Evolutionary Models of Market Structure

by

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"The organisms that can gain the new features faster are more variable. As a result, they gain advantages over other creatures. [...] Animals are higher than plants, because they are able to move consciously, go after food, find and eat useful things. [...] There are many differences between the animal and plant species, [...] First of all, the animal kingdom is more complicated. Besides, reason is the most beneficial feature of animals. Owing to reason, they can learn new things and adopt new, non-inherent abilities. For example, the trained horse or hunting falcon is at a higher point of development in the animal world. The first steps of human perfection begin from here. Such humans [probably anthropoid apes] live in the Western Sudan and other distant corners of the world. They are close to animals by their habits, deeds and behavior. [...] The human has features that distinguish him from other creatures, but he has other features that unite him with the animal world, vegetable kingdom or even with the inanimate bodies. [...] Before [the creation of humans], all differences between organisms were of the natural origin. The next step will be associated with spiritual perfection, will, observation and knowledge. [...] All these facts prove that the human being is placed on the middle step of the evolutionary stairway. According to his inherent nature, the human is related to the lower beings, and only with the help of his will can he reach the higher development level."

Nasir Tusi (a Persian Scholar (1201-1274) from the book "The Nasirean Ethics" translated from Persian by Wickens (2011, pp.43-48). In one chapter of this book Tusi developed a basic theory of evolution, foreshadowing the theories of European scientists like Lamarck (1809) and Darwin (1859) by more than 600 years.)

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Abstract

The dispute on oligopoly theory was commenced by Vega-Redondo (1997), in which he showed that long run outcome of a symmetric Cournot oligopoly game equals the competitive Walrasian. In this dissertation, first we extend an evolutionary game theoretic model to an asymmetric oligopolistic model where we obtain a form of equilibrium so-called *Walrasian in Expectation* with market price equal to average marginal costs. Then we analyse firms competition under relative payoffs maximizing (RPM) behaviour. RPM behaviour is implied by evolutionary stability. We consider a simple model of symmetric oligopoly where firms select a two dimensional strategy set of price and a non-price variable known as quality simultaneously. The role of cross-elasticities of demand will be shown in determining the evolutionary equilibrium. Finally, a nonparametric revealed preference approach will present an empirical content of the evolutionary oligopoly model. Testable restrictions are derived for an evolutionary model of asymmetric oligopoly in which firms have different cost functions to produce a homogenous good. A case study on the crude oil market with main producers is presented and we compare the rejection rates of both Cournot and evolutionary hypotheses.

Publications and Presentations

Former versions of Chapters 2, 3, and 4 appeared at the *Ruhr Economic Papers* publication series with numbers 497, 575, and 576 respectively.

Presentation based on Chapters 2 and 3 was done at the RGS Jamboree 2013 and 2014 and furthermore presentations based on Chapter 3 and 4 were given in TU Dortmund Brown Bag Seminar 2014 and 2015. In addition, chapters of this dissertation are presented at the international conferences.

Presentations of Chapter 3 at international conferences are as follows:

- International PhD Meeting of Thessaloniki in Economics, Greece (27-28 June 2014)
- Workshop on Behavioral Economics and Industrial Organization at Corvinus University of Budapest, Hungary (1-2 September 2014)
- 11th Spain-Italy-Netherlands Meeting on Game Theory (SING11) and European Meeting on Game Theory in St. Petersburg, Russia. (8 - 10 July 2015)

Presentations of Chapter 4 at international conferences are as follows:

- 49th Annual conference of Canadian Economic Association at Ryerson University, Toronto, Canada (29- 31 May 2015)
- Royal Economic Society PhD Meeting at the Westminster Business School, London, UK (8-9 January 2016)
- 5th World Congress of the Game Theory Society at Maastricht University, The Netherlands (24-28 July 2016)
- 69th European Meeting of the Econometric Society EEA-ESEM, Geneva, Switzerland (22 -26 August 2016)

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Abbreviations

APM	**A**bsolute **P**ayoffs **M**aximizing
BRC	**B**est **R**eachability **C**ondition
ESS	**E**volutionarily **S**table **S**trategy
FPESS	**F**inite **P**opulation **E**volutionarily **S**table **S**trategy
GSS	**G**lobally **S**urviving **S**trategy
LP	**L**inear **P**rogramming
MER	**M**onthly **E**nergy **R**eview
MWV	**M**ineraloel**W**irtschafts**V**erband (Association of the German Petroleum Industry)
NE	**N**ash **E**quilibrium
OPEC	**O**rganization of the **P**etroleum **E**xporting **C**ountries
IO	**I**ndustrial **O**rganization
QP	**Q**uadratic **P**rogramming
R&D	**R**esearch and **D**evelopment
RPM	**R**elative **P**ayoffs **M**aximizing
SSS	**S**tochastically **S**table **S**tate
WLC	**W**orst **L**eaving **C**ondition

Chapter 1

Introduction

Theory of oligopoly with a long history is placed at the heart of industrial organization. In 1838, Antoine Augustin Cournot published the book, "Researches on the Mathematical Principles of the Theory of Wealth", where he formally provided a mathematical foundation to explain the source of market power in oligopolistic markets. After almost half a century, Joseph Louis Franois Bertrand (1883) disapproved Cournot's work in which he discussed that if firms chose prices instead of quantities, then the competitive outcome with price equal to marginal cost would arise. Bertrand's conclusion then was proofed as well by Edgeworth (1889) in the case of a cost function under the law of diminishing returns. But, which of these two static oligopoly theories is right? This leads to what it known as Bertrand paradox in economics, i.e., Nash equilibria in quantity strategy and in price strategy are so different. Indeed, the number of firms in the industry is unrelated to the study of price competition and Bertrand paradox asserts that even oligopolists behave like competitive firms.

There are several lines of research in the literature of oligopoly theory that attempt to resolve the paradox by generalization or relaxing some assumptions of the model. First of all, by introducing capacity constraint in which firms are not capable of selling more than they can produce, it was Edgeworth (1889) that proposed the capacity constraint for remediation of this paradox. In modern literature, all models of capacity constraints justify noncompetitive prices. (See Levitan and Shubik (1972) in the case of symmetric capacities, Kreps and Scheinkman (1983) in the case asymmetric capacities, and other works as well in this topic for example Osborne and Pitchik (1986), Davidson and Deneckere (1986)). Whilst the literature of oligopoly theory shows us how price competition is soften using capacity constraint models, the second approach to resolve the paradox is using dynamic competition, i.e., when firms compete repeatedly. Based on the works of Friedman (1971), due to the occurrence of price war and retaliation between the players in the game, prices above marginal cost may be sustained in equilibrium. The idea behind a supergame theory of oligopoly is to characterize an optimal punishment strategy for obtaining the collusive outcome (See Abreu (1986) and Tirole (1988)). Obviously the literature of dynamic game is extensive and its development helped us to understand tacit collusion and coordinated effects in horizontal mergers. Third, price competition can be softened by relaxing of an assumption that firms produce a homogenous good. If products are differentiated, then consumers in Bertrand model may not switch entirely to the product with lower price. Then, a price above marginal cost can be sustained under product differentiation.

Even if the analysis extends from a homogeneous product to a differentiated product setup, the difference between two equilibria concepts, Bertrand and Cournot, still remains. The literature of modern industrial organization until the 1990s cannot reconcile this paradox. Indeed, as Shapiro (1989) stated:

"The various modern theories of oligopoly behavior are essentially a set of different games that have been analyzed; these games do not represent competing theories, but rather models relevant in different industries or circumstances. ... It is best to provide the reader with a word of warning. Unlike perfect competition or pure monopoly, there is no single "theory of oligopoly". ... Indeed, there has long been doubt the wisdom of seeking a single, universal theory of oligopoly, and I share this doubt." (Handbook of industrial organization 1989, Chapter 6 p.332)

Evolutionary game approach as a solution to Bertrand paradox

An alternative approach of evolutionary game theory, originated by Smith and Price (1973), provide us an answer for resolution of Bertrand paradox. The concept of evolutionarily stable strategy (ESS) is central in the analysis of evolutionary game theory. While A Nash strategy in a game is a strategy where it is not rational for any competitor to deviate from this strategy to other strategies, an ESS is instead a state of game dynamics where, in a very large (infinite) population of competitors, a new mutant strategy cannot successfully contest with the population to upset the prevailing dynamic. Schaffer (1988) extends ESS definition to a finite population game and further, in another paper (Schaffer (1989)), shows that evolutionary stable strategy (ESS) in a context of a symmetric duopoly with quantitiy-setting firms leads to Walrasian(competitive) equilibrium. This would raise the question why firms may adopt a competitive strategy attaining a lower absolute profit in an oligopoly setup (imperfect competition). In fact, a firm diminishes its own profit by adopting a competitive strategy in order to reduce the profit of its rival to an even larger extent. In the words of Schaffer:

"When firms have market power, the possibility of spiteful behavior exists: a firm may forgo profit-maximization and lower its profits and even its survival chances, but if the profits of its competitors are lowered still further, the spiteful firm will be the more likely survivor." (Schaffer (1989, p.44))

Consider a homogeneous good market composed of n firms.[1] Let $P(.)$ be the market demand function, a decreasing function of $x_1 + x_2 + ... + x_n$ where each x_i denotes the product quantity of firm $i = 1, 2, ..., n$ sold in the homogenous good market. Each firm i has the same differentiable and increasing cost function $C(.)$.

Suppose that firms involve in a game to make a choice on their quantities simultaneously. Then, we can define a symmetric Cournot-Nash equilibrium $(x^c, x^c, ..., x^c)$ in this game as follows:

[1] The analysis here follows Vega-Redondo (1996).

$$P(nx^c)x^c - C(x^c) \geq P((n-1)x^c + x)x - C(x) \qquad \forall x \geq 0 \tag{1.1}$$

Moreover, a symmetric Walrasian (competitive) output $(x^w, x^w, ..., x^w)$, i.e., each firm maximizes its profit taken the market-clearing price as given, is written off as the following condition

$$P(nx^w)x^w - C(x^w) \geq P(nx^w)x - C(x) \qquad \forall x \geq 0 \tag{1.2}$$

Consider now an ESS for the game among these n firms. Here we show that a strategy x^w is ESS, that is, if no mutant firm which chooses a different output than x^w, can obtain a higher profit than the other $n-1$ incumbent firms. Mathematically speaking, x^w is an ESS if the following condition holds:

$$P((n-1)x^w + x)x^w - C(x^w) > P((n-1)x^w + x)x - C(x) \qquad \forall x \neq x^w \tag{1.3}$$

To see whether the condition (1.3) is satisfied, note that since $P(.)$ is a decreasing function, we must have:

$$P((n-1)x^w + x)(x^w - x) > P(nx^w)(x^w - x) \tag{1.4}$$

Then we subtract the term $C(x^w) - C(x)$ from both sides of inequality (1.4) , it yields

$$\Big(P((n-1)x^w + x)x^w - C(x^w)\Big) - \Big(P((n-1)x^w + x)x - C(x)\Big) >$$

$$\Big(P(nx^w)x^w - C(x^w)\Big) - \Big(P(nx^w)x - C(x)\Big)$$

Knowing that the right hand side of above inequality is a non-negative number by (1.2), therefore this leads to the condition (1.3) as is required.

In fact, the previous argument shows that evolutionary stability in a quantity competition reproduces the same outcome as in Bertrand competition, that is, the Walrasian (competitive) outcome with price equals to the marginal cost, for a static oligopoly producing a homogenous good. Furthermore, Vega-Redondo (1997), in a dynamic stochastic framework, shows that competitive equilibrium is a long run (stochastically stable state) outcome of Cournot oligopoly game with a homogenous good market.

The evolutionary approach, indeed, was pioneered by Alchian (1950) (see also Nelson and Winter (1973) and Hirshleifer (1977)) where he suggests a modification to embed the principles of biological evolution and natural selection into economic analysis. Alchian argues that agents in economic environment might not be able to optimize their actions or choices due to the lack of perfect foresight, but rather they may possibly adopt a posteriori most appropriate actions that realize positive profit through relative efficiency. Then Schaffer (1989) reinforces Alchian's argument through an evolutionary model of economic natural selection which proves that evolutionary stability is hence based on relative payoffs. In chapter 3, we further lay emphasis on the relationship between ESS and relative success.

In the next chapter, we extend an evolutionary game theoretic model to an asymmetric oligopolistic industry composed of two groups of firms. In fact, here we reexamine the issue of evolutionary stability in asymmetric Cournot oligopoly with a homogenous product. Tanaka (1999) has shown that a celebrated result by Vega-Redondo (1997) for symmetric oligopoly, namely that the long-run evolutionary outcome of such a market equals the competitive Walrasian outcome (and not the Cournot-Nash outcome), can be extended to asymmetric oligopoly. We take issue with this extension and provided an alternative analysis of an asymmetric oligopoly game, which does not lead to marginal cost pricing and the competitive outcome in the long-run.[2]

Then chapter 3 draws our attention to the following important question in this literature. Do firms under relative payoffs maximizing (RPM) behavior always

[2]This chapter is taken from Leininger and M.Moghadam (2014), joint work with Wolfgang Leininger.

choose a strategy profile that results in tougher competition compared to firms under absolute payoffs maximizing (APM) behavior? We will address this issue through a simple model of symmetric oligopoly where firms select a two dimensional strategy set of price and a non-price variable known as quality simultaneously. Our results show that equilibrium solutions of RPM and APM are distinct. We further characterize the comparison between these two equilibrium concepts. In particular, RPM does not always lead to stricter competition compared to the Nash equilibrium (APM). In fact, the comparison between two equilibrium concepts is influenced by the parameters of the demand curve and the cost function. The derived conditions will determine under which circumstances RPM induces more competition or less competition w.r.t the price or non-price dimension.[3]

Finally chapter 4 presents a nonparametric approach to investigate an empirical content of the evolutionary oligopoly model. Using revealed preference approach introduced by Carvajal, Deb, Fenske, and Quah (2013, Econometrica), we derive testable conditions for an evolutionary model of asymmetric oligopoly setup where firms have different cost functions to produce a homogenous good. Therefore, without making any parametric assumption regarding to the demand curve and the cost function, this approach characterizes a set of conditions (restrictions) for an observational dataset to be consistent with the non-competitive evolutionary equilibrium. An empirical application to crude oil market with main producers is presented and we compare the rejection rates of both Cournot and evolutionary hypotheses.[4]

[3]This chapter proceeds from M. Moghadam (2015a).
[4]Chapter 4 is based on M. Moghadam (2015b).

Chapter 2

Evolutionary Model of Asymmetric Oligopoly

It is a widely known result that in terms of evolutionary stability the long-run outcome of a Cournot oligopoly market with finite number of firms approaches the perfectly competitive Walrasian market outcome (Vega-Redondo (1997)). However, in this chapter we show that an asymmetric structure in the cost functions of firms may change the long-run outcome. We show that the evolutionarily stable price in an asymmetric Cournot oligopoly need not equal the marginal cost, it may rather equal a weighted average of (different) marginal costs. We apply a symmetrization technique in order to transform the game with asymmetric firms into a symmetric oligopoly game and then extend Schaffer's definition (1988) of a finite population ESS (FPESS) to this setup. Moreover, we show that the FPESS in this game represents a stochastically stable state of an evolutionary process of imitation with experimentation.

2.1 Introduction

The establishment of evolutionary game models provides us with a new insight into the oligopoly theory. In a seminal paper, Vega-Redondo (1997) shows that Walrasian competitive equilibrium is the only stochastically stable state in a symmetric quantity oligopoly game with a homogenous product. Tanaka (1999) also considers an evolutionary game theoretic model for an asymmetric oligopolistic industry composed of two groups of firms, in which the output choices of low cost firms and high cost firms jointly determine market price of a homogeneous good. His main result shows that the finite population evolutionarily stable strategies (FPESS) of low cost and high cost firms are equal to the respective competitive (Walrasian) outputs in the two groups. Moreover, this static outcome is also the long-run equilibrium of a dynamic evolutionary model based on imitation and stochastic mutation. Tanaka's result is read as a generalization of the celebrated result of Vega-Redondo (1997), namely that FPESS in a *symmetric* oligopoly of identical firms selects the Walrasian outcome (and not the Cournot-Nash equilibrium), to asymmetric oligopoly.

The purpose of the present chapter is to reexamine this issue; more precisely, we will argue that Tanaka applies the finite population evolutionary argument in a particular parameterized way, which amounts to an evolutionary analysis of the two groups *in separation*. Otherwise it would not be possible to apply a symmetric solution concept like FPESS to the (full) asymmetric model. A standard procedure of evolutionary game theory (originating with Selten (1980)), when applying ESS to a multi-population model, is to symmetrize the given asymmetric game and then apply the evolutionary solution concept to the symmetrized game. We will show that if one follows this prescription in a simple alternative model, then the marginal cost pricing result will not be confirmed.

This chapter is organized as follows: in the next section we present our approach to evolutionary stability in asymmetric finite populations. Section 2.3 contains the analysis following from our approach. Section 2.4 considers dynamics explicitly and derives evolutionary equilibrium in the long run. Section 2.5 concludes.

2.2 Evolutionary stability in asymmetric oligopoly

In our model we employ an alternative conceptualization of evolutionarily stable strategies to asymmetric oligopoly markets. Recall that Tanaka applies the concept of a finite population evolutionarily stable strategy (FPESS) by Schaffer (1988) in which agents in economic and social environment adhere to *relative payoff maximizing* rather than absolute payoff maximizing behavior. Tanaka (1999) does so in the following way: each group of either low cost firms or high cost firms performs its relative payoff maximization under the assumption that the other group's behavior is *given*. The parametric treatment of the, respectively, other group's behavior allows for the symmetric treatment of firms in each single group; a prerequisite for the use of symmetric solution concept ESS. However, this parameterization also means that a mutation of firm strategies in the evolutionary process cannot occur in both groups simultaneously; in fact, the evolution of behavior in one group is *not* influenced by the evolution of behavior in the other group as the two evolutionary processes are studied in isolation of each other. And it is this fact that leads to the conclusion that the equilibrium price resulting from FPESS of the good must equal the marginal costs of low cost firms as well as high cost firms; accordingly, high cost firms supply a lower amount of the good than low cost firms in order to equalize marginal cost.

In contrast, we use Selten's (1980) approach to construct a symmetric monomorphic population game out of an asymmetric multi-population game. Firstly, we define a set of roles or information situations (high cost or low cost) where a firm must choose its action at each possible role. In addition, we have a role assignment map that assigns without replacement each of the $N = n_1 + n_2$ firms with probability of n_1/N as a low cost role and with probability n_2/N as a high cost role. Accordingly, firms carry out their actions (local strategies) based on the assigned role and a behavior strategy for a firm gives a local strategy for each role. Then Schaffer's (1988) definition of finite population ESS is applied onto these extended strategies in the now symmetric game. Note that this also amounts to a kind of "production uncertainty" as a firm's cost function might be high or low in any new play of the game.

Since a mutant strategy now may contain different behavior for *both* roles our result will differ from the one derived in Tanaka (1999). The analysis in Tanaka's paper does not allow for the possibility of instantaneous strategy mutation for both types of firms while, with this approach, mutation is allowed to take place simultaneously for both types.

The analysis of Vega-Redondo (1997) has been broadened on several grounds. Alós-Ferrer (2004) and Alós-Ferrer and Shi (2012) introduce memory capacity into the evolutionary dynamic model of imitation with mutation and prove that once firms remember not only the current actions and payoffs, but also those of the last periods, the set of stochastic stable states extends between the Walrasian and the Cournot Nash. The theoretical result by Apesteguia et al. (2010), related to the present paper, also show that Vega-Redondo's result does not remain robust to the smallest asymmetry in fixed costs. They consider an oligopoly set-up with a firm having a cost advantage compared to other firms in the market and as a result, a Walrasian state becomes no longer stochastically stable.

2.3 The model

Consider a game between N firms, which are divided into two groups, low cost firms and high cost firms. All firms produce a homogeneous good. The number of firms in the low cost group is n_1, and the number of firms in the high cost group is n_2 with $n_1 + n_2 = N$. Low cost firm i produces output level x_i and high cost firm i produces output level y_i.

The cost functions for the low cost firms and for the high cost firms are respectively given by:

$$C_l(x_i) = c(x_i) \quad and \quad C_h(y_i) = \gamma c(y_i) \quad where \quad \gamma > 1.$$

$c(.)$ represents an increasing, twice differentiable and convex function, i.e., with increasing marginal cost $c'(.)$. Let $P(.)$ be a differentiable and decreasing inverse demand function of the homogeneous product, whose argument is the total output in the market denoted by $X + Y$ where $X = x_1 + x_2 + ... + x_{n_1}$ is the total output of low cost firms and $Y = y_1 + y_2 + ... + y_{n_2}$ is total output of high cost firms.

Consequently, payoff functions for low cost firms and high cost firms are as follows,

$$\pi_i^l(x_i, X_{-i} + Y) = P(x_i, X_{-i} + Y)x_i - c(x_i), \quad i = 1, 2, , n_1$$

$$\pi_i^h(y_i, X + Y_{-i}) = P(y_i, X + Y_{-i})y_i - \gamma c(y_i), \quad i = 1, 2, , n_2$$

Where $X_{-i} = \sum_{j=1, j\neq i}^{n_1} x_j$ and $Y_{-i} = \sum_{j=1, j\neq i}^{n_2} y_j$.

Defining a concept of evolutionary stability requires a symmetric setup with identical players, but our game setup is asymmetric. We construct a symmetric monomorphic population game out of an asymmetric polymorphic-population game. To do this, first we need to define a set of roles or information situations. A firm may find itself in a number of roles (high cost or low cost) where it must choose its action at each possible role. In addition, we have a role assignment map that assigns without replacement each of the $N = n_1 + n_2$ firms with probability of n_1/N as a low cost role and with probability n_2/N as a high cost role. Firms then carry out their actions (local strategies) based on the assigned role. In fact, a firm contemplates behavior before it knows its assigned role. An action (local strategy) of firm $i \in \{1, .., N\}$ assigned at role high cost or low cost is to select a pure strategy of x_i or y_i and hence a behavior strategy for a firm i is a 2-tuple (x_i, y_i) giving a local strategy for each role. From this ex-ante point of view the game played in role-contingent strategies is symmetric (see Selten, 1980, pp. 97-8).

Definition 2.1. Let (x_i, y_i) and $[\mathbf{x}, \mathbf{y}]_{-\mathbf{i}} = [(x_1, y_1), ..., (x_{i-1}, y_{i-1}), (x_{i+1}, y_{i+1}), ..., (x_N, y_N)]$ be a behavior strategy for a firm i and behavior strategies of other firms respectively. Then local payoffs of firm i in roles low cost and high cost are defined as $\pi_i^l(x_i, [\mathbf{x}, \mathbf{y}]_{-\mathbf{i}})$ and $\pi_i^h(y_i, [\mathbf{x}, \mathbf{y}]_{-\mathbf{i}})$.

Correspondigly the total (expected) payoff function of firm i in the symmetrized monomorphic population game is

$$\Pi_i((x_i, y_i), [\mathbf{x}, \mathbf{y}]_{-\mathbf{i}}) = \frac{n_1}{N}\pi_i^l(x_i, [\mathbf{x}, \mathbf{y}]_{-\mathbf{i}}) + \frac{n_2}{N}\pi_i^h(y_i, [\mathbf{x}, \mathbf{y}]_{-\mathbf{i}}) \tag{2.1}$$

Consider now a finite population evolutionarily stable strategy (FPESS) of the game among these N firms. A strategy is evolutionary stable, if no mutant firm which chooses a different strategy than (x^*, y^*), say, can realize higher expected profits than the firms which employ the incumbent strategy (x^*, y^*). In other words, no mutant strategy (x, y) can invade a population of (x^*, y^*) strategists successfully.

Formally Schaffer‘s definition (1988) then reads

Definition 2.2. A strategy profile (x^*, y^*) in the symmetrized oligopoly game is a FPESS if

$$\Pi_i((x^*, y^*), (x^m, y^m), [\mathbf{x}^*, \mathbf{y}^*]_{-\mathbf{i}-\mathbf{j}}) > \Pi_j((x^m, y^m), [\mathbf{x}^*, \mathbf{y}^*]_{-\mathbf{j}}) \tag{2.2}$$

$\forall (x^m, y^m) \neq (x^*, y^*)$, and all $i \neq j$ where $(x_i, y_i) = (x^*, y^*)$ and $(x_j, y_j) = (x^m, y^m)$.

In a similar setup, Tanaka (1999) also considers an evolutionary game theoretic model for an asymmetric oligopoly. Tanaka applies the finite population evolutionary argument in a particular parameterized way, which amounts to an evolutionary analysis of the two groups in separation. Effectively, this approach postulates two separate independent mutation processes in the two groups. And this accounts for the marginal cost pricing result. The novelty of our definition, compared to Tanaka (1999), is that it allows the mutation to occur simultaneously for both types of firms. Hence evolution of behavior for one type depends on the evolution of behavior for the other type and vice versa and as we will show later this leads to a Walrasian result in expectation.

Particularly, Tanaka defines FPESS for low cost firms and high cost firms respectively, as follows:

x^* and y^* are a FPESS if the following two conditions (2.3)and (2.4) hold

$$\pi_i^l(x^*, x^m, X^*_{-i-j} + Y) > \pi_j^l(x^m, X^*_{-j} + Y) \tag{2.3}$$

$\forall x^m \neq x^*$ and all $i \neq j$ where $x_j = x^m$ and $x_i = x^*$. This requirement means that no mutant strategy of a low cost firm can yield higher profits than x^* *given the total output of the high cost firms* Y. Moreover,

$$\pi_i^h(y^*, y^m, X + Y^*_{-i-j}) > \pi_j^h(y^m, X + Y^*_{-j}) \tag{2.4}$$

$\forall y^m \neq y^*$ and all $i \neq j$ where $y_j = y^m$ and $y_i = y^*$. This requirement means that no mutant strategy for a high cost firm can yield higher profits than y^* *given the total output of the low cost firms* X. Hence Tanaka's model set-up is not really playing the field among N firms; it is rather playing the field among the low cost firms exclusively and the high cost firms exclusively and then coupling both results in a consistent way. His main result shows that the finite population evolutionarily stable strategies (FPESS) of low cost and high cost firms are equal to the respective competitive (Walrasian) outputs in the two groups.

Definition 2.3. A Walrasian outcome for N firms is given by a quantity profile (x^w, y^w) and a market price P^w that satisfy the set of following conditions:

$$P^w = c'(x_i^w) = c'(x^w) \quad \forall i = 1, , n_1 \tag{2.5}$$

$$P^w = \gamma c'(y_i^w) = \gamma c'(y^w) \quad \forall i = 1, , n_2 \tag{2.6}$$

Proposition 2.4. *In the symmetrized game of the asymmetric oligopoly market with two groups of low cost firms and high cost firms, any FPESS output of an individual firm does not conform to Walrasian behavior; neither for the low cost firm nor the high cost firm. But the equilibrium price equals a weighted average of marginal costs, where the weights are given by the population shares of high and low cost firms. Thus a Walrasian market outcome in expectation is obtained for the homogeneous good price and the total market output. Low cost firms supply a higher amount of output than high cost firms.*

Proof. In line with Schaffer (1989), we can find a FPESS as the solution of following optimization problem[1]

$$(x^*, y^*) = \arg\max_{x^m, y^m} \varphi = \Pi_j((x^m, y^m), [\mathbf{x}^*, \mathbf{y}^*]_{-\mathbf{j}}) - \Pi_i((x^*, y^*), (x^m, y^m), [\mathbf{x}^*, \mathbf{y}^*]_{-\mathbf{i}-\mathbf{j}}) \tag{2.7}$$

[1]Note that Tanaka (1999) derives FPESS as the solutions of the following two problems independently,

$$x^* = \arg\max_{x^m} \varphi^l = \pi_j^l(x^m, X_{-j}^* + Y) - \pi_i^l(x^*, x^m, X_{-i-j}^* + Y)$$

$$y^* = \arg\max_{y^m} \varphi^h = \pi_j^h(y^m, X + Y_{-j}^*) - \pi_i^h(y^*, y^m, X + Y_{-i-j}^*)$$

Values of $x^*(Y)$ and $y^*(X)$ are obtained given the total output of high cost firms and total output of low cost firms respectively. Hence a low cost firm only evaluates own payoff relative to the payoffs of other low cost firms; and a high cost firm only evaluates own payoff relative to payoffs of other high cost firms. And this lead to the marginal cost pricing result.

Hence we write down a mutant total payoff and an incumbent total payoff as follows:

$$\Pi_j((x^m,y^m),[\mathbf{x}^*,\mathbf{y}^*]_{-\mathbf{j}}) = \frac{n_1}{N}\pi^l_j(x^m,[\mathbf{x}^*,\mathbf{y}^*]_{-\mathbf{j}}) + \frac{n_2}{N}\pi^h_j(y^m,[\mathbf{x}^*,\mathbf{y}^*]_{-\mathbf{j}}) \tag{2.8}$$

$$\Pi_i((x^*,y^*),(x^m,y^m),[\mathbf{x}^*,\mathbf{y}^*]_{-\mathbf{i}-\mathbf{j}}) = \tag{2.9}$$

$\frac{n_1}{N}\left(\frac{n_1-1}{N-1}\pi^l_i(x^*,x^m,[\mathbf{x}^*,\mathbf{y}^*]_{-\mathbf{i}-\mathbf{j}})+\frac{n_2}{N-1}\pi^l_i(x^*,y^m,[\mathbf{x}^*,\mathbf{y}^*]_{-\mathbf{i}-\mathbf{j}})\right)+\frac{n_2}{N}\left(\frac{n_1}{N-1}\pi^h_i(y^*,x^m,[\mathbf{x}^*,\mathbf{y}^*]_{-\mathbf{i}-\mathbf{j}})+\right.$ $\left.\frac{n_2-1}{N-1}\pi^h_i(y^*,y^m,[\mathbf{x}^*,\mathbf{y}^*]_{-\mathbf{i}-\mathbf{j}})\right)$

The first term of $\Pi_j(.)$ indicates the potential role of the mutant as a member of low cost group, and the second term refers to its potential role as a member of high cost group. Accordingly, for incumbents $\Pi_i(.)$ the calculation is slightly more complicated and consists of four terms in order to account for the mutant's role. Consequently the FPESS can be established as the solution of following optimization problem

$(x^*,y^*) = \arg\max_{x^m,y^m}\varphi$

where $\varphi = \frac{n_1}{N}(P(x^m,[\mathbf{x}^*,\mathbf{y}^*]_{-\mathbf{j}})x^m - c(x^m)) + \frac{n_2}{N}(P(y^m,[\mathbf{x}^*,\mathbf{y}^*]_{-\mathbf{j}})y^m - \gamma c(y^m)) - (\frac{n_1}{N}\frac{n_1-1}{N-1})(P(x^*,x^m,[\mathbf{x}^*,\mathbf{y}^*]_{-\mathbf{i}-\mathbf{j}})x^*-c(x^*))-(\frac{n_1}{N}\frac{n_2}{N-1})(P(x^*,y^m,[\mathbf{x}^*,\mathbf{y}^*]_{-\mathbf{i}-\mathbf{j}})x^*-c(x^*))- (\frac{n_2}{N}\frac{n_1}{N-1})(P(y^*,x^m,[\mathbf{x}^*,\mathbf{y}^*]_{-\mathbf{i}-\mathbf{j}})y^* - \gamma c(y^*)) - (\frac{n_2}{N}\frac{n_2-1}{N-1})(P(y^*,y^m,[\mathbf{x}^*,\mathbf{y}^*]_{-\mathbf{i}-\mathbf{j}})y^* - \gamma c(y^*))$

First order conditions with respect to x^m, y^m respectively are as follows:

$$x^m: \quad \frac{n_1}{N}(P(.)+P'(.)x^m - c'(x^m)) - (\frac{n_1}{N}\frac{n_1-1}{N-1})P'(.)x^* - (\frac{n_2}{N}\frac{n_1}{N-1})P'(.)y^* = 0$$

$$y^m: \quad \frac{n_2}{N}(P(.)+P'(.)y^m - \gamma c'(y^m)) - (\frac{n_2}{N}\frac{n_2-1}{N-1})P'(.)y^* - (\frac{n_2}{N}\frac{n_1}{N-1})P'(.)x^* = 0$$

$P'(.)$ is the derivative of inverse demand function. Imposing symmetric condition($x^m = x^*, y^m = y^*$) and rearranging we obtain

$$P(X^*+Y^*) + (\frac{n_2}{N-1}(x^*-y^*))P'(X^*+Y^*) = c'(x^*) \tag{2.10}$$

$$P(X^* + Y^*) - (\frac{n_1}{N-1}(x^* - y^*))P'(X^* + Y^*) = \gamma c'(y^*) \tag{2.11}$$

From equations (2.10) and (2.11), it is obvious that the individual firm behavior does not conform to Walrasian equilibrium with price equal to marginal cost.

After multiplying both sides of equation (2.10) by n_1/N and equation (2.11) by n_2/N and then summing up both equations, the following expression for price is obtained

$$P(X^* + Y^*) = \frac{n_1}{N}c'(x^*) + \frac{n_2}{N}\gamma c'(y^*) \tag{2.12}$$

Further, subtracting (2.11) from (2.10) yields

$$c'(x^*) - \gamma c'(y^*) = \frac{N}{N-1}(x^* - y^*)P'(X^* + Y^*)$$

From this it is easily seen that $x^* > y^*$ must hold: assume the opposite; i.e. that $x^* \leq y^*$. Then the right-hand side of the above expression is non-negative as $P'(.) < 0$. But $c(.)$ is convex and $c'(.)$ hence increasing, which yields a contradiction to $\gamma > 1$. So FPESS output of a low cost firm is larger than that of a high cost firm. □

Equation (2.12) means that the price of the good in evolutionary equilibrium is equal to a convex combination of the marginal costs of low cost firms and high cost firms. The weights of marginal cost in this combination are identical with the population share of each group of firms. Here we call the FPESS price P^* as *Walrasian price in Expectation*. In fact, in an asymmetric oligopoly of homogeneous product, the market outcome converges to the competitive equilibrium. However, as we show in the following proposition (2.5), the output of high cost firms increases, while the output of low cost firms decreases and our Walrasian price in expectation moves firms' outputs in evolutionarily stable equilibrium closer to each other compared to the Walrasian equilibrium.

Proposition 2.5. *The following relationship between FPESS behavior* (x^*, y^*) *and Walrasian behavior* (x^w, y^w) *of individual firms holds:*

$$y^w < y^* < x^* < x^w$$

Proof. This relationship is equivalent with the following relationship $y^w < y^*$ and $x^* < x^w$ (since we already know that $y^* < x^*$). A proof by contradiction is employed. Lets assume that these inequalities do not hold. So we must have one the following cases.

1. $y^w = y^*$ and $x^* = x^w$
2. $y^w = y^*$ and $x^* > x^w$
3. $y^w = y^*$ and $x^* < x^w$
4. $y^w > y^*$ and $x^* = x^w$
5. $y^w > y^*$ and $x^* > x^w$
6. $y^w > y^*$ and $x^* < x^w$
7. $y^w < y^*$ and $x^* = x^w$
8. $y^w < y^*$ and $x^* > x^w$

First of all, we derive a useful condition for x^* and y^* in our solution of (2.10) and (2.11), which we employ in this proof. We have

$$P(X^* + Y^*) + (\frac{n_2}{N-1}(x^* - y^*))P'(X^* + Y^*) = c'(x^*)$$

$$P(X^* + Y^*) - (\frac{n_1}{N-1}(x^* - y^*))P'(X^* + Y^*) = \gamma c'(y^*)$$

Then, knowing that $\frac{n_i}{N-1}(x^* - y^*) > 0$, equations (2.10) and (2.11) imply that

$$P(X^* + Y^*) > c'(x^*), \quad P(X^* + Y^*) < \gamma c'(y^*)$$

By substituting equation (2.12)

$$\frac{n_1}{N}c'(x^*) + \frac{n_2}{N}\gamma c'(y^*) > c'(x^*), \quad \frac{n_1}{N}c'(x^*) + \frac{n_2}{N}\gamma c'(y^*) < \gamma c'(y^*)$$

And this implies that the values for x^* and y^* in our solution of (2.10) and (2.11) satisfies

$$\frac{c'(x^*)}{c'(y^*)} < \gamma \tag{2.13}$$

If case 1) were to apply, then we must have $\gamma c'(y^*) = \gamma c'(y^w)$ and $c'(x^*) = c'(x^w)$. Since $P(X^w + Y^w) = c'(x^w) = \gamma c'(y^w)$, this is obviously a contradiction to (2.13).

If case 2) were to apply, we obtain by convexity of the cost function $\gamma c'(y^*) = \gamma c'(y^w)$ and $c'(x^*) > c'(x^w)$. Since we have $P(X^w + Y^w) = c'(x^w) = \gamma c'(y^w)$, therefore $\gamma c'(y^*) = \gamma c'(y^w)$ and $c'(x^*) > c'(x^w)$ yield the following inequality $\gamma c'(y^*) < c'(x^*)$ which contradicts (2.13).

If case 3) were to apply, then we must have $P(X^* + Y^*) > P(X^w + Y^w)$ since $P(.)$ is decreasing. Then again, we have from equilibrium conditions $P(X^w + Y^w) = \gamma c'(y^w)$ and $P(X^* + Y^*) < \gamma c'(y^*)$. Therefore, $\gamma c'(y^*) = \gamma c'(y^w)$ (as $y^* = y^w$) leads to $P(X^* + Y^*) < P(X^w + Y^w)$ and this is a contradiction.

If case 4) were to apply, with the same reasoning like the case 2), as we have $P(X^w + Y^w) = c'(x^w) = \gamma c'(y^w)$, therefore $\gamma c'(y^w) > \gamma c'(y^*)$ and $c'(x^*) = c'(x^w)$ yield the following inequality $\gamma c'(y^*) < c'(x^*)$ which contradicts (2.13).

If case 5) were to apply, with the same reasoning like the case 2), we obtain by convexity of the cost function $c'(x^*) > c'(x^w)$ and $\gamma c'(y^*) < \gamma c'(y^w)$. Moreover, by $P(X^w + Y^w) = c'(x^w) = \gamma c'(y^w)$, these two inequalities yield the following inequality $\gamma c'(y^*) < c'(x^*)$ which contradicts (2.13).

If case 6) were to apply, then we must have $P(X^* + Y^*) > P(X^w + Y^w)$ as $P(.)$ is decreasing. Furthermore, we have from equilibrium conditions $P(X^w + Y^w) = \gamma c'(y^w)$ and $P(X^* + Y^*) < \gamma c'(y^*)$. Therefore, as $c'(.)$ is increasing, $\gamma c'(y^*) < \gamma c'(y^w)$ leads to $P(X^* + Y^*) < P(X^w + Y^w)$ and this is a contradiction.

If case 7) were to apply, then we must have $P(X^* + Y^*) < P(X^w + Y^w)$ as $P(.)$ is decreasing. Furthermore, we have from equilibrium conditions $P(X^w + Y^w) = c'(x^w)$ and $P(X^* + Y^*) > c'(x^*)$. Therefore, $c'(x^*) = c'(x^w)$ (as $x^* = x^w$) leads to $P(X^* + Y^*) > P(X^w + Y^w)$ and this is a contradiction.

If case 8) were to apply, we obtain $\gamma c'(y^*) > \gamma c'(y^w)$, $c'(x^*) > c'(x^w)$, and $P(X^* + Y^*) < P(X^w + Y^w)$. But knowing that $P(X^w + Y^w) = c'(x^w)$ and $P(X^* + Y^*) > c'(x^*)$ the first implication cannot hold. $\square$

Note that, one type of firm, namely the high cost firm sells at a price below its marginal cost. It is important to note that all firms - low cost firms and high cost firms- must make non-negative profit in FPESS equilibrium; otherwise high cost and low cost firms could not coexist in equilibrium. This requirement can indeed be met, if the cost function $c(.)$ is sufficiently convex. To see this in more detail, observe that the profit function of a high cost firm in FPESS equilibrium can be written as

$$\pi_i^h = P(X^* + Y^*)y^* - \gamma c(y^*) = (\frac{n_1}{N}c'(x^*) + \frac{n_2}{N}\gamma c'(y^*))y^* - \gamma c(y^*)$$

Let the cost function be of the following convex form:

$$c(q) = q^\rho \tag{2.14}$$

Proposition 2.6. *Assume that (2.14) holds. Then $\rho \geq \gamma$ implies both types of firms make positive profits in a FPESS equilibrium.*

Proof. By assuming (2.14), we obtain profit function of high cost firm as follow:

$$\pi_i^h = \Big(\frac{n_1}{N}\rho(x^*)^{\rho-1} + \frac{n_2}{N}\gamma\rho(y^*)^{\rho-1}\Big)y^* - \gamma(y^*)^\rho$$

We can decompose the term $\gamma(y^*)^\rho$ as follows:

$$\pi_i^h = \Big(\frac{n_1}{N}\rho(x^*)^{\rho-1} + \frac{n_2}{N}\gamma\rho(y^*)^{\rho-1}\Big)y^* - \Big(\frac{n_1}{N}\gamma(y^*)^{\rho-1} + \frac{n_2}{N}\gamma(y^*)^{\rho-1}\Big)y^* > 0$$

Since we have $x^* > y^*$, the condition $\rho \geq \gamma > 1$ implies that $\pi_i^h > 0$. □

Hence in evolutionary equilibrium, the high cost firm has to produce more than competitive level with the aim of surviving in the market and low cost firm benefit from a market power owing to its cost efficiency. We illustrate this in the following numerical example.

Example 2.1. *Let the number of firms in the low cost group and high cost group, respectively, be given by an integer $n \in [1, 10]$ and $m \in [1, 10]$. We use a linear demand function of the form $P = 1 - (\sum_{i=1}^{n} x_i + \sum_{i=1}^{m} y_i) = 1 - (X + Y)$. We specify the cost function to $c(q) = q^2$, i.e. $\rho = 2$. Consequently, $C_l(x_i) = c(x_i) = x_i^2$ and $C_h(y_i) = 2y_i^2$ with $\gamma = 2$. Then equilibrium outputs are obtained as follows:*

FPESS equilibrium:

$$X^* = \frac{n(5m + 5n - 4)}{-8 + 10m + 3m^2 + 6n + 8mn + 5n^2}, \quad Y^* = \frac{m(3m + 3n - 2)}{-8 + 10m + 3m^2 + 6n + 8mn + 5n^2}$$

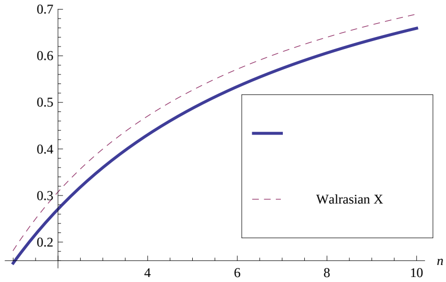
$$P^* = \frac{12m + 10n - 8}{-8 + 10m + 3m^2 + 6n + 8mn + 5n^2}$$

Walrasian equilibrium:

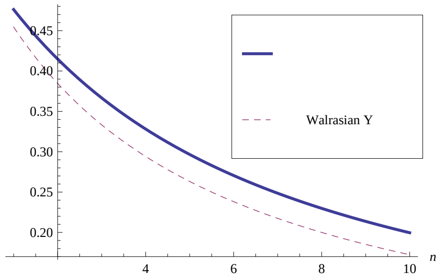
$$X^w = \frac{2n}{4 + m + 2n}, \quad Y^w = \frac{m}{4 + m + 2n}, \text{and} \quad P^w = \frac{4}{4 + m + 2n}$$

We can plot the above functions with n as the single variable for fixed values of m. For example, the following Figures 2.1 and 2.2 illustrate the result with $m = 5$. As we observe while the individual firms evolutionary stable outcomes are not the same with Walrasian outcomes, Walrasian price in expectation and total market output nevertheless converge to competitive equilibrium.

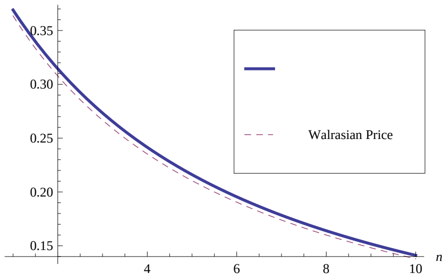
Our main result states that evolutionary forces in an oligopoly market with two types of firms determine the price in FPESS as equal to average marginal cost. It is not difficult to show that this Walrasian in expectation result generalizes to oligopolistic competition between K groups of firms with different cost functions.



(A) Total output of low cost firms

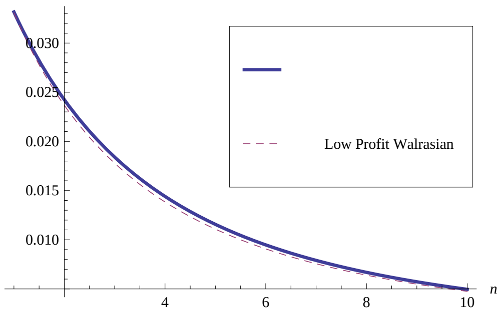


(B) Total output of high cost firms

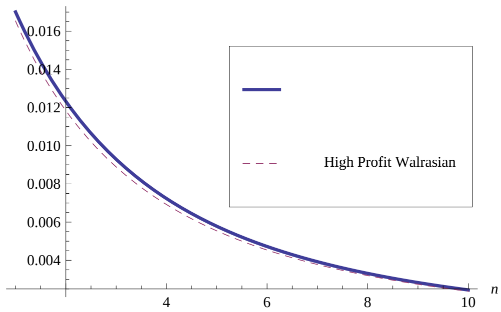


(C) Price

Figure 2.1: Comparison between FPESS and Walrasian



(A) Low cost firms



(B) High cost firms

Figure 2.2: Profit Comparison

Let' say that we have N firms that differ w.r.t. cost functions $c_1, ..., c_K$ and the number of firms in each group $k \in \{1, ..., K\}$ is n_k with $\sum_{k=1}^{K} n_k = N$.

Proposition 2.7. *In the symmetrized game of the asymmetric oligopoly market with K groups of firms that differ w.r.t. cost functions $c_1, ..., c_K$, the FPESS outputs do not correspond to the Walrasian outputs in each group of firms. However, the equilibrium price equals a weighted average of marginal costs.*

$$P(\sum_{k=1}^{K} X_k^*) + P'(\sum_{k=1}^{K} X_k^*)\Big(\frac{N}{N-1}(x_k^* - \overline{x^*})\Big) = c_k'(x_k^*) \quad \forall k = 1, ..., K$$

$$P(\sum_{k=1}^{K} X_k^*) = \sum_{k=1}^{K} \frac{n_k}{N} c_k'(x_k^*)$$

Proof. In this setup, the behavior strategy of firm i denoted by a vector $(x_{i1}, ..., x_{iK})$ gives a local strategy for each role $k \in \{1, ..., K\}$. The set of roles is identical with the set of cost functions $\{c_1, ..., c_K\}$. Let's define the local payoff functions for firm i in role k as a function of its local strategy x_{ik} and behavior strategies of other firms $[\mathbf{x}]_{-\mathbf{i}}$

$$\pi_i^k(x_{ik}, [\mathbf{x}]_{-\mathbf{i}}) = P(x_{ik}, [\mathbf{x}]_{-\mathbf{i}})x_{ik} - c_k(x_{ik}) \tag{2.15}$$

where

$$[\mathbf{x}]_{-\mathbf{i}} = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1K} \\ \vdots & \vdots & \ddots & \vdots \\ x_{i-1,1} & x_{i-1,2} & \dots & x_{i-1,K} \\ x_{i+1,1} & x_{i+1,2} & \dots & x_{i+1,K} \\ \vdots & \vdots & \ddots & \vdots \\ x_{N1} & x_{N2} & \dots & x_{NK} \end{bmatrix}$$

Therefore the total payoff of each firm in our symmetrized version of game is like

$$\Pi_i((x_{i1}, ..., x_{iK}), [\mathbf{x}]_{-\mathbf{i}}) = \sum_{k=1}^{K} \frac{n_k}{N} \pi_i^k(x_{ik}, [\mathbf{x}]_{-\mathbf{i}}) \quad \forall i = 1, .., N$$

Correspondingly the total payoffs for a mutant firm j and an incumbent firm i are as follows

$$\Pi_j((x_{j1}^m, ..., x_{jK}^m), [\mathbf{x}^*]_{-\mathbf{j}}) = \sum_{k=1}^{K} \frac{n_k}{N}\Big(P(x_{jk}^m, [\mathbf{x}^*]_{-\mathbf{j}})x_{jk}^m - c_k(x_{jk}^m)\Big)$$

$\Pi_i((x_{i1}^*, ..., x_{iK}^*), (x_{j1}^m, ..., x_{jK}^m), [\mathbf{x}^*]_{-\mathbf{i}-\mathbf{j}}) =$

$$\sum_{k=1}^{K} \frac{n_k}{N}\Bigg(\frac{n_k - 1}{N-1}\Big(P(x_{ik}^*, x_{jk}^m, [\mathbf{x}^*]_{-\mathbf{i}-\mathbf{j}})x_{ik}^* - c_k(x_{ik}^*)\Big) + \sum_{l\neq k}^{K} \frac{n_l}{N-1}\Big(P(x_{il}^*, x_{jk}^m, [\mathbf{x}^*]_{-\mathbf{i}-\mathbf{j}})x_{il}^* - c_l(x_{il}^*)\Big)\Bigg)$$

Then similar to optimization problem in (2.7), we drive the first order conditions with respect to x_{jk}^m

$$\frac{n_k}{N}\Big(P(.) + P'(.)x_k^m - c_k'(x_k^m)\Big) - \frac{n_k}{N}\frac{n_k - 1}{N-1}P'(.)x_k^* - \frac{n_k}{N}\sum_{l\neq k}^{K}\frac{n_l}{N-1}P'(.)x_l^* = 0$$

After imposing symmetry $x_k^m = x_k^*$ we obtain:

$$P(\sum_{k=1}^{K} X_k^*) + P'(\sum_{k=1}^{K} X_k^*)\Big(\frac{N - n_k}{N-1}x_k^* - \sum_{l\neq k}^{K}\frac{n_l}{N-1}x_l^*\Big) = c_k'(x_k^*)$$

Denote $\overline{x^*} = \sum_{k=1}^{K} \frac{n_k}{N} x_k^*$, the above equation can also be written as follow:

$$P(\sum_{k=1}^{K} X_k^*) + P'(\sum_{k=1}^{K} X_k^*)\Big(\frac{N}{N-1}(x_k^* - \overline{x^*})\Big) = c_k'(x_k^*) \tag{2.16}$$

By multiplying both sides of equations (2.16) in $\frac{n_k}{N}$ and summing over all equations, we get the weighted average of marginal costs pricing result.

$$P(\sum_{k=1}^{K} X_k^*) = \sum_{k=1}^{K} \frac{n_k}{N} c_k'(x_k^*) \tag{2.17}$$

$\square$

2.4 Stochastically stable state and long-run equilibrium

A stochastically stable state can be viewed as a refinement of the solution concept FPESS. One of the limitations of the latter as a static notion of equilibrium is that it need not capture the notion of long-run stability for an evolutionary process, if evolution is also exposed to stochastic influences. An FPESS is also stochastically stable if under vanishing stochastic influence the probability that the population is following it does not go to zero. We now show this property for the FPESS in our model with respect to an evolutionary process, which has been studied extensively in the literature.

We follow Vega-Redondo (1997) and other contributions to evolutionary oligopoly theory (e.g. Schenk-Hoppé (2000), Tanaka (2000), and Alos-Ferrer and Ania (2005)) who analyze a certain type of imitation dynamics between firms subject to a stochastic influence, called experimentation.

Consider that each firm $i \in \{1, ..., N\}$ must choose a pair of outputs (x_i, y_i) wherein both elements belong to a finite grid $\Gamma = \{0, \delta, 2\delta, ..., \nu\delta\}$ with grid step $\delta > 0$ and $\nu \in \mathbb{N}$. It also requires that any FPESS outputs x^* and y^* belong to this grid. The state space is then equal to Γ^{2N} and we denote it by Ω. The total number of states is equal to $(\nu+1)^2$ (including the state $(x = 0, y = 0)$). In the model, the N firms play a monomorphic oligopoly game in every period t. At each t, the state of the system is characterized by the vector $\omega(t) = \Big(\big(x_1(t), y_1(t)\big), ..., \big(x_N(t), y_N(t)\big)\Big)$ and associated with each vector $\omega(t)$, there is a vector of total payoff for all N players in the game, i.e., $\Pi(t) = (\Pi_1(t), ..., \Pi_N(t))$. Critical assumption here is that in the beginning of each period all firms are aware of the behavior strategies of other firms in the previous period so they are able to compute total payoffs. We follow a similar imitation rule as Vega-Redondo (1997) in which in period t each firm has a chance with independent and common probability $0 < p < 1$ to revise its behavior strategy to the another behavior strategy that attained the highest total profit among the strategies chosen by other firms in period $t-1$. Formally, each firm behavior strategy is chosen from the following set $B(t-1)$:

$$(x_i(t), y_i(t)) \in B(t-1) = \{(x,y) \in \Gamma^2 : \exists j \in \{1,..,N\} such \quad that \tag{2.18}$$

$$(x,y) = (x_i(t-1), y_i(t-1)) \quad \& \quad \forall k \neq j \in \{1,..,N\}, \quad \Pi_j(t-1) > \Pi_k(t-1)\}$$

In fact, in each time t of the process, each firm observes the behavior strategies of the other firms and the total profits associated with the behavior strategies of all firms in the market. Then it adopts(imitates) the behavior strategy that generated the highest total profit in the previous period. If all firms had chosen the same strategy in the previous period, no adjustments would occur. Yet, there is also the stochastic influence of "experimentation": each firm will change its strategy to an arbitrary other one with positive probability ϵ. In a suitably defined dynamic model usually with finitely many states such a process always converges to a stationary distribution of strategies. One is then interested in the strategies that form the support of the limit of these stationary distributions if the noisy experimentation probability ϵ approaches zero; i.e. those states are then stable against experimentation.

Note that we can use a result by Kandori et al. (1993) and Kandori and Rob (1995), which affirms that only monomorphic states in which all firms choose the same behavior strategy (x,y), i.e., symmetric strategy vectors $\omega_{(x,y)} = ((x,y), ..., (x,y))$, can occur in the limit set. Further we call the sate $\omega_{(x^*,y^*)}$ the FPESS state.

Proposition 2.8. *In the symmetrized game of the asymmetric oligopoly, the FPESS state $\omega_{(x^*,y^*)}$ is a stochastically stable state w.r.t. imitation dynamics with experimentation.*

Proof. To show that stochastic stability argument goes through in our model, we use a similar proof like in the theorem of Vega-Redondo (1997). As a matter of fact he showed that Walrasian state is a stochastically stable state in symmetric oligopoly for the reason that it satisfies two conditions, namely **best reachability**

condition (BRC) (corresponding to Vega-Redondo (Lemma 1, 1997)) and **worst leaving condition (WLC)** (corresponding to Vega-Redondo (Lemma 2, 1997)). BRC means that the number of experiments, i.e. stochastic shocks, needed for the process to reach a monomorphic state from other monomorphic states must be the minimal feasible one. Those states intuitively have the best chances to occur again and again as they are reached most easily. WLC means that the number of experiments needed for the process to leave them is higher than in other states. Those states, which are most easily reached and most difficultly left, are most frequently observed, which is the meaning of stochastic stability.

In fact, WLC requires that a strategy adopted by all firms, cannot be invaded by any single invader different from it, which is equivalent with the definition of FPESS (see Definition 2). Hence, if one can show that a FPESS state satisfies BRC then this FPESS state must be stochastically stable.

Obviously, the minimal number of experiments is one. However, this may not be the minimal feasible one for reachability of a certain state given the process. To check, one can operationalize the one-experiment requirement in the following way: suppose there is a strategy, which can invade successfully any monomorphically adopted; i.e. symmetric other strategy vector (recall, that there are only symmetric states in the limit). Then one experiment of one firm with this strategy would suffice to move the process to this invading strategy and the minimality requirement of best reachability would be met.[2]

[2] Such a strategy is called in the literature a Globally Surviving Strategy GSS (see Tanaka (2000)).Its relation to the notion of FPESS is the following: While a FPESS, if adopted by all firms, cannot be invaded by any single invader different from it, a GSS is defined as a most effective (single) invader: it can successfully invade any symmetrically adopted other strategy. This, in turn, is equivalent to the requirement that a GSS, if adopted by all firms, cannot be invaded by $(N-1)$ invaders with any commonly adopted other strategy. Hence, while a FPESS is stable against any single invader strategy, a GSS is stable against any $(N-1)$ identical invader strategies that occur simultaneously. (One can refer to this property as $(N-1)$-stability whereas FPESS demands 1-stability.) Consequently, if a GSS exists it is the only candidate for FPESS, because any other strategy, if adopted by all firms, could be invaded by it as a single invader. Hence any state different from a GSS can be left with a single experiment of a single firm, which experiments with the GSS strategy. If, in contrast, the GSS itself is also an FPESS, then FPESS state is the only state that cannot be left through any single experiment. Consequently, the WLC requirement is fulfilled as well.(Leininger (2006) examines global stability of an FPESS, which is an even stronger property as it demands m-stability for any m with $1 \leq m \leq n-1$, not just $m = 1$ and $m = n-1$ as is required here).

In other words, BRC for state $\omega_{(x^*,y^*)}$ holds; if the strategy (x^*, y^*) can invade any symmetric $\omega_{(x,y)} = ((x, y), ..., (x, y))$ state; i.e. if the following condition holds

$$\Pi_j\Big((x^*, y^*), (x, y), \overset{N-1}{...}, (x, y)\Big) - \Pi_i\Big((x^*, y^*), (x, y), \overset{N-1}{...}, (x, y)\Big) > 0 \quad \forall i \neq j \quad and \quad \forall (x, y) \neq (x^*, y^*) \tag{2.19}$$

Where $\Pi_j\Big((x^*, y^*), (x, y), ..., (x, y)\Big)$ denotes the total profit of a single mutant firm j if it expermints a (x^*, y^*) while the state of system is $\omega_{(x,y)}$.

$$\Pi_j\Big((x^*, y^*), (x, y), ..., (x, y)\Big) =$$

$$\frac{n_1}{N}\Big(P(x^* + (n_1 - 1)x + n_2 y)x^* - c(x^*)\Big) + \frac{n_2}{N}\Big(P(n_1 x + y^* + (n_2 - 1)y)y^* - \gamma c(y^*)\Big)$$

And total profits of the other firms $i \neq j$ are

$$\Pi_i\Big((x^*, y^*), (x, y), ..., (x, y)\Big) =$$

$$\frac{n_1}{N}\Big(\frac{n_1 - 1}{N - 1}\Big(P(x^* + (n_1 - 1)x + n_2 y)x - c(x)\Big) + \frac{n_2}{N - 1}\Big(P(n_1 x + y^* + (n_2 - 1)y)y - c(x)\Big)\Big)$$

$$+\frac{n_2}{N}\Big(\frac{n_1}{N - 1}\Big(P(x^* + (n_1 - 1)x + n_2 y)y - \gamma c(y)\Big) + \frac{n_2 - 1}{N - 1}\Big(P(n_1 x + y^* + (n_2 - 1)y)y - \gamma c(y)\Big)\Big)$$

Let define

$$\psi = \Pi_i\Big((x^*, y^*), (x, y), ..., (x, y)\Big) - \Pi_j\Big((x^*, y^*), (x, y), ..., (x, y)\Big)$$

A necessary and sufficient condition for (x^*, y^*) to be a most effective (single) invader is that it is a solution of the following problem

$$(x^*, y^*) = \arg\max_{x,y} \psi$$

Differentiating ψ with respect to x and y , we obtain the following first order conditions

$$\frac{n_1}{N}\Bigg(\frac{n_1 - 1}{N - 1}\Big((n_1 - 1)xP'(.) + P(.) - c'(x)\Big) + \frac{n_2}{N - 1}\Big(n_1 x P'(.) + P(.) - c'(x)\Big)\Bigg) +$$

$$\frac{n_2}{N}\Big(\frac{n_1}{N-1}\Big((n_1-1)yP'(.)\Big)+\frac{n_2-1}{N-1}\Big(n_1yP'(.)\Big)\Big)-\frac{n_1}{N}\Big((n_1-1)x^*P'(.)\Big)-\frac{n_2}{N}\Big(n_1y^*P'(.)\Big)=0$$

$$\frac{n_2}{N}\Big(\frac{n_1}{N-1}\Big(n_2yP'(.)+P(.)-\gamma c'(y)\Big)+\frac{n_2-1}{N-1}\Big((n_2-1)yP'(.)+P(.)-\gamma c'(y)\Big)\Big)+$$

$$\frac{n_1}{N}\left(\frac{n_1-1}{N-1}\Big(n_2xP'(.)\Big)+\frac{n_2}{N-1}\Big((n_2-1)xP'(.)\Big)\right)-\frac{n_1}{N}\Big(n_2x^*P'(.)\Big)-\frac{n_2}{N}\Big((n_2-1)y^*P'(.)\Big)=0$$

Symmetry imposes $x = x^*$ and $y = y^*$ which simplifies both above FOCs to

$$P(X^*+Y^*)+(\frac{n_2}{N-1}(x^*-y^*))P'(X^*+Y^*)=c'(x^*) \tag{2.20}$$

$$P(X^*+Y^*)-(\frac{n_1}{N-1}(x^*-y^*))P'(X^*+Y^*)=\gamma c'(y^*) \tag{2.21}$$

These are the same conditions as equation (2.10) and (2.11) in the previous section, which characterize the FPESS output. It means that the maximization problem of φ has the same solution as the maximization problem of ψ. Hence if an FPESS exists it must also be $(N-1)$-stable and we have shown that both conditions of BRC and WLC are satisfied for state $\omega_{(x^*,y^*)}$ and the proof is complete. □

2.5 Concluding remarks

In this chapter, we reexamined the issue of evolutionary stability in asymmetric Cournot oligopoly with a homogenous product. Tanaka (1999) has shown that a celebrated result by Vega-Redondo (1997) for symmetric oligopoly, namely that the long-run evolutionary outcome of such a market equals the competitive Walrasian outcome (and not the Cournot-Nash outcome), can be extended to asymmetric oligopoly. We took issue with this extension and provided an alternative analysis of an asymmetric oligopoly game, which shows that the individual firm outputs do not conform to Walrasian outputs, however the market total output and the price converge to Walrasian equilibrium in expectation.

We symmetrized the asymmetric evolutionary game in a particular way and then applied Schaffer (1988) definition of FPESS as a solution concept to the symmetrized game. As a first result, marginal cost pricing becomes incompatible with FPESS, both for the low cost firms as well as the high cost firms. Instead, a form of weighted average cost pricing results in equilibrium, where the weights are given by population shares of the high and low cost firms. We interpret this as Walrasian in expectation for market outcomes. One reason for a non-Walrasian individual firm outputs in our model is that we allow for *simultaneous* mutations in the behavior of a low cost and a high cost firm, while the analysis in Tanaka (1999) does not. Tanaka's evolutionary analysis treats the total number of firms effectively as two *separate* populations, each composed of identical firms, which do not interact with each other evolutionarily. This comes close to performing Vega-Redondo's symmetric analysis "doubly" and makes survival of both types of firms less surprising.

In contrast, in our model different firms *do* interact evolutionarily with each other and –perhaps surprisingly– both types of firms can survive in the long run *despite* their persistent differences in production cost. This, however, is not compatible anymore with marginal cost pricing.

Chapter 3

Price and Non-Price Competition in Evolutionary Oligopoly

Do firms under relative payoffs maximizing (RPM) behavior always choose a strategy profile that results in tougher competition compared to firms under absolute payoffs maximizing (APM) behavior? In this chapter, we will address this issue through a simple model of symmetric oligopoly where firms select a two dimensional strategy set of price and a non-price variable known as quality simultaneously. In conclusion, our results show that equilibrium solutions of RPM and APM are distinct. We further characterize the comparison between these two equilibrium concepts. In fact, this comparison is influenced by the parameters of the demand curve and the cost function. The conditions, derived in this chapter, determine under which circumstances RPM induces more competition or less competition w.r.t the price or non-price dimension.

3.1 Introduction

It is a well-known result by Schaffer (1988) and Schaffer (1989) that the concept of finite population evolutionary stable strategy (FPESS) can be characterized by *relative* payoff maximization and this solution concept is different from Nash equilibrium or absolute payoff maximization. As Schaffer explained agents in economic and social environment survive in the evolutionary process, if they can perform better than their opponents and so players adhere to *relative* payoffs maximizing (RPM) rather than *absolute* payoffs maximizing (APM) behavior. The behavior implied by RPM or spiteful behavior(Hamilton (1970)) leads to more competition between firms in a cournot oligopoly game and as Vega-Redondo (1997) revealed Walrasian equilibrium turns out to be unique stochastically stable state in the symmetric Cournot oligopoly.

Far ahead, the works by Tanaka (1999), Apesteguia et al. (2010) and Leininger and M.Moghadam (2014) investigate the equivalence of evolutionary equilibrium and Walrasian competitive equilibrium in an asymmetric Cournot oligopoly. While Tanaka (1999) shows that an asymmetric cost structure does not change the long-run outcome of Walrasian equilibrium in a homogenous oligopoly competition, Apesteguia et al. (2010) prove that Walrasian result of Vega-Redondo is sensitive to a cost asymmetry. They consider a setup where one firm has a small (fixed) cost advantage in comparison with other firms in the market. As a result of this cost‘s asymmetry, other quantities apart from Walrasian quantity will be chosen in long run outcomes of the game. In Tanaka's evolutionary analysis, indeed firms are only allowed to imitate each other from the same cost group. But as we have seen in previous chapter, using an alternative analysis of the asymmetric oligopoly game, a form of weighted average cost pricing so-called *Walrasian in expectation* rises in equilibrium where individual firm quantities do not correspond to Walrasian quantities.

In this chapter, we consider a symmetric oligopoly game where each firm has a two dimensional strategy set of price and non-price. Using a non-price strategy by firms in oligopoly competition is common since firms can be exceedingly competitive in price strategy. Therefore firms may also decide to compete in another dimension of non-price strategy to soften price competition. Hereafter we refer to this non-price strategy as quality. Moreover firm's cost function structure, considered in this model, follows from the literature in industrial organization. We assume that quality improvement requires fixed costs, while variable costs do not alter with quality[1], let's say a situation that firms invest in research and development activities to improve quality. (See e.g. Shaked and Sutton (1987), Banker et al. (1998) , Berry and Waldfogel (2010) , and Brécard (2010)). In the present chapter, we contribute to the literature of evolutionary game approach to oligopoly theory by showing the role of cross-elasticities of demand in determining the evolutionary equilibrium. Particularly, our analysis verifies that the market power is determined in Nash equilibrium by own elasticities of demand. Alternatively, in FPESS equilibrium, the market power is determined by not only the own elasticities of demand but also the cross-elasticities of demand. As a result, in the case of complement goods, firms under evolutionary equilibrium have more market power and behave less competitively. On the contrary, in the case of substitute goods, firms under evolutionary equilibrium have less market power and behave

[1]We obtain a similar result for a model of variable cost of quality improvement (See the working paper version M. Moghadam (2015a).

more competitively. The same interpretation applies for the quality improvement intensity, i.e., the ratio of quality cost over revenue. Nash equilibrium analysis demonstrates that if demand is somewhat more sensitive to changes in own quality compared to changes in own price, then quality improvement spending (or R&D expenditure) is a large percentage of revenue. Whereas in evolutionary equilibrium firm's quality improvement intensity is determined by both own elasticities of demand and cross elasticities of demand.

Related papers to this issue are Tanaka (2000), Hehenkamp and Wambach (2010), and Khan and Peeters (2015). Tanaka (2000) studies evolutionary game theoretical models for price-setting and for quantity-setting differentiated oligopoly with a linear demand function. Hehenkamp and Wambach (2010) investigate an evolutionary model of horizontal product differentiation in duopoly setup and show that minimum differentiation along all product characteristics, i.e., reposition to the center of product space, establishes the unique evolutionary equilibrium. Khan and Peeters (2015) show that Nash equilibrium outcomes, in a Salop circle model with firms choosing simultaneously price and quantity, coincide with outcomes in the stochastically stable state. The reason to obtain this result is allowing for a capacity constraint in their model that justify the NE (price above marginal cost) as the long run outcome of the evolutionary game.

The plan of this chapter is as follows: in the next section we explain the model and its assumptions. Section 3.3 analyzes the existence of FPESS and Nash equilibria and their distinctions in the model of quality improvement with fixed cost and further examines the link between these equilibrium concepts. Then section 3.5 concludes.

3.2 The model

In this section, we describe our oligopoly setup and further define two different types of equilibrium concepts, that is to say, standard Nash equilibrium and evolutionary equilibrium.

3.2.1 Nash equilibrium

We assume an industry of $i = 1, ..., n$ firms, each offering quantity amount x_i of a product that may vary in its quality q_i and its price p_i. It is also assumed that non-price variable or quality is a measurable attribute with values in the interval $[0, \infty)$. The quality level has a lower bound that is known as zero quality or minimum technologically feasible quality level.

Following Dixit (1979), demand functions for goods can be written as follow

$$x_i = D_i(\mathbf{p}, \mathbf{q}), i = 1, ..., n \tag{3.1}$$

where $\mathbf{p} = (p_1, p_2, ..., p_n)$ and $\mathbf{q} = (q_1, q_2, ..., q_n)$.

An increase in any p_j and q_j raises or lowers each x_i dependent on whether product pair (i, j) are complements or substitutes. Moreover we assume that the demand function D_i for each firm i is affected more by changes in its own price and quality than those of its competitors (see Tirole (1988)).

Assumption 3.1. *$D_i(\boldsymbol{p}, \boldsymbol{q})$ are continuous, twice differentiable and concave functions, and satisfy the following relations:*

$$\frac{\partial D_i}{\partial p_i} < 0, \text{ and } \left|\frac{\partial D_i}{\partial p_i}\right| > \left|\frac{\partial D_j}{\partial p_i}\right| \quad \forall i = 1, ..., n, j \neq i.$$
$$\frac{\partial D_i}{\partial q_i} > 0, \text{ and } \frac{\partial D_i}{\partial q_i} > \left|\frac{\partial D_j}{\partial q_i}\right| \quad \forall i = 1, ..., n, j \neq i.$$

$$\begin{cases} \frac{\partial D_j}{\partial p_i} < 0, \frac{\partial D_j}{\partial q_i} > 0 & \text{if } (i,j) \text{are complements goods} \\ \frac{\partial D_j}{\partial p_i} > 0, \frac{\partial D_j}{\partial q_i} < 0 & \text{if } (i,j) \text{are substitutes goods} \end{cases}$$

Concerning the cost function, on the one hand in the economic literature, Shaked and Sutton (1987) highlight that quality improvement requires increase in fixed costs or variable costs. More recently, also Berry and Waldfogel (2010) study product quality and market size. They consider two different types of industries including industries producing quality mostly with variable costs (like restaurant industry) and industries that produce quality mostly with fixed costs (like daily newspapers). Further, Brécard (2010) investigates also the effects of the introduction of a unit production cost in vertical model with fixed cost of quality improvement. On the other hand, in management literature, for instance Banker et al. (1998) assume that total cost function of firm i is affected by quality choice in both fixed cost and variable cost. Further they assert that it is a frequent phenomenon that variable production costs decline when quality is improved; e.g., when a higher quality (higher precision) product produced by robot have less needs of direct labor hours. In general, here we assume that the quality level selected by firm influences its cost through only fixed cost $f(.)$.[2]

Therefore, firm i faces a cost function $C_i(x_i, q_i) = f_i(q_i) + c_i(x_i)$. $f(.)$ and $c(.)$ are increasing and convex functions with respect to each of their arguments and all fixed costs that are not related to the quality, without loss of generality, are normalized to zero.

The firm i's profit function is then defined by

$$\pi_i(\mathbf{p}, \mathbf{q}) = p_i D_i(\mathbf{p}, \mathbf{q}) - C_i(D_i(\mathbf{p}, \mathbf{q}), q_i) \quad i = 1, 2, ..., n \tag{3.2}$$

The strategic variables are price and quality. Since the interaction between price and quality strategies of the firms only occur through the common demand function, price vector $\mathbf{p} = (p_1, p_2, ..., p_n)$ and quality vector $\mathbf{q} = (q_1, q_2, ..., q_n)$ can be written from the point of view of firm i, respectively as $(p_i, \mathbf{p}_{-i})$ and $(q_i, \mathbf{q}_{-i})$

[2]The analysis here leads to the same results when the cost of quality improvement requires only an increase in variable cost (see the discussion paper version M. Moghadam (2015a).

Let the number of firms n be fixed. Consider a simultaneous move game where each firm chooses a pair of quality and price (p_i, q_i). We assume that all firms produce a strictly positive quantity in equilibrium. So we have the following definition of standard Nash equilibrium.

Definition 3.2. A Nash equilibrium in an oligopoly competition is given by a price vector $\mathbf{p}^N$ and quality vector $\mathbf{q}^N$ such that each firm maximizes its profit; i.e.

$$(p_i^N, q_i^N) = \arg\max_{p_i, q_i} \pi_i(p_i, q_i, \mathbf{p}_{-i}^N, \mathbf{q}_{-i}^N) \quad \forall i = 1, ..., n \tag{3.3}$$

3.2.2 Evolutionary stability

In symmetric infinite population games, it is widely verified that the concept of evolutionary stable strategy is a refinement of Nash equilibrium. However, in finite population framework, the behavior implied by evolutionary stability may have distinctive features from Nash strategic behavior. The reason for this is as follows: when one player mutates from ante-adopted strategy to a new strategy in a population with small number of players, both incumbent and mutant players do not encounter a same population profile. In fact, mutant player confronts with a homogenous profile of $n - 1$ incumbent players and incumbent players face a profile of one single mutant and $n - 2$ other incumbent players.

Recall that firm's strategy choices are two dimensional including price and quality levels. Then consider a state of the system where all firms' strategy sets are the same and suppose that one firm experiments with a new different strategy. We say that a state is evolutionary stable, if no mutant firm which chooses a different strategy can realize higher profits than the firms which employ the incumbent strategy. In other words, no mutant strategy can invade a population of incumbent strategists successfully.

Formally, consider a state where all firms choose the same strategies (p^*, q^*). This state (p^*, q^*) is a *finite population evolutionarily stable strategy* (FPESS) when one mutant firm (an experimenter) chooses a different strategy $(p^m, q^m) \neq (p^*, q^*)$

its profit must be smaller than the profits of incumbent firms (the rest of firms). Formally speaking, we have the following definition by assuming firm i as mutant firm:

Definition 3.3. (p^*, q^*) is FPESS if $\pi_i(\mathbf{p}, \mathbf{q}) < \pi_j(\mathbf{p}, \mathbf{q}) \forall j \neq i$ and all $(p^m, q^m) \neq (p^*, q^*), i = 1, ..., n$. Where $\mathbf{p} = [p^*, ..., p^*, p_i = p^m, p^*, ..., p^*]$ and $\mathbf{q} = [q^*, ..., q^*, q_i = q^m, q^*, ..., q^*]$.[3]

3.2.3 Evolutionary stable strategies and relative payoffs

Classically, firms are assumed to be entities aiming at maximizing their payoffs. However except this standard behavior of absolute payoff maximizing, firms may engage in a competitive behavior of relative payoff maximizing. A firm may pursue a different behavior being ahead of its opponents making higher payoff than the others. According to Schaffer (1989), in a so-called *playing the field* game, we can also find a FPESS through solving a *relative* payoff function of firm i.

Definition 3.4. In a symmetric oligopoly, FPESS is obtained as the solution of following relative payoff optimization problem

$$(p^*, q^*) = \arg\max_{p^m, q^m} \varphi_i = \pi_i(\mathbf{p}, \mathbf{q}) - \pi_j(\mathbf{p}, \mathbf{q}) \tag{3.4}$$

Interpretation of this definition is as follows: the equilibrium condition of finite population evolutionary stable strategy in definition 3.3 is equivalent to saying when $(p^m, q^m) = (p^*, q^*)$, then $\pi_i(\mathbf{p}, \mathbf{q}) - \pi_j(\mathbf{p}, \mathbf{q})$ as a function of (p^m, q^m) approaches its maximum value of zero. In fact, FPESS concept can be characterized by *relative* payoff maximization and this solution concept is different from Nash equilibrium or absolute payoff maximization. However, note that also this means a FPESS is a Nash equilibrium for relative payoffs maximizing (RPM) firms.

[3] Clearly this definition includes any one-dimensional deviation (like (p^m, q^*) and (p^*, q^m)) by mutant.

3.3 Analysis

In addition to the strategic variable price p, firms often bring into play non-price strategic variables with the intention of softening the market competition. Firms may decide on for instance, how much to spend on quality improvement of their product or how much to invest on R&D to enhance new features to the basic product. Particularly, we will focus on the quality decision of the firm. In this section we are interested in analyzing the outcomes of Nash equilibrium and evolutionary equilibrium of this game and then we provide a comparison between the two equilibrium concepts and discuss the results.

Proposition 3.5. *Consider a symmetric oligopoly game where each firm has a two dimensional strategy set of price and quality. If the following two conditions* $\frac{\frac{\partial D_i}{\partial q_i}}{\frac{\partial D_i}{\partial p_i}} \neq \frac{\frac{\partial D_j}{\partial q_i}}{\frac{\partial D_j}{\partial p_i}}$ *and* $-2\frac{\partial D_i}{\partial p_i}\frac{\partial^2 f_i}{\partial q_i^2} > (\frac{\partial D_i}{\partial q_i})^2$ *hold, then FPESS equilibrium and symmetric Nash equilibrium both exist and are different.*

Proof. To prove so, we begin with analyzing through the lens of classical approach, i.e., maximization of absolute payoffs.

Using Definition 3.2 and profit function of firm i as specified in the previous section, that is

$$\pi_i(\mathbf{p}, \mathbf{q}) = p_i D_i(\mathbf{p}, \mathbf{q}) - f_i(q_i) - c_i(D_i(\mathbf{p}, \mathbf{q})) \quad i = 1, 2, ..., n$$

We derive the first order conditions for the Nash equilibrium with respect to p_i and q_i as follows:

$$\frac{\partial \pi_i}{\partial p_i} = D_i + \frac{\partial D_i}{\partial p_i} p_i - \frac{\partial c_i}{\partial D_i}\frac{\partial D_i}{\partial p_i} = 0 \tag{3.5}$$

$$\frac{\partial \pi_i}{\partial q_i} = \frac{\partial D_i}{\partial q_i} p_i - \frac{\partial f_i}{\partial q_i} - \frac{\partial c_i}{\partial D_i}\frac{\partial D_i}{\partial q_i} = 0 \tag{3.6}$$

Equation (3.5) is the familiar equality between marginal revenue and marginal cost. Besides equation (3.6) states that the marginal revenue related with one unit increase in quality level is equal to the marginal cost of producing this quality.

We get the following equality by combining both equations (3.5) and (3.6)

$$-\frac{\frac{\partial D_i}{\partial q_i}}{\frac{\partial D_i}{\partial p_i}} = \frac{\partial f_i}{\partial q_i}/D_i \tag{3.7}$$

Furthermore, for FPESS, by substitution of profit function in the optimization problem of definition 3.4, we obtain

$$\varphi_i = D_i(\mathbf{p},\mathbf{q})p_i - c_i(D_i(\mathbf{p},\mathbf{q})) - f_i(q_i) - D_j(\mathbf{p},\mathbf{q})p_j + c_j(D_j(\mathbf{p},\mathbf{q})) + f_j(q_j)$$

Given that $p_i = p^m$ and $q_i = q^m$, and $\forall j \neq i p_j = p^*$ and $q_j = q^*$.

Then the first order conditions for maximization of φ with respect to p_i and q_i are as follows:

$$\frac{\partial \varphi_i}{\partial p_i} = D_i + \frac{\partial D_i}{\partial p_i}p_i - \frac{\partial c_i}{\partial D_i}\frac{\partial D_i}{\partial p_i} - \frac{\partial D_j}{\partial p_i}p_j + \frac{\partial c_j}{\partial D_j}\frac{\partial D_j}{\partial p_i} = 0$$
$$\frac{\partial \varphi_i}{\partial q_i} = \frac{\partial D_i}{\partial q_i}p_i - \frac{\partial f_i}{\partial q_i} - \frac{\partial c_i}{\partial D_i}\frac{\partial D_i}{\partial q_i} - \frac{\partial D_j}{\partial q_i}p_j + \frac{\partial c_j}{\partial D_j}\frac{\partial D_j}{\partial q_i} = 0$$

In symmetric situations, by imposing $p_i = p_j, q_i = q_j$ and $\frac{\partial c_i}{\partial D_i} = \frac{\partial c_j}{\partial D_j}$, FOC's can be rewritten as

$$D_i + (p_i - \frac{\partial c_i}{\partial D_i})(\frac{\partial D_i}{\partial p_i} - \frac{\partial D_j}{\partial p_i}) = 0 \tag{3.8}$$

$$(p_i - \frac{\partial c_i}{\partial D_i})(\frac{\partial D_i}{\partial q_i} - \frac{\partial D_j}{\partial q_i}) = \frac{\partial f_i}{\partial q_i} \tag{3.9}$$

Then combining these two equations we obtain the following equality for FPESS

$$-\frac{\frac{\partial D_i}{\partial q_i} - \frac{\partial D_j}{\partial q_i}}{\frac{\partial D_i}{\partial p_i} - \frac{\partial D_j}{\partial p_i}} = \frac{\partial f_i}{\partial q_i}/D_i \tag{3.10}$$

By comparing with the solution from Nash equilibrium, i.e., equation (3.7), on condition that $\frac{\frac{\partial D_i}{\partial q_i}}{\frac{\partial D_i}{\partial p_i}} \neq \frac{\frac{\partial D_j}{\partial q_i}}{\frac{\partial D_j}{\partial p_i}}$, the solutions for Nash and FPESS equilibrium will be different.

Afterward, to ensure the existence of a unique equilibrium as well, it is required to check that second order conditions have negative definite Hessian matrix. So first we look at the solvability condition for the Nash equilibrium. Particularly, consider the following Hessian matrix

$$H_i = \begin{pmatrix} 2\frac{\partial D_i}{\partial p_i} + \frac{\partial^2 D_i}{\partial p_i^2}(p_i - \frac{\partial c_i}{\partial D_i}) - \frac{\partial^2 c_i}{\partial D_i^2}(\frac{\partial D_i}{\partial p_i})^2 & \frac{\partial D_i}{\partial q_i} + \frac{\partial^2 D_i}{\partial p_i \partial q_i}(p_i - \frac{\partial c_i}{\partial D_i}) - \frac{\partial^2 c_i}{\partial D_i^2}(\frac{\partial D_i}{\partial p_i}\frac{\partial D_i}{\partial q_i}) \\ \frac{\partial D_i}{\partial q_i} + \frac{\partial^2 D_i}{\partial p_i \partial q_i}(p_i - \frac{\partial c_i}{\partial D_i}) - \frac{\partial^2 c_i}{\partial D_i^2}(\frac{\partial D_i}{\partial p_i}\frac{\partial D_i}{\partial q_i}) & -\frac{\partial^2 f_i}{\partial q_i^2} + \frac{\partial^2 D_i}{\partial q_i^2}(p_i - \frac{\partial c_i}{\partial D_i}) - \frac{\partial^2 c_i}{\partial D_i^2}(\frac{\partial D_i}{\partial q_i})^2 \end{pmatrix}$$

It can be rewritten as follow

$$H_i = \begin{pmatrix} 2\frac{\partial D_i}{\partial p_i} & \frac{\partial D_i}{\partial q_i} \\ \frac{\partial D_i}{\partial q_i} & -\frac{\partial^2 f_i}{\partial q_i^2} \end{pmatrix} + (p_i - \frac{\partial c_i}{\partial D_i}) \begin{pmatrix} \frac{\partial^2 D_i}{\partial p_i^2} & \frac{\partial^2 D_i}{\partial p_i \partial q_i} \\ \frac{\partial^2 D_i}{\partial p_i \partial q_i} & \frac{\partial^2 D_i}{\partial q_i^2} \end{pmatrix} - \frac{\partial^2 c_i}{\partial D_i^2} \begin{pmatrix} (\frac{\partial D_i}{\partial p_i})^2 & \frac{\partial D_i}{\partial p_i}\frac{\partial D_i}{\partial q_i} \\ \frac{\partial D_i}{\partial p_i}\frac{\partial D_i}{\partial q_i} & (\frac{\partial D_i}{\partial q_i})^2 \end{pmatrix}$$

Since $D_i(.)$ is concave and $(p_i - \frac{\partial c_i}{\partial D_i}) > 0$ then it is required only that the first matrix be negative definite. That means we obtain the following condition

$$-2\frac{\partial D_i}{\partial p_i}\frac{\partial^2 f_i}{\partial q_i^2} > (\frac{\partial D_i}{\partial q_i})^2 \tag{3.11}$$

The above condition guarantee that $|H_i| < 0$ and solvability condition is satisfied.

Furthermore, to ensure the existence of a unique evolutionary equilibrium as well, it is required to check that second order conditions have negative definite Hessian matrix.

$$H_i = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

Where

$$a_{11} = 2\frac{\partial D_i}{\partial p_i} + \frac{\partial^2 D_i}{\partial p_i^2}(p_i - \frac{\partial c_i}{\partial D_i}) - \frac{\partial^2 D_j}{\partial p_i^2}(p_j - \frac{\partial c_j}{\partial D_j}) - \frac{\partial^2 c_i}{\partial D_i^2}(\frac{\partial D_i}{\partial p_i})^2 + \frac{\partial^2 c_j}{\partial D_j^2}(\frac{\partial D_j}{\partial p_i})^2$$

$$a_{12} = a_{21} = \frac{\partial D_i}{\partial q_i} + \frac{\partial^2 D_i}{\partial p_i \partial q_i}(p_i - \frac{\partial c_i}{\partial D_i}) - \frac{\partial^2 D_j}{\partial p_i \partial q_i}(p_j - \frac{\partial c_j}{\partial D_j}) - \frac{\partial^2 c_i}{\partial D_i^2}(\frac{\partial D_i}{\partial p_i}\frac{\partial D_i}{\partial q_i}) + \frac{\partial^2 c_j}{\partial D_j^2}(\frac{\partial D_j}{\partial p_i}\frac{\partial D_j}{\partial q_i})$$

$$a_{22} = -\frac{\partial^2 f_i}{\partial q_i^2} + \frac{\partial^2 D_i}{\partial q_i^2}(p_i - \frac{\partial c_i}{\partial D_i}) - \frac{\partial^2 D_j}{\partial q_i^2}(p_j - \frac{\partial c_j}{\partial D_j}) - \frac{\partial^2 c_i}{\partial D_i^2}(\frac{\partial D_i}{\partial q_i})^2 + \frac{\partial^2 c_j}{\partial D_j^2}(\frac{\partial D_j}{\partial q_i})^2$$

This matrix is the summation of following matrixes

$$H_i = \begin{pmatrix} 2\frac{\partial D_i}{\partial p_i} & \frac{\partial D_i}{\partial q_i} \\ \frac{\partial D_i}{\partial q_i} & -\frac{\partial^2 f_i}{\partial q_i^2} \end{pmatrix} + (p_i - \frac{\partial c_i}{\partial D_i}) \begin{pmatrix} \frac{\partial^2 D_i}{\partial p_i^2} & \frac{\partial^2 D_i}{\partial p_i \partial q_i} \\ \frac{\partial^2 D_i}{\partial p_i \partial q_i} & \frac{\partial^2 D_i}{\partial q_i^2} \end{pmatrix} - (p_j - \frac{\partial c_j}{\partial D_j}) \begin{pmatrix} \frac{\partial^2 D_j}{\partial p_i^2} & \frac{\partial^2 D_j}{\partial p_i \partial q_i} \\ \frac{\partial^2 D_j}{\partial p_i \partial q_i} & \frac{\partial^2 D_j}{\partial q_i^2} \end{pmatrix}$$

$$-\frac{\partial^2 c_i}{\partial D_i^2} \begin{pmatrix} (\frac{\partial D_i}{\partial p_i})^2 & \frac{\partial D_i}{\partial p_i}\frac{\partial D_i}{\partial q_i} \\ \frac{\partial D_i}{\partial p_i}\frac{\partial D_i}{\partial q_i} & (\frac{\partial D_i}{\partial q_i})^2 \end{pmatrix} + \frac{\partial^2 c_j}{\partial D_j^2} \begin{pmatrix} (\frac{\partial D_j}{\partial p_i})^2 & \frac{\partial D_j}{\partial p_i}\frac{\partial D_j}{\partial q_i} \\ \frac{\partial D_j}{\partial p_i}\frac{\partial D_j}{\partial q_i} & (\frac{\partial D_j}{\partial q_i})^2 \end{pmatrix}$$

Since demand system by assumption 3.1 is concave, it is sufficient that the first matrix be negative definite and we have also $\frac{\partial D_i}{\partial p_i} < 0$. Therefore solvability condition for FPESS is identical to condition (3.11) derived in Nash equilibrium, that is $-2\frac{\partial D_i}{\partial p_i}\frac{\partial^2 f_i}{\partial q_i^2} > (\frac{\partial D_i}{\partial q_i})^2$.

□

Note that, as we explained in section 3.2.3, the solutions for Nash and FPESS equilibrium can be considered as Nash equilibria of two different games. If we look at equations 3.5 and 3.6 and compare them with equations 3.8 and 3.9, then we immediately see that the role of the demand function D_i in 3.5 and 3.6 (the *absolute* Nash problem) is now taken by relative demand $D_i - D_j$ in 3.8 and 3.9 (a *relative* Nash problem). The two FOC have identical structure because both are FOCs of Nash problems; the first one (with D_i) stemming from absolute maximizers, the second (with $D_i - D_j$) stemming from relative maximizers. Since the concept of evolutionary stability is based on relative performance, the comparison between Nash equilibrium and FPESS crucially depends on the nature of goods; whether they are substitutes or complements. Hence we extend our analysis to investigate this issue.

To begin with, rephrasing both FOCs of equations (3.5) and (3.6) for the Nash equilibrium, we obtain

$$\frac{p_i - \frac{\partial c_i}{\partial D_i}}{p_i} = -\frac{D_i}{p_i \frac{\partial D_i}{\partial p_i}} = \frac{1}{\varepsilon_{D_i,p_i}} \tag{3.12}$$

$$\frac{p_i - \frac{\partial c_i}{\partial D_i}}{p_i} = \frac{\frac{\partial f_i}{\partial q_i}}{p_i \frac{\partial D_i}{\partial q_i}} = \frac{D_i}{q_i \frac{\partial D_i}{\partial q_i}} \frac{\frac{\partial f_i}{\partial q_i} q_i}{f_i} \frac{f_i}{p_i D_i} = \frac{\varepsilon_{f_i,q_i}}{\varepsilon_{D_i,q_i}} \frac{f_i}{p_i D_i} \tag{3.13}$$

Where $\varepsilon_{D_i,p_i} = -\frac{\partial D_i}{\partial p_i} \frac{p_i}{D_i}$ and $\varepsilon_{D_i,q_i} = \frac{\partial D_i}{\partial q_i} \frac{q_i}{D_i}$ are the own price elasticity of demand and the own quality elasticity of demand respectively. And $\varepsilon_{f_i,q_i} = \frac{\partial f_i}{\partial q_i} \frac{q_i}{f_i}$ denotes elasticity of fixed cost with respect to the quality.

Moreover, first order conditions of FPESS equilibrium i.e. equations (3.8) and (3.9), after some algebraic manipulation, can be rephrased as

$$\frac{p_i - \frac{\partial c_i}{\partial D_i}}{p_i} = \frac{1}{(-\frac{\partial D_i}{\partial p_i} \frac{p_i}{D_i} + \frac{\partial D_j}{\partial p_i} \frac{p_i}{D_j} \frac{D_j}{D_i})} = \frac{1}{(\varepsilon_{D_i,p_i} - \frac{D_j}{D_i} \varepsilon_{D_j,p_i})} \tag{3.14}$$

$$\frac{p_i - \frac{\partial c_i}{\partial D_i}}{p_i} = \frac{1}{(\frac{\partial D_i}{\partial q_i} \frac{q_i}{D_i} - \frac{\partial D_j}{\partial q_i} \frac{q_i}{D_j} \frac{D_j}{D_i})} \frac{\frac{\partial f_i}{\partial q_i} q_i}{f_i} \frac{f_i}{p_i D_i} = \frac{\varepsilon_{f_i,q_i}}{(\varepsilon_{D_i,q_i} - \frac{D_j}{D_i} \varepsilon_{D_j,q_i})} \frac{f_i}{p_i D_i} \tag{3.15}$$

Where $\varepsilon_{D_j,p_i} = -\frac{\partial D_j}{\partial p_i} \frac{p_i}{D_j}$ and $\varepsilon_{D_j,q_i} = \frac{\partial D_j}{\partial q_i} \frac{q_i}{D_j}$ are the cross price elasticity of demand and the cross quality elasticity of demand respectively.

Market power can be measured as the ability to raise the prices higher than the perfectly competitive level. As we know the market power can be assessed by the Lerner index, i.e., $L = \frac{p_i - \frac{\partial c_i}{\partial D_i}}{p_i}$. Therefore, our analysis demonstrate that the market power is determined in Nash equilibrium by own elasticities of demand whereas, in FPESS equilibrium, the market power is determined by not only the own elasticities of demand but also the cross-elasticities of demand.

On the one hand, consider the case of complement goods, i.e., ($\frac{\partial D_j}{\partial p_i} < 0$ or $\varepsilon_{D_j,p_i} = -\frac{\partial D_j}{\partial p_i}\frac{p_i}{D_j} < 0$) and ($\frac{\partial D_j}{\partial q_i} > 0$ or $\varepsilon_{D_j,q_i} = \frac{\partial D_j}{\partial q_i}\frac{q_i}{D_j} > 0$). In this case, Lerner index exceeds the inverse of the own demand elasticities. So firms under RPM choose a strategy set that leads them to having more market power and behave less competitively compared to the Nash equilibrium.

On the other hand, consider the case of substitute goods, i.e., ($\frac{\partial D_j}{\partial p_i} > 0$ or $\varepsilon_{D_j,p_i} = -\frac{\partial D_j}{\partial p_i}\frac{p_i}{D_j} > 0$) and ($\frac{\partial D_j}{\partial q_i} < 0$ or $\varepsilon_{D_j,q_i} = \frac{\partial D_j}{\partial q_i}\frac{q_i}{D_j} < 0$). In this case, Lerner index falls below of the inverse of the own demand elasticities. Thus, compared to Nash equilibrium, firms under RPM gain less market power and behave more competitively.

Moreover, combining two equations (3.12) and (3.13) for Nash equilibrium, we get the following interesting equality

$$\frac{f_i}{p_i D_i} = \frac{\varepsilon_{D_i,q_i}}{\varepsilon_{D_i,p_i}}\frac{1}{\varepsilon_{f_i,q_i}} \tag{3.16}$$

The ratio of quality cost f_i over revenue $R_i = p_i D_i$ is termed as *quality improvement intensity* that determines how much firm is willing to invest on quality improvement plans. In Nash equilibrium, equality (3.16) states that quality improvement intensity is equal to the ratio of the quality elasticity of demand over the price elasticity of demand multiplied by the inverse of quality elasticity of fixed cost. Our theoretical model suggests that if we want to measure the quality improvement intensity, it will require estimating the demand and cost functions. Therefore, if demand is somewhat more sensitive to changes in quality compared to changes in price, then quality improvement spending (or R&D expenditure) is a large percentage of revenue. Furthermore, quality improvement intensity is affected by the inverse of quality elasticity of fixed cost.

However, merging two equations (3.14) and (3.15) in the case of evolutionary equilibrium, we obtain a different equality for quality improvement intensity, that is

$$\frac{f_i}{p_i D_i} = \frac{(\varepsilon_{D_i,q_i} - \frac{D_j}{D_i}\varepsilon_{D_j,q_i})}{(\varepsilon_{D_i,p_i} - \frac{D_j}{D_i}\varepsilon_{D_j,p_i})}\frac{1}{\varepsilon_{f_i,q_i}} \tag{3.17}$$

This is summarized in the following proposition

Proposition 3.6. *In Nash equilibrium, firm's quality improvement intensity is determined by own elasticities of demand using the equality (3.16), i.e.,* $\frac{f_i}{p_i D_i} = \frac{\varepsilon_{D_i,q_i}}{\varepsilon_{D_i,p_i}}\frac{1}{\varepsilon_{f_i,q_i}}$ *, whereas in evolutionary equilibrium firm's quality improvement intensity is determined by both own elasticities of demand and cross elasticities of demand using the equality (3.17), i.e.,* $\frac{f_i}{p_i D_i} = \frac{(\varepsilon_{D_i,q_i} - \frac{D_j}{D_i}\varepsilon_{D_j,q_i})}{(\varepsilon_{D_i,p_i} - \frac{D_j}{D_i}\varepsilon_{D_j,p_i})}\frac{1}{\varepsilon_{f_i,q_i}}$.

In order to understand better, how these two equilibrium concepts are different, we need to make two assumptions about the structure of demand and cost functions. Firstly, we assume that fixed cost $f_i(q_i) = (f + \psi q_i^2)$ is increasing and convex in quality level q_i since improving product's quality level require an initial investment by firms. Without loss of generality, we normalize f to zero. So with a standard linear variable cost, the cost function for firm i is assumed like the following form

$$C_i(x_i, q_i) = \psi q_i^2 + \nu x_i \tag{3.18}$$

Second, let's assume also demand function has a linear form as follows:

$$x_i = a - p_i + q_i + \beta \sum_{j=1, j\neq i}^{n} p_j - \gamma \sum_{j=1, j\neq i}^{n} q_j \tag{3.19}$$

Where $|\beta| < 1$, $|\gamma| < 1$ and $\beta, \gamma \neq 0$.

The restrictions on β and γ are implied by assumption 3.1 in which we assume that the demand function for firm i is affected more by changes in its own price and quality than those of its competitors. Furthermore, we assume a typical assumption $a > \nu$ to ensure that quality level is non-negative in equilibrium.

In the following propositions, we characterize the comparison between two equilibrium concepts.

Proposition 3.7. *Suppose that cost function and linear demand function are like equations (3.18) and (3.19). Supposing besides that the following 3 assumptions A1, A2, and A3 hold*
A1: $a \geq (1-(n-1)\beta)\nu$
A2: $(4\psi - 1) + (n-1)(\gamma - 2\psi\beta) > 0$
A3: $(\frac{4\psi}{1+\gamma} - 1 + \gamma) + (n-2)(\gamma - \frac{2\psi\beta}{1+\gamma}) > 0$
then FPESS equilibrium leads to higher quality compared to the Nash equilibrium quality i.e.

$$q^* > q^N \quad \textit{if and only if} \quad \gamma > \bar{\gamma} = \tfrac{\beta}{2+\beta-n\beta}$$

Proof. See appendix A. □

Proposition 3.8. *Suppose that cost function and linear demand function are like equations (3.18) and (3.19). Supposing besides that the following 3 assumptions A1, A2, and A3 hold*
A1: $a \geq (1-(n-1)\beta)\nu$
A2: $(4\psi - 1) + (n-1)(\gamma - 2\psi\beta) > 0$
A3: $(\frac{4\psi}{1+\gamma} - 1 + \gamma) + (n-2)(\gamma - \frac{2\psi\beta}{1+\gamma}) > 0$
then FPESS equilibrium leads to lower price compared to the Nash equilibrium i.e.

$$p^* < p^N \quad \textit{if and only if} \quad \beta > \bar{\beta} = \tfrac{(1+\gamma-n\gamma)\gamma}{2\psi}$$

Proof. See appendix A. □

Assumptions A1, A2, and A3 will guarantee that we have positive quality in equilibrium. The interpretation of Propositions 3.7 and 3.8 is as follows: RPM firm engages in more price (quality) competition if the price (quality) effect of other competitors on the demand of good i, i.e., $\beta(\gamma)$ is greater than the threshold $\bar{\beta}(\bar{\gamma})$. The conditions, derived in these two propositions, determine under which circumstances FPESS equilibrium induce more competition or less competition w.r.t the price or non-price dimension. Note that thresholds $\bar{\beta}(\bar{\gamma})$ are decreasing function of n ($\frac{\partial\bar{\beta}}{\partial n} = -\frac{\gamma^2}{2\psi} < 0$ and $\frac{\partial\bar{\gamma}}{\partial n} = -\frac{\beta^2}{(2+\beta-n\beta)^2} < 0$). This means that the higher number of firm in market the lower are these thresholds. Therefore, in case of β and $\gamma > 0$, the conditions on β and γ will disappear for n big enough and FPESS equilibrium for all ranges of β and γ leads to more competition in both dimensions of price and quality compared to Nash equilibrium.

Therefore, the comparison between two equilibrium concepts is influenced by the parameters of demand curve and cost function. To see in more detail dissimilarities concerning both strategic behavior and evolutionary behavior, we further study the following numerical example. This numerical example helps us to recognize better how variations in β, γ and market size n can influence the comparison between FPESS and Nash equilibrium. Note that if the price effect of good j on the demand of good i (β) or the quality effect of good j on the demand of good i (γ) are both positive, the goods of the firms are substitutes and if β and γ are negative, each two goods are complements.

Example 3.1. *This numerical example is performed with fixed scenario* $\psi = 1/2$ *but varying* β, γ *and* n. *Feasible ranges in case of substitute goods are* $0.01 < \beta < 1, 0.01 < \gamma < 1$ *and in case of complements goods are* $-1 < \beta < -0.01, -1 < \gamma < -0.01$ *and* $1 < n < 20$. *We plot the regions that satisfy the conditions of propositions 3.7 and 3.8 simultaneously in order to see what sort of parameters comply with the following circumstances:*

a) $p^* > p^N$ *and* $q^* > q^N$

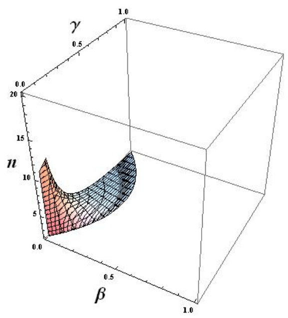
b) $p^* < p^N$ *and* $q^* < q^N$

c) $p^* > p^N$ *and* $q^* < q^N$

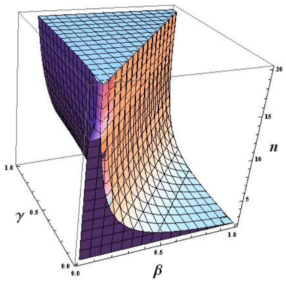
d) $p^* < p^N$ *and* $q^* > q^N$

3D plots in the figures 3.1 and 3.2 illustrate all above circumstances (a - d) for substitute goods and complements goods respectively. We obtain the following results.

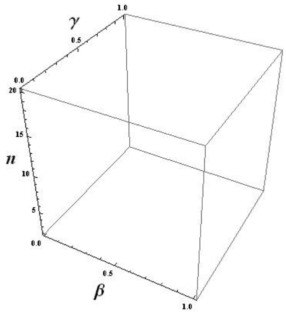
Result 3.9. *If goods of firms are substitutes, FPESS equilibrium cannot lead to less price competition and less quality competition compared to Nash equilibrium.*



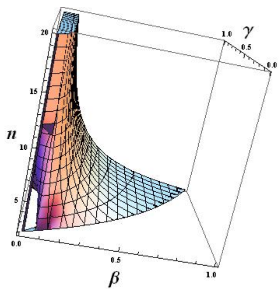
(A) $p^* > p^N$ and $q^* > q^N$



(B) $p^* < p^N$ and $q^* < q^N$

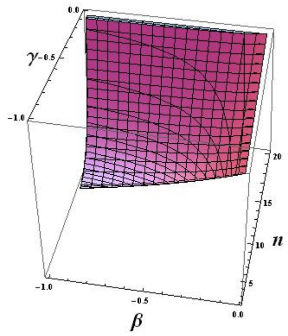


(C) $p^* > p^N$ and $q^* < q^N$

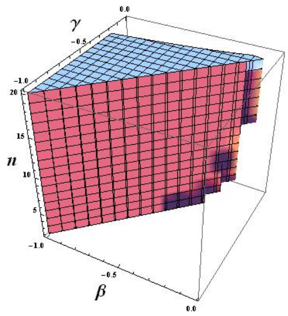


(D) $p^* < p^N$ and $q^* > q^N$

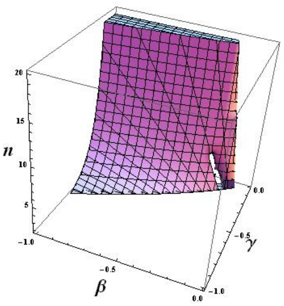
Figure 3.1: In Case of Substitute Goods



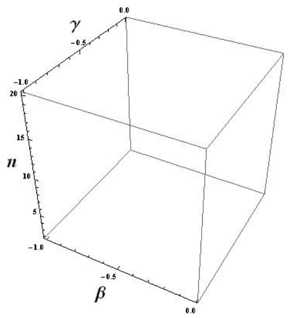
(A) $p^* > p^N$ and $q^* > q^N$



(B) $p^* < p^N$ and $q^* < q^N$



(C) $p^* > p^N$ and $q^* < q^N$



(D) $p^* < p^N$ and $q^* > q^N$

Figure 3.2: In Case of Complement Goods

Result 3.10. *If goods of firms are complements, FPESS equilibrium cannot lead to more price competition and more quality competition compared to Nash equilibrium.*

Results 3.9 and 3.10 are demonstrated by figures 3.1-(C) and 3.2-(D) respectively. Since FPESS is a Nash equilibrium for relative payoff maximizing (RPM) firms, therefore one interpretation of these results is that, if goods are substitutes, less price competition (higher price) and less quality competition (lower quality) are not feasible for a firm under RPM behavior. However if goods are complements, more price competition and more quality competition are not feasible for a firm under RPM behavior.

Note that, in our model with the existence of non-price strategy, when goods are substitutes ($\beta, \gamma > 0$) a RPM firm may choose less price competition i.e. $p^* > p^N$ (see Figure 3.1-(A); in fact it uses a non-price strategy to soften price competition). And when goods are complements ($\beta, \gamma < 0$), a RPM firm may possibly decide on more price competition i.e. $p^* < p^N$ (see Figure 3.2-(B)).

Our evolutionary analysis can be directly applied to a different setup like an oligopoly-technology model of price competition with technology choice rather than quality choice, (e.g. see Vives (2008) and Acemoglu and Jensen (2013)). In this type of games, firms decide about technology choice besides setting output or price. In fact, firm i incurs a similar cost of $C_i(x_i, a_i) = f_i(a_i) + c_i(a_i, x_i)$ by choosing technology a_i together with the quantity x_i but the demand is not affected by the technology choice a_i.

Notice that here it is not required to study the dynamic concept of evolutionary stability. Alos-Ferrer and Ania (2005), based on a result of Ellison (2000), show that in a symmetric N-player game with finite strategy set, a strictly globally stable ESS is the unique stochastic stable state of the imitation dynamics with experimentation. Further, Leininger (2006) shows that any ESS of a quasi-submodular generalized aggregative game is strictly globally stable. Since the oligopoly game under analysis is an aggregative game and by assuming that our payoff function satisfies quasi-submodularity property then a static solution of FPESS would be sufficient in this context.

3.4 Conclusion

In the present chapter, we have applied a concept of finite population evolutionarily stable strategy (FPESS) by Schaffer (1989), in which agents in economic and social environment adhere to *relative* payoff maximizing rather than *absolute* payoff maximizing behavior, in an oligopoly framework. A simple model of firm competition with simultaneous price and quality choices has been analyzed with the aim of comparing FPESS and Nash equilibria in oligopoly market. In general, the notion of FPESS and Nash equilibrium are not identical. Ania (2008) and Hehenkamp et al. (2010) study this relationship in different classes of games and particularly in the framework of Bertrand oligopoly with homogenous product.

The ratio of quality cost over revenue, so called as *quality improvement intensity*, determines how much firm is willing to invest on quality improvement plans. Our Nash equilibrium analysis shows that quality improvement intensity is affected by the ratio of the own quality elasticity of demand over the own price elasticity of demand. However, evolutionary equilibrium analysis is evidence for using both own elasticities of demand and cross elasticities of demand in order to determine firm's quality improvement intensity. Moreover, a similar result has been obtained for market power (measured here by Lerner index) in which Nash analysis proves that the market power is only determined by own price elasticity of demand. While, instead in evolutionary equilibrium, the market power is determined by not only the own price elasticity of demand but also the cross price elasticity of demand. Thus, in the case of complement goods, the firms market power is higher under evolutionary equilibrium. Quite the reverse, in the case of substitute goods, firms under evolutionary equilibrium have less market power and behave more competitively compared to the Nash equilibrium.

Chapter 4

The Nonparametric Approach to Evolutionary Oligopoly

Based on the results of chapter 2, this chapter presents a Walrasian equilibrium in expectation for an evolutionary model in the asymmetric oligopoly setup where firms have different cost functions to produce a homogenous good. Then, using revealed preference approach introduced by Carvajal, Deb, Fenske, and Quah (2013, Econometrica 2013), I derive the testable conditions of the evolutionary oligopoly model. Therefore, without making any parametric assumption regarding to the demand curve and the cost function, this approach characterizes a set of conditions (restrictions) on observational dataset to be consistent with the non-competitive evolutionary equilibrium. An empirical application to crude oil market with main producers is presented and I compare the rejection rates of both Cournot and evolutionary hypotheses.

4.1 Introduction

Revealed preference analysis is a practical and widely used instrument to test empirically the consistency of a theoretical model of consumer behavior with an observational dataset. This method has also been applied to analyze the firm behavior. For example, Afriat (1972), Hanoch and Rothschild (1972) and Varian (1984) characterize a data consistency test for production analysis of profit maximization and cost minimization models. Most recently Carvajal et al. (2013, 2014) and Cherchye et al. (2013) derive testable conditions in the Cournot oligopoly model of firm competition. Both papers by Carvajal et al. (2013, 2014) apply a revealed preference approach in a single-product and multi-product oligopoly while Cherchye et al. (2013) proposes a differential approach where the equilibrium price and quantities are functions of exogenous demand and supply shifters.

On the other hand, evolutionary oligopoly theory arises from the seminal papers of Schaffer (1988, 1989). Mainly these two papers, in contrast to Friedman's 1953 conjecture, argue that firm survival condition does not follow absolute payoff maximizing (APM) behavior rather it tracks a relative payoff maximizing (RPM) behavior. In particular, Schaffer (1989) verifies that firm survival is better demonstrated through an evolutionary model with relative payoff maximization rather than absolute payoff maximization. The appropriate evolutionary setup for firm competition is playing the field where all finitely many players in oligopoly game compete with each other instantaneously. So the behavior implied by RPM or spiteful behavior (Hamilton (1970)) leads to more competition between firms in a quantity oligopoly game. Furthermore, Vega-Redondo (1997) shows that the Walrasian equilibrium turns out to be the unique stochastically stable state in the symmetric Cournot oligopoly. However, in the present paper we show that evolutionary stability leads to a *non-Walrasian* outcome in an asymmetric oligopoly.

The standard evolutionary game theory applies for identical players (with respect to their cost type) in a symmetric setup. Here, first we construct an evolutionary model in the asymmetric oligopoly where firms have different cost functions. An equilibrium concept of *finite population evolutionary stable strategy* (FPESS), defined by Schaffer (1988), is applied. Defining a concept of evolutionary stability requires a symmetric setup with identical players. So we apply a symmetrization technique in order to transform the game with asymmetric firms into a symmetric oligopoly game and then extend Schaffer's definition of a FPESS to this setup. As a result, we identify a non-Walrasian solution concept for evolutionary stability in the asymmetric oligopoly.

Thereafter we study the consistency of evolutionary oligopoly model with an observational dataset. In particular, we attempt to answer the following questions. Whether a given set of observations is consistent with the evolutionary oligopoly model or not? Without making any parametric assumption on demand function, how can we recuperate the marginal costs from an observed behavior? In principal, inverse demand function and cost functions are not observable however we are able to observe equilibrium price and equilibrium quantities of all firms in the market. Suppose that we are given with an observed dataset on the behavior of an industry consisting of K firms producing a homogenous good. Consider a set of observations $\{p_t^*, (q_{k,t}^*)_{k\in\mathcal{K}}\}_{t\in\mathcal{T}}$ where p_t^* is an observed price of homogenous market good for each $t \in \mathcal{T} = \{1, ..., T\}$ and $q_{k,t}^*$ is observed output quantity of each firm

$k \in \mathcal{K} = \{1, ..., K\}$. For each time t, total output of industry $Q_t^* = \sum_{k=1}^{K} q_{k,t}^*$ is observed. Following Carvajal et al. (2013), we will derive conditions on this set of observations to be consistent with the evolutionary oligopoly model. In general these testable conditions take the form of a linear programing (LP) problem.

Motivating Example. Consider a homogenous good market with two firms 1 and 2. Assume that we observe the produced quantities of each firm and the market price of good in the two sequential periods as follows:

$$\begin{aligned} &\text{At observation } t, \quad p_t^* = 100, q_{1,t}^* = 6, and \quad q_{2,t}^* = 30. \\ &\text{At observation } t', \quad p_{t'}^* = 50, q_{1,t'}^* = 9, and \quad q_{2,t'}^* = 30. \end{aligned}$$

This example mimics the recent falling trend of crude oil price in which OPEC (here firm 2) as the main producer in the oil market did not alter its production level in response to US oil production increase (here firm 1). Annual average of US crude oil production due to the new extraction techniques of Shale oil raised from 5,65 million barrels per day in 2011 to 8,68 million barrels per day in 2014 despite the fact that annual average of OPEC oil production was round 32 million barrels per day without an intense variation during this period. In this example, firm 1 competes spitefully to increase its production levels and this leads to an abrupt fall of the market good price at time t'. With an easy calculation, we observe that there is a 25% decrease in US oil revenue ($p_t^* q_{k,t}^*$) (from 600 to 450) while at the same time OPEC oil revenue was decreased by 50% (from 3000 to 1500). Why did OPEC not reduce its output in response to the introduction of the US Shale oil technology? As Schaffer (1989) explains if firms have market power, profit maximizers are not necessarily the best survivors because of the possibility of spiteful behavior. This spiteful behavior, where one player harms itself in order to harm another more, cannot be explained by Cournot competition. Is the evolutionary behavior the answer? And how we can test for this behavior?

This chapter is organized as follows. The next section presents the evolutionary oligopoly model. Then, section 4.3 discusses the data consistency test of evolutionary oligopoly model. Section 4.4 examines a case study in the crude oil market and concluding remarks are offered in section 4.5.

4.2 The evolutionary oligopoly model

Consider K firms where all firms engage simultaneously to play an oligopoly stage game in each period t. Number of firms is constant for all t. The strategies for firms are their output quantities. Here we consider a static setup in which a player has the same strategy for all periods t. In evolutionary game theory, we assume that all players inherited their strategies and they cannot change their strategies. Though, analogous to biology, there is a chance for mutation or experimenting of a new strategy. As Schaffer (1989) explained the mutation can be perceived as the following situation in which the owner of one of the K firms hires a new manager and then this new manager may with some positive probability choose a different strategy for its firm.

We define the payoff function of each firm k, that produces q_k given that all other firms in the market produce $Q_{-k} = \sum_{l \neq k}^{K} q_l$, as following:

$$\pi_k(q_k, Q_{-k}) = P(q_k, Q_{-k})q_k - c_k(q_k), \ \forall k \in \mathcal{K} \tag{4.1}$$

$P(.)$ is an inverse demand function and decreasing in its arguments and $c_k(.)$ represents an increasing, twice differentiable and convex cost function.

In such a game, where players have different cost functions, defining a concept of evolutionary stability requires a symmetric setup with identical players. In chapter 2, we have identified a solution concept of Walrasian in expectation for evolutionary stability in an asymmetric oligopoly game where players are not identical with respect to their cost functions. In the model, Selten's approach is applied to construct a symmetric monomorphic population game out of an asymmetric polymorphic-population game. Proposition 2.7 generalizes the asymmetric oligopoly setup from two groups of high cost and low cost firms to K groups of firms that differ w.r.t. cost functions. In the present chapter, we adopt an analogous asymmetric setup, nevertheless with only one firm in each group, i.e., K firms with K different cost functions.

In order to construct a symmetric monomorphic population game out of an asymmetric game with non-identical players, we define a set of roles or information situations as follows

Definition 4.1. A firm may find itself in a number of roles or information situations $i \in \{1, ..., K\}$ where it must choose its action at each possible role (information situation).[1]

The set of roles here is identical with the set of cost functions $\{c_1, ..., c_k, ..., c_K\}$. Further we need a role assignment like so

Definition 4.2. A role assignment is a map that assigns without replacement each of roles $i \in \{1, ..., K\}$ to one of the K firms.

Firm $k \in \{1, ..., K\}$ is chosen by our role assignment with probability of $1/K$ as a firm with cost c_k. In this set-up, a firm contemplates behavior before it knows its assigned role $c_k \in \{c_1, ..., c_K\}$, an action (local strategy) of firm k assigned at role i is to select a pure strategy of q_{ki} and hence

Definition 4.3. A behavior strategy for a firm k is a vector $\mathbf{q_k} = [q_{k1}, ..., q_{ki}, ..., q_{kK}]$ giving a local strategy q_{ki} for each role $i \in \{1, ..., K\}$.

From this ex-ante point of view the game played in role-contingent strategies is symmetric (see Selten (1980, pp. 97-8.)). Let $\pi_{ki}(q_{ki}, [\mathbf{q}]_{-k})$ be a local payoff of firm k in role i when the other firms play their behavior strategies of $[\mathbf{q}]_{-k} = [\mathbf{q_1}, ..., \mathbf{q_{k-1}}, \mathbf{q_{k+1}}, ..., \mathbf{q_K}]$. Therefore one can define the total (expected) payoff function of each firm as follows

[1] We use a dissimilar notation i for a role to be not confused with player's notation k.

Definition 4.4. Let $\mathbf{q_k}$ and $[\mathbf{q}]_{-\mathbf{k}}$ be a behavior strategy for a firm k and behavior strategies of other firms respectively. The total (expected) payoff function for each firm k in the monomorphic population game is

$$E\pi_k([q_{k1}, ..., q_{ki}, ..., q_{kK}], [\mathbf{q}]_{-k}) = \sum_{i=1}^{K} \frac{1}{K} \pi_{ki}(q_{ki}, [\mathbf{q}]_{-k}) \quad \forall k = 1, ..., K \tag{4.2}$$

Consider now a finite population evolutionarily stable strategy of the game among these K firms. A strategy is evolutionary stable, if no mutant firm $l \neq k$ which chooses a different behavior strategy than $\mathbf{q_k^*} = [q_{k1}^*, ..., q_{ki}^*, ..., q_{kK}^*]$, say, can realize higher total profits than the firms which employ the incumbent behavior strategy $\mathbf{q}^*$. In other words, no mutant behavior strategy $\mathbf{q}^m$ can invade a population of $\mathbf{q}^*$ strategists successfully. Formally Schaffer's (1988) definition then reads

Definition 4.5. A behavior strategy profile $\mathbf{q}^*$ is a FPESS if

$$E\pi_k(\mathbf{q_k^*}, \mathbf{q_l^m}, [\mathbf{q}^*]_{-k-l}) > E\pi_l(\mathbf{q_l^m}, [\mathbf{q}^*]_{-l}) \quad \forall \mathbf{q_l^m} \neq \mathbf{q_k^*}, and \quad \forall l \neq k. \tag{4.3}$$

Therefore we write mutant's total payoff and incumbent's total payoff respectively as follows:

$$E\pi_l([q_{l1}^m, ..., q_{lK}^m], [\mathbf{q}^*]_{-l}) = \sum_{i=1}^{K} \frac{1}{K} P(q_{li}^m, [\mathbf{q}^*]_{-l}) q_{li}^m - c_i(q_{li}^m) \tag{4.4}$$

$$E\pi_k([q_{k1}^*, ..., q_{kK}^*], [q_{l1}^m, ..., q_{lK}^m], [\mathbf{q}]_{-k-l}) = \sum_{i=1}^{K} \frac{1}{K} (\sum_{j \neq i}^{K} \frac{1}{K-1} (P(q_{kj}^*, q_{li}^m, [\mathbf{q}^*]_{-k-l}) q_{kj}^* - c_j(q_{kj}^*)) \tag{4.5}$$

$E\pi_l(.)$, the mutant's expected payoff consists of K local payoffs where the mutant assigned to the role i with the uniform probability function of $1/K$. Accordingly, the calculation of incumbent's expected payoff $E\pi_k(.)$ is slightly more complicated and consists of other terms in order to account for the mutant's role. For example, when a mutant is assigned to the role i with the probability of $1/K$ then the incumbent's role $j \neq i$ can be assigned from the $K-1$ possibility with the probability of $1/(K-1)$.

Theorem 4.6. *In the symmetrized game of the asymmetric oligopoly market with K firms that differ w.r.t. cost functions, there exists an evolutionary equilibrium where the FPESS quantities satisfy the following equations*

$$P(\sum_{k=1}^{K} q_k^*) + P'(\sum_{k=1}^{K} q_k^*)(q_k^* - \sum_{l\neq k}^{K} \frac{1}{K-1} q_l^*) = c_k'(q_k^*) \quad \forall k \in \{1, ..., K\}. \tag{4.6}$$

Proof. According to Schaffer (1989), in a playing the field game, we can find a FPESS as the solution of following optimization problem

$$(q_{l1}^*, ..., q_{lK}^*) = \underset{q_{l1}^m, ..., q_{lK}^m}{\arg\max}\, \varphi = E\pi_l([q_{l1}^m, ..., q_{lK}^m], [\mathbf{q}^*]_{-l}) - E\pi_k([q_{k1}^*, ..., q_{kK}^*], [q_{l1}^m, ..., q_{lK}^m], [\mathbf{q}]_{-k-l}) \tag{4.7}$$

First order conditions with respect to q_{li}^m respectively are as follows:

$$\frac{1}{K}(P(.) + P'(.)q_{li}^m - c_i'(q_{li}^m)) - \frac{1}{K}\sum_{j\neq i}^{K} \frac{1}{K-1} P'(.)q_{kj}^* = 0 \quad \forall i \in \{1, ..., K\}$$

$P'(.)$ is the derivative of inverse demand function. For the reason that the solution must be symmetric in players and satisfies definition (4.5), we impose $q_{li}^m = q_{ki}^* = q_i^*$. Then after rearranging, the following set of equations are obtained

$$P(\sum_{i=1}^{K} q_i^*) + P'(\sum_{i=1}^{K} q_i^*)(q_i^* - \sum_{j\neq i}^{K} \frac{1}{K-1} q_j^*) = c_i'(q_i^*) \quad \forall i \in \{1, ..., K\}$$

The set of roles can be identified with the set of players in our asymmetric setup and the proof is complete. □

In fact the strategy that survives in economic natural selection under playing the field conditions is the relative, not absolute, payoff maximizing strategy. A firm needs to beat the average of expected payoffs over all firms rather than to maximize its absolute expected payoff to be evolutionarily successful.

This theorem represents that, in a homogenous good market with asymmetric cost modeling, the equilibrium price is determined such that a low cost firm obtains a positive markup over its marginal cost while a high cost firms sells in a price lower than its marginal cost. Note that, as it has shown in Proposition 2.5, all types of firms can coexists and make a positive profit if the cost functions are sufficiently convex.

It is important to note that the evolutionary equilibrium in the asymmetric setup is different from both Walraisan equilibrium and Cournot equilibrium. The FPESS quantities do not correspond to the Walrasian quantities for each individual firm. However, the evolutionary equilibrium price so-called as *Walrasian price in expectation* equals an average of marginal costs.

4.3 Testing data consistency of evolutionary oligopoly model (characterization)

Deriving testable implications from an evolutionary model of oligopoly is important because it better explains conditions for firm survival in the market. Particularly, we are able to test empirically whether the model described in the previous section effectively holds or not and it also permits us to compare whether it is distinguishable from the competitive Walrasian behavior and the best reply Cournot Nash model of firm behavior. In principal, the inverse demand function and cost functions are not observable however we are able to observe the equilibrium quantities of all firms and the market equilibrium price. Suppose that we are given with the observed dataset on the behavior of an industry or a market with K firms producing a homogenous single good. Consider a set of observations $\{p_t^*, (q_{k,t}^*)_{k\in\mathcal{K}}\}_{t\in\mathcal{T}}$ where $p_t^* > 0$ is an observed price of homogenous market good at time $t \in \mathcal{T} = \{1, ..., T\}$ and $q_{k,t}^* > 0$ is observed output quantity of each firm $k \in \mathcal{K} = \{1, ..., K\}$ in every period of t. Total output of industry $Q_t^* = \sum_{k=1}^{K} q_{k,t}^*$ is also observed for each time t. Market demand of this single good is determined by a continuous and differentiable inverse demand function P_t at each time t and we assume that it is decreasing in its argument. In addition, each firm k has also a continuous and increasing function of c_k.

Before addressing the data consistency test for the evolutionary oligopoly model, we first give the definitions for the rationalization of an observational dataset for the Cournot Nash model and perfect competition (Walrasian) model of firm behavior. Carvajal et al. (2013) formally define and characterize the consistency of a dataset with the Cournot model under convex cost functions[2] as follows:

Definition 4.7 (Cournot rationalizability with convex cost functions)**.** Consider a set of observations $\{p_t^*, (q_{k,t}^*)_{k\in\mathcal{K}}\}_{t\in\mathcal{T}}$. This dataset is Cournot rationalizable if there exist convex cost functions $c_k(.)\forall k \in \mathcal{K}$ and decreasing inverse demand functions $P_t(.)$ for each $t \in \mathcal{T}$ that satisfy the following two conditions

1. $P_t(Q_t^*) = p_t^*$
2. $P_t(Q_t^*) + P_t'(Q_t^*)q_{k,t}^* = c_k'(q_{k,t}^*)$

The first condition connects unobserved inverse demand function evaluated at total output of industry Q_t^* to observed prices in each time t and the second condition states that $q_{k,t}^*$, given the output of other firms (best responses of other firms $q_{l,t}^*, l \neq k$), must solve the first order condition of firm k's profit maximization problem at each time t.

Note that the approach explained here, without making any parametric assumption about demand curve and cost functions, checks the consistency of Cournot model with a set of observations. Similarly we can define the rationalization of our evolutionary oligopoly model with an observational dataset as follows:

Definition 4.8 (Evolutionary rationalizability)**.** Consider a set of observations $\{p_t^*, (q_{k,t}^*)_{k\in\mathcal{K}}\}_{t\in\mathcal{T}}$. This dataset is evolutionary rationalizable if there exist convex cost functions $c_k(.)\forall k \in \mathcal{K}$ and decreasing inverse demand functions $P_t(.)$ for each $t \in \mathcal{T}$ that satisfy the following two conditions

1. $P_t(Q_t^*) = p_t^*$

[2]As it has been argued in the paper Cournot rationalizability on its own does not impose operational constraints on the observation set across time and it requires further assumptions e.g. assuming a convexity property for cost functions.

2. $P_t(Q_t^*) + P_t'(Q_t^*)\widehat{q_{k,t}^*} = c_k'(q_{k,t}^*)$ where $\widehat{q_{k,t}^*} = (q_{k,t}^* - \sum_{l\neq k}^{K} \frac{1}{K-1} q_{l,t}^*)$

The difference between Definition 4.7 and Definition 4.8 comes from the second condition which it is inferred directly from equation 4.6. Here the relative quantity terms of $\widehat{q_{k,t}^*}$ are substituted in the markup term as opposed to the absolute quantity terms of $q_{k,t}^*$. In a relative contest implied by evolutionary successfulness, the relative position of firm k at time t in the market, i.e. $\widehat{q_{k,t}^*}$, is determined by its observed quantity $q_{k,t}^*$ subtracted from the average quantity of the rest of firms in the market $\sum_{l\neq k}^{K} \frac{1}{K-1} q_{l,t}^*$. That is why, in an evolutionary oligopolistic competition, a low cost firm chooses larger quantity benefiting a higher markup over its marginal cost and it pushes a high cost firm to produce a smaller quantity at a price level even lower than its marginal cost.

To compare with Cournot model and evolutionary model, we further consider the perfect competition model and provide also a definition of Walrasian rationalizability. Price taking behavior Walrasian (Perfect competition) model sets the marginal cost of each firm k at time t equal to market price at time t.

Definition 4.9 (Walrasian rationalizability)**.** Consider a set of observations $\{p_t^*, (q_{k,t}^*)_{k\in\mathcal{K}}\}_{t\in\mathcal{T}}$. This dataset is Walrasian rationalizable if there exist convex cost functions $c_k(.) \forall k \in \mathcal{K}$ and decreasing inverse demand functions $P_t(.)$ for each $t \in \mathcal{T}$ that satisfy the following condition $P_t(Q_t^*) = p_t^* = c_k'(q_{k,t}^*)$.

Analogous to Carvajal et al. (2013), we define $c_k'(q_{k,t}^*)$ as a set of subgradients of $c_k(.)$ at $q_{k,t}^*$ and $P_t'(Q_t^*)$ as a set of gradients of inverse demand function $P_t(.)$ at Q_t^* and assume that the set of observations $\{p_t^*, (q_{k,t}^*)_{k\in\mathcal{K}}\}_{t\in\mathcal{T}}$ is consistent with evolutionary oligopoly model. Suppose there exists a set of numbers $x_{k,t} \geq 0$ and $y_t \leq 0$ that belong to the subsequent sets of $c_k'(q_{k,t}^*)$ and $P_t'(Q_t^*)$ that satisfy the first order condition 2) in definition 4.8 of firm k at each time t. Then after substituting condition 1) into the condition 2), we obtain the following property

$$y_t = \frac{x_{1,t} - p_t^*}{\widehat{q_{1,t}^*}} = \frac{x_{2,t} - p_t^*}{\widehat{q_{2,t}^*}} = ... = \frac{x_{K,t} - p_t^*}{\widehat{q_{K,t}^*}} \leq 0 \tag{4.8}$$

We denote equation 4.8 as the joint demand slope property[3] if it is satisfied for each $t \in \mathcal{T}$. Moreover we have another type of restrictions imposed by convexity of cost functions, that is,

$$if \quad q^*_{k,t'} < q^*_{k,t} \quad then \quad x_{k,t'} \leq x_{k,t} \quad \forall k \in \mathcal{K}$$

This set of restrictions, the so-called *co-monotone property*, imposes across time for each firm k and it can also be expressed as

$$(q^*_{k,t'} - q^*_{k,t})(x_{k,t'} - x_{k,t}) \geq 0 \tag{4.9}$$

This says that the set of $\{x_{k,t}\}_{(\forall t \in \mathcal{T} \& \forall k \in \mathcal{K})}$ obeys increasing marginal costs. So we say that a non-increasing (decreasing) inverse demand function $P_t(.)$ and convex cost functions $c_k(.)$ *evolutionarily* rationalize the dataset if the set of $\{x_{k,t}\}_{(\forall t \in \mathcal{T} \& \forall k \in \mathcal{K})}$ satisfies the above two properties. Hence the following theorem summarizes the above discussion.

Theorem 4.10. *The set of observations $\{p^*_t, (q^*_{k,t})_{k \in \mathcal{K}}\}_{t \in \mathcal{T}}$ is consistent with our evolutionary model under convex cost functions if and only if there exist two number sets of $\{y_t \leq 0\}_{(\forall t \in \mathcal{T})}$ and $\{x_{k,t} \geq 0\}_{(\forall t \in \mathcal{T} \& \forall k \in \mathcal{K})}$ that satisfy the following properties.*

1. $y_t = \frac{x_{k,t} - p^*_t}{\widehat{q^*_{k,t}}} \leq 0 \quad where \quad \widehat{q^*_{k,t}} = (q^*_{k,t} - \sum_{l \neq k}^{K} \frac{1}{K-1} q^*_{l,t}) \quad \forall t \in \mathcal{T} \quad and \quad \forall k \in \mathcal{K}$

2. $(q^*_{k,t'} - q^*_{k,t})(x_{k,t'} - x_{k,t}) \geq 0 \quad \forall t, t' \in \mathcal{T} and \quad \forall k \in \mathcal{K}$

Proof. Assume that the set of observations is consistent with cost functions $\{c_k\}_{\forall k \in \mathcal{K}}$ and demand functions $\{P_t\}_{\forall t \in \mathcal{T}}$ then we have already proved that there exist $x_{k,t} \in c'_k(q^*_{k,t}) \& y_t \in P'_t(Q^*_t)$ that satisfy the properties of 1 and 2.

To show the reverse, first of all, it is required to show that if we have positive scalars $\{x_{k,t}\}_{(\forall t \in \mathcal{T})}$ that are increasing with $q^*_{k,t}$ for some firm k; then there exist

[3]This property is known by Carvajal et al. (2013) as common ratio property.

a convex cost function c_k with $x_{k,t} \in c'_k(q^*_{k,t})$. The proof of this statement follows exactly from Carvajal et al. (2013). (See Carvajal et al.(2013, Lemma 2)).

Secondly, suppose that there are $\{x_{k,t}\}_{(\forall t \in \mathcal{T} \& \forall k \in \mathcal{K})}$ such that the joint demand slope property and co-monotone property hold and moreover there are convex cost functions c_k with $x_{k,t} \in c'_k(q^*_{k,t})$. Then, we show that $\{(q^*_{k,t})_{k\in\mathcal{K}}\}_{t\in\mathcal{T}}$ form an evolutionary equilibrium if there exist a non-increasing demand function $P_t(.)$ such that $P_t(Q^*_t) = p^*_t$ and with firms having cost functions c_k.

Following Carvajal et al.(2013, Lemma 1), define P_t by $P_t(Q) = \alpha_t + \beta_t Q$, where $\beta_t = \frac{x_{k,t}-p^*_t}{\widehat{q^*_{k,t}}}$ and we can choose α_t such that $P_t(Q^*_t) = p^*_t$. A mutant firm l in our symmetrized version of evolutionary game selects a different behavior strategy $\mathbf{q^m_{l,t}} = [q^m_{l1,t}, ..., q^m_{lK,t}]$ from other incumbent firms $[\mathbf{q}^*]_{-l,t} = [\mathbf{q^*_{1,t}}, ..., \mathbf{q^*_{l-1,t}}, \mathbf{q^*_{l+1,t}}, ..., \mathbf{q^*_{K,t}}]$ at time t. Here a mutant firm l chooses a local strategy $q^m_{li,t} \geq 0$ to maximize the following relative total payoff (equation 4.7) at each role i and time t

$$\varphi_{l,t} = \sum_{i=1}^{K} \frac{1}{K} P_t(q^m_{li,t}, [\mathbf{q}^*]_{-l,t}) q^m_{li,t} - c_i(q^m_{li,t}) - \sum_{i=1}^{K} \frac{1}{K} (\sum_{j\neq i}^{K} \frac{1}{K-1} (P_t(q^*_{kj,t}, q^m_{li,t}, [\mathbf{q}^*]_{-k-l,t}) q^*_{kj,t} - c_j(q^*_{kj,t}))$$

Since $\varphi_{l,t}$ is concave, $q^m_{li,t}$ is optimal if and only if it satisfies the following FOC evaluated at $q^m_{li,t} = q^*_{ki,t} = q^*_{i,t}$

$$P(Q^*_t) + q^*_{i,t} P'(Q^*_t) - \sum_{j\neq i}^{K} \frac{1}{K-1} q^*_{j,t} P'(Q^*_t) - c'_i(q^*_i) = 0 \quad \forall i \in \{1, ..., K\}$$

As the set of roles is equivalent with the set of players and we also have $q^*_{k,t} - \sum_{j\neq i}^{K} \frac{1}{K-1} q^*_{l,t} = \widehat{q^*_{k,t}}, x_{k,t} \in c'_k(q^*_{k,t})$ and $P'(Q^*_t) = \beta_t = \frac{x_{k,t}-p^*_t}{\widehat{q^*_{k,t}}}$

$$p^*_t + \widehat{q^*_{k,t}} (\frac{x_{k,t} - p^*_t}{\widehat{q^*_{k,t}}}) - x_{k,t} = 0$$

Therefore we have proven that $q^*_{k,t}$ constitute an evolutionary equilibrium for firm k at observation t and this also completes the proof that the set of observations $\{p^*_t, (q^*_{k,t})_{k\in\mathcal{K}}\}_{t\in\mathcal{T}}$ for all t constitutes the evolutionary equilibrium. $\square$

Note that the relative quantity terms of $\widehat{q^*_{k,t}}$ may be negative or positive and knowing that the slope of demand curve is negative at each time t, we must have $x_{k,t} < p^*_t$ if $\widehat{q^*_{k,t}} > 0$ and $x_{k,t} > p^*_t$ if $\widehat{q^*_{k,t}} < 0$. So this condition can be summarized in the following form $(x_{k,t} - p^*_t)\widehat{q^*_{k,t}} < 0$.

In general, these properties impose linear restrictions with unknowns $x_{k,t}$ that can be checked by linear programming (LP) or quadratic programming (QP) methods. Therefore the test takes the form of mathematical optimization problem in which the consistency of a dataset with the evolutionary model would be verified if the linear constraints could produce a convex feasible region of possible values for those unknowns. This feasible region is a convex polytope that formed as the intersection of finitely many half spaces defined by the following sets of linear restrictions of i-iii

i. $\frac{-x_{k,t}}{\widehat{q^*_{k,t}}} + \frac{x_{l,t}}{\widehat{q^*_{l,t}}} = \frac{-p^*_t}{\widehat{q^*_{k,t}}} + \frac{p^*_t}{\widehat{q^*_{k,t}}}$ $\quad \forall k, l \in \mathcal{K}, k \neq l$ and $\forall t \in \mathcal{T}$

ii. $\widehat{q^*_{k,t}} x_{k,t} < p^*_t \widehat{q^*_{k,t}}$ $\quad \forall k \in \mathcal{K}$ and $\forall t \in \mathcal{T}$

iii. $(q^*_{k,t'} - q^*_{k,t})(x_{k,t'} - x_{k,t}) \geq 0$ $\quad \forall k \in \mathcal{K}, \forall t, t' \in \mathcal{T}$ and $t \neq t'$

where $\widehat{q^*_{k,t}} = \frac{K}{K-1} q^*_{k,t} - \frac{\sum_{i=1}^{K} q^*_{k,t}}{K-1}$ Note that Cournot rationalizability imposes a different set of restrictions of iv-v on the dataset nevertheless the co-monotone condition iii is the same.

iv. $\frac{-x_{k,t}}{q^*_{k,t}} + \frac{x_{l,t}}{q^*_{l,t}} = \frac{-p^*_t}{q^*_{k,t}} + \frac{p^*_t}{q^*_{k,t}}$ $\quad \forall k, l \in \mathcal{K}, k \neq l$ and $\forall t \in \mathcal{T}$

v. $q^*_{k,t} x_{k,t} < p^*_t q^*_{k,t}$ $\quad \forall k \in \mathcal{K}$ and $\forall t \in \mathcal{T}$

iii. $(q^*_{k,t'} - q^*_{k,t})(x_{k,t'} - x_{k,t}) \geq 0$ $\quad \forall k \in \mathcal{K}, \forall t, t' \in \mathcal{T}$ and $t \neq t'$

Example 4.1 illustrates a dataset that is not consistent with the Cournot model but can be rationalized by the evolutionary model. In this example, firm 1 competes spitefully to increase its production levels and this leads to an abrupt fall of the market good price at time 2. This spiteful behavior, where one player harms itself in order to harm another more, cannot be explained by Cournot competition.

Furthermore this example explains the recent dropping trend of crude oil price in which OPEC as the main oil markets producer did not alter its production level in response to growing of USA production caused by new extraction techniques of shale oil. This observed behavior cannot be rationalized by the Cournot competition model but rather by the relative payoff maximizers of the evolutionary model.

Example 4.1. *Consider a homogenous good market with two firms 1 and 2 and assume that we observe the produced quantities of each firm and the market price of good in the two sequential periods as follows:*

$$\begin{aligned} &\textit{At observation } t, \quad p_t^* = 100, q_{1,t}^* = 6, \textit{ and } q_{2,t}^* = 30. \\ &\textit{At observation } t', \quad p_{t'}^* = 50, q_{1,t'}^* = 9, \textit{ and } q_{2,t'}^* = 30. \end{aligned}$$

To see whether this dataset is Cournot rationalizable with convex cost function, it is required to find a set of numbers assigned to marginal costs i.e. $x_{1,t}, x_{1,t'}, x_{2,t}, x_{2,t'} \geq 0$ that satisfy the restrictions (iii-iv). So we have

$$\begin{gathered} 5x_{1,t} - x_{2,t} = 400, \quad 10x_{1,t'} - 3x_{2,t'} = 350 \\ x_{1,t}, x_{2,t} \leq 100, \quad x_{1,t'}, x_{2,t'} \leq 50 \\ x_{1,t} \leq x_{1,t'} \end{gathered}$$

Note that co-monotone property does not impose a restriction on firm 2. So it is straightforward to check that the solution space defined by these restrictions does not have a feasible region. (Since $5x_{1,t} - x_{2,t} = 400$ does not intersect with the region $0 \leq x_{1,t} \leq 50, 0 \leq x_{2,t} \leq 100$.) As a result, this dataset cannot be rationalized by Cournot model.

However the evolutionary rationalizability (linear restrictions of i-iii) leads to

$$\begin{gathered} x_{1,t} + x_{2,t} = 200, \quad x_{1,t'} + x_{2,t'} = 100 \\ x_{1,t} \geq 100, x_{2,t} \leq 100, \quad x_{1,t'} \geq 50, x_{2,t'} \leq 50 \\ x_{1,t} \leq x_{1,t'} \end{gathered}$$

Solving for a feasible region, the set of candidate solutions narrows down as follows

$$x_{1,t} = 100, x_{2,t} = 100, y_t = 0$$
$$x_{1,t'} = 100, x_{2,t'} = 0, y_{t'} = -50/21$$

Along to the identification of marginal costs, the slopes of demand curve in each time i.e. $y_t, y_{t'} \leq 0$ have been identified by formula $y_t = \frac{x_{k,t} - p_t^*}{\widehat{q_{k,t}^*}}$.

4.4 A case study in the oil market

In this section we apply the consistency test explained in the section 4.3 to a dataset of the crude oil market. We want to test both Cournot and evolutionary hypothesis among three major players in the oil market that is, OPEC total production as a single unit, Russia and USA. As a matter of fact, OPEC countries produce approximately at least 40 percent of the world's oil since its formation in 1960 and this share is even more for the internationally traded oil. OPEC as a major player in oil market together with the two next largest producers, USA and Russia,[4] are accounted for over than 85% of the world's oil production in 1973 and over than 66% in 2014. We use a dataset which contains annual series of crude oil production in thousands of barrels per day by these three players in the market. The data sources include oil production series from Monthly Energy Review (MER) U.S. Energy Information Administration and also price series of annual averages of selected OPEC crude oils (OPEC basket) published by association of German petroleum industry (MWV) from 1973 until 2015.

We split up the whole dataset into several subsets such that each subset is made of W sequential years as time windows ($W = 2, 3, ..., 6$) and I numbers of countries ($I = 2, 3$). The rejection rates are calculated in the vein of Carvajal et al. (2013) where first we test whether each subset is consistent with the Cournot model or

[4] Russia (formerly Soviet Union) is a major oil producer and regarded as a one unit during time period of the study, since most of oil production in Soviet Union (around more than 95 percent) was produced in the present-day territory of Russia.

evolutionary model, i.e., whether a mathematical optimization program consisted of linear restriction of i-iii or iii-v has a feasible region. So the test is repeated on all subsets of the dataset and then, we report rejection rate as a proportion of subsets that fail the test and cannot be rationalized by each model (See Appendix B for for the data and Appendix C for the Matlab code). Table 1 illustrates the rejection rates for the subsets of this data (with the number of countries $I = 2, 3$ and windows W from 2 years up to 6 years). Comparing the evolutionary model to the Cournot model, we see that the rejection rates jump down for the evolutionary model. For example, in case of I=2 countries and T=2 years window, the drop in rejection rates is more than 50 percent (from 0.398 to 0.187). Put side by side both Cournot and evolutionary models, we conclude that evolutionary oligopoly model explains better the dataset of main producers (OPEC, Russia and USA) in the oil market.

TABLE 4.1: Rejection rates of Cournot and evolutionary models with main oil producers

OPEC, Russia and USA, Annual datasets				
W	Cournot model		Evolutionary model	
	I=2	I=3	I=2	I=3
2 years	0.398	0.683	0.183	0.439
3 years	0.608	0.850	0.333	0.750
4 years	0.744	0.923	0.487	0.872
5 years	0.833	0.974	0.596	0.921
6 years	0.919	1.000	0.694	0.946

OPEC basket price series are applied.

4.5 Conclusion

The contributions of present study are twofold. Firstly we show that a static evolutionary model offers a different solution than a competitive Walrasian equilibrium. In fact, here we take issue with the result by Vega-Redondo (1997) that the imitation of successful strategies leads to the competitive equilibrium outcome in the symmetric quantity game of a homogenous good market. Apesteguia et al. (2010) also show that Vega-Redondo's result is not robust to the slightest asymmetry in fixed costs. Then secondly we design for practical purposes a revealed preference test to check the consistency of developed model with a generic set of observations based on the work of Carvajal et al. (2013). Therefore, contrary to the typical empirical literature in industrial organization without making any parametric assumption regarding to the demand curve and the cost function, this approach characterizes a set of conditions (restrictions) on observational dataset to be consistent with evolutionary oligopoly model. Finally, this nonparametric revealed preference test has been applied to a dataset for oil market and we conclude that the behavior of top oil producers in the market is more consistent with an evolutionary game than a Cournot game.

Chapter 5

Discussion and future researches

This study investigates competition using evolutionary game models when the number of players in the market are finite and they have market power. Here we deliberate why it makes sense to use an evolutionary game theory approach in order to analyse an oligopolistic market. In fact, as we know a rational individual selection leads to Cournot equilibrium in the quantity-setting oligopoly game (as opposed to a rational group selection that results in the collusive outcome). However, a bounded rational Darwinian selection through a mechanism of imitation and mutation turns out to be the competitive Walrasian outcome. The evolutionary oligopoly literature was initiated by Alchian (1950), who were the first to argue bounded rationality in economic behavior. In Alchian's own words:

"Profit maximization is meaningless as a guide to specifiable action. ... Observable patterns of behavior and organization are predictable in terms of their relative probabilities of success or viability if they are tried." (Alchian (1950, pp. 211-220))

Then Schaffer (1989) is the pioneer to develop an evolutionary model of economic natural selection in the oligopolistic market. Schaffer shows that a firm choosing an evolutionary stable quantity strategy survives longer than a firm that selects a different strategy (like a Nash strategy) in the market. In a seminal paper, Vega-Redondo (1997) proves that imitation of the most profitable firm in a dynamic setup leads to the Walrasian competitive equilibrium as the long run outcome of oligopoly game.

Our contributions to this literature are as follows:

First of all, in chapter 2, we have shown that Darwinian selection, specifically evolutionary stability, in an asymmetric setup with finite number of firm having different cost structure, give rise to a form of equilibrium so-called *Walrasian in expectation.* Characteristic of this equilibrium is such that it includes a semi competitive market outcome with price equal to average marginal costs while individual firms outcomes do not correspond with their counterpart competitive outcomes.

Second, in the chapter 3, we have focused on explaining the behaviour of RPM firms and comparing with rational APM firms in a symmetric oligopoly setup, nevertheless extending their strategic choice in an additional dimension i.e. price and non-price variable. While APM captures the behaviour of self-interested rational players, RPM perceive the spiteful behaviour of players that involve in a relative game. As a result, the market power measured by Lerner index is influenced in evolutionary equilibrium by not only the own elasticities of demand but also the cross-elasticities of demand.

Finally, testable conditions for the evolutionary oligopoly model have been derived in chapter 4. A revealed preference approach has been applied. This non-parametric methodology (as it does not impose a functional form on market demand and firms cost structure) allow us to test the consistency of the evolutionary model with a generic observational dataset of a homogenous good oligopolistic market. This test takes the form of a linear programing composed of observable restrictions and the consistency of a dataset with the evolutionary model would be verified if the linear programming could be solved for a feasible region. To end, we undertook a case study including a dataset on crude oil market for three major oil producers, i.e., OPEC, Russia and USA. Comparing the rejection rates between evolutionary model and Cournot model, we conclude that the behaviour of major oil producers is better explained by evolutionary equilibrium rather than a Cournot Nash equilibrium.

A strand of experimental literature (see e.g. Huck et al. (1999) and Offerman et al. (2002)) examine an evolutionary model hypothesis in a quantity setting oligopoly and they find a robust evidence for supporting the competitive Walrasian equilibrium. The treatment variable in these studies is the level of information availability regarding to other players quantity choices and profits. When subjects are provided with information about other competitors profits then the rest point converge to the competitive outcome. Abbink and Brandts (2008) also find a similar result in the context of Bertrand price competition where prices converge to the Walrasian outcome in the long run. Furthermore, in a Salop circle model of price competition, Selten and Apesteguia (2005)) have also approved that the subjects behaviour can be explained by the imitation equilibrium.

For future researches, first we suggest to study further the oligopoly theory using learning models and adopting different assumptions regarding to players information on profits and strategies of other players. Additionally, a thought-provoking line for forthcoming researches might be to derive the testable restrictions for the other different evolutionary oligopoly models using revealed preference approach. The literature in the evolutionary game theory of oligopoly is narrow. Specially it requires to investigate more on the empirical testing of firm behaviour in the market. Exploring more case studies on observational datasets across industries may help us to understand the complex behaviour of firms in the market competition.

Appendix A

The proof of propositions 3.7 and 3.8

Proof. Assuming the cost function (3.18) and the linear demand function (3.19), equations (3.5) and (3.6) can be rewritten like the following

$$a - (1 - (n-1)\beta)p^N + (1 - (n-1)\gamma)q^N = p^N - \nu$$

$$p^N = \nu + 2\psi q^N$$

Rearranging above equations,they yield q^N and p^N as follow:

$$q^N = \frac{a - (1 - (n-1)\beta)\nu}{2\psi(2 - (n-1)\beta) - (1 - (n-1)\gamma)}$$

$$p^N = \nu + \frac{a - (1 - (n-1)\beta)\nu}{(2 - (n-1)\beta) - \frac{((1-(n-1)\gamma)}{2\psi}}$$

Likewise, equations (3.8)and (3.9) can be also inscribed along these lines:

$$(-1-\beta)(p^*-\nu)+a-(1-(n-1)\beta)p^*+(1-(n-1)\gamma)q^*=0$$

$$p^*=\nu+\frac{2\psi}{(1+\gamma)}q^*$$

After solving for q^* and p^* , we obtain

$$q^*=\frac{a-(1-(n-1)\beta)\nu}{\frac{2\psi}{1+\gamma}(2-(n-2)\beta)-(1-(n-1)\gamma)}$$

$$p^*=\nu+\frac{a-(1-(n-1)\beta)\nu}{(2-(n-2)\beta)-\frac{((1-(n-1)\gamma)(1+\gamma)}{2\psi}}$$

First of all, to ensure a positive quality in equilibrium, it is sufficient that we assume the following 3 assumptions
A1: $a \geq (1-(n-1)\beta)\nu$
A2: $2\psi(2-(n-1)\beta)-(1-(n-1)\gamma)=(4\psi-1)+(n-1)(\gamma-2\psi\beta)>0$ and
A3: $\frac{2\psi}{1+\gamma}(2-(n-2)\beta)-(1-(n-1)\gamma)=(\frac{4\psi}{1+\gamma}-1+\gamma)+(n-2)(\gamma-\frac{2\psi\beta}{1+\gamma})>0$.

Next, we verify that these assumptions A1, A2, and A3 are not mutually exclusive. To do this, first we check for A2 and A3.

On the one hand, by solvability condition in proposition 3.5 i.e. $-2\frac{\partial D_i}{\partial p_i}\frac{\partial^2 f_i}{\partial q_i^2} > (\frac{\partial D_i}{\partial q_i})^2$, we have $4\psi-1>0$. Moreover, we know that n is greater equal than 2 in oligopoly game ($n \geq 2$), therefore a sufficient but not necessary condition for A2 to be hold is that $\gamma-2\psi\beta>0$ or $2\psi\beta<\gamma$.
On the other hand, the first term of A3, i.e., $(\frac{4\psi}{1+\gamma}-1+\gamma)$ is also positive, since $\psi > \frac{1-\gamma^2}{4}$ always holds (knowing that $\psi > 1/4$ and $0<\gamma^2<1$). Therefore a sufficient but not necessary condition for A3 to be hold is that $\gamma-\frac{2\psi\beta}{1+\gamma}>0$ or $2\psi\beta<\gamma(1+\gamma)$.

So, if $2\psi\beta < \gamma$ satisfies, it will imply that $2\psi\beta < \gamma(1+\gamma)$ must be satisfied (knowing that $\gamma < \gamma + \gamma^2$).

Furthermore, for assumption A1 to hold we must have $\beta \geq (1 - a/\nu)/(n-1)$, where the RHS is negative (as $a > \nu$). Hence we have the following inequalities $2\psi(1 - a/\nu)/(n-1) \leq 2\psi\beta < \gamma$. And since we can always find a range for each parameter such that $2\psi(1-a/\nu)/(n-1) \leq 2\psi\beta < \gamma$ fulfils, then it is verified that A1, A2, and A3 are not mutually exclusive.

Then, we have $q^* > q^N$ if and only if $2\psi(2-(n-1)\beta) > \frac{2\psi}{1+\gamma}(2-(n-2)\beta)$.

Simplifying this inequality, we get the condition of proposition 3.7

$$\gamma > \bar{\gamma} = \frac{\beta}{2 + \beta - n\beta}$$

To see that $2\psi(1 - a/\nu)/(n-1) \leq 2\psi\beta < \gamma$ do not exclude the condition $\gamma > \frac{\beta}{2+\beta-n\beta}$, this condition can be rewritten as $\beta < 2\gamma + \gamma\beta(1-n)$. Moreover, the inequality $2\psi\beta < \gamma$ can be rephrased like $\beta < \gamma/2\psi$. Since we have also $\psi > 1/4$, that means that we must have $\beta < 2\gamma$. Therefore, it does not exclude our condition $\beta < 2\gamma + \overset{+}{\gamma}\beta(1 \overset{-}{-} n)$, since the term $\gamma\beta(1-n)$ is negative (Note that β and γ have the same sign. They are both positive in the case of substitute goods and both negative in the case of complement goods).

And we have $p^* < p^N$ if and only if $(2-(n-2)\beta) - \frac{((1-(n-1)\gamma)(1+\gamma)}{2\psi} > (2-(n-1)\beta) - \frac{((1-(n-1)\gamma)}{2\psi}$ and this inequality leads to the following condition

$$\beta > \bar{\beta} = \frac{(1+\gamma-n\gamma)\gamma}{2\psi}$$

Note that in this case this inequality can be also rephrased as $2\psi\beta > \gamma - (n-1)\gamma^2$ and it is obivious that $\gamma > \gamma - (n-1)\gamma^2$. Therefore, our assumptions do not exclude this condition. But clearly under our assumptions the region of relevant parameters becomes very small for γ. $\square$

Appendix B

Data

Table B.1: Average prices for OPEC crude oil in U.S. dollars per barrel

Annual Average	
1973	2,7
1974	11
1975	10,43
1976	11,6
1977	12,5
1978	12,79
1979	29,19
1980	35,52
1981	34
1982	32,38
1983	29,04
1984	28,2
1985	27,01
1986	13,53
1987	17,73
1988	14,24
1989	17,31
1990	22,26
1991	18,62
1992	18,44
1993	16,33
1994	15,53
1995	16,86
1996	20,29
1997	18,86
1998	12,28
1999	17,44
2000	27,6
2001	23,12
2002	24,36
2003	28,1
2004	36,05
2005	50,59
2006	61
2007	69,04
2008	94,1
2009	60,86
2010	77,38
2011	107,46
2012	109,45
2013	105,87
2014	96,29
2015	49,49

Table B.2: Crude Oil Production: OPEC, Russia and USA (Thousand Barrels per Day)

Annual Average	Total OPEC	Former U.S.S.R	Russia	United States
1973	29660,781	8324	Not Available	9207,953
1974	29322,1	8912	Not Available	8774,205
1975	25789,858	9523	Not Available	8374,737
1976	29118,427	10060	Not Available	8131,639
1977	29584,4	10603	Not Available	8244,562
1978	28161,789	11105	Not Available	8707,441
1979	29351	11384	Not Available	8551,534
1980	25383	11706	Not Available	8596,626
1981	21217	11850	Not Available	8571,573
1982	17772	11912	Not Available	8648,534
1983	16561,26	11972	Not Available	8687,668
1984	16496	11861	Not Available	8878,951
1985	15367,071	11585	Not Available	8971,378
1986	17587	11895	Not Available	8680,142
1987	17731,321	12050	Not Available	8348,978
1988	19741	12053	Not Available	8139,689
1989	21406,586	11715	Not Available	7613,077
1990	22498,068	10975	Not Available	7355,307
1991	22421,951	9992	Not Available	7416,545
1992	23729,03	Not Available	7631,937	7171,125
1993	24464,858	Not Available	6730,009	6846,665
1994	24876,024	Not Available	6135	6661,579
1995	25499,576	Not Available	5995	6559,639
1996	26003,462	Not Available	5850	6464,527
1997	27274,086	Not Available	5920	6451,592
1998	28346,405	Not Available	5854	6251,833
1999	27199,487	Not Available	6078,948	5881,458
2000	28944,094	Not Available	6479,202	5821,604
2001	28129,158	Not Available	6917	5801,401
2002	26465,189	Not Available	7408,173	5744,078
2003	27977,097	Not Available	8132,199	5649,3
2004	30431,939	Not Available	8804,708	5440,995
2005	31897,157	Not Available	9043,082	5181,369
2006	31606,639	Not Available	9247,205	5087,767
2007	31354,082	Not Available	9437,063	5077,047
2008	32722,919	Not Available	9356,784	4999,719
2009	31044,989	Not Available	9495,365	5349,78
2010	32003,437	Not Available	9694,114	5481,81
2011	32229,324	Not Available	9773,518	5644,927
2012	33402,149	Not Available	9921,609	6496,738
2013	32460,082	Not Available	10053,844	7463,517
2014	32446,109	Not Available	10107,088	8680,09

Appendix C

Matlab Code

```
%%%%The following codes are modified version from Carvajal, Deb, Fenske, and Quah
    (Econometrica 2013)
%%%%Quadratic Programming for Homogeneous Product Evolutionary Oligopoly

function results = testconvex(p,q)


%p is a tx1 vector of prices
%q is a txk vector of quantities for the N countries

p=p';            % Transposed Price and Quantities
q=q';
[N, T]=size(q);  %Number of countries, number of periods
ceq=0;           %number of equality constraints (Joint Demand Slope Property)
c=0;             %number of inequality constraints (Co-monotone Property)

b=zeros(1,1);    %Make the vectors of RHS constants empty to begin with
beq=zeros(1,1);

%Calculating the Relative Quantities

rq=zeros(N,T);
su=zeros(N);
su=sum(q);
for t=1: T
    for i=1 : N
rq(i,t)=(N/(N-1))*q(i,t)-su(t)/(N-1);
    end
end

%The vector of unknowns
```

```
%[x_11,...,x_N1,...,x1T...,xNT,e11,...,e_N1,...,e1T...,eNT]

%First set of constraints i.e. Joint Demand Slope Property

%Constraints:-x(i,t)/rq(i,t)+x(j,t)/rq(j,t)=-p(t)/rq(i,t)+p(t)/rq(j,t)

for t=1 : T
    for i=1:1
        for j=i+1:N
        ceq=ceq+1;
        Aeq(ceq,N*(t-1)+i)=-1/rq(i,t);       %Coefficient on x^it
        Aeq(ceq,N*(t-1)+j)=1/rq(j,t);        %Coefficient on x^jt
        beq(ceq)=-p(t)/rq(i,t)+p(t)/rq(j,t);  %RHS constant
        end
    end
end

%Second set of constraints, i.e. rq(i,t)x(i,t)< rq(i,t)p(t)

  for t=1 : T
    for i=1:N
        c=c+1;
        A(c,N*(t-1)+i)=rq(i,t);      %Coefficient on x^it
        b(c)=p(t)*rq(i,t);
    end
  end

%Third set of constraints are inequalities:
%i.e. Co-monotone Property.
%with x minus epsilon
%Also, the signs are reversed because quadprog accepts only "less than"
%constraints: -(q(i,t)-q(i,tt))(x(i,t)-x(i,tt))<0
for i=1:N
    for t=1 : T
        for tt=t+1:T
        c=c+1;
        A(c,N*(t-1)+i)=-(q(i,t)-q(i,tt));        %Coefficient on x^it
        A(c,N*(tt-1)+i)=q(i,t)-q(i,tt);          %Coefficient on x^itt
        b(c)=0;
        end
    end
end

%The objective function
H=[zeros(N*T)];                                  %x'*H*x should be 0: No errors in this test
f=zeros(N*T,1)  ;                                %f'*x should be 0
```

```
UB=Inf*ones(N*T,1)   ;                            %No upper bound on the
    parameters
LB=zeros(N*T,1)    ;                              %Lower bounds are set to be zero

[x,L,exitflag] = quadprog(H,f,A,b,Aeq,beq,LB,UB) ;
results.exit=exitflag ;
results.reject=(exitflag~=1) ;

%x is the vector of unknowns
%L is the test statistic
%exitflag tells me if it worked
%The program accepts Ax<b constraints, but our constraints are
%Ax>b, so enter it as -Ax<-b
%Equality constraints are fine as is

 end



clc;
clear;
close all
tic;

cd('C:\Users\Win7ADM\Desktop\Evolutionary-totalopec-russia-us')
load('dataset.mat')

%%%Table I:Rejection Rates of Evolutionary Hypothesis
windows=[2;3;4;5;6];

%%%Table I: Convex Costs: Rejection Rates for Major Oil Producers
%First Column: Combinations of OPEC-USA,USA-Russia, OPEC-Russia
column=1;
table=1;
pricevector=oilpricenominalWTI;
quantmatrix=datasetoru;
ncountries=2;

Ncountries=size(quantmatrix,2);
rejectionrate=0;
output=[0,0,0];
for w=1:length(windows);
    window=windows(w);
    ntests = (43+1-window)*nchoosek(Ncountries,ncountries);
    rejections = [];
        if rejectionrate>=0.99
        rejectionrate=1;
        end
```

```
    if rejectionrate<0.99
    for i1=1 : Ncountries-1
        for i2=i1+1:Ncountries
         qinterest=[quantmatrix(:,i1),quantmatrix(:,i2)];
         nyears=size(qinterest,1);
         nperiods=nyears-window;
             for y=1 : nperiods
                 m0=y;
                 mEnd=m0+window-1;
                 testconvex(pricevector(m0:mEnd,:),qinterest(m0:mEnd,:));
                 rejections=[rejections;ans.reject];
                 progress=[table,column,window,100*length(rejections)/ntests]
             end
        end
    end
    rejectionrate=mean(rejections);
    end
    output=[output;ncountries,window,rejectionrate];
end
    save rejection_evolutionary_2countries output


%Second Column: Combination of all Countries i.e. total opec, USA and Russia
column=2;
pricevector=oilpricenominalWTI;
quantmatrix=datasetoru;
ncountries=3;

Ncountries=size(quantmatrix,2);
rejectionrate=0;
output=[0,0,0];
for w=1:length(windows);
    window=windows(w);
    ntests = (43+1-window)*nchoosek(Ncountries,ncountries);
    rejections = [];
        if rejectionrate>=0.99
        rejectionrate=1;
        end
    if rejectionrate<0.99
         qinterest=quantmatrix;
         nyears=size(qinterest,1);
         nperiods=nyears-window;
             for y=1 : nperiods
                 m0=y;
                 mEnd=m0+window-1;
                 testconvex(pricevector(m0:mEnd,:),qinterest(m0:mEnd,:));
                 rejections=[rejections;ans.reject];
                 progress=[table,column,window,100*length(rejections)/ntests]
```

```
                end
        rejectionrate=mean(rejections);
        end
        output=[output;ncountries,window,rejectionrate];
end
        save rejection_evolutionary_allcountries output


%%%%Quadratic Programming for Homogeneous Product Cournot Oligopoly

function results = testconvex(p,q)

%p is a tx1 vector of prices
%qin is a txk vector of quantities for the k firms

p=p';            %Program only works when these are transposed
q=q';
[N T]=size(q);   %Number of firms, number of periods
ceq=0;           %number of equality constraints (common ratio property)
c=0;             %number of inequality constraints (

b=zeros(1,1);    %Make the vectors of RHS constants empty to begin with
beq=zeros(1,1);

%The vector of unknowns
%[x_11,...,x_N1,...,x1T...,xNT,e11,...,e_N1,...,e1T...,eNT]

%First set of constraints: equalities of the (P-x)/Q terms
    %i.e. common ratio property
    %Constraints are:-x(i,t)/q(i,t)+x(j,t)/q(j,t)=-p(t)/q(i,t)+p(t)/q(j,t)
for t=1 : T
    for i=1:1
        for j=i+1:N
        ceq=ceq+1;
        Aeq(ceq,N*(t-1)+i)=-1/q(i,t);      %Coefficient on x^it
        Aeq(ceq,N*(t-1)+j)=1/q(j,t);       %Coefficient on x^jt
        beq(ceq)=-p(t)/q(i,t)+p(t)/q(j,t); %RHS constant
        end
    end
end

%Second set of constraints are the (P-x)/Q>0 inequalities
%But quadprog only accepts "less than" constraints
%so this becomes x(i,t)<p(t)
  for t=1 : T
    for i=1:N
        c=c+1;
        A(c,N*(t-1)+i)=1;      %Coefficient on x^it
```

```
        b(c)=p(t);             %The equality constraint within a single year
    end
  end

%Third set of constraints are inequalities:
%i.e. x is increasing in q.
%with x minus epsilon
%Also, the signs are reversed because quadprog accepts only "less than"
%constraints: -(q(i,t)-q(i,tt))(x(i,t)-x(i,tt))<0
for i=1:N
    for t=1 : T
        for tt=t+1:T
        c=c+1;
        A(c,N*(t-1)+i)=-(q(i,t)-q(i,tt));       %Coefficient on x^it
        A(c,N*(tt-1)+i)=q(i,t)-q(i,tt);         %Coefficient on x^itt
        b(c)=0;                                 %The sum must be negative
        end
    end
end

%Define the objective function
H=[zeros(N*T)];                                  %x'*H*x should be 0: No errors in this test
f=zeros(N*T,1)  ;                                %f'*x should be 0
UB=Inf*ones(N*T,1)  ;                            %No upper bound on the parameters
LB=zeros(N*T,1)   ;                              %x must be positive

[x,L,exitflag] = quadprog(H,f,A,b,Aeq,beq,LB,UB) ;
results.exit=exitflag ;
results.reject=(exitflag~=1) ;

%x is the vector of unknowns
%L is the test statistic
%exitflag tells me if it worked
%The program accepts Ax<b constraints, but our constraints are
%Ax>b, so enter it as -Ax<-b
%Equality constraints are fine as is

end

clc;
clear;
close all
tic;

cd('C:\Users\Win7ADM\Desktop\data and programs-totalopec -russia-us')
load('dataset.mat')
```

```

%%%Table I: Rejection Rates of Cournot hypothesis
windows=[2;3;4;5;6];

%%%Table I: Convex Costs: Rejection Rates for Major Oil Producers
%First Column: Combinations of OPEC-USA,USA-Russia, OPEC-Russia
column=1;
table=1;
pricevector=oilpricenominalWTI;
quantmatrix=datasetoru;
ncountries=2;

Ncountries=size(quantmatrix,2);
rejectionrate=0;
output=[0,0,0];
for w=1:length(windows);
    window=windows(w);
    ntests = (43+1-window)*nchoosek(Ncountries,ncountries);
    rejections = [];
        if rejectionrate>=0.99
        rejectionrate=1;
        end
    if rejectionrate<0.99
    for i1=1 : Ncountries-1
        for i2=i1+1:Ncountries
         qinterest=[quantmatrix(:,i1),quantmatrix(:,i2)];
         nyears=size(qinterest,1);
         nperiods=nyears-window;
             for y=1 : nperiods
                 m0=y;
                 mEnd=m0+window-1;
                 testconvex(pricevector(m0:mEnd,:),qinterest(m0:mEnd,:));
                 rejections=[rejections;ans.reject];
                 progress=[table,column,window,100*length(rejections)/ntests]
             end
        end
    end
    rejectionrate=mean(rejections);
    end
    output=[output;ncountries,window,rejectionrate];
end
    save rejection_cournot_2countries output


%Second Column: Combination of all Countries i.e. total opec, USA and Russia
column=2;
pricevector=oilpricenominalWTI;
quantmatrix=datasetoru;
```

```
ncountries=3;

Ncountries=size(quantmatrix,2);
rejectionrate=0;
output=[0,0,0];
for w=1:length(windows);
    window=windows(w);
    ntests = (43+1-window)*nchoosek(Ncountries,ncountries);
    rejections = [];
        if rejectionrate>=0.99
        rejectionrate=1;
        end
    if rejectionrate<0.99
         qinterest=quantmatrix;
         nyears=size(qinterest,1);
         nperiods=nyears-window;
             for y=1 : nperiods
                 m0=y;
                 mEnd=m0+window-1;
                 testconvex(pricevector(m0:mEnd,:),qinterest(m0:mEnd,:));
                 rejections=[rejections;ans.reject];
                 progress=[table,column,window,100*length(rejections)/ntests]
             end
    rejectionrate=mean(rejections);
    end
    output=[output;ncountries,window,rejectionrate];
end
    save rejection_cournot_allcountries output
```

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