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Some aspects of thinking of Jakub Kresa for development of school mathematics

This article makes an excursion into the history of mathematics, which is used in school mathematics. We would like to describe some components of mathematical notions developed by Jakub Kresa (1648-1715). We would like to compare his work with another two big mathematicians Isaac Newton (1643 -1727) and René Descartes (1596-1650), because it is possible to see some similarities in topics of their works. Isaac Newton wrote separately arithmetic and algebra in *Arithmetica universalis*, but in his work *Philosophiae naturalis principia mathematica* he used only geometry. René Descartes integrates the knowledge from algebra, arithmetic and geometry in his work *La Geometrie* and in a similar way, Jakub Kresa described the mathematical theory in his work *Analysis speciose trigonometriae sphaericae*.

1. Introduction

Jakub Kresa lived in the time in which arithmetic has geometrical representations. René Descartes wrote in his book *La Geometrie* that "any problem in geometry can easily be reduced to such terms that a knowledge of the lengths of certain straight lines is sufficient for its construction. Just as arithmetic consisting of only four or five operations, namely, addition, subtraction, multiplication, division and the extraction of roots."

Now let AB taken as unity (see Figure 1), and let be required to multiply BD by BC. Descartes argues that "I only have to join the points A and C, and draw DE parallel to CA, then BE is the product of BD and BC. If it be required to divide BE by BD, I join E and D, and draw AC parallel to DE, then BC is the result of the division (see Descartes (1954)).

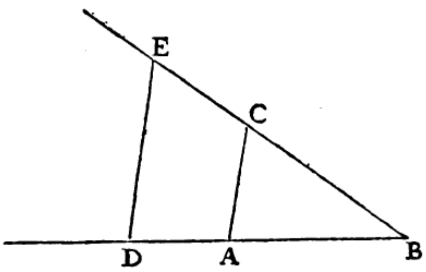


Figure 1

Isaac Newton wrote separately arithmetic and algebra in *Arithmetica universalis,* but in his work *Philosophiae naturalis principia mathematica* he used only geometry. He didn't use in *Philosophiae naturalis principia mathematica* arithmetical and algebraic approach, because this book was written in style of Euclidean Elements and Italian algebra by Cardano (words cub., quad.). We can see in the next figure modern transcription of *Principia mathematica* from Isaac Newton (see Newton (2009)):

$PQq.$ vel (per Lem. VII.) $PRq.$ ad $QTq.$ & ex natura circuli rectangulum $QR \times RN + QN$ æquale est PR quadrato. Coeuntibus autem punctis P, Q fit $RN + QN$ æqualis $2PM$. Ergo est $CP\ quad.$ ad $PM\ quad.$ ut $QR \times 2PM$ ad $QT\ quad.$ adeoq; $\frac{QT\ quad.}{QR}$ æquale $\frac{2PM\ cub.}{CP\ quad.}$, & $\frac{QT\ quad. \times SP\ quad.}{QR}$ æquale $\frac{2PM\ cub. \times SP\ quad.}{CP\ quad.}$ Est ergo (per Corol. Theor. V.) vis centripeta reciproce ut $\frac{2PM\ cub. \times SP\ quad.}{CP\ quad.}$ hoc est (neglecta ratione determinata $\frac{2SP\ quad.}{CP\ quad.}$) reciproce ut $PM\ cub.$ $Q.\ E.\ I.$

Figure 2

We cannot find in this book algebraic notation used by Descartes, however it could bring many advantages and he used in his other book *Arithmetica universalis* this notation. He wrote this book later than *Principia* and it is possible to see development of notation and style. We can see in the next figure fragment of Newton *Arithmetica universalis* (published in 1761 after death of Newton, see Newton (1761)):

$$x^4 - x^3 - 19xx + 49x - 30 = 0,$$

unitas eſt radix, quoniam ſcripta pro x producit

$$1 - 1 - 19 + 49 - 30,$$

hoc eſt nihil. Sed æquationis ejuſdem plures eſſe poſſunt radices. Nam ſi in hac eadem æquatione

$$x^4 - x^3 - 19xx + 49x - 30 = 0,$$

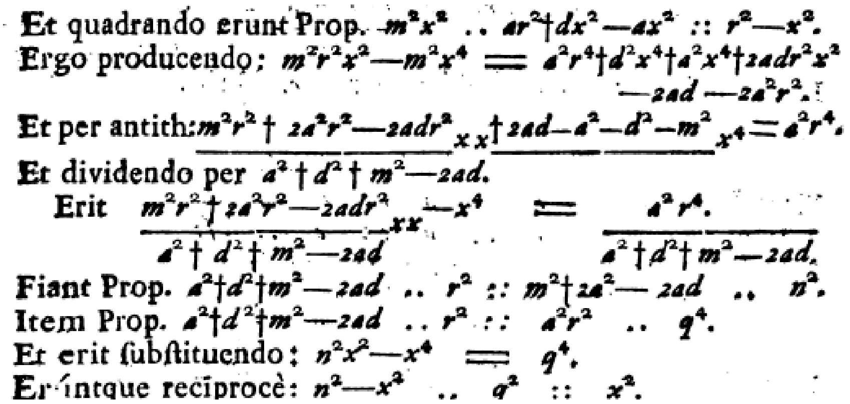
pro x ſcribas numerum 2, & pro poteſtatibus x ſimiles poteſtates numeri 2, producetur

$$16 - 8 - 76 + 98 - 30,$$

Figure 3

2. Jakub Kresa and his work *Analysis speciose trigonometriae sphaericae*

Jakub Kresa in his work *Analysis speciose trigonometriae sphaericae* used at that time very modern notation similar to Descartes and Newton from *Arithmetica universalis*. We can see in the next figure fragment of Jakub Kresas´ book *Analysis speciose trigonometriae sphaericae* (published in 1720 after death of Jakub Kresa, see Kresa (1720)):

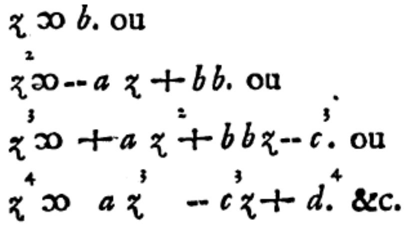


Et quadrando erunt Prop. m^2x^2 .. $ar^2 \dagger dx^2 - ax^2$ :: $r^2 - x^2$.
Ergo producendo: $m^2r^2x^2 - m^2x^4 = a^2r^4 \dagger d^2x^4 \dagger a^2x^4 \dagger 2adr^2x^2 - 2ad - 2a^2r^2$.
Et per antith: $\overline{m^2r^2 \dagger 2a^2r^2 - 2adr^2}\, xx \dagger \overline{2ad - a^2 - d^2 - m^2}\, x^4 = a^2r^4$.
Et dividendo per $a^2 \dagger d^2 \dagger m^2 - 2ad$.
Erit $\frac{m^2r^2 \dagger 2a^2r^2 - 2adr^2}{a^2 \dagger d^2 \dagger m^2 - 2ad} xx - x^4 = \frac{a^2r^4}{a^2 \dagger d^2 \dagger m^2 - 2ad}$.
Fiant Prop. $a^2 \dagger d^2 \dagger m^2 - 2ad$ .. r^2 :: $m^2 \dagger 2a^2 - 2ad$ .. n^2.
Item Prop. $a^2 \dagger d^2 \dagger m^2 - 2ad$ .. r^2 :: a^2r^2 .. q^4.
Et erit ſubſtituendo: $n^2x^2 - x^4 = q^4$.
Er-intque reciprocè: $n^2 - x^2$ .. q^2 :: x^2.

Figure 4

We can see in this figure, that the publisher of Kresa's work has problems with mathematical signs. Newton's *Arithmetica universalis* was published in bigger quality, but the publisher of Kresa's book needed to find instead of mathematical sign some similar other signs. For example, instead of + the cross † was used, which signifies the date of death.

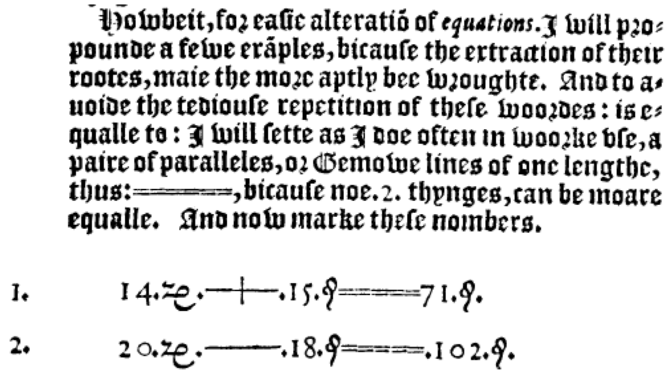
We can compare also style of writing of the sign = "equal" by these authors Newton and Kresa used the style from nowadays and Descartes had special signs. We can see in the next figure fragment of Descartes' book *La Geometrie* as an appendix to Discovery about method (see Descartes (1954)):



$z \propto b$. ou
$z^2 \propto - a z + bb$. ou
$z^3 \propto + a z^2 + bbz - c^3$. ou
$z^4 \propto a z^3 - c^3 z + d^4$. &c.

Figure 5

First author, who used the sign =, was Robert Record in the book the Whetstone of Witte in the year 1557. We can see in the next figure fragment of this book (see Record (1557)):



Howbeit, for easie alteratiō of *equations*. I will propounde a fewe exāples, bicause the extraction of their rootes, maie the more aptly bee wroughte. And to auoide the tediouse repetition of these woordes: is equalle to: I will sette as I doe often in woorke use, a paire of paralleles, or Gemowe lines of one lengthe, thus: ═══════, bicause noe. 2. thynges, can be moare equalle. And now marke these nombers.

1. 14.ꝛ.—+—.15.ꝭ═════71.ꝭ.
2. 20.ꝛ.———.18.ꝭ═════.102.ꝭ.

Figure 6

3. Conclusions

Jakub Kresa lived in the 17th century in which was created the language of modern algebra, so the signs in mathematics were not stable at that time. Many books used style of old ancient Euclidean geometry combine with style of Italian cosits group from 15th or 16th century (Cardano, Tartaglia). Descartes brings the new style of writing of mathematical texts, which was used also by Jakub Kresa.

It is very important for mathematical education in schools nowadays and also in math teacher training programs, that students develop their ability to explain every problem in more ways, they also need to find other solutions than their teacher. Jakub Kresa solved in his work mostly practical problems and he would like to explain his findings for people, who didn't have any study of mathematics.

In future it will be possible that parts of mathematics by Jakub Kresa and other authors will be presented in modern form through ICT tools and educational software, because a lot of original historical mathematical works is possible to find in electronic form in internet. These activities can help in popularisation of mathematics in every kind of school.

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