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Abstract

We apply a heterogenous coefficient spatial autoregressive panel model to explore competition/cooperation by duopoly pairs of German fueling stations in setting prices for diesel and E5 fuel. We rely on a Markov Chain Monte Carlo (MCMC) estimation methodology applied with non-informative priors, which produces estimates equivalent to those from (quasi-) maximum likelihood. We explore station-level pricing behavior using pairs of proximately situated fueling stations with no nearby neighbors. Our sample data represents average daily diesel and e5 fuel prices, and refinery cost information covering more than 487 days.

The heterogeneous coefficients spatial autoregressive panel data model uses the large sample of daily time periods to produce spatial autoregressive model estimates for each fueling station. These estimates provide information regarding the price reaction function of *each station* to its duopoly rival station. This is in contrast to conventional estimates of price reaction functions that average over the entire cross-sectional sample of stations. We show how these estimates can be used to infer competition versus cooperation in price setting by *individual stations*.

KEYWORDS: Spatial panel data models, Markov Chain Monte Carlo, spatial autoregressive model, observation-level spatial interaction.

JEL: C11, C23, D43

1 Introduction

There is a great deal of literature on regional tax competition between local governments within a country (Allers and Elhorst, 2005, Elhorst and Fréret, 2009), gas station pricing (Pennerstorfer, 2009, Kihm et al. 2016), hospital pricing (Mobley, 2003), research activity competition between economics departments (Elhorst and Zigová, 2014), and so on. Empirical investigations often rely on spatial econometric methods developed to analyze spatially dependent cross-sectional and panel data.

The basic methodology involves comparing behavioral outcomes in one region (or observational unit such as gas station, hospital, etc.) to actions taking place in neighboring regions, or behavioral reactions taken by an individual or institution to actions taken by a more general type of neighbor, a peer group or a set of peer institutions. Spatial autoregressive processes/models represent a parsimonious way to specify a *global* relationship between a sample of (say N) regions/institutions/individuals and the average behavior of neighboring regions/insitutions/peers in the sample. By global, we mean that the sample of size N produces a scalar parameter indicating the average strength, sign, and statistical significance of reaction, where averaging takes place over the sample of size N. This allows an inference regarding the presence or absence of a positive/negative/insignificant reaction by the *typical* observational unit (region, institution, or individual) to actions of neighbors. For example, we might be able to conclude that on average over the sample of N regions we see statistical evidence of a negative and significant reaction function involving tax rates set by the typical region to average tax rates set by neighboring regions. This could be interpreted as evidence in favor of tax competition between regions in our sample.

A more ideal situation would allow inference regarding how each of the *individual* observational units $i = 1, \ldots, N$ react to actions taken by each unit i's neighboring units. Some regions/institutions/individuals might exhibit competitive reactions vis à vis their neighbors, while others react in a cooperative fashion, or do not react at all. This is an ideal situation because competition/cooperation reflect outcomes of institutional/individual decisions, which we might expect to vary across the sample of observational units.

As comprehensively summarized in recent surveys by Eckert (2013) and Noel (2016), the majority of studies that address competition in the retail gasoline market averages over the sample of stations, not allowing for the possibility that some stations interact with their neighbors in a collusive manner while others interact in a competitive manner. Much focus has instead been directed at questions of station-level price dispersion (Eckert and West, 2005; Lewis, 2008; Atkinson et al., 2009) and the extent to which stations pass through taxes and upstream cost shocks (Chouinard and Perloff, 2004; Bello and Contin-Pilart, 2012). With regard to the latter stream, several papers have addressed the influence of the oil price on the retail gas price, interpreting differential responses to oil price increases and decreases as evidence for market power or collusion among gas stations (Borenstein et al., 1997; Bachmeier and Griffin, 2003; Galeotti et al., 2003; Verlinda, 2008).

Aquaro et al. (2015) make the observation that space-time panel data samples covering longer time spans are becoming increasingly prevalent. If we let N denote the number of spatial units in the sample and T the number of time periods, panel data sets with sufficiently large T allow us to exploit sample data along the time dimension to produce spatial autoregressive parameter estimates for all N spatial units. In our setting, where we wish to examine competition/cooperation between price setting behavior of individual fueling stations, individual estimates relating to the reaction function for a duopoly pair of stations to their single neighboring stations' pricing actions hold a great deal of intuitive appeal.

Although use of georeferenced station-level data and spatial explanatory variables is common in this literature, studies that employ spatial econometric methods to the question of gasoline prices are relatively few in number. Exceptions include Pennerstofer's (2009) application of a spatial lag model to study price competition using cross-section data on gasoline stations in Austria and, more recently, Filippini and Heimsch's (2015) analysis of the impact of CO2 taxes on gas demand, which employs a spatial autoregressive model with autoregressive disturbances on panel data from Switzerland. Firgo et al. (2015) also applies a spatial-autoregressive model to examine the importance of centrality using a measure of network centrality based on the locations of gasoline stations in the road network of retail gasoline stations in Vienna, Austria. Their results show that prices of gasoline stations are more strongly correlated with prices of central competitors. Pinkse et al. (2002) apply a semiparametric spatial autoregressive estimator to data from U.S. wholesale gasoline markets and find that competition is highly localized. A common feature of these empirical studies is that estimates and inferences reflect averages over either a cross-section or static panel of stations.

Our heterogeneous coefficient spatial autoregressive model is in contrast to conventional static spatial panel models where a single (scalar) dependence parameter is estimated that relates the NT decision outcomes in the vector y and the $NT-$vector of spatial lags $(I_T \otimes W)y$, representing a linear combination of neighboring unit decisions. The scalar dependence parameter averages the relationship over all N fueling stations and T time periods.[1] It is reasonable to surmise that there are a large number of situations where the level of interaction between observational units differs greatly when considering spatial interaction patterns, and our fueling station price setting interaction represents one such situation.

Section 2 develops a model of (station-level) duopoly price reaction functions involving daily prices of each station and their neighbor to changes in refinery cost.

In section 3 we adapt the heterogeneous coefficient SAR model from Aquaro et al. (2015) to our case of duopoly station pairs, and discuss how it will be applied to our examination of German fueling station price setting behavior. Section 3 also discusses interpretation of the model estimates, a topic not covered in Aquaro et al. (2015). Section 4 describes a Markov Chain Monte Carlo (MCMC) procedure for estimation of the model parameters.

In section 5 we apply the model to a sample of plausible duopoly pairs of fueling stations located around Germany.[2] Station pairs were selected such that: (1) the stations were within 1000 meters of each other, (2) had no other neighboring stations within 4000 meters, and (3) consisted of different brands. This resulted in a sample of 188 duopoly pairs of stations (376 stations).

Section 6 contains concluding remarks and discusses areas for future research.

2 Station-level fuel pricing decisions

We focus on the duopoly case where we have two stations labeled i and j, and will construct a sample of duopoly stations to empirically implement the model described in what follows.

Station i's optimization problem is given by:

[1] Of course, the conventional static space-time panel model can allow for station-specific and time-specific fixed effects in an attempt to ameliorate the restrictiveness of the model. This, however, amounts to allowing for station-specific and time-specific differences in the model intercept.

[2] The fueling stations in our sample represent all German stations that operated continuously between June 1, 2014 and September 30, 2015.

$$
\begin{aligned}
\max_{(p_i)} \quad & \pi_i(p_i, p_j, q_i, c) = p_i q_i - 2\gamma_i c q_i \\
\text{s.t.} \quad & q_i = \alpha_i - \lambda_i p_i + \theta_i p_j \\
& p_i > 0; p_j > 0; q_i \geq 0; c > 0,
\end{aligned}
$$

where c is marginal (refinery) cost assumed constant across stations, but varying over time, and $2\gamma_i$ is a station-specific rate at which refinery costs translate into marginal operating cost that we denote $c_i = 2\gamma_i c$.[3] Demand of station i (at time t) responds to own-price p_i as well as a single neighboring station's price p_j.

The first-order condition for station i's optimization problem is:

$$
\frac{\partial \pi_i}{\partial p_i} = \alpha_i - 2\lambda_i p_i + \theta_i p_j + 2\lambda_i \gamma_i c = 0. \tag{1}
$$

Rearranging (11), and letting $\psi_i = \theta_i / 2\lambda_i$, and $\tilde{\alpha}_i = \alpha_i / (2\lambda_i)$, station i's reaction function becomes:

$$
\begin{aligned}
p_i &= \frac{\theta_i}{2\lambda_i} p_j + \gamma_i c + \tilde{\alpha}_i \\
\Rightarrow R_i(p_j) : p_i &= \psi_i p_j + \gamma_i c + \tilde{\alpha}_i + \varepsilon_i,
\end{aligned} \tag{2}
$$

where we use $R_i(p_j)$ to denote reaction of station i price to neighboring station j, and we add an error term ε_i. For the duopoly case we also have a reaction function for station j:

$$
\Rightarrow R_j(p_i) : p_j = \psi_j p_i + \gamma_j c + \tilde{\alpha}_j + \varepsilon_j, \tag{3}
$$

These reaction functions can be viewed as spatial autoregressive relationships based on a single nearest neighbor. Numerous other authors have arrived at similar expressions, e.g., Pinske et al. (2002) consider a more general case where each station $i = 1, \ldots, N$ reacts to a linear combination of neighboring station prices. This can be specified using

[3] The term $2\gamma_i$ simplifies results without loss of generality.

an $N \times N$ row-normalized spatial weight matrix W, as: $p_{it} = \psi_i w_i p_t + x_{it}\beta_i + \varepsilon_{it}$, where p_t is a vector of all prices, and the 1×2 vector $x_{it} = \begin{pmatrix} 1 & c \end{pmatrix}$, and $\beta_i' = \begin{pmatrix} \tilde{\alpha}_i & \gamma_i \end{pmatrix}$, and the $1 \times N$ vector w_i represents the ith row of the matrix W, with non-zero elements representing weights assigned to neighboring station prices, and zero elements for non-neighboring stations. The matrix-vector product $w_i p_t$ produces a linear combination of prices from neighboring stations with weights determined by the non-zero elements in row i of the matrix W. The scalar ψ_i measures the extent of (spatial) dependence of station i's price on that of the neighboring station(s). Spatial autoregressive models rely on row-normalization of the matrix W so row-sums are unity, resulting in the parameters $\psi_i, \psi_j < 1$.

The duopoly reaction functions represent a special case of the more general spatial autoregressive relationship, where the ith row of the matrix W contains a single non-zero element equal to one for station j in (2), and the jth row of the matrix W for station i in (3).

The reaction function for station i can be rearranged as:

$$p_i = \frac{\beta_i}{1-\psi_i\psi_j}x_i + \frac{\psi_i\beta_j}{1-\psi_i\psi_j}x_j + \frac{\varepsilon_i + \psi_i\varepsilon_j}{1-\psi_i\psi_j} \tag{4}$$

If we assume the (refinery) cost (c_{it}, c_{jt}) facing both stations at time t is the same, we can rearrange the expression in (4) for time t as:

$$p_{it} = \frac{1}{1-\psi_i\psi_j}(\gamma_i + \psi_i\gamma_j)c_t + \frac{1}{1-\psi_i\psi_j}(\tilde{\alpha}_i + \psi_i\tilde{\alpha}_j) + \frac{\varepsilon_i + \psi_i\varepsilon_j}{1-\psi_i\psi_j} \tag{5}$$

The coefficient estimate from the relationship between p_{it} and c_t that reflects the change in (expected) price as a result of changes in cost will consist of two components shown in (6) and (7), where E is the expectation operator.

$$\frac{\partial E(p_{it})}{\partial c_t} = \frac{\gamma_i}{1-\psi_i\psi_j} \tag{6}$$

$$+ \frac{\psi_i\gamma_j}{1-\psi_i\psi_j} \tag{7}$$

We note that if $\psi_i = 0$, so station i does not react to price changes by station j,[4] then we have the usual independent relationship where: $\partial p_{it}/\partial c_t = \gamma_i$. This represents a situation where station i's price reaction to cost changes does not take into consideration neighboring station j.

In cases where $0 < \psi_i, \psi_j < 1$, reflecting dependence between the two stations (positively sloped reaction functions), the expression in (6) indicates that positive reaction function slopes $0 < \psi_i, \psi_j < 1$ will have a magnifying impact on the first component of station i's price change in response to a cost change, since $1/(1 - \psi_i\psi_j) > 1$, when $0 < \psi_i, \psi_j < 1$. The second component in (7) indicates that the slope of station i's reaction function (ψ_i) as well as station j's price sensitivity to cost changes (γ_j) will also impact the observed changes in station i's price in response to cost changes.

In the duopoly case we have a symmetric expression for firm j:

$$\frac{\partial E(p_{jt})}{\partial c_t} = \frac{\gamma_j}{1 - \psi_i\psi_j} \tag{8}$$

$$+ \frac{\psi_j\gamma_i}{1 - \psi_i\psi_j} \tag{9}$$

Figure 1 shows two positively sloped reaction functions for stations i and j (R_i^1, R_j^1) in p_i, p_j space, intersecting at point A reflecting p_i^1, p_j^1. A change in cost c will shift both reaction functions upward, where $\gamma_i, \gamma_j > 0$, determines the magnitude of shift. A new intersection of the reaction functions R_i^2, R_j^2 based on the cost change occurs at point B, associated with p_i^2, p_j^2.

The two components of the station i coefficient from (6) and (7) are labeled $D.E.$ and $S.I.$ in the figure, since these can be viewed as representing a *direct effect* and *spillin effect*. The spillin effect represents the impact on station i's price change attributable to its duopoly dependence on the price set by station j. There is also a *spillout effect* labeled $S.O.$ in the figure that reflects the second expression associated with station j's coefficient in (9), which measures the impact of cost changes on station j price changes. The spillout effect represents the impact on station j's price change attributable to its duopoly dependence on the price set by station i.

Figure 2 shows a case where $\psi_i, \psi_j < 0$, so that each station reacts negatively to price

[4] The slope of the reaction function $\psi_i = 0$.

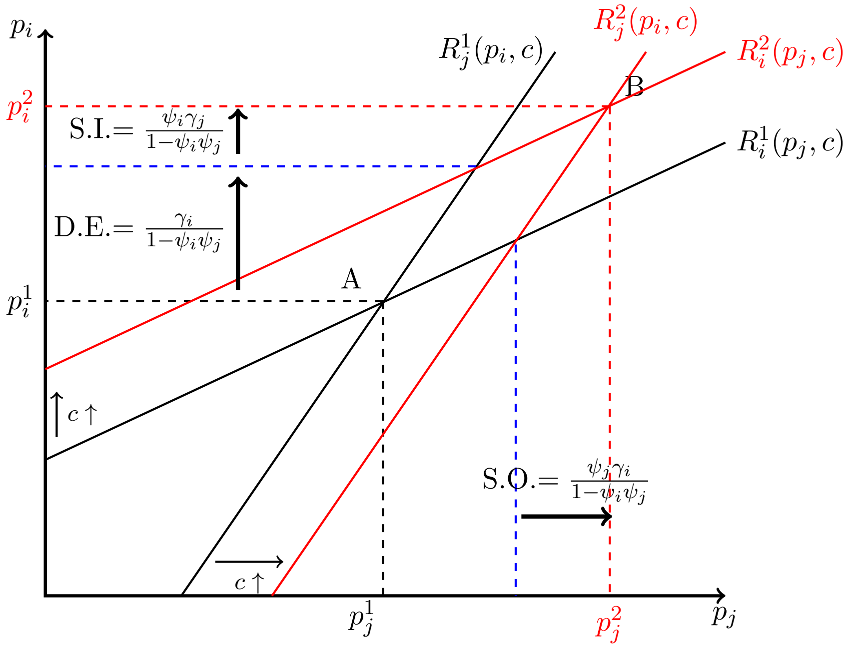

Figure 1: Case 1: Cooperation

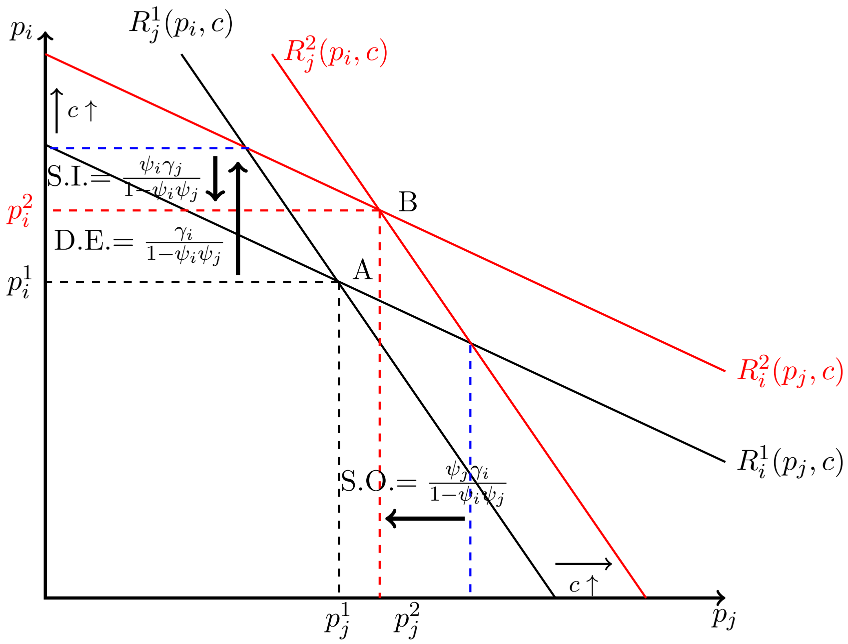

Figure 2: Case 2: Competition

changes by the rival. This situation results in a positive cost change moving us from equilibrium point A to B, so prices are higher ($p_i^2 > p_i^1, p_j^2 > p_j^1$). There is a positive direct effect when $\gamma_i, \gamma_j > 0$, as we would expect. The spillin and spillout effects are however negative, resulting in an ameliorating impact on the resulting price changes.

The empirical implications of the duopoly model are that positive (and significant) spillin and spillout effects for both stations imply cooperation between stations i and j. Similarly, negative (and significant) spillin and spillout effects for both stations suggest competition. It seems reasonable to require that *both stations* exhibit a significant reaction (significant spillin and spillout effects) for us to draw an inference of cooperation or competition by the two stations.

There are however other possible outcomes, for example where: $\psi_i = 0, \psi_j > 0, \gamma_i, \gamma_j > 0$, suggests a scenario where station i is a price leader, and station j a follower. In this case, station j reacts to station i's prices, while the converse is not the case. Also, this results in a positive spillout effect from the price leader i to station j the follower, while the spillin effect is zero. Of course, there is a symmetric case where station j is the price leader.

This scenario is shown in Figure 3, where station i is the leader, and $\psi_i = 0$ implies a horizontal reaction function. The magnitude of change in price for this case should be less than that for the case of cooperation.

Given estimates for the model parameters ψ_i, ψ_j and estimates of the spillin and spillout effects constructed from these parameters as well as estimates for γ_i, γ_j, we can enumerate the scenarios set forth in Table 1. Estimates for the parameters γ_i, γ_j for all stations in our empirical application were positive, or not significantly different from zero, with none of these negative and significant. Scenarios set forth in the table are based on positive or zero values for γ_i, γ_j. It was also the case that all but one of the estimates for ψ_i, ψ_j were positive or zero, allowing us to ignore negative ψ_i, ψ_j scenarios in Table 1 as well.

The numerous other scenarios represent cases where either the estimates of price dependence ψ_i, ψ_j are insignificant, or the spillin, spillout effects are not significant. Since our conclusions regarding competition, price leadership and cooperation rely on non-zero estimates for either ψ_i, ψ_j or one or the other spillin and spillout effects, we cannot draw conclusions in the face of these outcomes for the model estimates. We can conclude that a lack of price dependence between stations exists when ψ_i, ψ_j are not significantly different from zero, and this would also lead to small and likely insignificant spillin and spillout

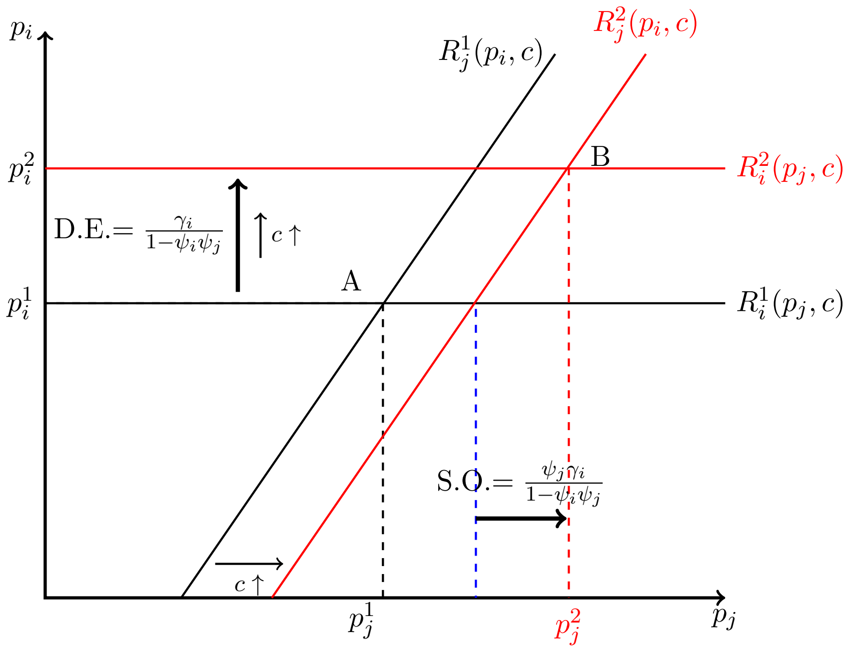

Figure 3: Case 3: Price Leader(firm i)

effects (see (7) and (9)).

3 The heterogenous spatial autoregressive model

The heterogeneous SAR model of Aquaro et al. (2015) (which we label HSAR hereafter) can be adapted to the special case of our sample of duopoly station pairs. Focusing on two stations that we label $i = 1$ and $j = 2$, we can write the relationships as in (10) and (11), where w_{12} represents the $1, 2$ element of a row-normalized spatial weight matrix with $w_{11} = w_{22} = 0$, and $w_{12} = w_{21} = 1$.

$$p_{1t} = \psi_1 w_{12} p_{2t} + c_t\gamma_1 + \alpha_1 + \varepsilon_{1t}, t = 1, 2, \ldots, T \tag{10}$$

$$p_{2t} = \psi_2 w_{21} p_{1t} + c_t\gamma_2 + \alpha_2 + \varepsilon_{2t}, t = 1, 2, \ldots, T \tag{11}$$

The disturbances $\varepsilon_{1t}, \varepsilon_{2t}$ are assumed distributed independently, and for our purposes we can assume independent normal distributions, $\varepsilon_{kt} \sim N(0, \sigma_k^2), k = 1, 2.$[5]

[5]Since we rely on the same explanatory variable c_t for both equations, there is no gain in efficiency from allowing for non-zero covariance between the two disturbance terms.

	Cooperation scenarios			
Cooperation:	$\psi_i > 0$	$\psi_j > 0$	spillin > 0	spillout > 0
i as Price leader	$\psi_i = 0$	$\psi_j > 0$	spillin $= 0$	spillout> 0
j as Price leader	$\psi_i > 0$	$\psi_j = 0$	spillin > 0	spillout$= 0$
	Competition scenario			
Competition:	$\psi_i < 0$	$\psi_j < 0$	spillin < 0	spillout < 0
	Other non-cooperation/competition scenarios (for $\psi_i, \psi_j \geq 0$)			
	$\psi_i = 0$	$\psi_j = 0$	spillin $= 0$	spillout$= 0$
	$\psi_i > 0$	$\psi_j > 0$	spillin $= 0$	spillout$= 0$
	$\psi_i > 0$	$\psi_j > 0$	spillin $= 0$	spillout> 0
	$\psi_i > 0$	$\psi_j > 0$	spillin > 0	spillout$= 0$
	$\psi_i > 0$	$\psi_j > 0$	spillin $= 0$	spillout> 0
	$\psi_i > 0$	$\psi_j = 0$	spillin $= 0$	spillout$= 0$
	$\psi_i = 0$	$\psi_j > 0$	spillin $= 0$	spillout$= 0$

Table 1: Possible estimation outcomes given positive or zero estimates for γ_i, γ_j

The refinery cost explanatory variable c_t is the same for both stations, and assumed exogenous, and we require that covariance matrices $E(c_t c_s), \forall\ t, s$ are time-invariant and finite as well as non-singular. The requirement of time-invariance arises because we are using the time dimension of the sample data to estimate parameters for each station. Our econometric specification relies on changes expressed as $\Delta p_{it} = p_{it} - p_{it-7}$, where we transform to changes from the same day last week to eliminate day-of-the-week pricing variation. The difference transformation is also applied to $\Delta c_t = c_t - c_{t-7}$, to meet the time-invariance requirement.

The HSAR model (in difference form) can be written in matrix notation shown in (12) by stacking stations, where we note that the parameters Ψ, γ do not change from those of the levels form.

$$
\begin{aligned}
\Delta p_t &= \Psi W \Delta p_t + \gamma \Delta c_t + \varepsilon_t && (12)\\
W &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} && (13)\\
\Delta p_t &= (\Delta p_{1t}, \Delta p_{2t})' \\
\Delta c_t &= \begin{pmatrix} \Delta c_t & 0 \\ 0 & \Delta c_t \end{pmatrix} \\
\gamma &= (\gamma_1, \gamma_2)' \\
\Psi &= \begin{pmatrix} \psi_1 & 0 \\ 0 & \psi_2 \end{pmatrix} \\
\varepsilon_t &= (\varepsilon_{1t}, \varepsilon_{2t})' \\
\varepsilon_{1t}, \varepsilon_{2t} &\sim N(0, \sigma_k^2), k = 1, 2
\end{aligned}
$$

The data generating process for the HSAR model can be written as:

$$
\Delta p_t = (I_2 - \Psi W)^{-1}(\gamma \Delta c_t + \varepsilon_t),\ t = 1, \ldots, T \qquad (14)
$$

We note that the partial derivatives of the reduced form HSAR model relationship from (14), shown in (15) reflect those considered earlier for our pair of duopoly stations. Since the parameters Ψ, γ are the same as those from the price and cost levels form of the relationship, $\partial \Delta p / \partial \Delta c = \partial p / \partial c$. These expressions take into account the fact that Ψ, γ do not change over time. The partial derivatives show how price changes respond to changes in cost, taking into account dependence on rival stations.

$$
\begin{aligned}
\partial p / \partial c &= \begin{pmatrix} \partial p_1 / \partial c_1 & \partial p_1 / \partial c_2 \\ \partial p_2 / \partial c_1 & \partial p_2 / \partial c_2 \end{pmatrix} \\
&= (I_N - \Psi W)^{-1} I_2 \gamma && (15)
\end{aligned}
$$

Expression (15) is an 2×2 matrix, since a change in station $i = 1$ price could (potentially) impact the price of station $j = 2$, with the strength of this impact determined by the levels of dependence between stations (Ψ).

The main diagonal of the matrix in (15) represents own-partial derivatives ($\partial p_k/\partial c_k$, k=1,2), while the off-diagonal elements are cross-partial derivatives ($\partial p_j/\partial c_i$) showing impacts of each station on the other station $j \neq i$.

The model provides estimates of $\gamma_k, \psi_k, \sigma_k^2$ for each station $k = 1, 2$. We can interpret the parameters γ_k as station-specific sensitivity to cost changes, and we note that application of the differences transformation eliminates station-level fixed effects (that would be captured by a station-specific intercept) as well as day-of-the-week pricing effects. Station-level fixed effects represent time-invariant differences across stations that might arise from: branding or location advantages, station-specific cost factors, traffic access patterns, etc. We note also that separate variance scalar estimates for each station accommodate heteroscedasticity.

The matrix inverse: $(I_2 - \Psi W)^{-1}$ can be written as an infinite series:

$$(I_2 - \Psi W)^{-1} = I_2 + \Psi W + (\Psi W)^2 + (\Psi W)^3 \ldots$$

For the case of duopoly, the matrix W takes a special form, as do the matrices $W^2, W^3, \ldots$. To illustrate this, consider two duopoly stations, where stations 1 and 2 are rivals, leading to:

$$W = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$
$$W^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The matrix powers $W^3, W^5, W^7, \ldots = W$ and the matrices $W^4, W^6, W^8 \ldots = W^2 = I_2$.

In the spatial econometrics literature, the matrix W identifies neighbors to each station, while the matrix W^2 indicates neighbors to each station's neighbors, and W^3 the neighbors to the neighbors of the neighbors, and so on for higher-order powers. The matrix $W^2 = I_2$ because station 1 is a neighbor to its neighbor (rival) station 2, while station 2 is a neighbor to its neighbor (rival) station 1. The matrix W^3 identifies neighbors to the neighbors of the neighbors to each station. Since there is only a single neighbor in our duopoly case, we have that $W^3 = W$.

The diagonal terms of the matrix inverse: $(\Psi W)^2, (\Psi W)^4, (\Psi W)^6, \ldots$ capture *feedback effects* arising from the rivalry, which decay in magnitude for higher-order terms, since $\psi_k < 1, k = 1, \ldots, 2$. Reactions taken by station 2 to actions of station 1 produce a reaction of station 1 to the reaction of station 2, and so on. Of course, the limit of an infinite series involving two stations with associated dependence parameters ψ_i, ψ_j is: $1/(1 - \psi_i\psi_j)$ as indicated in expressions (6) and (8).

The off-diagonal terms of the matrix inverse: $\Psi W, (\Psi W)^3, (\Psi W)^5, \ldots$ capture spillover or rivalry effects, specifically the reaction of station j to station i, and that of station i to station j. The limiting expressions for these off-diagonal terms in the case of two rival stations will be: $\psi_i/(1 - \psi_i\psi_j)$ and $\psi_j/(1 - \psi_i\psi_j)$ as indicated in expressions (7) and (9).

4 MCMC estimation of the model

We develop a Bayesian MCMC approach to estimating the model, where Bayesian estimation requires that prior distributions be assigned for the model parameters. However, we use normal priors for the parameter $\gamma_k, \psi_k, k = 1, 2$, with zero prior means and extremely large variances, to produce posterior estimates equivalent to those from maximum likelihood estimation.[6] We also rely on uninformative priors for the parameters $\sigma_k, k = 1, 2$, as prior information for these is unlikely to be available in applied modeling situations. As is traditional, we assume the priors for the parameters $\gamma_i, \psi_i, \sigma_i^2$ are independent.

Basically, MCMC estimation decomposes a complicated problem involving 2×1 parameter vectors $\gamma_k, \psi_k, \sigma_k^2$ into a sequence of simpler problems involving conditional distributions that are typically simple. Our MCMC estimation proceeds by sequentially sampling from the complete sequence of conditional distributions for: the 2 different parameters γ_k, the 2 different scalar parameters ψ_k and 2 different scalar noise variances σ_k^2. A single pass through of the sampler involves evaluating only 6 different conditional distributions. Each of these conditional distributions is relatively simple to sample from, and involves calculations based on matrices or vectors of small dimensions.

[6]Use of a normal prior for the parameters ψ_k might be viewed as problematical given a theoretical upper bound of unity for this parameter. However, during MCMC estimation we reject candidate values of ψ_k that exceed unity inside our Metropolis-Hastings sampling scheme.

The likelihood function for our special case of the more general model set forth in Aquaro et al. (2015) is shown in (16), where $\theta = (\psi_1, \psi_2, \gamma_1, \gamma_2, \sigma_1^2, \sigma_2^2)$, the model parameters.

$$\begin{aligned}
\ln \mathrm{L}(\theta) &= -T\ln(2\pi) - \frac{T}{2}(\ln\sigma_1^2 + \ln\sigma_2^2) + T\ln|I_2 - \Psi W| \\
&- \frac{1}{2}\sum_{k=1}^{2}(\Delta p_k - \psi_k \Delta p_k^* - \gamma_k \Delta c)'(\Delta p_k - \psi_k \Delta p_k^* - \gamma_k \Delta c)/\sigma_k^2 \qquad (16) \\
\Delta p_k^* &= \Delta p_j, j \neq k
\end{aligned}$$

Markov Chain Monte Carlo estimation consists of sampling draws from the complete sequence of conditional posterior distributions. These are derived from the log likelihood in (16), considering each parameter sequentially while assuming all others are known. In our case, where extremely large prior variances are used, the prior distributions do not play a material role in the posterior estimates or the conditional distributions. We are simply using MCMC as an alternative to maximizing the likelihood function.

Sampling begins with arbitrary values for the parameters θ, which are updated using sequential passes through the MCMC estimation procedure. These involve producing (in sequence) draws for the parameters γ_k, σ_k^2 and $\psi_k, k = 1, 2$, from the conditional distribution for these parameters.

A number m of such passes are carried out, with draws from some initial number of passes b discarded to allow the sampler to "burn-in". Posterior means, standard deviations, and other summary statistics for these distributions of the parameters are analyzed using the sample of $m - b$ retained draws. The number of passes usually is in the thousands to produce an adequate sample of size $m - b$ on which to base posterior inference.

Despite the apparent computational intensity of evaluating these 6 conditional distributions thousands of times, the conditional posteriors for the parameters γ_k, σ_k^2 take distributional forms that are known, and easy to sample from since they involve matrices/vectors of small dimension.

Specifics regarding the 6 conditional posterior distributions are presented. It should also be noted that several simplifications arise in the conditional distributions because of our use of zero prior means and very large prior variance settings. This essentially allows us to ignore the prior distributions when constructing conditional distributions.

We calculate the mean and variance-covariance for $\gamma_k, k = 1, 2$ using the conditional posterior based on arbitrary starting values, that we label $\psi_{k(0)}, \sigma^2_{k(0)}$ in (17), where we let p^*_k represent $p_j, j \neq k$.

$$
\begin{aligned}
p(\gamma_k, k = 1, 2 | \psi_{k(0)}, \sigma^2_{k(0)}) &\sim N(\gamma_{k^*}, \Sigma_{k^*}) \\
\gamma_{k^*} &= (\Delta c' \Delta c)^{-1} (\Delta p_k - \psi_{k(0)} \Delta p^*_k) \\
\Sigma_{k^*} &= \sigma^2_{k(0)} (\Delta c' \Delta c)^{-1}
\end{aligned}
\tag{17}
$$

An updated value that we label $\gamma_k^{(1)}$ can be obtained from a univariate normal distribution with mean γ^* and variance equal to Σ^*. The updated values $\gamma_k^{(1)}$ will be used in place of $\gamma_k^{(0)}$ when calculating the conditional posterior for updating $\sigma^2_{k(0)}$ based on the inverse Gamma distribution shown in (18).

$$
\begin{aligned}
p(\sigma^2_k, k = 1, 2 | \gamma_k^{(1)}, \psi_k^{(0)}) &\sim IG(a_1, b_1) \\
a_1 &= T/2 \\
b_1 &= (\Delta p_k - \psi_k^{(0)} p^*_k - \gamma_k^{(1)} \Delta c)' (p_k - \psi_k^{(0)} p^*_k - \gamma_k^{(1)} \Delta c)/2
\end{aligned}
\tag{18}
$$

We label the updated value produced by this draw $\sigma^2_{k(1)}$ which replaces the initial value $\sigma^2_{k(0)}$ in the conditional posterior expression for $\psi_k, k = 1, 2$.

While the conditional posteriors for the parameters γ_k, σ^2_k take known distributional forms that are easy to sample from, the conditional posterior for the parameters ψ_k does not have this property (LeSage and Pace, 2009). A Metropolis-Hastings (M-H) approach is used to sample these parameters based on the conditional posterior. For (M-H) sampling we require a *proposal distribution* from which we generate a candidate value for the parameter ψ_k, which we label $\tilde{\psi}_k$.

We use a normal distribution as the proposal distribution along with a *tuned random-walk procedure* suggested by Holloway et al. (2002) to produce the candidate values $\tilde{\psi}_k$. The procedure involves use of the current value ψ_k, a random deviate drawn from a standard normal distribution, and a tuning parameter z as shown in (19).

$$\tilde{\psi}_k = \psi_k + z \cdot N(0,1) \tag{19}$$

Expression (19) should make it clear why this type of proposal generating procedure is labeled a random-walk procedure. The goal of tuning the proposals coming from the normal proposal distribution is to ensure that the M-H sampling procedure *moves* over the entire conditional distribution. We would like the proposal to produce draws from the dense part of this distribution and avoid a situation where the sampler is stuck in a very low density part of the conditional distribution where the support is low.

To achieve this goal, the tuning parameter z in (19) is adjusted based on monitoring the acceptance rates from the M-H procedure during the MCMC drawing procedure. Specifically, if the acceptance rate falls below 40%, we adjust $z' = z/1.1$, which decreases the variance of the normal random deviates produced by the proposal distribution, so that new proposals are more closely related to the current value ψ_k. This should lead to an increased acceptance rate. If the acceptance rate rises above 60%, we adjust $z' = (1.1)z$, which increases the variance of the normal random deviates so that new proposals range more widely over the domain of the parameter ψ_k. This should result in a lower acceptance rate. The goal is to achieve a situation where the tuning parameter settles to a fixed value resulting in an acceptance rate between 40 and 60 percent. At this point, no further adjustments to the tuning parameter take place and we continue to sample from the normal proposal distribution using the resulting tuned value of z.

Aquaro et al. (2015) provide theoretical bounds on the parameters ψ_i in the more general case involving a sample of $i = 1, \ldots, N$ observations over T time periods. In our case where the matrix W takes the simple form shown in (13), the theoretical bounds on ψ_1, ψ_2 are -1 and 1. These bounds also ensure that $(I_2 - \Psi W)$ is invertible.

The MCMC algorithm was coded to reject candidate values that fell outside the (-1,1) range, and draw a new candidate value in these cases until a value within the (-1,1) interval arose. However, as a practical matter, estimation did not produce candidate values outside the (-1,1) interval, so there appears to be no issue regarding inference at the boundary of the parameter space.[7]

The candidate value $\tilde{\psi}_k$ as well as the current value ψ_k are evaluated in the expression

[7] The proportion of MCMC draws outside the (-1,1) interval can be interpreted an estimate of the posterior probability that ψ_k lies outside the interval.

for the (logged) conditional posterior in (20). Note that we use updated sampled values for γ_k and σ_k^2 when evaluating the conditional posterior in (20). Since $\Psi = \text{diag}(\psi_1, \psi_2)$ in (20), the conditional distribution for ψ_1 depends on ψ_2 and that for ψ_2 on the parameter ψ_1.

$$\begin{aligned} \ln(p(\psi_k)|\psi_j, j \neq k, \gamma_k, \sigma_k^2) &= -T\ln\pi\sigma_k^2 + T\ln|I_2 - \Psi W| - \ln(e'e/2\sigma_k^2) \\ e &= (\Delta p_k - \psi_k \Delta p_k^* - \gamma_k \Delta c) \end{aligned} \tag{20}$$

If $(\ln p(\tilde{\psi}_k) - \ln p(\psi_k)) > \exp(1)$, we accept the candidate value $\tilde{\psi}_k$ as an update for the current parameter ψ_k. If this condition is not true, we compare $\nu(\psi_k, \tilde{\psi}_k)$ calculated using:

$$\nu(\psi_k, \tilde{\psi}_k) = \min\left[1, \frac{p(\tilde{\psi}_k|\gamma_k, \sigma_k^2)}{p(\psi_k|\gamma_k, \sigma_k^2)}\right] \tag{21}$$

with a uniform random deviate (say r), and decide acceptance based on: $r < \nu(\psi^k, \tilde{\psi}_k)$ (accept), set $\psi_k^{(1)} = \tilde{\psi}_k$, otherwise (reject). If we reject the candidate value, we simply set $\psi_k^{(1)} = \psi_k$, that is, we stay with the current value of ψ_k.

Having completed one pass through of the MCMC sampler updating all parameters, $\theta^{(1)}$, we return to sample a second update of the parameters, sampling from the sequence of conditional distributions as outlined above. This produces a new set of draws, $\theta^{(2)}$, and the process is continued making m passes through the sampler to produce $m - b$ sets of draws for the parameters, where values from the first b (burn-in) passes are discarded to allow the sampler to achieve a steady-state and begin sampling from high density regions of the conditional posterior distributions of the parameters. The set of parameter draws $\theta^{(m-b)}$ can be used to calculate posterior means and standard deviations for the parameters. These draws reflect not conditional distributions of the parameters but rather the joint posterior distribution from which we draw inferences.

5 The model applied to German fueling stations

Since September 2013, stations in Germany are legally obligated to post every price change, the precise time stamp, the geographic coordinates of the station, the operating hours and brand on an online portal, the so-called Market Transparency Unit for Fuel (Haucap et al.

2015). To access these data, a script was used to continuously retrieve entries from the site and store these on a server (Frondel et al. 2015). From the raw data, a balanced panel of daily station-level prices was created for over 14,000 filling stations in Germany, operating over the period from June 1, 2014 to September 30, 2015, or $T = 487$ days. Prices are in nominal terms and include excise and value-added taxes. To measure the cost variable, we use the daily refined diesel and gas prices reported in Rotterdam, where one of the major pipelines into Germany originates.

A 2012 report by the International Energy Agency[8] describes Germany as having a deregulated oil market, with a large number of independents in the refining and retail sectors. The German government does not have an ownership stake in any of the companies operating in the oil sector. The IEA (2012) lists five major brands as having the highest market shares, with Aral (BP) and Shell commanding 22.5% and 21% of fuel sales respectively, followed by Jet (ConocoPhillips Germany) with 10.5%, and Total and Esso with 7.5% each. Nevertheless, the report also notes that numerous other refinery companies and independent and medium-sized oil companies are active on the fuel market, including Avia, Westfalen and Freie Tankstellen (bft). While the IEA characterizes the German market as being largely competitive, a report of Germany's Federal Cartel Office (Bundeskartellamt 2011) strikes a more critical tone. This report singles out the five major brands for their role in exercising market-dominating influence as oligopolists, which it argues leads to higher gas prices than would otherwise prevail under perfect competition.

5.1 The sample data

Starting with more than 14,000 stations, a sample was constructed to represent plausible duopoly pairs of stations. We selected from the sample of all German stations in continuous operation over the 487 days covered by our sample those station-pairs that were: (1) within 1000 meters of each other (2) have no other neighboring stations within 4000 meters, and (3) are different brands. The resulting sample size is 188 duopoly pairs of stations (376 stations) after eliminating 10 pairs of Autobahn stations. One reason for the small number of Autobahn station-pairs is that Autobahn stations are often of the same brand, located across the highway from each other. Since we required different brands for the station-pairs, most Autobahn station-pairs were eliminated from the sample. Both price (excluding taxes)

[8] Oil & Gas Security Emergency Response of IEA Countries, which we reference as IEA (2012).

and cost were log-transformed, then differenced from the same day of the previous week.

5.2 Model estimates

	Results based on 99 percent credible intervals				
	Cooperation scenarios				
Cooperation:	$\psi_i > 0$	$\psi_j > 0$	spillin > 0	spillout > 0	118
i or j as Price leader	$\psi_i = 0$ or $\psi_i > 0$	$\psi_j > 0$ or $\psi_j = 0$	spillin $= 0$ or spillin > 0	spillout> 0 or spillout $= 0$	32
	Competition scenario				
Competition:	$\psi_i < 0$	$\psi_j < 0$	spillin < 0	spillout < 0	0
	Other non-cooperation/competition scenarios				
	$\psi_i = 0$	$\psi_j = 0$	spillin $\leq, > 0$	spillout$\leq, > 0$	22
	$\psi_i > 0$	$\psi_j > 0$	spillin $= 0$	spillout$= 0$	4
	$\psi_i > 0$	$\psi_j > 0$	spillin $= 0$	spillout> 0	1
	$\psi_i > 0$	$\psi_j = 0$	spillin $= 0$	spillout$= 0$	5
	$\psi_i = 0$	$\psi_j > 0$	spillin $= 0$	spillout$= 0$	6
	Results based on 95 percent credible intervals				
	Cooperation scenarios				
Cooperation:	$\psi_i > 0$	$\psi_j > 0$	spillin > 0	spillout > 0	150
i or j as Price leader	$\psi_i = 0$ or $\psi_i > 0$	$\psi_j > 0$ or $\psi_j = 0$	spillin $= 0$ or spillin > 0	spillout> 0 or spillout $= 0$	23
	Competition scenario				
Competition:	$\psi_i < 0$	$\psi_j < 0$	spillin < 0	spillout < 0	0
	Other non-cooperation/competition scenarios				
	$\psi_i = 0$	$\psi_j = 0$	spillin $\leq, > 0$	spillout$\leq, > 0$	10
	$\psi_i > 0$	$\psi_j > 0$	spillin $= 0$	spillout$= 0$	4
	$\psi_i > 0$	$\psi_j > 0$	spillin $= 0$	spillout> 0	0
	$\psi_i > 0$	$\psi_j = 0$	spillin $= 0$	spillout$= 0$	3
	$\psi_i = 0$	$\psi_j > 0$	spillin $= 0$	spillout$= 0$	2

Table 2: Classification outcomes based on estimates for diesel fuel 188 station pairs

Estimates of the model parameters were based on a set of 4,000 retained draws, obtained from two separate MCMC runs of 2,500 draws, with the first 500 discarded to allow for burn-in of the sampler. Output from the two runs was used to determine that the MCMC sampling process converged, since the posterior means and standard deviations of the parameter draws from the two runs were nearly identical. The set of 4,000 retained draws were used to construct summary statistics for the parameters reported in this section, as well as empirical credible intervals that were used to draw inferences regarding the statistical significance of the parameter estimates.

Results are presented in Table 2 for the sample of 188 diesel fuel prices, using the classification system set forth in Table 1 of section 2. Estimates of ψ_i, ψ_j in conjunction with the spillin and spillout effects were used to produce classifications. Results vary depending on whether we use 95 or 99 percent credible intervals to test for significance of ψ_i, ψ_j and the spillin and spillout effects, so both are presented in the table.

Based on use of the 99 percent credible intervals, the diesel fuel classification results indicate that 118 of the 188 stations would be classified as engaging in cooperative diesel fuel price setting behavior, and another 32 of the 188 stations reflect a price leadership outcome. Together, this constitutes nearly 80 percent of the stations engaging in some type of non-competitive pricing behavior. This may not be surprising given the large cost associated with price wars that could arise in the case of duopoly stations. Using the looser 95 percent credible intervals, there are 150 stations classified as engaging in cooperative pricing and 23 in price leadership, for a total of 173 of 188 or around 92 percent of stations involved in non-competitive pricing behaviors. There are no cases of stations engaged in competitive pricing based on use of either the 99 or 95 percent intervals.

	Results based on 99 percent credible intervals				
	Cooperation scenarios				
Cooperation:	$\psi_i > 0$	$\psi_j > 0$	spillin > 0	spillout > 0	44
i or j as Price leader	$\psi_i = 0$	$\psi_j > 0$	spillin $= 0$	spillout> 0	139
	or $\psi_i > 0$	or $\psi_j = 0$	or spillin > 0	or spillout $= 0$	
	Competition scenario				
Competition:	$\psi_i < 0$	$\psi_j < 0$	spillin < 0	spillout < 0	0
	Other non-cooperation/competition scenarios				
	$\psi_i = 0$	$\psi_j = 0$	spillin $\geq, < 0$	spillout$\geq, < 0$	4
	$\psi_i < 0$	$\psi_j > 0$	spillin < 0	spillout> 0	1
	Results based on 95 percent credible intervals				
	Cooperation scenarios				
Cooperation:	$\psi_i > 0$	$\psi_j > 0$	spillin > 0	spillout > 0	79
i or j as Price leader	$\psi_i = 0$	$\psi_j > 0$	spillin $= 0$	spillout> 0	104
	or $\psi_i > 0$	or $\psi_j = 0$	or spillin > 0	or spillout $= 0$	
	Competition scenario				
Competition:	$\psi_i < 0$	$\psi_j < 0$	spillin < 0	spillout < 0	0
	Other non-cooperation/competition scenarios				
	$\psi_i = 0$	$\psi_j = 0$	spillin $\geq, < 0$	spillout$\geq, < 0$	1
	$\psi_i < 0$	$\psi_j > 0$	spillin < 0	spillout> 0	4

Table 3: Classification outcomes based on estimates for e5 fuel 188 station pairs

Classification of the 188 stations based on e5 fuel price estimates using the 99 and 95 percent credible intervals are shown in Table 3 in an identical format to those in Table 2. Based on the 99 percent intervals, we see 44 cases of cooperation and 139 price leadership results, for a total of 183 of 188 or over 97 percent of stations engaged in some type of non-competitive pricing behavior. Using the 95 percent intervals, we have 79 cooperative pricing stations and 104 price leadership cases for the same total of 183 of 188 stations. Using the looser 95 percent credible intervals produces a shift of some of the price leadership classifications into the cooperative behavior category, but does not change the total number of non-competitive cases. There are no cases of competitive pricing based on use of either the 99 or 95 percent intervals.

Comparing results from the diesel and e5 fuels shown in the two tables, one result that emerges is that non-competitive pricing behavior (defined as both cooperative pricing as well as price leadership) is slightly more prevalent in the case of e5 fuel than for diesel. Specifically, for the 99 and 95 percent intervals we have 80 and 92 percent of stations (respectively) engaging in non-competitive pricing of diesel fuel, compared to 97 percent of stations (respectively) in the case of e5 fuel.

Another distinction is that cooperative pricing behavior is more prevalent than price leadership for diesel fuel, while the opposite is true of e5 fuel, where we see more stations classified as involved in price leadership situations.

These differences might be explained by the longer term trends in demand for diesel versus e5 fuels. In this regard, the IEA (2012, page 10) notes that in Germany the demand for diesel increased by around 16% between 2001 and 2011 while demand for gasoline dropped by nearly 30% during the same period. Rotemberg and Saloner (1986) argue that during periods of rising demand and falling cost (the case of diesel fuel), the gains from cheating become larger, which would make successful cooperation more profitable for both stations. Cheating might be more likely to arise in a price leadership situation with frequently changing prices (in our case, most station prices change every day).

In the next sections, we analyze characteristics such as distance between station pairs, brands of the two stations, magnitude of the direct, spillin and spillout effects estimates and price markup magnitudes, based on the classification of stations into cooperation versus price leadership. Of course, our classification scheme produces different results based on use of the 99 or 95 percent credible intervals. Our analysis is carried out using both sets of classifications for stations in an attempt to see whether conclusions are robust in this regard.

5.3 Distances between stations based on classifications

Table 4: Summary of duopoly station pair distances

	Mean distance in miles	Median distance in miles	std deviation
Based on 99 percent credible intervals			
Diesel sample of 188 station pairs			
118 cooperating stations	0.2693	0.2486	0.1836
32 price leadership stations	0.2833	0.2518	0.1886
e5 sample of 188 station pairs			
44 cooperating stations	0.2543	0.1900	0.2005
139 price leadership stations	0.2867	0.2712	0.1810
Based on 95 percent credible intervals			
Diesel sample of 188 station pairs			
150 cooperating stations	0.2745	0.2674	0.1817
23 price leadership stations	0.2681	0.2283	0.1814
e5 sample of 188 station pairs			
79 cooperating stations	0.2643	0.2297	0.1891
104 price leadership stations	0.2865	0.2692	0.1823

The mean, median and standard deviation of distances between the pairs of cooperating and price leadership stations are shown in Table 4 for both diesel and e5 fuels. In the case of diesel fuel no clear pattern emerges that is consistent for the 99 and 95 percent intervals. Based on the 99 percent interval classifications, cooperating stations appear closer than stations engaged in price leadership, but this pattern is reversed when considering distances between stations based on the 95 percent interval classifications. It is also the case that given the standard deviations reported in the table, the differences in mean and median distances are not statistically significant.

A pattern emerges for the case of e5 fuels, where cooperating stations appear to be on average closer than stations engaged in price leadership, based on classifications constructed from both 99 and 95 percent intervals. However, given the standard deviations reported in the table, these differences are not statistically significant.

5.4 Effects estimates for stations based on classifications

Table 5 reports direct as well as spillin and spillout effects estimates averaged over the cooperative and price leadership stations. From the table, we see larger direct effects for cooperating stations in the case of both diesel and e5 fuels for classifications based on both 99 and 95 percent intervals. Given the standard deviations reported in the table, these are statistically significant differences.

The direct effects measure the elasticity response of price changes to changes in refinery costs because of the log transformation applied to both price and cost. The results reported suggest that stations engaged in cooperative pricing behavior are more sensitive to cost changes than those engaged in price leadership schemes. This hold true for both diesel and e5 fuels, and for classifications of stations based on both 99 and 95 percent intervals.

A point to note is that the magnitude of direct effects was not used when classifying stations into cooperative versus price leadership categories.

There are also larger spillin and spillout effects for cooperating stations versus price leadership stations in the case of both diesel and e5 fuels, and for both 99 and 95 percent intervals. However, unlike the case of direct effects, the classification criterion for cooperative pricing behavior involves positive and statistically significant spillin and spillout effects as well as positive and significant ψ_i, ψ_j. This almost guarantees this type of outcome. Since zero spillin or spillout effects result in a price leadership classification (in the presence of the correct set of positive and zero dependence parameters ψ_i, ψ_j), we would expect to see larger spillin/spillout effects for the case of stations engaged in cooperative pricing behavior. These differences are significant given the reported standard deviations.

5.5 Brand configurations for stations based on classifications

Table 6 shows brand configurations for the cooperative versus price leadership pricing station pairs. The table summarizes brands into three categories, one representing a station pair involving two name brands (Aral, Esso, Jet, Shell or Total), another where both stations are independents (not the five large brands), and the third situation where one station is an independent and the other a name brand.[9]

Comparing column elements from the table, we see that cooperative pricing versus price leadership using 99 percent level appears more likely between two name brands than price leadership behavior. This result holds true for both diesel and e5, and based on classifications using both 99 and 95 percent intervals. Of course, name brand stations may exhibit more sophisticated pricing behavior than independents, and may benefit from coordination efforts at the company level.

Two independents are less likely to cooperate than result in price leadership, for both diesel and e5. However, this result is the same for e5 fuel across both the 99 and 95 percent interval classifications, but not for diesel fuel, where independents are more likely to cooperate than engage in price leadership based on the 95 percent interval classifications.

One name brand and one independent appear more likely to cooperate than engage in price leadership, another result that is not robust over both the 99 and 95 percent classifications.

Comparisons across row elements of the table are problematical. For example, row elements in the table suggest that cooperation and price leadership involving two name brands is the rarest outcome, occurring in between 6 and 22 percent of the cases. However, this is due to criteria used to construct the sample of duopoly stations. As noted, close physical proximity, relative locational isolation, as well different brand stations were requirements for a station pair to enter our duopoly sample. Since the larger name brand stations tend not to operate in relative locational isolation, these represent a smaller proportion of our sample of stations.

[9] As noted in the introduction to this section, numerous other refinery companies and independent and medium-sized oil companies are active on the fuel market, including Avia, Westfalen and Freie Tankstellen (bft). So, use of the term "independent" is used somewhat loosely.

5.6 Price markups for stations based on classifications

The model predicts larger price changes in response to changes in refinery cost for stations engaged in cooperative pricing than those involved in price leadership scenarios (compare Figure 1 and Figure 3). These changes are estimated by the total derivative: $\partial \Delta price / \partial \Delta cost$, which consists of the sum of the direct plus spillin plus spillout effects.

The model also indicates that in the absence of a reaction to a rival's price setting, the price change response to cost changes would be equal to the coefficients γ_i, γ_j for each duopoly station pair. Therefore, the price markup due to cooperation or price leadership behavior relative to competition is reflected by the difference between these estimated parameters.

Table 7 shows estimates of the price markups due to non-competitive pricing behavior relative to the hypothetical estimate of price changes in a competitive situation. Recall, the price versus cost relationship was estimated using a log transformation applied to both. The competitive situation estimate reflects an average (median, standard deviation) of the 188 pairs of estimates for the parameters γ_i, γ_j.

The first three rows in each segment of the table show total derivatives for firms classified as engaging in cooperative and price leadership behavior as well as the competitive estimate. The last two rows show the difference (markup) between the two non-competitive scenarios and the third row representing our estimate of a competitive market. The standard deviations reported in the table indicate that differences in total price change responses to cost changes for the cooperative versus price leadership behavior are significant.

Results are consistent across classifications of stations based on both the 99 and 95 percent intervals. The table indicates that both cooperative and price leadership diesel price markups relative to competition are greater than those for e5 fuel. For example using classifications based on both 99 and 95 percent intervals, the diesel price cooperative markup over competition is around 0.14, versus 0.11 for e5 prices.

It seems plausible that demand for diesel is less elastic than that for e5 fuel, due to the use of diesel by trucking fleets involved in commercial transportation. Since a component of e5 fuel demand is associated with driving for recreational purposes, this should result in a larger elasticity relative to diesel fuel. Larger price differences due to non-competitive behavior would be expected for the more inelastic diesel fuel, consistent with our results.

The results also consistently indicates higher price markups for both fuels arising from cooperative pricing behavior relative to price leadership, which is a prediction of our model. For example, cooperative pricing behavior results in a markup of around 0.14 for diesel at both the 99 and 95 percent interval classifications, whereas price leadership results in a markup of around 0.08 for diesel for both of these interval classifications.

6 Conclusion

We apply a heterogeneous coefficient spatial autoregressive (HSAR) panel model that is capable of producing *station-level estimates* of duopoly gas station price rivalry. Our approach allows for inferences regarding how each *individual* station reacts to price actions taken by its rival. This contrasts with conventional homogeneous coefficient panel models that would average over the sample of all duopoly station pairs to produce estimates that reflect the typical station's behavior. It seems reasonable to expect pricing behavior to vary across the sample of station pairs, with some stations reacting in a cooperative fashion, while others engage in price leadership-follower arrangements, or in competitive pricing behavior.

We derive a duopoly model of station pair price reaction functions and show that this takes the form of a special case of the more general HSAR specification. A Markov Chain Monte Carlo approach to estimation is set forth for our duopoly station pairs. This approach to estimation based on prior distributions that have zero prior means with very large prior variances produces results equivalent to those from quasi maximum likelihood estimation set forth in Aquaro et al. (2015).

Partial derivatives that allow classification of station pairs into categories of price cooperative behavior, price leader-follower behavior as well as competition are derived.

The empirical classification results show that non-competitive pricing behavior (defined as both cooperative pricing as well as price leadership) is prevalent in the case of e5 and diesel fuel. Using 95 percent credible intervals for our estimates as a basis for classification, we find that 92 percent of stations engaged in non-competitive pricing of diesel fuel and 97 percent of stations in the case of e5 fuel. Other findings were that cooperative pricing behavior is more prevalent than price leadership for diesel fuel, while the opposite is true of e5 fuel, where we see more stations classified as involved in price leadership situations.

The study also analyzes characteristics such as distance between station pairs, brands of the two stations, magnitude of the direct, spillin and spillout effects estimates and price markup magnitudes, based on the classification of stations into cooperation versus price leadership categories.

One area for future exploration would be computational improvements for the HSAR model. Although our MCMC approach applied to two stations was relatively fast, applying the method to the sample of 188 station pairs was time consuming. Extending the estimation method to more general cases involving a larger sample such as the 14,000 stations operating in Germany would pose computational challenges. Another point is that observation-level estimates for samples that involve a large number of observations pose some interesting challenges for presentation of estimation results to readers. How to summarize and present estimates for multiple parameter estimates for each observation when the number of observations is large may require some interesting data visualization tools.

There has been a stream of gas station pricing literature that addresses the question of asymmetric responses of retail gasoline prices to increases versus decreases in cost, (e.g., Verlinda (2008)). The HSAR model may have potential use in examining this type of issue, since station-level responses of price to cost increases versus decreases could be estimated.

References

Allers, M.A. and J.P. Elhorst (2005) Tax mimicking and yardstick competition among local governments in the Netherlands. *International Tax and Public Finance*, 12(4), 493-513.

Aquaro, M., N. Bailey and M.H. Pesaran (2015) Quasi Maximum Likelihood Estimation of Spatial Models with Heterogeneous Coefficients. USC-INET Research Paper, (15-17).

Atkinson, B., A. Eckert and D.S. West (2009) Retail gasoline price cycles: Evidence from Guelph, Ontario using bi-hourly, station-specific retail price data. *Energy Journal*, 30, 85-109.

Backmeier, L.J. and J.M. Griffin (2003) New evidence on asymmetric gasoline price responses. *Review of Economics and Statistics*, 85(3), 772-776.

Barron, J.M., Taylor, B.A. and Umbeck, J.R. (2004) Number of sellers, average prices and price dispersion. *International Journal of Industrial Organization* 22, 10411066.

Bello, A and Ignacio Contin-Pilart (2012) Taxes, cost and demand shifters as determinants in the regional gasoline price formation process: Evidence from Spain. *Energy Policy* 48, 439448.

Bundeskartellamt (2011) Sektoruntersuchung Kraftstoffe. Berlin, www.bundeskartellamt.de/SharedD
%20Abschlussbericht.html/

Chouinard, H. and J.M. Perloff (2004) Incidence of federal and state gasoline taxes. *Economics Letters*, 83, 55-60.

Borenstein, S., A.C. Cameron and R. Gilbert (1997) Do gasoline prices respond asymmetrically to crude oil price changes? *Quarterly Journal of Economics*, 112(1), 305-339.

Eckert, A. (2013) Empirical Studies of gasoline retailing: A guide to the literature *Journal of Economic Surveys*, 27(1), 140-166.

Eckert, A. and D.S. West (2005) Price uniformity and competition in a retail gasoline market. *Journal of Economic Behavior and Organization*, 38, 25-46.

Elhorst, J.P. and S. Fréret (2009) Evidence of political yardstick competition in France using a two-regime spatial Durbin model with fixed effects. *Journal of Regional Science*, 49(5), 931-951.

Elhorst, J.P. and K. Zigová (2014) Competition in research activity among economic departments: Evidence by negative spatial autocorrelation. *Geographical Analysis*, 46, 104-125.

Firgo, M., D. Pennerstorfer and C.R. Weiss (2015) Centrality and pricing in spatially differentiated markets: The case of gasoline, *International Journal of Industrial Organization*, 40, 81-90.

Filippini, M. and F. Heimisch (2015) The regional impact of a CO2 tax on gasoline demand: A spatial econometric approach. Center of Economic Research at ETH Zurich, Working Paper 15/210.

Frondel, M., C. Vance and A. Kihm (2015) Time lags in the pass-through of crude oil prices: Big data evidence from the German gasoline market. *Applied Economics Letters*, Nov 11, 1-5.

Galeotti, M., Lanza A. and Manera M. (2003) Rockets and feathers revisited: An international comparison on European gasoline markets. *Energy Economics*, 25(2), 175-190.

Haucap, J., U. Heimeshoff and M. Siekmann (2015) Price dispersion and station heterogeneity on German retail gasoline markets (No. 171). DICE Discussion Paper.

Holloway, G., B. Shankar, S. Rahman (2002) Bayesian spatial probit estimation: A primer with an application of HYV rice adoption. *Agricultural Economics*, 27, 383-402.

IEA (2012) Oil & Gas Security Emergency Response of IEA Countries (2012) International Energy Agency, www.iea.org/publications/freepublications/publication/iea-response-system-for-oil-supply-emergencies-2012.html

Kihm, A, N. Ritter and C. Vance (2016) Is the German retail gas market competitive? A spatial-temporal analysis using quantile regression. *Land Economics*, forthcoming

LeSage, J.P. and R.K. Pace. (2009) *Introduction to spatial econometrics*. Boca Raton: CRC Press.

Lewis, M. (2008) Price dispersion and competition with differentiated sellers, *Journal of Industrial Economics* 56, 654678.

Mobley, L.R. (2003) Estimating hospital market pricing: an equilibrium approach using spatial econometrics. *Regional Science and Urban Economics*, 33(4), 489-516.

Noel, M.D. (2016) Retail gasoline markets *Handbook on economics of retailing and distribution*, Edward Elgar Publishing, 392-412.

Pennerstorfer, D. (2009) Spatial price competition in retail gasoline markets: Evidence from Austria. *The Annals of Regional Science*, 43(1), 133-158.

Pinske, J., M. Slade and C. Brett (2002) Spatial price competition: A semiparametric approach, *Econometrica*, 70(3), 1111-1153.

Rotemberg, J.J. and G. Saloner (1986) A Supergame-Theoretic Model of Price Wars during Booms *The American Economic Review*, 76(3), 390-407

Verlinda, J.A. (2008) Do rockets rise faster and feathers fall slower in an atmosphere of local market power? Evidence from the retail gasoline market. *Journal of Industrial Economics*, 56(3), 581-612.

Table 5: Summary effects estimates for cooperative and price leadership station pairs

Based on 99 percent credible intervals			
Diesel sample of 188 station pairs			
Direct effects estimates			
	mean	median	std. deviation
118 Cooperating stations	0.1122	0.1146	0.0177
32 Price leadership stations	0.0935	0.0965	0.0194
Spillin/spillout effects estimates			
Spillin	mean	median	std. deviation
118 Cooperating stations	0.0556	0.0536	0.0208
32 Price leadership stations	0.0326	0.0304	0.0175
Spillout	mean	median	std. deviation
118 Cooperating stations	0.0630	0.0613	0.0209
32 Price leadership stations	0.0436	0.0434	0.0181
e5 sample of 188 station pairs			
Direct effects estimates			
	mean	median	std. deviation
44 Cooperating stations	0.1244	0.1266	0.0171
139 Price leadership stations	0.1072	0.1104	0.0206
Spillin/spillout effects estimates			
Spillin	mean	median	std. deviation
44 Cooperating stations	0.0259	0.0228	0.0095
139 Price leadership stations	0.0074	0.0077	0.0100
Spillout	mean	median	std. deviation
44 Cooperating stations	0.0704	0.0688	0.0166
139 Price leadership stations	0.0667	0.0675	0.0161
Based on 95 percent credible intervals			
Diesel sample of 188 station pairs			
Direct effects estimates			
	mean	median	std. deviation
150 Cooperating stations	0.1091	0.1120	0.0186
23 Price leadership stations	0.0895	0.0893	0.0197
Spillin/spillout effects estimates			
Spillin	mean	median	std. deviation
150 Cooperating stations	0.0515	0.0492	0.0213
23 Price leadership stations	0.0305	0.0248	0.0190
Spillout	mean	median	std. deviation
150 Cooperating stations	0.0586	0.0555	0.0214
23 Price leadership stations	0.0342	0.0330	0.0185
e5 sample of 188 station pairs			
Direct effects estimates			
	mean	median	std. deviation
79 Cooperating stations	0.1223	0.1222	0.0168
104 Price leadership stations	0.1034	0.1069	0.0213
Spillin/spillout effects estimates			
Spillin	mean	median	std. deviation
79 Cooperating stations	0.0228	0.0213	0.0089
104 Price leadership stations	0.0044	0.0040	0.0075
Spillout	mean	median	std. deviation
79 Cooperating stations	0.0692	0.0694	0.0160
104 Price leadership stations	0.0656	0.0665	0.0174

Table 6: Summary of brand pairings for cooperative and price leadership stations

Based on 99 percent credible intervals			
Diesel sample of 188 station pairs			
	Two name brands	Two independents	One of each
118 Cooperating stations	0.1610†	0.4153	0.4237
32 Price leadership stations	0.0625	0.5313	0.4063
e5 fuel sample of 188 station pairs			
	Two name brands	Two independents	One of each
44 Cooperating stations	0.2273	0.3636	0.5000
139 Price leadership stations	0.1151	0.4748	0.4101
Based on 95 percent credible intervals			
Diesel sample of 188 station pairs			
	Two name brands	Two independents	One of each
150 Cooperating stations	0.1400	0.4400	0.4200
23 Price leadership stations	0.1304	0.4348	0.4348
e5 fuel sample of 188 station pairs			
	Two name brands	Two independents	One of each
79 Cooperating stations	0.1772	0.4051	0.4177
104 Price leadership stations	0.0673	0.4519	0.4808

† indicates the proportion of cooperative or price leadership stations by fuel type

Table 7: Duopoly station price markups relative to competition

Based on 99 percent credible intervals

Diesel sample of 188 station pairs	Mean	Median	std deviation
$\partial\Delta price/\partial\Delta cost$ 118 cooperating stations	0.2308	0.2279	0.0359
$\partial\Delta price/\partial\Delta cost$ 32 price leadership stations	0.1697	0.1752	0.0299
$\partial\Delta price/\partial\Delta cost$ 188 competitive stations	0.0835	0.0854	0.0022
cooperative pricing markup	0.1473	0.1425	
price leadership markup	0.0862	0.0898	

e5 sample of 188 station pairs	Mean	Median	std deviation
$\partial\Delta price/\partial\Delta cost$ 118 cooperating stations	0.2208	0.2251	0.0274
$\partial\Delta price/\partial\Delta cost$ 32 price leadership stations	0.1813	0.1855	0.0337
$\partial\Delta price/\partial\Delta cost$ 188 competitive stations	0.1059	0.1075	0.0013
cooperative pricing markup	0.1149	0.1176	
price leadership markup	0.0754	0.0780	

Based on 95 percent credible intervals

Diesel sample of 188 station pairs	Mean	Median	std deviation
$\partial\Delta price/\partial\Delta cost$ 118 cooperating stations	0.2192	0.2175	0.0404
$\partial\Delta price/\partial\Delta cost$ 32 price leadership stations	0.1542	0.1487	0.0288
$\partial\Delta price/\partial\Delta cost$ 188 competitive stations	0.0835	0.0854	0.0022
cooperative pricing markup	0.1357	0.1321	
price leadership markup	0.0707	0.0633	

e5 sample of 188 station pairs	Mean	Median	std deviation
$\partial\Delta price/\partial\Delta cost$ 118 cooperating stations	0.2142	0.2180	0.0270
$\partial\Delta price/\partial\Delta cost$ 32 price leadership stations	0.1734	0.1756	0.0322
$\partial\Delta price/\partial\Delta cost$ 188 competitive stations	0.1059	0.1075	0.0013
cooperative pricing markup	0.1083	0.1105	
price leadership markup	0.0675	0.0681	