is incompatible with the goal to enforce GCPV.

GCPV. Overall the potential reads

$$
\begin{aligned}
&V(H,\phi,\varphi,\theta) = m_H^2 HH^\dagger + m_\varphi^2 \varphi\varphi^\dagger + m_\theta^2 \theta\theta^\dagger \\
&+ \lambda_H (HH^\dagger)^2 + \lambda_\varphi (\varphi\varphi^\dagger)^2 + \lambda_\theta (\theta\theta^\dagger)^2 + \lambda_{\varphi\theta} (\varphi\varphi^\dagger)(\theta\theta^\dagger) \\
&+ \left(\lambda_{\varphi H} \varphi\varphi^\dagger + \lambda_{\theta H} \theta\theta^\dagger\right)\left(HH^\dagger\right) \\
&+ m_\phi^2 \left[\phi_i \phi_i^\dagger\right] + \lambda_1 \left[(\phi_i\phi_i^\dagger)^2\right] + \lambda_2 \left(\phi_1\phi_1^\dagger\phi_2\phi_2^\dagger + \phi_2\phi_2^\dagger\phi_3\phi_3^\dagger + \phi_3\phi_3^\dagger\phi_1\phi_1^\dagger\right) \\
&+ \lambda_3 \left(\phi_1\phi_2^\dagger\phi_1\phi_3^\dagger + \phi_2\phi_3^\dagger\phi_2\phi_1^\dagger + \phi_3\phi_1^\dagger\phi_3\phi_2^\dagger + \text{h.c.}\right) \\
&+ \left(\lambda_{H\phi} HH^\dagger + \lambda_{\varphi\phi} \varphi\varphi^\dagger + \lambda_{\theta\phi} \theta\theta^\dagger\right)\left[\phi_i\phi_i^\dagger\right] . \qquad (3.35)
\end{aligned}
$$

Indeed, one can easily verify that this potential only contains one phase dependent term, i.e. the one associated with λ_3. Combined with the prerequisite $\lambda_3 > 0$ it effectively enforces GCPV with ϕ acquiring a VEV of the form presented in equation (3.8). The magnitudes of the VEVs of H, ϕ, φ and θ are controlled by the various mass terms and quartic couplings. Again, the scales are not required to be the same. After the breaking of the FN symmetry $\mathrm{U}(1)_F$, the flavor symmetry $\Delta(27)$ and EW symmetry through the VEVs of the respective scalar fields, finally viable masses and mixing of all fermions arise at low energy.

3.3.5. Conclusions

Building on the foundation of a viable quark sector including a FN mechanism, we discussed the common issues related to the structures of the lepton part and presented a fully compatible set. For different frameworks composed from this set, we performed an extensive phenomenological scan and found several regions of parameter space where viable lepton masses and mixing can be obtained. With viable solutions readily available, we proceed to construct concrete models for two of these with an appealing, natural structure of parameters. The related scalar potential leads to GCPV with the desired VEV structure $\propto (\omega, 1, 1)$. Overall, this amounts to the presentation of a first existence proof of models of quark and lepton masses and mixing that feature GCPV.

4. A generic class of R-parity violating models

In the previous two chapters we discussed models based on concrete flavor symmetries, which employed the seesaw mechanism to generate small neutrino masses. Further, these models considered the nowadays required large mixing angle θ_{13} arising as some kind of modification of TBM or a combination with some of its eigenvectors. However this is not the only viable guiding principle for lepton mixing. For instance see e.g. [119] for recent considerations of how $\theta_{13} \approx 9^\circ$ can be seen a a *reduction* of an even larger guiding value. Another possibility, which is often used as some kind of benchmark, is that the neutrino mass matrix has random entries. This is usually referred to as neutrino mass anarchy [120, 121, 122, 123, 124, 125].

In this chapter we are going to discuss a generic class of flavor models which feature the concept of RPV (c.f. subsections 1.2.2 and 1.2.3) to generate small neutrino masses. Recall that in this context, one of the main challenges to be addressed by the flavor symmetry is to explain the hierarchy of the RPV couplings. By just requiring some simple relations of the charge assignment, we demonstrate how this arises and further achieve a significant reduction in the number of independent couplings.

In order to address the above mentioned ambiguity of TBM and other principles, we will illustrate how both, TBM and realistic models are constructed and constrained in this framework. This will serve as an example in which realistic models can not be seen as a deviation from TBM. We will further investigate the constraints for the absolute mass scale and the discrimination between NH and IH.

The basic idea for the flavor hypothesis was already sketched in the diploma thesis:

Daniel Pidt
Neutrino Masses and Lepton Flavor Violating Decays in the Minimal Supersymmetric Standard Model without R-Parity

Except for this starting point everything else was developed later on and this chapter is based on the previously published article [1]:

Gautam Bhattacharyya, Heinrich Päs, and Daniel Pidt
R-Parity violating flavor symmetries, recent neutrino data and absolute neutrino mass scale
Phys.Rev. D84 (2011), p.113009
arXiv:1109.6183

4.1. A simple flavor hypothesis

The basis of the framework is the MSSM with RPV. The latter leads to the inclusion of the superpotential terms given by equation (1.37). Since the structure of these terms will be important for the following discussion we repeat it in this place for the sake of convenience.

$$W_{\mathrm{RPV}} = -\mu_i L_i H_2 + \frac{1}{2}\lambda_{ijk} L_i L_j \overline{E}_k + \lambda'_{ijk} L_i Q_j \overline{D}_k + \frac{1}{2}\lambda''_{ijk} \overline{U}_i \overline{D}_j \overline{D}_k. \tag{4.1}$$

As previously stressed, the most general approach means the introduction of 48 a priori complex couplings to the superpotential, leading to a loss of predictivity. One solution for this problem is to forbid/suppress and relate this couplings. As already demonstrated in chapter 3 for the case of GCPV, flavor symmetries are a very appealing candidate to achieve this. In the following we will show how the large set of RPV couplings can be brought down to six independent lepton number L violating ones. The only requirement is the assumption of a simple flavor hypothesis leading to a class of interesting, generic models. The remaining couplings are sufficient to construct viable Majorana neutrino masses and mixing which will be detailed in section 4.2.

But first we have to state our hypothesis. We assume that the Yukawa structure leading to the masses and mixing of quarks and charged leptons is fixed by some unspecified global symmetry. This symmetry also ensures baryon number conservation. There is a second global symmetry (X), an abelian horizontal symmetry, which is at the centre of our attention. *Only* leptons are charged under X, such that for each generation i,

$$Q_X(L_i) = -Q_X(\overline{E}_i)\,. \tag{4.2}$$

We assume that the Q_X charges of different generations are all positive. The horizontal symmetry is explicitly broken by a small parameter $\varepsilon < 1$, whose charge under X is $Q_X(\varepsilon) = -1$. If the total charge of a given superpotential term is n, then the term is suppressed by ε^n. As an example, if $X = \mathsf{Z}_N$, then the suppression would be $\varepsilon^{n(\mathrm{mod}N)}$ [126].

Now we look at the consequences of equation (4.2) for the 48 RPV couplings of equation (4.1). Since B-number is conserved, all the λ'' couplings vanish right away. Since only leptons are charged under X, it follows that $Q_X(L_i Q_j \overline{D}_k) = Q_X(L_i H_u) = Q_X(L_i)$, and hence $\lambda'_i \equiv \lambda'_{ijk} \simeq \tilde{\mu}_i \equiv \mu_i/\mu$, where the supersymmetry preserving μ parameter is assumed to be of the same order as the supersymmetry breaking soft masses ($\tilde{m}$). Turning our attention to the $L_i L_j \overline{E}_k$ operator, we notice that when $j = k$, the same argument as above leads to $\lambda_{ijj} \simeq \lambda'_{ijk} \simeq \tilde{\mu}_i$. Thus 39 *a priory* independent L-violating couplings basically boil down to only six

$$\tilde{\mu}_i \left(\simeq \lambda'_{ijk} \simeq \lambda_{ijj}\right),\ \lambda_{123},\ \lambda_{132},\ \lambda_{231}\,. \tag{4.3}$$

Thanks to the flavor symmetry, the L-violating bilinear soft parameters B_i would be aligned to the corresponding superpotential parameters μ_i as well, i.e. $\tilde{B}_i \equiv B_i/\tilde{m}^2 \simeq \tilde{\mu}_i$. It should be noted that when we say that two couplings are related, we mean that they have a common

Our couplings	Related to	Existing limits (Sources)	Refs.
$\tilde{\mu}_i$	$\tilde{\mu}_i, \lambda'_{ijk}, \lambda_{ijj}$	$1.5 \times 10^{-6} \left(M_{\tilde{\chi}}\right)$ $[m_{\nu_i}]$	[127]
λ_{123}	λ_{123}	0.03 $(\tilde{\tau}_R)$ $[V_{ud}]$	[128]
λ_{132}	λ_{132}	0.03 $\left(\tilde{\mu}_R\right)$ $[R_\tau]$	[128]
λ_{231}	λ_{231}	0.05 $(\tilde{e}_R)$ $[R_\tau]$	[128]

Table 4.1.: The list of the six independent couplings and the standard couplings they are related to by the flavor symmetry X. The three $\tilde{\mu}_i$ couplings are of the same order of magnitude as 36 out of 39 *a priori* independent RPV couplings. A mass of 100 GeV is assumed for the superparticles exchanged in the processes involved. These superparticles are indicated within first bracket right after the bounds (the weak gaugino mass M_χ and the three scalar leptons $\tilde{\ell}_R$). The entries in the square brackets specify the different observables from which the bounds originate. Here, $R_\tau = \Gamma(\tau^- \to \mu^- \overline{\nu}_\mu \nu_\tau)/\Gamma(\mu^- \to e^- \overline{\nu}_e \nu_\mu)$.

suppression factor ϵ^{Q_X}. Indeed, there are order-one uncertainties in the actual coefficients of the operators, for which reason we have used a 'near-equality' sign in equation (4.3). Now we come to the relative sizes of the L-violating couplings. The suppression would depend on the sum of Q_X charges of the associated lepton fields as a power of ε. More specifically,

$$\tilde{\mu}_i \simeq \tilde{B}_i \simeq \lambda'_i \sim \varepsilon^{Q_X(L_i)}, \;\; \lambda_{ijk} \sim \varepsilon^{\left\{Q_X(L_i)+Q_X(L_j)+Q_X(\overline{E}_k)\right\}}. \tag{4.4}$$

Eventually, we shall provide a specific demonstration with $X = \mathsf{Z}_{N_1} \times \mathsf{Z}_{N_2}$ [126], which means there are all together 6 charges for the three lepton generations.

Many RPV couplings which are not so strongly constrained may now be related by equation (4.4) to the ones which are severely constrained by experiments. The existing bounds on the individual and product couplings can be found in the reviews [129, 130, 131, 132, 40].

4.2. Relevant neutrino mass terms and mixing

One of the appealing aspects of R-parity violation is that it generates neutrino masses and mixing through a perfectly renormalizable interaction without the need of introducing any right-handed neutrino. This has already been studied at various levels of detail [133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 127, 147]. In this work we will follow the notation of [147]. The neutrino masses, in the basis in which all the sneutrino vacuum expectation values vanish, can be written as

$$\begin{aligned} m_{ij} \approx \; & \frac{\cos^2\beta}{\tilde{m}}\mu_i\mu_j + \frac{g^2}{64\pi^2\cos^2\beta}\frac{B_iB_j}{\tilde{m}^3} + \frac{g^2}{64\pi^2\cos\beta}\frac{\mu_iB_j+\mu_jB_i}{\tilde{m}^2} \\ & + \sum_k \frac{3}{16\pi^2} g m_{d_k} \frac{\mu_i\lambda'_{jkk}+\mu_j\lambda'_{ikk}}{\tilde{m}} + \sum_k \frac{1}{16\pi^2} g m_{e_k} \frac{\mu_i\lambda_{jkk}+\mu_j\lambda_{ikk}}{\tilde{m}} \\ & + \sum_{l,k} \frac{3}{8\pi^2}\lambda'_{ilk}\lambda'_{jkl}\frac{m_{d_l}m_{d_k}}{\tilde{m}_q^2}\mu\tan\beta + \sum_{l,k}\frac{1}{8\pi^2}\lambda_{ilk}\lambda_{jkl}\frac{m_{e_l}m_{e_k}}{\tilde{m}^2}\mu\tan\beta, \end{aligned} \tag{4.5}$$

where m_{d_i} (m_{e_i}) denote the masses of the down-type quarks (charged leptons). The associated diagrams can be found in section A.1 in the appendix. A comment on the approximations made above is in order. We have denoted the squark masses by $\tilde{m}_q$ and assumed them to be somewhat heavier than a common mass scale $\tilde{m}$ assumed for the sleptons and weak gauginos/Higgsinos. This approximation may be crude but is good enough for our order-of-magnitude estimate of the RPV couplings. In equation (4.5), the first line accounts for the tree level and one loop contributions from bilinear couplings only, the second line represents the one loop contributions from both bilinear and trilinear couplings, while the last line stands for one loop contributions from trilinear couplings only. The possibility of large left-right squark/slepton mixing which may be induced by large $\tan\beta(\equiv v_u/v_d)$ has been taken into account in the purely trilinear loop dynamics. The tree level $\mu_i\mu_j$ contribution generates a rank-one mass matrix and therefore yields only one mass eigenvalue. Since, in our case, B_i, λ'_i, λ_{ijj} are all proportional to μ_i, even after including their contributions the rank-one nature of the mass matrix does not change. What breaks the alignment and yields more non-vanishing eigenvalues is the contribution from the purely trilinear loops involving $\lambda_{ijk}(i \neq j \neq k)$, since these couplings are not aligned with μ_i. This leaves us with the remaining three couplings, namely λ_{123}, λ_{132} and λ_{231}, no two indices of which are the same, for generating the second *mandatory* and the third *optional* nonvanishing neutrino masses and the three mixing angles (two large and one small). Note that the existing bounds on λ_{ijk} with $i \neq j \neq k$ are relatively less stringent – see table 4.1.

Different low energy processes, especially some lepton flavor violating decays, yield important constraints on trilinear product couplings [148, 149, 150]. Due to the smallness of *most* of the couplings as shown in the first row of table 4.1, these constraints are in almost all cases easily satisfied. The bounds emerging from the non-observation of $K_L^0 \to \mu\overline{e}/e\overline{\mu}$ [149, 150], namely,

$$\lambda_{ijk}\lambda'_{lmn} < 6.7\times10^{-9}\; m^2_{\tilde{\nu}_{L3}}\;/\;(100\,\mathrm{GeV})^2\,, \tag{4.6}$$

with the combinations $(ijk)(lmn)$: (312)(312), (312)(321), (321)(312), (321)(321), play a crucial role in neutrino mass/mixing model building in our scenario, as we shall see later. Due to the specific inter-connections among RPV couplings owing to the flavor symmetry, equation (4.6) leads to the following limits:

$$\lambda_{132}\lambda'_3,\;\; \lambda_{231}\lambda'_3 < 6.7\times10^{-9}\; m^2_{\tilde{\nu}_{L3}}\;/\;(100\,\mathrm{GeV})^2\,. \tag{4.7}$$

If we set the numerical values of the couplings near their upper limits (see table 4.1), they turn out to be large enough to offset the loop suppression factors. The mass matrix entries can then be written with only six RPV couplings as

$$m_{ee} \approx a\mu_1\mu_1 + \frac{1}{8\pi^2}\lambda_{123}\lambda_{132}\frac{m_\mu m_\tau}{\tilde{m}^2}\mu\tan\beta, \tag{4.8}$$

$$m_{e\mu} \approx a\mu_1\mu_2 + \frac{1}{8\pi^2}\lambda_{123}\lambda_{232}\frac{m_\tau m_\mu}{\tilde{m}^2}\mu\tan\beta + \frac{1}{8\pi^2}\lambda_{213}\lambda_{131}\frac{m_\tau m_e}{\tilde{m}^2}\mu\tan\beta, \tag{4.9}$$

$$m_{e\tau} \approx a\mu_1\mu_3 + \frac{1}{8\pi^2}\lambda_{132}\lambda_{323}\frac{m_\mu m_\tau}{\tilde{m}^2}\mu\tan\beta + \frac{1}{8\pi^2}\lambda_{312}\lambda_{121}\frac{m_\mu m_e}{\tilde{m}^2}\mu\tan\beta, \tag{4.10}$$

$$m_{\mu\mu} \approx a\mu_2\mu_2 + \frac{1}{8\pi^2}\lambda_{231}\lambda_{213}\frac{m_e m_\tau}{\tilde{m}^2}\mu\tan\beta, \tag{4.11}$$

$$m_{\mu\tau} \approx a\mu_2\mu_3 + \frac{1}{8\pi^2}\lambda_{231}\lambda_{313}\frac{m_\tau m_e}{\tilde{m}^2}\mu\tan\beta + \frac{1}{8\pi^2}\lambda_{321}\lambda_{212}\frac{m_\mu m_e}{\tilde{m}^2}\mu\tan\beta, \tag{4.12}$$

$$m_{\tau\tau} \approx a\mu_3\mu_3 + \frac{1}{8\pi^2}\lambda_{312}\lambda_{321}\frac{m_\mu m_e}{\tilde{m}^2}\mu\tan\beta, \tag{4.13}$$

with

$$a = \frac{\cos^2\beta}{\tilde{m}} + \sum_k \frac{3g m_{d_k}}{8\pi^2\tilde{m}^2} + \sum_k \frac{g m_{e_k}}{8\pi^2\tilde{m}^2} + \sum_{k,l} \frac{3m_{d_l} m_{d_k}}{8\pi^2\mu\tilde{m}_q^2}\tan\beta. \tag{4.14}$$

With this mass matrix we try to reproduce the neutrino oscillation data, namely, the two mass-squared differences (Δm^2_{21} and Δm^2_{31}) and the three mixing angles (θ_{12}, θ_{23} and θ_{13}). For simplicity we assume that all the phases in the neutrino mixing matrix are zero. Since neutrino oscillation analysis probes only the mass-squared differences and not their absolute values, we need to assume the hierarchy (normal/inverted) of the masses and the size of the smallest eigenvalue to fix the other two masses. There is no lower limit on the smallest neutrino mass eigenvalue, it can still be zero.

We take the best fit values of the neutrino mass-squared differences from [151]: $\Delta m^2_{21} = 7.59\times10^{-5}$ eV^2, $\Delta m^2_{31}(\text{IH}) = -2.40\times10^{-3}$ eV^2, $\Delta m^2_{31}(\text{NH}) = 2.51\times10^{-3}$ eV^2. The two mixing angles θ_{12} and θ_{23} are set to their TBM values (using best fit values instead does not lead to significant changes). On the other hand, nowadays it is an established experimental fact, that the value of θ_{13} deviates from its TBM value which demands $\theta_{13} = 0$. Depending on the included measurements and reactor fluxes it seems to be the case that it is rather close to $\theta_{13} \approx 9°$. We are now at a point were the field of neutrino oscillation parameters reaches the stage of precision measurements. Future tasks will for example include the determination of the Dirac CPV phase δ. For some very recent illustrations of the current work in progress, see e.g. [152].

However, the focus of this work is to demonstrate the principal model building challenges and strategies of this field. Especially in this chapter we would like to point out the difference to the previous frameworks, which mostly used TBM as a guiding principle to yield viable masses and mixing patterns, compared to other ones in which this is unrelated. Thus in the following section we will discuss two benchmark scenarios of neutrino mixing. The first one will be TBM and it will be confronted with a *realistic* setup which assumes $\theta_{13} = 9°$. We shall later see, that this will have interesting consequences for another important question of the neutrino puzzle – NH or IH?

4.3. Classes of models

4.3.1. Models of TBM

The TBM structure immediately implies that $|m_{e\mu}| = |m_{e\tau}|$ and $|m_{\mu\mu}| = |m_{\tau\tau}|$, regardless of whether the lightest mass eigenvalue is vanishing or not, and also irrespective of whether the neutrino mass hierarchy is normal or inverted. For our couplings this can be comfortably realized by setting $|\lambda_{123}| = |\lambda_{132}|$ and $|\mu_2| = |\mu_3|$ – see equations (4.8)-(4.13). This means that we can parametrize the mass matrix with four independent RPV parameters instead of six, which of course would improve the predictivity of the model. Dropping the terms in the loop contribution proportional to the electron mass, we obtain

$$m_{e\mu} \approx m_{e\tau} \approx a\mu_1\mu_2 - \frac{1}{8\pi^2}\lambda_{123}\mu_3\frac{m_\tau m_\mu}{\tilde{m}^2}\tan\beta. \tag{4.15}$$

Clearly, under this situation, the absolute values for the tree-level contributions to $m_{\mu\mu} \sim a\mu_2\mu_2$, $m_{\mu\tau} \sim a\mu_2\mu_3$ and $m_{\tau\tau} \sim a\mu_3\mu_3$ are the same. Setting all CP-violating phases to zero, the TBM mixing matrix takes the form already displayed in equation (1.20). To fix the numerical values of the mass matrix from $m = V_{\mathrm{TBM}}^{\top} \times \mathrm{diag}(m_1, m_2, m_3) \times V_{\mathrm{TBM}}$, all we need to decide is the mass hierarchy (normal or inverted) and the smallest mass eigenvalue.

Inverted hierarchy: We first consider the case of inverted hierarchy with $m_3 = 0$. This choice additionally demands $m_{\mu\tau} = -m_{\mu\mu}$. One obtains

$$m = \begin{pmatrix} 4.92\times10^{-2} & 2.56\times10^{-4} & -2.56\times10^{-4} \\ 2.56\times10^{-4} & 2.47\times10^{-2} & -2.47\times10^{-2} \\ -2.56\times10^{-4} & -2.47\times10^{-2} & 2.47\times10^{-2} \end{pmatrix} \mathrm{eV}\ (\mathrm{IH, TBM}, m_3 = 0). \tag{4.16}$$

By setting $\mu_2 = -\mu_3$ and keeping $\lambda_{231} \lesssim \lambda_{123}$, we obtain a rough analytical solution using equations (4.4) and (4.8)-(4.13):

$$|\mu_2| = |\mu_3| \approx \sqrt{a^{-1} m_{\mu\mu}}, \tag{4.17}$$

$$\lambda_{123} \approx \sqrt{\frac{4\pi^2 m_{ee}\tilde{m}^2}{m_\mu m_\tau \mu \tan\beta}} \tag{4.18}$$

$$\mu_1 \approx \frac{m_{e\mu} + \mu_2\lambda_{123} m_\tau m_\mu \tan\beta / \left(8\pi^2\tilde{m}^2\right)}{(a\mu_2)}. \tag{4.19}$$

Putting $\tilde{m} = \mu = 100\,\text{GeV}$ and $\tilde{m}_q = 300\,\text{GeV}$ in equations (4.17)-(4.19) we obtain a solution (with $\mu_2 = -\mu_3$, and $\lambda_{132} = -\lambda_{123}$)

$$\tilde{\mu}_1 = 1.9 \times 10^{-8}\,, \quad \tilde{\mu}_2 = -4.7 \times 10^{-6}\,, \tag{4.20}$$

$$\lambda_{231} \sim 10^{-4}, \quad \lambda_{123} = -3.2 \times 10^{-4}, \tag{4.21}$$

for $\tan\beta = 10$.

To illustrate how this coupling pattern can arise from a flavor symmetry we are providing an exemplary flavor group for this case. However, since this choice is not necessarily unique and our conclusions do not depend on the specific flavor group, we omit this exercise for the other scenarios. In this case, the required relative suppression can be reproduced by a family symmetry $X = \mathsf{Z}_4 \times \mathsf{Z}_8$ with a breaking parameter ε. The necessary charge assignments are given by

$$Q_X(L_1) = (2,5)\,,\; Q_X(L_2) = (0,5)\,,\; Q_X(L_3) = (3,2)\,, \tag{4.22}$$

which imply

$$Q_X(L_1L_2E_3^C) = (3,0)\,,\; Q_X(L_1L_3E_2^C) = (1,2)\,,\; Q_X(L_2L_3E_1^C) = (1,2)\,. \tag{4.23}$$

These assignments lead exactly to the required suppression of the couplings with $\varepsilon \approx 0.1$ as

$$\mu_2(=\mu_3) \sim \varepsilon^5\,,\; \mu_1 \sim \varepsilon^7\,,\; \lambda_{123}(=\lambda_{132}) \sim \varepsilon^3,\; \lambda_{231} \sim \varepsilon^3\,. \tag{4.24}$$

In the above example, the near equality of the magnitude of the entries in the $\mu-\tau$ block is ensured by saturating them with the tree level contributions, while keeping the loop contributions suppressed. If, within the TBM framework, we now consider m_3 to be slightly above zero, then $m_{\mu\mu} = m_{\tau\tau} > |m_{\mu\tau}|$. To obtain $m_3 = 0.001$ eV with $\tilde{m} = \mu = 100$ GeV, we need $\tilde{\mu}_1 = 1.9 \times 10^{-8}$, $\tilde{\mu}_2 = -4.6 \times 10^{-6}$, $\tilde{\mu}_3 = 4.7 \times 10^{-6}$, $\lambda_{123} = -3.1 \times 10^{-4}$, $\lambda_{132} = -3.3 \times 10^{-4}$, $\lambda_{231} = 2.7 \times 10^{-3}$. We should note two important things: (i) These choices imply $\lambda_{231}\lambda'_3 = 1.3 \times 10^{-8}$, which mildly overshoots the $K_L \to \mu e$ bound as shown in equation (4.7). If we increase m_3 further, the disagreement with the K_L bounds deepens. (ii) The 'four parameter' scenario with $|\mu_2| = |\mu_3|$ and $|\lambda_{123} = \lambda_{132}|$ is not compatible with a non-vanishing absolute neutrino mass scale, i.e. we cannot fit the data assuming these 'equalities' with $m_3 > 0$, because of the hierarchical nature of the charged lepton masses which appear in equations (4.8)-(4.13).

Normal hierarchy: We now consider normal hierarchy of neutrino masses. In this case the smallest mass eigenvalue is m_1. Within the TBM structure if we keep $m_1 = 0$, it follows that $m_{\mu\mu} = m_{\tau\tau} > |m_{\mu\tau}|$. The numerical values of the mass matrix entries are

$$m = \begin{pmatrix} 2.90 \times 10^{-3} & 2.90 \times 10^{-3} & -2.90 \times 10^{-3} \\ 2.90 \times 10^{-3} & 2.80 \times 10^{-2} & 2.21 \times 10^{-2} \\ -2.90 \times 10^{-3} & 2.21 \times 10^{-2} & 2.80 \times 10^{-2} \end{pmatrix} \text{eV (NH, TBM, } m_1 = 0). \tag{4.25}$$

The couplings needed to fit these entries are $\tilde{\mu}_1 = -5.2 \times 10^{-7}$, $\tilde{\mu}_2 = 3.9 \times 10^{-6}$, $\tilde{\mu}_3 = 5.0 \times 10^{-6}$, $\lambda_{123} = -4.4 \times 10^{-3}$, $\lambda_{132} = -1.2 \times 10^{-6}$, $\lambda_{231} = 1.0 \times 10^{-3}$. Although we are within the $K_L \to \mu e$ bound, the requirement $m_{e\mu} = -m_{e\tau}$ is realized quite differently. The relative signs of the tree-level couplings invariably imply $m_{e\mu}^{\text{tree}} \approx +m_{e\tau}^{\text{tree}}$. This difference between the experimental requirement and the tree-level contribution cannot be resolved, even keeping in mind that signs of the RPV couplings can be chosen at will and also each neutrino field can be redefined to absorb a sign. Therefore, a sign adjustment for one of the entries $(e\mu)$ via a large loop contribution is needed, while the loop contribution to the other one $(e\tau)$ becomes negligible. This is reflected in the large hierarchy between λ_{123} and λ_{132}. We recall that such a sign adjustment was not required in the case of inverted hierarchy (TBM, $m_3 = 0$). If we now increase the value of m_1 (from zero) and try to fit normal hierarchy within the TBM framework, the $K_L \to \mu e$ bound haunts us like in the case of inverted hierarchy with $m_3 > 0$. Therefore, our most robust conclusion is the tight constraint for the smallest mass eigenvalue. Thus, inverted hierarchy can be fit with four parameters, while normal hierarchy requires six parameters and a sign altering large loop correction.

4.3.2. Realistic models

In the light of the present experimental status quo, we now study how flexible we are to accommodate a large mixing angle θ_{13}, which is close to the value of 9.0°. Further we will again explore both relevant neutrino mass orderings, NH and IH. Unlike in the case of TBM which guarantees $|m_{e\mu}| = |m_{e\tau}|$ and $m_{\mu\mu} = m_{\tau\tau}$, it is not possible to fit the data with 4 parameters when $\theta_{13} \neq 0$.

Inverted hierarchy: First we consider the case $m_3 = 0$. The numerical entries of the mass matrix are given by

$$m = \begin{pmatrix} 4.80 \times 10^{-2} & -5.13 \times 10^{-3} & -5.63 \times 10^{-3} \\ -5.13 \times 10^{-3} & 2.53 \times 10^{-2} & -2.41 \times 10^{-2} \\ -5.63 \times 10^{-3} & -2.41 \times 10^{-2} & 2.54 \times 10^{-2} \end{pmatrix} \text{eV} \,(\text{IH}, \theta_{13} = 9.0^\circ, m_3 = 0)\,. \tag{4.26}$$

This can be fit with $\tilde{\mu}_1 = -1.1 \times 10^{-6}$, $\tilde{\mu}_2 = -4.5 \times 10^{-6}$, $\tilde{\mu}_3 = 4.8 \times 10^{-6}$, $\lambda_{123} = 9.3 \times 10^{-3}$, $\lambda_{132} = 1.1 \times 10^{-5}$, $\lambda_{231} = -1.1 \times 10^{-4}$. Two things are worth noting: (i) The magnitudes of λ_{123} and λ_{132} are separated by nearly three orders, while they assumed identical numerical values in the case of TBM. (ii) The tree-level contribution to $m_{e\mu}$ has the wrong sign like in the case of NH with θ_{13}=0. Again a large sign adjusting loop contribution is needed to be in agreement with the experimental data. If we now increase the value of m_3, the required magnitude for λ_{231} becomes larger, and eventually beyond $m_3 = 0.01$ eV the $K_L \to \mu e$ bound overshoots.

Normal hierarchy: For $m_1 = 0$, the mass matrix entries are given by

$$m = \begin{pmatrix} 4.06 \times 10^{-3} & 8.02 \times 10^{-3} & 2.29 \times 10^{-3} \\ 8.02 \times 10^{-3} & 2.67 \times 10^{-2} & 2.16 \times 10^{-2} \\ 2.29 \times 10^{-3} & 2.16 \times 10^{-2} & 2.80 \times 10^{-2} \end{pmatrix} \text{eV} \,(\text{NH}, \theta_{13} = 9.0^\circ, m_1 = 0)\,. \tag{4.27}$$

This can be reproduced with $\tilde{\mu}_1 = 4.1 \times 10^{-7}$, $\tilde{\mu}_2 = 3.8 \times 10^{-6}$, $\tilde{\mu}_3 = 5.0 \times 10^{-6}$, $\lambda_{123} = -5.3 \times 10^{-3}$, $\lambda_{132} = -1.5 \times 10^{-6}$, $\lambda_{231} = 8.3 \times 10^{-4}$. Note that λ_{231} is small enough to satisfy the $K_L \to \mu e$ bound. Contrary to the case of inverted hierarchy, now no large sign-flipping correction for $m_{e\mu}$ is needed. However, the difference between the values of $m_{e\mu}$ and $m_{e\tau}$ still leads to a hierarchy in the λ-couplings. Just like in the previous cases, the $K_L \to \mu e$ bound begins to be relevant as soon as m_1 increases to around 0.005 eV (which requires $\lambda_{231} = 1.5 \times 10^{-3}$). The main conclusion for non-zero θ_{13} is again that the smallest mass eigenvalue is required to be almost vanishing in both hierarchies. But contrary to the TBM case, now IH requires a sign adjustment, while NH does not.

4.4. Collider signatures and conclusion

The LHC signatures of the λ_{ijk} couplings have e.g. been explored in [153]. In our scenario, only three couplings $\lambda_{ijk}(i \neq j \neq k)$ are relatively large ($10^{-3} - 10^{-4}$), the rest are of order 10^{-6}. The large couplings are small enough to make sure that the RPV vertex is numerically relevant only at the end of a supersymmetry cascade when the lightest neutralino decays via a λ_{ijk} interaction, $\tilde{\chi}_1^0 \to l^\pm l^\mp \nu$. The λ_{ijk} couplings thus give rise to $l_i l_k$ or $l_j l_k$ final states plus missing energy. Depending on the numerical values of of the corresponding λ_{ijk} couplings the branching ratios into the $l_i l_k$ or $l_j l_k$ channel will scale as $|\lambda_{ijk}|^2$. Thus both invariant mass distributions and number counting of the final state leptons should be a part of the search method. However, other decay channels like $\tilde{\chi}_1^0 \to W^\pm l^\mp$ and $\tilde{\chi}_1^0 \to Z\nu$ are available due to the presence of the bilinear couplings. Their role has been investigated in detail in [154]. Therefore, a detailed study of neutralino decays is important to test this and other RPV models and differentiate between them. Unfortunately no sign of SUSY has been observed so far by ATLAS and CMS. For a recent overview of this topic see e.g. [155]. This indicates a somewhat heavier squark mass scale than the one we chose. However, scaling the slepton masses accordingly can balance this out. The task to generate a viable SUSY mass spectrum is another challenge itself and there is no unique structure arising from the different theoretical concepts, especially in the lepton sector. However, this is beyond the scope of this work and does not lead to any significant changes related to our previous discussions.

Overall, in this chapter we have studied a generic and simple flavor model which reduces the number of independent couplings from 39 to 6, i.e. μ_i ($i = 1,2,3$), λ_{123}, λ_{132} and λ_{231}. This results in an extremely predictive framework, which can reproduce the correct neutrino masses and mixings while satisfying all other low energy bounds.

The simplest incarnation of this scenario would be the TBM case. It can be built from only four parameters and prefers inverse hierarchy. We illustrated for this case how a specific flavor model could be constructed, via a $\mathrm{Z}_4 \times \mathrm{Z}_8$ symmetry. A non-vanishing mixing angle θ_{13} necessarily requires a six parameter realization and arises in a different manor.

A general conclusion of all possible realizations is an almost vanishing absolute mass scale for neutrinos, i.e. an essentially massless lightest neutrino. This feature is tightly related to the non-observation of $K_L \to \mu e$ which affects many important coupling products in this framework. As a consequence, any positive signal in one of the upcoming neutrinoless double beta decay experiments would imply an inverted neutrino mass hierarchy, since for the combination of normal hierarchy and an almost vanishing absolute mass scale, the resulting $|m_{ee}|$ is beyond their sensitivity. In other words, since a large value of θ_{13} is now established, our scenario is only able to accommodate a positive signal of neutrinoless double beta decay at the expense of large sign-flipping correction to one of the off-diagonal elements of the mass matrix. Moreover, the flavor structure proposed here can lead to specific decays of a neutralino LSP at the LHC.

5. Conclusion

At the moment, the field of neutrino physics is in a phase transition from the era of rough concepts to the age of precision physics. Not a long time ago, the established knowledge could have been summarized by stating that three light neutrinos exist and that we have a general idea of how they mix. But this has begun to change rapidly. On the one hand, each new publication from one of the LHC collaborations so far constrained the parameter space for new heavy particles around the TeV scale further and further. On the other hand, astrophysical experiments and experiments dedicated to specific neutrino observables continue to reduce the error bars on the known mixing parameters and push the upper bounds for neutrino masses to lower and lower scales. They even promise further to yield first bounds for former unconstrained parameters, like the Dirac phase. The models presented in this work are located at the link between these two phases and chapter 1 provided a short glance at some theoretical concepts and experimental results in this field, necessary to understand these.

In chapter 2 we revisited two models for leptons based on the symmetry A_4, which were proposed before the exclusion of exact tribimaximal-mixing. Of these one did not stand the test of time while the other one is now under severe pressure. Minimal ultraviolet completions for these models previously existed, and we investigated how next to minimal completions and modifications of the flavon content fare in addressing their present issues. For the first framework, composed of $A_4 \times Z_3 \times U(1)_{FN}$, we found that modifications of the minimal ultraviolet completion do not work very well in this respect. Instead we found a different appealing solution. It consists of the existing minimal completion, preserving its increase in predictivity, in combination with a non-trivial singlet flavon. This new model is additionally able to explain realistic mixing parameters, yielding a viable, elegant description of the lepton sector. A similar result was found for the second class of $A_4 \times Z_4$ models. Again we proceeded by exploring possible modifications of the completion and the field content and arrived at the conclusion, that a liaison between the the minimal completion and the addition of a non-trivial singlet flavon is the most attractive remedy. While in this case the original model is still viable, the involved parameters started to require more and more finetuning which is alleviated by our proposed solution that leads to a natural order of these. In the context of both models, we were able to highlight the virtues of ultraviolet completions compared to effective models. We emphasized how they indeed lead to an increase in predictivity and illustrated how they rule out several options in the transition from tribimaximal-mixing to realistic scenarios.

Following this, we shifted our focus from ultraviolet completions to the topic of spontaneous CP violation in chapter 3. The symmetry A_4 is replaced by $\Delta(27)$. We sketched that this framework is the minimal basis to implement geometrical CP violation which is a very appealing implementation of spontaneous CP violation featuring calculable phases. Building on the foundation of the first viable model of the quark sector for this setup, we propose some improved version, which addresses two of the original shortcomings. The Yukawa hierarchy is explained via the Froggatt-Nielsen mechanism or a Z_N symmetry and the calculable phases are protected since the nontrivial $\Delta(27)$ singlet of our implementation is charged under the additional symmetry. Afterwards we tackled the challenge to provide a more complete picture. To arrive at a first consistent description of all fermions, we had to modify the field structure again. Contrary to the improved model of the quark sector H transforms as a $\Delta(27)$ singlet in this framework and the associated flavon transforms as a 3_{01}. This decouples the scales of symmetry breaking, which has the disadvantage to lose the interesting scalar phenomenology but on the other hand provides the opportunity for a higher scale of CP breaking. This might be useful for future models building on our work, investigating additional topics like leptogenesis. For different implementations of this general framework we performed thorough scans of the available parameter space and found multiple regions leading to viable masses and mixing. We further investigated the finetuning necessary in this context and constructed examples of natural models perfectly compatible with working structures for the quark sector. This completed the first presented proof of existence of a model realistically describing all fermion and masses mixing, featuring geometrical CP violation.

In the final chapter 4 we traded the concrete flavor symmetries of the previous ones for a more general concept, the assumptions of a simple flavor hypothesis. This is used to provide a predictive description of neutrino masses and mixing based on R-parity violation instead of the seesaw mechanism. While the other two classes of frameworks discussed in this work still used the tribimaximal-mixing paradigm as a starting point, we showed in this chapter how realistic models may arise in a different way. This was done by exploring and comparing coupling patterns leading to both, realistic and tribimaximal-mixing. We further highlighted the differences between inverse hierarchy and normal hierarchy. This discrimination turned out to have a relevant impact too. Overall, while we provided a simple, concrete four parameter implementation of a model for one tribimaximal-mixing case, we concluded that the realistic descriptions required six new couplings. As a general conclusion, all realistic models in this context shared one common feature, an almost vanishing smallest neutrino mass eigenvalue, making this framework falsifiable by the next generation of neutrinoless double beta decay experiments.

All in all, this work covered many aspects of the earlier mentioned transition phase of neutrino physics. We explored its roots in the paradigm of tribimaximal-mixing, proposed models altering this to adapt to present developments, and provided building blocks for future models addressing even more challenging tasks. Indeed, interesting times lie ahead and there is still much work to be done.

A. Appendix

A.1. Additional diagrams for the A_4 models

This section of the appendix presents the superdiagrams for the charged lepton mass terms of the A_4 models discussed in chapter 2 which are the same as in [60]. Figure A.1 displays the the diagrams generating the e, μ and τ masses in the AF models. Figure A.2 illustrates the modifications of these diagrams due to the presence of the operator $\{\phi_l \chi_\tau \chi_\tau^c\}$ in the renormalizable superpotential. These can be easily absorbed by a redefinition of the charged lepton masses and are thus irrelevant. The repeated latin indices are used to indicate the A_4 invariant contractions.

Figures A.1 and A.4 show the same diagrams for the AM models. Note that the τ diagram is absent, since it remains unchanged compared to the AF models. The effects of the operator $\xi\{\chi_\tau \chi_\tau^c\}$ can again be accounted for by a redefinition of the charged lepton masses. The operator $\{\phi_\nu \chi_\tau \chi_\tau^c\}$ however, enables non-diagonal entries in the charged lepton mass matrix.

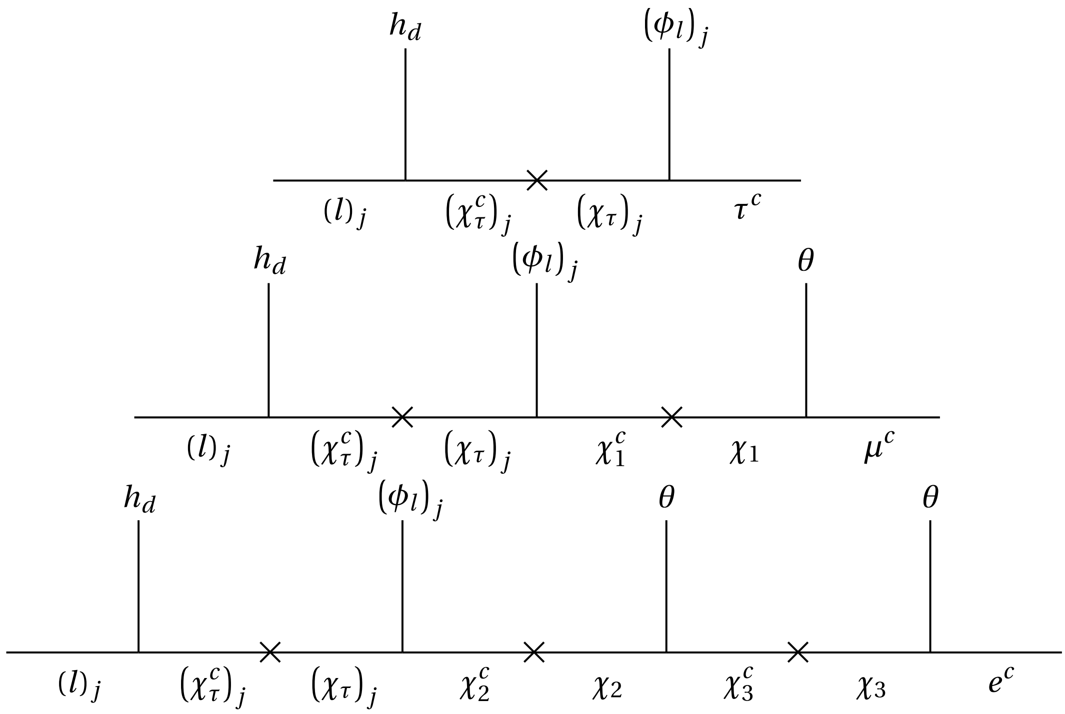


Figure A.1.: Superdiagrams to generate the effective e, μ and τ mass terms in the AF models.

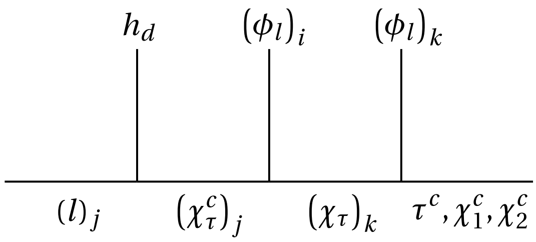


Figure A.2.: Generic modifications to the charged lepton mass terms enabled by the superpotential operator $\{\phi_l \chi_\tau \chi_\tau^c\}$.

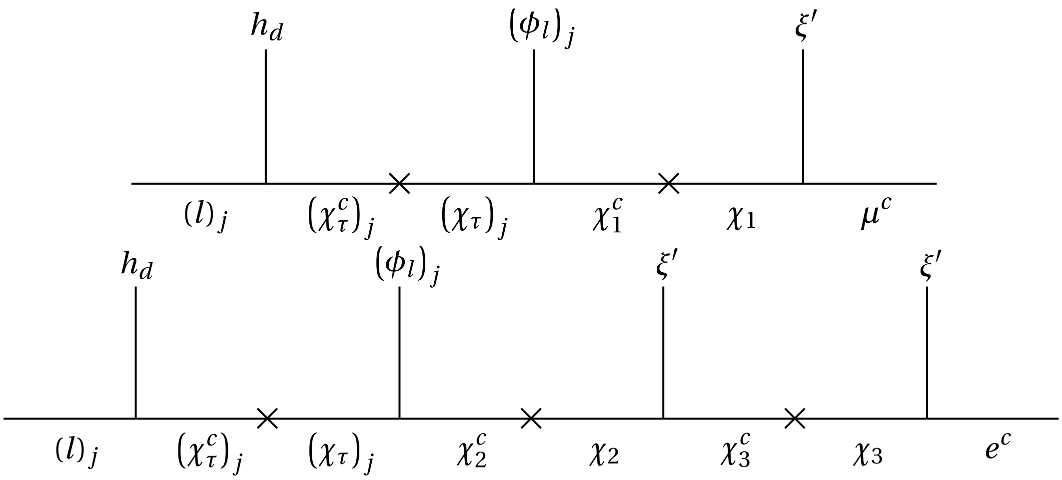


Figure A.3.: Superdiagrams to generate the effective μ and τ mass terms in the AM models. The e mass term remains unchanged compared to the AF models.

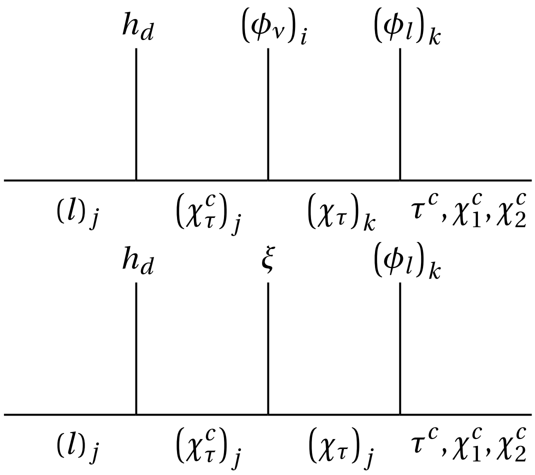


Figure A.4.: Modifications to the charged lepton mass terms enabled by the superpotential operators $\{\phi_\nu \chi_\tau \chi^c_\tau\}$ and $\xi\,\{\chi_\tau \chi^c_\tau\}$ respectively.

A.2. Tables of $\Delta(27)$ coupling sets

In this section of the appendix we present viable coupling sets for all classes of $\Delta(27)$ models discussed in chapter 3. All considered models were built with a set of six parameters, chosen from the the pool of the four coupling pairs (z_1, z_4), (z_2, z_5), (z_3, z_6) and (A, B) corresponding to the two different contractions of the invariants $H^\dagger H^\dagger \xi (L_i L_i \phi_i)_{00}$, $H^\dagger H^\dagger \xi' (L_i L_i \phi_i)_{02}$, $H^\dagger H^\dagger \xi'' (L_i L_i \phi_i)_{01}$, respective the combination $(H^\dagger H^\dagger (L\phi^\dagger)_{00} (L\phi^\dagger)_{00}, H^\dagger H^\dagger (L\phi^\dagger)_{01} (L\phi^\dagger)_{02})$.

The tables contain sample numerical values for the used parameters. Table A.1 corresponds to models with the full set of 6 z_i parameters, whereas tables A.2, A.3 and A.4 have some examples for each of the three classes with (A, B). Table A.5 has examples where (z_1, z_4) is larger by one order of magnitude. For all classes of models we were able to identify large parts of viable parameterspace leading to mixing angles and mass squared differences within the 3-σ bounds of [21]. All models cover both neutrino mass orderings, IH and NH.

In order to have an idea of the fine-tuning, we have relied on the procedure discussed in [156]: we use d_{FT} as a quantitative measure of the fine-tuning, a dimensionless quantity defined as the sum of the absolute values of ratios between all parameters and respective errors, where these errors are themselves defined as the deviation in that parameter that leads to an increase of χ^2 by 1 (while the other parameters remain at their fitted values). In [156], another similar quantity d_{Data} is introduced, defined simply the sum of the absolute values of ratios between the data and respective errors - for comparison, $d_{\text{Data}} = 39.4773$. As can be seen in the tables below, for the hits we display, d_{FT} is no more than one or two orders of magnitude higher than d_{Data}.

z_1	z_2	z_3	z_4	z_5	z_6	χ^2	d_{FT}
-0.00554161	-0.00340302	-0.00227104	-0.0141038	0.0175277	0.0170266	0.46992	640.595
0.00967825	-0.0118758	-0.00670678	-0.0160151	-0.00629062	-0.00653824	0.187884	246.89

Table A.1.: Sample hits for the 6 z_i model. First row is for IH, second row for NH.

A	B	z_1	z_4	z_2	z_5	χ^2	d_{FT}
-0.00353652	0.00107432	-0.0524306	-0.00585345	-0.00696862	0.0118005	1.161	516.293
0.0109313	-0.0215866	0.0172491	0.0154776	-0.00496799	-0.00163566	0.247987	1480.21
0.00907931	-0.0256511	-0.00227895	0.0142952	0.00284323	0.00881012	0.389833	1074.57

Table A.2.: Sample hits for the contractions ξ and ξ' in the A,B class of models.

A	B	z_1	z_4	z_3	z_6	χ^2	d_{FT}
-0.00941157	0.00922199	0.0190035	-0.0103075	-0.0277012	-0.045596	0.791304	3546.25
-0.0141045	0.0349938	-0.00117646	-0.00713608	-0.00700718	0.00389799	0.277284	2234.42

Table A.3.: Sample hits for the contractions ξ and ξ'' in the A,B class of models.

A	B	z_2	z_5	z_3	z_6	χ^2	d_{FT}
0.0099768	-0.0387091	-0.00543329	0.00613746	-0.0101139	0.0284193	0.607939	4035.83
0.00438667	-0.0049329	0.00337298	-0.00261386	0.0616697	-0.0127218	0.132725	1301.91
0.00476606	-0.00529681	0.0608295	-0.0129415	0.00381149	-0.00135987	0.915034	1269.28

Table A.4.: Sample hits for the contractions ξ' and ξ'' in the A,B class of models.

A	B	z_1	z_4	z_2	z_5	χ^2	d_{FT}
0.00245874	-0.00750093	0.0561966	0.0143339	0.0045167	-0.00187831	0.901573	725.276

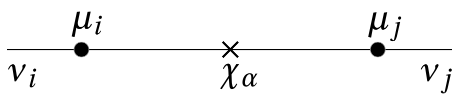
A	B	z_1	z_4	z_3	z_6	χ^2	d_{FT}
0.00153913	-0.00573201	0.0427374	0.0325508	0.00544977	-0.0013738	0.249697	640.799

Table A.5.: A sample hit for the contractions ξ and ξ' (top) and ξ and ξ'' (bottom) in the A,B class of models matching the natural hierarchies associated with the FN charges listed in Table 3.5.

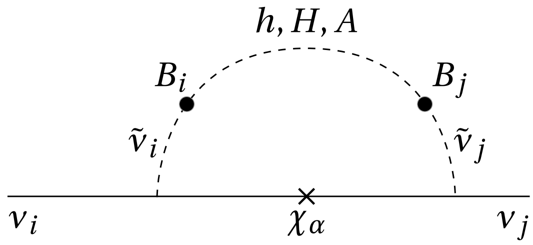
A.3. Neutrino mass diagrams for the RPV models

This section of the appendix lists the neutrino mass terms and associated topologies used in chapter 4. In the diagrams, a circle is used to indicate mixing while a cross represents a mass insertion.

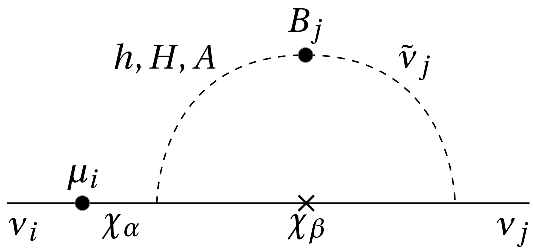
- $m^{\nu}_{ij} \supset \frac{\cos^2\beta}{\tilde{m}}\mu_i\mu_j$



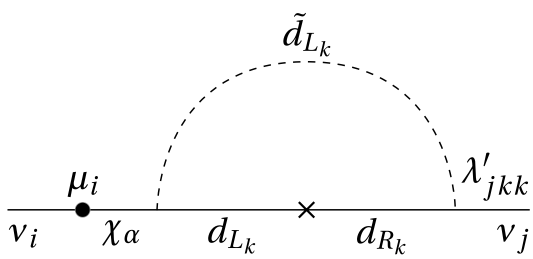
- $m^{\nu}_{ij} \supset \frac{g^2}{64\pi^2\cos^2\beta}\frac{B_iB_j}{\tilde{m}^3}$



- $m^{\nu}_{ij} \supset \frac{g^2}{64\pi^2\cos\beta}\frac{\mu_iB_j+\mu_jB_i}{\tilde{m}^2}$

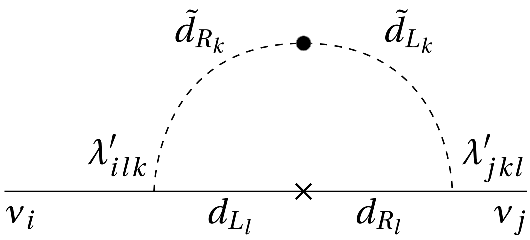


- $m^{\nu}_{ij} \supset \sum_k \frac{3}{16\pi^2}gm_{d_k}\frac{\mu_i\lambda'_{jkk}+\mu_j\lambda'_{ikk}}{\tilde{m}} + \sum_k \frac{1}{16\pi^2}gm_{e_k}\frac{\mu_i\lambda_{jkk}+\mu_j\lambda_{ikk}}{\tilde{m}}$



(The diagrams $\propto \mu\lambda$ are obtained by replacing the (s)quarks with the according (s)leptons.)

- $m^{\nu}_{ij} \supset \sum_{l,k} \frac{3}{8\pi^2} \lambda'_{ilk} \lambda'_{jkl} \frac{m_{d_l} m_{d_k}}{\tilde{m}_q^2} \mu \tan\beta + \sum_{l,k} \frac{1}{8\pi^2} \lambda_{ilk} \lambda_{jkl} \frac{m_{e_l} m_{e_k}}{\tilde{m}^2} \mu \tan\beta$



(The diagrams $\propto \lambda\lambda$ are obtained by replacing the (s)quarks with the according (s)leptons.)

Acronyms

Altarelli-Feruglio (AF)

Referring to the names of the involved physicists. 17, 19, 20, 24, 25, 28, 29, 31, 69

Altarelli-Meloni (AM)

Referring to the names of the involved physicists. 17, 19, 29, 69

beyond the Standard Model (BSM)

A generic term covering all particle physics not described by the standard model. 1, 10, 17, 40

CP violation (CPV)

Referring to the invariance under the combinations of charge conjugation symmetry C and and parity inversion P. 7, 39–41, 61, 80

electromagnetic (EM)

Referring to the electromagnetic force, observable below the scale of electroweak symmetry breaking. 2, 4, 6

electroweak (EW)

Referring to the unification of $\mathsf{SU}(2)_L$ and $\mathsf{U}(1)_Y$. 4, 6, 48, 55

electroweak symmetry breaking (EWSB)

The breaking of electroweak symmetry by the vacuum expectation value of a scalar field. 1, 5, 6, 13, 14, 40, 47

Froggatt-Nielsen (FN)

Referring to the names of the involved physicists. 19, 20, 28, 43, 44, 47, 48, 50, 52–55, 68

geometrical CP violation (GCPV)

SCPV with precisely calculable phases of the involved complex VEVs. 2, 39–43, 47, 48, 50, 52, 54, 55, 58, 67, 68

Glashow-Weinberg-Salam (GWS)

Referring to the names of the involved physicists. 3, 4

inverse hierarchy (IH)

Referring to the order $m_3 \ll m_1 < m_2$ of the three light neutrino mass eigenvalues. 2, 8, 53, 57, 62, 64, 65, 68, 72

leading order (LO)

The leading terms in the context of perturbation theory. 17–19, 28

Minimal supersymmetric Standard Model (MSSM)

A general supersymmetric model requiring the smallest number of fields. 11, 13, 14, 19, 58

next to leading order (NLO)

C.f. leading order. 18, 36

normal hierarchy (NH)

Referring to the order $m_1 < m_2 \ll m_3$ of the three light neutrino mass eigenvalues. 2, 8, 53, 57, 62, 64, 65, 68, 72

Pontecorvo–Maki–Nakagawa–Sakata (PMNS)

Referring to the names of the involved physicists. 7

Quantum Chromodynamics (QCD)

The theory of the strong interaction, described by the gauge group $\mathsf{SU}(3)_C$. 3, 4

R-parity violation (RPV)

Referring to the quantum number $R_P = (-1)^{3(B-L)+2S}$ defined by baryon number B, lepton number L and spin S. 2, 14, 15, 57–62, 64, 65, 68

spontaneous CP violation (SCPV)

The implementation of CPV via the spontaneous breakdown of a symmetry. 39–42, 67, 68, 79

Standard Model (SM)

A local quantum gauge field theory, describing most observed particle physics. 1, 2, 4–6, 9–13, 39, 40, 43, 44, 47, 49, 50

Supersymmetry (SUSY)

A symmetry relating bosons and fermions. The only possible extension of the Poincaré symmetry of the S-matrix. 1, 2, 10, 11, 13, 16, 19, 40, 65

tribimaximal-mixing (TBM)

A special three generation neutrino mixing pattern. 1, 2, 8, 17–20, 24–27, 29, 32, 37, 38, 51, 52, 57, 61–65, 67, 68

ultraviolet (UV)

In this work generally referring to high energy. 1, 17–20, 22, 25, 28, 32, 33, 36, 37, 39, 44, 49, 67

vacuum expectation value (VEV)

The vacuum expectation value of a (complex) scalar field. 4–6, 13, 19, 20, 22, 23, 25, 26, 28, 31, 33, 35, 36, 40–43, 46–50, 55, 79

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