Distributions of Age at Death from Roman Epitaph Inscriptions: An Application of Data Mining

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Abstract:
Thousands of age at death inscriptions from Roman epitaphs are statistically analyzed. The Gompertz distribution is used to estimate survivor functions. The smoothed distributions are classified according to the estimation results. Similarities and differences can be detected more easily. Parameters such as mean, mode, skewness, and kurtosis are calculated. Cluster analysis provides three typical distributions. The analysis of the force of mortality function of the three clusters yields that the epigraphic sample is not representative of the mortality in the Roman Empire. However, the data is not worthless. It can be used to show and to explain the differences in the burial and commemorative processes. Finally, the bias due to a growing population is discussed. A simple formula is proposed for estimating the growth rate. The paper also discusses some special parameter constellations of the Gompertz distribution, since in this special application it cannot be approximated by the Gumbel distribution (as is often done in life table analysis).

Keywords: Gompertz distribution, data analysis, cluster analysis, mortality, life table, Roman demography

1. Introduction

Thousands of inscriptions from epitaphs of the Roman Empire that record the age at death of the individual have been collected (e.g., Beloch 1886, Harkness 1896, Macdonnell 1913, Russell 1958, Szilágyi 1961, 1962, 1963). The life expectancy at age x is calculated as the ratio of the sum of total years lived to the total number of individuals of that age. It has long been thought that this number reflects the life expectancy of Roman men and women. However, since Durand's (1959) and Hopkins's (1966) work it has been known that these epigraphic samples are not representative of the mortality in the Roman Empire, even if one assumes a stationary population. Infant mortality is always underestimated, and old age mortality is generally underestimated. Epitaphs of elderly deceased individuals are sometimes evidence of a remarkable longevity. Even in the middle age groups of, say, 10 to 60 years, the tombstone inscriptions do not, in general, give an accurate record of mortality (Parkin 1992, p 7). These findings are clearly shown in, for example, Figure 1, where force of mortality functions of 6,008 Roman males and 3,972 Roman females have been calculated. The force of mortality function shows, approximately, the number of individuals dying at age x as a percentage of those surviving to that age. These functions are compared with the force of mortality function of the Suessmilch life table, which represents the mortality of the eighteenth century (Suessmilch 1775). Johann Peter Suessmilch (1707-1767), one of the founding fathers of demography in Germany, published a life table with a life expectation at birth of about 29 years.

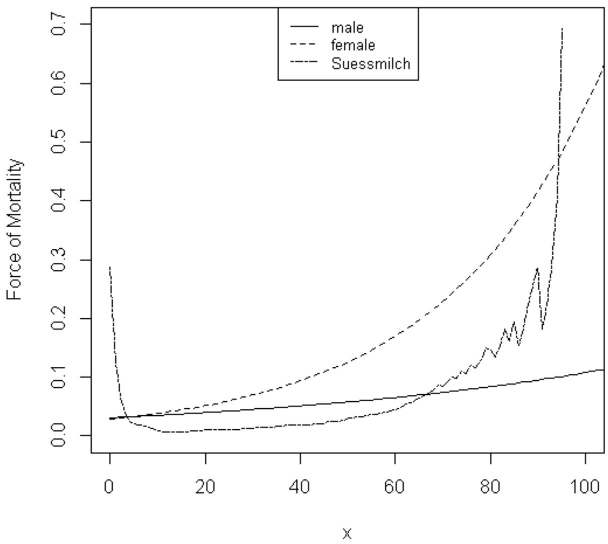

Figure 1: Force of mortality functions (Rome)

Other sources of error and bias are mentioned by Clauss (1973), who offers the most detailed demonstration, or Parkin (1992, pp. 6-18): age rounding by multiples of 5,[1] a gender bias, as men are more likely than women to have an epitaph with an age inscription (which is reflected by a high sex ratio (see Figure 2)), and a serious class bias (not all classes are represented, because inscribed tombstones were not cheap; Burn (1953) states that tombstones with ages were found primarily in the middle-class and lower middle-class urban population, whereas members of the upper classes generally did not give ages on tombstones (Burn, p. 7)). Figure 3 shows as an example the distributions of age at death in the castrum Mogontiacum, a military camp, precursor to the German city of Mainz. The shape of the distribution is very different from the shape of the distribution in Rome (see Figure 2), because of the high proportion of soldiers in the age information (88%, see Clauss 1973, p. 399). In sum, we cannot use the data to calculate demographic parameters, such as life expectancy, of the Roman population. Nevertheless, the data are not worthless. They can be used to show and to explain the differences in the burial and commemorative processes. For example, the considerable preponderance of boys points to the high value attached to male offspring in a male-oriented society (Laes 2007, p. 33). “The ages reflect something of the structure of these societies, and attitudes towards age and the life course. They reflect the way in which different age groups are judged differently, and the interplay between age and gender” (see Revell, 2005, p. 46).
In this paper Roman funerary data of the Italian cities and European provinces of the Roman Empire is re-examined. Age at death distributions are analyzed and categorized.

[1] The phenomenon of a preference for ages ending in 0 and 5 was extensively analyzed by Duncan-Jones (1990, 79-92). However, age rounding is a minor problem. Smoothing the distributions with a Gompertz distribution (see Appendix 1) reduces the biases.

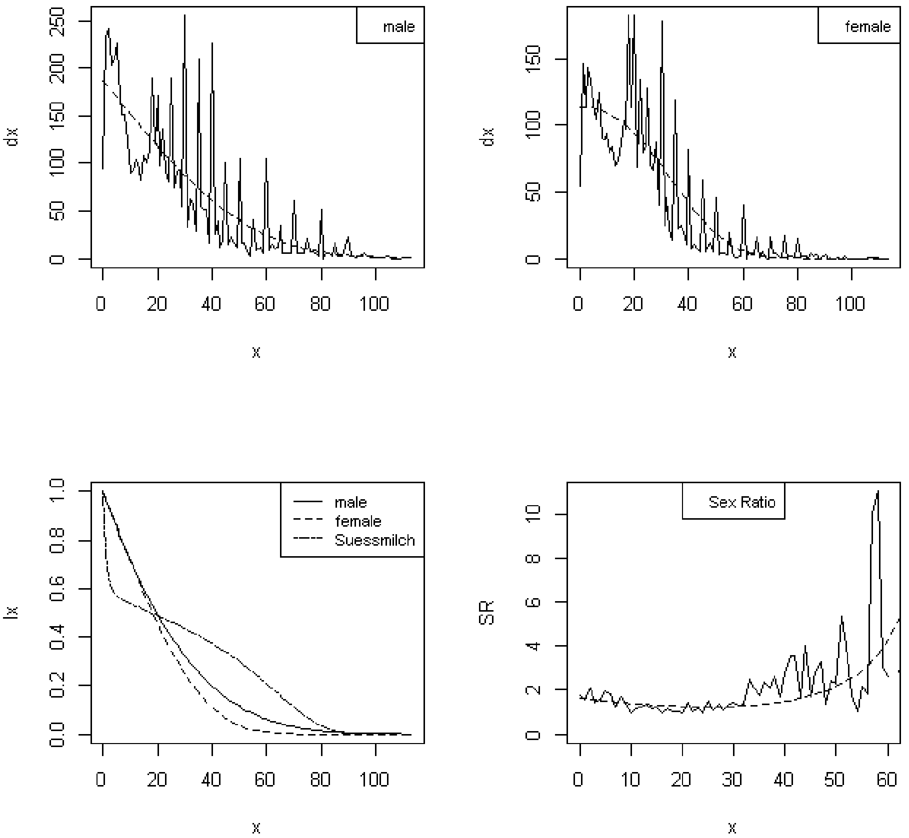

Figure 2: Recorded and smoothed age at death distributions (male and female), survivor functions lx, and recorded and smoothed age specific sex ratios (Rome)

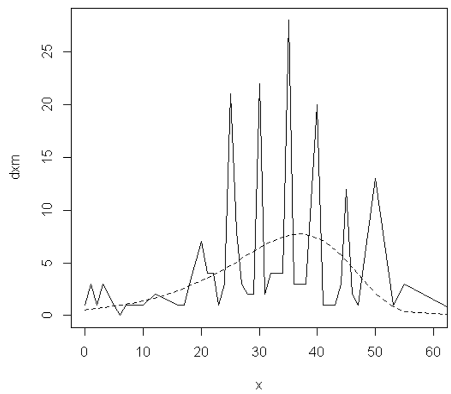

Figure 3: Recorded and smoothed age at death distribution (male), sex ratio is 209:33 (Mogontiacum)

2. Data and Distributions

The inscriptions number 24,854 (15,173 males, 9,681 females), and were collected by the Hungarian scholar Szilágyi (1961, 1962, 1963). The data comes from 48 cities and provinces of the European part of the Roman Empire between the first and the seventh centuries (see Table A2 in Appendix 2). Minor addition errors in Szilágyi's data were corrected. The age at death distributions were smoothed, because the dominance of ages that are multiples of 5 hides the essential shape of the distributions. The smoothing function was the Gompertz distribution (see Appendix 1), which was fitted to the corresponding survivor functions by non-linear least squares. As a result, we can represent the age at death distributions by a two-parameter function with A and k. From Figure 4 it can be seen that the survivor functions can be well represented by survivor functions of the Gompertz distribution. The smoothed age at death distributions shown in Figure 5 seem, at first glance, very different. We have distributions that are skewed to the left, symmetrical distributions, and distributions that are skewed to the right. The mean ranges between 21 and 47 (see Table A3). The mean age and the other parameters have been calculated from the fitted Gompertz distribution by numerical integration. The statistics presumably represent normal or average conditions of mortality during a period of several centuries. The last three columns of Table A2 represent the proportion of individuals, *l*(25), who are at least 25 years old. The difference F(25)=1-*l*(25) is the proportion of inscriptions for those whom we may conventionally consider young, as Laes (2007) did. He concluded in his investigation that this proportion is about 61% (see Laes, 2007, p. 28).

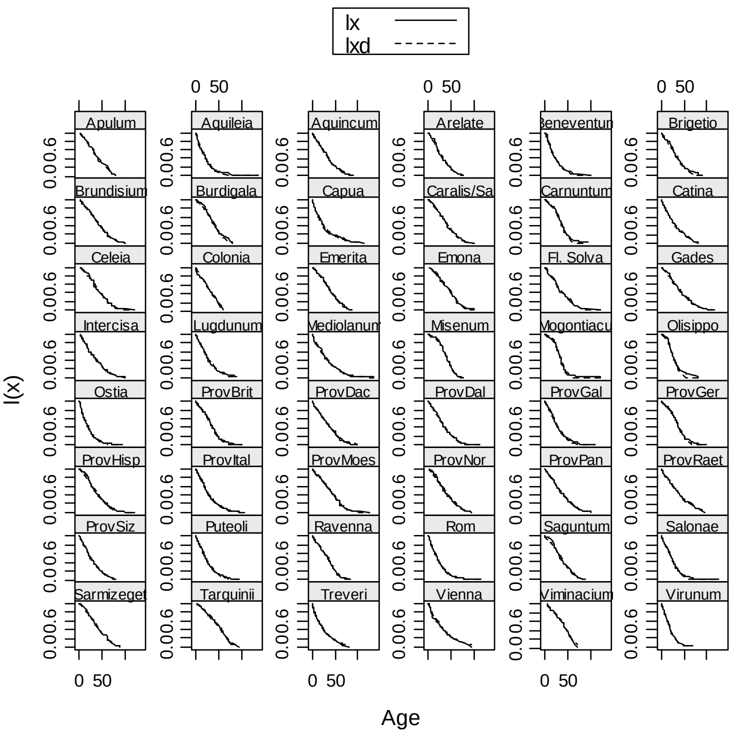

Figure 4: Survivor functions (recorded=lx, fitted=lxd)
(Fl. Solva: Flavia Solva; ProvBrit: Province Britannia; Dac: Dacia; Ger: Germania; Gal: Gallia; Hisp: Hispania; Ital: Italia; Moes: Moesia, Nor: Noricum; Pan: Pannonia; Raet: Raetia; Siz: Sizilia)

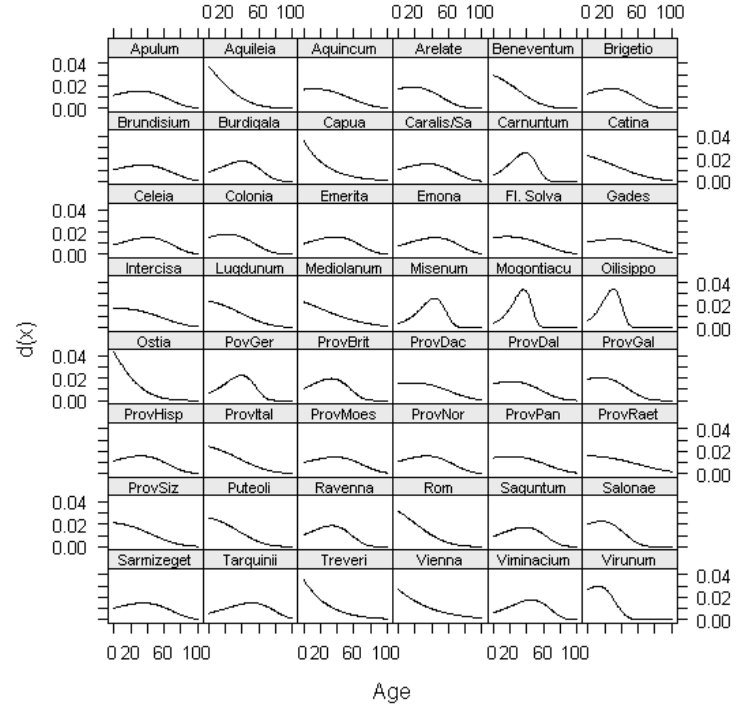

Figure 5: Fitted age at death distributions
(Fl. Solva: Flavia Solva; ProvBrit: Province Britannia; Dac: Dacia; Ger: Germania; Gal: Gallia; Hisp: Hispania; Ital: Italia; Moes: Moesia, Nor: Noricum; Pan: Pannonia; Raet: Raetia; Siz: Sizilia)

The shapes of the distributions are determined by the parameters A and k of the Gompertz distribution, which are shown as a scatter plot in Figure 6. We can identify groups or clusters of similar parameters. The points in the lower right corner are characterized by high values of k and low values of A, whereas the points in the upper left corner show low k values and high A values. The typical A-k constellation of a real life table has very low A values, and k values ranging between 0.05 and 0.13. The parameters of the Suessmilch life table, for example, are A=0.00081 and k=0.065, whereas the parameters of the German life table (female) 2007-2009 are A=0.00000199 and k=0.125. The graph is divided into three segments by the two straight lines A=k and $A = \frac{1}{6.4339} \cdot k$ (skew=0). Constellations in the upper left corner form distributions that are strongly skewed to the right (in fact the Gompertz distribution tends to the exponential distribution if k tends to zero, and we can approximate the Gompertz distribution by the simpler exponential distribution, see Appendix 1), whereas constellations in the lower right corner form distributions that are skewed to the left. Measure of skewness (and kurtosis) are given in Table A3. Distributions with constellations of A and k that are near the line A=k can be approximated by the LHR-distribution (linear hazard rate, see Appendix 1). Since the median of the Gompertz distribution is $x_{0.5} = \frac{\ln\left(\frac{A + k \cdot \ln 2}{A}\right)}{k}$, we can solve this equation for A and obtain $A = \frac{\ln 2}{\exp\left(k \cdot x_{0.5}\right) - 1} \cdot k$. If we set

$x_{0.5} = 20, 25, 30, 40$, we obtain the so called iso-median-lines, which represent points of equal median values, and are shown in Figure 6.[2] A distribution that is skewed to the right (skew>0) typically has a median smaller than its mean.

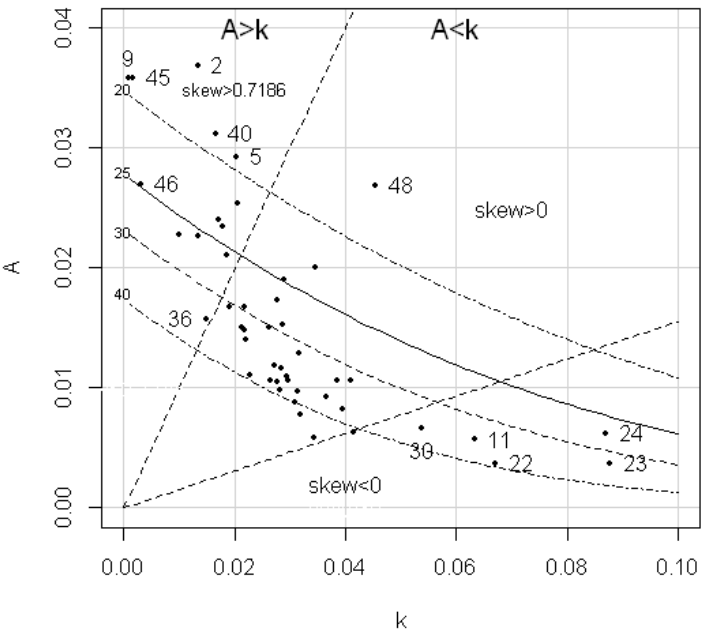

Figure 6: Parameters A and k of the theoretical distributions, and iso-median-lines
23: Mogontiacum, 24: Olisoppo, 22: Misenum, 11: Carnuntum, 30: Province Germania,
2: Aquileia, 45: Treveri, 9: Capua, 40: Rome, 5: Beneventum, 46: Vienna
48: Virunum, 36 Province Raetia

3. Cluster Analysis

In the next step, a cluster analysis is applied in order to group the different smoothed distributions into similar categories. We perform k-means clustering on a data matrix that contains each of the 35 values of A and k and the proportion of military epitaphs p_{mil}.[3] The method requires one to specify the number of clusters to be extracted. After some trials with different numbers, we concluded that the number of clusters should be 3. The results are seen in Figure 7, Table 1 and Table A4. Averages are not weighted. Because of the high distribution of males, only small differences between the set of the graphs dx and dxm at the top of Figure 7 can be seen. We therefore restrict our explanation to the male (dxm) and female (dxw) age distributions (but see Fig A1 in the Appendix 2). We get a distribution with a very high proportion of military persons (n=3), a distribution with a medium proportion of military persons (n=7), and a distribution with mostly civilians (n=25 or n=24). Since the k-value for the females of Olisippo was regarded as an outlier, it was omitted, and the cluster analysis was carried out with only 24 cities.

[2] Iso-mean-lines are not drawn because there is no simple relationship between A and k.
[3] The proportion of military epitaphs was given for 35 cities (see Clauss 1973, pp. 415-416). In the cluster analysis, Olisippo was removed for the female population, because of its outlier value of k.

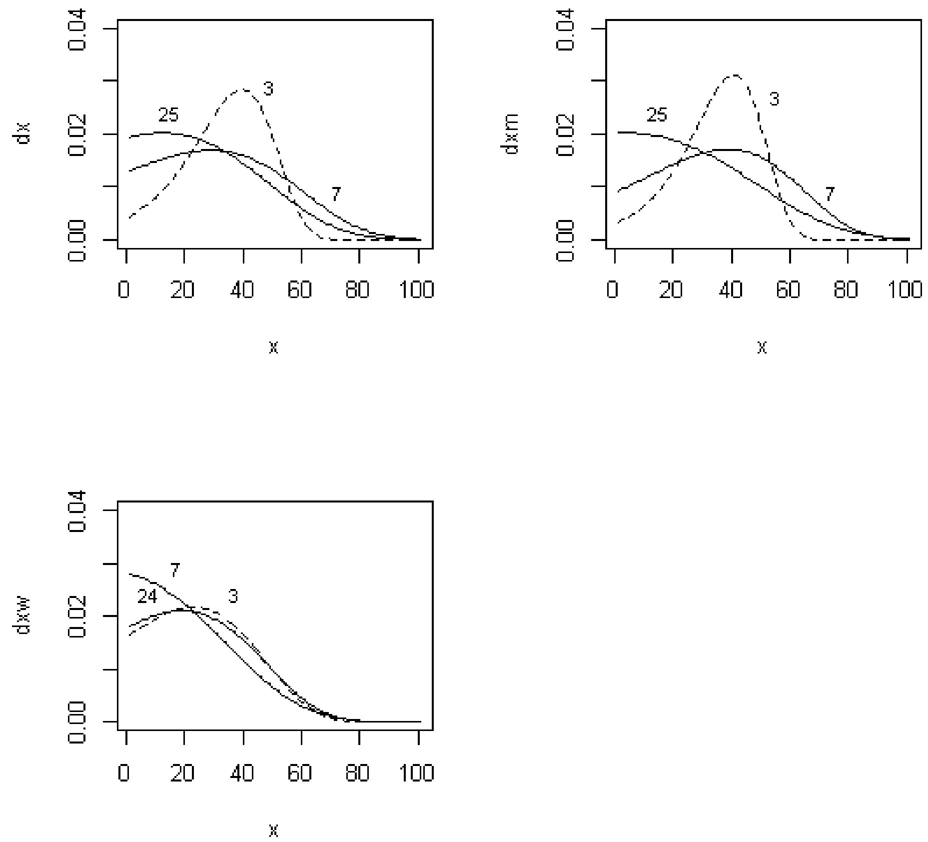

Figure 7: Age at death distributions in different clusters (numbers represent cluster size, dx=all, dxm=male, dxw=female)

In essence, we can identify three distributions for the males. It should be clear that the theoretical distributions cannot be used for very low ages (see Figure 2). In the “military” cluster with n=3 are the military camps from different regions, Carnuntum (Pannonia), Misenum (Italia), and Mogontiacum (Germania). The mode in the cluster is nearly 40 years. The distribution is slightly skewed to the left, which means that the proportion of young ages is small. Indeed, l(25)=0.771, which means that only 32.9% of the inscriptions with an age indication can be considered as those of young people. The next largest cluster (n=7) contains six cities from the Danube provinces of Dacia, Pannonia, and Moesia, and Ravenna in Italy (see Table A4). In these cities, the proportion of civilians and non-civilians is equal. The male age distribution is similar to that of the previous “military” cluster, but the variance is much larger. The civilian population means that the proportion of tombstones with young and old age inscriptions is higher. In the “civilian male” cluster are 25 cities (see Table A4) with a very low proportion of military inscriptions. The distribution is strongly skewed to the right, with a heavy right tail (mode=3.1, mean=29). The ratio of young inscriptions is 49%. This is less than the percentage of 61.1% which is reported in Laes (2007, p. 28). The difference can be explained by the selected cities. On the other hand, the percentage in Rome for the male population is 60%, and in Ostia it is 72.5% (see Table A2). Remember that in our cluster analysis the results are not weighted by the number of inscriptions. The shape can be partly explained by two factors (see Hopkins 1966, p. 263). Sons were commemorated by their parents earlier and until much greater ages than daughters, because of son preferences, and since husbands were in general older than wives and were survived by them, wives could commemorate husbands at much later ages than husbands commemorated wives. The “civilian” cluster for females (n=24), in contrast, is unimodal with a mode of 18 years. The proportion of very old and very young ages is smaller than in the “civilian” male cluster, for the reasons that have been just mentioned. The modal value could reflect a higher mortality during the reproductive period. But as Hopkins (1966, p. 262) points out, a young wife can be commemorated by her husband and by their parents. “Wives who died young had a greater chance of being commemorated.” The pattern of female mortality is more or less independent of the proportion of military inscriptions in Carnuntum, Misenum, and Mogontiacum, as can be seen in Figure 7. The age distributions for females (n=7) in the six cities in the Danube provinces and Ravenna, where proportion of military inscriptions is 50%, exhibit a rather different pattern. The distribution is strongly skewed to the right, with a modal value of zero. Daughters are relatively more likely to be commemorated than sons. Is this shape a result of the small sample size, which is only 197, or is it an indication of higher infant mortality of girls in these provinces, or does the low sex ratio reflect an increased social importance of daughters in these cities? In contrast to other cities, the sex ratio in the age ranges 0-10 and 0-20 is less than 1. In Aquincum, for example, the sex ratio in the age group 0-10 (0-20) is only 0.88 (0.9), whereas in Rome it is 1.58 (1.36).

Table 1: Parameters of the different clusters

all		A	k	Mean	Mode	StD	Skew	Kurt	n	l25	p_{mil}
	1	0.0044	0.0727	33.4	38.7	13.7	-0.30	-0.52	3	0.732	0.853
	2	0.0129	0.0297	33.7	28.0	20.4	0.38	-0.56	7	0.62	0.505
	3	0.0194	0.0261	27.8	11.5	18.6	0.60	-0.26	25	0.505	0.064
male											
	1	0.0032	0.0812	34.5	39.8	13.0	-0.42	-0.36	3	0.771	0.853
	2	0.0092	0.0360	36.8	38.0	20.1	0.18	-0.70	7	0.689	0.505
	3	0.0201	0.0215	29.0	3.1	20.2	0.69	-0.08	25	0.514	0.064
female											
	1	0.0165	0.0384	26.2	22.0	15.8	0.37	-0.56	3	0.5	0.853
	2	0.0281	0.0233	22.5	0	16.3	0.80	0.16	7	0.385	0.505
	3	0.0181	0.0331	26.4	18.3	16.7	0.47	-0.45	24	0.495	0.064

p_{mil}=proportion of military epitaphs among all epitaphs

Mode $m = -\frac{\ln\left(\frac{A}{k}\right)}{k}$ if m>0, else m=0; StD=Standard deviation, Kurt=Kurtosis. Mean, standard deviation, skewness and kurtosis have been calculated by numerical integration.

Figure 8 exhibits the force of mortality functions in the three clusters for the male and female populations. The functions of the epitaph populations are compared with the force of mortality function of the Suessmilch life table from the eighteenth century.

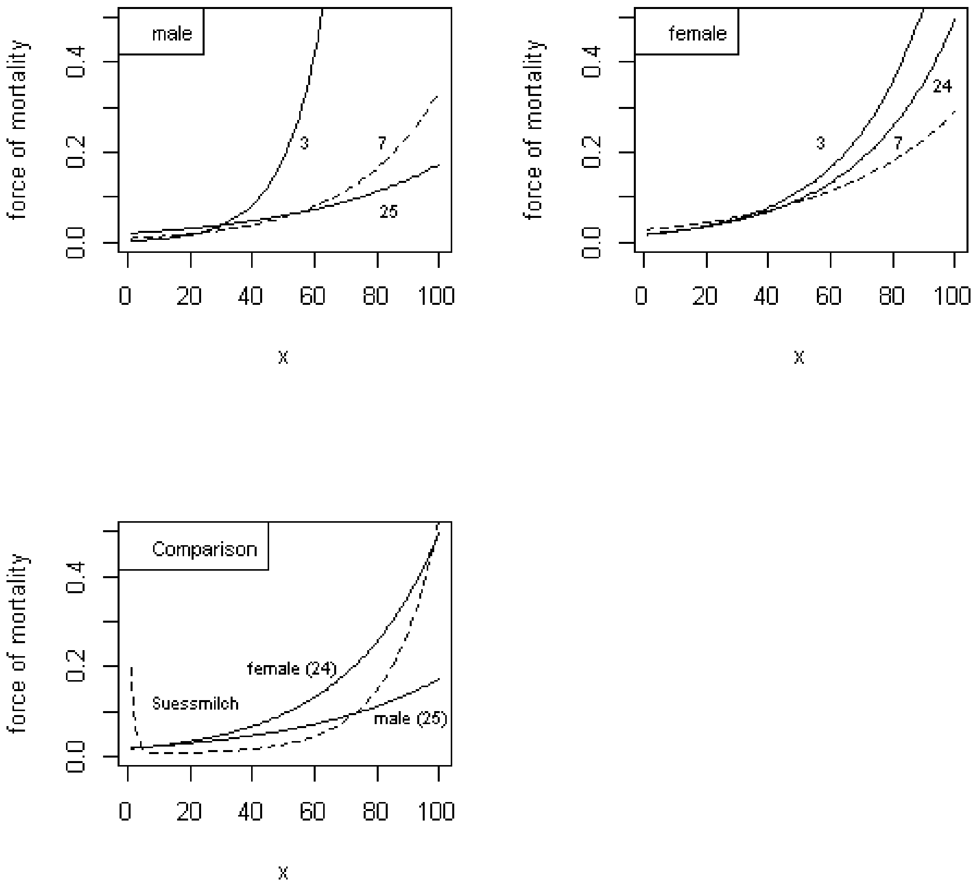

Figure 8: Force of mortality functions in different clusters (n=cluster size)

Similar results for the male population (n=24) were obtained by Hopkins (1966, p. 256). There is an underestimation of mortality at young ages and very old ages, and an overestimation in the middle age classes. Hopkins uses as a reference UN model life tables with a life expectancy of 20 and 30 years. Since Hopkins' functions are not sufficiently smoothed, the comparison between the different functions is more difficult. However, in contrast to Hopkins, the force of mortality in the upper age classes of the female epitaph population in our investigation is much higher than in real or model life tables with a high mortality (Hopkins 1966, p. 257). The pattern of mortality from the age of 40 for military personnel and civilians in Hopkins (1966, p. 258) is similar to our results: there is a steep increase for military persons and a shallow increase for mostly civilian persons from age 40.
We can summarize our findings and, of course, those of Hopkins with a concise quote from Scheidel (2007 p. 8): "The resultant statistics merely reveal the average death [or more general the mortality pattern] of those individuals who happened to be commemorated in stone: far from generating demographically representative samples of actual populations, commemorative practices were shaped by a variety of factors such as

geographical provenance, class, religion, language, gender and, most crucially, age. Because of these manifold distortions, age distributions derived from epigraphic samples do not normally match any demographically creditable pattern, except very occasionally by chance."

4. The influence of the population growth rate

Finally, we will analyze the influence of the population growth rate on the estimates (see also Durand 1959, pp. 370 ff.). In general, a stationary and closed population is assumed: the numbers of yearly births and deaths are equal and constant, and the age structure is not affected by migration (see, e.g. Pflaumer 2015a). Willcox (1938) has already remarked that the large numbers of deaths in early adulthood in some areas, like Rome, are not only the result of premature mortality but also the result of high migration of young persons to the capital city. In demography it has long been known that the age distribution of deaths depends on the population growth rate (see, e.g., Keyfitz 1977). The mean is a decreasing function of the growth rate. The proportion of deaths in the younger age groups will increase, whereas the proportion of deaths in the older age groups will decrease, if the growth rate is positive. The opposite would be true if the population is decreasing.

One obtains the stationary distribution of the age at death from a discrete stable population growing by a factor q>1 simply by multiplying the number of deaths at age x by the factor q^x , x=0, 1, 2, …This result was first found by Euler (1760).

We now assign this result to our continuous density distributions.

If f(x) is the density of the observed deaths, which is biased by a positive growth rate r, then we get the density of the stationary population by $f_S(x) = \frac{f(x)\cdot e^{r\cdot x}}{\int_0^\infty f(x)\cdot e^{r\cdot x}dx}$. Assuming a Gompertz distribution, it is easy to show with calculus that the modal value m of the stationary density is given by

$m = -\frac{\ln\left(\frac{A}{k+r}\right)}{k}$. Solving the equation for r yields $r = Ae^{k\cdot m} - k$.

This equation gives an estimate of the unknown population growth rate if the modal m value is known. The age at death distributions of Rome in Figure 2 are strongly skewed to the left. There is a predominance of deaths at young ages. The Roman age distribution falls in cluster 3 (see Table 1), where the modal values are 11.5 years (all), 3.1 years (male), and 18.3 years (female). If we assume these modal values for Rome as well, then we estimate a yearly growth rate of about 2% with the parameters A and k that are given in Table A2. With this result, we could adjust the distribution of age at death in Rome so as to eliminate the effect of population growth.

5. Conclusion

Age at death distributions can be fitted by a relatively simple function, the Gompertz distribution. It is somewhat surprising that only two parameters need to be known in order to model these distributions sufficiently for all the cities and provinces in the Roman Empire. This fit removes the irregularities of age rounding and enables a clearer analysis. Although earlier results that epitaph material is not suitable for calculating life tables for the Roman population could only be confirmed, the different types of distribution can give evidence about commemorative habits in the Roman Empire. Similarities and differences can be detected more easily. In particular, possible missing values at age x (e.g., sex ratio) or biases due to population growth can be estimated by the model. The theoretical distributions simplify the comparison and facilitate the search for differences and similarities with advanced statistical methods.

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Appendix:

1. Formulas of the Gompertz distribution:

(see also, e.g., Pflaumer 2011 or Pollard 1991)

Force of mortality: $\mu(x)=A\cdot e^{k\cdot x}$

Survivor function: $l(x)=\exp\left(-\int_0^x \mu(u)du\right)=\exp\left(\frac{A}{k}-\frac{A}{k}\cdot e^{k\cdot x}\right)$

Density function: $f(x)=-\frac{dl(x)}{dx}=\mu(x)\cdot l(x)$

Table A1: Skewness and kurtosis of the Gompertz distribution

Parameter	Ratio (A>0, k>0)	Skewness	Kurtosis	Remarks
k=0	$\frac{k}{A}=0$	2	6	Exponential distribution $l(x)=e^{-A\cdot x}$
k<A	$\frac{k}{A}=\frac{1}{6.433902376}$	1.4601	2.5662	
k=A	$\frac{k}{A}=1$	0.7186	-0.0174	Approximation by the LHR distribution
k>A	$\frac{k}{A}=6.433902376$	0	-0.7237	
	$\frac{k}{A}=52.64917497$	-0.5924	0	
	$\frac{k}{A}=4808$	-1.0879	1.9806	German life table 2007/09 (male)
	$\frac{k}{A}=62424$	-1.1305	2.3070	German life table 2007/09 (female)
	A<<k	-1.1395	2.4	Approximation: Gumbel distribution: $l(x)=\exp\left(-\frac{A}{k}\cdot e^{k\cdot x}\right)$

Values of the Gompertz distribution have been obtained by numerical integration

If 0<A<<k, then the Gompertz distribution can be approximated by a Gumbel distribution, whose moments can be determined analytically. This is the case for modern life tables with low mortality. If k<0, then we obtain the negative Gompertz distribution (see Marshall and Olkin 2007, p. 368) with an exponentially decreasing force of mortality function. Its skewness exceeds 2. It is an improper distribution, since $\lim_{x\to\infty} l(x)>0$. If A=k, the Gompertz distribution can be approximated for small values of x by the linear hazard rate (LHR) distribution, whose force of mortality function is $\mu(x)=A+A^2x$. Its survivor function is $l(x)=\exp\left(1-e^{A\cdot x}\right)$. Mean, variance and skewness are:

$E(X)=\frac{0.6557}{A}$, $Var(X)=\frac{0.2587}{A^2}$, and sk(X)=1.089.

2. Tables and Figures

Table A2: Estimation results and parameters

	Unit	Province	km	Am	kw	Aw	k	A	Nm	Nw	N	SR	l25	l25m	l25w
1	Apulum	Dacia	0.0274	0.0099	0.0364	0.0137	0.0271	0.0119	40	21	61	1.90	0.655	0.701	0.572
2	Aquileia	Italia	0.0070	0.0393	0.0268	0.0318	0.0133	0.0369	141	95	236	1.48	0.335	0.342	0.322
3	Aquincum	Pannonia	0.0291	0.0108	0.0191	0.0314	0.0217	0.0167	110	52	162	2.12	0.575	0.671	0.366
4	Arelate	Gallia	0.0197	0.0180	0.0446	0.0148	0.0278	0.0173	53	40	93	1.33	0.536	0.560	0.506
5	Beneventum	Italia	0.0136	0.0322	0.0378	0.0226	0.0204	0.0293	72	28	100	2.57	0.385	0.384	0.391
6	Brigetio	Pannonia	0.0334	0.0109	0.0351	0.0155	0.0316	0.0129	65	36	101	1.81	0.611	0.654	0.539
7	Brundisium	Italia	0.0280	0.0097	0.0248	0.0118	0.0265	0.0106	124	89	213	1.39	0.687	0.704	0.665
8	Burdigala	Gallia	0.0360	0.0086	0.0460	0.0075	0.0395	0.0082	101	78	179	1.29	0.704	0.705	0.704
9	Capua	Italia	-0.0022	0.0347	0.0113	0.0347	0.0008	0.0358	88	59	147	1.49	0.405	0.430	0.367
10	Caralis/Sa	Italia	0.0280	0.0103	0.0339	0.0107	0.0296	0.0106	104	74	178	1.41	0.675	0.690	0.657
11	Carnuntum	Pannonia	0.0706	0.0045	0.0287	0.0162	0.0633	0.0057	167	37	204	4.51	0.707	0.737	0.553
12	Catina	Italia	0.0172	0.0205	0.0092	0.0255	0.0135	0.0227	53	47	100	1.13	0.509	0.527	0.488
13	Celeia	Noricum	0.0272	0.0093	0.0373	0.0079	0.0308	0.0088	115	91	206	1.26	0.717	0.716	0.721
14	Colonia	Germania	0.0395	0.0100	-0.0048	0.0691	0.0286	0.0153	47	10	57	4.70	0.571	0.654	0.196
15	Emerita	Hispania	0.0224	0.0104	0.0532	0.0067	0.0313	0.0097	73	72	145	1.01	0.693	0.706	0.705
16	Emona	Pannonia	0.0289	0.0090	0.0357	0.0065	0.0319	0.0078	66	50	116	1.32	0.741	0.718	0.768
17	Flavia Solva	Noricum	0.0132	0.0166	0.0410	0.0109	0.0213	0.0150	42	33	75	1.27	0.609	0.612	0.622
18	Gades	Hispania	0.0214	0.0112	0.0249	0.0109	0.0228	0.0111	75	62	137	1.21	0.688	0.691	0.686
19	Intercisa	Pannonia	0.0215	0.0123	0.0245	0.0210	0.0190	0.0167	50	39	89	1.28	0.586	0.665	0.485
20	Lugdunum	Gallia	0.0079	0.0247	0.0459	0.0174	0.0178	0.0234	127	98	225	1.30	0.478	0.505	0.442
21	Mediolanum	Italia	0.0063	0.0220	0.0169	0.0225	0.0100	0.0228	72	67	139	1.07	0.523	0.551	0.496
22	Misenum	Italia	0.0754	0.0025	0.0543	0.0139	0.0671	0.0037	212	32	244	6.63	0.788	0.831	0.477
23	Mogontiacum	Germania	0.0977	0.0026	0.0323	0.0194	0.0878	0.0037	209	33	242	6.33	0.717	0.754	0.474
24	Olisippo	Hispania	0.0709	0.0090	0.1345	0.0019	0.0870	0.0062	58	22	80	2.64	0.572	0.538	0.671
25	Ostia	Italia	0.0031	0.0497	0.0276	0.0332	0.0121	0.0428	387	265	652	1.46	0.286	0.275	0.302
26	ProvBrit	Britannia	0.0514	0.0073	0.0227	0.0201	0.0410	0.0106	152	69	221	2.20	0.629	0.689	0.509
27	ProvDac	Dacia	0.0177	0.0151	0.0327	0.0134	0.0217	0.0148	177	96	273	1.84	0.612	0.623	0.596
28	ProvDal	Dalmatia	0.0295	0.0132	0.0220	0.0180	0.0263	0.0150	348	230	578	1.51	0.588	0.614	0.550
29	ProvGal	Gallia	0.0221	0.0202	0.0483	0.0152	0.0290	0.0190	290	168	458	1.73	0.497	0.509	0.478
30	ProvGer	Germania	0.0594	0.0055	0.0358	0.0117	0.0537	0.0066	197	52	249	3.79	0.706	0.729	0.624
31	ProvHisp	Hispania	0.0275	0.0106	0.0322	0.0123	0.0284	0.0116	1066	827	1893	1.29	0.656	0.684	0.623
32	ProvItal	Italia	0.0156	0.0227	0.0242	0.0242	0.0171	0.0240	2045	1305	3350	1.57	0.474	0.500	0.435
33	ProvMoes	Moesia	0.0321	0.0075	0.0246	0.0150	0.0282	0.0098	284	136	420	2.09	0.699	0.749	0.595
34	ProvNor	Noricum	0.0265	0.0122	0.0335	0.0094	0.0293	0.0110	207	149	356	1.39	0.668	0.650	0.693
35	ProvPan	Pannonia	0.0228	0.0126	0.0226	0.0165	0.0221	0.0140	379	204	583	1.86	0.626	0.653	0.575
36	ProvRaet	Raetia	0.0155	0.0142	0.0149	0.0182	0.0148	0.0157	54	31	85	1.74	0.621	0.648	0.576
37	ProvSiz	Italia	0.0201	0.0213	0.0170	0.0207	0.0185	0.0211	114	86	200	1.33	0.511	0.501	0.524
38	Puteoli	Italia	0.0141	0.0294	0.0301	0.0206	0.0206	0.0254	358	268	626	1.34	0.435	0.415	0.464
39	Ravenna	Italia	0.0520	0.0059	0.0341	0.0267	0.0386	0.0106	94	30	124	3.13	0.641	0.740	0.348
40	Rome	Italia	0.0124	0.0311	0.0297	0.0286	0.0165	0.0312	6008	3972	9980	1.51	0.381	0.401	0.346
41	Saguntum	Hispania	0.0277	0.0107	0.0506	0.0071	0.0365	0.0092	72	61	133	1.18	0.686	0.679	0.699
42	Salonae	Dalmatia	0.0322	0.0200	0.0387	0.0200	0.0345	0.0201	341	236	577	1.44	0.450	0.463	0.431
43	Sarmizeget	Dacia	0.0250	0.0099	0.0368	0.0104	0.0277	0.0105	44	30	74	1.47	0.685	0.710	0.654
44	Tarquinii	Italia	0.0362	0.0052	0.0316	0.0069	0.0343	0.0059	69	60	129	1.15	0.792	0.808	0.770
45	Treveri	Gallia	0.0030	0.0318	0.0017	0.0420	0.0017	0.0358	78	48	126	1.63	0.401	0.438	0.342
46	Vienna	Gallia	0.0015	0.0255	0.0061	0.0285	0.0032	0.0270	68	55	123	1.24	0.496	0.522	0.464
47	Viminacium	Moesia	0.0490	0.0044	0.0190	0.0192	0.0415	0.0063	41	9	50	4.56	0.760	0.805	0.541
48	Virunum	Noricum	0.0381	0.0293	0.0586	0.0231	0.0455	0.0269	36	29	65	1.24	0.286	0.294	0.270

(N = number of cases, SR=Sex Ratio, l25=survival rate to age 25, m=male, w=female)

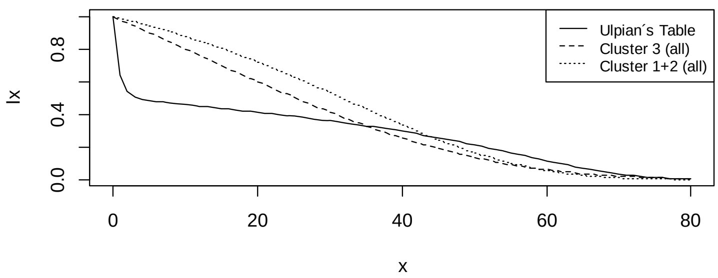
Table A3: Parameters: mean, standard deviation, and skewness

Unit	all				male				female			
	mean	sdv	skew	kurtosis	mean	sdv	skew	kurtosis	mean	sdv	skew	kurtosis
Apulum	36.7	22.3	0.38	-0.55	40.4	23.6	0.31	-0.62	29.8	17.5	0.32	-0.61
Aquileia	21.0	17.2	1.14	1.21	22.0	19.4	1.41	2.34	19.8	14.3	0.79	0.15
Aquincum	32.7	22.0	0.61	-0.23	37.5	22.1	0.32	-0.62	21.9	16.7	0.93	0.51
Arelate	29.1	18.9	0.52	-0.37	32.2	22.3	0.68	-0.10	26.0	15.0	0.27	-0.65
Beneventum	22.7	16.9	0.87	0.35	23.3	18.7	1.08	0.98	22.0	14.1	0.51	-0.40
Brigetio	32.8	19.6	0.35	-0.58	35.0	20.1	0.27	-0.65	28.2	17.2	0.39	-0.55
Brundisium	39.5	23.6	0.35	-0.59	40.5	23.5	0.29	-0.64	38.3	23.6	0.41	-0.52
Burdigala	37.1	19.5	0.10	-0.72	38.1	20.5	0.15	-0.71	35.6	17.8	0.02	-0.73
Capua	27.3	26.8	1.88	5.09	30.9	33.6	2.69	15.55	22.8	18.9	1.19	1.37
Caralis/Sa	37.6	22.0	0.30	-0.63	39.2	23.0	0.31	-0.62	35.1	20.0	0.26	-0.66
Carnuntum	33.2	14.7	-0.18	-0.64	33.8	14.0	-0.28	-0.54	29.9	19.0	0.48	-0.43
Catina	30.5	23.3	0.94	0.53	30.7	22.3	0.79	0.15	30.4	24.9	1.14	1.21
Celeia	40.7	22.7	0.22	-0.68	42.0	24.3	0.29	-0.64	39.0	20.5	0.11	-0.72
Colonia	31.0	19.5	0.46	-0.46	33.7	18.4	0.17	-0.70	15.7	17.2	2.86	19.75
Emerita	38.4	21.8	0.25	-0.67	43.0	26.4	0.40	-0.53	34.5	16.3	-0.07	-0.70
Emona	42.5	23.0	0.16	-0.71	41.5	23.6	0.25	-0.67	43.7	22.3	0.05	-0.73
Flavia Solva	35.2	23.3	0.57	-0.29	38.5	28.2	0.81	0.20	31.7	17.5	0.19	-0.70
Gades	41.1	25.4	0.42	-0.51	42.0	26.3	0.45	-0.47	40.0	24.3	0.38	-0.55
Intercisa	34.2	23.6	0.66	-0.13	39.6	25.3	0.49	-0.42	27.0	18.5	0.65	-0.15
Lugdunum	27.7	20.4	0.83	0.25	32.1	26.7	1.19	1.39	23.5	13.9	0.33	-0.61
Mediolanum	32.6	26.1	1.06	0.93	36.7	30.9	1.24	1.56	28.9	21.3	0.84	0.27
Misenum	37.4	15.0	-0.32	-0.49	39.2	14.2	-0.46	-0.28	24.4	13.3	0.18	-0.70
Mogontiacum	31.3	11.9	-0.40	-0.38	32.3	11.2	-0.51	-0.17	25.6	16.5	0.51	-0.40
Olisippo	26.3	11.1	-0.25	-0.58	25.8	12.2	-0.07	-0.70	27.9	8.6	-0.66	0.16
Ostia	18.9	15.9	1.24	1.58	19.0	18.0	1.72	3.99	19.0	13.8	0.80	0.16
ProvBrit	32.2	17.6	0.18	-0.70	33.9	16.5	-0.03	-0.72	28.5	19.7	0.67	-0.12
ProvDac	35.3	23.2	0.56	-0.32	37.5	25.7	0.65	-0.15	31.6	18.9	0.36	-0.58
ProvDal	32.5	20.7	0.49	-0.43	33.4	20.3	0.39	-0.55	31.0	21.1	0.63	-0.19
ProvGal	27.0	17.7	0.54	-0.34	28.7	19.9	0.68	-0.10	24.7	14.1	0.25	-0.66
ProvGer	34.5	16.3	-0.08	-0.70	35.0	15.6	-0.17	-0.65	32.7	18.7	0.27	-0.65
ProvHisp	36.5	21.8	0.35	-0.58	38.9	23.0	0.33	-0.60	33.4	19.7	0.33	-0.61
ProvItal	27.5	20.5	0.86	0.32	29.4	22.0	0.88	0.36	24.6	17.4	0.72	-0.02
ProvMoes	40.1	23.3	0.29	-0.64	43.2	23.2	0.14	-0.71	33.3	21.5	0.51	-0.39
ProvNor	37.0	21.8	0.32	-0.61	36.5	22.4	0.40	-0.53	37.8	21.0	0.21	-0.69
ProvPan	36.2	23.6	0.53	-0.36	38.2	24.2	0.47	-0.44	32.4	21.6	0.59	-0.27
ProvRaet	38.7	27.5	0.74	0.04	40.8	28.3	0.68	-0.09	34.9	25.4	0.80	0.17
ProvSiz	29.5	21.2	0.77	0.11	28.5	20.3	0.74	0.04	30.6	22.3	0.80	0.17
Puteoli	25.0	18.3	0.81	0.18	24.8	19.6	1.03	0.81	25.4	16.7	0.56	-0.31
Ravenna	33.1	18.4	0.20	-0.69	36.8	17.1	-0.10	-0.69	20.6	13.9	0.62	-0.22
Rome	22.9	17.8	0.99	0.68	24.4	19.7	1.10	1.07	20.6	14.4	0.70	-0.05
Saguntum	36.6	19.9	0.17	-0.71	38.6	22.8	0.33	-0.60	34.6	16.8	-0.03	-0.72
Salonae	24.4	15.6	0.50	-0.41	25.2	16.3	0.52	-0.38	23.4	14.6	0.45	-0.48
Sarmizeget	39.0	23.0	0.33	-0.61	42.1	25.1	0.34	-0.59	34.3	19.1	0.21	-0.69
Tarquinii	46.7	23.6	0.03	-0.73	47.9	23.3	-0.03	-0.72	45.3	24.0	0.12	-0.72
Treveri	26.7	25.6	1.77	4.34	28.9	26.8	1.61	3.38	22.9	22.1	1.80	4.53
Vienna	33.4	30.5	1.55	3.01	37.1	35.3	1.73	4.07	29.6	25.7	1.35	2.03
Viminacium	40.8	20.1	-0.01	-0.72	42.9	19.0	-0.18	-0.64	31.2	22.0	0.72	-0.01
Virunum	18.4	11.8	0.50	-0.41	18.6	12.5	0.61	-0.23	18.0	10.7	0.34	-0.60

Table A4: Cluster results

ID	Unit	p_{mil}	Province	Cluster	ID	Unit	p_{mil}	Province	Cluster
11	Carnuntum	0.734	Pannonia	1	20	Lugdunum	0.097	Gallia	3
22	Misenum	0.889	Italia	1	21	Mediolanum	0.035	Italia	3
23	Mogontiacum	0.937	Germania	1	24	Olisippo	0.012	Hispania	3
1	Apulum	0.377	Dacia	2	25	Ostia	0.027	Italia	3
14	Colonia	0.438	Germania	2	38	Puteoli	0.041	Italia	3
19	Intercisa	0.584	Pannonia	2	40	Rome	0.083	Italia	3
39	Ravenna	0.581	Italia	2	41	Saguntum	0.015	Hispania	3
3	Aquincum	0.463	Pannonia	2	42	Salonae	0.09	Dalmatia	3
47	Viminacium	0.6	Moesia	2	43	Sarmizeget	0.216	Dacia	3
6	Brigetio	0.49	Pannonia	2	44	Tarquinii	0.015	Italia	3
10	Caralis/Sa	0.067	Italia	3	45	Treveri	0.031	Gallia	3
12	Catina	0.01	Italia	3	48	Virunum	0.153	Noricum	3
13	Celeia	0.136	Noricum	3	4	Arelate	0.032	Gallia	3
15	Emerita	0.062	Hispania	3	5	Beneventum	0.02	Italia	3
16	Emona	0.034	Pannonia	3	7	Brundisium	0.023	Italia	3
17	Flavia Solva	0.148	Noricum	3	8	Burdigala	0.022	Gallia	3
18	Gades	0.007	Hispania	3	9	Capua	0.068	Italia	3
2	Aquileia	0.148	Italia	3					

Female Population without Olisippo (Outlier); p_{mil}=proportion of military epitaphs



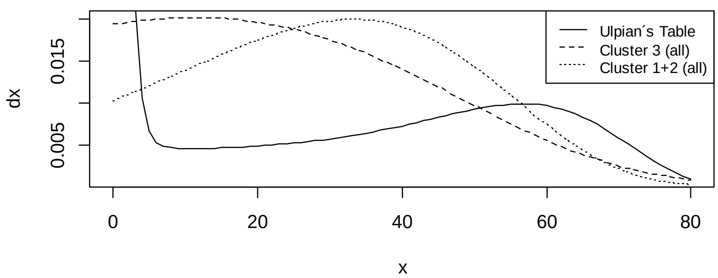


Figure A1: Comparison of life table and death density functions
(Ulpian´s Table: see Pflaumer, 2015b, Table 3; Cluster 3 and Cluster 1+2 (weighted average): see Table 1)