

## Correspondence Article: Counterexample for suspension-aware schedulability analysis of EDF scheduling

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## 1 Introduction

Self-suspension behavior has been demonstrated to appear in complex cyber-physical real-time systems, e.g., multiprocessor locking protocols, computation offloading, and multicore resource sharing, as demonstrated in (Chen et al. (2019), Section 2). Although the impact of self-suspension behavior has been investigated since 1990, the literature of this research topic has been flawed as reported in the review by Chen et al. (2019).

Although the review by Chen et al. (2019) provides a comprehensive survey of the literature, two unresolved issues are listed in the concluding remark. One of them is regarding the "correctness of Theorem 8 in (Devi (2003), Section 4.5) ... supported with a rigorous proof, since self-suspension behavior has induced several non-trivial phenomena". This paper provides a counterexample of Theorem 8 in (Devi (2003), Section 4.5) and disproves the schedulability test.

We consider a set of implicit-deadline periodic tasks  $\mathbb{T} = \{\tau_1, \dots, \tau_n\}$ , in which each task  $\tau_i$  has its period  $T_i$ , worst-case self-suspension time  $S_i$ , and worst-case execution time  $C_i$ . The relative deadline  $D_i$  is set to  $T_i$ . There are two main models of self-suspending tasks: the *dynamic* self-suspension and *segmented* (or *multi-segment*) self-suspension models. Devi's analysis in Devi (2003) considers the dynamic self-suspension model. That is, a task instance (job) released by a task  $\tau_i$  can suspend arbitrarily as long as the total amount of suspension time of the job is not more than  $S_i$ .

Devi's analysis for implicit-deadline task systems is rephrased as follows:

mario.guenzel@tu-dortmund.de

Department of Informatics, TU Dortmund University, Dortmund, Germany



Jian-Jia Chen jian-jia.chen@cs.uni-dortmund.de
Mario Günzel

**Theorem 1** (Devi 2003) Let  $\mathbb{T} = \left\{ \tau_1, \tau_2, \dots, \tau_n \right\}$  be a system of n implicit-deadline periodic tasks, arranged in order of non-decreasing periods. The task set  $\mathbb{T}$  is schedulable using preemptive EDF if for all k with  $1 \le k \le n$  inequality  $\frac{B_k + B_k'}{T_k} + \sum_{i=1}^k \frac{C_i}{T_i} \le 1$  holds, where  $B_k = \sum_{i=1}^k \min\{S_i, C_i\}$  and  $B_k' = \max_{1 \le i \le k} \left( \max\{0, S_i - C_i\} \right)$ .

Note that the notation follows the survey paper by Chen et al. (2019) instead of the original paper by Devi (2003). Moreover, Devi considered arbitrary-deadline task systems with asynchronous arrival times. Our counterexample is valid by considering two implicit-deadline periodic tasks released at the same time and disproves also the general case.

## 2 Counterexample for Devi's analysis

The following task set  $\mathbb{T}$  with only two tasks provides a counterexample for Devi's analysis:

$$\begin{array}{lll} - & \tau_1: (T_1=D_1=6, C_1=5, S_1=1) \text{ and} \\ - & \tau_2: (T_2=D_2=8, C_2=\epsilon, S_2=0), \text{ for any } 0<\epsilon \leq 1/3. \end{array}$$

The test of Theorem 1 is as follows:

- When k = 1, we have  $B_1 = 1$  and  $B'_1 = 0$ . Therefore, when k = 1, we obtain  $\frac{B_k + B'_k}{T_k} + \sum_{i=1}^k \frac{C_i}{T_i} = 1.$
- When k = 2, we have  $B_2 = 1$  and  $B'_2 = 0$ . Therefore, when k = 2, we obtain  $\frac{B_k + B'_k}{T_k} + \sum_{i=1}^k \frac{C_i}{T_i} = \frac{1}{8} + \frac{\epsilon}{8} + \frac{5}{6} = \frac{23 + 3\epsilon}{24} \le 1$ , since  $\epsilon \le 1/3$ .

Therefore, Devi's schedulability test concludes that the task set is feasibly scheduled by preemptive EDF. But, a concrete schedule as demonstrated in Figure 1 shows that one of the jobs of task  $\tau_1$  misses its deadline even when both tasks release their first jobs at the same time.

The example in Fig. 1 shows that a job of task  $\tau_1$  may be blocked by a job of task  $\tau_2$ , which results in a deadline miss of the job of task  $\tau_1$ . The counterexample only requires task  $\tau_1$  to suspend once. It shows that applying Devi's analysis in Devi

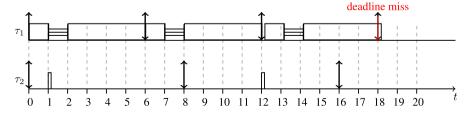


Fig. 1 A concrete EDF schedule with a deadline miss

(2003) is unsafe even for the segmented self-suspension model under EDF scheduling. We note that the above counterexample is only for Theorem 8 in Devi (2003). We do not examine any other schedulability tests in Devi (2003).

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Mario Günzel received his M.Sc. degree in 2019 from the Faculty of Mathematics at the University of Duisburg-Essen in Germany. Since that time he is a Ph.D. student at the chair for Design Automation of Embedded Systems at TU Dortmund University in Germany, where he is supervised by Prof. Dr. Jian-Jia Chen.His research interest is in the area of embedded and real-time systems. Currently he focuses his research on the schedulability analysis of self-suspending task sets.





Jian-Jia Chen is Professor at Department of Informatics in TUDortmund University in Germany. He was Juniorprofessor at Departmentof Informatics in Karlsruhe Institute of Technology (KIT) in Germanyfrom May 2010 to March 2014. He received his Ph.D. degree fromDepartment of Computer Science and Information Engineering, NationalTaiwan University, Taiwan in 2006. He received his B.S. degree fromthe Department of Chemistry at National Taiwan University 2001.Between Jan. 2008 and April 2010, he was a postdoc researcher at ETHZurich, Switzerland. His research interests include real-time systems, embedded systems, energy-efficient scheduling, power-aware designs,temperatureaware scheduling, and distributed computing. He received the European Research Council (ERC) Consolidator Award in 2019. He hasreceived more than 10 Best Paper Awards and Outstanding Paper Awardsand has involved in Technical Committees in many international conferences.

