

Distributions of Age at Death from Roman Epitaph Inscriptions in North Africa¹

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Abstract:

Thousands of inscriptions of age at death from Roman epitaphs in North Africa are statistically analyzed. The Gompertz distribution is used to estimate survivor functions. The smoothed distributions are classified according to the estimation results. Similarities and differences can be detected more easily. Parameters such as mean, mode, skewness, and kurtosis are calculated. Cluster analysis provides three typical distributions. The analysis of the force of mortality function of the three clusters shows that the epigraphic sample is not representative of the mortality in North Africa. The results are compared with data from epitaphs from the European provinces. Africa is quite different. The general mortality level is much lower. The African cluster is much more homogeneous than the European cluster. The distributions are determined by three factors: mortality levels, commemorative processes, and population growth rates.

Keywords: Gompertz distribution, data analysis, cluster analysis, mortality, life table, Roman demography

1. Introduction

This paper continues an investigation by Pflaumer (2016) in which ages at death from epitaphs of the European provinces of the Roman Empire are statistically analyzed. Age at death distributions are analyzed and categorized. “Roman North Africa is altogether different. Unlike in Europe, African epitaphs almost invariably give decedents’ age at death” (Frier 2000, p. 791). It was customary in Rome to record the age of death of young decedents, but the age of elderly decedents was not recorded so often (Parkin 1992, p. 9). These differences in the practice of commemoration are one reason for the much higher mean ages of death in the African provinces. The epigraphic samples are not representative of the mortality in the African provinces, as we can see, for example, in Figure 1. Infant mortality is always underestimated, but old age mortality is also underestimated. Epitaphs of elderly deceased individuals sometimes describe a remarkable longevity. We observe age rounding by multiples of five, and a gender bias, as men are more likely than women to have an epitaph with an age inscription, and this is reflected in a high sex ratio. The survivor functions are compared with that of the Suessmilch life table, which represents mortality in the eighteenth century (Suessmilch 1775). Johann Peter Suessmilch (1707-1767), one of the founding fathers of demography in Germany, published a life table with a life expectation at birth of about 29 years.

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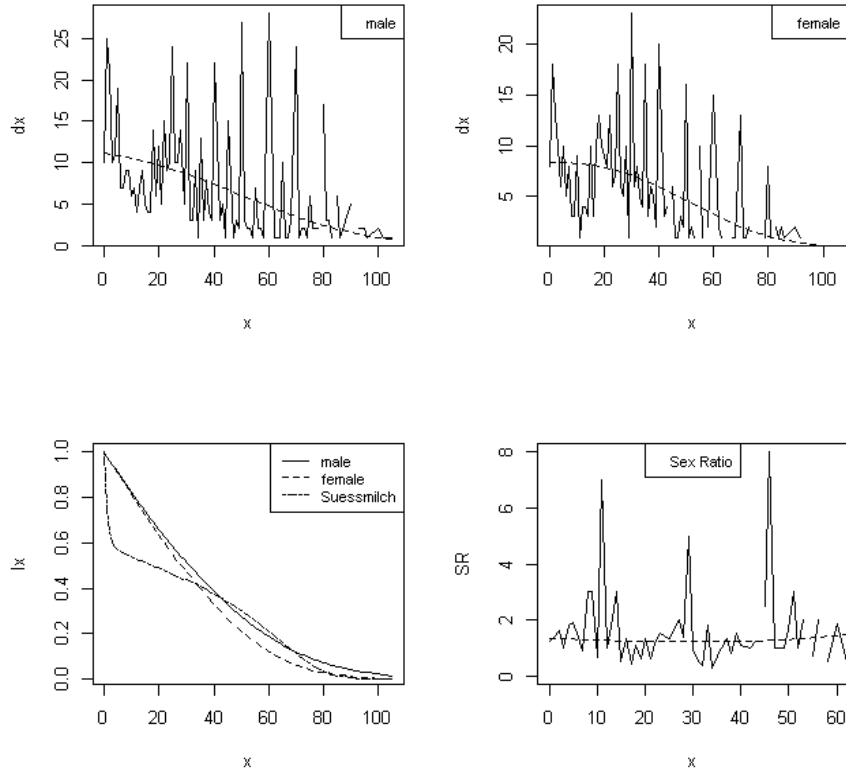


Figure 1: Recorded and smoothed age at death distributions dx (male and female), survivor functions lx , and recorded and smoothed age specific sex ratios SR (Carthage)

2. Data and Distributions

The inscriptions number 18,056 (10,410 males, 7,646 females), and were collected by the Hungarian scholar Szilágyi (1965, 1966, 1967). The data collection comes from 31 cities and provinces of the African part of the Roman Empire between the first and the seventh centuries (see Table A1 in Appendix 2). Minor addition errors in Szilágyi's data were corrected. The age at death distributions were smoothed, because the dominance of ages that are multiples of five hides the essential shape of the distributions. The smoothing function was the Gompertz distribution (see, e.g., Pollard 1991), which was fitted to the corresponding survivor functions by non-linear least squares. As a result, we can represent the age at death distributions by a two-parameter function with A and k . From Figure 3 it can be seen that the survivor functions can be well represented by survivor functions of the Gompertz distribution. The smoothed age at death distributions shown in Figure 2 seem, at first glance, very different. We have distributions that are skewed to the left, symmetrical distributions, and distributions that are skewed to the right. The mean ranges between 29.7 in Caesarea and 60.6 in Thagaste (see Table A2). These means are significantly higher than in the European provinces, where they are between 21 and 47 years (see Figure 4). The mean age and the other parameters have been calculated from the fitted Gompertz distribution by numerical integration. Presumably the statistics represent normal or average conditions of mortality during a period of several centuries.

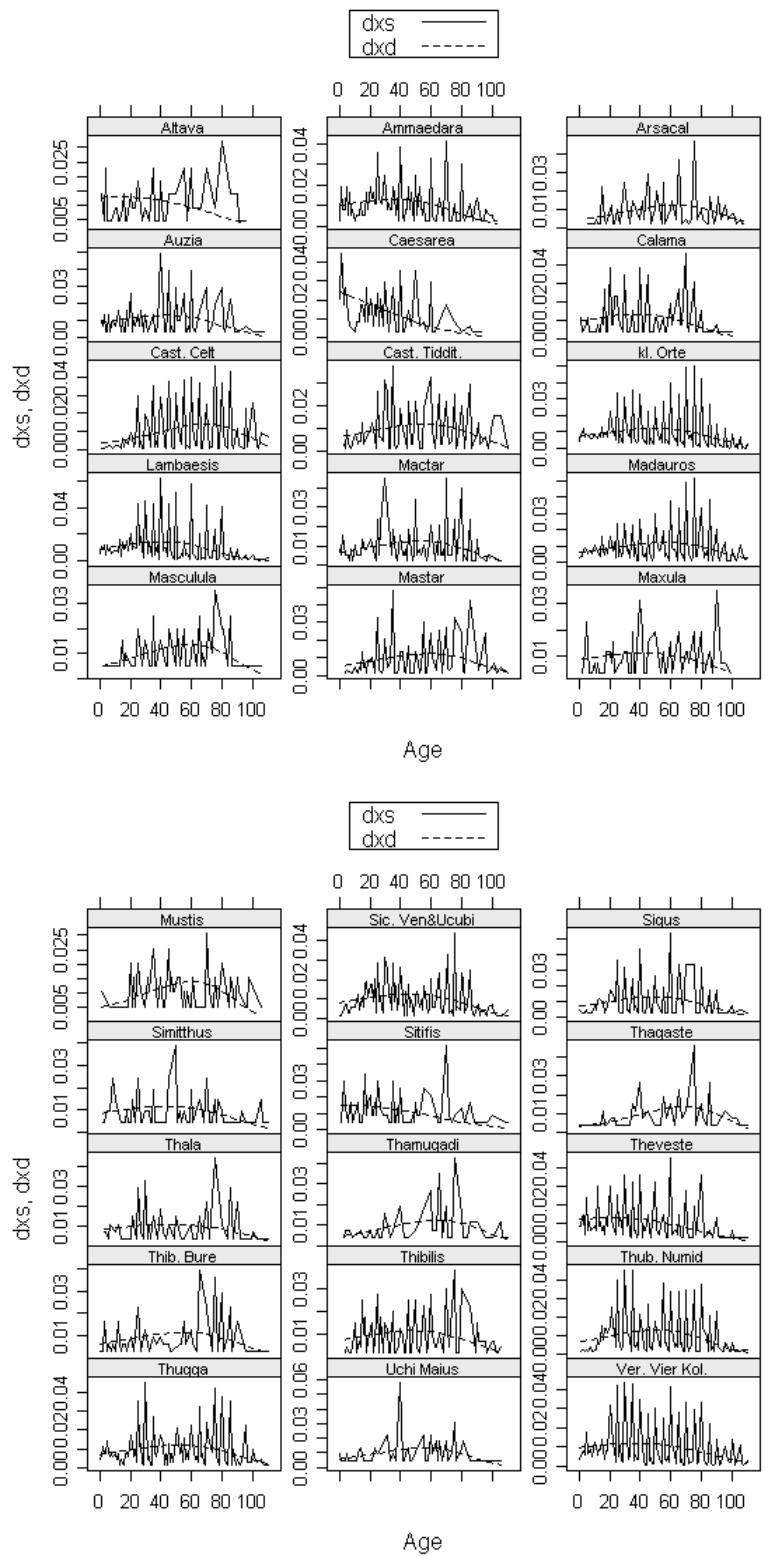


Figure 2: Recorded (dxs) and fitted (dxd) age at death distributions without Carthage (Fig. 1); Abbreviations of the cities are explained in Appendix 2

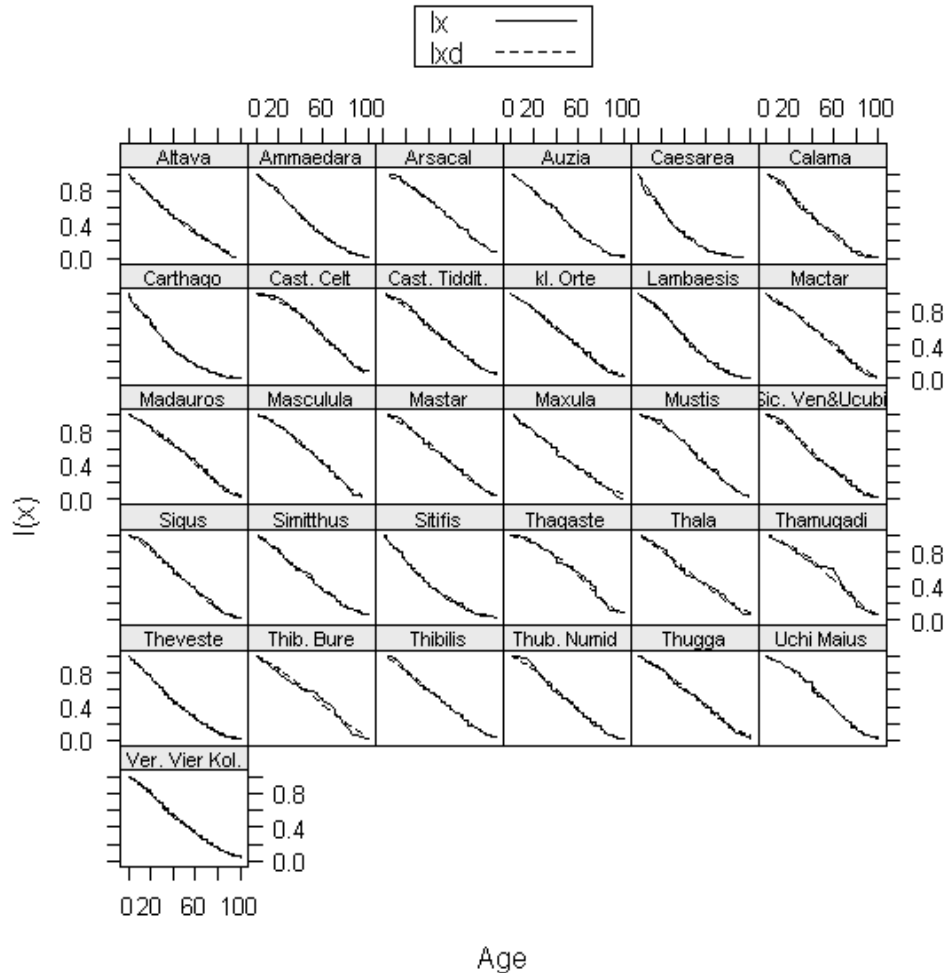


Figure 3: Survivor functions (recorded= l_x , fitted= l_{xd})

The accuracy of the fit is also seen in Figure 4, where the theoretical means from the smoothed data are compared with the original means from the unsmoothed data, calculated by Szilágyi. The shapes of the distributions are determined by the parameters A and k of the Gompertz distribution, which are shown as a scatter plot in Figure 4. We can identify groups or clusters of similar parameters. The points in the lower right corner are characterized by high values of A and low values of k , whereas the points in the upper left corner show low A values and high k values. The typical A - k constellation of a real life table has very low A values, and k values ranging between 0.05 and 0.13. For example, the parameters of the Suessmilch life table are $A=0.00081$ and $k=0.065$, whereas the parameters of the German life table (female) 2007-2009 are $A=0.00000199$ and $k=0.125$. For our data, apart from a few outliers, the values of A do not exceed 0.01, and the values of k are mostly between 0.02 and 0.035. Compared with the parameter constellations of the European provinces, the variation of A and k in Africa is small (see Figure 5). The African distributions form a special cluster that is much more homogeneous than the European clusters (see Figure 6).

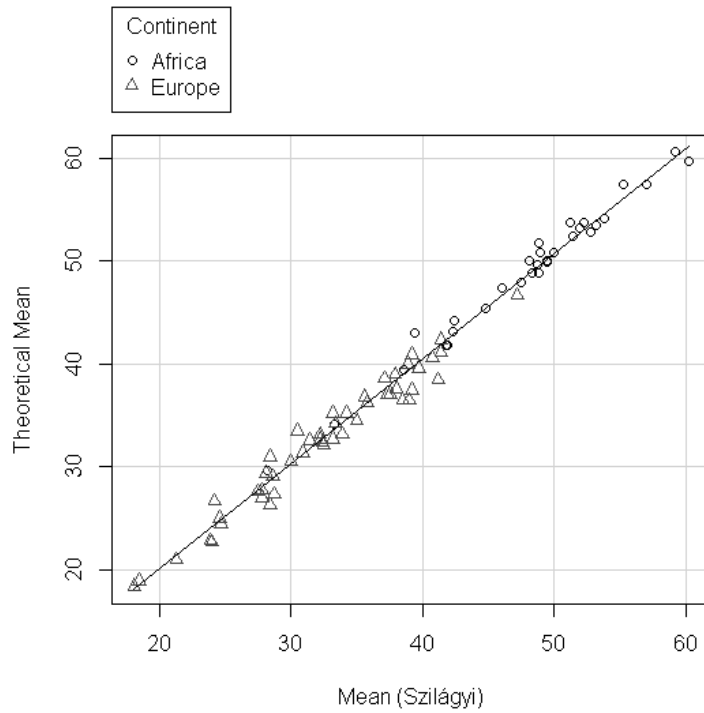


Figure 4 : Means of the data set and the theoretical distributions
 (Correlation coefficients: 0.9932 (all; n=79), 0.9808 (Europe; n=48), 0.9910 (Africa; n=31))

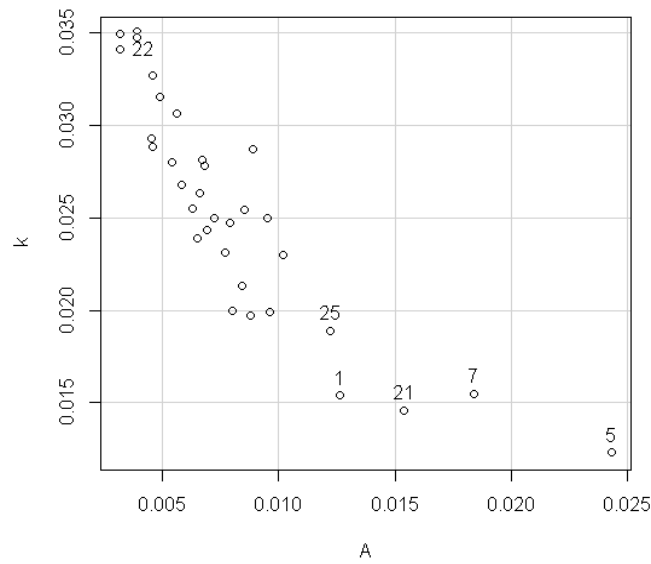


Figure 5: Parameters A and k for the theoretical distributions
 (8: Castellum Celtianum; 22: Thagaste; 25: Theveste; 1: Altava, 21: Sitifis; 7: Carthago; 5: Caesarea)

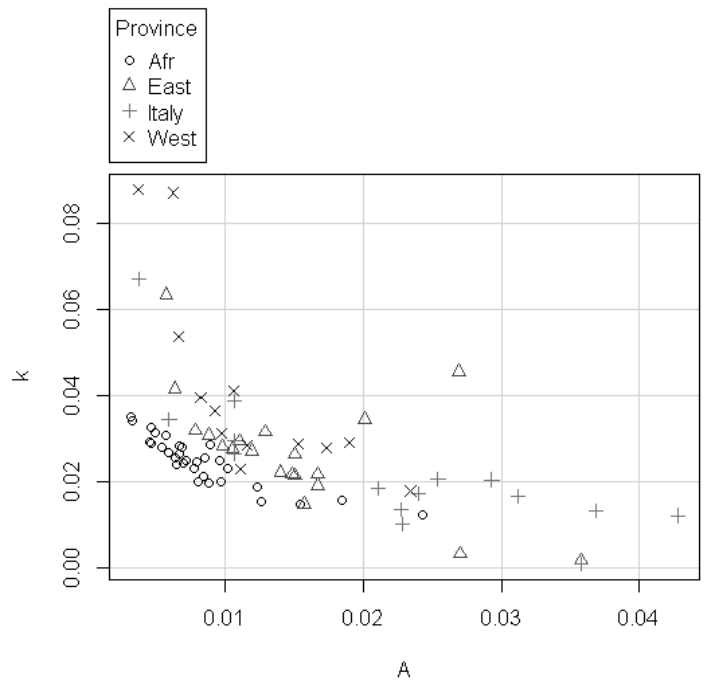


Figure 6: Parameters A and k for the theoretical distributions in different Roman provinces

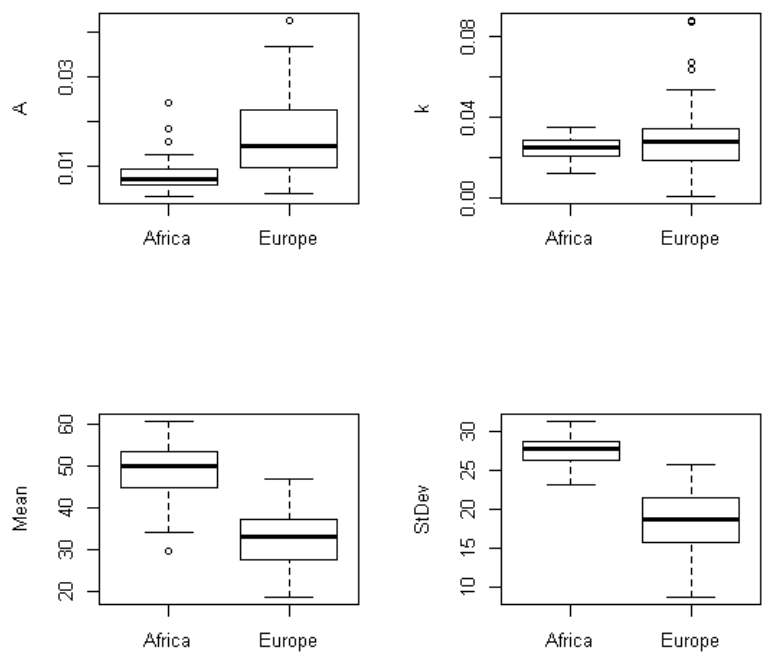


Figure 7a: Box plots for the parameters A and k, mean and standard deviation (Africa: n=31; Europe: n=4)

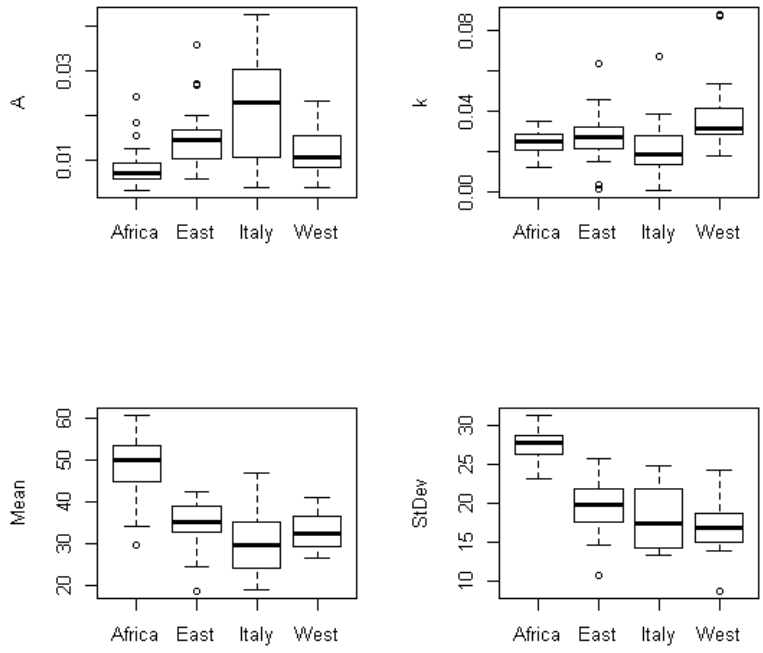


Figure 7b: Box Plots for the parameters A and k, mean and standard deviation (Africa: n=31; East: n=20; Italy: n=15; West: n=13)

The differences between Europe and Africa can be clearly seen in the series of box plots in Figures 7a and 7b and in Appendix 3. The main difference is in A, not in k. A, which represents the general mortality level that is independent of the age x , is much lower in Africa, whereas k, the senescent component, is more or less the same. In Europe, the variance of k is higher. Since $\frac{dl(x)}{dA} < 0$, a smaller value of A leads to a higher mean.² The smaller A is, the better are the external living conditions (e.g., better climatic conditions, and fewer epidemics and famines). The quotient $Q = \frac{A}{k}$ is smaller in Africa than in Europe. Therefore, the distributions of age at death are more skewed to the right in Europe (see also Pflaumer 2016, Appendix 1).

3. Cluster Analysis

In the next step, a cluster analysis was applied in order to group the different smoothed distributions into similar categories. We performed k-means clustering on a data matrix containing each of the 31 values of A and k. The method requires one to specify the number of clusters to extract. After some trials with different numbers, we concluded that the number of clusters should be three. The results are seen in Figure 8, Table 1 and Table A3 in Appendix 2. Averages are not weighted. Because of the high distribution of

² See also Appendix 3.

males, only small differences between the set of graphs dx and dxm at the top of Figure 8 can be seen. We therefore restrict our explanation to the male (dxm) and female (dxw) age distributions.

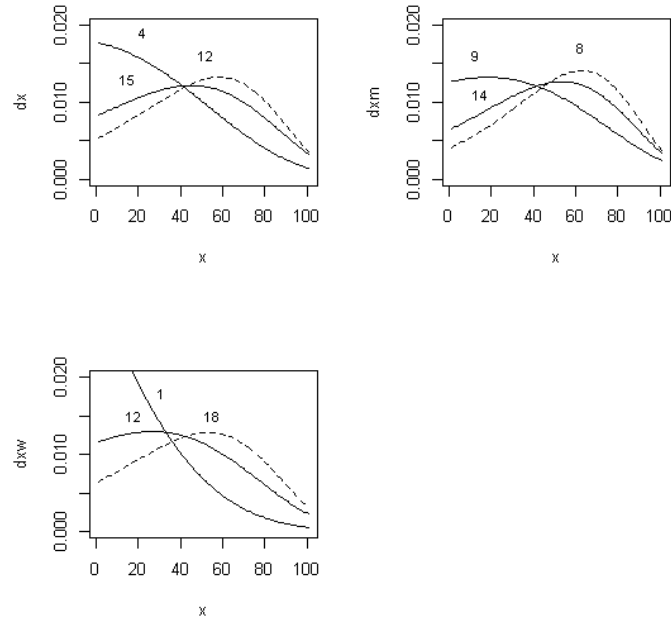


Figure 8: Age at death distributions in different clusters (numbers represent cluster size, dx=all, dxm=male, dxw=female)

In essence, we can identify three distributions for the males, with two clusters differing only slightly (B and C), and two clusters for the females, if we neglect the outlier cluster A that has only one observation (see Table 1). The results point to a different attitude to age and age commemoration. The most popular ages for men and women to be commemorated are between 40 and 60 (B and C). There is less emphasis on the commemoration of those who die young (A).

Table 1: Parameters of the different clusters

| Cluster | A | k | n | Modus | Mean | Stand.Dev. | Skew | Kurtosis |
|---------|--------|--------|----|-------|------|------------|--------|----------|
| all | | | | | | | | |
| A | 0.0177 | 0.0145 | 4 | 0 | 35.8 | 26.1 | 0.802 | 0.174 |
| B | 0.0083 | 0.0231 | 15 | 44.4 | 48.1 | 28.1 | 0.305 | -0.626 |
| C | 0.0054 | 0.0301 | 12 | 57.4 | 52.2 | 26.6 | 0.050 | -0.728 |
| male | | | | | | | | |
| A | 0.0127 | 0.0172 | 9 | 17.7 | 42.3 | 28.3 | 0.592 | -0.262 |
| B | 0.0065 | 0.0270 | 14 | 52.6 | 50.6 | 27.3 | 0.155 | -0.711 |
| C | 0.0042 | 0.0339 | 8 | 62.0 | 54.5 | 25.7 | -0.076 | -0.701 |
| female | | | | | | | | |
| A | 0.0313 | 0.0082 | 1 | 0 | 26.2 | 22.3 | 1.271 | 1.706 |
| B | 0.0117 | 0.0193 | 12 | 26.1 | 42.6 | 27.5 | 0.512 | -0.391 |
| C | 0.0065 | 0.0277 | 18 | 52.4 | 49.9 | 26.8 | 0.145 | -0.714 |

Figure 9 exhibits the force of mortality functions in the three clusters for the male and female populations. The functions of the epitaph populations are compared with the force of mortality functions of the Suessmilch life table from the eighteenth century. The force of mortality approximately shows the number of individuals dying at age x as a percentage of those surviving to that age.

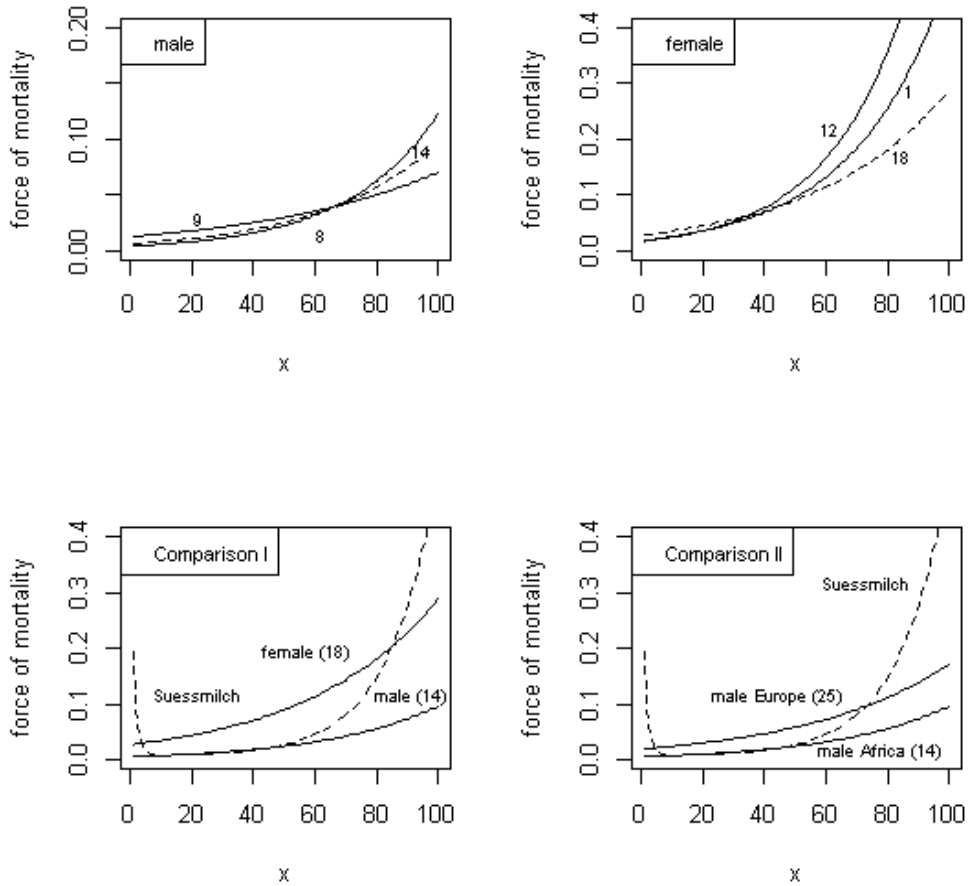


Figure 9: Force of mortality functions in different clusters (n =cluster size)

We can summarize the findings with a slightly changed quote of Frier (2000, p. 791): “Although the African epitaphs under represent juveniles, they produce credible mortality rates for males and females [in the middle age classes]; and, ...these mortality levels are reasonably comparable to Model South, level 2, in which both sexes have a life expectancy at birth of about 22.5 years. In later ages, however, mortality rates are artificially lowered owing to age exaggeration, a phenomenon that apparently begins somewhat later for women than for men.” However, Hopkins (1966, p. 257) mentioned for the garrison town Lambaesis: “An almost equal number of military personnel and civilians, each with equally improbable but opposite recorded characteristics, have been amalgamated to make an average, which on first sight appears reasonable”. But even if the mortality rates are credible in the middle age classes, we are not able to fit a mortality function to the whole age span, without knowing the pattern for the young and old ages. If we compare the pattern with the Suessmilch life table, which is well justified (see

Figure 9), then we obtain an estimated life expectancy of 29 years, which is much higher than Frier's estimation of 22.5 years.

4. Conclusion

In Africa, the distributions of age at death have higher means and variances than those in Europe (see Fig. 10). The analysis of the force of mortality function of the three clusters shows that the epigraphic sample is not representative of the mortality in North Africa.

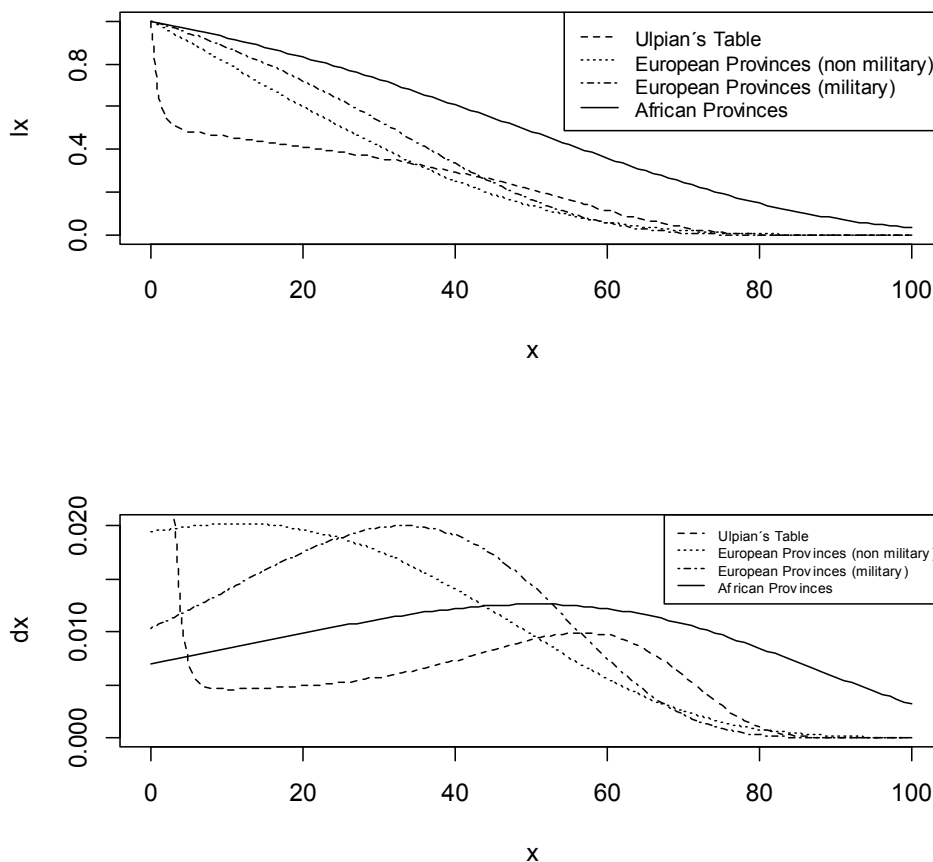


Figure 10: Comparison of life table and death density functions (European Provinces (male and female): see Pflaumer 2016, Table 1; Ulpien's Table: see Pflaumer, 2015, Table 3; African provinces are represented by a weighted average of the parameters of the clusters B and C (male and female) in Table 1)

The distributions and their parameters depend on the three factors: Mortality, commemorative processes, and growth rate of the population. Using the Gompertz distribution, we find that the general mortality parameter A is much lower in Africa than in the European provinces. Thus, the higher mean in the distributions reflects a higher life expectancy in the African provinces, without contesting the influence of commemorative processes. The reasons are speculative: better climatic conditions, fewer pandemic

diseases, or fewer famines. The skewed distribution of Carthage (see Figure 1) indicates an influence of the population growth rate due to migration, which could also be observed in other populous cities, e.g., Rome (see Pflaumer 2016, section 4 or Durand 1959).

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Appendix

1. The influence of A and k on mean, median, and modus

An increase of A and k leads to a decrease in both the mean and the median, since the derivatives $\frac{dl(x)}{dA}$ and $\frac{dl(x)}{dk}$ are negative. But A and k act contrary regarding the rectangularization of a life table. An increase of k yields a higher degree, whereas an increase of A yields a lower degree of rectangularization, if we measure the degree, e.g., with the “Prolate Rectangularity Index of Eakin and Witten” or its modification³. An

increase of A also reduces the modus $m = -\frac{\ln\left(\frac{A}{k}\right)}{k}$, since $\frac{dlm}{dA} = -\frac{1}{A \cdot k} < 0$. However, an

increase of k reduces the modus only, if $k > A \cdot e$, since $\frac{dlm}{dk} = \frac{1 + \ln\left(\frac{A}{k}\right)}{k^2}$.

³ see Pflaumer, P (2010): Measuring the Rectangularization of Life Tables Using the Gompertz Distribution, *JSM Proceedings, Social Statistics Section*. Alexandria, VA: American Statistical Association: 664-674.

2. Tables

Table A1: Estimation Results

| ID | Region | k | A | km | Am | kw | Aw | nm | nw | n |
|----|------------------|--------|--------|--------|--------|--------|--------|------|------|------|
| 1 | Altava | 0.0154 | 0.0126 | 0.0168 | 0.0121 | 0.0139 | 0.0130 | 55 | 47 | 102 |
| 2 | Ammaedara | 0.0230 | 0.0102 | 0.0264 | 0.0082 | 0.0199 | 0.0127 | 171 | 148 | 319 |
| 3 | Arsacal | 0.0288 | 0.0046 | 0.0293 | 0.0046 | 0.0282 | 0.0046 | 123 | 89 | 212 |
| 4 | Auzia | 0.0254 | 0.0085 | 0.0289 | 0.0071 | 0.0211 | 0.0107 | 133 | 87 | 220 |
| 5 | Caesarea | 0.0123 | 0.0243 | 0.0156 | 0.0205 | 0.0082 | 0.0313 | 158 | 92 | 250 |
| 6 | Calama | 0.0250 | 0.0095 | 0.0201 | 0.0104 | 0.0337 | 0.0080 | 97 | 74 | 171 |
| 7 | Carthago | 0.0155 | 0.0184 | 0.0137 | 0.0180 | 0.0198 | 0.0183 | 620 | 453 | 1073 |
| 8 | Cast. Celtianum | 0.0349 | 0.0032 | 0.0372 | 0.0029 | 0.0325 | 0.0035 | 680 | 578 | 1258 |
| 9 | Cast. Tiddit. | 0.0255 | 0.0063 | 0.0281 | 0.0060 | 0.0230 | 0.0066 | 229 | 199 | 428 |
| 10 | kl. Orte | 0.0250 | 0.0072 | 0.0256 | 0.0067 | 0.0243 | 0.0078 | 3249 | 2218 | 5467 |
| 11 | Lambaesis | 0.0287 | 0.0089 | 0.0300 | 0.0077 | 0.0284 | 0.0105 | 811 | 555 | 1366 |
| 12 | Mactar | 0.0263 | 0.0066 | 0.0319 | 0.0050 | 0.0197 | 0.0091 | 153 | 112 | 265 |
| 13 | Madauros | 0.0268 | 0.0058 | 0.0264 | 0.0057 | 0.0277 | 0.0060 | 461 | 323 | 784 |
| 14 | Masculula | 0.0315 | 0.0049 | 0.0286 | 0.0057 | 0.0350 | 0.0041 | 61 | 54 | 115 |
| 15 | Mastar | 0.0280 | 0.0054 | 0.0310 | 0.0051 | 0.0247 | 0.0057 | 209 | 150 | 359 |
| 16 | Maxula | 0.0197 | 0.0088 | 0.0150 | 0.0113 | 0.0250 | 0.0065 | 74 | 63 | 137 |
| 17 | Mustis | 0.0327 | 0.0046 | 0.0338 | 0.0039 | 0.0312 | 0.0059 | 71 | 46 | 117 |
| 18 | Sic. Ven & Ucubi | 0.0247 | 0.0079 | 0.0231 | 0.0084 | 0.0266 | 0.0073 | 368 | 293 | 661 |
| 19 | Sigus | 0.0278 | 0.0068 | 0.0313 | 0.0055 | 0.0241 | 0.0087 | 165 | 125 | 290 |
| 20 | Simitthus | 0.0213 | 0.0084 | 0.0264 | 0.0060 | 0.0182 | 0.0108 | 55 | 58 | 113 |
| 21 | Sitifis | 0.0146 | 0.0154 | 0.0137 | 0.0149 | 0.0166 | 0.0158 | 101 | 68 | 169 |
| 22 | Thagaste | 0.0341 | 0.0032 | 0.0367 | 0.0026 | 0.0328 | 0.0038 | 64 | 50 | 114 |
| 23 | Thala | 0.0200 | 0.0080 | 0.0194 | 0.0076 | 0.0216 | 0.0083 | 82 | 56 | 138 |
| 24 | Thamugadi | 0.0293 | 0.0045 | 0.0346 | 0.0031 | 0.0232 | 0.0068 | 53 | 44 | 97 |
| 25 | Theveste | 0.0189 | 0.0122 | 0.0207 | 0.0098 | 0.0184 | 0.0151 | 215 | 173 | 388 |
| 26 | Thib. Bure | 0.0239 | 0.0065 | 0.0260 | 0.0058 | 0.0218 | 0.0073 | 80 | 74 | 154 |
| 27 | Thibilis | 0.0231 | 0.0077 | 0.0262 | 0.0065 | 0.0199 | 0.0090 | 136 | 127 | 263 |
| 28 | Thub. Numid | 0.0281 | 0.0067 | 0.0282 | 0.0063 | 0.0285 | 0.0072 | 391 | 302 | 693 |
| 29 | Thugga | 0.0243 | 0.0069 | 0.0247 | 0.0066 | 0.0238 | 0.0074 | 367 | 249 | 616 |
| 30 | Uchi Maius | 0.0306 | 0.0056 | 0.0346 | 0.0051 | 0.0252 | 0.0064 | 73 | 48 | 121 |
| 31 | Ver. Vier Kol. | 0.0199 | 0.0096 | 0.0195 | 0.0094 | 0.0207 | 0.0098 | 905 | 691 | 1596 |

m: male, w: female

Cast. Tiddit.: Castellum Tidditanorum

kl. Orte: small cities in African provinces without Egypt (see Szilágyi 1967, p. 25)

Sic. Ven & Ucubi: Sicca Veneria and Ucubi

Thib. Bure: Thibursicum Bure

Thub. Numid: Thubursicum Numidarum

Ver. Vier Kol.: United four colonies: Quattuor Coloniae-Cirta, Chullu, Milev, Rusicade

Table A2: Parameters

| Region | ID | all | | | | male | | | | female | | | |
|------------------|----|------|------|-------|-------|------|------|-------|-------|--------|------|-------|-------|
| | | mean | std | skew | kurt | mean | std | skew | kurt | mean | std | skew | kurt |
| Altava | 1 | 44.2 | 30.1 | 0.63 | -0.19 | 44.0 | 29.3 | 0.58 | -0.28 | 44.9 | 31.2 | 0.69 | -0.08 |
| Ammaedara | 2 | 43.0 | 26.1 | 0.39 | -0.55 | 45.5 | 25.8 | 0.25 | -0.67 | 40.1 | 26.1 | 0.53 | -0.36 |
| Arsacal | 3 | 57.5 | 28.6 | 0.01 | -0.73 | 56.9 | 28.2 | 0.00 | -0.72 | 58.1 | 29.0 | 0.02 | -0.73 |
| Auzia | 4 | 45.4 | 26.2 | 0.28 | -0.65 | 46.8 | 25.4 | 0.16 | -0.71 | 43.4 | 27.1 | 0.44 | -0.49 |
| Caesarea | 5 | 29.7 | 23.2 | 1.00 | 0.74 | 31.6 | 23.3 | 0.83 | 0.25 | 26.2 | 22.3 | 1.27 | 1.71 |
| Calama | 6 | 43.1 | 25.5 | 0.33 | -0.61 | 45.0 | 28.2 | 0.45 | -0.48 | 40.8 | 22.0 | 0.15 | -0.71 |
| Carthago | 7 | 34.2 | 24.8 | 0.79 | 0.15 | 36.0 | 26.5 | 0.83 | 0.25 | 31.8 | 22.1 | 0.69 | -0.09 |
| Cast. Celt. | 8 | 59.7 | 26.6 | -0.17 | -0.64 | 59.6 | 25.6 | -0.22 | -0.60 | 60.2 | 27.6 | -0.12 | -0.68 |
| Cast. Tiddit. | 9 | 52.9 | 28.7 | 0.16 | -0.71 | 51.5 | 27.2 | 0.11 | -0.72 | 54.4 | 30.4 | 0.22 | -0.68 |
| kl. Orte | 10 | 49.9 | 27.9 | 0.22 | -0.68 | 51.2 | 28.1 | 0.19 | -0.70 | 48.6 | 27.8 | 0.26 | -0.66 |
| Lambaesis | 11 | 41.9 | 23.8 | 0.25 | -0.67 | 44.1 | 24.1 | 0.18 | -0.70 | 38.5 | 22.6 | 0.32 | -0.62 |
| Mactar | 12 | 50.9 | 27.7 | 0.17 | -0.71 | 52.3 | 25.9 | 0.00 | -0.72 | 49.0 | 30.1 | 0.40 | -0.53 |
| Madauros | 13 | 53.7 | 28.4 | 0.12 | -0.72 | 54.6 | 28.8 | 0.12 | -0.72 | 51.9 | 27.4 | 0.12 | -0.72 |
| Masculula | 14 | 53.2 | 26.3 | 0.00 | -0.72 | 52.3 | 27.2 | 0.09 | -0.73 | 54.0 | 25.2 | -0.09 | -0.69 |
| Mastar | 15 | 54.2 | 28.0 | 0.08 | -0.73 | 52.7 | 26.4 | 0.02 | -0.73 | 56.5 | 30.2 | 0.14 | -0.72 |
| Maxula | 16 | 50.0 | 30.4 | 0.39 | -0.55 | 47.9 | 32.1 | 0.60 | -0.25 | 52.6 | 28.8 | 0.18 | -0.70 |
| Mustis | 17 | 53.5 | 25.9 | -0.03 | -0.72 | 56.3 | 26.2 | -0.10 | -0.69 | 49.1 | 25.3 | 0.07 | -0.73 |
| Sic. Ven & Ucubi | 18 | 47.9 | 27.3 | 0.26 | -0.66 | 47.8 | 28.0 | 0.31 | -0.62 | 48.1 | 26.7 | 0.20 | -0.69 |
| Sigus | 19 | 48.8 | 26.4 | 0.16 | -0.71 | 50.7 | 25.7 | 0.04 | -0.73 | 46.0 | 26.9 | 0.31 | -0.62 |
| Simitthus | 20 | 49.6 | 29.5 | 0.34 | -0.59 | 53.2 | 28.4 | 0.13 | -0.72 | 45.8 | 29.4 | 0.50 | -0.40 |
| Sitifis | 21 | 39.4 | 28.0 | 0.74 | 0.03 | 41.1 | 29.3 | 0.75 | 0.06 | 37.1 | 25.9 | 0.70 | -0.06 |
| Thagaste | 22 | 60.6 | 27.1 | -0.17 | -0.65 | 62.6 | 26.4 | -0.25 | -0.57 | 57.9 | 27.0 | -0.10 | -0.69 |
| Thala | 23 | 52.4 | 31.2 | 0.35 | -0.59 | 54.6 | 32.5 | 0.34 | -0.60 | 49.6 | 29.3 | 0.33 | -0.60 |
| Thamugadi | 24 | 57.5 | 28.4 | 0.00 | -0.72 | 60.8 | 26.9 | -0.18 | -0.64 | 53.3 | 29.9 | 0.23 | -0.68 |
| Theveste | 25 | 41.9 | 27.3 | 0.54 | -0.35 | 46.0 | 28.3 | 0.41 | -0.52 | 37.0 | 25.2 | 0.64 | -0.18 |
| Thib. Bure | 26 | 53.8 | 29.8 | 0.20 | -0.69 | 54.5 | 29.0 | 0.13 | -0.72 | 52.9 | 30.5 | 0.28 | -0.65 |
| Thibilis | 27 | 50.1 | 28.8 | 0.28 | -0.65 | 51.4 | 27.9 | 0.17 | -0.71 | 49.1 | 30.0 | 0.39 | -0.54 |
| Thub. Numid | 28 | 48.8 | 26.3 | 0.15 | -0.71 | 50.2 | 26.7 | 0.13 | -0.72 | 46.8 | 25.5 | 0.17 | -0.71 |
| Thugga | 29 | 51.8 | 28.9 | 0.22 | -0.69 | 52.5 | 28.9 | 0.19 | -0.70 | 50.4 | 28.6 | 0.25 | -0.67 |
| Uchi Maius | 30 | 50.9 | 26.0 | 0.06 | -0.73 | 49.6 | 24.2 | -0.02 | -0.72 | 52.8 | 28.8 | 0.17 | -0.70 |
| Ver. Vier Kol | 31 | 47.4 | 29.3 | 0.42 | -0.51 | 48.3 | 29.9 | 0.42 | -0.51 | 46.0 | 28.3 | 0.41 | -0.52 |

Parameters have been calculated by numerical integration.

Cast. Tiddit.: Castellum Tidditanorum

kl. Orte: small cities in African provinces without Egypt (see Szilágy 1967, p. 25)

Sic. Ven & Ucubi: Sicca Veneria and Ucubi

Thib. Bure: Thibursicum Bure

Thub. Numid: Thubursicum Numidarum

Ver. Vier Kol.: United four colonies: Quattuor Coloniae-Cirta, Chullu, Milev, Rusicade

Table A3: Clusters

| male | | | female | | |
|------|------------------|---------|--------|------------------|---------|
| ID | Region | Cluster | ID | Region | Cluster |
| 1 | Altava | A | 1 | Altava | B |
| 16 | Maxula | A | 12 | Mactar | B |
| 21 | Sitifis | A | 2 | Ammaedara | B |
| 23 | Thala | A | 20 | Simitthus | B |
| 25 | Theveste | A | 21 | Sitifis | B |
| 31 | Ver. Vier Kol | A | 23 | Thala | B |
| 5 | Caesarea | A | 25 | Theveste | B |
| 6 | Calama | A | 26 | Thib. Bure | B |
| 7 | Carthago | A | 27 | Thibilis | B |
| 10 | kl. Orte | B | 31 | Ver. Vier Kol | B |
| 11 | Lambaesis | B | 4 | Auzia | B |
| 13 | Madauros | B | 7 | Carthago | B |
| 14 | Masculula | B | 10 | kl. Orte | C |
| 18 | Sic. Ven & Ucubi | B | 11 | Lambaesis | C |
| 2 | Ammaedara | B | 13 | Madauros | C |
| 20 | Simitthus | B | 14 | Masculula | C |
| 26 | Thib. Bure | B | 15 | Mastar | C |
| 27 | Thibilis | B | 16 | Maxula | C |
| 28 | Thub. Numid | B | 17 | Mustis | C |
| 29 | Thugga | B | 18 | Sic. Ven & Ucubi | C |
| 3 | Arsacal | B | 19 | Sigus | C |
| 4 | Auzia | B | 22 | Thagaste | C |
| 9 | Cast. Tiddit. | B | 24 | Thamugadi | C |
| 12 | Mactar | C | 28 | Thub. Numid | C |
| 15 | Mastar | C | 29 | Thugga | C |
| 17 | Mustis | C | 3 | Arsacal | C |
| 19 | Sigus | C | 30 | Uchi Maius | C |
| 22 | Thagaste | C | 6 | Calama | C |
| 24 | Thamugadi | C | 8 | Cast. Celt. | C |
| 30 | Uchi Maius | C | 9 | Cast. Tiddit. | C |
| 8 | Cast. Celt. | C | 5 | Caesarea | A |

Cast. Tiddit.: Castellum Tidditanorum

kl. Orte: small cities in African provinces without Egypt (see Szilágy 1967, p. 25)

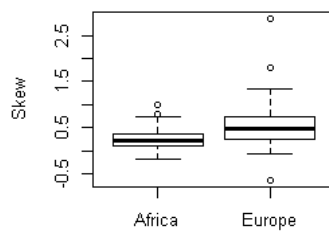
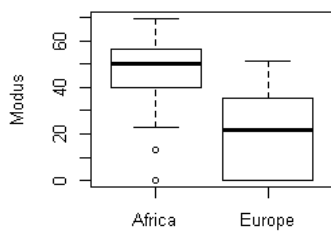
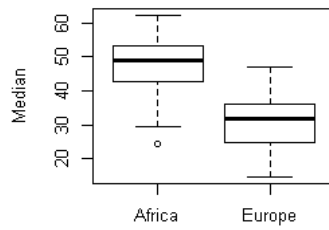
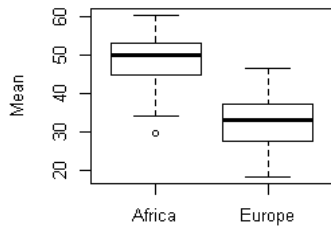
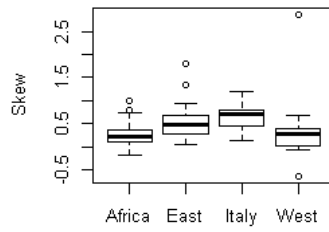
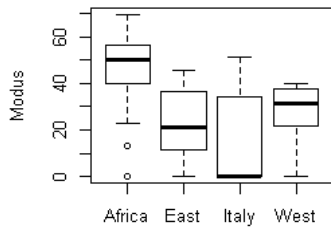
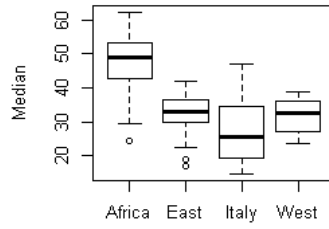
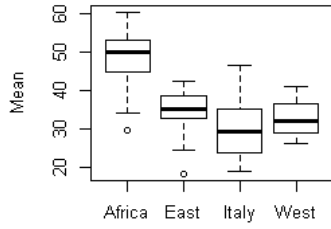
Sic. Ven & Ucubi: Sicca Veneria and Ucubi

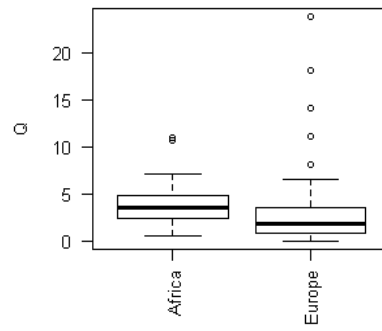
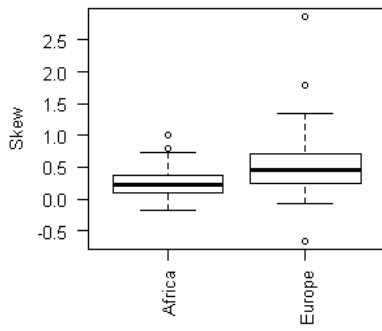
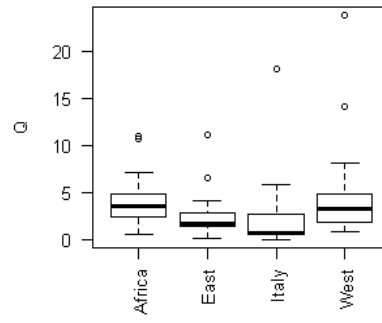
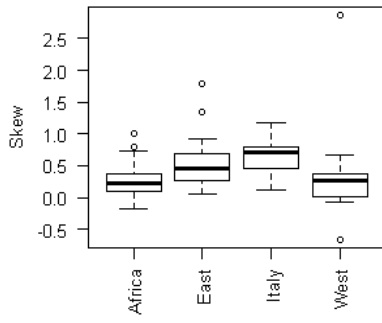
Thib. Bure: Thibursicum Bure

Thub. Numid: Thubursicum Numidarum

Ver. Vier Kol.: United four colonies: Quattuor Coloniae-Cirta, Chullu, Milev, Rusicade

3. Box Plots





$Q=A/k$