Essays on Information Economics, Political Economy and Network Goods

Dominik Mark Maxara, M.Sc.

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Introduction

This dissertation explores the intricate interplay between signaling mechanisms in electoral contexts, the impact of misinformation on democratic polarization, and the dynamic improvement of quality in network goods, particularly within the gaming industry in three essays.

The first essay models and analyzes the role of primary elections as a vital mechanism for voter learning. By employing a Bayesian game framework, I reveal how these electoral processes enable voters to discern candidates' true ideological positions and learn the true state of the world amidst uncertainty. This analysis not only fills a gap in existing literature concerning candidate signaling but also provides valuable insights into enhancing democratic representation.

The second essay addresses a pressing contemporary issue: the rise of polarization driven by political failures and fake news which is overlooked in the literature and instead solely attributed to fake news. Through the combination of multi-armed bandit theory and fundamental political economy concepts, I illustrate how misinformation disrupts belief formation processes and erodes trust in media credibility. The findings underscore the necessity for restoring confidence in reliable information sources to stabilize democratic systems facing increasing extremism.

The third essay investigates update cycles for network goods in a dynamic setup by deriving testable predictions through a trackable model embedded in the network good literature. We extend the existing literature by analyzing strategic quality improvements for network goods over time while navigating competitive pressures. To test the predictions from the

model we provide empirical evidence from the gaming industry. The results highlight that maintaining user engagement relies heavily on continuous innovation and responsiveness to market changes.

By examining these themes—-voter behavior in elections, the effects of misinformation on democracy, and strategic quality management in network goods—-this dissertation contributes to our understanding of how information shapes economic and political landscapes today. As we navigate these complex dynamics, it is essential to recognize that effective governance and sustainable market practices hinge upon informed decision-making rooted in credible information.

Chapter 1

Signaling in Elections: Voter Learning Through Primaries - unpublished

Abstract

This paper presents a novel study of primary elections and their role in informing voters, employing an innovative Bayesian game framework. By modeling a two-stage spatial election game, I demonstrate how primary elections facilitate voters' learning about candidates' ideological bliss points amidst uncertainty about the state of the world. My findings reveal that despite candidates' inherent biases and personal preferences, voters can fully learn critical information about the underlying political landscape–including candidates' ideal points and state of the world–through candidate signals during primaries. Notably, I establish that lying costs are essential for preventing misleading signals; however, my alternative mechanism with weakly binding campaign pledges demonstrates how informative equilibria can be achieved without stringent assumptions. The implications of this research extend beyond theoretical insights. They contribute to electoral design discussions focused on enhancing democratic representation and mitigating polarization within political systems. Overall, this work underscores the significance of primaries as pivotal mechanisms for voter learning and engagement in contemporary electoral contexts.

Keywords: Primary Elections, Voter Learning, Candidate Signaling, Bayesian Game Theory, Political Representation, Electoral Dynamics

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1.1 Introduction

In representative democracies, primary elections play a crucial role in shaping political representation by determining which candidates advance to general elections. These electoral mechanisms not only facilitate candidate selection but also serve as vital platforms to inform voters. Despite extensive discussions surrounding primary elections and their impact on polarization and extremism and discussions concerning misinformation and the spreading of fake news (Carey and Polga-Hecimovich (2006), Sides et al. (2018), Hirano et al. (2010)), there remains a gap in the literature regarding the systematic analysis of signaling mechanisms within these contexts. This paper addresses a critical gap in the literature by introducing a novel Bayesian game framework to model the interaction between candidates and voters during primary elections. Through a two-stage spatial election game, it provides an in-depth analysis of how primary elections enable voters to learn essential information about candidates' ideal points, even amidst uncertainties related to external developments and candidates' personal biases. By explicitly modeling both the primary and general election phases within a unified framework this research offers fresh insights into how platform announcements during primaries shape voter decision-making in subsequent general elections. This comprehensive perspective advances our understanding of electoral dynamics and highlights the pivotal role of primary elections in democratic processes.

The significance of understanding how primary elections influence voter behavior is underscored by recent events in the U.S. electoral system. For instance, during the 2020 and 2016 Democratic primaries, candidates like Bernie Sanders and Elizabeth Warren proposed progressive platforms to attract left-leaning voters (Collins (2015), Otterbein and Thompson (2019)), demonstrating how platform announcements deviate from the classic median voter theory and rather serve informational purposes amidst ideological diversity. Similarly, Donald Trump's unconventional campaign strategies during the 2016 Republican primaries

as well as his more recent 2024 strategies showcased how candidates signal their positions on critical issues to resonate with specific voter bases. His statements often diverged from traditional Republican rhetoric, which polarized opinions but also clarified his position for voters seeking radical change (Orr (2019), Payne (2024), Isenstadt et al. (2024)). These cases illustrate how primary elections can expose candidate biases and preferences, allowing voters to discern true ideological positions and the state of the world.

In this context we can also take a look at the significant impact of COVID-19 on Primary Elections in 2020. In particular, how candidates communicated their responses to public health crises. Candidates had to navigate complex issues surrounding healthcare and economic recovery while signaling their readiness to lead through uncertain times. Biden's focus on science-based policies contrasted with Trump's approach, providing clear signals that shaped voter perceptions during the primaries (Bisbee and Honig (2022)).

At the core of these examples lies a fundamental observation: Political platforms exhibit significant variability, particularly during primary elections, often reflecting more extreme positions that tend to moderate as candidates transition to the general election. Despite the ambiguity and noise inherent in these signals, voters are able to learn valuable insights regarding current economic pressures, emerging issues, and potential challenges. This highlights the importance of primaries as critical periods for voter learning and engagement as well as the importance to shed light on this specific mechanism.

Additionally, there is another important aspect to the detailed analysis of primary elections and candidate bias. Following the development of the electoral system in the US universally adopted after the 1968 Democratic Convention in Chicago and the increasing adaptation of primary elections in Latin America (Carey and Polga-Hecimovich (2006)), the discussion of whether the introduction of this system was beneficial for electoral participation has been debated extensively over the years. Most prominently discussed is the claim that primary elections are exacerbating polarization and contributing to extremist candidates. However, this has been falsified (Sides et al. (2018)). When faced with primaries, data suggest that

voters will opt for a candidate that best represents their own personal beliefs, while the general election exerts pressure towards more moderate platforms (Hirano et al. (2010)). Potential primary models should capture these observations.

Consequently, I propose a theory in which candidates first compete in a primary election before the winning candidates compete in the general election. I assume that candidates have private information about the state of the world while at the same time are biased and have their own personal bliss,points unknown to voters. To achieve this, I introduce the state of the world as an exogenous shift parameter, as, for example, elaborated and detailed in Cukierman and Tommasi (1998), unidirectionally affecting the utility of all individuals. This concept captures the idea that while political viewpoints may differ relative to one another, they are similarly influenced by major global events that shift these viewpoints in a common direction. A recent example is the start of Russia's invasion of Ukraine. Prior to this event, a majority of people in Europe opposed increased military spending. However, this significant development shifted public opinion towards advocating for higher defense budgets. Despite this collective shift, individual political perspectives relative to each other remained unchanged, resulting in varying opinions regarding the extent of military spending or its overarching objectives.

I model candidates as policy–motivated who-due to their information advantage–can sway voters' beliefs about the state of the world. I analyze then the resulting competition and take a look at the voters' potential to decompose the biased signals.
I follow the classical literature on electoral competition and implement a two-stage election game allowing for different platform announcements during these stages. The stages represent the primary and the general election. While electoral promises during the primaries are not necessarily binding, I assume full commitment in the general stage, namely, the implementation of the finally winning platform.

This assumption is reasonable, as evidenced by the work of Thomson et al. (2018), who found that approximately $60\% - 80\%$ of campaign pledges are implemented in parliamentary democracies with majority governments. They achieve similar results in their earlier work which also takes a closer look at the US and UK (Thomson et al. (2017)). This high rate of fulfillment can be attributed to voter accountability, which incentivizes candidates to adhere to their platforms. Additionally, stable political contexts and credible signaling during campaigns further reinforce this connection. However, it is important to note that these factors are highly context-dependent and can vary significantly across different political systems.

As a consequence of full commitment, I will endogenously rationalize that voters will select candidates based on perceived ideological proximity during the primary which is truthfully reported due to high lying cost. Candidates competing for the right to represent their respective political wing during the general election have to consider this. During the second stage pressure by median voter preferences will moderate initially more extreme platforms. The resulting separating Bayesian equilibrium therefore generally follows observations of US elections.

My findings reveal that while candidates possess inherent biases stemming from personal preferences, effective competition during primaries allows voters to disentangle noisy signals and gain insights into true ideological stances as well as the state of the world. Notably, I demonstrate that lying costs are crucial for preventing misleading communications; however, I propose an alternative mechanism involving weakly binding campaign pledges that can achieve informative equilibria without this stringent assumption and therefore completely disregarding lying costs. In the literature, this is also introduced as flip-flopping-cost (Hummel (2010)). The insights drawn from this analysis not only deepen the understanding of electoral dynamics but also provide valuable lessons for contemporary political systems grappling with challenges related to transparency and representation.

The remainder of this paper is organized as follows: Section 1.2 situates my research within the broader academic discourse on primary elections, signaling mechanisms and voter behavior by reviewing existing work. I briefly discuss relevant theoretical frameworks and key findings and highlight the significance of my approach. Section 1.3 provides a detailed exposition of the theoretical framework used to analyze the signaling structure. I present the Bayesian game that forms the basis of my analysis, specifying key components such as candidate behavior, voter preferences, and the strategic interactions between them as well as discuss all assumptions made. Afterwards I analyze the model in its general form in Section 1.4 and derive all implications formally differentiating between linear and non-linear functional forms. Section 1.5 presents an alternative mechanism to the benchmark model which allows me to draw similar results but with relaxed assumptions. I conclude in Section 1.6 and discuss the implications as well as potential additions and changes to the model. All proofs are relegated to the Appendix A.

1.2 Related literature

This chapter reviews the existing literature on primary elections and information signaling to contextualize my research within the broader academic discourse. While elections, including primaries, have been extensively studied, there is currently no two-stage analysis of electoral systems that incorporates the rich signaling structure examined here. This study specifically focuses on candidates as senders who have an information advantage, thereby exploring the information rationale behind primaries presenting a novel contribution in this field.

Jackson et al. (2007) build a simple discrete model with complete information of primary elections that demonstrates how these processes can lead to non-median voter outcomes. Similarly, Amorós et al. (2016) discuss how variations in types of primaries and caucuses across the U.S. impact electoral dynamics.

Owen and Grofman (2006) further contribute to this field by exploring a two-stage electoral process, analyzing how electoral dynamics align with empirical realities. Their framework supports the understanding of the strategic interactions between candidates during primaries and general elections. In this context Coleman (1971) laid foundational insights into the institutional impact on party positions.

Agranov (2016) presents a two-stage incomplete information model where platforms are not fully observable. Contrary to my approach platforms are however exogenously given and not chosen by candidates and in consequence cannot serve as signals. Still this study complements my findings regarding candidates pandering to their party base during primaries while shifting toward more centrist positions in general elections.

Hummel (2010) examines a two-stage model incorporating flip-flopping costs, illustrating that candidates first compete in primaries before transitioning to general elections; this concept parallels my notion of weakly binding campaign pledges while I introduce rich informational structure into a similar setup.

Callander and Wilson (2007) analyze the effects of turnout and sequential primaries on electoral competition, highlighting how voter participation shapes candidate strategies during these critical phases.

Hummel (2012) justifies sincere voting under simultaneous or early rounds of sequential voting without knife-edge assumptions, lending credence to my assertion that voters make informed choices based on perceived ideological proximity.

Crutzen et al. (2010) emphasize nomination processes as tools for informing poorly informed voters about candidate positions in internal nomination processes. This underscores the importance of effective signaling during primary elections in enhancing voter knowledge.

Hortala-Vallve and Mueller (2010) discuss the selection process for candidates within parties, emphasizing how primaries serve a unifying role among party members underlying their importance.

Casas (2020) explores open versus closed primaries influenced by ideological factors within

endogenous party affiliations. The findings resonate with my analysis by illustrating the impact of primary structures on candidate positioning and voter alignment. Furthermore, Grofman et al. (2016) examine two-dimensional two-party electoral competition with respect to the existence of equilibria and implications arising from open versus closed primaries.

In addition to these contributions, I adopt insights from Meirowitz (2005) and Meirowitz and Shotts (2009), who focus on candidates' information allocation during primaries specifically linking them to voter preferences as signals for candidates' beliefs about median voter preferences. However, I argue that low turnout during primaries as described by Gerber et al. (2017) may weaken this claim regarding representation.

The use of informative primaries is central to my model; here, platform announcements serve as mediums through which voters learn about candidates' true ideologies amidst noise in political messaging. The concept of signaling through platform announcements is further explored by Martinelli (2001), who highlights that parties often possess better information than voters themselves in a single election model.

While valence signaling literature such as Andreottola (2021) and Dodlova and Zudenkova (2021) opens up additional dimensions affecting voter utility directly, I maintain that candidate positions remain noisy due to inherent uncertainties in policy announcements. This aligns with Snyder and Ting (2002) conclusions regarding party affiliation as another layer in signaling mechanisms.

To achieve a robust informational structure within my model drawing from Cukierman and Tommasi (1998), I incorporate personal preferences alongside a state of the world while introducing a two-stage framework involving multiple strategic players rather than the common single-stage scenarios.

Ultimately, my goal is to explore the benefits of primary elections for voters. Specifically, how information is disclosed in such settings enabling public learning of candidates' true ideologies alongside external conditions influencing policy decisions. Mechanisms such as lying costs or weakly binding campaign promises are integral components established in ex-

isting political competition literature as evident in Banks (1990)'s general election models or González et al. (2021)'s reputational cost frameworks within party contexts.

As I construct a classic spatial model of political competition, the established literature including Wittman (1983), Wittman and Arrow (1990), Alesina and Cukierman (1990), Roemer (1994) are to be mentioned as well in this context. My results also align with results from both primary and general election stages with empirical data presented by Carey and Polga-Hecimovich (2006), Sides et al. (2018) and Hirano et al. (2010), collectively suggesting that extreme positions observed during primaries tend towards more moderate positions in the general election.

1.3 Model

In this political competition model, the ultimate goal is to determine policy platform choices. I assume a one-dimensional feasible policy space over $\mathbb{R}$ for the final implementation by the winning candidate, as well as for platform announcements made during both primary and general elections.[1]

To effectively capture voters' preferences and illustrate how the policies determine their payoffs, I introduce an individual personal bliss point $b_j \in \mathbb{R}$. This bliss point represents private information unique to each voter j and reflects their ideal policy outcome. Accordingly, voter j's utility function is formulated based on classic models of ideological preferences

$$u_{v_j}(y) = -|y - (b_j + s^e)| \tag{1.1}$$

The variable y represents the policy ultimately implemented following the elections. Consequently, voter j's utility is determined by the distance between the implemented policy

[1]While this policy space could be further specified or restricted such as limiting it to a specific interval, as seen in some parts of the literature (Downs (1957)), I will maintain the most general assumptions possible without introducing additional constraints.

and their ideal point, $(b_j + s^e)$. As previously mentioned, b_j captures their personal political bliss point, whereas s^e denotes the expected state of the world based on the voter's beliefs. The actual value of s is drawn at the beginning of the game and is unobserved by voters. I assume that voters share the same prior that s follows a normal distribution, denoted as $s \sim \mathcal{N}(0, \sigma_s^2)$. This assumption is however not crucial for my results. Following Cukierman and Tommasi (1998), I assume that all voters are similarly affected by an external event in a uniform direction, preserving their relative positions within the political spectrum. This framework acknowledges that while individual preferences vary among voters, significant events potentially obscured from public awareness can lead to shifts in their political views. Importantly, this shift affects all voters uniformly.

It is worth noting that in this model, voters are considered risk-neutral, aligning with common trends in existing literature. Empirical evidence suggests that voters tend to exhibit risk neutrality in mass elections (Tiemann (2019)).

Given the utility function (1.1), voter j exhibits single-peaked preferences symmetric around their ideal point $(b_j + s^e)$. To account for the continuum of voters, I will assume sincere voting, which is a standard approach in the literature (Downs (1957); Agranov (2016)). Consequently, the median voter theorem applies under majority voting. For any sufficiently large electorate, the election outcome under majority voting is determined by the preferences of the median voter.

In this framework, I represent the voting population through a median voter's personal bliss point b_j and its associated distribution $F_j : \mathbb{R} \to [0, 1]$. I restrict this distribution to be symmetric, unimodal, and log-concave with existing means. This minor restriction aligns with classical approaches in the literature that typically utilize uniform or bell-shaped distributions (Cukierman and Tommasi (1998)). Notably, symmetry is not essential for any of the general results and can be relaxed if necessary.

Furthermore, it can be demonstrated that for any voting population comprising n voters, drawing n bliss points from a uniform distribution results in a median that follows the beta distribution $\mathbb{B}(n+1, n+1)$. This observation adheres to my original restrictions on log-concavity and should adequately encompass all relevant cases.

The timing consists of two stages and captures competition during primary and general elections. In each stage, candidates first announce platforms, voters observe the announcements and update their beliefs about candidates' ideal policies and the state of the world, and then the election takes place determining a winner and ultimately the implemented policy.

As I already mentioned, the perfect signaling of voters' bliss points is unrealistic. Consequently, I assume that during the primary election, the information gained by candidates through the voting behavior of the participating population is highly limited. Therefore, candidates are myopic and three median voters are drawn to capture the electoral system throughout both stages. One median voter is drawn for the right-wing primary, one median voter is drawn for the left-wing primary and one median voter is drawn for the general election. During the primary, a candidate as representative of the left political wing and a candidate as representative of the right political wing are selected. I assume that the left median voter's bliss point (b_{Lm}) follows $b_{Lm} \sim F_L$ with mean μ_L and the right median voters' bliss point (b_{Rm}) will follow $b_{Rm} \sim F_R$ with mean μ_R. The ideal policy preferences of the electorate who participates in the elections is unknown until the final election's result. Consequently, the private bliss point of the median voter during the general election (b_{Gm}) is also unknown and drawn according to $b_{Gm} \sim F_G$ with mean μ_G. I assume that: $\mu_L < \mu_G < \mu_R$. Note that the choice of two median voters for the primary allows to disentangle the primaries of the left- and right-wings. This is a realistic assumption due to the U.S. having a multitude of primary systems, from open to closed ones, and, together with low turnout rates, a relevant interaction between voters of each wing is neglectable (Ferrer and Thorning (2023)).

While voters' utility (1.1) is driven by the finally implemented policy, their choice of candi-

date during the first stage still matters as I allow for varying platform announcements during primary and general elections. I discuss the case of weakly binding campaign promises later as an alternative mechanism. Platform announcements by each candidate during the primary and general elections are always publicly observed by voters. These announcements serve as signals to voters, which allow them to update their beliefs about the state of the world and the ideal policy of each candidate. In consequence, voters will support a candidate whose expected bliss point is closest to their own, choosing the best possible representative under the sincere voting assumption. I also rationalize this behavior in the primary using results from the general election. During the primary stage, only left-wing candidates compete with each other and similarly only right-wing candidates compete with each other. Therefore a winning candidate of each wing emerges for the general election. Winning platforms announced in the general election are to be implemented in the end, as I assume full commitment in the general election. Therefore, in the general election, it is rational for voters to sincerely support the candidate whose announced platform gives them the highest possible utility given their beliefs about the state of the world s.

I assume there are four candidates, denoted as $L1$, $L2$, $R1$, $R2$. As the name might suggest, $L1$ and $L2$ are two left-wing candidates competing first over voter support during the left-wing primary, and $R1$ and $R2$ are two right-wing candidates competing over support during the right-wing primary.

Candidates are policy-motivated, and want the implemented policy to be close to their own bliss points. Candidates' preferences are represented by Euclidean distance utility functions. Contrary to voters who are modeled as risk neutral here maximizing their utility from potential outcomes, candidates are risk averse minimizing potential losses. Hence candidate

i's objective utility is given by:

$$u_i(x_i, y) = -g(|y - (\epsilon_i + s)|) - C_1(|x_i - (\epsilon_i + s)|) - C_2(|y_i - (\epsilon_i + s)|) \tag{1.2}$$

Where $C_1 : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ is linear and I distinguish between the following two cases for g, $C_2 : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ in the further analysis:

1. $-g(.)$ and $-C_2(.)$ are linear functions.
2. $-g(.)$ and $-C_2(.)$ are non-linear functions which satisfy Inada conditions (concave, strictly increasing, continuous and zero in the origin $g(0) = C_2(0) = g'(0) = C_2'(0) = 0$)

Due to this, u_i is concave. The finally implemented policy os denoted by y, and $s \sim \mathcal{N}(\mu_s, \sigma_s^2)$ is the state-of-the-world shift parameter, which is known by all candidates. I assume that the mean shift is zero and therefore $\mu_s = 0$. However, this assumption is not critical for the results. Along the same line, $\epsilon_i \sim \mathcal{N}(\mu_{\epsilon_i}, \sigma_\epsilon^2)$ represents candidate i's personal bliss point. These bliss points are private information of the candidates with voters sharing the same priors concerning the distribution of ϵ_i. The information advantage of candidates is justified by their knowledge of the political system and its members, and in consequence, their competitors. Additionally, candidates are supported by a campaign team and should be fully informed about current and upcoming events that might affect their political stances better than the general population as well as the stances of their political rivals. Last but not least, x_i denotes candidate i's platform announcement during the primary election and y_i denotes candidate i's platform announcement during the general election. Therefore, $C_1(.)$ and $C_2(.)$ capture the cost of deviating from one's bliss, such as lying or posture, while $g(.)$ captures the policy motivation of the candidate. It is reasonable to assume that plausible deception is connected to high costs.

Candidate i announces a platform x_i for the primary and, should he win, a possibly differing platform y_i for the general election. Under full commitment, y_i has to be implemented if i

wins, resulting in $y_i = y$. The announced platforms $x = (x_{L1}, x_{L2}, x_{R1}, x_{R2})$ and $y = (y_L, y_R)$ are observed publicly turning this political competition model into a signaling game.

With this in mind, denote as $P_{i2}(y_i, y_j)$ the probability that candidate i wins if he announces platform y_i and his opponent announces platform y_j in the general election phase. The expected utility of candidate k in the second stage amounts to:

$$\begin{aligned}\Pi_k^2(y_i, y_j) = & P_{i2}(y_i, y_j)\left(-g(|y_i - (\epsilon_k + s)|)\right) \\ & + (1 - P_{i2}(y_i, y_j))\left(-g(|y_j - (\epsilon_k + s)|)\right) \qquad \forall\, i, j, k \in \{L1, L2, R1, R2\} \\ & - \mathbb{1}_{\{i,j\}}(k)\, C_2\left(|y_k - (\epsilon_k + s)|\right)\end{aligned} \tag{1.3}$$

where the indicator function $\mathbb{1}_{\{i,j\}}(k)$ s given by:

$$\mathbb{1}_{\{i,j\}}(k) = \begin{cases} 1 & \text{if } k \in \{i, j\} \\ 0 & \text{otherwise} \end{cases}$$

Denote as $P_{i,1}(x) = P_{i,1}(x_{L1}, x_{L2}, x_{R1}, x_{R2})$ the probability that candidate i wins during the primary against his direct competitor if he announces platform x_i and the other candidates announce platforms x_{-i} during the first phase. Consequently, I specify the overall expected utility of candidate k, including the primary election phase, given that he knows all candidates' private bliss points ϵ_j as well as the state of the world s:

$$\begin{aligned}\Pi_k(x, y) = & P_{L1,1}(x)\, P_{R1,1}(x)\, \Pi_k^2(y_{L1}, y_{R1}) \\ & + P_{L1,1}(x)\, P_{R2,1}(x)\, \Pi_k^2(y_{L1}, y_{R2}) \\ & + P_{L2,1}(x)\, P_{R1,1}(x)\, \Pi_k^2(y_{L2}, y_{R1}) \qquad \forall k \in \{L1, L2, R1, R2\} \\ & + P_{L2,1}(x)\, P_{R2,1}(x)\, \Pi_k^2(y_{L2}, y_{R2}) \\ & - C_1(|x_k - (\epsilon_k + s)|)\end{aligned} \tag{1.4}$$

Due to the wing-specific rivalry, these probabilities can be further simplified.

It holds: $P_{L2,1}(x) = 1 - P_{L1,1}(x)$ and $P_{R2,1}(x) = 1 - P_{R1,1}(x)$.

The timing of this political competition model, illustrated in Figure 1, is as follows:

Step 1: First, the state of the world $s \sim \mathcal{N}(0, \sigma_s^2)$ and candidates' bliss points $\epsilon_i \sim \mathcal{N}(\mu_{\epsilon_i}, \sigma_\epsilon^2)$ are realized. Candidates learn about their own and their competitors' bliss points as well as the-state-of-the-world shift parameter. This information is, however, not observed by voters. Similarly, voters' bliss points $b_{Lm} \sim F_L, b_{Rm} \sim F_R$ and $b_{Gm} \sim F_G$ are drawn and learned by the voters but not observed by the candidates.

Step 2: Secondly, all candidates simultaneously and non-cooperatively announce platforms for the primary election $x_i \in \mathbb{R}$.

Step 3: Third, voters observe these announcements and update their beliefs about s and $\epsilon = (\epsilon_{L1}, \epsilon_{L2}, \epsilon_{R1}, \epsilon_{R2})$. Then, a winner of the left-wing and a winner of the right-wing primary elections are determined by vote. Voters elect candidates whose expected bliss points are closer to their own.

Step 4: Fourth, the winning candidates simultaneously and non-cooperatively announce platforms $y_i \in \mathbb{R}$ for the general election. Again, the voters update their beliefs, and a winner is determined by vote. Voters cast votes for their most preferred policy, given their own bliss points and their beliefs about the state of the world.

Step 5: Finally, winning candidate i implements his announced policy $y_i = y$.

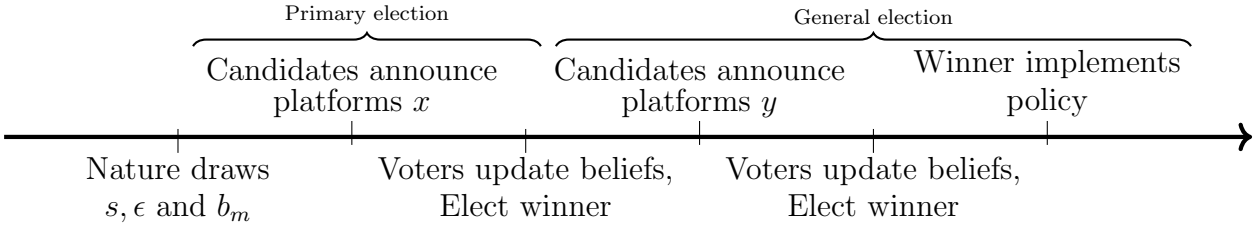

Figure 1.1: Sequence of events

I characterize most informative perfect Bayesian Nash equilibria in this signaling game in which voters fully learn about the state of the world s and candidates' private bliss points ϵ. In this context, beliefs μ in each information set for each player are consistent (updated by Bayes' rule) and all strategies satisfy sequential rationality (optimality in expectation given the beliefs).

1.4 Analysis

Starting with the analysis of the general election stage, I concentrate on cases where ideal point of the left-wing candidates are to the left from ideal points of right-wing candidates, namely: $\epsilon_i \leq \epsilon_j$. This assumption ensures that a potential right-wing candidate does not run in a left-wing primary. It is realistic to assume that candidates run for the party they identify with. See, for example, Snyder and Ting (2002).

In the most informative equilibrium, during the second stage, either all information is finally fully revealed, resulting in $s^e = s$, or no additional information is revealed. This means, in terms of Bayesian updating for s^e, that: $\frac{\partial(s^e|x)}{\partial y_i} = \frac{\partial(s^e|x)}{\partial y_j} = 0$. The reason for this is that if the primary stage has been already informative, I show that any additional information during the general stage provided by either candidate is sufficient for voters to fully learn about the state of the world and candidates' true bliss points. Going forth, I calculate the winning probability $P_{i2}(y_i, y_j)$ by finding the critical value, which leaves the median voter indifferent between voting for either candidate following the classic literature, for example, as in Cukierman and Tommasi (1998). Without loss of generality, assume y_i is smaller than y_j, and hence the indifferent voter with bliss point b_{ind} fulfills: $y_i < b_{ind} + s^e < y_j$. It then holds that:

$$u_{v_{ind}}(y_i) \stackrel{!}{=} u_{v_{ind}}(y_j)$$

$$\Leftrightarrow -(b_{ind} + s^e - y_i) = -(y_j - (b_{ind} + s^e))$$
$$\Leftrightarrow b_{ind} = \frac{y_i + y_j - 2s^e}{2} =: \theta^e$$

Given the median voter's distribution F_G candidate i then wins if the median voter's bliss point is to the left of the indifferent voter's bliss point:

$$P_{i2}(y_i, y_j) = P(b_{Gm} < b_{ind}) = P\left(b_{Gm} < \frac{y_i + y_j - 2s^e}{2}\right) = F_G\left(\frac{y_i + y_j - 2s^e}{2}\right) = F_G(\theta^e)$$

It follows that second stage candidates i and j expected utilities are given by:

$$\begin{aligned}\Pi_i^2(y_i, y_j) =& F_G\left(\theta^e\right)\left(-g(|y_i - (\epsilon_i + s)|)\right) \\ &+ \left(1 - F_G\left(\theta^e\right)\right)\left(-g(|y_j - (\epsilon_i + s)|)\right) - C_2\left(|y_i - (\epsilon_i + s)|\right)\end{aligned} \tag{1.5}$$

$$\begin{aligned}\Pi_j^2(y_i, y_j) =& F_G\left(\theta^e\right)\left(-g(|(\epsilon_j + s) - y_i|)\right) \\ &+ \left(1 - F_G\left(\theta^e\right)\right)\left(-g(|(\epsilon_j + s) - y_j|)\right) - C_2\left(|(\epsilon_j + s) - y_j|\right)\end{aligned} \tag{1.6}$$

With the next two lemmata, I first show that a best-response platform announcement is always closer to the candidates own ideal point than the opponent's platform announcement is. This means I show that for a left-wing candidate, it is also always optimal to propose a platform to the left of his right-wing opponent, and vice versa. After that, I show that extreme positions to the left of a left-wing candidate or to the right of a right-wing candidate are unattractive and will not be announced during the general election. This result is supported by observations of Sides et al. (2018) and others. The proofs of this and following lemmata and propositions are relegated to the Appendix A.

Lemma 1.4.1.

Given that $\epsilon_i < \epsilon_j$ it holds:

$$y_i^* = \underset{y_i \in \mathbb{R}}{arg\,max}\ \Pi_i^2(y_i, y_j) \Rightarrow |y_i^* - (\epsilon_i + s)| \leq |y_j - (\epsilon_i + s)|$$

$$y_j^* = \underset{y_j \in \mathbb{R}}{arg\,max}\ \Pi_j^2(y_i, y_j) \Rightarrow |y_i - (\epsilon_j + s)| \geq |y_j^* - (\epsilon_j + s)|$$

This means if candidate j proposes a platform closer to candidate i's personal bliss point, i can strictly increase his expected utility by deviating towards y_j. In consequence this means i's platform will always represent his bliss point best among all platforms proposed.

Lemma 1.4.2.

Given $\epsilon_i < \epsilon_j$, it holds that $\Pi_i^2(y_i, y_j)$ is strictly increasing in $y_i < \epsilon_i + s$ and $\Pi_j^2(y_i, y_j)$ is strictly decreasing in $y_j > \epsilon_j + s$.

Lemma 1.4.2 shows that it is optimal to place your platform to the right of your personal bliss point if you are a left-wing candidate and to the left of your personal bliss point if you are a right-wing candidate. This should not be too surprising as the pressure by the median voter aims towards the political center.

Using previous results and applying the mean value theorem, the next proposition shows the existence of maxima within the interval specified by Lemma 1.4.1 and Lemma 1.4.2 of the candidates' utility functions.

Proposition 1.4.3.

Given $\epsilon_i < \epsilon_j$, it holds that $\Pi_i^2(y_i, y_j)$ and $\Pi_j^2(y_i, y_j)$ assume their respective local maxima in y_i^* and y_j^* which fulfill the following:

$$\exists y_i^* \in (\epsilon_i + s,\ y_j) \qquad with \qquad \frac{\partial \Pi_i^2(y_i, y_j)}{\partial y_i}\Big|_{y_i = y_i^*} = 0 \quad \forall y_j$$

$$\exists y_j^* \in (y_i,\ \epsilon_j + s) \qquad with \qquad \frac{\partial \Pi_j^2(y_i, y_j)}{\partial y_j}\Big|_{y_j = y_j^*} = 0 \quad \forall y_i$$

Summing up, this allows us to resolve all absolute values as we learn that candidates will place their optimal platform between the opponent's platform and their own personal bliss. Using Proposition 1.4.3, the absolute values of the candidates' expected utility functions (1.5) and (1.6) can be resolved. Consequently, it follows that the respective first-order conditions for $\Pi_i^2(y_i, y_j)$ and $\Pi_j^2(y_i, y_j)$ can be written as follows:

$$\begin{aligned}\frac{\partial \Pi_i^2(y_i, y_j)}{\partial y_i} =& \frac{1}{2} f_G\left(\theta^e\right)\left(g(y_j - (\epsilon_i + s)) - g(y_i - (\epsilon_i + s))\right) \\ & - F_G\left(\theta^e\right) g'(y_i - (\epsilon_i + s)) - C_2'\left(y_i - (\epsilon_i + s)\right) \stackrel{!}{=} 0\end{aligned} \tag{1.7}$$

$$\begin{aligned}\frac{\partial \Pi_j^2(y_i, y_j)}{\partial y_j} =& \frac{1}{2} f_G\left(\theta^e\right)\left(g((\epsilon_j + s) - y_j) - g((\epsilon_j + s) - y_i)\right) \\ & + \left(1 - F_G\left(\theta^e\right)\right) g'((\epsilon_j + s) - y_j) + C_2'\left((\epsilon_j + s) - y_j\right) \stackrel{!}{=} 0\end{aligned} \tag{1.8}$$

Given the existence of at least one maximizer as shown in Proposition 1.4.3 and therefore the best response function in the general stage, it remains to be shown that any maximizer is unique, which I deduce in the next proposition using single-crossing properties from Quah and Strulovici (2012).

Proposition 1.4.4.

$\Pi_i^2(y_i, y_j)$ *and* $\Pi_j^2(y_i, y_j)$ *from* (1.5) *and* (1.6) *have a unique respective maximizer and are quasi-concave.*

The existence of a unique maximizer under either full information revelation or no additional information revelation verifies the existence of unique best response functions in the general election stage. In other words, there exists an equilibrium in which both candidates announce platforms that are best responses to each other. It remains to be shown whether this best response is type-dependent informing the voters and resulting in a fully revealing equilibrium in the end. For further analysis, I now distinguish between two cases, as already foreshadowed in the model description. First, I analyze the more general case of risk-averse candidates with $-g(\cdot)$ and $-C_2(\cdot)$ being non-linear concave functions that satisfy Inada conditions (symmetry about the y-axis, and therefore the first derivatives are zero in the origin).

Afterwards, I analyze the linear risk-neutral candidates case. I show that this distinction matters as the results are fundamentally different.

1.4.1 General election with risk-averse candidates

In the case of risk-averse candidates, it is not possible to derive a reaction or best response function in closed form. However, by analyzing the winning probability, I show the informing nature of the announced platforms and characterize the equilibrium as well as provide comparative statics.

Recall that I showed in Proposition 1.4.3 that the equilibrium is characterized by the first-order conditions (1.7) and (1.8) and in Proposition 1.4.4 that their respective solutions are unique. It follows that, equations (1.7) and (1.8) can be divided by each other, giving an expression for the winning equilibrium probability under mutual best responses:

$$F_G(\theta^e) = \frac{1}{2} + \frac{1}{2}\underbrace{\left(\frac{C_2'((\epsilon_j+s)-y_j)}{g'((\epsilon_j+s)-y_j)} - \frac{C_2'(y_i-(\epsilon_i+s))}{g'(y_i-(\epsilon_i+s))}\right)}_{(i)} + \frac{1}{4} f_G(\theta^e) \cdot \underbrace{\left(\frac{(g(y_j-(\epsilon_i+s))-g(y_i-(\epsilon_i+s)))}{g'(y_i-(\epsilon_i+s))} - \frac{(g(\epsilon_j+s-y_i)-g(\epsilon_j+s-y_j))}{g'(\epsilon_j+s-y_j)}\right)}_{(ii)} \tag{1.9}$$

In the case of risk-neutral candidates, (i) and (ii) will cancel out due to their respective functional properties. This already hints to the finding that, in contrast to an informative equilibrium, a pooling non-informative equilibrium arises in the linear case. I discuss this further in Section 1.4.2.

In the non-linear case, canceling out is only possible for term (i) if $C_2(\cdot) = c_2 \cdot g(\cdot)$ with $c_2 \in \mathbb{R}$, but not for term (ii) unless ϵ_i and ϵ_j are symmetric around the median voter's expected value μ_G. In this case the equilibrium probability is $\frac{1}{2}$ and term (i) and term (ii) cancel out as $(y_i^* - (\epsilon_i + s)) = ((\epsilon_j + s) - y_j^*)$.

This means in general, the winning probability in the case of risk-averse candidates will be

shaped according to the candidates' relative true ideological positions towards each other under mutual best responses. As a result, platform announcements during the general election differ in these relative positions. In other words, candidates take their own and their opponent's true ideology into account when placing their platforms. This mechanism provides voters with additional information and, therefore, full revelation in combination with the first stage as I show later. Also in contrast to the classical medium voter theory, the resulting winning probability will differ from $\frac{1}{2}$. Decomposing the winning probability allows to make additional observations.

Term (i) captures the relative factor between cost and policy motivation, which can be constant and possibly vanish as described earlier depending on the model specifications.

Due to the monotonicity and convexity of $g(\cdot)$, term (ii) is negative for $|\epsilon_i - \mu_G| > |\epsilon_j - \mu_G|$ and positive for $|\epsilon_i - \mu_G| < |\epsilon_j - \mu_G|$, increasing the winning probability of the candidate who is closer to the expected median voter in equilibrium. This is supported by the fact that the left-hand side of term (ii) decreases while the right-hand side increases if ϵ_i decreases. This is further amplified as $y_i^*(y_j^*)$ decreases with ϵ_i. Applying the envelop theorem emphasizes this claim: $\Pi_i^2(y_i, y_j)$ and $\Pi_j^2(y_i, y_j)$, and therefore their corresponding value functions are monotonic in ϵ_i and ϵ_j, respectively. Hence, the distance between the announced platform and personal bliss point changes, and therefor the winning probability as described above:

$$\begin{aligned}\frac{\partial \Pi_i^2(y_i, y_j)}{\partial \epsilon_i} =& F_G\left(\theta^e\right) g'(y_i - (\epsilon_i + s)) \\ &+ \left(1 - F_G\left(\theta^e\right)\right) g'(y_j - (\epsilon_i + s)) + C_2'\left(y_i - (\epsilon_i + s)\right) > 0\end{aligned} \tag{1.10}$$

$$\begin{aligned}\frac{\partial \Pi_j^2(y_i, y_j)}{\partial \epsilon_j} =& - F_G\left(\theta^e\right) g'((\epsilon_j + s) - y_i) \\ &- \left(1 - F_G\left(\theta^e\right)\right) g'((\epsilon_j + s) - y_j) - C_2'\left((\epsilon_j + s) - y_j\right) < 0\end{aligned} \tag{1.11}$$

Consequently, this shows that in the non-liner case, candidates choose platforms not solely based on the median voter's position but also on their own bliss point relative to their competitor's one. A change in the private bliss point ϵ_i of candidate i away from the expected median voter's private bliss point μ_G reduces i's winning probability as the announced platform also moves away from the median following the candidate's private bliss point. This holds even if there is no lying cost as equation (1.9) shows due to the full commitment assumption. Taking this into account, it rationalizes the fact that voters should choose a candidate in the first, primary stage who they believe is close to their own political bliss point under sincere voting. This choice will increase their political representation during the general election. At the same time, this candidate behavior provides information about the relative positions to the voters.

Example with risk-averse candidates:

For illustration, I provide in the appendix A.1 an example with $g(x) = x^2$, $C_2(x) = c_2 \cdot x^2$ and $b_{Gm} \sim \mathcal{U}[-\frac{1}{2}, \frac{1}{2}]$. It underlines former results and I show that solving the game for $s = 0$ is sufficient to extend the results to $s \neq 0$. The winning probability 1.9 amounts then to:

$$F_G\left(\theta^e\right) = \frac{1}{2} + \frac{1}{8}(y_j(y_i) - y_i(y_j))^2 \left(\frac{1}{y_i(y_j) - \epsilon_i} - \frac{1}{\epsilon_j - y_j(y_i)}\right) \tag{1.12}$$

This winning probability only incorporates respective distances between best response platform announcements and bliss points in this case. These vary, however, based on relative ideological candidate positions. Candidates with a bliss point closer to the center will in consequence benefit from a higher winning probability.

Before analyzing the game in its primary stage, I next provide a solution to the linear case with risk-neutral candidates in the general election to show the differences.

1.4.2 General election with risk-neutral candidates

In the case of risk-neutral candidates, I focus on cases where candidate bliss points sufficiently diverge from the expected median voter's position. This allows them to announce equilibrium platforms that are more moderate than their bliss points and will prevent platforms from converging towards the center due to the linear structure with linear Euclidean distances. Therefore, I postulate that the following holds:

$$|\epsilon_k - \mu_G| > |s^e - s| + \frac{c_2 + \frac{1}{2}}{f(\mu_G)} \tag{1.13}$$

If this condition is violated candidates' platforms are given by $y_i^* = y_j^* = \mu_G + s^e$. In the linear case, it is sufficient to use $g(x) = x$ as the identity and $C_2(x) = c_2 \cdot x$ with $c_2 \in \mathbb{R}^+$ as it is always possible to normalize. Then the following Corollary holds:

Corollary 1.4.5.

In the general election with risk-neutral candidates the following platform announcements constitute best responses to each other:

$$y_i^*(y_j^*) = \mu_G + s^e - \frac{c_2 + \frac{1}{2}}{f(\mu_G)} \qquad\qquad y_j^*(y_i^*) = \mu_G + s^e + \frac{c_2 + \frac{1}{2}}{f(\mu_G)}$$

Note that the resulting announcements only depend on s^e and not on ϵ_k. In other words, only the belief of the voters prior about s is used. Therefore, no new information can be transmitted to the voter in the general election, as both candidates, independently of their own preferred positions, announce platforms to win with equal expected probability in expectation. These platforms reflect only their political wing affiliations. Hence, s^e is ultimately only based on prior beliefs formed in the primary election, leaving only this first stage to inform voters. This is a major difference from the non-linear case, which transmits additional information in the general election. Nevertheless, it is essential to demonstrate that this behavior in the general election is supported by informative signals during the

primary, reinforcing the information rationale and significance of primaries.

1.4.3 Primary Election with risk-averse candidates

Building upon my earlier discussions regarding the general election, I now consider how candidates strategically position themselves during primaries to appeal to their respective party bases while navigating the complexities of voter preferences. The primary is a crucial component of the electoral process that significantly influences candidate selection and voter perception of candidates. This section examines how platform announcements made during this stage act as signals to voters, allowing them to assess candidates' alignment with their own ideological preferences or stances.

As shown in (1.4) the utility of candidates in the primary stage can be written as:

$$\begin{aligned}\Pi_k(x,y) =& P_{L1,1}(x)\, P_{R1,1}(x)\, \Pi_k^2(y_{L1}, y_{R1}) \\ &+ P_{L1,1}(x)\, P_{R2,1}(x)\, \Pi_k^2(y_{L1}, y_{R2}) \\ &+ P_{L2,1}(x)\, P_{R1,1}(x)\, \Pi_k^2(y_{L2}, y_{R1}) \qquad \forall k \in \{L1, L2, R1, R2\} \\ &+ P_{L2,1}(x)\, P_{R2,1}(x)\, \Pi_k^2(y_{L2}, y_{R2}) \\ &- C_1(|x_k - (\epsilon_k + s)|)\end{aligned}$$

The lying cost function $C_1(.)$ captures the notion that candidates' platform choices x_i during the primary elections, although not strictly bound by commitments for the general election, must still reflect their genuine positions within the electoral landscape. This concept underscores the importance of authenticity in candidate messaging; it is intuitively challenging for a candidate to convincingly portray themselves as fundamentally different from their true beliefs without incurring significant costs.

When candidates attempt to mislead voters by adopting platforms that starkly contrast their

actual positions, they risk alienating their core supporters and undermining their credibility. Such discrepancies can lead to perceptions of insincerity, which may ultimately diminish voter trust and support. As a result, candidates are incentivized to present platforms that are at least somewhat representative of their political ideologies, even in the absence of formal commitments for future elections. While explicitly modeling this mechanism is beyond the scope of this paper, it is the likeliest micro-foundation to rationalize costs.

This alignment between primary platform announcements and candidates' true preferences is essential for maintaining electoral viability. It reflects a strategic balance where candidates must navigate the pressures of appealing to party bases while also considering how these positions will be perceived by a broader electorate in subsequent phases of the electoral process. The lying cost function thus serves as a critical mechanism that influences candidates' behavior during primaries, ensuring that their announced platforms resonate with potential voters.

The only possible equilibrium, however, is then the truthful bliss point platform to prevent any deviations. This will also sustain an informative equilibrium. For it to hold, assume linearity in lying costs $C_1(|x_i - (\epsilon_i + s)|) = c_1 \cdot |x_i - (\epsilon_i + s)|$ and a sufficiently high value of c_1. I first calculate Bayesian updating under the corresponding signals in lemma 1.4.6 and then a lower bound for c_1 in lemma 1.4.7.

Lemma 1.4.6.

Upon receiving signals $q_1 := s + \epsilon_1,\ ...,\ q_4 := s + \epsilon_4$ *it then holds:*

$$s^e = E[s|q_1, ..., q_4] = \frac{4\sigma_s^2 \overline{s'} + \sigma_\epsilon^2 \mu_s}{4\sigma_s^2 + \sigma_\epsilon^2} \quad \textit{with } \overline{s'} = \frac{1}{4}\sum_{i=1}^{4}(q_i - \mu_{\epsilon_i})$$

And analogously or by linearity of expectation:

$$\epsilon_i^e = E[\epsilon_i|q_1, ..., q_4] = \frac{4\sigma_s^2 \overline{S'} + \sigma_\epsilon^2 \mu_s'}{4\sigma_s^2 + \sigma_\epsilon^2}$$

$$with\ \overline{S'} = \frac{1}{4}\sum_{j=1}^{4}(q_i - q_j + \mu_{\epsilon_j}),\ \mu'_s = q_i - \mu_s$$

Lemma 1.4.7.

The equilibrium, is sustained for:

$$c_1 > max\left\{0, \frac{2\sigma_s^2 + \sigma_\epsilon^2}{8\sigma_s^2 + 2\sigma_\epsilon^2} f_L\left(\frac{\epsilon_{L1}^e + \epsilon_{L2}^e}{2}\right)\left(F_R\left(\frac{\epsilon_{R1}^e + \epsilon_{R2}^e}{2}\right)\overline{\Pi} + \Pi_i^2(y_{L1}, y_{R2}) - \Pi_i^2(y_{L2}, y_{R2})\right)\right.$$
$$\left. - \frac{2\sigma_s^2}{8\sigma_s^2 + 2\sigma_\epsilon^2} f_R\left(\frac{\epsilon_{R1}^e + \epsilon_{R2}^e}{2}\right)\left(F_L\left(\frac{\epsilon_{R1}^e + \epsilon_{R2}^e}{2}\right)\overline{\Pi} + \Pi_i^2(y_{L2}, y_{R1}) - \Pi_i^2(y_{L2}, y_{R2})\right)\right\}$$

with: $\overline{\Pi} = \Pi_i^2(y_{L1}, y_{R1}) - \Pi_i^2(y_{L1}, y_{R2}) - \Pi_i^2(y_{L2}, y_{R1}) + \Pi_i^2(y_{L2}, y_{R2})$

Under Lemma 1.4.7, all candidates announce their ideal platforms based on the state of the world and their private bliss points so that it holds: $x_i^* = \epsilon_i + s$. This one-dimensional signal based on a two-dimensional typesetting cannot be disentangled by voters, leading to Bayesian updating as described in Lemma 1.4.6. However, this is perfectly in line with the literature on noisy signaling like Crawford and Sobel (1982). These platforms during the primary are more extreme compared to the platforms proposed in general election as shown in 1.4.3. This is in line with the theoretical and empirical literature outlined in Section 1.2 for most models analyzing primaries and also in the data.

Having derived the optimal platform announcements for the general and for the primary stage, I now show that with the information provided to the voter all uncertainty can be resolved. The primary platforms, together with the platforms in the general election, provide voters with a total of six unique informative signals while faced with five unknowns before having to cast a vote during the general election. As all signals relate to the same set of unknowns but are different in composition, this overdetermined equation system is perfectly solvable, allowing full information to be revealed. Due to the specific form of the primary platforms, I give a closed-form solution for this in the following proposition.

Proposition 1.4.8.

In the non-linear game under sufficient lying cost, all information is fully revealed via the primary platform announcements and general election platform announcements, and it holds:

$$s = \frac{y_i^* + y_j^*}{2} - F^{-1}(\overline{K_3}) \text{ and } \epsilon_k = x_k^* - \frac{y_i^* + y_j^*}{2} + F^{-1}(\overline{K_3}) \text{ for } k \in \{i, j\} \text{ and } \overline{K_3} \text{ constant}$$

The overdetermination of the equation system also allows for the following implication: a deviation by a candidate only in one stage lies off the equilibrium path. Due to only five unknowns but six equations the calculation of the true values based on the remaining five equations, excluding the false one is still possible, as long as the deviating agent and stage are known, simply because the equation system will not be truthfully solvable for the remaining sixth signal. This also means that the deviating agent can be punished by assigning off-path beliefs, which decreases this agent's winning probability in the general election. This is in line with the assignment of arbitrary beliefs off the equilibrium path for perfect Bayesian equilibria. Doing so under the assumption that the deviant is known allows to drop the restrictive cost constraint of Lemma 1.4.7. However, it hinges at the assumption that the deviant can be identified which is a strong premise. Deviation in both stages is not feasible due to Proposition 1.4.4 and the full commitment assumption of the second stage. Consequently, any candidate that would deviate in the first stage would like to return to his own optimal platform in the second general election stage.

1.4.4 Primary Election with risk-neutral candidates

Due to the non-revealing nature of the general election case of risk-neutral candidates, the primary stage differs slightly from the case of risk-averse candidates. Only a partial informing outcome can be achieved under a sufficient lower bound for the lying cost.

Similarly, to the non-linear case, a strong assumption on the lying cost function is necessary to sustain equilibrium. Under such assumptions, the result is the same in terms of the platforms chosen, and the following corollary holds:

Corollary 1.4.9.

In the primary election with risk-neutral candidates the following platform announcements are optimal:

$$x_i^* = \epsilon_i + s \qquad \forall i \in \{L1, L2, R1, R2\}$$

This holds for:

$$\begin{aligned} c_1 > & \frac{\sigma_s^2}{4\sigma_s^2 + \sigma_\epsilon^2} + \frac{2\sigma_s^2 + \sigma_\epsilon^2}{8\sigma_s^2 + 2\sigma_\epsilon^2} f_L\left(\frac{\epsilon_k^e(x) + \epsilon_{-k}^e(x)}{2}\right) c_2 |y_k - (\epsilon_k + s)| \\ & + F_L\left(\frac{\epsilon_k^e(x) + \epsilon_{-k}^e(x)}{2}\right) \frac{c_2 \sigma_s^2}{4\sigma_s^2 + \sigma_\epsilon^2} \end{aligned}$$

where:

$$|y_k - (\epsilon_k + s)| = min\left\{\left|\mu_G + s^e - \frac{c_2 + \frac{1}{2}}{f(\mu_G)} - (\epsilon_k + s)\right|, \left|\mu_G + s^e + \frac{c_2 + \frac{1}{2}}{f(\mu_G)} - (\epsilon_k + s)\right|\right\}$$

Due to these results in the general election, the primary in the model with risk-neutral candidates serves as the only possibility for voters to acquire some information. Without a primary no information would be revealed which underlines the importance of primaries as an information acquisition mechanism.

1.5 Alternative mechanism

The lying cost mechanism might partially seem unrealistic due to the high-cost assumption outlined in Lemma 1.4.7 or the cost linearity. Therefore, I propose an alternative mechanism

that gives similar results but holds without such a strong assumption. So far primary and general stages are disconnected from each other except for the learned information. Primaries solely serve as some form of candidate presentation. This leads to voters disregarding the actual platforms proposed in the primary and voting based on the expected personal bliss points of the candidates. In the following, I relax this assumption and introduce a weakly binding campaign promises property instead or so-called flip-flopping cost as presented in Hummel (2010). This means that promises made during the primary will be weakly binding in the general election and deviation from an a priori promised position will be costly. Under this property, lying costs can be completely disregarded.

This mechanism reflects the observation that candidates enter the primary race with specific promises and commitments, while actively defending their political positions against opponents while attempting to persuade voters of their viability. During this phase, candidates present platforms that reflect their ideological stances and policy priorities, which serve as key signals to the electorate regarding their intentions and values.
If a candidate were to completely abandon these platforms in the transition to the general election, it would likely lead to significant voter disappointment and disillusionment. Such a drastic shift could be perceived as a betrayal of trust, undermining the candidate's credibility in the eyes of voters who initially supported them based on their declared positions.

As a consequence, this loss of reliability can have detrimental effects on a candidate's electoral prospects. Voters are more inclined to support candidates they perceive as consistent and trustworthy; thus, abandoning previously stated commitments can alienate core supporters and diminish overall public confidence. The literature supports this claim. For example, Thomson et al. (2018), show that approximately $60\% - 80\%$ of campaign pledges are implemented in parliamentary democracies with majority governments.

To capture this, I introduce an overarching cost function $C(.)$, which represents the reputation loss, dropping the former lying cost mechanism. Candidates' objective utility function (1.3) then amounts to:

$$\overline{u}_i(x_i, y) = -g(|y - (\epsilon_i + s)|) - C(|x_i - y_i|) \tag{1.14}$$

where $-C : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ satisfies Inada conditions (a concave, strictly increasing, and continuous function that is zero at the origin ($C_1(0) = 0$)). Due to this, u_i is concave. The expected utility during the general stage becomes accordingly:

$$\begin{aligned}\overline{\Pi}_i^2(y_i, y_j, x_i) =& F_G\left(\theta^e\right)\left(-g(|y_i - (\epsilon_i + s)|)\right) \\ &+ \left(1 - F_G\left(\theta^e\right)\right)\left(-g(|y_j - (\epsilon_i + s)|)\right) - C\left(|y_i - x_i|\right)\end{aligned} \tag{1.15}$$

$$\begin{aligned}\overline{\Pi}_j^2(y_i, y_j, x_j) =& F_G\left(\theta^e\right)\left(-g(|(\epsilon_j + s) - y_i|)\right) \\ &+ \left(1 - F_G\left(\theta^e\right)\right)\left(-g(|(\epsilon_j + s) - y_j|)\right) - C\left(|x_j - y_j|\right)\end{aligned} \tag{1.16}$$

Similar to the lying cost mechanism, I will show existence and uniqueness of best responses in the general stage.

Proposition 1.5.1.
$\overline{\Pi}_i^2(y_i, y_j)$ *and* $\overline{\Pi}_j^2(y_i, y_j)$ *from* (1.15) *and* (1.16) *have a unique respective maximizer, defied by the first-order condition, and are quasi-concave.*

The winning probability then forms similar to (1.9). However, signs might vary now, as depending on $C'(\cdot)$ and $g'(\cdot)$, it does not necessarily hold that $y_i^* > x_i$ and $y_i^* > \epsilon_i + s$, as well as $y_j^* < x_j$ and $y_j^* < \epsilon_j + s$. In the following, I focus on the case where $C'(\cdot)$ is sufficiently low and non-binding so that the aforementioned conditions hold. In consequence I assume

the following:

$$\begin{aligned}\frac{\partial\,\Pi_i^2(y_i,y_j)}{\partial y_i} =&\frac{1}{2}f_G\left(\theta^e\right)\left(g(|(\epsilon_i+s)-y_j|)-g(\epsilon_i+s-y_i)\right)\\ &+F_G\left(\theta^e\right)g'(\epsilon_i+s-y_i)+sgn(x_i-y_i)C'\left(|x_i-y_i|\right)>0\end{aligned} \tag{1.17}$$

$$\begin{aligned}\frac{\partial\,\Pi_j^2(y_i,y_j)}{\partial y_j} =&\frac{1}{2}f_G\left(\theta^e\right)\left(g((\epsilon_j+s)-y_j)-g(|(\epsilon_j+s)-y_i|)\right)\\ &-\left(1-F\left(\theta^e\right)\right)g'(y_j-(\epsilon_j+s))-sgn(y_j-x_j)C'\left(|y_j-x_j|\right)<0\end{aligned} \tag{1.18}$$

As mentioned in Section 1.2, the literature documents that candidates are more moderate during the general election. Hence, it makes sense to assume (1.17) and (1.18); therefore, it holds:

$$\begin{aligned}F_G\left(\theta^e\right)=&\frac{1}{2}+\frac{1}{2}\left(\frac{C'\left(x_j-y_j\right)}{g'((\epsilon_j+s)-y_j)}-\frac{C'\left(y_i-x_i\right)}{g'(y_i-(\epsilon_i+s))}\right)+\frac{1}{4}f_G\left(\theta^e\right)\\ &\cdot\left(\frac{(g(y_j-(\epsilon_i+s))-g(y_i-(\epsilon_i+s)))}{g'(y_i-(\epsilon_i+s))}-\frac{(g(\epsilon_j+s-y_i)-g(\epsilon_j+s-y_j))}{g'(\epsilon_j+s-y_j)}\right)\end{aligned} \tag{1.19}$$

Again, in the case of risk-neutrality, both candidates win with equal probability, as in the lying cost mechanism, while the case of risk-averse candidates is dependent on their relative positions. The alternative mechanism therefore does not breach my findings but preserves them. The primary stage under weakly binding campaign promises cost becomes:

$$\begin{aligned}\Pi_k(x,y)=&P_{L1,1}(x)\,P_{R1,1}(x)\,\overline{\Pi}_k^2(y_{L1},y_{R1},x_k)\\ &+P_{L1,1}(x)\,P_{R2,1}(x)\,\overline{\Pi}_k^2(y_{L1},y_{R2},x_k)\\ &+P_{L2,1}(x)\,P_{R1,1}(x)\,\overline{\Pi}_k^2(y_{L2},y_{R1},x_k)\\ &+P_{L2,1}(x)\,P_{R2,1}(x)\,\overline{\Pi}_k^2(y_{L2},y_{R2},x_k)\end{aligned}\qquad \forall k\in\{L1,L2,R1,R2\} \tag{1.20}$$

Note that the choice of a policy platform x_i during the primary now also directly effects the expected payoffs during the general election, and by that, the general election platform choice

y_i, such that candidates now need to take their actions in the primary election into account in the general election. As the finally implemented policy is also affected by the primary, it is even more reasonable for voters to base their primary vote now on the announced platform during the primary. Equivalently to the general election stage the respective critical values are:

$$\theta_L^e := \frac{x_{L1} + x_{L2} - 2s^e}{2} \qquad \theta_R^e := \frac{x_{R1} + x_{R2} - 2s^e}{2}$$

Using these I derive the following final result.

Proposition 1.5.2.

Given (1.17) and (1.18), $\Pi_k(x, y)$ *has a unique maximizer* x_k^* *for all* $k \in \{L1, L2, R1, R2\}$ *and is quasiconcave.*

Proposition 1.5.2 together with Proposition 1.5.1 shows that the alternative mechanism generates a similar fully revealing outcome as the benchmark setting. However, without strong assumptions on lying cost, as before.

1.6 Conclusion

In this study, I have developed a novel framework to explore the intricate dynamics of primary elections and their pivotal role in informing voters. My findings illuminate how primary elections serve not only as a mechanism for candidate selection but also as an essential tool for voter learning. By modeling the interactions between candidates and voters within a two-stage spatial election game, I have demonstrated that even amidst inherent biases and uncertainties, voters can discern critical information about the political landscape via primaries.

The implications of my research extend beyond theoretical insights into practical considerations for electoral design. I have established that mechanisms such as lying costs play a

crucial role in ensuring the integrity of candidate signals during primaries. Furthermore, my alternative mechanism involving weakly binding campaign pledges presents an approach to achieve informative equilibria without imposing stringent assumptions on candidates' behavior.

My analysis reveals that while extreme positions may dominate during primary elections, the electoral platforms tend to moderate as candidates transition to the general election. This phenomenon underscores the importance of primaries as critical periods for voter engagement and learning, allowing constituents to make informed choices that align with their ideological preferences.

Looking ahead, future research could expand upon my findings by investigating how varying structures of primary elections such as open versus closed systems impact voter behavior and candidate strategies across different political contexts. Additionally, examining the effects of external factors such as media influence or social movements on candidate signaling could provide further insights into enhancing democratic representation.

In conclusion, this study significantly contributes to our understanding of electoral dynamics by highlighting the signaling function of primaries in contemporary political systems. As democracies continue to grapple with challenges related to polarization and misinformation, ensuring transparent communication between candidates and voters is essential for reinforcing democratic processes.

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A

Appendix

A.1 Example with risk-averse candidates

For illustration, I provide a specific non-linear application to make calculations more accessible. First, rewrite the winning probability (1.9) using the extended mean value theorem. For term (ii) it holds that $\exists \xi_i \in [\epsilon_i + s, y_j]$, $\exists \xi_j \in [y_i, \epsilon_j + s]$:

$$\begin{aligned}(ii) =&(y_j - y_i) + \frac{1}{2}\frac{(y_j - y_i)^2 g''(\xi_i)}{g'(y_i - (\epsilon_i + s))} - ((y_j - y_i) + \frac{1}{2}\frac{(y_j - y_i)^2 g''(\xi_j)}{g'((\epsilon_j + s) - y_j)}) \\ =&\frac{1}{2}(y_j - y_i)^2 \left(\frac{g''(\xi_i)}{g'(y_i - (\epsilon_i + s))} - \frac{g''(\xi_j)}{g'((\epsilon_j + s) - y_j)}\right)\end{aligned}$$

The distance in platform announcements, adjusted for the relative changes in utility, illustrated using quadratic functions follows:

Let $g(x) = x^2$, $C_2(x) = c_2 \cdot x^2$ and $b_{Gm} \sim \mathcal{U}[-\frac{1}{2}, \frac{1}{2}]$, then it holds:

$$F_G\left(\theta^e\right) = \frac{1}{2} + \frac{1}{8}(y_j - y_i)^2 \left(\frac{1}{y_i - (\epsilon_i + s)} - \frac{1}{(\epsilon_j + s) - y_j}\right) \tag{A.1}$$

As (A.1) shows, the winning probability is now solely defined by the respective distances between platform announcements and bliss points, which vary based on relative ideological candidate positions. Furthermore, the first-order conditions can now be written as:

$$\begin{aligned}\frac{\partial \Pi_i^2(y_i, y_j)}{\partial y_i} =&\frac{1}{2}((y_j - (\epsilon_i + s))^2 - (y_i - (\epsilon_i + s))^2) \\ &- (2\theta^e + 1)\left(y_i - (\epsilon_i + s)\right) - 2c_2\left(y_i - (\epsilon_i + s)\right) \stackrel{!}{=} 0\end{aligned} \tag{A.2}$$

$$\begin{aligned}\frac{\partial \Pi_j^2(y_i, y_j)}{\partial y_j} =&\frac{1}{2}(((\epsilon_j + s) - y_j)^2 - ((\epsilon_j + s) - y_i)^2) \\ &+ (2 - (2\theta^e + 1))\left((\epsilon_j + s) - y_j\right) + 2c_2\left((\epsilon_j + s) - y_j\right) \stackrel{!}{=} 0\end{aligned} \tag{A.3}$$

Note that any best responses y_i^0, y_j^0 derived from (A.2) and (A.3) for $s = 0$ can be used to solve the game for $s \neq 0$ with $y_i^s = y_i^0 + s$ and $y_j^s = y_j^0 + s$. Hence, it is sufficient to solve the problem for $s = 0$. Which results in the following reaction functions:

$$\begin{aligned} y_i(y_j) =& \frac{1}{3}\sqrt{1 + 4\epsilon_i^2 + 4c_2^2 + 2y_j + 4y_j^2 + 4c_2(1 + y_j) + 2\epsilon_i(1 + 2c_2 - 2y_j)} \\ & - \frac{1}{3}(1 - 2\epsilon_i + 2c_2 + y_j) \end{aligned} \tag{A.4}$$

$$\begin{aligned} y_j(y_i) =& -\frac{1}{3}\sqrt{1 + 4\epsilon_j^2 + 4c_2^2 - 2y_i + 4y_i^2 + 4c_2(1 - y_i) - 2\epsilon_j(1 + 2c_2 + 2y_i)} \\ & + \frac{1}{3}(1 + 2\epsilon_j + 2c_2 - y_i) \end{aligned} \tag{A.5}$$

There exists no simple closed-form solution unless further assumptions on parameters are made. However, it is possible to derive the reaction to changes in ϵ_i or ϵ_j and its implications by calculating the respective derivatives, which are non-constant, supporting the former claim.

$$\frac{\partial y_i(y_j)}{\partial \epsilon_i} = \frac{1}{3}\frac{1 + 2c_2 - 2y_j + 4\epsilon_i}{\sqrt{1 + 4\epsilon_i^2 + 4c_2^2 + 2y_j + 4y_j^2 + 4c_2(1 + y_j) + 2\epsilon_i(1 + 2c_2 - 2y_j)}} + \frac{2}{3} > 0$$

$$\frac{\partial y_j(y_i)}{\partial \epsilon_j} = \frac{1}{3}\frac{1 + 2c_2 + 2y_i - 4\epsilon_j}{\sqrt{1 + 4\epsilon_j^2 + 4c_2^2 - 2y_i + 4y_i^2 + 4c_2(1 - y_i) - 2\epsilon_j(1 + 2c_2 + 2y_i)}} + \frac{2}{3} > 0$$

This shows that if the personal bliss of a candidate moves closer to the center, so will his political platform announcement in the general election and vice versa. The candidate whose bliss point is closer to the median voter preferred policy has the highest winning probability.

A.2 Proofs

Lemma 1.4.1.
Given that $\epsilon_i < \epsilon_j$ it holds:

$$\begin{aligned} y_i^* &= \underset{y_i \in \mathbb{R}}{arg\,max}\ \Pi_i^2(y_i, y_j) \Rightarrow |y_i^* - (\epsilon_i + s)| \leq |y_j - (\epsilon_i + s)| \\ y_j^* &= \underset{y_j \in \mathbb{R}}{arg\,max}\ \Pi_j^2(y_i, y_j) \Rightarrow |y_i - (\epsilon_j + s)| \geq |y_j^* - (\epsilon_j + s)| \end{aligned}$$

Proof. Assuming $|y_i - (\epsilon_i + s)| > |y_j - (\epsilon_i + s)|$ and $\epsilon_i < \epsilon_j$ and applying the monotonicity of $g(.)$ and $C_2(.)$ and the properties of CDF's, the expected utility of candidate i, $\Pi_i^2(y_i, y_j)$

has the following upper bound:

$$\begin{aligned}\Pi_i^2(y_i, y_j) &= F_G\left(\theta^e\right)\left(-g(|y_i - (\epsilon_i + s)|)\right) \\ &\quad + \left(1 - F_G\left(\theta^e\right)\right)\left(-g(|y_j - (\epsilon_i + s)|)\right) - C_2\left(|y_i - (\epsilon_i + s)|\right) \\ &< \left(-g(|y_j - (\epsilon_i + s)|)\right) - C_2\left(|y_j - (\epsilon_i + s)|\right) \\ &= \frac{1}{2}\left(-g(|y_j - (\epsilon_i + s)|)\right) + \frac{1}{2}\left(-g(|y_j - (\epsilon_i + s)|)\right) - C_2\left(|y_j - (\epsilon_i + s)|\right) \\ &= \Pi_i^2(y_j, y_j)\end{aligned}$$

Analogously it holds for $\Pi_j^2(y_i, y_j)$ that $\Pi_j^2(y_i, y_j) < \Pi_j^2(y_i, y_i)$.

□

Lemma 1.4.2.
Given $\epsilon_i < \epsilon_j$, it holds that $\Pi_i^2(y_i, y_j)$ is strictly increasing in $y_i < \epsilon_i + s$ and $\Pi_j^2(y_i, y_j)$ is strictly decreasing in $y_j > \epsilon_j + s$.

Proof. If $\epsilon_i < \epsilon_j$ and $y_i < \epsilon_i + s$ and accordingly $y_j > \epsilon_j + s$ for $\Pi_j^2(y_i, y_j)$, using the monotonicity of $g(.)$ and $C_2(.)$ as well as the properties of CDFs, it holds for the following first derivatives:

$$\begin{aligned}\frac{\partial\, \Pi_i^2(y_i, y_j)}{\partial y_i} =& \frac{1}{2} f_G\left(\theta^e\right) \underbrace{\left(g(|(\epsilon_i + s) - y_j|) - g(\epsilon_i + s - y_i)\right)}_{>0} \\ &+ F_G\left(\theta^e\right) g'(\epsilon_i + s - y_i) + C_2'\left(\epsilon_i + s - y_i\right) > 0 \\ \frac{\partial\, \Pi_j^2(y_i, y_j)}{\partial y_j} =& \frac{1}{2} f_G\left(\theta^e\right) \underbrace{\left(g((\epsilon_j + s) - y_j) - g(|(\epsilon_j + s) - y_i|)\right)}_{<0} \\ &- \left(1 - F\left(\theta^e\right)\right) g'(y_j - (\epsilon_j + s)) - C_2'\left(y_j - (\epsilon_j + s)\right) < 0\end{aligned}$$

□

Proposition 1.4.3.
Given $\epsilon_i < \epsilon_j$, it holds that $\Pi_i^2(y_i, y_j)$ and $\Pi_j^2(y_i, y_j)$ assume their respective local maxima in y_i^* and y_j^* which fulfill the following:

$$\begin{aligned}&\exists y_i^* \in (\epsilon_i + s,\ y_j) \qquad \textit{with} \quad \frac{\partial \Pi_i^2(y_i, y_j)}{\partial y_i}\Big|_{y_i = y_i^*} = 0 \quad \forall y_j \\ &\exists y_j^* \in (y_i,\ \epsilon_j + s) \qquad \textit{with} \quad \frac{\partial \Pi_j^2(y_i, y_j)}{\partial y_j}\Big|_{y_j = y_j^*} = 0 \quad \forall y_i\end{aligned}$$

Proof. As $\Pi_i^2(y_i, y_j)$ is continuous and differentiable on the respective intervals, this follows directly from lemmata 1.4.1 and 1.4.2. Evaluating the first derivatives in the respective points gives:

$$\begin{aligned}
\frac{\partial \Pi_i^2(y_i, y_j)}{\partial y_i}\Big|_{y_i=\epsilon_i+s^e} =& \frac{1}{2} f_G\left(\frac{\epsilon_i + y_j - s^e}{2}\right)(g(y_j - (\epsilon_i + s)) - g(0)) \\
& - F_G\left(\frac{\epsilon_i + y_j - s^e}{2}\right) g'(0) - C_2'(0) \\
=& \frac{1}{2} f_G\left(\frac{\epsilon_i + y_j - s^e}{2}\right) g(y_j - (\epsilon_i + s)) > 0 \\
\frac{\partial \Pi_j^2(y_i, y_j)}{\partial y_j}\Big|_{y_j=\epsilon_j+s^e} =& \frac{1}{2} f_G\left(\frac{y_i + \epsilon_j - s^e}{2}\right)(g(0) - g((\epsilon_j + s) - y_i)) \\
& + \left(1 - F_G\left(\frac{y_i + \epsilon_j - s^e}{2}\right)\right) g'(0) + C_2'(0) \\
=& -\frac{1}{2} f_G\left(\frac{y_i + \epsilon_j - s^e}{2}\right) g((\epsilon_j + s) - y_i) < 0 \\
\frac{\partial \Pi_i^2(y_i, y_j)}{\partial y_i}\Big|_{y_i=y_j} =& -F_G(y_j - s^e)\, g'(y_j - (\epsilon_i + s)) - C_2'(y_j - (\epsilon_i + s)) < 0 \\
\frac{\partial \Pi_j^2(y_i, y_j)}{\partial y_j}\Big|_{y_j=y_i} =& (1 - F_G(y_i - s^e))\, g'((\epsilon_j + s) - y_i) + C_2'((\epsilon_j + s) - y_i) > 0
\end{aligned}$$

By applying the mean value theorem, the proposition follows directly.

□

Proposition 1.4.4.
$\Pi_i^2(y_i, y_j)$ *and* $\Pi_j^2(y_i, y_j)$ *from* (1.5) *and* (1.6) *have a unique respective maximizer and are quasi-concave.*

Proof. Following proposition 1.4.3 second derivatives for $\Pi_i^2(y_i, y_j)$ and $\Pi_j^2(y_i, y_j)$ in $\epsilon_i + s \leq y_i \leq y_j \leq \epsilon_j + s$ can be written as follows:

$$\begin{aligned}
\frac{\partial^2 \Pi_j^2(y_i, y_j)}{(\partial y_i)^2} =& \frac{1}{4} f_G'(\theta^e)\,(g(y_j - (\epsilon_i + s)) - g(y_i - (\epsilon_i + s))) \\
& - f_G(\theta^e)\, g'(y_i - (\epsilon_i + s)) - F_G(\theta^e)\, g''(y_i - (\epsilon_i + s)) \\
& - C_2''(y_i - (\epsilon_i + s)) \\
\frac{\partial^2 \Pi_j^2(y_i, y_j)}{(\partial y_j)^2} =& \frac{1}{4} f_G'(\theta^e)\,(g((\epsilon_j + s) - y_j) - g((\epsilon_j + s) - y_i)) \\
& - f_G(\theta^e)\, g'((\epsilon_j + s) - y_j) - (1 - F_G(\theta^e))\, g''((\epsilon_j + s) - y_j) \\
& - C_2''((\epsilon_j + s) - y_j)
\end{aligned}$$

First, consider the case of $C_2(x) = 0\ \forall x \in \mathbb{R}$. The critical points of the first-order conditions from (1.7) and (1.8) can then be rewritten as:

$$\frac{1}{2}\frac{g(y_j - (\epsilon_i + s)) - g(y_i^* - (\epsilon_i + s))}{g'(y_i^* - (\epsilon_i + s))} = \frac{F_G\left(\theta^{e*}\right)}{f_G\left(\theta^{e*}\right)}$$

$$\frac{1}{2}\frac{(g((\epsilon_j + s) - y_j^*) - g((\epsilon_j + s) - y_i))}{g'((\epsilon_j + s) - y_j^*)} = -\frac{1 - F_G\left(\theta^{e*}\right)}{f_G\left(\theta^{e*}\right)}$$

Evaluating the second derivatives under their respective critical points, rearranging and applying log-concavity of F_G yields:

$$\frac{\partial^2 \Pi_j^2(y_i, y_j)}{(\partial y_i)^2}\Big|_{y_i = y_i^*} = \underbrace{\frac{1}{2}\frac{f_G'\left(\theta^{e*}\right) F_G\left(\theta^{e*}\right)}{f_G^2\left(\theta^{e*}\right)} - 1}_{<0} - \frac{F_G\left(\theta^{e*}\right)}{f_G\left(\theta^{e*}\right)}\frac{g''(y_i^* - (\epsilon_i + s))}{g'(y_i^* - (\epsilon_i + s))} < 0$$

$$\frac{\partial^2 \Pi_j^2(y_i, y_j)}{(\partial y_j)^2}\Big|_{y_j = y_j^*} = \underbrace{-\frac{1}{2}\frac{f_G'\left(\theta^{e*}\right)\left(1 - F_G\left(\theta^{e*}\right)\right)}{f_G^2\left(\theta^{e*}\right)} - 1}_{<0} - \frac{\left(1 - F_G\left(\theta^{e*}\right)\right) g''((\epsilon_j + s) - y_j^*)}{f_G\left(\theta^{e*}\right) g'((\epsilon_j + s) - y_j^*)} < 0$$

Following this, I can conclude that $\Pi_k^2(y_i, y_j)$ is concave in the critical point of the first-order conditions, making it its unique global maximum for the case $C_2(x) = 0\ \forall x \in \mathbb{R}$. Therefore, the first derivative of $\Pi_k^2(y_i, y_j)$ fulfills the single crossing property in accordance with Quah and Strulovici (2012). Note, however, that I analyze a switch from a positive slope to a negative slope following Proposition 1.4.3. Hence, I need single crossing of the first derivative of a decreasing function. This is achieved by substituting $f(s)$ in Quah and Strulovici (2012)'s original sense by $-f(s)$, maintaining the validity of the property and reversing inequalities. Signed-ratio monotonicity is then fulfilled by the first derivative, and the following holds for $y_i' < y_i''$ and respectively $\theta^{e'} < \theta^{e''}$:

$$\frac{\frac{1}{2} f_G(\theta^{e'})(g(y_j - (\epsilon_i + s)) + g(y_i' - (\epsilon_i + s)))}{F_G(\theta^{e'}) g'(y_i' - (\epsilon_i + s))} \geq \frac{\frac{1}{2} f_G(\theta^{e''})(g(y_j - (\epsilon_i + s)) + g(y_i'' - (\epsilon_i + s)))}{F_G(\theta^{e''}) g'(y_i'' - (\epsilon_i + s))}$$

Given that this holds, consider now that $C_2(x)$ is convex and monotonic. Including the lying cost now only increases the denominator in this inequality from the left side to the right side as $C_2'(y_i' - (\epsilon_i + s)) < C_2'(y_i'' - (\epsilon_i + s))$, and it follows directly, that:

$$\frac{\frac{1}{2} f_G(\theta^{e'})(g(y_j - (\epsilon_i + s)) + g(y_i' - (\epsilon_i + s)))}{F_G(\theta^{e'}) g'(y_i' - (\epsilon_i + s)) + C_2'(y_i' - (\epsilon_i + s))} \geq \frac{\frac{1}{2} f_G(\theta^{e''})(g(y_j - (\epsilon_i + s)) + g(y_i'' - (\epsilon_i + s)))}{F_G(\theta^{e''}) g'(y_i'' - (\epsilon_i + s)) + C_2'(y_i'' - (\epsilon_i + s))}$$

This proves the single crossing property for the general problem.

□

Corollary 1.4.5.
In the general election with risk-neutral candidates the following platform announcements constitute best responses to each other:

$$y_i^*(y_j^*) = \mu_G + s^e - \frac{c_2 + \frac{1}{2}}{f(\mu_G)} \qquad\qquad y_j^*(y_i^*) = \mu_G + s^e + \frac{c_2 + \frac{1}{2}}{f(\mu_G)}$$

Proof.
First, I rewrite the second-stage candidates' expected utilities from (1.5) and (1.6):

$$\begin{aligned}\Pi_i^2(y_i, y_j) = &- F_G\left(\theta^e\right)|y_i - (\epsilon_i + s)| \\ &- \left(1 - F_G\left(\theta^e\right)\right)|y_j - (\epsilon_i + s)| - c_2|y_i - (\epsilon_i + s)|\end{aligned} \tag{A.6}$$

$$\begin{aligned}\Pi_j^2(y_i, y_j) = &- F_G\left(\theta^e\right)|(\epsilon_j + s) - y_i| \\ &- \left(1 - F_G\left(\theta^e\right)\right)|(\epsilon_j + s) - y_j| - c_2|(\epsilon_j + s) - y_j|\end{aligned} \tag{A.7}$$

Following this, the first-order conditions in (1.7) and (1.8) simplify to:

$$\frac{\partial \Pi_i^2(y_i, y_j)}{\partial y_i} = \frac{1}{2} f_G\left(\theta^e\right)\left(y_j - y_i\right) - F_G\left(\theta^e\right) - c_2 \stackrel{!}{=} 0 \tag{A.8}$$

$$\frac{\partial \Pi_j^2(y_i, y_j)}{\partial y_j} = \frac{1}{2} f_G\left(\theta^e\right)\left(y_i - y_j\right) + \left(1 - F_G\left(\theta^e\right)\right) + c_2 \stackrel{!}{=} 0 \tag{A.9}$$

Dividing (A.8) and (A.9) by each other gives a solution for the equilibrium probability of winning under mutual best responses, as already shown generally in (1.9). As F_G is symmetric and unimodal with existing mean, it holds:

$$F_G\left(\theta^e\right) = \frac{1}{2} \;\Leftrightarrow\; \theta^e = \frac{y_i + y_j - 2s^e}{2} = \mu_G \tag{A.10}$$

Applying this to the first-order conditions, I obtain the following expressions:

$$\frac{1}{2} f_G\left(\mu_G\right)\left(y_j - y_i\right) - \frac{1}{2} - c_2 \stackrel{!}{=} 0 \tag{A.11}$$

$$\frac{1}{2} f_G\left(\mu_G\right)\left(y_i - y_j\right) + \frac{1}{2} + c_2 \stackrel{!}{=} 0 \tag{A.12}$$

Using (A.10), (A.11) and (A.12), I derive the following optimal platform announcements in the linear case:

$$y_i^* = \mu_G + s^e - \frac{c_2 + \frac{1}{2}}{f(\mu_G)} \tag{A.13}$$

$$y_j^* = \mu_G + s^e + \frac{c_2 + \frac{1}{2}}{f(\mu_G)} \tag{A.14}$$

□

Lemma 1.4.6.
Upon receiving signals $q_1 := s + \epsilon_1,\ ...,\ q_4 := s + \epsilon_4$ *it then holds:*

$$s^e = E[s|q_1, ..., q_4] = \frac{4\sigma_s^2 \overline{s'} + \sigma_\epsilon^2 \mu_s}{4\sigma_s^2 + \sigma_\epsilon^2} \quad with\, \overline{s'} = \frac{1}{4}\sum_{i=1}^{4}(q_i - \mu_{\epsilon_i})$$

And analogously or by linearity of expectation:

$$\epsilon_i^e = E[\epsilon_i|q_1, ..., q_4] = \frac{4\sigma_s^2 \overline{S'} + \sigma_\epsilon^2 \mu_s'}{4\sigma_s^2 + \sigma_\epsilon^2}$$
$$with\, \overline{S'} = \frac{1}{4}\sum_{j=1}^{4}(q_i - q_j + \mu_{\epsilon_j}),\ \mu_s' = q_i - \mu_s$$

Proof.
Bayes' theorem: $Posterior = \frac{likelihood * prior}{evidence}$
Prior: $P(s) = f_s(s) = \frac{1}{\sqrt{2\pi\sigma_s^2}} exp\left(-\frac{(s-\mu_s)^2}{2\sigma_s^2}\right)$
Likelihood: $P(q_1, ..., q_4|s) = P(q_1|s)P(s_2|s)P(s_3|s)P(q_4|s)$

$$P(q_1|s) = f_{\epsilon_i}(q_i - s) = \frac{1}{\sqrt{2\pi\sigma_\epsilon^2}} exp\left(-\frac{(q_i - s - \mu_{\epsilon_i})^2}{2\sigma_\epsilon^2}\right)$$

Define: $q_i' = q_i - \mu_{\epsilon_i}$

$$P(q_1, ..., q_4|s) = \frac{1}{(2\pi\sigma_\epsilon^2)^2} exp\left(-\frac{\sum_{i=1}^{4}(q_i' - s)^2}{2\sigma_\epsilon^2}\right)$$

Likelihood x prior: $P(q_1, ..., q_4|s)P(s)$

$$= \frac{1}{\sqrt{2\pi\sigma_s^2}(2\pi\sigma_\epsilon^2)^2} exp\left(-\frac{\sigma_\epsilon^2(s-\mu_s)^2 + \sigma_s^2 \sum_{i=1}^{4}(q_i'-s)^2}{2\sigma_s^2\sigma_\epsilon^2} \right)$$

Evidence:

$$P(q_1, ..., q_4) = \int_{-\infty}^{+\infty} f_s(x) \prod_{i=1}^{4} f_{\epsilon_i}(x - q_i) dx$$

Posterior:

$$P(s|q_1, ..., q_4) \propto exp\left(-\frac{\sigma_\epsilon^2(s-\mu_s)^2 + \sigma_s^2 \sum_{i=1}^{4}(q_i'-s)^2}{2\sigma_s^2\sigma_\epsilon^2} \right)$$

Define:
$\overline{s'} = \frac{1}{4}\sum_{i=1}^{4} q_i'$ (empirical mean)
$\tilde{s}' = \frac{1}{4}\sum_{i=1}^{4}(q_i' - \overline{s'})^2$ (empirical variance)
It holds:

$$\sum_{i=1}^{4}(q_i'-s)^2 = 4\tilde{s}' + 4(s-\overline{s'})^2$$

$$\begin{aligned} \Rightarrow P(s|q_1, ..., q_4) &\propto exp\left(-\frac{\sigma_\epsilon^2(s-\mu_s)^2 + \sigma_s^2\left(4\tilde{s}' + 4(s-\overline{s'})^2\right)}{2\sigma_s^2\sigma_\epsilon^2} \right) \\ &\propto exp\left(-\frac{\sigma_\epsilon^2(s-\mu_s)^2 + 4\sigma_s^2(s-\overline{s'})^2}{2\sigma_s^2\sigma_\epsilon^2} \right) \end{aligned}$$

Expanding and rearranging yields:

$$\begin{aligned} P(s|q_1, ..., q_4) &\propto exp\left(-\frac{s^2 - 2s\frac{4\sigma_s^2\overline{s'} + \sigma_\epsilon^2\mu_s}{4\sigma_s^2+\sigma_\epsilon^2} + \frac{\sigma_\epsilon^2\mu_s^2 + 4\sigma_s^2\overline{s'}^2}{4\sigma_s^2+\sigma_\epsilon^2}}{2\frac{\sigma_s^2\sigma_\epsilon^2}{4\sigma_s^2+\sigma_\epsilon^2}} \right) \\ &\propto exp\left(-\frac{(s-s^e)^2}{2\sigma_{s^e}^2} \right) \end{aligned}$$

□

Lemma 1.4.7.

The equilibrium, is sustained for:

$$c_1 > max\left\{0, \frac{2\sigma_s^2+\sigma_\epsilon^2}{8\sigma_s^2+2\sigma_\epsilon^2} f_L\left(\frac{\epsilon_{L1}^e+\epsilon_{L2}^e}{2}\right)\left(F_R\left(\frac{\epsilon_{R1}^e+\epsilon_{R2}^e}{2}\right)\overline{\Pi}+\Pi_i^2(y_{L1},y_{R2})-\Pi_i^2(y_{L2},y_{R2})\right)\right.$$
$$\left. - \frac{2\sigma_s^2}{8\sigma_s^2+2\sigma_\epsilon^2} f_R\left(\frac{\epsilon_{R1}^e+\epsilon_{R2}^e}{2}\right)\left(F_L\left(\frac{\epsilon_{R1}^e+\epsilon_{R2}^e}{2}\right)\overline{\Pi}+\Pi_i^2(y_{L2},y_{R1})-\Pi_i^2(y_{L2},y_{R2})\right)\right\}$$

with: $\overline{\Pi} = \Pi_i^2(y_{L1},y_{R1}) - \Pi_i^2(y_{L1},y_{R2}) - \Pi_i^2(y_{L2},y_{R1}) + \Pi_i^2(y_{L2},y_{R2})$

Proof.

Applying Bayesian-Updating for $q_i = f^{-1}(x_i) = \epsilon_i + s$ with monotonic $f(\cdot)$ allows belief updating according to Lemma 1.4.6. Calculating bliss points of indifferent voters and their respective derivatives based on this gives:

$$\frac{\epsilon_i^e+\epsilon_j^e}{2} = \frac{\sigma_\epsilon^2(q_i+q_j-2\mu_s)+\sigma_s^2\left(2q_i+2q_j-2q_k-2q_l+2\sum_{k=1}^{4}\mu_{\epsilon_k}\right)}{8\sigma_s^2+2\sigma_\epsilon^2}$$

$$\frac{\partial\left(\frac{\epsilon_i^e+\epsilon_j^e}{2}\right)}{\partial x_i} = \frac{2\sigma_s^2+\sigma_\epsilon^2}{8\sigma_s^2+2\sigma_\epsilon^2}\frac{\partial f^{-1}(x_i)}{\partial x_i}, \qquad \frac{\partial\left(\frac{\epsilon_j^e+\epsilon_k^e}{2}\right)}{\partial x_i} = \frac{-\sigma_s^2}{4\sigma_s^2+\sigma_\epsilon^2}\frac{\partial f^{-1}(x_i)}{\partial x_i}$$

Assume that, without loss of generality, $\epsilon_{L1} < \epsilon_{L2}$ and $\epsilon_{R1} < \epsilon_{R2}$. The winning probabilities can then be written using CDFs for the respective median voter position and the matching bliss points of indifferent voters:

$$P_{L1,1}(x) = F_L\left(\frac{\epsilon_{L1}^e+\epsilon_{L2}^e}{2}\right), \qquad P_{R1,1}(x) = F_R\left(\frac{\epsilon_{R1}^e+\epsilon_{R2}^e}{2}\right)$$

The first derivative of (1.4) then amounts to:

$$\begin{aligned}\frac{\partial\Pi_i(x,y)}{\partial x_i} =& \left(\frac{\partial P_{L1,1}(x)}{\partial x_i}P_{R1,1}(x) + P_{L1,1}(x)\frac{\partial P_{R1,1}(x)}{\partial x_i}\right)\Pi_i^2(y_{L1},y_{R1})\\ &+\left(\frac{\partial P_{L1,1}(x)}{\partial x_i} - \frac{\partial P_{L1,1}(x)}{\partial x_i}P_{R1,1}(x) - P_{L1,1}(x)\frac{\partial P_{R1,1}(x)}{\partial x_i}\right)\Pi_i^2(y_{L1},y_{R2})\\ &+\left(\frac{\partial P_{R1,1}(x)}{\partial x_i} - \frac{\partial P_{L1,1}(x)}{\partial x_i}P_{R1,1}(x) - P_{L1,1}(x)\frac{\partial P_{R1,1}(x)}{\partial x_i}\right)\Pi_i^2(y_{L2},y_{R1})\\ &+\left(-\frac{\partial P_{L1,1}(x)}{\partial x_i} + \frac{\partial P_{L1,1}(x)}{\partial x_i}P_{R1,1}(x) + P_{L1,1}(x)\frac{\partial P_{R1,1}(x)}{\partial x_i} - \frac{\partial P_{R1,1}(x)}{\partial x_i}\right)\\ &\cdot\Pi_i^2(y_{L2},y_{R2}) - \frac{\partial C_1(|x_i-(\epsilon_i+s)|)}{\partial x_i}\end{aligned}$$

This allows to find a lower bound for c_1 by rearranging and plugging in. To prevent deviations, it has to hold that:

$$c_1 > max\left\{0, \frac{2\sigma_s^2+\sigma_\epsilon^2}{8\sigma_s^2+2\sigma_\epsilon^2} f_L\left(\frac{\epsilon_{L1}^e+\epsilon_{L2}^e}{2}\right)\left(F_R\left(\frac{\epsilon_{R1}^e+\epsilon_{R2}^e}{2}\right)\overline{\Pi}+\Pi_i^2(y_{L1},y_{R2})-\Pi_i^2(y_{L2},y_{R2})\right)\right.$$
$$\left. - \frac{2\sigma_s^2}{8\sigma_s^2+2\sigma_\epsilon^2} f_R\left(\frac{\epsilon_{R1}^e+\epsilon_{R2}^e}{2}\right)\left(F_L\left(\frac{\epsilon_{R1}^e+\epsilon_{R2}^e}{2}\right)\overline{\Pi}+\Pi_i^2(y_{L2},y_{R1})-\Pi_i^2(y_{L2},y_{R2})\right)\right\}$$
$$\text{where: } \overline{\Pi} = \Pi_i^2(y_{L1},y_{R1})-\Pi_i^2(y_{L1},y_{R2})-\Pi_i^2(y_{L2},y_{R1})+\Pi_i^2(y_{L2},y_{R2})$$

□

Proposition 1.4.8.
In the non-linear game under sufficient lying cost, all information is fully revealed via the primary platform announcements and general election platform announcements, and it holds:

$$s = \frac{y_i^*+y_j^*}{2} - F^{-1}(\overline{K_3}) \text{ and } \epsilon_k = x_k^* - \frac{y_i^*+y_j^*}{2} + F^{-1}(\overline{K_3}) \text{ for } k \in \{i,j\} \text{ and } \overline{K_3} \text{ constant}$$

Proof.
For the general stage under non-linearity, I showed that the optimal policy platforms y_i^* and y_j^* are unique and defined by (1.7) and (1.8). Therefore, it holds:

$$\begin{aligned}&\frac{1}{2} f\left(\frac{y_i^*+y_j^*-2s}{2}\right)\left(g(y_j^*-(\epsilon_i+s))-g(y_i^*-(\epsilon_i+s))\right)\\ &-F\left(\frac{y_i^*+y_j^*-2s}{2}\right) g'(y_i^*-(\epsilon_i+s)) - C_2'\left(y_i^*-(\epsilon_i+s)\right) \stackrel{!}{=} 0\end{aligned} \tag{A.15}$$

$$\begin{aligned}&\frac{1}{2} f\left(\frac{y_i^*+y_j^*-2s}{2}\right)\left(g((\epsilon_j+s)-y_j^*)-g((\epsilon_j+s)-y_i^*)\right)\\ &+\left(1-F\left(\frac{y_i^*+y_j^*-2s}{2}\right)\right) g'((\epsilon_j+s)-y_j^*) + C_2'\left((\epsilon_j+s)-y_j^*\right) \stackrel{!}{=} 0\end{aligned} \tag{A.16}$$

For the primary stage, under sufficient lying costs, I showed that candidates optimal primary platforms are given as follows:

$$x_i^* = \epsilon_i + s \tag{A.17}$$
$$x_j^* = \epsilon_j + s \tag{A.18}$$

Due to proposition 1.4.3 together with (1.9), (1.10) and (1.11), the following independence properties hold additionally:

$$x_i^* \neq y_i^* \qquad x_j^* \neq y_j^* \qquad y_j^* \neq y_i^*$$

Plugging (A.17) into (A.15) gives:

$$f\left(\frac{y_i^*+y_j^*-2s}{2}\right)\frac{g(y_j^*-x_i^*)-g(y_i^*-x_i^*)}{2}=F\left(\frac{y_i^*+y_j^*-2s}{2}\right)g'(y_i^*-x_i^*)+C_2'\left(y_i^*-x_i^*\right)$$

$$\Leftrightarrow\frac{1}{2}f\left(\frac{y_i^*+y_j^*-2s}{2}\right)=\frac{F\left(\frac{y_i^*+y_j^*-2s}{2}\right)g'(y_i^*-x_i^*)+C_2'\left(y_i^*-x_i^*\right)}{g(y_j^*-x_i^*)-g(y_i^*-x_i^*)}$$

$$=F\left(\frac{y_i^*+y_j^*-2s}{2}\right)\cdot\overline{K_1}+\overline{K_2}$$

$$\text{with }\overline{K_1}=\frac{g'(y_i^*-x_i^*)}{g(y_j^*-x_i^*)-g(y_i^*-x_i^*)}\in(0,\infty)\text{ and }\overline{K_2}=\frac{C_2'\left(y_i^*-x_i^*\right)}{g(y_j^*-x_i^*)-g(y_i^*-x_i^*)}\in(0,\infty)$$

Plugging those together with (A.18) into (A.16) yields:

$$\left(F\left(\frac{y_i^*+y_j^*-2s}{2}\right)\overline{K_1}+\overline{K_2}\right)\left(g(x_j^*-y_j^*)-g(x_j^*-y_i^*)\right)+g'(x_j^*-y_j^*)$$

$$=F\left(\frac{y_i^*+y_j^*-2s}{2}\right)g'(x_j^*-y_j^*)-C_2'\left(x_j^*-y_j^*\right)$$

$$\Leftrightarrow F\left(\frac{y_i^*+y_j^*-2s}{2}\right)=\frac{\overline{K_2}(g(x_j^*-y_i^*)-g(x_j^*-y_j^*))+g'(x_j^*-y_j^*)+C_2'\left(x_j^*-y_j^*\right)}{g'(x_j^*-y_j^*)-\overline{K_1}(g(x_j^*-y_i^*)-g(x_j^*-y_j^*))}=:\overline{K_3}$$

Applying the inverse function for $F(.)$ yields:

$$\frac{y_i^*+y_j^*-2s}{2}=F^{-1}(\overline{K_3})$$

Hence it holds:

$$s=\frac{y_i^*+y_j^*}{2}-F^{-1}(\overline{K_3})\text{ and }\epsilon_k=x_k^*-\frac{y_i^*+y_j^*}{2}+F^{-1}(\overline{K_3})\text{ with }k\in\{i,j\}$$

□

Corollary 1.4.9.

In the primary election with risk-neutral candidates the following platform announcements are optimal:

$$x_i^*=\epsilon_i+s\qquad\forall i\in\{L1,L2,R1,R2\}$$

This holds for:

$$c_1 > \frac{\sigma_s^2}{4\sigma_s^2+\sigma_\epsilon^2} + \frac{2\sigma_s^2+\sigma_\epsilon^2}{8\sigma_s^2+2\sigma_\epsilon^2} f_L\left(\frac{\epsilon_k^e(x)+\epsilon_{-k}^e(x)}{2}\right) c_2|y_k-(\epsilon_k+s)| + F_L\left(\frac{\epsilon_k^e(x)+\epsilon_{-k}^e(x)}{2}\right)\frac{c_2\sigma_s^2}{4\sigma_s^2+\sigma_\epsilon^2}$$

where:

$$|y_k-(\epsilon_k+s)| = min\left\{\left|\mu_G+s^e-\frac{c_2+\frac{1}{2}}{f(\mu_G)}-(\epsilon_k+s)\right|, \left|\mu_G+s^e+\frac{c_2+\frac{1}{2}}{f(\mu_G)}-(\epsilon_k+s)\right|\right\}$$

Proof.
Given the optimal strategies calculated in Corollary 1.4.5, I first calculate the expected payoffs of (1.3) for $i,j,k \in \{L1,L2,R1,R2\}$.

$$\begin{aligned}\Pi_k^2(y_i,y_j) &= -\frac{1}{2}\left|\mu_G+s^e-\frac{c_2+\frac{1}{2}}{f(\mu_G)}-(\epsilon_k+s)\right| \\ &\quad -\frac{1}{2}\left|\mu_G+s^e+\frac{c_2+\frac{1}{2}}{f(\mu_G)}-(\epsilon_k+s)\right| \\ &\quad -\mathbb{1}_{\{i,j\}}(k)\, c_2|y_k-(\epsilon_k+s)| \\ &= -|\mu_G+s^e-(\epsilon_k+s)| - \mathbb{1}_{\{i,j\}}(k)\, c_2|y_k-(\epsilon_k+s)|\end{aligned}$$

The sufficient distance assumption (1.13) allows to resolve the absolute values and the calculation of the overall expected utility across both stages from (1.4). Primary stage expected payoffs then can be written as follows:

$$\Pi_k(x,y) = -P_{k,1}(x)\cdot c_2|y_k-(\epsilon_k+s)| - |\mu_G+s^e-(\epsilon_k+s)| - c_1|x_k-(\epsilon_k+s)|$$

Plugging in the probability using the critical value and the CDF yields:

$$\begin{aligned}\Pi_k(x,y) = &- F_L\left(\frac{\epsilon_k^e(x)+\epsilon_{-k}^e(x)}{2}\right) c_2|y_k-(\epsilon_k+s)| - |\mu_G+s^e(x)-(\epsilon_k+s)| \\ &- c_1|x_k-(\epsilon_k+s)|\end{aligned}$$

Calculating the first derivative and solving for c_1 gives the following upper bound:

$$\begin{aligned}c_1 > &\frac{\sigma_s^2}{4\sigma_s^2+\sigma_\epsilon^2} + \frac{2\sigma_s^2+\sigma_\epsilon^2}{8\sigma_s^2+2\sigma_\epsilon^2} f_L\left(\frac{\epsilon_k^e(x)+\epsilon_{-k}^e(x)}{2}\right) c_2|y_k-(\epsilon_k+s)| \\ &+ F_L\left(\frac{\epsilon_k^e(x)+\epsilon_{-k}^e(x)}{2}\right)\frac{c_2\sigma_s^2}{4\sigma_s^2+\sigma_\epsilon^2}\end{aligned} \tag{A.19}$$

where:

$$|y_k - (\epsilon_k + s)| = \min\left\{\left|\mu_G + s^e - \frac{c_2 + \frac{1}{2}}{f(\mu_G)} - (\epsilon_k + s)\right|, \left|\mu_G + s^e + \frac{c_2 + \frac{1}{2}}{f(\mu_G)} - (\epsilon_k + s)\right|\right\}$$

□

Proposition 1.5.1.
$\overline{\Pi}_i^2(y_i, y_j)$ *and* $\overline{\Pi}_j^2(y_i, y_j)$ *from* (1.15) *and* (1.16) *have a unique respective maximizer, defied by the first-order condition, and are quasi-concave.*

Proof. Using Lemma 1.4.1, Lemma 1.4.2, Proposition 1.4.3 and Proposition 1.4.4, I showed that $\overline{\Pi}_i^2(y_i, y_j, x_i)$ and $\overline{\Pi}_j^2(y_i, y_j, x_j)$, respectively, fulfill the single-crossing property for $C(.) = 0$, and therefore the following holds for $y_i' < y_i''$ and, respectively, $\theta^{e'} < \theta^{e''}$:

$$\begin{aligned}&\frac{\frac{1}{2}f_G(\theta^{e'})\Delta(y_i') + \mathbb{1}_{[y_i' \leq \epsilon_i + s]}(y_i')F_G(\theta^{e'})g'(|y_i' - (\epsilon_i + s)|)}{\mathbb{1}_{[y_i' \leq \epsilon_i + s]}(y_i') + \mathbb{1}_{[y_i' > \epsilon_i + s]}(y_i')F_G(\theta^{e'})g'(|y_i' - (\epsilon_i + s)|)} \\ &\quad \geq \frac{\frac{1}{2}f_G(\theta^{e''})\Delta(y_i'') + \mathbb{1}_{[y_i'' \leq \epsilon_i + s]}(y_i'')F_G(\theta^{e'})g'(|y_i'' - (\epsilon_i + s)|)}{\mathbb{1}_{[y_i'' \leq \epsilon_i + s]}(y_i'') + \mathbb{1}_{[y_i'' > \epsilon_i + s]}(y_i'')F_G(\theta^{e''})g'(|y_i'' - (\epsilon_i + s)|)}\end{aligned}$$

where $\mathbb{1}_{[A]}(x)$ is the indicator function and $\Delta(y_i) = g(|y_j - (\epsilon_i + s)|) + g(|y_i - (\epsilon_i + s)|)$. Consider now including the weakly binding campaign costs $C(.) \geq 0$. Note that the sign of the derivative depends on whether $y_i > x_i$ or $y_i < x_i$. This results in the following expression:

$$\begin{aligned}&\frac{\frac{1}{2}f_G(\theta^{e'})\Delta(y_i') + \mathbb{1}_{[y_i' \leq \epsilon_i + s]}(y_i')F_G(\theta^{e'})g'(|y_i' - (\epsilon_i + s)|) + \mathbb{1}_{[y_i' \leq x_i]}(y_i')C'(x_i - y_i')}{\mathbb{1}_{[y_i' \leq \epsilon_i + s]}(y_i') + \mathbb{1}_{[y_i' > \epsilon_i + s]}(y_i')F_G(\theta^{e'})g'(|y_i' - (\epsilon_i + s)|) + \mathbb{1}_{[y_i' > x_i]}(y_i')C'(y_i' - x_i)} \\ &\quad \geq \frac{\frac{1}{2}f_G(\theta^{e''})\Delta(y_i'') + \mathbb{1}_{[y_i'' \leq \epsilon_i + s]}(y_i'')F_G(\theta^{e'})g'(|y_i'' - (\epsilon_i + s)|) + \mathbb{1}_{[y_i'' \leq x_i]}(y_i'')C'(x_i - y_i'')}{\mathbb{1}_{[y_i'' \leq \epsilon_i + s]}(y_i'') + \mathbb{1}_{[y_i'' > \epsilon_i + s]}(y_i'')F_G(\theta^{e''})g'(|y_i'' - (\epsilon_i + s)|) + \mathbb{1}_{[y_i'' > x_i]}(y_i'')C'(y_i'' - x_i)}\end{aligned} \tag{A.20}$$

Adding the costs yields three cases to analyze under which the original inequality still holds due to the monotonicity and convexity of $C(.)$:

Case 1 $[y_i' \leq x_i \wedge y_i'' \leq x_i]$: The numerator on the left of (A.20) increases more than on the right for $x_i \geq y_i'' \geq y_i'$ and therefore $C'(x_i - y_i') \geq C'(x_i - y_i'')$.

Case 2 $[y_i' > x_i \Rightarrow y_i'' > x_i]$: The denominator on the right of (A.20) increases more than on the left for $y_i'' \geq y_i' \geq x_i$ and therefore $C'(y_i' - x_i) \leq C'(y_i'' - x_i)$.

Case 3 $[y_i' \leq x_i \wedge y_i'' > x_i]$: The numerator on the left of (A.20) increases and the denominator on the right increases for $y_i' \leq x_i \leq y_i''$.

Consequently, (A.20) holds in all three cases, and signed-ratio monotonicity is fulfilled by the first derivatives of (1.15) and (1.16), which therefore fulfill the single crossing property.

□

Proposition 1.5.2.
Given (1.17) and (1.18), $\Pi_k(x, y)$ *has a unique maximizer* x_k^* *for all* $k \in \{L1, L2, R1, R2\}$ *and is quasiconcave.*

Proof. The product of two log-concave distributions is log-concave again. The best response derived in the primary from Proposition 1.5.1 is continuous and bijective due to its separating nature making it monotonic. It follows that (1.20) behaves equivalent to (1.15) and (1.16). Therefore, the existence of a unique respective maximizer follows directly from Proposition 1.5.1 and the proof is analogues.

□

Chapter 2

Democracy at a Crossroads: How Political Failures and Fake News Drive Polarization - unpublished

Abstract

This paper examines the dynamics of polarization of beliefs within democratic systems, focusing on the interplay between political failures and fake news. By integrating rational voter theory into an enriched information framework, it demonstrates how signal jamming effects from misinformation disrupt belief formation processes and erode trust in media credibility. Using a dynamic spatial voting model with elements from multi-armed bandit theory, this study explores how voters' beliefs evolve across multiple electoral cycles under conditions of governmental failure and media disruptions. Unlike prior studies that focus primarily on irrationality or entirely novel mechanisms, I bridge gaps in the existing literature by incorporating established principles such as Bayesian updating and median voter theorem concepts into a complex information structure. My findings reveal that fake news primarily undermine reliable informational channels rather than directly persuading voters toward specific ideological positions. The results are ideological entrenchment and polarization of beliefs. This analysis underscores the critical need for restoring trust in credible information sources and delivering clear governmental success signals to stabilize democratic systems amidst rising extremism worldwide.

Keywords: Fake News, Belief Polarization, Misinformation, Belief Formation, Trust in Media, Democratic Systems

JEL Codes: D72, D83, Z13

2.1 Introduction

Democracy, the cornerstone of modern governance, has been facing unprecedented challenges in recent years. Across Europe and the United States, political landscapes are increasingly marked by polarization, extremism, and heated discourse that undermine consensus-building processes (Coi (2024)). From Germany's Alternative für Deutschland (AfD) (Christian Edwards and Schmidt (2024)) and Die Linke (Sahlberg and Lunday (2025)) to Italy's Giorgia Meloni led government (Politico (2022)) and France's left-wing coalition (Momtaz (2020)), similar trends are observable across Poland, the U.S.–taking into account Donald Trump and the MAGA movement (Samaras (2025))- and also other nations (Colelaere and Harnández-Morales (2025)). These developments signal a growing fragmentation within democratic societies that threatens their ability to address pressing issues effectively. Polarization has fostered ideological entrenchment, making constructive dialogue rare and exacerbating divisions between political factions.

A notable feature of this phenomenon is the rise of extreme parties on both ends of the spectrum, left or right, which have gained traction among electorates disillusioned with traditional centrist politics. The resulting aggressive discourse often prioritizes confrontation over collaboration, leaving little room for compromise or collective problem-solving (Anderlini (2024)). Acemoglu and Robinson (2024) aptly capture this sentiment: "Democracy is in crisis around the world... These crises appear to be rooted at least in part in a growing belief that democracy has failed to deliver on its promises."

While these alarming trends have sparked widespread concern among scholars and policymakers alike, much of the existing literature attributes them solely to fake news or more general misinformation disseminated with the intent to deceive public opinion. This explanation often relies on models grounded in Bayesian updating theory, which assumes rational

voters adjust their beliefs based on new information signals. However, these models fail to account for the observed dynamics; under rational updating alone, fake news would not possess sufficient persuasive power to drive such dramatic shifts toward extremism or polarization (Sanna and Lagnado (2025), Thaler (2024), Brody (2021), Brody and Meier (2021)).

In response to this theoretical gap, alternative mechanisms have been proposed in recent studies ranging from cognitive biases to emotional manipulation to explain how fake news might influence voter behavior beyond Bayesian frameworks (Sanna and Lagnado (2025), Brody (2021)). While these approaches offer valuable insights into individual decision-making processes under misinformation conditions, they overlook a critical aspect of fake news: its impact on broader information ecosystems rather than direct persuasion.

I argue that fake news do not primarily aim to sway public beliefs toward specific positions but instead undermine the credibility of reliable information sources available to voters. By reducing trust in media outlets and jamming informational channels necessary for belief updating about current world affairs, fake news erode society's ability to build consensus around effective policies. In democratic systems where governments fail repeatedly to solve pressing problems such as economic stagnation or migration challenges (Hudson (2016)) this lack of credible signals compounds into deeper polarization.

The absence of decisive governmental success signals leaves citizens grappling with uncertainty about optimal solutions while simultaneously exposed to fragmented narratives amplified by fake news networks. As consensus building falters due to diminished informational value from media sources, voters increasingly gravitate toward extreme positions–a phenomenon observable across many democracies today.

Unlike previous studies that propose entirely new mechanisms for understanding polarization

under misinformation conditions, I take a unique approach by incorporating basic rational voter theory into an enriched information framework. The result demonstrates how increased polarization of beliefs emerges naturally when signal jamming effects disrupt belief formation processes tied closely with media credibility and governmental performance failures.

To address this dynamics, I employ a dynamic modeling approach rooted in spatial voting frameworks (Downs (1957)) and enriched with elements drawn from multi-armed bandit theory (Che and Mierendorff (2019)). By structuring elections as iterative feedback loops, the model captures how voters' beliefs about optimal policies evolve across multiple electoral cycles. These feedback loops incorporate updating of beliefs driven by governmental successes or failures, signals from media outlets, and later disruptions caused by fake news. This framework enables a structured examination of how voter perceptions interact with policy effectiveness and external influences over time.

Unlike prior studies that primarily focus on irrationality or entirely novel mechanisms to explain polarization under misinformation conditions (Gentzkow and Shapiro (2006),Sanna and Lagnado (2025), Brody (2021)), this paper integrates the rational voter theory into a complex information structure. The analysis demonstrates that heightened polarization can emerge naturally when signal jamming disrupts belief formation processes closely tied to media credibility and governmental performance. In doing so, it aligns theoretical findings with real-world observations of democracies.

By bridging gaps in existing literature with arguments within established principles such as Bayesian updating and median voter theorem concepts, I offer new insights into why democracy appears increasingly fragile amidst rising extremism worldwide and what role signal jamming through fake news plays in exacerbating these challenges.

The remainder of this paper is organized as follows: Section 2.2 reviews related literature across three major themes: multi-armed bandit theory, economic theory of voter behavior and polarization and the influence of fake news on electoral processes and information economics in the context of Bayesian updating. I highlight key differences between existing work and my approach. Section 2.3 introduces the model specifications built upon classic spatial voting theory while integrating elements from multi-armed bandit theory to analyze how voters' beliefs evolve across multiple electoral periods. Section 2.4 presents a detailed analysis of the model's implications for belief polarization dynamics, including the emergence of extreme beliefs over time. Finally, Section 2.5 concludes with a discussion of broader implications for democratic systems alongside potential extensions or modifications to the model. All proofs are relegated to the Appendix B.

2.2 Related literature

This paper relates to several strands of literature, including multi-armed bandit theory, economic models of voter behavior and polarization, fake news and information economics, and Bayesian updating under misinformation. By bridging insights from these areas, the paper provides a novel framework that integrates rational voter theory into an enriched information structure to explain polarization dynamics in democratic systems.

The dynamic modeling approach employed in this paper is closely related to multi-armed bandit theory, which explores iterative learning processes through experimentation and feedback loops. Recent advancements in rich learning models include Che and Mierendorff (2019), Fudenberg et al. (2018), and Ke and Villas-Boas (2019). These studies provide foundational insights into decision-making under uncertainty but face limitations when applied to multidimensional or continuous state spaces like the one used here. Consequently, I deviate from the multi-armed bandit literature and work with myopic agents. This concept is

widely used in the literature concerning fake news and updating (Fudenberg et al. (2018)).

Additionally, this work complements studies on media choice under Bayesian principles (Calvert (1985); Suen (2004)), which suggest that voters may optimally consume biased news sources. In contrast to studies, this paper assumes unbiased media signals to isolate the effects of fake news signal jamming without introducing additional bias into the model.

In terms of general Economic Theory of Voter Behavior and polarization, the foundational work by Downs (1957) introduces spatial voting models and the median voter theorem, both of which are central to this paper's theoretical framework. The interplay between political processes and economic outcomes is further explored by Persson and Tabellini (2002), who summarize various political decision-making models relevant for understanding how voter beliefs shape policy decisions over time. I contribute to this literature strand with the novel framework analyzed in this paper.

Glaeser et al. (2007) underline the importance of education as a crucial role in democratic systems by influencing susceptibility to misinformation. Their model highlights how education fosters civic engagement and reduces vulnerability to manipulation. This insight complements the analysis presented here regarding media credibility as an essential component of consensus-building.

Fake news has emerged as a critical factor undermining trust in reliable information sources. The seminal work by Gentzkow and Shapiro (2006) analyzes media bias as an equilibrium outcome shaped by consumer expectations about quality. Their findings provide theoretical grounding for understanding how fake news impacts belief formation through signal jamming mechanisms.

Similarly, Edmond (2013) explores the dynamics of information manipulation in the context of political regime change, offering further insight into how distorted signals influence collec-

tive decision-making. His framework highlights how misinformation not only disrupts belief formation but also shapes broader social and political outcomes by altering coordination dynamics complementing this study.

Empirical evidence from Allcott and Gentzkow (2017) highlights the prevalence of fake news during elections, particularly its role in reinforcing ideological segregation via social media networks. These observations underscore how misinformation exacerbates polarization, a phenomenon I model and analyze.

Acemoglu and Ozdaglar (2011) extend these discussions with a theoretical exploration of opinion dynamics within social networks. They examine whether social learning aggregates dispersed information or amplifies incorrect beliefs. This question is central to understanding how fake news disrupts consensus formation processes over time and complements general Bayesian frameworks.

While traditional Bayesian updating frameworks assume rational agents adjust beliefs based on new signals (Milgrom and Roberts (1986)), recent studies show that such assumptions fail to account for observed shifts toward extremism under misinformation conditions (Sanna and Lagnado (2025); Thaler (2024); Brody (2021); Brody and Meier (2021)). I extend these findings by demonstrating how fake news erodes informational channels necessary for effective belief updating rather than directly persuading voters toward specific positions which has been shown to be ineffective for purely Bayesian agents, sparking an increase in different updating mechanisms to show polarization of failed updating. In this context, self-confirming equilibria introduced by Fudenberg and Levine (1993) highlight scenarios where agents fail to fully update beliefs due to cognitive biases.

2.3 Model

This section introduces a repeated election game to analyze how voter beliefs about the state of the world and therefore their perception of the best suited policy evolves over multiple periods in a spatial voting framework.

Consider the state of the world s which determines the optimal policy and the payoff arrival rate λ which determines the difficulty of generating payoffs from the state of the world. Both the state of the world and payoff arrival rate are unknown to voters and are private information of political candidates and news outlets.

By incorporating candidates' policy platforms, economic conditions, and information signals from news outlets during electoral campaigns, this model explores how electoral outcomes are shaped by myopic agents and iterative feedback loops. The timing of events within each election period follows a structured sequence: candidates propose policies, voters cast their votes in elections, winning candidates implement their platforms, payoffs are realized, and voters update their beliefs about the state of the world and the arrival rate based on outcomes and media signals which are sent in the aftermath of the incumbent's performance realization.

My dynamic model is based on a classic one-dimensional spatial voting setup in the Downsian sense. I assume a policy space on the real line normalized to $[0, 1]$. I analyze the development of electorate beliefs over time in multiple elections and build a model of repeated elections. Each period, two competing candidates $i \in \{1, 2\}$ propose their policy platforms as tuples $p_{1t} = (a_{1t}, b_{1t})$ and $p_{2t} = (a_{2t}, b_{2t})$. In this sense, a_{ti} and b_{ti} form the lower and upper bounds of the policy space feasible to candidate i in period t. Consequently, a policy platform in this context is a subset of the entire policy space $[a_{it}, b_{it}] \subseteq [0, 1]$. These platforms are subject to full commitment, meaning that an announced platform is also to be implemented should the respective candidate win the election. In this context, I will also implement the following alternative notation for platforms using the interval's center c_{it} and distance d_{it} which will

come in handy later.

$$p_{it} = (a_{it}, b_{it}) = \left(c_{it} - \frac{d_{it}}{2}, c_{it} + \frac{d_{it}}{2} \right) \tag{2.1}$$

I assume that candidates are office motivated meaning that they receive a payoff of $r > 0$ from holding office. The expected utility of candidate i in period t is therefore given by:

$$E[u_{it}(p_{it}, p_{jt})] = r \cdot Prob(i \text{ wins against } j \text{ in period } t) \tag{2.2}$$

While office motivation might seem to be a strong assumption at first, it will later become evident that it plays a limited role in the broader picture and policy motivation or any convex combinations of these two will yield the same results. In general, candidates' motivations and beliefs do not drastically affect the results.

The electorate consists of a single representative median voter. Her utility in each period depends on underlying economic conditions, formalized by an unknown state of the world denoted by $s \in [0, 1]$. In particular, the state of the world represents an unsolved problem or economic situation the country is currently facing.

The representative median voter has an initial prior belief about the state of the world $F_0 : [0, 1] \to [0, 1]$, where $F_0(x)$ denotes the probability that the state of the world s is located to the left of x.

The assumption about a single representative voter has little to no impact on the results due to the median voter theorem. Assuming instead a unit mass of voters with heterogeneous single peaked beliefs about s on $[0, 1]$ and therefore single peaked preferences is equally feasible if all voters are affected uniformly each period by success or failure of the incumbent and elections are decided by majority vote and, in case of tie, by a fair coin toss. The voter is assumed to be myopic and therefore prioritizes immediate concerns over long-term strategic considerations.

Should a candidate with policy platform p_{it} be successful during incumbency, a payoff of Π is generated for the electorate. In addition, the voter learns the true value of s and the profit arrival rate λ in this case. Otherwise, the payoff is 0.

I assume specifically that the likelihood of profit generation during incumbency is contingent upon the distance between the chosen policy platform and the actual true state of the world. In particular, I formalize the probability of a successful policy in period t given a platform p_{it} and the true state s as follows:

$$\text{Prob}(p_{it} \text{ is successful}|s) = \int_{a_{it}}^{b_{it}} \lambda(1-(y-s)^2)\frac{1}{g(b_{it}-a_{it})}\Delta t\, dy \tag{2.3}$$

In the context of 2.3, profit generation is Poisson breakthrough distributed (Poisson distributed with a fully revealing signal as in Che and Mierendorff (2019)) with a profit arrival rate of $\lambda > 0$ within an incumbency period of length Δt. Similarly, to the state of the world, I assume the arrival rate is unknown and can take two values, either high λ_h or low λ_l. This factor captures the difficulty of dealing with the state of the world. The lower λ the more difficult is profit generation. The voter's prior belief in period t about $\lambda = \lambda_h$ is denoted as q_t. I assume that: $1 > \lambda_h > \lambda_l > 0$.

The probability of government success then consequently depends on the quadratic distance between the true state of the world s and the policy platform as well as λ: $\lambda(1-(y-s)^2)$. The closer the policy platform is to the state of the world, the larger is the success probability. This is, however, weighted by $\frac{1}{g(b_{it}-a_{it})}$ capturing a negative impact of large policy spaces on the success probability, with $g(\cdot)$ being a continuous function.

As for the choice of quadratic distances as metric, it should be noted that generally a variety of metrics can be used here. Most commonly used in the literature is the absolute value. However, this mainly introduces a great variety of case distinctions which need to be analyzed separately at the benefit of sharper probability decline around the state of the

world. Since this does not offer additional insights, I assume quadratic distance to produce continuous and trackable results allowing smoother changes in probability as policies move closer to or farther from the true state.

The provided representation overall aligns with real-world scenarios where there is often a optimal solution to various issues, be they economic, social, or environmental. The further away your proposals are from this optimal solution, the less likely is actual success.
In democratic systems, in particular, competing political parties strive to attract voters by proposing policies that resonate with their constituents' needs and preferences. The closer a party's proposed platform aligns with the true best response to the problems represented by s the more likely it is to garner support from the population. Voters generally favor policies they believe will effectively address their concerns and enhance their well-being. Thus, if one party presents a platform that closely matches the believed optimal policy, it stands a better chance of being perceived as credible and effective. At the same time, however, trying to cover the entire policy space is not feasible since trying to implement multiple different solutions at the same time will sabotage ultimately the positive results since resources are usually limited.

As already mentioned in cases where an incumbent successfully generates profits, voters gain access to information revealing the true state s and the arrival rate λ. This is a simplifying assumption which increases the conclusiveness of a successful policy. It is, however, not unrealistic to assume that a successful policy allows everybody to learn how it worked and therefore reveal all information.
Conversely, if the incumbent fails to generate profits, the voter updates her belief from her prior $(F_t(x), q_t)$ at the beginning of period t to a posterior belief $(F_{t+\frac{1}{2}}(x), q_{t+\frac{1}{2}})$. In this context, the notation: t to $t+\frac{1}{2}$ keeps apart the payoff signal from the government stage and the media signal from the electoral campaign ($t+\frac{1}{2}$ to $t+1$). Following the govern-

ment stage, electoral campaigning begins. This then leads to new policy proposals at the beginning of the next period and a new election. During electoral campaigning, after payoffs were realized, I introduce a news outlet or media signal $n \in \{0, 1\}$. News outlets will report that the policy platform of the incumbent was ill advised ($n = 1$) or abstain from any harsh critique ($n = 0$). The probability of these signals is dependent on the implemented policy platform p_{it} and the true state of the world s similarly to profit generation, weighted by a constant $k \in \mathbb{R}^+$. As news outlets know the state of the world and the profit arrival rate λ, it holds:

$$\begin{aligned} \text{Prob}(n = 1|s) &= \begin{cases} 1, & \text{if } k(c_{it} - s)^2 > 1 \\ k(c_{it} - s)^2, & \text{otherwise} \end{cases} \\ \text{Prob}(n = 0|s) &= \begin{cases} 0, & \text{if } k(c_{it} - s)^2 > 1 \\ 1 - k(c_{it} - s)^2, & \text{otherwise} \end{cases} \end{aligned} \tag{2.4}$$

Based on this signal the voter can update her belief from $(F_{t+\frac{1}{2}}(x), q_{t+\frac{1}{2}})$ to $(F_{t+1}(x), q_{t+1})$ forming a new prior for the next period's election. The media signal will mainly help to disentangle potential government failure due to bad policy ($(c_{it} - s)^2$ large) from failure due to a hard problem ($\lambda = \lambda_l$).

Note that assuming truthful reveal under success and the concentration of additional news on the policy center without additional noise by the policy size simply clears up these signals. These assumptions are not strong in the dynamic framework of this model, as they only determine the magnitude of the impact on the beliefs caused by the signals.

In Section 2.4.3, I relax this assumption and introduce news signals that are no longer conclusive, exploring the resulting implications.

In general, the inclusion of news outlets as an information source in this spatial setup is highly realistic. News media plays a crucial role in modern democracies especially during

electoral campaigns by shaping voter perceptions, providing additional signals about policy effectiveness, and acting as watchdogs over incumbents. Their ability to critique policies complements voters' direct observations, especially when outcomes are ambiguous or failures occur.

Furthermore, it is particularly plausible that media coverage criticizes policies based on their distance from the true state s. Policies far from s are more likely to be critiqued due to their evident inadequacy. The salience of problems amplifies this dynamic, as poorly aligned policies tend to attract attention by failing to address observable issues. In addition, there is an alignment between public interest and media incentives thus, highlighting inadequacies in policies distant from s reinforces their role as watchdogs holding incumbents accountable. In summary, incorporating news signals into this spatial model enhances its realism by capturing key dynamics such as problem salience and public interest alignment. These factors collectively explain why news outlets are more likely to criticize ill-advised policy choices when they diverge significantly from the true state of the world.

Given a prior $(F_t(x), q_t)$ at the beginning of period t based on past observations during periods $0, 1, ..., t-1$, the representative voter maximizes her expected utility. She therefore prefers candidate i's platform over candidate j's platform if the following condition holds:

$$\Pi \cdot \text{Prob}(p_{it} \text{ is successful}|s) > \Pi \cdot \text{Prob}(p_{jt} \text{ is successful}|s) \tag{2.5}$$

with Π being the payoff from problem solving. Importantly, however, the representative voter is assumed to be myopic and does not account for future elections when making decisions. This contrasts sharply with frameworks such as multi-armed bandit theory, where actions are viewed as costly experiments designed to optimize long-term outcomes. In this model, elections are framed as active choices aimed at maximizing problem solving within the cur-

rent period. Incorporating future choices and possibly experimenting with different policies ties directly into Multi-armed Bandit Theory which has made however limited progress in multidimensional models. Continuous models like the one presented here are not tractable and beyond the current state of the literature (Che and Mierendorff (2019)).

Nevertheless, this assumption is reasonable as voters often prioritize immediate concerns over long-term strategic considerations due to limited information. Additionally, in current democratic systems, it is extremely unusual to forecast future period electoral dynamics or policy impacts as basis for current voting decisions.

Second, elections inherently emphasize short-term accountability; candidates campaign on promises that address pressing issues rather than distant uncertainties. This focus aligns with voter behavior that seeks tangible improvements in their immediate circumstances.

Third, myopic expectations reflect real-world voting behavior shaped by psychological and informational limitations. Voters may lack access to comprehensive data or expertise necessary to evaluate platforms beyond their immediate utility, reinforcing the plausibility of this assumption (Fair (1978), Kramer (1971), William (1975)).

Summarizing the structure of the model, its timing in period t can be outlined as follows:

Step 1: Candidates 1 and 2 announce policy platforms p_{1t} and p_{2t}.

Step 2: An election takes place decided by the representative voter. If the representative voter is indifferent, a fair coin toss determines the outcome.

Step 3: The winning candidate implements her proposed policy for the government stage.

Step 4: At the end of period t's government stage payoffs are realized.

Step 5: The representative voter updates her beliefs $(F_{t+\frac{1}{2}}(x), q_{t+\frac{1}{2}})$ about s and λ depending on the payoff (either Π or 0).

Step 6: News outlets signal either $n = 1$ or $n = 0$ and the voter updates her beliefs again taking the media signal into account $(F_{t+1}(x), q_{t+1})$. The game then proceeds to period $t + 1$.

The game starts in period $t = 0$ with the representative voter's initial prior $(F_0(x), q_0)$. News outlets and candidates are perfectly informed about the state of the world and the profit arrival rate λ.

This iterative process creates a feedback loop across multiple periods whereby each round influences subsequent elections through evolving voter beliefs which I will analyze in the following section.

2.4 Analysis

2.4.1 Policy Platforms

Building on the dynamic model outlined in the previous chapter, I first determine the representative voter's preferred policy platform.

Corollary 2.4.1.

Given a prior belief $F_t(x)$ at the beginning of period t, the representative voter's preferences are single peaked and she prefers the following policy platform $p_t^* = (a_t^*, b_t^*)$ to be implemented:

$$p_t^* = \left(c_t^* - \frac{d_t^*}{2}, c_t^* + \frac{d_t^*}{2}\right) \qquad \textit{with } c_t^* = E_t(s)$$

$$d_t^* \textit{ solves: } \frac{1 - \frac{d_t^2}{4} - Var_t(s)}{d_t\left(1 - \frac{d_t^2}{12} - Var_t(s)\right)} \frac{g(d_t)}{g'(d_t)} = 1$$

Where, $E_t(s)$ is the expected value of the state of the world s based on the belief $F_t(x)$ and $Var_t(s)$ is the variance of the state of the world s based on the belief $F_t(x)$.

Intuitively, voters prefer policy platforms centered around their expectation of the state of the world. Since the success probability of a candidate's policy depends on how closely their platform aligns with the true state s and s is unknown to voters, centering the platform around its expected value ensures targeting the most likely realization of s based on current beliefs and maximizing success probability.

The optimal platform range d_t is however highly dependent on the impact of $g(d_t)$. While broader platforms may attempt to cover larger portions of the policy space in hopes of appealing to diverse voters or addressing multiple issues simultaneously, this approach reduces success probabilities. This means strictly increasing functions like $g(x) = x$ or $g(x) = x^2$ encourage highly centralized platforms with $d_t = 0$ and focused solutions when designing policy platforms. Since the representative voter is hinged at one dimensional policy spaces, I assume $g(x) = x$ in all examples. Note, however, that functions like $g(x) = kx^4 - x^2 + 1$ encourage broader policy spaces[1].

For $g(x) = x$ the representative voter dictates the proposed policy for office motivated candidates meaning that both candidates will propose and, in the end, implement the representative voters preferred policy $p_t = (E_t(s), E_t(s))$ in period t. Note that this result is also independent of candidates knowledge advantage about the realization of s.

This reasoning mirrors real-world political strategies. Candidates often propose policies aimed at addressing prevalent issues based on public perception—essentially reflecting collective expectations about what solutions are most needed or feasible. For example, consider the implementation of protectionist trade policies based on the widespread but incorrect belief that outsourcing and free trade are the primary causes of domestic job losses. While these policies might resonate with voters who feel economically disenfranchised, they can lead to unintended consequences such as higher consumer prices, retaliatory tariffs from trading partners, and reduced economic growth. In this case, the policy is shaped more by

[1]Broader policy spaces do not necessarily support the median voter theorem which in turn means that a single representative voter might be insufficient to capture all dynamics. For linear $g(x)$, however, voter preferences are single peaked.

catering to misconceptions than by addressing the actual root causes of unemployment, such as automation or shifts in industry demand.

2.4.2 Beliefs

Having established the optimal policy platforms that candidates propose based on voter's beliefs, I now turn attention to how these beliefs evolve over time. This subsection analyzes the dynamic process through which voters update their prior beliefs about the true state s as elections unfold across multiple periods. These updates are influenced by both electoral outcomes and external signals, such as media coverage critiquing policy effectiveness. Especially media signals are crucial for disentangling bad policies from λ (bad times).

I first analyze how voter beliefs evolve solely through the government success or failure channel, setting aside the influence of news signals for now. This allows to isolate the direct impact of electoral outcomes and policy effectiveness on voter perceptions. Once this foundation is established, I will extend the analysis by incorporating the role of media coverage and its implications for belief updates. While government success signals the true state of the world s, beliefs will evolve according to the following proposition and lemma in the absence of breakthrough payoff.

Lemma 2.4.2.

Voter's belief is a martingale under Bayesian updating.

Proposition 2.4.3.

In the absence of breakthrough payoff under policy p_t, the voter's belief about the state of the world s, F_t will evolve after government's success or failure according to: $F_{t+\frac{1}{2}}(x) =$

$F_t(x) + \dot{F}_t(x)\Delta t$ *with:*

$$\dot{F}_t(x) = \frac{\lambda^e \, d_t}{g(d_t)} \left(\int_0^x (c_t - y)^2 f_t(y) \, dy - F_t(x) \int_0^1 (c_t - y)^2 f_t(y) \, dy \right)$$

with $\lambda^e = q_t\lambda_h + (1 - q_t)\lambda_l$.

Having determined the change in belief for the state of the world s by analyzing how its cumulative distribution function (CDF) evolves, we can now proceed to compute the corresponding probability density function (PDF). Since the PDF is the derivative of the CDF with respect to the random variable, it follows from Proposition 2.4.3 that:

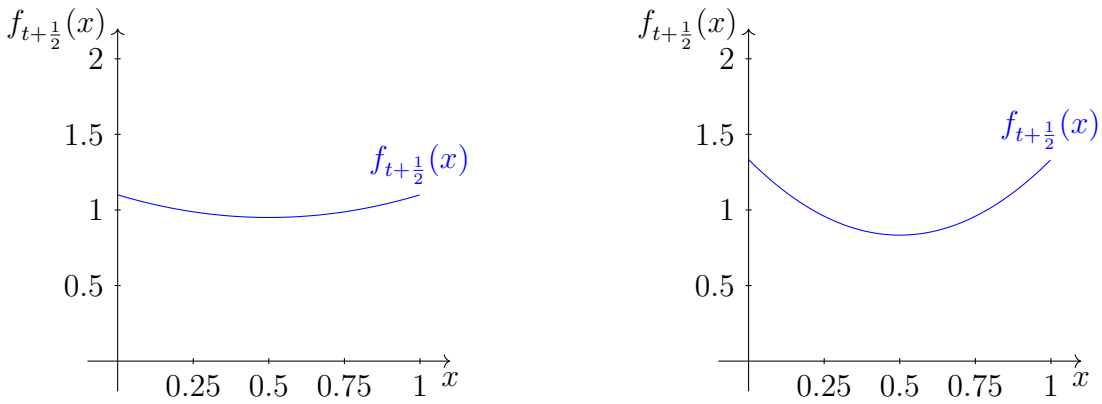
$$\dot{f}_t(x) = \frac{f_t(x)\lambda^e \, d_t}{g(d_t)} \left((c_t - x)^2 - \int_0^1 (c_t - y)^2 f_t(y) \, dy \right) \tag{2.6}$$

This expression represents how the voter's belief about the relative likelihood of $x \in [0, 1]$ being the true state of the world s has changed. In this context, $\dot{f}_t(x)$ in (2.6) measures the average squared deviation from c_t relative to the individual quadratic deviation of x from c_t. This means that the PDF will decrease in points close to the center of the policy platform c_t. The margin of decrease is higher the closer a point lies to the center. While the relative likelihood of points which are further away will increase in turn. For these points it holds that $\pm x \geq \sqrt{\int_0^1 (c_t - y)^2 f_t(y) \, dy} \pm c_t$. The rate of increase and decrease is weighted by the prior $f_t(x)$ and the arrival rate λ as well as the covered space of the policy d_t. Taking Corollary 2.4.1 into account, as well as $g(x) = x$, this simplifies to:

$$\dot{f}_t(x) = f_t(x)\lambda^e \left((E_t(s) - x)^2 - Var_t(s) \right) \tag{2.7}$$

Example:

Assume the prior $f_t(x)$ follows a Uniform distribution on $[0, 1]$. The voter's beliefs for this case are depicted in the figures below.



(a) $\lambda = 0.3$ (b) $\lambda = 1$

Figure 2.1: Effect of Government failure on voter's beliefs

This implies that the voter's belief shifts away from unsuccessful policies, which, in turn, also depends on her belief about the arrival rate, namely, difficulty of the problem λ. In addition to the updating of beliefs about the state of the world s presented in Proposition 2.4.3, the voter's belief about the Payoff arrival rate will develop as follows:

Corollary 2.4.4.

In the absence of breakthrough payoff under policy p_t, the voter's belief q_t about the arrival rate λ after the government stage will evolve as follows:

$$q_{t+\frac{1}{2}} = \frac{(1-\lambda_h l_t(1))q_t}{(1-\lambda_h l_t(1))q_t + (1-\lambda_l l_t(1))(1-q_t)}$$

with $l_t(x) = \frac{d_t}{g(d_t)}\left(F_t(x) + \frac{b_t a_t - c_t^2}{3} - \int_0^1 (c_t - y)^2 f_t(y)\, dy\right)$.

It is immediately clear that in the absence of breakthrough payoff, the likelihood of $\lambda_t = \lambda_h$ decreases and therefore the voter concludes that the problem is likely to be hard to solve. As a result, $q_{t+\frac{1}{2}} < q_t$ from upwards estimation of Corolarry 2.4.4 due to $q_t \leq 1$, $l_t(1) \leq 1$ and $1 \geq \lambda_h > \lambda_l$.

I now turn to the impact of news sources during electoral campaigns in preparation for

the next period's election at the end of period t. News sources evaluate the incumbency and shape the voter's belief by either being critical ($n = 1$) or not ($n = 0$), as formalized in (2.4). The following Corollary represents the development of voter's beliefs in light of the news signal during electoral campaigns:

Corollary 2.4.5.

In the presence of news critique ($n = 1$) concerning the incumbents policy platform p_{it}, the voter's belief $f_{t+\frac{1}{2}}$ about the state of the world will evolve as follows:

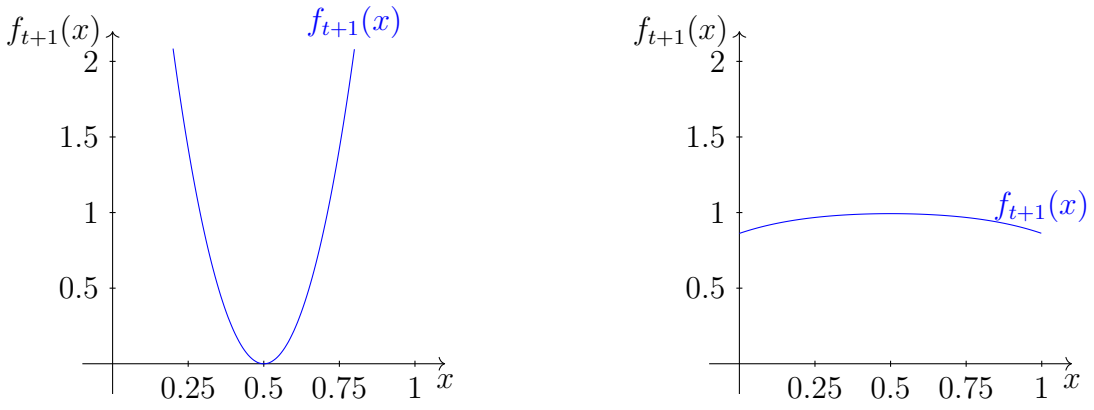
$$f_{t+1}(x) = \frac{(c_{it} - x)^2 f_{t+\frac{1}{2}}(x)}{\int_0^1 (c_{it} - y)^2 f_{t+\frac{1}{2}}(y)\, dy}$$

and in the absence of news critique ($n = 0$):

$$f_{t+1}(x) = \frac{(1 - k(c_{it} - x)^2) f_{t+\frac{1}{2}}(x)}{1 - k\int_0^1 (c_{it} - y)^2 f_{t+\frac{1}{2}}(y)\, dy}$$

It follows that in the presence of news critique concerning the incumbent's policy platform, the voter's belief about the mismatch of the policy platform is amplified. Not only did the government with this policy fail to resolve the problem and generate profits but also draw critique from informed source. As a result, the impact of the unknown arrival rate and therefore difficulty to solve the problem on the voter's beliefs about the state of the world is reduced.

In contrast, the absence of critique will ensure the voter that the government's recent failure is not so much due to an ill-advised policy platform but due to the difficulty of the problem at hand. Hence, her belief will converge towards the platform, reducing average quadratic deviation from c_{it}. Revisiting the previous example 2.4.2 from earlier for $\lambda = 0.3$, the absence and presence of the news signal lead to the following shift in the voter's belief:



(a) Presence of news

(b) Absence of news

Figure 2.2: Effect of news on the voter's beliefs

In this context, I focus on the game in which the government fails to solve the problem for some periods and in which then failure and news sources determine the development of the voter's beliefs, as otherwise the true state s is revealed.

Assume that the problem at hand appears during an incumbency with a left-wing policy $c_0 = \frac{1}{16}$ and the initial belief is uniformly distributed. The government however fails to solve the problem in the same period it appeared in, which shifts the voter's belief to the right and discourages a left policy. An example for this would be the breakout of the Russo-Ukrainian War which coincided with a left coalition in Germany. Alternatively one can assume the initial prior belief about s is strongly slanted towards a right belief, e.g.: $f_0(x) \approx \lambda^e(\frac{25}{35} + (1/16 - x)^2)$. Similarly results can be generated for initial uniform beliefs and $c_0 = \frac{1}{2}$ or any other arbitrary prior. To illustrate more of the dynamics, I will therefore start with a non-uniform prior belief.

Belief updating under government failure and news in the case described above results in the following beliefs, with blue depicting the belief at $t = 0$ and other colors depicting the belief in subsequent periods:

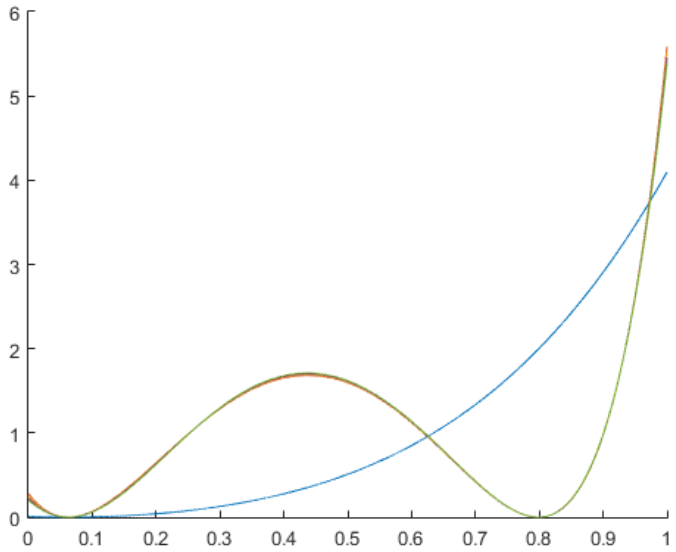


Figure 2.3: Effect of news on the voter's belief in the absence of breakthrough payoff.

As figure 2.3 shows, the news channel has a consolidating effect on the voter's belief leading to a fairly centered distribution with an outlier to the right due to the initial strongly left oriented policy platform.

The corresponding policy platforms, news signals and government success probabilities for $\lambda = \lambda_l = 0.1$, $s = 0.6$, $k = 10$ are given by:

Period 0: $c_0 = 0.06250$; $n_0 = 1$; Prob(p_0 is successful$|s$) $= 0.071$;
Prob($n = 1|s$) $= 1$

Period 1: $c_1 = 0.78378$; $n_1 = 1$; Prob(p_1 is successful$|s$) $= 0.097$;
Prob($n = 1|s$) $= 0.34$

Period 2: $c_2 = 0.59990$; $n_2 = 0$; Prob(p_2 is successful$|s$) $= 0.1$;
Prob($n = 1|s$) $= 0.0001$

Period 3: $c_3 = 0.59773$; $n_3 = 0$; Prob(p_2 is successful$|s$) $= 0.1$;
Prob($n = 1|s$) $= 0.0001$

Period 4: $c_4 = 0.59522$; $n_4 = 0$; Prob(p_4 is successful$|s$) $= 0.1$;

Prob$(n = 1|s) = 0.0001$

I show in the following how Fake News affect the news media signal and consequently the belief formation of the representative voter over time.

2.4.3 Fake News

While the broader literature predominantly focuses on the persuasive aspects of fake news (Sanna and Lagnado (2025), Thaler (2024), Brody (2021), Brody and Meier (2021)), it concludes that under Bayesian updating and rational receivers, its impact remains highly limited. However, an important aspect that has been largely overlooked is how the presence of fake news renders affected signal channels inconclusive. This, in turn, influences the ability of the voter to draw useful information from the media signal.

To address this aspect, I introduce fake critique. Think about fake news and their target. They usually aim at mentioning the inability or unwillingness of the government to solve specific problems like, for example, potential migration issues (Scott and Cerulus (2019)). Given this fake critique, when observing the news channel during the electoral campaign the voter needs to account that it might be fake news. Assume the fake signal arrival rate is given by $1 > \gamma > 0$. It follows:

Corollary 2.4.6.

Given a fake news arrival rate of $1 > \gamma > 0$*, the voter's belief about the state of the world evolves as follows:*

$$f_{t+1}(x) = (1-\gamma)\frac{(c_{it}-x)^2 f_{t+\frac{1}{2}}(x)}{\int_0^1 (c_{it}-y)^2 f_{t+\frac{1}{2}}(y)\,dy} + \gamma\frac{(1-k(c_{it}-x)^2) f_{t+\frac{1}{2}}(x)}{1-k\int_0^1 (c_{it}-y)^2 f_{t+\frac{1}{2}}(y)\,dy}$$

As a result, the impact of the news channel diminishes and even if the likelihood of the news critique being false is really high, the centralizing effect of news disappears. The following figure illustrates this argument:

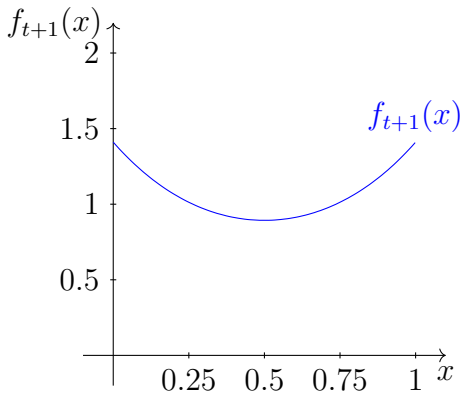


Figure 2.4: Effect of false news on the voter's beliefs ($\gamma = 0.9$).

In conclusion, the presence of fake news even if anticipated significantly undermines the ability of media to consolidate public beliefs. As a result, beliefs diverge towards extreme positions when the government fails to address the underlying issues. However, extreme platforms do not manifest in equilibrium due to the single representative voter and under a unit mass of voters due to the implications of the median voter theorem, which influences electoral outcomes. Consequently, even when the true state of the world is known to the government, their capacity to enhance the likelihood of successful governance is diminished. Revisiting the previous example corresponding to Figure (2.3) and using numerical calculations to illustrates this dynamic shows:

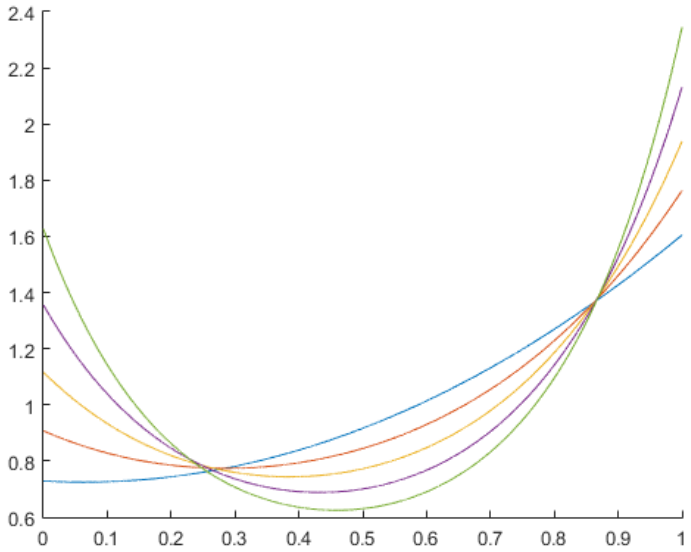


Figure 2.5: Effect of fake news on the voter's beliefs in the absence of breakthrough payoff.

Figure 2.5 illustrates how fake news influences voter beliefs over time, particularly when governments fail to match the state of the world. As noted in Corollary 2.4.6, fake news undermine the effectiveness of media signals, leading to a divergence in voter beliefs rather than consolidation around a central position.

However, the policy platforms centralize while beliefs diverge:

Period 0: $c_0 = 0.0625$

Period 1: $c_1 = 0,5730$

Period 2: $c_2 = 0,5653$

Period 3: $c_3 = 0,5575$

Period 4: $c_4 = 0,5498$

This allows some central observations: with an increasing rate of fake news γ, voters are less inclined to trust media critiques as reliable indicators of policy inadequacy. This results in their beliefs becoming more polarized within the policy space $[0, 1]$. Instead of converging

toward an optimal or informed understanding of the true state s, voters remain uncertain. This means that under fake news and government failure voters become increasingly polarized which reflects accurately current real-world dynamics where misinformation fosters ideological entrenchment rather than constructive discourse (Christian Edwards and Schmidt (2024), Sahlberg and Lunday (2025), Politico (2022), Momtaz (2020), Samaras (2025), Colelaere and Harnández-Morales (2025)).

However, polarization of beliefs is not the only consequence. In addition, the representative voter enforces policy platforms which might be more insufficient to solve current problems than optimal once. This happens even though candidates and news are fully informed about the state of the world as they are forced to cater towards the voter to win the election and converge towards his expectations.

2.5 Conclusion

This paper has examined the dynamics of polarization of beliefs in democratic systems through the lens of rational voter theory, enriched by a complex information framework. It demonstrates that political failures and fake news interact to disrupt belief formation processes, leading to natural belief polarization under the myopic voter assumption. By modeling elections as iterative feedback loops within a spatial voting framework and incorporating elements from multi-armed bandit theory, this study captures how voters' beliefs evolve under conditions of governmental success or failure and media credibility disruptions.

The central finding is that fake news in this setting undermine the credibility of reliable informational channels necessary for belief updating. This signal jamming effect erodes trust in media sources, amplifies uncertainty about policy effectiveness, and drives ideological entrenchment among voters. In environments where governments fail repeatedly to

address pressing problems such as economic stagnation, migration crises or other challenges, these effects mix, leaving societies fragmented and polarized. While electoral outcomes remain constrained by the voter's expectations, resulting in centralized policy platforms, the divergence in public beliefs highlights a growing problem within democratic systems.

From an economic perspective, these results underscore the importance of credible information flows for consensus-building processes essential to democracies. The erosion of trust in media due to fake news weakens the ability of democratic systems to identify optimal policies and implement solutions efficiently. Furthermore, the interaction between government performance failures and signal jamming effects foster ideological divisions that hinder cooperative problem-solving.

This study highlights that restoring trust in reliable information sources is critical for mitigating belief polarization driven by misinformation. Governments should also prioritize delivering clear success signals through effective policymaking on urgent issues to reduce uncertainty among voters. These interventions could help stabilize democratic systems by addressing structural vulnerabilities revealed through this analysis.

Several extensions could provide additional insights into the dynamics of polarization and voter behavior. Introducing additional candidates' incentives to the model might allow for strategic candidate decisions that favor extreme political positions, thereby relaxing the constraints imposed by the representative voter. Analyzing voter turnout within this setup could offer valuable perspectives on how participation rates interact with polarization and electoral outcomes. Modifying the news mechanism to incorporate noisy signals, as suggested by (Gentzkow and Shapiro (2006); Brody (2021); Brody and Meier (2021)), would align more closely with real-world information environments and deepen the understanding of misinformation effects on belief updating. Finally, relaxing the assumption of a single rep-

resentative voter and introducing multiple heterogeneous voters with individual beliefs and distinct news channels would enable a richer analysis of collective decision-making processes in this setup further enhancing the understanding of polarization dynamics while broadening the applicability of this framework.

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B

Appendix

B.1 Proofs

Corollary 2.4.1.
Given a prior belief $F_t(x)$ at the beginning of period t, the representative voter's preferences are single peaked and she prefers the following policy platform $p_t^* = (a_t^*, b_t^*)$ to be implemented:

$$p_t^* = \left(c_t^* - \frac{d_t^*}{2}, c_t^* + \frac{d_t^*}{2}\right) \qquad \textit{with } c_t^* = E_t(s)$$

$$d_t^* \textit{ solves: } \frac{1 - \frac{d_t^2}{4} - Var_t(s)}{d_t\left(1 - \frac{d_t^2}{12} - Var_t(s)\right)} \frac{g(d_t)}{g'(d_t)} = 1$$

Where, $E_t(s)$ is the expected value of the state of the world s based on the belief $F_t(x)$ and $Var_t(s)$ is the variance of the state of the world s based on the belief $F_t(x)$.

Proof.
Using (2.3), under any arbitrary implemented policy $p_t = (a_t, b_t)$ in period t and beliefs $F_t(x)$, the voter's expected utility is given by:

$$E[v_t(p_t)] = \Pi \int_{a_t}^{b_t} \int_{0}^{1} \lambda^e (1 - (x-y)^2) \frac{1}{g(b_t - a_t)} f_t(y) \Delta t \, dy \, dx \tag{B.1}$$

Rearranging yields:

$$\begin{aligned} E[v_t(p_t)] =& \frac{\Pi \lambda^e \Delta t}{g(b_t - a_t)} \left(\int_{a_t}^{b_t} \int_{0}^{1} f_t(y) \, dy \, dx - \int_{a_t}^{b_t} \int_{0}^{1} (x-y)^2 f_t(y) \, dy \, dx \right) \\ =& \frac{\Pi \lambda^e \Delta t}{g(b_t - a_t)} \left((b_t - a_t) - \int_{a_t}^{b_t} \int_{0}^{1} (x-y)^2 f_t(y) \, dy \, dx \right) \end{aligned}$$

$$
\begin{aligned}
&\overset{\text{Fubini}}{=}\frac{\Pi\lambda^e\Delta t}{g(b_t-a_t)}\left((b_t-a_t)-\int_0^1\left(\frac{b_t^3-a_t^3}{3}-(b_t^2-a_t^2)y+y^2(b_t-a_t)\right)f_t(y)\,dy\right)\\
&=\frac{\Pi\lambda^e\Delta t(b_t-a_t)}{g(b_t-a_t)}\left(1-\frac{b_t^3-a_t^3}{3(b_t-a_t)}+\int_0^1\left((b_t+a_t)y-y^2\right)f_t(y)\,dy\right)\\
&=\frac{\Pi\lambda^e\Delta t\,d_t}{g(d_t)}\left(1-\left(c_t^2+\frac{d_t^2}{12}\right)+2c_tE_t(s)-\int_0^1 y^2f_t(y)\,dy\right)
\end{aligned}
$$

where $E_t(s)$ is the expected value of the state of the world s based on the belief $F_t(x)$. The first-order condition with respect to c_t then yields:

$$
\begin{aligned}
&\frac{\partial E[v_t(p_t)]}{\partial c_t}\overset{!}{=}0\\
\Leftrightarrow\ &\frac{\Pi\lambda^e\Delta t\,d_t}{g(d_t)}\left(2E_t(s)-2c_t\right)\overset{!}{=}0\\
\Rightarrow\ &c_t^*=E_t(s)
\end{aligned}
$$

Similarly for the optimal distance d_t^* it holds at $c_t^*=E_t(s)$:

$$
\begin{aligned}
E[v_t(p_t)]=&\frac{\Pi\lambda^e\Delta t\,d_t}{g(d_t)}\left(1-\frac{d_t^2}{12}+(E_t(s))^2-\int_0^1 y^2f_t(y)\,dy\right)\\
=&\frac{\Pi\lambda^e\Delta t\,d_t}{g(d_t)}\left(1-\frac{d_t^2}{12}-\mathrm{Var}_t(s)\right)\\
\Rightarrow&\frac{\partial}{\partial d_t}\left(\frac{d_t-\frac{d_t^3}{12}-d_t\mathrm{Var}_t(s)}{g(d_t)}\right)\overset{!}{=}0\\
\Leftrightarrow&\left(1-\frac{d_t^2}{4}-\mathrm{Var}_t(s)\right)g(d_t)-\left(d_t-\frac{d_t^3}{12}-d_t\mathrm{Var}_t(s)\right)g'(d_t)=0
\end{aligned}
$$

where $\mathrm{Var}_t(s)$ is the variance of the state of the world s based on the belief $F_t(x)$.
Note that the first derivative might be decreasing. In these cases, it holds for the optimal distance: $d_t=0$. For $g(d_t)=d_t$ it holds:

$$
\begin{aligned}
&\frac{\partial}{\partial d_t}\left(\frac{d_t-\frac{d_t^3}{12}-d_t\mathrm{Var}_t(s)}{g(d_t)}\right)<0\\
\Leftrightarrow\ &\left(1-\frac{d_t^2}{4}-\mathrm{Var}_t(s)\right)g(d_t)<\left(d_t-\frac{d_t^3}{12}-d_t\mathrm{Var}_t(s)\right)g'(d_t)
\end{aligned}
$$

$$\Leftrightarrow \left(1 - \frac{d_t^2}{4} - \mathrm{Var}_t(s)\right) d_t < \left(d_t - \frac{d_t^3}{12} - d_t \mathrm{Var}_t(s)\right)$$
$$\Leftrightarrow \frac{1}{12} < \frac{1}{4}$$

Hence for $g(d_t) = d_t$ it holds that $\frac{\partial E[v_t(p_t)]}{\partial d_t} < 0$.
Also, for the second derivative it holds:

$$\frac{\partial^2 E[v_t(p_t)]}{\partial c_t^2} = \frac{\partial}{\partial c_t} \Pi \lambda^e \Delta t (2E_t(s) - 2c_t) = -2$$

Therefore, the second-order conditions is satisfied.

□

Lemma 2.4.2.
Voter's belief is a martingale under Bayesian updating.

Proof.
Let $F_t(x) = P(s \leq x | h_t)$ for the random history h_t and p_t denote the policy platform in t. In addition, denote: $P(p_t \text{ is successful}) \equiv P(\xi = 1)$ and $P(p_t \text{ fails}) \equiv P(\xi = 0)$. Then it holds:

$$\begin{aligned}
E[F_{t+1}(x)|h_t] &= P(\xi = 1)P(s \leq x | h_t, \xi = 1) + P(\xi = 0)P(s \leq x | h_t, \xi = 0) \\
&= P(\xi = 1)\frac{P(\xi = 1 | h_t, s \leq x)P(s \leq x | h_t)}{P(\xi = 1)} + P(\xi = 0)\frac{P(\xi = 0 | h_t, s \leq x)P(s \leq x | h_t)}{P(\xi = 0)} \\
&= P(\xi = 1 | h_t, s \leq x)P(s \leq x | h_t) + P(\xi = 0 | h_t, s \leq x)P(s \leq x | h_t) \\
&= P(s \leq x | h_t) \\
&= F_t(x)
\end{aligned}$$

Hence the martingale property is fulfilled.

□

Proposition 2.4.3.
In the absence of breakthrough payoff under policy p_t, the voter's belief about the state of the world s, F_t will evolve after government's success or failure according to: $F_{t+\frac{1}{2}}(x) = F_t(x) + \dot{F}_t(x)\Delta t$ with:

$$\dot{F}_t(x) = \frac{\lambda^e \, d_t}{g(d_t)} \left(\int_0^x (c_t - y)^2 f_t(y) \, dy - F_t(x) \int_0^1 (c_t - y)^2 f_t(y) \, dy \right)$$

with $\lambda^e = q_t\lambda_h + (1-q_t)\lambda_l$.

Proof.
Belief updating is a martingale. Hence using martingale properties yields:

$$F_t(x) = h_t(x)\Delta t + \left[F_t(x) + \dot{F}_t(x)\Delta t\right](1 - h_t(1)\Delta t) \tag{B.2}$$

with $h(x)$ being the probability to receive breakthrough payoff under the true state of the world $s \in [0, x]$ during an incumbency period t of length Δt. It follows:

$$h_t(x) = \int_{a_t}^{b_t}\int_{0}^{x} \lambda^e(1-(z-y)^2) f_t(y) \frac{1}{g(b_t - a_t)}\, dy\, dz$$

Analogously to the proof of Corollary 2.4.1:

$$h_t(x) = \frac{\lambda^e}{g(b_t-a_t)}\left(F_t(x)(b_t-a_t) - \int_0^x \left(\frac{b_t^3-a_t^3}{3} - (b_t^2-a_t^2)y + y^2(b_t-a_t)\right) f_t(y)\, dy\right) \tag{B.3}$$

Rearranging (B.2) yields:

$$\begin{aligned}
&F_t(x) = h(x)\Delta t + \left[F_t(x) + \dot{F}_t(x)\Delta t\right](1 - h_t(1)\Delta t)\\
\Leftrightarrow\ &F_t(x) = h_t(x)\Delta t + F_t(x) + \dot{F}_t(x)\Delta t - F_t(x)h_t(1)\Delta t - \dot{F}_t(x)\Delta t\, h_t(1)\Delta t\\
\Leftrightarrow\ &\dot{F}_t(x)\Delta t = F_t(x) - h_t(x)\Delta t - F_t(x) + F_t(x)h_t(1)\Delta t + \dot{F}_t(x)\Delta t\, h_t(1)\Delta t\\
\Leftrightarrow\ &\dot{F}_t(x)\Delta t = F_t(x)h_t(1)\Delta t - h_t(x)\Delta t + \dot{F}_t(x)\Delta t\, h_t(1)\Delta t\\
\Leftrightarrow\ &\dot{F}_t(x) = F_t(x)h_t(1) - h_t(x) + \dot{F}_t(x)\, h_t(1)\Delta t\\
\underset{\Delta t\to 0}{\Rightarrow}\ &\dot{F}_t(x) = F_t(x)h_t(1) - h_t(x)
\end{aligned} \tag{B.4}$$

Plugging (B.3) into (B.4) yields:

$$\begin{aligned}
\dot{F}_t(x) =& F_t(x)\frac{\lambda^e}{g(b_t-a_t)}\left((b_t-a_t) - \int_0^1 \left(\frac{b_t^3-a_t^3}{3} - (b_t^2-a_t^2)y + y^2(b_t-a_t)\right) f_t(y)\, dy\right)\\
&- \frac{\lambda^e}{g(b_t-a_t)}\left(F_t(x)(b_t-a_t) - \int_0^x \left(\frac{b_t^3-a_t^3}{3} - (b_t^2-a_t^2)y + y^2(b_t-a_t)\right) f_t(y)\, dy\right)\\
=& F_t(x)\frac{\lambda^e}{g(b_t-a_t)}\left((b_t-a_t) - \frac{b_t^3-a_t^3}{3} - \int_0^1 \left((b_t^2-a_t^2)y - y^2(b_t-a_t)\right) f_t(y)\, dy\right)
\end{aligned}$$

$$
\begin{aligned}
&- \frac{\lambda^e}{g(b_t - a_t)} \left(F_t(x)(b_t - a_t) - F_t(x)\frac{b_t^3 - a_t^3}{3} + \int_0^x \left((b_t^2 - a_t^2)y - y^2(b_t - a_t)\right) f_t(y)\, dy \right) \\
=&\frac{\lambda^e}{g(b_t - a_t)} \left(F_t(x) \int_0^1 \left((b_t^2 - a_t^2)y - y^2(b_t - a_t)\right) f_t(y)\, dy \right) \\
&- \frac{\lambda^e}{g(b_t - a_t)} \left(\int_0^x \left((b_t^2 - a_t^2)y - y^2(b_t - a_t)\right) f_t(y)\, dy \right) \\
=&\frac{\lambda^e\, d_t}{g(d_t)} \left(F_t(x) \int_0^1 \left(c_t^2 - (c_t - y)^2\right) f_t(y)\, dy - \int_0^x \left(c_t^2 - (c_t - y)^2\right) f_t(y)\, dy \right) \\
=&\frac{\lambda^e\, d_t}{g(d_t)} \left(F_t(x)c_t^2 - F_t(x) \int_0^1 (c_t - y)^2 f_t(y)\, dy + \int_0^x (c_t - y)^2 f_t(y)\, dy - F_t(x)c_t^2 \right) \\
=&\frac{\lambda^e\, d_t}{g(d_t)} \left(\int_0^x (c_t - y)^2 f_t(y)\, dy - F_t(x) \int_0^1 (c_t - y)^2 f_t(y)\, dy \right)
\end{aligned}
$$

□

Corollary 2.4.4.

In the absence of breakthrough payoff under policy p_t, the voter's belief q_t about the arrival rate λ after the government stage will evolve as follows:

$$q_{t+\frac{1}{2}} = \frac{(1 - \lambda_h l_t(1))q_t}{(1 - \lambda_h l_t(1))q_t + (1 - \lambda_l l_t(1))(1 - q_t)}$$

with $l_t(x) = \frac{d_t}{g(d_t)} \left(F_t(x) + \frac{b_t a_t - c_t^2}{3} - \int_0^1 (c_t - y)^2 f_t(y)\, dy \right)$.

Proof.

According to Proposition 2.4.3, the probability to receive a breakthrough payoff under the true state of the world $s \in [0, x]$ during an incumbency period t of length Δt is given by:

$$h_t(x) = \int_{a_t}^{b_t} \int_0^x \lambda^e (1 - (z - y)^2) f_t(y) \frac{1}{g(b_t - a_t)}\, dy\, dz$$

It follows that:

$$\text{Prob(No Payoff}|\lambda) = 1 - \int_{a_t}^{b_t} \int_0^1 \lambda^e (1 - (z - y)^2) f_t(y) \frac{1}{g(b_t - a_t)}\, dy\, dz$$

$$
\begin{aligned}
&= 1 - \frac{\lambda^e}{g(d_t)} \left(d_t - \int_0^1 \left(\frac{b_t^3 - a_t^3}{3} - (b_t^2 - a_t^2)y + y^2(b_t - a_t) \right) f_t(y)\, dy \right) \\
&= 1 - \frac{\lambda^e dt}{g(d_t)} \left(1 - \frac{4c_t^2 - b_t a_t}{3} + \int_0^1 \left(2c_t y - y^2 \right) f_t(y)\, dy \right) \\
&= 1 - \frac{\lambda^e dt}{g(d_t)} \left(1 - \frac{4c_t^2 - b_t a_t}{3} - \int_0^1 (y - c_t)^2 f_t(y)\, dy + c_t^2 \right) \\
&= 1 - \frac{\lambda^e dt}{g(d_t)} \left(1 + \frac{b_t a_t - c_t^2}{3} - \int_0^1 (y - c_t)^2 f_t(y)\, dy \right) \\
&= 1 - \lambda^e l_t(1)
\end{aligned}
$$

Applying Bayes' rule yields:

$$
\begin{aligned}
q_{t+\frac{1}{2}} =& \text{Prob}(\lambda = \lambda_h | \text{No Payoff}) \\
=& \frac{\text{Prob}(\text{No Payoff} | \lambda = \lambda_h)\text{Prob}(\lambda = \lambda_h)}{\text{Prob}(\text{No Payoff} | \lambda = \lambda_h)\text{Prob}(\lambda = \lambda_h) + \text{Prob}(\text{No Payoff} | \lambda = \lambda_l)\text{Prob}(\lambda = \lambda_l)}
\end{aligned}
$$

□

Corollary 2.4.5.
In the presence of news critique ($n = 1$) concerning the incumbents policy platform p_{it}, the voter's belief $f_{t+\frac{1}{2}}$ about the state of the world will evolve as follows:

$$
f_{t+1}(x) = \frac{(c_{it} - x)^2 f_{t+\frac{1}{2}}(x)}{\int_0^1 (c_{it} - y)^2 f_{t+\frac{1}{2}}(y)\, dy}
$$

and in the absence of news critique ($n = 0$):

$$
f_{t+1}(x) = \frac{(1 - k(c_{it} - x)^2) f_{t+\frac{1}{2}}(x)}{1 - k \int_0^1 (c_{it} - y)^2 f_{t+\frac{1}{2}}(y)\, dy}
$$

Proof.
Applying Bayes' rule using (2.4) analogously to Corollary 2.4.4.

□

Corollary 2.4.6.
Given a fake news arrival rate of $1 > \gamma > 0$, the voter's belief about the state of the world

evolves as follows:

$$f_{t+1}(x) = (1-\gamma)\frac{(c_{it}-x)^2 f_{t+\frac{1}{2}}(x)}{\int_0^1 (c_{it}-y)^2 f_{t+\frac{1}{2}}(y)\,dy} + \gamma\frac{(1-k(c_{it}-x)^2) f_{t+\frac{1}{2}}(x)}{1-k\int_0^1 (c_{it}-y)^2 f_{t+\frac{1}{2}}(y)\,dy}$$

Proof.

The proof follows from Corollary 2.4.5 under Bayes' rule.

□

Chapter 3

Optimizing Update Cycles for Network Goods: Evidence from the Gaming Industry - unpublished

This Chapter is joint work with:
Patrick Hufschmidt[1], Simon Ostendorf[2] and Gereon Riepe[3]

Abstract

This paper develops a dynamic model of quality investment in network goods under myopic consumer expectations. Firms choose quality updates over time, balancing increasing marginal costs against the declining marginal benefit from network growth. The model yields two main predictions: (i) quality improvements exhibit diminishing marginal growth over time as products' quality approaches a steady state; and (ii) in response to rising competitive pressure, captured by an increase in the value of the outside option, firms that remain in the market respond with additional quality investment, though not enough to fully offset network losses. We provide the first empirical evidence on this topic based on a novel dataset, as we scrape detailed patch histories as well as daily and monthly measures of player engagement and viewership on Steam and Twitch. While our analysis is based on a restricted set of free-to-play multiplayer first-person shooters to ensure comparability, the results are broadly consistent with the model's predictions and highlight the strategic role of dynamic quality investment in sustaining user engagement in network goods.

Keywords: Network goods, quality investment, dynamic, multiplayer games, panel data

JEL Codes: L13, L15, L86, D83, C23

[1]TU Dortmund University (patrick.hufschmidt@tu-dortmund.de)
[2]Ruhr University Bochum (simon.ostendorf@rub.de)
[3]Ruhr University Bochum (gereon.riepe@rub.de)

3.1 Introduction

The rapid expansion of internet connectivity and digital infrastructure has fundamentally transformed platform markets, embedding network effects as a defining feature of modern digital economic markets. In these markets, the value derived from a product is no longer contingent solely upon its intrinsic quality but increasingly depends on how many other consumers adopt and regularly consume it. Whether in social media platforms, messaging apps, or free-to-play multiplayer video games, network goods are distinguished by their ability to leverage interdependent consumer demand to sustain growth and engagement.

For firms operating in such environments, managing product quality becomes a central strategic challenge. Unlike traditional markets where pricing serves as the primary lever for competition, many digital platforms, particularly those adopting free-to-play models, cannot rely on direct pricing mechanisms to attract or retain users. Instead, they must invest in ongoing quality improvements that enhance user experience while simultaneously fostering and preserving network effects. This shift underscores the need for new theoretical frameworks that capture the interplay between dynamic quality investment and evolving network size.

This paper addresses this gap by developing a dynamic model of quality development tailored to free-to-use network goods under myopic consumer expectations. Firms in our model face increasing marginal costs of quality investment and diminishing marginal returns in terms of network growth over time. Our approach differs from classical literature (Easley and Kleinberg (2010), Shy (2001), Belleflamme and Peitz (2021)) focused primarily on static pricing strategies or duopolistic competition by situating firm behavior within fragmented and competitive markets. Consumers now face a large and diverse set of substitutable platforms, and firms must respond to changes in aggregate market pressure rather than engage in duopoly contest.

We model this environment by treating the outside option as exogenous and competitive,

reflecting the utility from an alternative platform. Firms do not engage in strategic anticipation but face outside competition (Goh et al. (2023)). This reflects the reality of many free-to-use digital ecosystems, where pricing plays a limited role, and competitive advantage is sustained through continuous quality improvements and network growth (Sánchez-Cartas (2022)).

The video game industry provides an ideal empirical setting to test the dynamics in these markets due to its remarkable growth trajectory and reliance on strong network effects. As of 2024, the global gaming market's revenue was valued at approximately 455 billion U.S. dollars and is projected to grow to 733 billion U.S. dollars by 2030 (Clement (2024)). In this context, multiplayer games have become emblematic of digital network goods: their success hinges not only on gameplay mechanics but also on their ability to sustain vibrant player communities through regular updates such as patches, balance adjustments, and content expansions. Many contemporary games are distributed for free, with developers earning revenue through optional in-game purchases, such as cosmetic upgrades or seasonal battle passes. This free-to-play model decouples quality from price and makes ongoing product development (i.e., quality updates) a key strategic variable. In addition, these free-to-play games heavily rely on multiplayer engagement, emphasizing network effects.

Our contribution is twofold: First, we develop a theoretical framework that predicts how firms optimize their update cycles under competitive pressure while accounting for convex adjustment costs and declining marginal benefits from additional investments. The model yields several key insights:

(i) Products' quality and networks converge toward steady-state levels based on critical thresholds of quality.

(ii) Firms cease updating once marginal costs outweigh benefits resulting in a stable long-run quality level.

(iii) Quality improvements are front-loaded due to convex adjustment costs and decrease

over time.

(iv) Rising competitive pressure prompts upward adjustments in quality without fully offsetting potential user loss.

Second, we provide empirical evidence using novel data scraped from Steam API, Twitch API, as well as publicly accessible historical data from SteamDB.com and TwitchTracker.com for free-to-play multiplayer first person shooters–a genre heavily reliant on network effects. By analyzing detailed patch histories alongside daily measures of player activity across multiple titles, we provide empirical support for our theoretical predictions: successful products converge quality- and network-wise to steady states at positive adoption levels through iterative updates while less competitive offerings phase out under insufficient investment or external shocks.

This paper contributes both theoretically and empirically to understanding dynamic strategies for sustaining user engagement in fragmented platform markets beyond traditional price-based competition frameworks. While our focus lies primarily within the video game industry due to its rich data availability and pronounced reliance on networks effects, our findings hold broader implications for other digital ecosystems including social media platforms, online education tools, communication apps, live-streaming services—and potentially any market where user utility depends jointly on intrinsic product characteristics and externalities arising from collective adoption.

The remainder of this paper is organized as follows. Section 3.2 provides a comprehensive overview of existing research in this field while establishing the context of our approach and novelty in contrast to existing work. Section 3.3 provides a detailed exposition of the theoretical framework used to analyze the network good in a dynamic quality setup. We first introduce the network mechanism under quality improvements following the existing literature on price competition and then introduce sequential decision making on the producer side over time. We analyze this framework in Section 3.3.3 and derive four main predictions for the empirical analysis. Section 3.4 contains an overview of the collected data as well

as first descriptive evidence for the validity of our approach. Applying ARIMA and Cox proportional hazards models in Section 3.4.2, we derive preliminary results that support our theoretical model. These results are finally discussed in Section 3.5 and we conclude in Section 3.6. All proofs are relegated to the Appendix C.

3.2 Related literature

We build on the extensive literature on network goods, platform competition, and dynamic user adoption. Our model is situated in a growing strand of research that analyzes how network effects shape firm strategies and consumer demand in digital platform markets.

Classical contributions include the early formalization of interdependent demand and network principles by Rohlfs (1974), Artle and Averous (1973), and later by Arthur (1990), Arthur (1996) and Economides (1996), who modeled the positive externalities embedded in user networks. Shy (2001) provides a comprehensive conceptual framework of network goods, while Easley and Kleinberg (2010) and Belleflamme and Peitz (2021) offer accessible yet rigorous treatments of network formation, adoption dynamics, and platform effects, which serve as central references for our model.

Several studies have explicitly modeled competition in network markets, often under duopoly or oligopoly settings. Katz and Shapiro (1985) analyze a simple oligopoly model with demand-side externalities and fulfilled expectations equilibria, laying the groundwork for subsequent theoretical studies. Pricing-focused contributions include Shapiro and Varian (2008), who examine pricing strategies under network competition, and Griva and Vettas (2011), who develop a vertically and horizontally differentiated duopoly model under price competition and expectation formation.

Our work departs from this tradition by focusing on product quality rather than price. We consider environments where the network good is free to use and monetized indirectly, for example via optional in-game purchases or premium features. This class of models is related

to Wu et al. (2013), who study accessory pricing in network goods and provide empirical support for monetization via non-essential upgrades.

While most existing models emphasize static or one-shot decisions, our approach is explicitly dynamic. We analyze how quality evolves over time in response to competition and network size development. In doing so, we incorporate the notion of myopic expectations, following Pollak (1975), who defines myopic behavior as decision-making that ignores the future consequences of present choices, a concept widely used in network diffusion literature (e.g., Jackson (2005)).

Importantly, while much of the prior work focuses on strategic interactions between a small number of platforms, we model a setting with a competitive outside option. As argued by Cennamo and Santalo (2013) and Zhu and Iansiti (2012), modern digital markets such as messaging apps, video games, and social media are increasingly shaped by diffuse competition among many alternatives rather than classical rivalries (Goh et al. (2023)). This motivates our use of an exogenous outside option instead of explicit strategic behavior.

Our focus on the video game industry aligns with several empirical and theoretical studies that highlight its platform characteristics and strong network effects. Clements and Ohashi (2005) and Marchand and Hennig-Thurau (2013) emphasize the role of platform-software variety, innovation, and user adoption dynamics in video game markets. Montag et al. (2019) discusses how freemium (offering basic services for free while charging for premium features) and multiplayer games adapt continuously to shifting user expectations and market trends. While the literature has extensively explored pricing strategies and static equilibrium analysis of network goods, the role of dynamic quality development remains underexplored. Our paper contributes to filling this gap by modeling quality as a dynamic control variable in a competitive platform market. We show how firms update quality over time in response to network feedback and external competition, and we derive several empirically testable predictions regarding steady-state adoption, quality convergence, and adaptation to shocks. Our model is complemented by empirical applications to the video game industry, where

both quality updates and network size evolution are observable at high frequency.

3.3 Model

We analyze the dynamic product quality for a single digital network good in a competitive environment. Assume a competitive market price $p = 0$. Note that the results hold for any exogenous price $p \geq 0$. Furthermore, we argue that freemium does not influence the initial adoption decision and ignore it for now. Thus, consider the following utility function U_i of a consumer i from consuming the network product:

$$U_i(Q_t, n_t) = v_i(Q_t) + g_i(n_t) \tag{3.1}$$

Where, $v_i(Q_t)$ is the quality-dependent value function, measuring consumer i's utility from using the product of quality Q_t at time t. The second term, $g_i(n_t)$, captures the positive network externalities, i.e., valuation for network size. This formulation follows the notion of interdependent demand for network goods, as discussed by Belleflamme and Peitz (2021), but departs from standard models by treating quality, rather than price, as the firm's dynamic control variable in a zero-price competitive market.

We assume that quality valuation and network benefits are additively separable, following Belleflamme and Peitz (2021). However, this assumption is not essential for our results; similar dynamic behavior can also emerge under multiplicative utility forms, as shown by Easley and Kleinberg (2010).

Following the literature (Shy (2001), Easley and Kleinberg (2010), Belleflamme and Peitz (2021)), we normalize the total number of consumers to a unit mass. The firm offers the network good at a quality level $Q_t \geq 0$. We assume that consumers have homogeneous preferences over quality $v_i(Q_t)$ but heterogeneous valuations of the network size $g_i(n_t)$.[4] In

[4]Note that Easley and Kleinberg (2010) present a complementary model in which consumers differ in their valuation of quality while network effects are homogeneous, leading to tractable demand expressions under one-shot pricing games.

this setting, it is sufficient to assume that $v_i(Q_t)$ is strictly increasing and concave, and that $g_i(n_t)$ is concave in n_t, to ensure interior solutions under general heterogeneity in network preferences. Let $v_i(Q_t) = Q_t$ and $g_i(n_t) = w_i n_t$. Here, w_i represents consumer i's idiosyncratic valuation of network participation and is assumed to be uniformly distributed on the unit interval, i.e., $w_i \sim U[0, 1]$. Thus, the utility of consumer i at time t is given by:

$$U_i(Q_t, n_t) = Q_t + w_i n_t \tag{3.2}$$

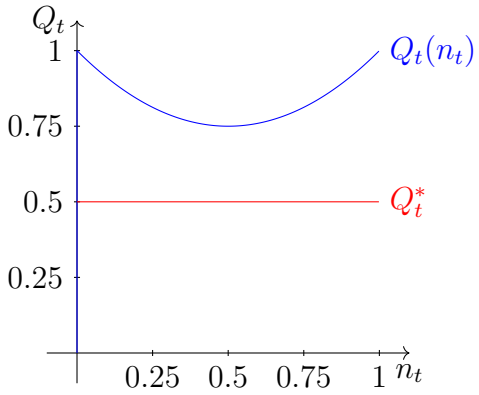
To model aggregate product demand, we introduce an exogenous outside option θ, representing the utility a consumer obtains from alternative digital products. In contrast to classical models of network competition involving a small number of strategic firms, we assume that the firm under consideration faces a large and diffuse set of competing products (e.g., other social media platforms, video games, or communication tools), whose quality and adoption evolve independently and are not directly observable or predictable by the firm. This setting reflects the empirical reality of modern digital markets, where consumers can choose among a wide array of functionally overlapping services (Goh et al. (2023)). This modeling approach is consistent with the treatment of competitive environments in Belleflamme and Peitz (2021) and aligns with observed firm behavior in zero-price network industries. Given the outside option θ, consumer j is indifferent between network product and outside option if and only if:

$$Q_t + w_i n_t = \theta \tag{3.3}$$

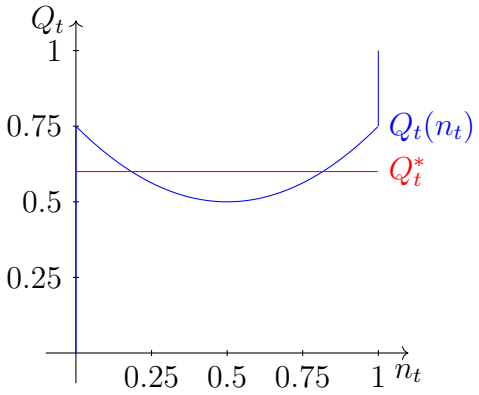
It follows from (3.3) that all users i with higher network valuation $w_i > w_j$ will consume the product. With unit mass of consumers and uniformly distributed preferences w_i, this is fulfilled for all users allocated to the right-hand side of the indifferent user j. Hence, it holds $n_t = 1 - w_j$. This yields the inverse demand curve equivalent expression for a given θ under quality competition:

$$Q_t(n_t) = \theta - n_t(1 - n_t) \tag{3.4}$$

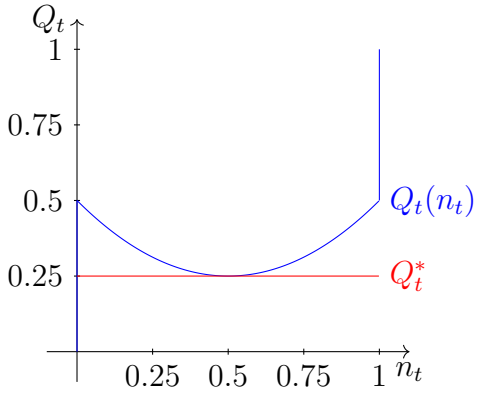
Equation (3.4) describes a convex relationship between network size and produce quality. It can be interpreted as an inverted version of the classic demand curve under network effects first introduced by Rohlfs (1974). Specifically, 3.4 is strictly decreasing for $n_t < \frac{1}{2}$ and strictly increasing for $n_t > \frac{1}{2}$. Intuitively, increasing n_t has two opposing effects on the quality. A positive impact on Q_t due to the network effect of interdependent demand and a negative impact due to the growing demand on quality to convince those consumers with lower network preferences. For $n_t < \frac{1}{2}$, the positive network effect dominates, and increasing adoption allows for lower quality. For $n_t > \frac{1}{2}$, quality must rise to attract and retain users with lower network valuations. A quality-network equilibrium in this context is a pair (Q_t^*, n_t^*) such that the firm's chosen quality Q_t^* is consistent with the inverse demand defined by (3.4). See Figure 3.1 for illustration.



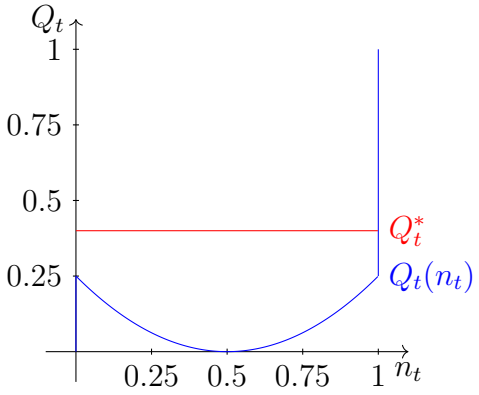
(a) $\theta = 1$, $Q_t = 0.5$
One equilibrium: (0,0.5)



(b) $\theta = 0.75$, $Q_t = 0.6$
Three equilibria: (0,0.6), (0.18,0.6), (0.82,0.6)



(c) $\theta = 0.5$, $Q_t = 0.25$
Two equilibria: (0,0.25), (0.5,0.25)



(d) $\theta = 0.25$, $Q_t = 0.4$
One equilibrium: (1,0.4)

Figure 3.1: Illustration of possible scenarios: Demand (blue), Supply (red)

Similarly to Belleflamme and Peitz (2021), there can exist at most three possible equilibria for any given parameters. The trivial ones are the corner cases of no user ($n_t = 0$), which occurs when quality and network size are too low to compete with the outside option, and all users ($n_t = 1$), which results when quality is sufficiently high to attract all users regardless of network valuation. The remaining interior equilibria are derived by solving Equation (3.4) for n_t. This yields:

$$n^*_{2,3} = \frac{1}{2} \pm \sqrt{\frac{1}{4} + Q_t - \theta} \qquad \text{for} \quad \theta - Q_t \in \left(0, \frac{1}{4}\right) \tag{3.5}$$

These interior solutions (3.5) exist only when $\theta - Q_t \in \left(0, \frac{1}{4}\right)$. Otherwise, the product is either too low in quality to generate adoption or so dominant that it captures the entire market. This poses, however, no problem for our model, as the optimal quality resulting from dynamic optimization will lie within this interval. Intuitively, the firm must offer a minimum quality level Q_t to attract initial adoption. However, providing too much quality relative to the outside option θ would result in excessive investment. Hence, meaningful dynamics emerge only when Q_t is sufficiently high to induce user adoption but not so high as to trivially dominate the market if the marginal return from investment is decreasing.

3.3.1 Network Dynamic

When transitioning our static analysis to a dynamic setup, there are several factors to take into account. Following Jackson (2005), Easley and Kleinberg (2010) and Belleflamme and Peitz (2021) note that in a dynamic setup for any consumer i the network size is not static. In other words, every consumer forms expectations about the future network size, while his own possible impact on the consumption behavior of other potential consumers is limited due to the continuum of consumers. Consequently, in contrast to (3.2), every consumer maximizes his expected utility depending on the expected network size n_t^e:

$$EU_i(Q_t, n_t^e) = Q_t + w_i n_t^e \tag{3.6}$$

In line with the classic literature, we assume that individuals form rational expectations about the current network size. This behavioral assumption reflects a form of bounded foresight often referred to as *myopic expectations*, as introduced by Pollak (1975). Therefore, individuals will not anticipate future growth or decline of the product's network size but instead use their current observation as decision basis to join, leave or not join the network. Given the large consumer base and heterogeneous preferences, it is reasonable to disregard the influence of any single individual as well as detailed market knowledge. An equilibrium,

presented in (3.5), has to be consistent with the expectations formed by consumers. We thus restrict ourselves to *fulfilled expectations equilibria* (FEE), following Katz and Shapiro (1985), Jackson (2005), and Belleflamme and Peitz (2021), among others. A fulfilled expectations equilibrium is characterized by consumers forming correct beliefs about network size, such that $n_t^e = n_t$ and no consumer holds incorrect expectations about the number of adopters. Under myopic expectations and FEE, quality-network equilibria as shown in figure (3.1) can be analyzed in a dynamic context. It follows from (3.3) that the indifferent consumer j given the belief that n_t^e users are in the network gains the same utility from consuming the firm's product or opting for the outside option θ.

$$Q_t + w_j n_t^e = \theta \tag{3.7}$$

Under myopic expectations all consumers share the same belief about the expected network size n_t^e. As a result, every user with valuation $w_i \in [w_j, 1]$ will consume the product. Given the uniform distribution of w_i, this implies that the realized network size is $n_t = 1 - w_j$. We can derive the number of users in the network in the next period $t+1$ as a function of the expected network size n_t^e in the current period, the outside option θ, and product quality Q_t by rearranging as follows:

$$n_{t+1}(n_t^e) = \begin{cases} 1 & \text{if } Q_t > \theta \\ 1 - \dfrac{\theta - Q_t}{n_t^e} & \text{if } 0 \leq \theta - Q_t \leq n_t^e \\ 0 & \text{otherwise} \end{cases} \tag{3.8}$$

In this context, the condition $\theta - Q_t \leq n_t^e$ ensures that the product quality is sufficiently high relative to the outside option such that at least the consumer with the highest valuation ($w_i = 1$) consumes the network product ($\theta \leq Q_t + n_t^e$), given the network size expectation n_t^e. The functional form of (3.8) allows us to analyze the dynamic behavior of the network size by plotting $n_{t+1}(n_t^e)$ against the 45-degree line representing fulfilled expectations equilibria

(FEE), where $n_t^e = n_t$. See Figure 3.2 for an illustration.

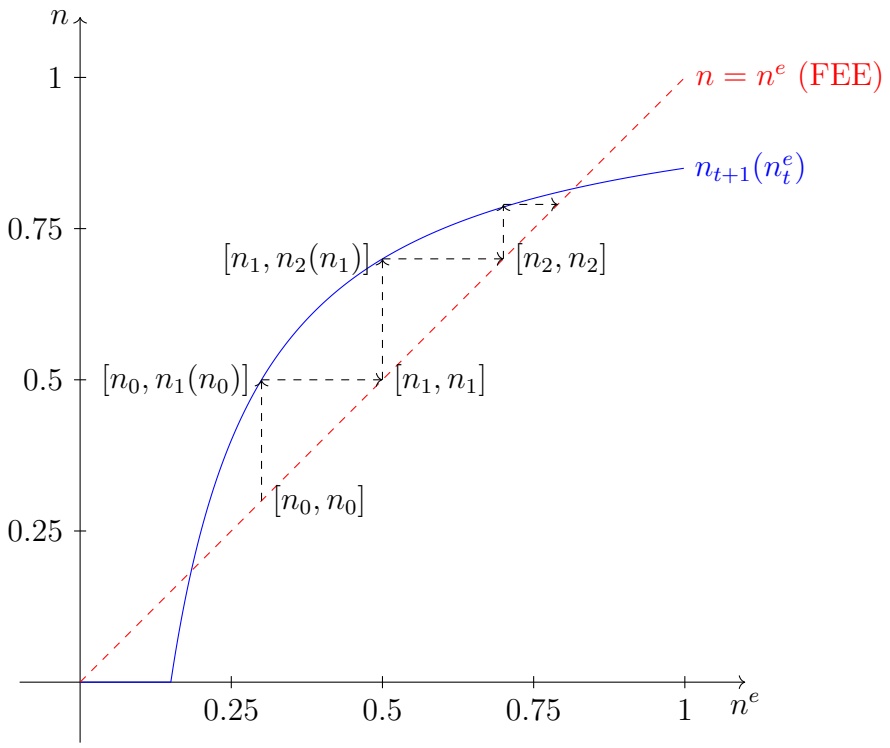
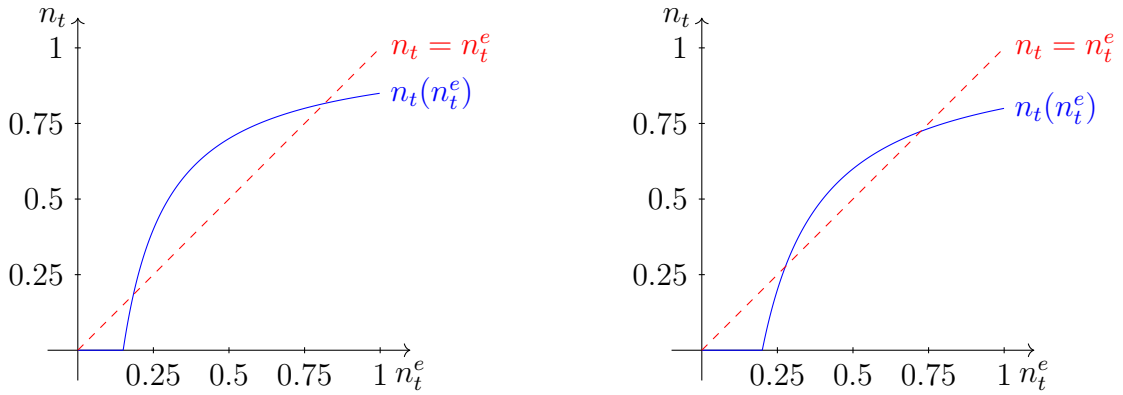


Figure 3.2: Example with $n_0 = 0.3$, $\theta = 0.75$ and $Q_t = 0.6$

Similarly to Jackson (2005), Easley and Kleinberg (2010), and Belleflamme and Peitz (2021), Figure 3.2 illustrates all possible equilibria as intersections between the fulfilled expectations line (FEE) and the network development function $n_{t+1}(n_t^e)$ outlined in (3.8). The network evolves dynamically from an initial population size n_0 as follows: under myopic expectations, consumers form the expectation $n_0^e = n_0$. Based on this expectation, a share of $n_1 = n_{t+1}(n_0)$ users join the network. Consumers then update their expectation to $n_1^e = n_1$, and the process repeats. This iterative adjustment continues until the network converges to a fixed point, an equilibrium where expectations are fulfilled. In the sense of Jackson (2005), stable equilibrium means that the network size will converge towards this equilibrium given any starting network size above or below this equilibrium point. In contrast, an unstable equilibrium can only exist if the starting network size is exactly the equilibrium point as any network size larger or smaller will diverge from it. In general, it holds that as long as

$n_{t+1}(n_t^e)$ lies above $n = n^e$, the network size is increasing while if it is below, the network size is decreasing. Consequently, the first equilibrium point $n_1^* = 0$ and the third equilibrium point $n_3^* = \frac{1}{2} + \sqrt{\frac{1}{4} + Q_t - \theta}$ are stable while the second $n_2^* = \frac{1}{2} - \sqrt{\frac{1}{4} + Q_t - \theta}$ is unstable (see (3.5)). This means that as long as the producer of the network good has an interest to keep his network product in the market he aims to surpass the second equilibrium point. Otherwise, the network shrinks and all consumers opt for the outside option.

The distance between n_3^* and n_2^* follows directly from (3.5) and therefore the difference in product quality versus outside option $Q_t - \theta$. See figure (3.3) for an illustration:



(a) $\theta = 0.75$, $q_t = 0.6$ (b) $\theta = 0.8$, $q_t = 0.6$

Figure 3.3: Network equilbria

3.3.2 Quality Dynamic

Having established the dynamics of consumer adoption, we now turn to the supplier's problem. As noted in the introduction, the product is free to use and competes with other offerings based on quality and network size. Nonetheless, the supplier can still generate revenue through monetization mechanisms embedded within the product, such as advertising or optional paid content.

One common approach is to offer enhanced user experiences or individualization features at an additional charge. These mechanisms typically target consumers' willingness to pay

for customization or premium access. Wu et al. (2013) provide a convincing model of such internal monetization strategies. However, since our focus lies on the network, quality interaction rather than the monetization structure itself, we abstract from those details.

For simplicity, we assume the existence of an exogenous per-period price p_c, which represents the price of optional paid items within the network good. This assumption is particularly realistic in the context of free-to-play video games, where items such as seasonal Battle Passes are almoust always consistently priced at fixed amount of 9.99 \$. Other prominent examples include the purchase of verification badges (e.g., the blue checkmark on $\mathbb{X}$), paid feature extensions in Microsoft Teams or Zoom. Naturally not all consumers pay or use these functions. Hence, we assume that a share $s \in (0, 1)$ of the network population pays p_c in every period, i.e., $s \cdot p_c$ represents the average revenue per user in the network. Consequently, supplier's revenue then follows:

$$p_c \cdot s \cdot n_t(n_{t-1}, Q_{t-1}, \theta) \tag{3.9}$$

Note that the exogenous parameter θ enters linearly in $n_t(.)$. Therefore, it doesn't matter whether we formulate beliefs about θ and it enters as an expected value or we assume perfect foresight. For the sake of simplicity and tractability we use the latter.

While the revenue is highly network size dependent, the provision and development of the product is costly. We will disregard linear cost in this context due to its limited applicability and simple slope comparison and use convex cost. This assumption is most common in the literature due to increasing marginal cost and also fits well with our data (see, for example, Tirole (1988)).

We assume that costs are generated during the production of the original product and then each period the decision to increase the quality is made. Fix cost and maintenance cost can be disregarded concerning the actual optimization problem. Hence, we assume cost must depend and convexly increase in the current quality level of the product and also be convex

in each periods update. Therefore, it is more costly to increase the products quality within one period alone compared to splitting it into smaller parts. Given time constraints and limited staff for product development, this is a feasible assumption. To generate explicit predictions, we specify the cost function as follows:

$$C(Q_t, q_t) = c_1 \cdot Q_t \cdot q_t + c_2 \cdot q_t^2 \qquad \text{with } c_1, c_2 > 0 \tag{3.10}$$

While Q_t represents the current quality of the network product in period t, q_t is the marginal increase or quality update of the product decided on in period t. It follows from (3.9) and (3.10) that the dynamic problem can be formulated as a Bellman equation, where $\beta \in [0, 1]$ denotes the discount rate.

$$V(n_t, Q_t) = \max_{q_t \in \mathbb{R}^+} p_c \cdot s \cdot n_t - c_1 \cdot Q_t \cdot q_t - c_2 \cdot q_t^2 + \beta V(n_{t+1}, Q_{t+1}) \tag{3.11}$$

s.t.

$$n_{t+1} = 1 - \frac{\theta - (Q_t + q_t)}{n_t} \tag{3.12}$$

$$Q_{t+1} = Q_t + q_t \tag{3.13}$$

The value function (3.11) is the profit maximization problem while discounting future profits. Constraint (3.12) is the network size development which we deduced in (3.8) and specifies the law of motion. Note, however, that contrary to (3.8), (3.12) depends on $Q_{t+1} = Q_t + q_t$ as we assume that Q_t is the quality state given at the beginning of period t which is then updated in t by q_t. Based on this the new network size n_{t+1} for period $t+1$ then forms. This has no noteworthy impact on the results and is adopted solely to align with the temporal structure inherent in the characteristics of the Bellman equation. Constraint (3.13) is the increase of the products current quality induced by the update in period t.

Proposition 3.3.1 (The Euler equation)**.**

Solving the Bellman equation yields the following optimal quality update policy function:

$$q_t(q_{t+1}) = \frac{\frac{\beta \cdot p_c \cdot s}{n_t} - c_1 \cdot Q_t - \beta \cdot c_1 \cdot q_{t+1}}{2c_2} \tag{3.14}$$

The Euler equation (3.14) characterizes the firm's optimal dynamic path of quality updates. It balances the marginal benefit of raising quality—through its effect on future network size and revenue—against the marginal cost of increasing quality in the present and near future. The term $\frac{\beta \cdot p_c \cdot s}{n_t}$ reflects the discounted marginal revenue from expanding the network, while the terms $c_1 Q_t$ and $c_1 q_{t+1}$ capture how current quality levels and anticipated future updates increase the linear cost of quality improvement. Crucially, the denominator $2c_2$ reflects the convex component of adjustment costs: the larger c_2, the more costly it is to implement large quality updates within a single period. As a result, the firm optimally spreads quality improvements over time, leading to smooth rather than abrupt adjustment paths.

Using the derived network-quality equilibria under FEE (3.5), the quality dynamics defined by the policy function from Proposition (3.3.1), and the dynamic network development specified in 3.8, we derive a variety of testable predictions for the data in the following.

3.3.3 Results

Prediction 1: The network size reaches a positive steady state or declines to zero.

According to (3.5) there exist only two stable network equilibria under myopic expectations. This means that in our dynamic setup the network will assume one of two possible steady states corresponding to the stable equilibria derived in (3.5). The first one is the trivial zero state which satisfies $n_{t+1} = n_t = 0$. The zero state realizes when the product is not updated and shocks to n_t or θ drive the network size below the unstable point $n_2^* = \frac{1}{2} - \sqrt{\frac{1}{4} - \theta + Q_t}$. After that network dynamics induced by myopic expectations move the

consumer base towards zero. The second steady state is the stable network point $n_3^* = \frac{1}{2} + \sqrt{\frac{1}{4} - \theta + Q_t}$. Therefore, we predict a stable network size, ceteris paribus, for a successful product. An unsuccessful product will not attract a sufficiently large network and always tend towards zero, while an abandoned product drops towards 0 under sufficient shocks. Figures (3.4) and (3.5) illustrate this argument. Intuitively, this happens because once a certain number of users adopted the product, it reaches a point where growth stabilizes and most potential users are already part of the network. Due to this saturation of the primary target audience, growth will slow down and reach the steady state. A product with no updates, however, slowly starts to lose its users as more and more consumers realize the diminishing network.

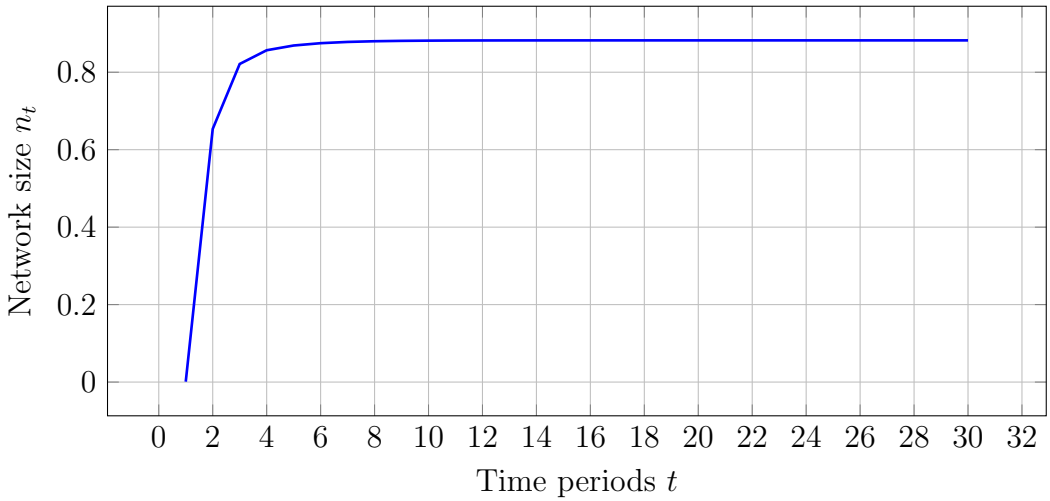


Figure 3.4: Network size steady state for $\beta = 0.95$, $c_1 = 1$, $c_2 = 2$, $p_c = 2$, $s = 0.3$, $\theta = 0.75$.

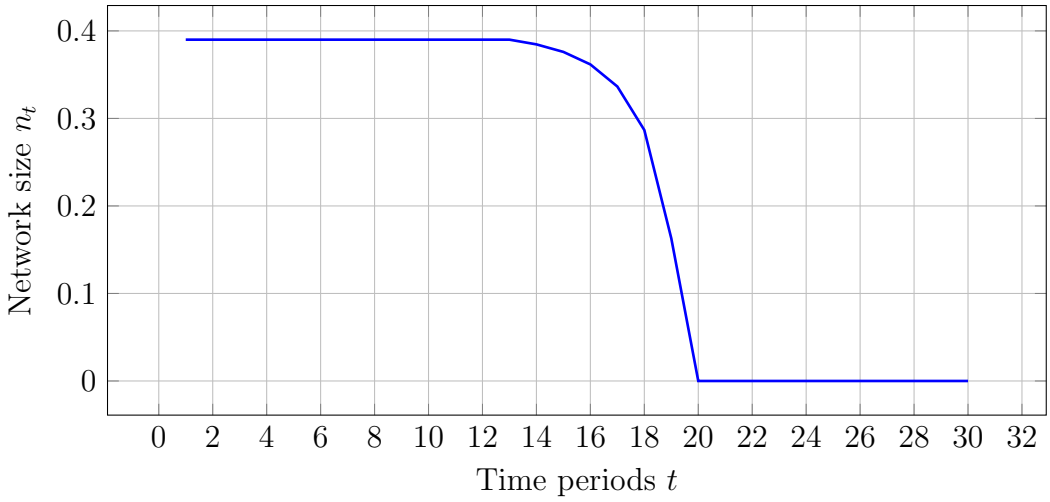


Figure 3.5: Network phasing out for $Q^* = 0.5$, $\theta = 0.74$, $n_1 = 0.39$.

Prediction 2: The total product quality assumes a steady state and is not updated further past that, ceteris paribus.

Following directly from proposition (3.3.1) and prediction 1 there exists an optimal product quality that is implemented. After it is reached, we then expect no further quality changes.

Proposition 3.3.2 (Final product quality)**.**

The final produce quality Q^* is implicitly characterized by:

$$\frac{Q^*}{2} + Q^* \sqrt{\frac{1}{4} - \theta + Q^*} = \frac{\beta \cdot p_c \cdot s}{c_1}$$

Note that there exists no closed form solution to this and numerical solvers are advised in concrete cases to determine Q^*. Proposition (3.3.2) shows the product's total quality reaches a steady state. Beyond this stage, ceteris paribus, there are no further quality improvements. As for prediction 1, the reaching of a steady state is driven by several reasons. Due to diminishing marginal returns, once a product reaches a quality level that satisfies most users, incentives to invest in new significant upgrades decline. Additionally, achieving higher quality leads to further increased costs and complexity, with each incremental improvement requiring more resources than earlier stages. If improvements at the same time do not

significantly enhance customers experience, resulting in noteworthy network growth, they are not valuable to the supplier. As a result, companies may shift their attention from upgrading the existing product to developing an entirely new product that caters to emerging market trends. Companies might then focus on aspects like customer service instead of continuous enhancements for products that already reached a steady state. These aspects however extend beyond the scope of this paper. Figure (3.6) and (3.7) illustrate the quality update dynamics.

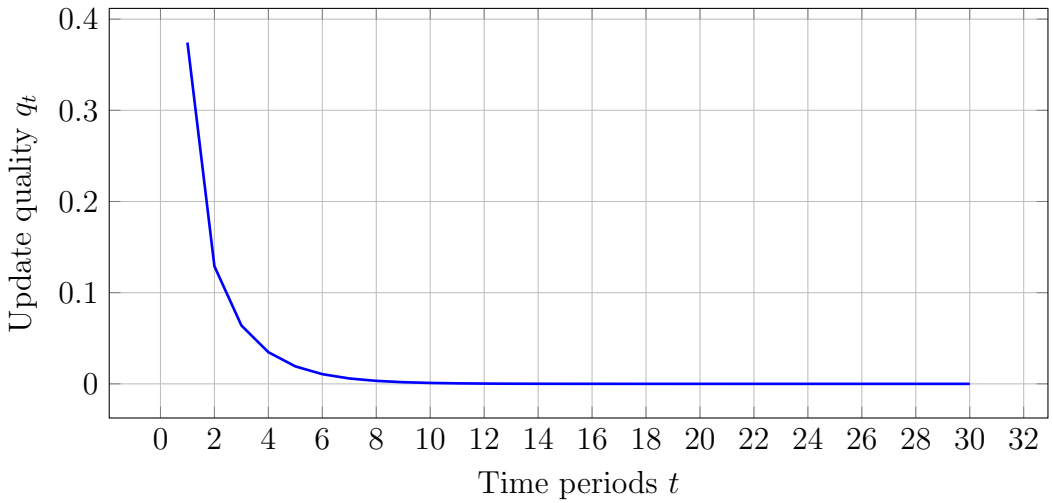


Figure 3.6: Quality update over t for: $\beta = 0.95,\ c_1 = 1,\ c_2 = 2,\ p_c = 2,\ s = 0.3,\ \theta = 0.75$.

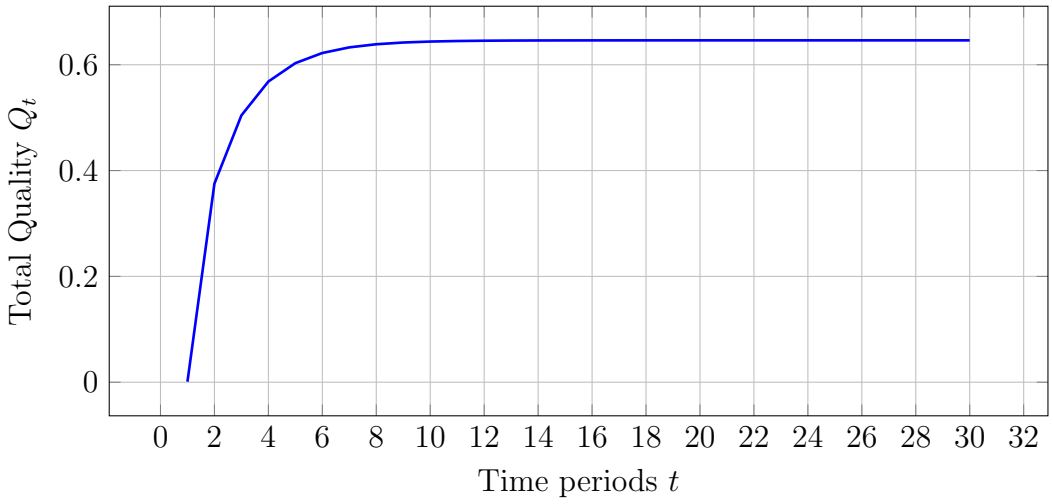


Figure 3.7: Total Quality over t for: $\beta = 0.95,\ c_1 = 1,\ c_2 = 2,\ p_c = 2,\ s = 0.3,\ \theta = 0.75$.

Prediction 3: The Quality growth exhibits diminishing marginal growth.

According to the Euler equation (3.14) which is decreasing in n_t and Q_t, product quality updates are large at the beginning and become smaller over time until the steady state is reached. This concave behavior of product development over time can be attributed to several key factors. First, due to the increase in development cost, larger updates become less attractive for higher quality levels. Realistically, earlier improvements may require less investment and complexity compared to later stages gradually flattening the curve. Additionally, the future steady network state described in Prediction 1 does not respond as strong to further quality improvements relative to the cost involved. Hence, it is most profitable to reach a sufficient quality level early to start a quick dynamic growth of the consumer base caused by the products network effect under myopic expectations. Figures (3.6) and (3.7) also illustrate this argument.

Prediction 4: A lifetime support network product adapts to increased competition through quality innovation.

Under growing competition, namely an increase in the outside option θ, producers will increase their products quality. This need for innovation is meant to retain the product's market share and is driven by the threat of a potential customer loss given increased user expectations. Another driver of innovation is the technological advancement which might have been introduced into the market by competitors. During COVID-19 pandemic, this was mainly the introduction and improvement of reliant online conferencing systems for companies as practiced, for example, by Cisco Webex and Microsoft Teams. However, the quality increase cannot fully compensate potential network loss. It follows from Proposition (3.3.2) that θ enters negatively into the left-hand side but at a disproportionate rate. Figures (3.8), (3.9) and (3.10) illustrate this argument.

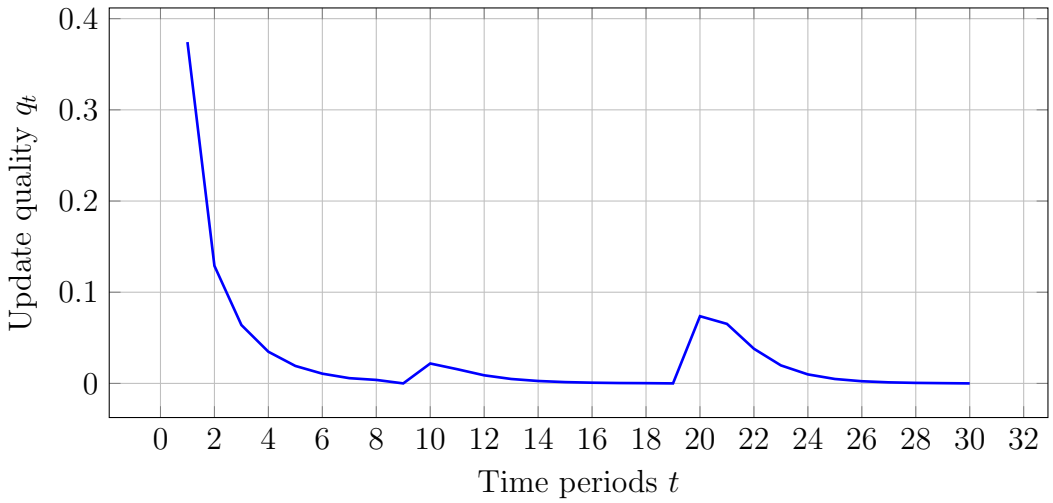


Figure 3.8: Quality Update dynamics under shocks for: $\beta = 0.95$, $c_1 = 1$, $c_2 = 2$, $p_c = 2$, $s = 0.3$, $\theta = 0.75$, shocks (permanent increases to θ) at $t = 10$ ($\theta = 0.85$) and $t = 20$ ($\theta = 1.15$).

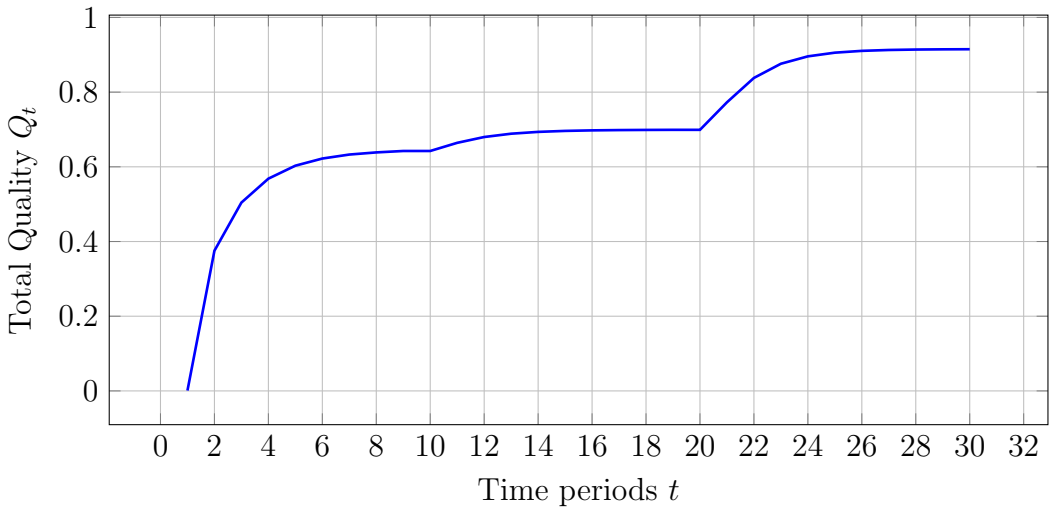


Figure 3.9: Quality dynamics under shocks for: $\beta = 0.95$, $c_1 = 1$, $c_2 = 2$, $p_c = 2$, $s = 0.3$, $\theta = 0.75$.
Shocks (permanent increases to θ) at $t = 10$ ($\theta = 0.85$) and $t = 20$ ($\theta = 1.15$).

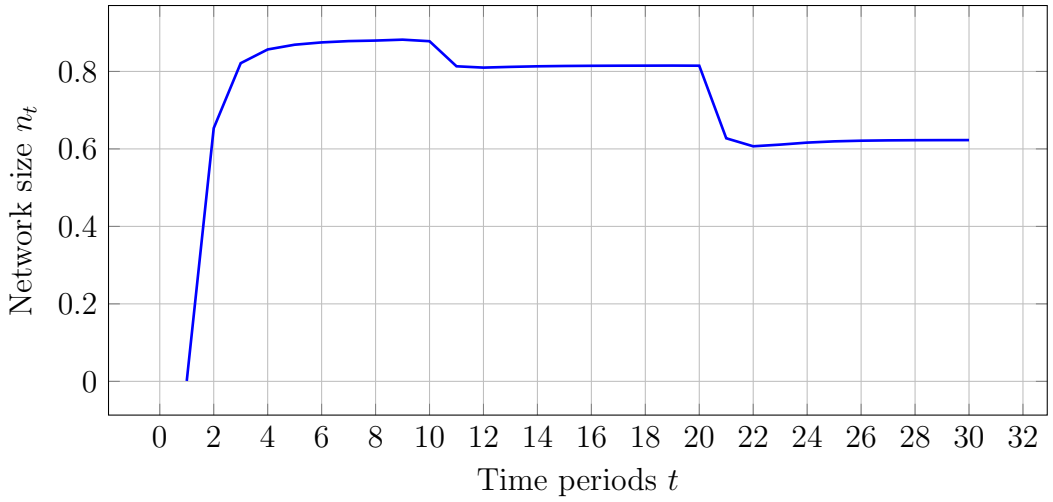


Figure 3.10: Network size under shocks for: $\beta = 0.95, c_1 = 1, c_2 = 2, p_c = 2, s = 0.3, \theta = 0.75$. Shocks (permanent increases to θ) at $t = 10$ ($\theta = 0.85$) and $t = 20$ ($\theta = 1.15$).

3.4 Empirical Analysis

The developed theoretical model applies to digital network goods, where user utility depends both on intrinsic product quality and on the presence of other users and profit is generated by additional optional product dependent goods. Multiplayer video games offer a particularly suitable empirical setting to test these dynamics: they are network goods by design, with user experience shaped by the size and composition of the active player base and usually offer additional optional consumption in form of cosmetics or season passes. Quality evolves endogenously through patches, balance updates, and content expansions. Moreover, user engagement in this domain is highly dynamic and directly observable.

To test the model's predictions, we construct a panel dataset centered on the *Steam* platform. Note that the data collection is still ongoing; the results presented here should be interpreted as preliminary and may be subject to revision. At this stage, we focus on the most prominent PC-based multiplayer titles, combining market-level patterns with a detailed case study.

Our data sources include the official Steam API and Twitch API, as well as publicly acces-

sible historical data from *SteamDB.com* and *TwitchTracker.com*.[5] For 49 major multiplayer titles—most of which follow a free-to-play model—we collect monthly player numbers, daily user reviews, patch histories, and detailed game-level metadata (e.g., release year, price, number of DLCs (Downloadable Conten), hardware requirements, and genre tags). This data allows us to track both network size and evolving product quality over time.

To capture broader market dynamics beyond platform-specific engagement, we complement the Steam data with Twitch viewership statistics. Twitch is a platform-agnostic streaming service that aggregates attention across the gaming industry. We include 11 additional multiplayer games with strong Twitch visibility but no corresponding Steam data,[6] resulting in a total sample of 58 titles. Steam-based activity is available from 2014 (or from a game's release), while Twitch data begins in 2017.

3.4.1 Descriptive Evidence

Before turning to formal prediction testing, we present descriptive statistics that highlight the dynamic structure of the multiplayer game market and introduce the core empirical variables.

Figure 3.11 shows the monthly share of active players for the ten most-played games on Steam. Shares are normalized to reflect relative composition, abstracting from growth in the platform's total user base. While Steam-exclusive titles such as *Counter-Strike 2* and *Dota 2* remain persistently dominant, their market positions fluctuate substantially over time. This underscores the need to model network goods as evolving and competitive environments, shaped by feedback between quality and user base.

The sharp rise in market share following the release of *PUBG* in early 2017 illustrates

[5]To ensure transparency and reproducibility, all data were collected via official APIs, available downloads, or manual extraction from publicly accessible HTML pages. No automated scraping tools were used.

[6]These 11 titles are exclusive to distribution platforms such as the Epic Games Store, Battle.net (Blizzard), or Riot Client. On the Steam side, *Counter-Strike 2*, *Dota 2*, and *Team Fortress* are likewise distribution platform-exclusive. The remaining titles in our sample are cross-platform. While this asymmetry offers an interesting context for comparative platform strategy, our focus lies on quality–network feedback for individual products.

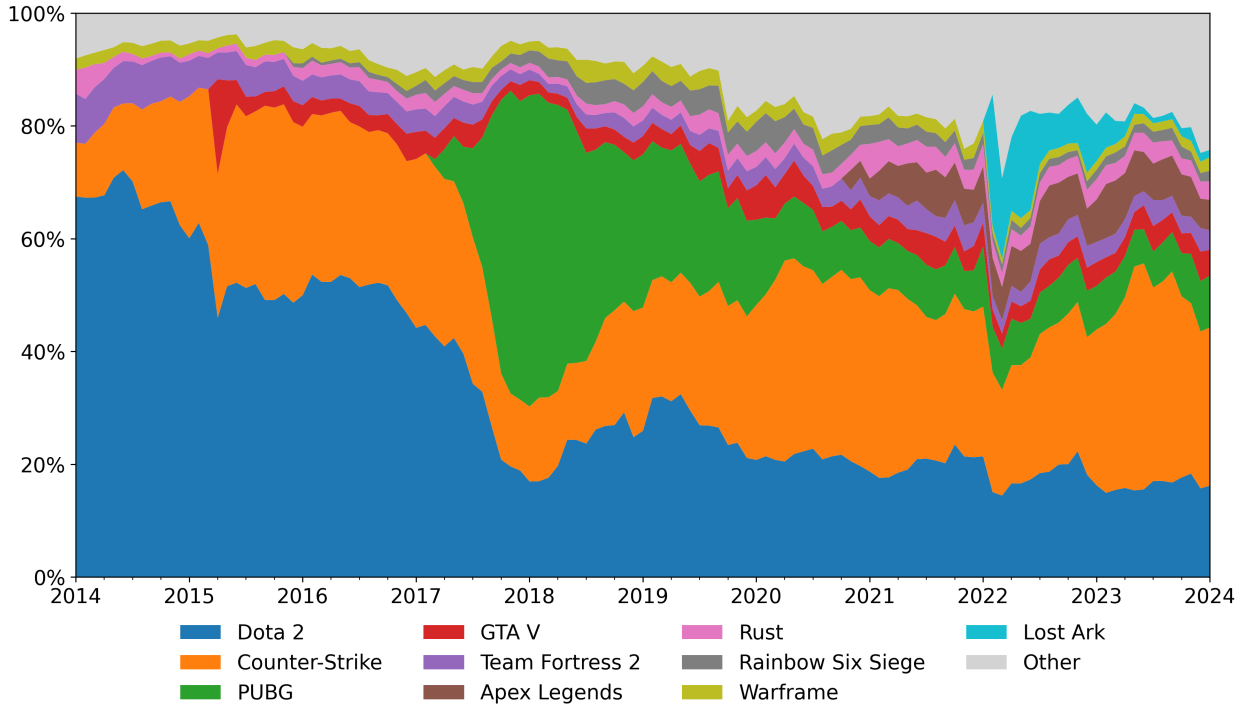


Figure 3.11: Monthly Share of Players for the Top 10 Games on Steam: with time on the x-axis and share of players on the y-axis.

the disruptive potential of innovation. Similar, though smaller, shocks are visible after the releases of *GTA V*, *Lost Ark*, and *Apex Legends*. While the figure depicts relative shares rather than absolute numbers, many of these shifts appear to cross genre boundaries—for instance, from a MOBA (Multiplayer Online Battle Arena) like *Dota 2* to a battle royale shooter like *PUBG*, or from a tactical shooter like *Counter-Strike* to an MMO like *Lost Ark*. Comparable shifts in user attention are observable in Twitch viewership (see Appendix C.2). These dynamics support our claim that competition in network markets is not confined to a static duopoly or clear oligopoly.

To illustrate the core variables used in the empirical analysis, we next turn to a representative case study. Figure 3.12 plots monthly player numbers and user reviews for *Counter-Strike*, one of the most persistent and competitively relevant titles in our sample.

The time series in Figure 3.12 reveals several notable dynamics. Sharp increases in review activity follow major updates in October 2017, December 2018 (became free-to-play), May

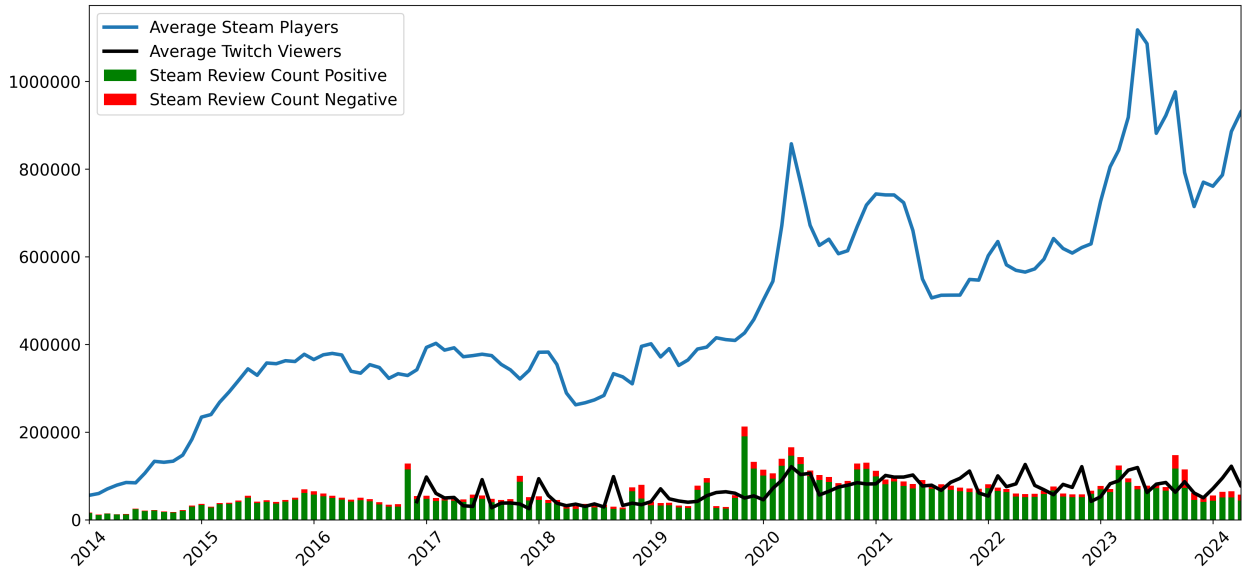


Figure 3.12: Monthly Players and Reviews for Counter-Strike: with time on the x-axis and absolute values on the y-axis.

2019, November 2019, December 2020, September 2021, and—most prominently—September 2023, which marks the release of *Counter-Strike 2*. In addition, the general surge in early 2020 coincides with the COVID-19 pandemic and mirrors broader increases in online activity.

For *Counter-Strike* the Pearson correlation between average monthly Steam players and average Twitch viewers is $r = 0.58$ ($p < 0.001$), suggesting that Twitch can serve as a meaningful proxy for visibility and aggregate engagement.

Overall, these empirical patterns align with the core mechanisms of our model. Product quality, rather than price, drives user engagement in the presence of network effects, and that both quality and network size evolve jointly over time. Contrary to the assumptions of much of the static network literature, competition in such settings is not necessarily restricted to a stable oligopoly. Rather, we argue that dynamic quality competition, amplified by connectivity and attention spillovers, is central to understanding engagement in digital network goods. While this paper focuses on multiplayer games, the same logic applies more broadly to markets such as social media, online education, online dating and live-streaming platforms, where quality and network size co-evolve.

3.4.2 Case studies

To ensure comparability across network structures and to address limitations from incomplete data collection, we focus our preliminary evidence on a sample of selected free-to-play multiplayer titles within the commercially significant and network-intensive first-person shooter (FPS) genre and provide correlational evidence.

The selected games include: *Destiny 2*, *Apex Legends™*, *Halo Infinite*, *THE FINALS*, *Overwatch® 2*, *Paladins®*, *World of Tanks Blitz*, *PUBG: BATTLEGROUNDS*, and *Counter-Strike 2*. Given the limited sample size, our analysis takes the form of a case study.[7]

We test Prediction 1 using ARIMA models to capture time-series dynamics in player engagement. Predictions 2 through 4 are evaluated using Cox proportional hazards models, which allow us to analyze the timing and determinants of user churn in competitive online environments.

Prediction 1

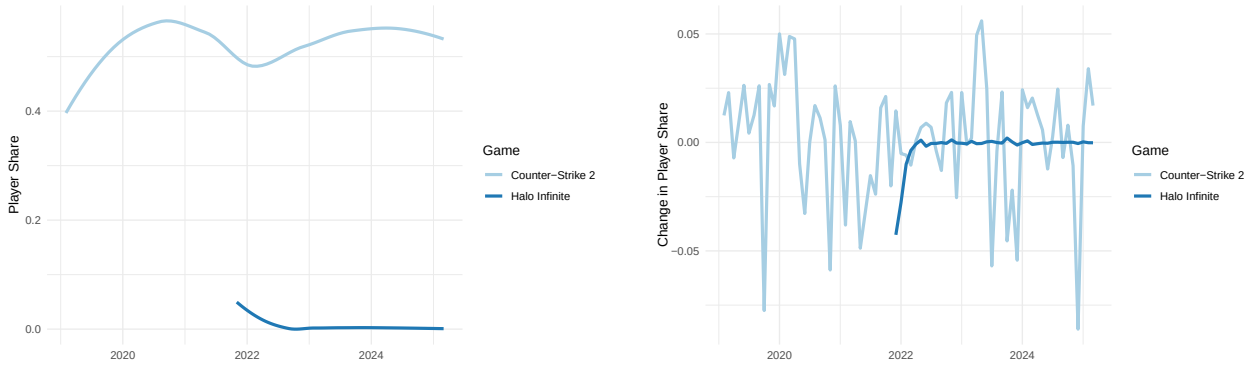
To empirically assess the stability of platform-based gaming networks, we analyze monthly user engagement data for two prominent free-to-play multiplayer shooter titles: *Counter-Strike 2* and *Halo Infinite*. Rather than modeling absolute player counts, which are subject to a general upward trend in the multiplayer shooter genre, we focus on the player share of each game—defined as the proportion of players each title captures relative to the total player base within the genre. This normalization allows us to isolate game-specific dynamics from broader market growth effects.

Figure 3.13 illustrates the evolution of these player shares (panel a) and their corresponding monthly percentage point changes (panel b). According to our theoretical predictions, each game's network size is expected to reach a stable, positive steady state or decline toward zero. In both scenarios, the percentage point change in player share is expected to

[7] A more comprehensive and rigorous analysis will be possible in future work once we have scraped more patch- and game-level data.

converge to zero over time. Panel a) of Figure 3.13 shows that *Counter-Strike 2* maintains a consistently high player share, suggesting convergence to a positive steady state. In contrast, *Halo Infinite's* player share steadily declines toward zero, indicating a failure to establish a stable player network.

Panel b) confirms that while the monthly changes in player share exhibit considerable short-term volatility, these fluctuations appear to oscillate around zero—consistent with the model's prediction. To formally test this, we estimate separate ARIMA$(1,0,1)(1,0,1)_{12}$ models (i.e., with one autoregressive and one moving average term in both seasonal and non-seasonal components, with seasonality of 12 months) for the differenced player share time series of each game shown in panel b). In line with our theoretical expectations, these models are expected to exhibit a mean of zero, reflecting that the monthly changes in player share tend to average out over time.



(a) Smoothed monthly player share by game.

(b) Monthly change in player share (percentage points).

Figure 3.13: Player share trends for *Counter-Strike* and *Halo Infinite*. The time series begin at the release date of *Halo Infinite* and, for *Counter-Strike*, at the release date of its free-to-play version.

The ARIMA estimates provide a reduced-form representation of network dynamics. The autoregressive term (AR(1)) captures the degree of intrinsic momentum, i.e., how much past user activity predicts current engagement. The mean parameter (μ) reflects the long-run expected value of the process, conditional on stationarity. Seasonal autoregressive and moving average terms capture periodic fluctuations due to recurrent events (e.g., product

updates, tournaments). A high AR(1) coefficient alongside a positive mean and minimal seasonal dependence suggests a network that self-sustains in the absence of shocks, consistent with convergence to a non-trivial equilibrium. In contrast, low or imprecise autoregressive feedback, heavy reliance on seasonal effects, and mean reversion to zero are indicative of a fragile or "dying" network. We estimate the following empirical model:

$$\begin{aligned} y_t = \mu + \phi_1 y_{t-1} + \Phi_1 y_{t-12} - \phi_1 \Phi_1 y_{t-13} \\ + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \Theta_1 \varepsilon_{t-12} + \theta_1 \Theta_1 \varepsilon_{t-13} \end{aligned}$$

where y_t denotes the observed value of the time series (e.g., normalized user activity) at time t, and μ is the unconditional mean of the stationary process. The parameters ϕ_1 and θ_1 capture the short-run (non-seasonal) autoregressive and moving average dynamics at lag 1, respectively. Seasonal components are modeled through Φ_1 and Θ_1, which represent the autoregressive and moving average terms at the seasonal lag of 12 periods (e.g., months). The term ε_t is an independently and identically distributed white noise error, with zero mean and constant variance. The interaction terms $\phi_1 \Phi_1$ and $\theta_1 \Theta_1$ account for the effect of joint lag structures at period $t - 13$.

For *Counter-Strike 2*, we find a strong and statistically significant autoregressive component (AR(1) = 0.747, $p < 0.01$), paired with a moderately sized moving average effect and a small, statistically insignificant seasonal structure. The estimated long-run mean is positive (0.0020), though not significantly different from zero. These results suggest a network characterized by strong internal momentum and a tendency to self-sustain in the absence of external shocks. The absence of strong seasonal dependencies indicates that user activity does not rely heavily on recurrent events or exogenous interventions. This pattern is consistent with theoretical predictions of convergence to a non-trivial, stable equilibrium.

By contrast, *Halo Infinite* exhibits markedly weaker dynamic structure. The AR(1) co-

Table 3.1: ARIMA Model Estimates for *Counter-Strike 2* and *Halo Infinite*

	Counter-Strike 2	*Halo Infinite*
AR(1)	0.747***	0.296
	(0.145)	(0.693)
MA(1)	-0.461**	-0.411
	(0.200)	(0.645)
Seasonal AR(1) [12]	-0.001	-0.795
	(0.419)	(0.721)
Seasonal MA(1) [12]	0.154	0.656
	(0.403)	(0.872)
Mean	0.0020	0.0000
	(0.0059)	(0.0002)
Residual SD $\hat{\sigma}$	0.0294	0.000921
Log-likelihood	293.27	208.33
AIC	-574.55	-404.67
BIC	-556.99	-395.00

Notes: Estimates from ARIMA(1,0,1)(1,0,1)[12] models using normalized monthly time series data. Standard errors are reported in parentheses below each coefficient. Significance levels: $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$. Both models assume trend and seasonal stationarity. *Counter-Strike 2* exhibits strong short-run persistence with statistically significant dynamics. *Halo Infinite* shows weaker and noisier dynamics but more pronounced seasonal effects. Residual standard deviations ($\hat{\sigma}$) reflect model fit on the training data.

efficient is smaller (0.296) and statistically imprecise, and the estimated mean is effectively zero. Seasonal components are more pronounced in magnitude but remain statistically insignificant, implying that observed fluctuations are primarily driven by noise rather than systematic periodic forces. These findings suggest a fragile network dynamic that lacks the intrinsic persistence necessary for long-run stability. The empirical evidence here points toward the zero state.

Together, these estimates support the hypothesis that platform-based multiplayer networks tend to evolve toward one of two stable equilibria: (i) a self-sustaining positive steady state driven by strong autoregressive feedback, or (ii) a decay to near-zero activity characterized by weak internal dynamics and heavy reliance on exogenous stimuli.

Predictions 2-4

To empirically investigate the timing of quality-enhancing updates in networked products, we estimate Cox proportional hazards models using patch-level data from leading free-to-play multiplayer games in the first-person shooter genre. In particular, we include the following titles for the period from January 2016 to November 2024: *Destiny 2*, *Apex Legends™*, *THE FINALS*, *Overwatch® 2*, *Paladins®*, *World of Tanks Blitz*, *PUBG: BATTLEGROUNDS*. Cox proportional hazards models are a standard method in survival analysis for analyzing how covariates influence the timing of events (Shenyang 2009; Heckman and Singer 1984).

In our setting, we are interested in predicting the survival of game versions or network states. Therefore, these models are well-suited to empirically test the paper's further predictions, namely, (2) product quality eventually reaches a steady state, such that the likelihood of updates declines over time; (3) quality improvements exhibit diminishing marginal returns, with earlier updates arriving more frequently than later ones; and (4) products respond to increased competitive pressure through innovation, proxied by a higher hazard of updates following competitor activity.

Furthermore, the Cox model's semi-parametric structure is particularly appropriate for

modeling the timing of updates without requiring strong assumptions about the baseline hazard. To account for unobserved heterogeneity across games, we include a frailty term at the game level. This is preferable to fixed effects in a predictive context, particularly given the small number of networks in the sample, as it allows for partial pooling across units and improves generalization while controlling for game-specific baseline hazard variation.

Hence, we estimate the hazard of a new patch release as a function of product age and recent competitor activity using a Cox proportional hazards model specified as follows:

$$h_{ij}(t) = h_0(t) \cdot u_j \cdot \exp\big(\beta_1 \cdot \text{Patches}_{0-14d,ij} + \beta_2 \cdot \text{Patches}_{15-28d,ij} + \beta_3 \cdot \text{Patches}_{29-42d,ij} + \beta_4 \cdot \text{Patches}_{43-56d,ij} + \beta_5 \cdot \text{DaysSinceRelease}_{ij}\big) \quad (3.15)$$

where $h_{ij}(t)$ denotes the hazard of patch i in game j at time t (monthly data), and $u_j \sim \text{Gamma}(\theta, \theta)$ is a game-level frailty term capturing persistent unobserved differences in patching propensity. The covariates count the number of patches released by competing games in the respective time windows preceding the focal patch, and $\text{DaysSinceRelease}_{ij}$ is the age of the game in days at the time of patch i.

Table 3.2 reports the results. We find evidence that the timing of patch releases is influenced by both the age of the product and recent competitor activity. The coefficient on *Days Since Release* is negative and highly significant, indicating that patch frequency declines as the game matures. Quantitatively, each additional day since release reduces the hazard of a new patch by approximately 0.06%, implying a decline of about 6% over the course of 100 days. This supports the hypothesis that quality improvements are more frequent after launch, when marginal returns are highest, and diminish over time as the network approaches a steady state, which supports Prediction 3.

Recent competitor activity is positively associated with patching hazard in the short term. Specifically, the likelihood of a patch is significantly positively associated with competitor

Table 3.2: Cox Proportional Hazards Estimates of Patch Timing

	(1) Cox PH	(2) Cox PH + Frailty
Competitor Patches (0–14d)	0.129** (0.046)	0.181*** (0.051)
Competitor Patches (15–28d)	0.115** (0.045)	0.214*** (0.050)
Competitor Patches (29–42d)	0.206*** (0.061)	0.200*** (0.064)
Competitor Patches (43–56d)	0.028 (0.051)	0.037 (0.052)
Days Since Release	-0.0006*** (0.0001)	-0.0007*** (0.0001)
Frailty Variance (Game ID)	—	0.221
Frailty Variance p-value	—	0.00007
Log-likelihood	-766.3	-755.9
Observations	186	186

Notes: Dependent variable is the hazard rate of a new patch. Coefficients are estimated with Cox proportional hazards models. Robust standard errors in parentheses. Model (2) includes a shared gamma-distributed frailty term at the game level. The frailty variance is statistically significant ($\chi^2 = 27.58$, $p = 0.00007$), indicating unobserved heterogeneity in baseline patching rates across games. $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$.

patches released, specifically within the last 42, with the strongest effects observed in the 15-28 day window. Each additional competitor patch in this window is associated with an approximate 12% increase in the hazard, while patches in the 29-42 day window raise the hazard by 20%. These findings suggest that firms actively monitor the update activity of rivals and accelerate their own releases in response, particularly in the short term.

However, competitor patches older than 42 days show no significant correlation, suggesting the impact of competition on patch timing is time-limited. This pattern aligns with the notion of responsiveness to dynamic competitive threats.

Lastly, the significant frailty variance across games suggests the presence of unobserved heterogeneity in baseline patching behavior. Including a shared frailty term allows the model to partially account for such differences, helping to reduce potential bias in the estimated effects of age and competition. While the frailty term does not capture all sources of game-specific variation, its inclusion improves robustness by addressing latent factors that may influence patching frequency across titles.

3.5 Discussion

A central distinction from traditional network goods literature in our approach lies in the choice of quality (rather than price) as the strategic variable, complemented by a dynamic rather than static modeling framework. This shift is motivated by the empirical reality that many network goods (such as video games, social media platforms, and dating apps) are offered free of charge and are continuously updated with new content and features. In such markets, monetary price loses its role as an allocation mechanism. Instead, firms compete by adjusting the quality of their products, broadly defined to include technical stability, content innovation, and overall user experience.

We further argue that the competitive landscape for network goods has fundamentally changed due to the rise of connectivity and the pervasive influence of digital networks. Traditional oligopolistic structures have given way to markets characterized by intense competition among a multitude of (partially) substitutable offerings. In the case of video games, dozens or even hundreds of titles within the same genre or style vie for user engagement. A similar dynamic applies to social media and online dating, where new entrants regularly emerge and contest market share. To model this environment, we introduce a competitive framework centered on an outside option parameter θ, representing the attractiveness of alternative products. This modeling choice abstracts from direct strategic interaction between firms and instead emphasizes adaptive behavior: firms respond to observed market changes rather than anticipating the actions of their competitors. This assumption is particularly well-suited to fragmented markets, and it is supported by our empirical findings: we observe that the likelihood of a quality update peaks around 30 days after a competitor released an update, indicating a reactive rather than proactive pattern of firm behavior.

At the case study level, the empirical results largely support the theoretical predictions derived from the model presented in Section 3.3.3. However, there are several reasons why the theoretical and empirical findings do not fully converge. First, it is difficult to empirically

prove the two steady state conditions (zero state, network steady state) for some networks in our sample. While games such as Counter-Strike and Halo Infinite offer exemplary evidence for this, other games do not show a steady state pattern. This may be as we only observe a few games which are among the most popular in this genre and suffer therefore under intensive competition. Second, due to the small number of observations, the empirical evidence should be interpreted with caution. Moreover, we do not yet leverage the full richness of our dataset, for example, by omitting potentially important control variables such as game-specific features or marketing intensity. Third, the analysis is currently limited to a single genre (first-person shooters), and we do not explore potential heterogeneities across game types, monetization strategies, or user demographics.

Despite these limitations, this paper represents the first attempt to model competitive network dynamics in a major digital industry both theoretically and empirically. The empirical results suggest that update cycles are interdependent and that responses to competitors' quality improvements, proxied by patch releases, are relatively prompt. Similarly, the steady state assumption is supported by our preliminary empirical evidence. By incorporating quality discounting over time into the model we can also account for the repeated occurrence of smaller technical patches in the data which disturb the steady state prediction. In general, it is reasonable to account for the ever present demand of technical support for lifetime service games given rapidly changing hardware and operating systems.

Concerning the lagged arrival of content updates for games, a priori, one would expect competitive responses in the live-service gaming market to occur within a relatively short time frame, given the strategic importance of player attention and engagement in a highly competitive environment, such as free-to-play multiplayer titles. While the specific timing of responses depends on development pipelines and content modularity, studios are generally capable of releasing minor updates (e.g., balance adjustments or limited-time events) within a few weeks. Larger feature patches may require longer lead times but can also be rescheduled in response to external developments. Accordingly, it is reasonable to observe that the

hazard of a patch increases in the weeks following competitor activity, as studios seek to retain players and signal continued support. Prediction 4 is therefore supported by our preliminary case study analysis.

3.6 Conclusion

This paper presents a dynamic model of quality investment in network goods, with a focus on free-to-play multiplayer games in the video gaming industry. By shifting the strategic emphasis from pricing to quality updates, we address the realities of modern platform markets where user engagement is driven by intrinsic product quality and network effects rather than prices.

Our theoretical framework highlights key dynamics such as diminishing marginal returns to quality improvements, steady-state convergence of network size and product quality, and adaptive responses to competitive pressure through innovation. The preliminary case study analysis supports our predictions from the theoretical framework. The ARIMA model reveals that successful products tend toward stable positive equilibria, while weaker networks decay under competitive pressure or insufficient updates. Similarly, Cox proportional hazards models demonstrate that firms actively respond to competitor activity through patch releases and that update frequency declines as products mature. This is precisely consistent with diminishing returns and steady-state dynamics.

Overall, our approach highlights the importance of quality as a dynamic strategic variable in competitive network markets to sustain user engagement. While this study focuses on free-to-play multiplayer games, its insights extend to other digital network goods including social media platforms, online education tools, and live-streaming services. This paper contributes both theoretically and empirically to our understanding of dynamics in these markets by extending the existing literature. Future research could expand on these findings by incorporating additional control variables (e.g., marketing intensity or genre-specific

factors), exploring cross-genre differences, or examining long-term monetization strategies alongside quality dynamics.

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C

Appendix

C.1 Proofs

Proposition 3.3.1 (The Euler equation)**.**
Solving the Bellman equation yields the following optimal quality update policy function:

$$q_t(q_{t+1}) = \frac{\frac{\beta \cdot p_c \cdot s}{n_t} - c_1 \cdot Q_t - \beta \cdot c_1 \cdot q_{t+1}}{2c_2} \tag{3.14}$$

Proof. The first-order condition for (3.11) is given by:

$$\frac{\partial V(n_t, Q_t)}{\partial q_t} = -c_1 \cdot Q_t - 2c_2 \cdot q_t + \beta \frac{\partial V(n_{t+1}, Q_{t+1})}{\partial n_{t+1}} \frac{1}{n_t} + \beta \frac{\partial V(n_{t+1}, Q_{t+1})}{\partial Q_{t+1}} = 0 \tag{C.1}$$

Applying the Envelope-Theorem yields:

$$\frac{\partial V(n_t, Q_t)}{\partial n_t} = p_c \cdot s \tag{C.2}$$

$$\frac{\partial V(n_t, Q_t)}{\partial Q_t} = -c_1 \cdot q_t \tag{C.3}$$

Lagging everything by one period and plugging (C.2) and (C.3) into (C.1) yields:

$$-c_1 \cdot Q_t - 2c_2 \cdot q_t + \frac{\beta \cdot p_c \cdot s}{n_t} - \beta \cdot c_1 \cdot q_{t+1} = 0$$

Rearranging then yields:

$$q_t(q_{t+1}) = \frac{\frac{\beta \cdot p_c \cdot s}{n_t} - c_1 \cdot Q_t - \beta \cdot c_1 \cdot q_{t+1}}{2c_2}$$

□

Proposition 3.3.2 (Final product quality)**.**

The final produce quality Q^* is implicitly characterized by:

$$\frac{Q^*}{2} + Q^* \sqrt{\frac{1}{4} - \theta + Q^*} = \frac{\beta \cdot p_c \cdot s}{c_1}$$

Proof. The steady state to the Euler equation derived in proposition (3.3.1) is defined by the following criteria:

- The network product will not receive any further updates: $q_t = q_{t+1} = 0$.
- The network size is **stable** in n_3^* and will not change anymore ceteris paribus :

$$n_3^* = \frac{1}{2} + \sqrt{\frac{1}{4} - \theta + Q_t}$$

Plugging this into (3.14) and rearranging yields:

$$\frac{Q^*}{2} + Q^* \sqrt{\frac{1}{4} - \theta + Q^*} = \frac{\beta \cdot p_c \cdot s}{c_1} \tag{C.4}$$

Taking the derivative with respect to Q^* on the left-hand side of (C.4) yields:

$$\frac{1}{2} + \sqrt{\frac{1}{4} - \theta + Q^*} + \frac{Q^*}{\sqrt{1 - 4\theta + 4Q^*}} > 0 \qquad \text{for } Q^* \geq 0$$

As the left-hand side of (C.4) is strictly increasing in Q^* while the right-hand side remains constant, there exists a unique quality solution to the equation.

□

C.2 Graphs

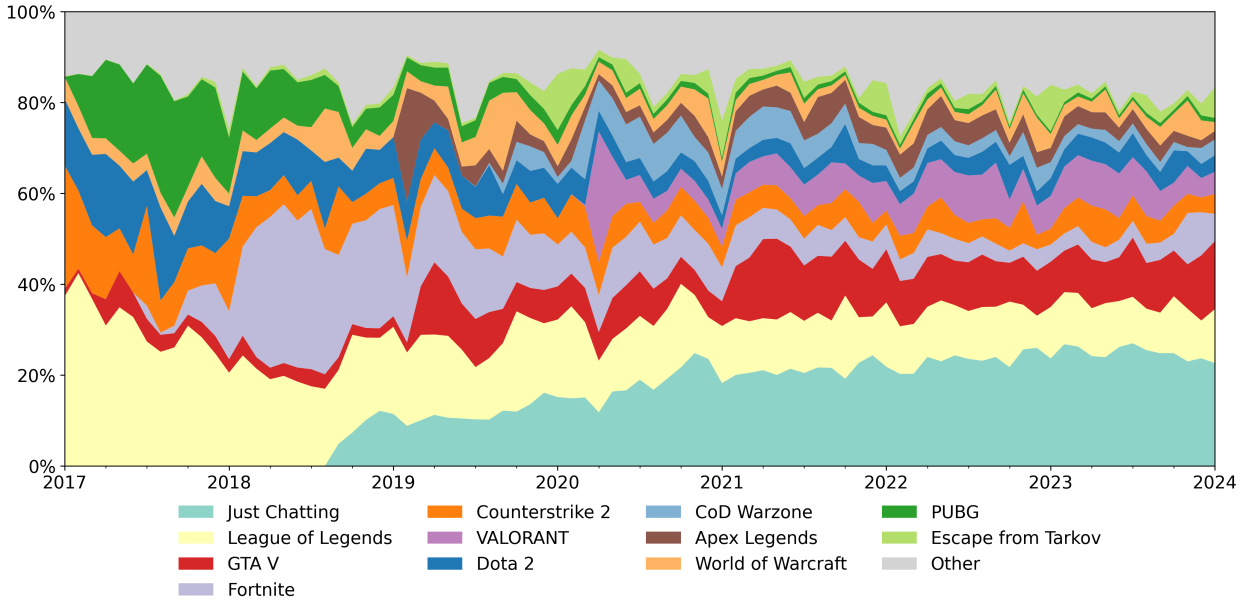

Figure C.1: Monthly share of viewers for the top 12 games on Twitch: with time on the x-axis and share of players on the y-axis.

Conclusion

This cumulative dissertation comprises three comprehensive essays on information economics, political economy, and network goods through three comprehensive essays. The first essay delves into the signaling mechanisms in primary elections, demonstrating how voter learning can be enhanced through strategic candidate behavior amidst uncertainties. The second essay addresses the pressing issue of belief polarization within democracies, highlighting how political failures and misinformation disrupt belief formation processes through media sources. Finally, the third essay investigates the optimization of update cycles for network goods within the gaming industry, emphasizing the importance of continuous quality improvements to sustain user engagement.

Collectively, these studies contribute valuable insights to the existing literature, in particular, into how information shapes economic and political landscapes today. They underscore the critical need for effective governance and transparent communication to mitigate challenges posed by misinformation and polarization. As digital platforms continue to evolve, understanding these dynamics will be essential for fostering informed decision-making and enhancing democratic representation.

Hinweis auf die Nutzung von Künstlicher Intelligenz

Während der Vorbereitung dieser Dissertation habe ich ChatGPT 4o und 4o mini zur Verbesserung der Grammatik und Lesbarkeit verwendet. Nach der Nutzung dieses Tools habe ich den Inhalt sorgfältig überprüft und überarbeitet und übernehme die volle Verantwortung für den Inhalt der Dissertation.

Eidesstattliche Versicherung

Hiermit erkläre ich an Eides statt, dass ich die Dissertation selbstständig verfasst und alle in Anspruch genommenen Quellen und Hilfen in der Dissertation vermerkt habe. Die den herangezogenen Werken wörtlich oder sinngemäß entnommenen Stellen sind als solche gekennzeichnet. Diese Dissertation ist weder in der gegenwärtigen oder in einer anderen Fassung oder in Teilen an der TU Dortmund noch an einer anderen Hochschule in Zusammenhang mit einer staatlichen oder akademischen Prüfung bereits vorgelegt worden.

Dortmund, 01.07.2025

Dominik Mark Maxara